This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2024.3416700

Performance Analysis of FAS-aided Backscatter Communications

Farshad Rostami Ghadi, *Member, IEEE*, Masoud Kaveh, Kai-Kit Wong, *Fellow, IEEE*, and Yangyang Zhang

***Abstract*—In this letter, we study the performance of backscatter communications (BC) over the position-flexible fluid antenna system (FAS) technology. We consider a situation where a single fixed-antenna source sends information to a FAS reader through the wireless forward (i.e., source-to-tag) and backscatter (i.e., tag-to-reader) channels. We first derive the cumulative distribution function (CDF) of the equivalent channel at the FAS receiver, and obtain closed-form expressions for the outage probability (OP) and the delay outage rate (DOR) assuming a correlated Rayleigh distribution. To gain more insights into the system performance, we present analytical expressions of the diversity order, OP and DOR in the high signal-to-noise ratio (SNR) regime. Numerical results reveal that FAS at the reader can significantly improve the performance of BC compared with a fixed-antenna reader.**

***Index Terms*—Backscatter communication, correlated fading channel, fluid antenna system, outage probability.**

I. Introduction

ONE CRUCIAL aspect of the sixth-generation (6G) wireless networks is to support low-power communication, useful particularly for Internet-of-Things (IoT) devices. Given the applications of evolving technologies such as Radio Frequency Identification (RFID) systems and IoT, emphasis has been paid to backscatter communication (BC) in recent years [1]. BC is a cost-effective wireless approach that enables low-power devices to send data by reflecting or modulating existing radio frequency (RF) signals in the environment.

Independently, fluid antenna system (FAS) has emerged as a technology to obtain a new degree of freedom (dof) through antenna position flexibility recently [2]. The concept of FAS was first introduced by Wong *et al.* [3], thanks to the advances in reconfigurable antennas such as liquid-based antennas and pixel-based antennas and etc [4]. In brief, FAS represents all forms of movable and non-movable flexible-position antenna systems [5]. Early results on FAS have presented promising results in terms of diversity order and outage probability (OP), e.g., [6], [7], [8], [9]. Copulas have also been shown to be useful in the performance analysis for FAS [10]. Additionally, it was illustrated that reconfigurable intelligent surface (RIS) could combine with FAS for amazing performance gains [11]. The channel estimation problem for FAS is challenging but has recently been tackled in [12], [13]. Furthermore, FAS has found applications in multiple access [14], [15].

BC and FAS are attractive in their own rights, but integrating BC with the dynamic and reconfigurable properties of FAS is not well understand and can potentially be synergistic. FAS is able to adaptively modify their radiating structures based on environmental conditions or network demands; this adaptability, when coupled with BC, allows for dynamic adjustments in the reflection and modulation of RF signals. Motivated by this, the objective of this letter is to evaluate the performance of BC when backscatter devices take advantage of FAS.

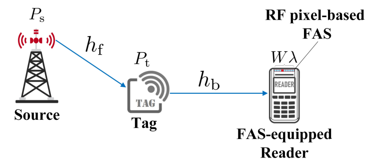


Fig. 1. The system model of the FAS-aided BC.

In particular, we consider a single fixed-antenna source that intends to transmit data to a FAS-equipped reader through the forward (i.e., source-to-tag) and backscatter (i.e., tag-to-reader) channels. Our contributions are summarized as follows: (i) we derive the cumulative distribution function (CDF) of the equivalent channel at the FAS-equipped reader with K preset positions, i.e., the CDF of the maximum of K random variables (RVs), each of which is a product of the forward and backscatter channels, using copulas; (ii) we obtain the OP and delay outage rate (DOR) in closed-form expressions under correlated Rayleigh fading channels; (iii) we derive the diversity order and the asymptotic expressions of the OP and DOR in the high signal-to-noise ratio (SNR) regime; and (iv) we present numerical results for the FAS-aided BC, showing that the FAS reader can significantly enhance the system performance compared to a single fixed-antenna reader.

II. System Model

Consider a wireless FAS-aided BC as illustrated in Fig. 1, where a single fixed-antenna source aims to send information x to a reader that is equipped with a FAS through the forward and backscatter channels. Hence, the instantaneous received signal power at the tag is given by $P_{\rm t} = P_{\rm s} L_{\rm s} G_{\rm f}$, in which $P_{\rm s}$ denotes the transmit power by the source, $L_{\rm s}$ includes the gains of the transmit and receive antennas and frequency-dependent propagation losses, and $G_{\rm f} = |h_{\rm f}|^2$ is the fading channel gain between the source and the tag, where $h_{\rm f}$ denotes the corresponding forward fading channel coefficient. On the reader side, the FAS can freely switch to one of the K preset positions (i.e., ports) evenly distributed on a linear space of length $W\lambda$ where λ is the wavelength. In this letter, the switching delay is assumed to be negligible, thanks to the pixel-based FAS [4]. Also, it is assumed that the FAS consists of only one RF chain, and only one port can be activated for communication.[1] Under such assumptions, the received signal at the k-th port of the reader is given by

$$y_k = h_{\rm f} h_{{\rm b},k} x + z_k, \tag{1}$$

[1]To enhance network connectivity and data rate, this model can be extended to the compact ultra massive antenna array (CUMA), where, instead of activating only one port at the reader, an ultra massive number of ports are activated at the reader for reception [15].

The work of F. Rostami Ghadi and K. K. Wong was supported in part by the Engineering and Physical Sciences Research Council (EPSRC) under grant EP/W026813/1.

F. Rostami Ghadi and K.-K. Wong are with the Department of Electronic and Electrical Engineering, University College London, London WC1E 6BT, U.K. (e-mail: {f.rostamighadi, kai-kit.wong}@ucl.ac.uk). He is also affiliated with Yonsei Frontier Laboratory, Yonsei University, Seoul, 03722, Korea.

Masoud Kaveh is with the Department of Information and Communication Engineering, Aalto University, 02150 Espoo, Finland.

Y. Zhang is with Kuang-Chi Science Limited, Hong Kong SAR, China.

Corresponding author: F. Rostami Ghadi.

where $h_{\mathrm{b},k}$ is the backscatter channel coefficient between the tag and the k-th port of the FAS reader with the respective fading channel gain $G_{\mathrm{b},k} = |h_{\mathrm{b},k}|^2$ and z_k is the independent identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and variance σ^2 at each port. Without loss of generality, we assume that $\mathbb{E}[G_\mathrm{f}] = \mathbb{E}[G_{\mathrm{b},k}] = 1$, where $\mathbb{E}[\cdot]$ denotes the expectation operator.

We further assume that the FAS always switches to the best port with the strongest signal for communication, i.e.,

$$G_{\mathrm{FAS}} = \max\{G_{\mathrm{p},1} \dots, G_{\mathrm{p},K}\}, \tag{2}$$

where $G_{\mathrm{p},k} = G_\mathrm{f} G_{\mathrm{b},k}$ denotes the product channel gain of the forward and backscatter links. It is worth noting that $G_{\mathrm{p},k}$ for $k \in \{1, \dots, K\}$ are spatially correlated,[2] and the correlation matrix $\mathbf{R}$ can be characterized by Jake's model as [16]

$$\mathbf{R} = [\mu_{k,l}] = \omega J_0\left(\frac{2\pi(k-l)}{K-1}W\right), \tag{3}$$

where $\mu_{k,l}$ denotes the correlation parameter that can control the dependency between two arbitrary fluid antenna ports k and l, ω is the large-scale fading effect, and $J_0(\cdot)$ represents the zero-order Bessel function of the first kind. The received SNR at the reader is defined as

$$\gamma = \frac{P_\mathrm{t} G_{\mathrm{FAS}}}{\sigma^2} = \bar{\gamma} G_{\mathrm{FAS}}, \tag{4}$$

in which $\bar{\gamma} = \frac{P_\mathrm{t}}{\sigma^2}$ is the average SNR.

III. Performance Analysis

Here, we derive the CDF of the equivalent channel gain at the FAS reader, and then obtain closed-form expressions for the OP and DOR. Also, we derive the diversity order and the asymptotic expressions of the OP and DOR in the high SNR regime. Furthermore, we provide some insights into the OP and DOR behavior when W and/or K are sufficiently large.

A. Statistical Characterization

From (2), we can see that the CDF of the equivalent fading channel gain at the FAS reader is defined as the CDF of the maximum of K correlated RVs that each includes the product of two independent RVs. Assuming that all the fading channels undergo Rayleigh distribution, the CDF of G_{FAS} is derived as the following proposition.

Proposition 1: The CDF of $G_{\mathrm{FAS}} = \max\{G_{\mathrm{p},1} \dots, G_{\mathrm{p},K}\}$ for the considered FAS-aided BC is given by

$$F_{G_{\mathrm{FAS}}}(r) = \left[K\left(1 - 2\sqrt{r}\mathcal{K}_1\left(2\sqrt{r}\right)\right)^{-\theta} - K + 1\right]^{\frac{-1}{\theta}}, \tag{5}$$

in which $\mathcal{K}_1(\cdot)$ denotes the first-order modified Bessel function of the second kind and $\theta \in (0, \infty)$ is the dependence parameter so that $\theta \to 0$ represents the independent case.

Proof: By using the CDF definition, $F_{G_{\mathrm{FAS}}}(r)$ can be mathematically expressed as

$$\begin{aligned} F_{G_{\mathrm{FAS}}}(r) &= \Pr\left(\max\{G_{\mathrm{p},1} \dots, G_{\mathrm{p},K}\} \le r\right) \\ &= F_{G_{\mathrm{p},1} \dots, G_{\mathrm{p},K}}(r, \dots, r) \\ &\overset{(a)}{=} C\left(F_{G_{\mathrm{p},1}}(r), \dots, F_{G_{\mathrm{p},K}}(r)\right), \end{aligned} \tag{6}$$

in which (a) is obtained by using Sklar's theorem [17, Thm. 2.10.9] and $C(\cdot) : [0,1]^d \to [0,1]$ denotes the copula function that is a joint CDF of d random vectors on the unit cube $[0,1]^d$ with uniform marginal distributions, i.e.,

$$C(u_1, \dots, u_d; \theta) = \Pr(U_1 \le u_1, \dots, U_d \le u_d), \tag{7}$$

where $u_i = F_{s_i}(s_i)$ and s_i denotes any arbitrary RV for $i \in \{1, \dots, d\}$ and θ represents the dependence parameter which can measure the linear/non-linear correlation between arbitrary correlated RVs. From (6), we now need to find the CDF of the product channel, i.e., $F_{G_{\mathrm{p},k}}(r)$. Assuming Rayleigh fading channels, we can derive the CDF of $G_{\mathrm{p},k} = G_\mathrm{f} G_{\mathrm{b},k}$ as

$$\begin{aligned} F_{G_{\mathrm{p},k}}(r) &= \Pr\left(G_\mathrm{f} G_{\mathrm{b},k} \le r\right) \\ &= \int_0^\infty f_{G_\mathrm{f}}(g_\mathrm{f}) F_{G_{\mathrm{b},k}}\left(\frac{r}{g_\mathrm{f}}\right) \mathrm{d}g_\mathrm{f} \qquad (8) \\ &\overset{(b)}{=} 1 - 2\sqrt{r}\mathcal{K}_1\left(2\sqrt{r}\right), \qquad (9) \end{aligned}$$

in which (b) is obtained by solving the integral in (8) with the help of [18, 3.471.9]. By inserting (9) into (6), the CDF of G_{FAS} is obtained for any arbitrary choice of copulas. However, in order to analyze the performance of the considered system model, it is required to select a copula that can describe the spatial correlation between FAS ports. For this purpose, we exploit the Clayton copula because it can accurately describe the tail dependence between correlated RVs. It should be noted that an outage mainly occurs in deep fading conditions, where knowing the behavior of the tail dependence of fading coefficients is necessary; therefore, this choice is justified. As a result, by substituting the Clayton copula definition from [10, Def. 3] into (6) and considering the similar marginal CDF, (5) is obtained, and the proof is completed. ■

It is worth pointing out that since, in the copula definition, the non-linear transformations are applied to the considered RVs, the linear correlation cannot be maintained anymore. In other words, the dependence parameter θ does not necessarily represent the linear correlation between the correlated RVs. Therefore, rank correlation coefficients should be considered for the copula-based analysis since they are preserved under any monotonic transformation. Consequently, they are able to describe the structure of dependency beyond linear correlation. To tackle this issue, we use Spearman's ρ correlation coefficient that is identical to Pearson's product moment correlation coefficient for a pair of continuous RVs, i.e., $\mathbf{R} = [\mu_{k,l}] = [\rho_{k,l}]$ [17, Sec. 5.1.2]. Therefore, ρ between two arbitrary correlated RVs is mathematically defined as

$$\rho = 12 \iint_{[0,1]^2} u_1 u_2 \mathrm{d}C(u_1, u_2) - 3. \tag{10}$$

By plugging the Clayton copula into (10) and computing the integral, the Spearman's ρ for the Clayton copula can be approximated as $\rho \approx \frac{3\theta}{2(\theta+2)}$ [19]. Then, considering that the linear correlation coefficient is identical to Spearman's ρ, θ can be expressed in terms of the defined correlation matrix as

$$\theta \approx \frac{4\mathbf{R}}{3 - 2\mathbf{R}}, \tag{11}$$

where we have abused the notation of $\mathbf{R}$ to mean its element. As a result, by substituting (11) into (5), the CDF of G_{FAS} in Proposition 1 can be defined in terms of Jake's model.

[2] The equivalent correlated channel is modeled by using eigenvalue decomposition to obtain the correlation matrix $\mathbf{R} = [\mu_{k,l}] = \mathbf{V\Lambda V}^H$, where $\mathbf{V}$ is a $K \times K$ matrix whose k-th column, denoted by v_k, is the eigenvector of $\mathbf{R}$ and $\Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_K)$ denotes a $K \times K$ diagonal matrix whose k-th diagonal entry is the corresponding eigenvalue of v_k [8].

This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2024.3416700

B. OP Analysis

OP refers to the probability of the instantaneous SNR γ below the required SNR threshold $\gamma_{\rm th}$, i.e., $P_{\rm o} = \Pr\left(\gamma \leq \gamma_{\rm th}\right)$. Therefore, the OP is derived as the following proposition.

Proposition 2: The OP for the considered FAS-assisted BC under correlated Rayleigh fading channels is given by

$$P_{\rm o} = \left[K\left(1-2\sqrt{\frac{\gamma_{\rm th}}{\bar{\gamma}}}\mathcal{K}_1\left(2\sqrt{\frac{\gamma_{\rm th}}{\bar{\gamma}}}\right)\right)^{\frac{4\mathbf{R}}{2\mathbf{R}-3}} - K + 1\right]^{\frac{2\mathbf{R}-3}{4\mathbf{R}}}, \tag{12}$$

in which $\gamma_{\rm th}$ is the SNR threshold and $\mathbf{R}$ is defined in (3).

Proof: By inserting the SNR of the considered FAS-aided BC from (4) into the OP definition, we have

$$P_{\rm o} = \Pr\left(G_{\rm FAS} \leq \frac{\gamma_{\rm th}}{\bar{\gamma}}\right) = F_{G_{\rm FAS}}\left(\frac{\gamma_{\rm th}}{\bar{\gamma}}\right). \tag{13}$$

Now, by applying the CDF of $G_{\rm FAS}$ from Proposition 1 into (13), the proof is accomplished. ■

C. DOR Analysis

DOR is a momentous metric in wireless networks to evaluate the performance of ultra-reliable and low-latency communications (URLLC) which is defined as the probability that the transmission delay for a certain amount of data R in a wireless channel with a bandwidth B exceeds a certain predefined threshold $T_{\rm th}$, i.e., $\Pr\left(T_{\rm dt} > T_{\rm th}\right)$, in which $T_{\rm dt} = \frac{R}{B\log_2(1+\gamma)}$ defines the delivery time [20]. Thus, the DOR for our system model can be obtained using the following proposition.

Proposition 3: The DOR for the considered FA-aided BC under correlated Rayleigh fading channels is given by

$$P_{\rm dor} = \left[K\left(1-2\sqrt{\frac{\hat{T}_{\rm th}}{\bar{\gamma}}}\mathcal{K}_1\left(2\sqrt{\frac{\hat{T}_{\rm th}}{\bar{\gamma}}}\right)\right)^{\frac{4\mathbf{R}}{2\mathbf{R}-3}} - K + 1\right]^{\frac{2\mathbf{R}-3}{4\mathbf{R}}}, \tag{14}$$

where $\hat{T}_{\rm th} = {\rm e}^{\frac{R\ln 2}{BT_{\rm th}}}$.

Proof: By substituting the delivery time into the DOR definition, we have

$$\begin{aligned} P_{\rm dor} &= \Pr\left(\frac{R}{B\log_2\left(1+\bar{\gamma}G_{\rm FAS}\right)} > T_{\rm th}\right) \\ &= \Pr\left(G_{\rm FAS} \leq \frac{{\rm e}^{\frac{R\ln 2}{BT_{\rm th}}}}{\bar{\gamma}}\right) = F_{G_{\rm FAS}}\left(\frac{\hat{T}_{\rm th}}{\bar{\gamma}}\right). \end{aligned} \tag{15}$$

Now, by inserting $\hat{T}_{\rm th} = {\rm e}^{\frac{R\ln 2}{BT_{\rm th}}}$ into the CDF of $G_{\rm FAS}$ from Proposition 1, the proof is completed. ■

It is noteworthy that the theoretical expressions in (12) and (14) can provide intuitive insights into the OP and DOR performance against the channel parameters thanks to the modified Bessel function $\mathcal{K}_1(r)$ and the Clayton copula properties. The following remark represents some example of such insights.

Remark 1: Under a fixed correlation setup (i.e., constant $\mathbf{R}$*), it is mathematically understandable that the OP and DOR values decrease as the average SNR* $\bar{\gamma}$ *grows for a fixed SNR threshold* $\gamma_{\rm th}$ *and fixed delivery threshold* $\hat{T}_{\rm th}$*, which is in alignment with the wireless communication concept that channel quality improves as* $\bar{\gamma}$ *increases. Moreover, it can be found that under a fixed* $\mathbf{R}$ *and* $\bar{\gamma}$*, increasing* $\gamma_{\rm th}$ *and* $\hat{T}_{\rm th}$ *leads to an increase in the OP and DOR values, respectively, which aligns with the fact that transmission becomes more challenging at very high rates. Such behaviors are obtained due to the modified Bessel function* $\mathcal{K}_1(r)$ *properties, i.e., decreasing (increasing)* r *leads to an increase (decrease) in the value of* $\mathcal{K}_1(r)$*. Therefore,* $F_{G_{{\rm p},k}}(r)$ *in terms of* $r = \{\frac{\gamma_{\rm th}}{\bar{\gamma}}, \frac{\hat{T}_{\rm th}}{\bar{\gamma}}\}$ *decreases (increases) and the overall expressions, i.e., OP in (12) and DOR in (14), decrease (increase) in terms of the corresponding parameter. Such intuitive insights are also provided in Section IV through numerical results, which validate the accuracy of the theoretical analysis.*

D. Asymptotic Analysis

Although the derived OP and DOR in Propositions 2 and 3 are in simple closed-form expressions, we are interested in the asymptotic behavior of the obtained metrics in the high SNR regime (i.e., $\gamma \to \infty$) to gain more insights into the system performance. To do so, by exploiting the series expansion of the Bessel function $\mathcal{K}_1(r)$ when $r \to 0$, we have

$$F^{\infty}_{G_{{\rm p},k}}(r) \approx r\left(1 - 2\zeta - \log r\right), \tag{16}$$

where ζ is the Euler-Mascheroni constant [21]. Hence, the asymptotic expressions of the OP and DOR for the considered system model can be obtained in the following corollary.

Corollary 1: The asymptotic expressions of the OP and DOR for the considered FAS-equipped BC in the high SNR regime, i.e., $\bar{\gamma} \to \infty$ are respectively given by

$$P^{\infty}_{\rm o} \approx \left[K\left(\frac{\gamma_{\rm th}}{\bar{\gamma}}\left[1-2\zeta-\log\left(\frac{\gamma_{\rm th}}{\bar{\gamma}}\right)\right]\right)^{\frac{4\mathbf{R}}{2\mathbf{R}-3}} - K + 1\right]^{\frac{2\mathbf{R}-3}{4\mathbf{R}}}, \tag{17}$$

$$P^{\infty}_{\rm dor} \approx \left[K\left(\frac{\hat{T}_{\rm th}}{\bar{\gamma}}\left[1-2\zeta-\log\left(\frac{\hat{T}_{\rm th}}{\bar{\gamma}}\right)\right]\right)^{\frac{4\mathbf{R}}{2\mathbf{R}-3}} - K + 1\right]^{\frac{2\mathbf{R}-3}{4\mathbf{R}}}. \tag{18}$$

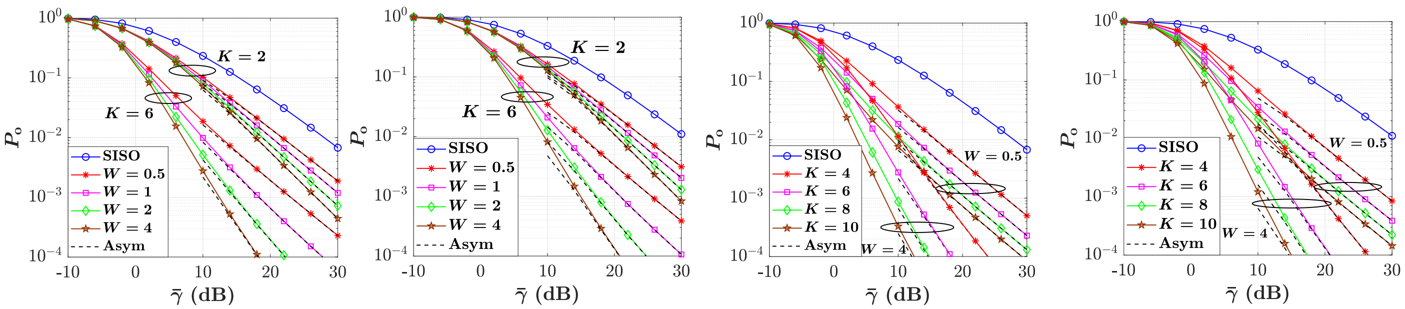
Proof: In the high SNR regime (i.e., $\bar{\gamma} \to \infty$), we have $\frac{\eta}{\bar{\gamma}} \to 0$, where $\eta \in \left\{\gamma_{\rm th}, \hat{T}_{\rm th}\right\}$. Therefore, by substituting (16) into (12) and (14), the proof is accomplished. ■

Similar to Remark 1, the asymptotic expressions in (17) and (18) can provide intuitive insights into the OP and DOR performance in the high SNR regime. Such mathematical visions are achieved through the logarithm function $\log(r)$ features, i.e., increasing (decreasing) r leads to an increase (decrease) in the value of $\log(r)$. As such, $F^{\infty}_{G_{{\rm p},k}}(r)$ in terms of $r = \{\frac{\gamma_{\rm th}}{\bar{\gamma}}, \frac{\hat{T}_{\rm th}}{\bar{\gamma}}\}$ increases (decreases) and the overall expressions increase (decrease) against the corresponding parameter.

Furthermore, the asymptotic behavior of the OP and DOR for sufficiently large W and K, i.e., $W \to \infty$ and/or $K \to \infty$ can provide deep insights into the derived expressions.

Remark 2: Given (3), when $W \to \infty$*, for a fixed* K*, the Bessel function* $J_0(\cdot)$ *reaches* 0*, and thus,* $\mathbf{R}$ *is* 0*. Then, by considering* $\mathbf{R} = 0$ *in (12) and (14), the OP and DOR reach* 0 *when* $W \to \infty$*. This trend is also confirmed by the copula dependence parameter* θ *because, given (11), if* $\mathbf{R} = 0$*,* θ *reaches* 0*, which corresponds to the independent case based on the Clayton copula definition. In other words, it shows that*

This article has been accepted for publication in IEEE Wireless Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LWC.2024.3416700



(a) OP (b) DOR (c) OP (d) DOR

Fig. 2. Performance of OP and DOR versus average SNR $\bar{\gamma}$ for (a) and (b) different FAS size W; and for (c) and (d) different number of FAS ports K.

the spatial separation between the fluid antenna ports is large enough and the spatial correlation becomes negligible.

Remark 3: Given (3), when $K \to \infty$ for a fixed W, the Bessel function $J_0(\cdot)$ is 1, and thus, $\mathbf{R}$ reaches ω. Then, by inserting $\mathbf{R} = 1$ (e.g., without loss of generality, let $\omega = 1$) in (12) and (14), the OP and DOR converge to constant values when $K \to \infty$. This trend is mainly due to the fact that under such an assumption, the ports become too close to each other and the spatial correlation effects dominate; hence, diversity gain reduces after reaching a certain point and the decrease in OP and DOR slows down so that they eventually saturate. This is also validated by the copula dependence parameter θ because, given (11), if $\mathbf{R} = 1$, then θ is equal to 4, which means a weak dependence structure scenario since, regarding the Clayton copula definition $\theta \to 0$ and $\theta \to \infty$ represent the independent and strong correlation cases, respectively.

Furthermore, another effective metric for analyzing system performance in the high SNR regime is diversity order, which is mathematically defined as $D_{\text{FAS}} = -\lim_{\bar{\gamma}\to\infty} \frac{\log P_o(\bar{\gamma})}{\log \bar{\gamma}}$.

Proposition 4: The diversity order for the FAS-aided BC under correlated Rayleigh fading channels is given by

$$D_{\text{FAS}} \approx \min\left(K, K'\right), \tag{19}$$

where K' denotes the numerical rank of correlation matrix $\mathbf{R}$ when $K \to \infty$ for a fixed W.

Proof: The details of proof are given in Appendix A.

IV. Numerical Results

Here, we present numerical results to assess the considered system performance in terms of the OP and DOR, which have been double-checked by the Monte-Carlo simulation method. In the simulations, we have set the parameters as $\gamma_{\text{th}} = 0$ dB, $R = 5$ kbits, $B = 2$ GHz, $T_{\text{dt}} = 3$ ms, $\bar{\gamma} = 20$ dB, $W = \{0.5, 1, 2, 4, 6\}$, and $N = \{2, 4, 6, 8, 10\}$.

Figs. 2(a) and 2(b) respectively illustrate the behavior of OP and DOR in terms of the average SNR $\bar{\gamma}$ for given values of FAS size W under correlated Rayleigh fading channels. As expected, the OP and DOR decrease as $\bar{\gamma}$ increases, which is reasonable since the channel condition improves (see Remark 1). Moreover, it can be observed that by increasing the FAS size W for a fixed number of ports K, the performance of OP and DOR improves. The reason for this behavior is that increasing the spatial separation between the FAS ports by increasing W for a fixed K can reduce the spatial correlation between FAS ports (see Remark 2). Additionally, we can

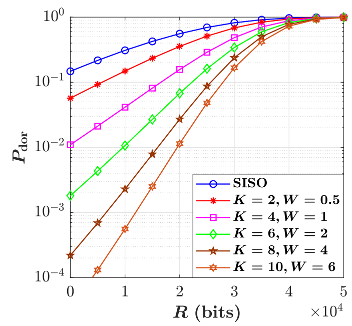


Fig. 3. DOR versus amount of data R for selected of values W and K.

clearly observe that such an improvement is more noticeable when K is large. The performance of OP and DOR in terms of $\bar{\gamma}$ for given values of K under correlated Rayleigh fading channels is presented in Figs. 2(c) and 2(d), respectively. We can see that as the number of FAS ports K grows, lower values of the OP and DOR can be obtained. The main reason is that although increasing K for a fixed value of W raises the spatial correlation between FAS ports, it can potentially improve the channel capacity, diversity gain, and spatial multiplexing at the same time. Hence, this can help mitigate fading and improve the overall link quality (see Remark 3).

Furthermore, as we can see in Fig. 2, considering a FAS reader instead of a single fixed-antenna reader can significantly enhance the performance of BC in terms of OP and DOR. In order to evaluate how the FAS reader affects the DOR performance in terms of transmitted data R over BC, we present Fig. 3 for selected values of W and K. First, we can observe that as W and K increase simultaneously, the spatial correlation between the FAS ports becomes balanced; hence, lower values of the OP and DOR are reached. Additionally, as expected, it can be noticed that as R increases, the DOR performance becomes worse, such that transmitting a high amount of data (e.g., $R = 3$ kbits) with low delay is almost impractical when a single fixed-antenna reader or a FAS reader with small W and K are considered. However, thanks to the FAS reader, a large amount of information with a small delay can be sent when the W and K are large enough.

V. Conclusion

This letter addressed the performance of BC with the aid of FAS. Specifically, we assumed that a fixed-antenna source sends information to a FAS-aided reader via wireless forward and backscatter channels. We first derived the CDF of the equivalent channel (i.e., the maximum of K correlated RVs such that each is a product of the forward and backscatter channels) for the reader by exploiting the copula technique. Then we derived closed-form expressions for the OP and DOR assuming correlated Rayleigh fading channels. Furthermore, we obtained the diversity order and asymptotic expressions of the OP and DOR in the high SNR regime. Our analytical results revealed that the FAS reader can provide a remarkable performance in terms of the OP and DOR compared with the conventional single fixed-antenna reader over BC.

Appendix A
Proof of Proposition 4

Given that directly solving the limit in the diversity order definition is almost mathematically intractable, we utilize the method provided in [22] to approximate the PDF of the equivalent channel at the FAS-equipped reader. Consequently, the PDF of $|h_{\rm FAS}|$ can be approximated as

$$f_{|h_{\rm FAS}|}(r) \approx 2\kappa r^{2M+1} + o\left(r^{2M+1}\right). \tag{20}$$

Thus, based on (20), the CDF at high SNR can be derived as

$$F_{|h_{\rm FAS}|}(r) \approx \frac{\kappa}{M+1} r^{2(M+1)} + o\left(\frac{1}{\bar{\gamma}^{M+1}}\right). \tag{21}$$

Due to the correlation of complex channel coefficients $\boldsymbol{h}_{\rm p} = [h_{{\rm p},1}, \dots, h_{{\rm p},K}]^T$, the PDF of $\boldsymbol{h}_{\rm p}$ in terms of its amplitude $|h_{{\rm p},K}|$ and phase β_K can be written as [23]

$$f_{|\boldsymbol{h}_{\rm p}|,\boldsymbol{\beta}}\left(|h_{{\rm p},1}|, \beta_1 \dots, |h_{{\rm p},K}|, \beta_K\right) = \prod_{k=1}^{K} \frac{|h_{{\rm p},k}| H_{{\rm p},k}}{\pi^K \det\left(\mathbf{R}\right)}, \tag{22}$$

in which $H_{{\rm p},k} = {\rm e}^{-\frac{N_{k,k}\left|h_{{\rm p},k}\right|^2 + 2\sum_{l=k+1}^{K} N_{k,l}\left|h_{{\rm p},k}\right|\left|h_{{\rm p},l}\right|\cos(\beta_l-\beta_k)}{\det(\mathbf{R})}}$, where $N_{k,l}$ defines the (k,l)-th entry of the correlation matrix's co-factor $\boldsymbol{N}$ and $\det\left(\mathbf{R}\right)$ is the determinant of $\mathbf{R}$. Now, by considering (22) and using Leibniz integral, the approximated PDF of FAS at high SNR can be derived as

$$\begin{aligned} f_{|h_{\rm FAS}|}(r) = &\frac{Kr}{\pi^K \det\left(\mathbf{R}\right)} \int_0^{2\pi} \cdots \int_0^{2\pi} H_{{\rm p},K} \int_0^r |h_{{\rm p},K-1}| \\ &\times \left(H_{{\rm p},K-1} \left(\int_0^r |h_{{\rm p},2} H_{{\rm p},2}| \left(|h_{{\rm p},1}|\, H_{{\rm p},1} {\rm d}\,|h_{{\rm p},1}| \right) {\rm d}\,|h_{{\rm p},2}| \right) \right. \\ &\left. \times\, {\rm d}\,|h_{{\rm p},K-1}| \right) {\rm d}\beta_1, \dots, {\rm d}\beta_K. \end{aligned} \tag{23}$$

Next, the integral $I = \int_0^\infty |h_{{\rm p},k}|\, H_{{\rm p},k} {\rm d}\,|h_{{\rm p},k}|$ can be solved by applying Taylor series approximation at around zero [24], i.e., $I = \frac{r^2}{2} + o(r^2)$ for $k = \{1, \dots, K-1\}$. Additionally, the Taylor series approximation of $H_{{\rm p},k}$ at zero is $H_{{\rm p},k} = 1 + o(1)$. Now, by inserting the above results into (23), we have

$$f_{|h_{\rm FAS}|}(r) = \frac{2K}{\det(\mathbf{R})} r^{2K-1} + o\left(r^{2K-1}\right), \tag{24}$$

$$F_{|h_{\rm FAS}|}(r) \approx \frac{1}{\det(\mathbf{R})} r^{2K} + o\left(\frac{1}{\bar{\gamma}^K}\right). \tag{25}$$

Therefore, under $W \to \infty$, it is straightforward from (25) that the diversity order of this system is given by $D_{\rm FAS} = M + 1 = K$. However, if W is finite, $\mathbf{R}$ might be near to being singular. In this regard, consider $K \to \infty$ for a finite W. Thus, the correlation between the k-th port and $(k+1)$-th port is $\mathbf{R} = \lim_{K\to\infty} \omega J_0\left(\frac{2\pi W}{K-1}\right) = \omega J_0(0)$, and we have $h_{{\rm p},k+1} = h_{{\rm p},k}$. Thus, the joint CDF of $h_{{\rm p},k}$ and $h_{{\rm p},k+1}$ is $F_{h_{{\rm p},k+1},h_{{\rm p},k}}(r_1, r_2) = F_{h_{{\rm p},k}}(\min\{r_1, r_2\})$, which implies that they reduce to singularity. Since there are many such ports, we can use a finite K' ports to approximate the channels of FAS with K ports, where K' is the numerical rank of $\mathbf{R}$ such that $\mathbf{R}'$ is covariance with $K \to \infty$ for a fixed W. As a result, the diversity gain of FAS is limited by $\min\{K, K'\}$.

References

[1] S. J. Nawaz, S. K. Sharma, B. Mansoor, M. N. Patwary and N. M. Khan, "Non-coherent and backscatter communications: Enabling ultra-massive connectivity in 6G wireless networks," *IEEE Access*, vol. 9, pp. 38144–38186, Feb. 2021.

[2] K.-K. Wong, K.-F. Tong, Y. Shen, Y. Chen, and Y. Zhang, "Bruce Lee-inspired fluid antenna system: Six research topics and the potentials for 6G," *Frontiers Commun. Netw.*, vol. 3, no. 853416, Mar. 2022.

[3] K.-K. Wong, A. Shojaeifard, K.-F. Tong, and Y. Zhang, "Fluid antenna systems," *IEEE Trans. Wirel. Commun.*, vol. 20, no. 3, pp. 1950–1962, Mar. 2021.

[4] L. Jing, M. Li, and R. Murch, "Compact pattern reconfigurable pixel antenna with diagonal pixel connections," *IEEE Trans. Antennas & Propag.*, vol. 70, no. 10, pp. 8951–8961, Oct. 2022.

[5] L. Zhu and K. K. Wong, "Historical review of fluid antenna and movable antenna," *arXiv preprint* arXiv:2401.02362v2, 2024.

[6] K. K. Wong, A. Shojaeifard, K.-F. Tong, and Y. Zhang, "Performance limits of fluid antenna systems," *IEEE Commun. Lett.*, vol. 24, no. 11, pp. 2469–2472, Nov. 2020.

[7] M. Khammassi, A. Kammoun, and M.-S. Alouini, "A new analytical approximation of the fluid antenna system channel," *IEEE Trans. Wirel. Commun.*, vol. 22, no. 12, pp. 8843–8858, Dec. 2023.

[8] W. K. New, K.-K. Wong, H. Xu, K.-F. Tong, and C.-B. Chae, "Fluid antenna system: New insights on outage probability and diversity gain," *IEEE Trans. Wirel. Commun.*, vol. 23, no. 1, pp. 128–140, Jan. 2024.

[9] F. R. Ghadi *et al.*, "A Gaussian copula approach to the performance analysis of fluid antenna systems," *arXiv preprint* arXiv:2309.07506, 2023.

[10] F. R. Ghadi, K.-K. Wong, F. J. Lopez-Martinez, and K.-F. Tong, "Copula-based performance analysis for fluid antenna systems under arbitrary fading channels," *IEEE Commun. Lett.*, vol. 27, no. 11, pp. 3068–3072, Nov. 2023.

[11] F. R. Ghadi *et al.*, "On performance of RIS-aided fluid antenna systems," *IEEE Wirel. Commun. Lett.* doi:10.1109/LWC.2024.3405636, 2024.

[12] C. Skouroumounis and I. Krikidis, "Fluid antenna with linear MMSE channel estimation for large-scale cellular networks," *IEEE Trans. Commun.*, vol. 71, no. 2, pp. 1112–1125, Feb. 2023.

[13] Z. Zhang, J. Zhu, L. Dai, and R. W. Heath Jr, "Successive bayesian reconstructor for channel estimation in fluid antenna systems," *arXiv preprint*, arXiv:2312.06551v3, Jan. 2024.

[14] K.-K. Wong and K.-F. Tong, "Fluid antenna multiple access," *IEEE Trans. Wirel. Commun.*, vol. 21, no. 7, pp. 4801–4815, Jul. 2022.

[15] K.-K. Wong, C.-B. Chae, and K.-F. Tong, "Compact ultra massive antenna array: A simple open-loop massive connectivity scheme," *IEEE Trans. Wirel. Commun.*, early access, doi:10.1109/TWC.2023.3330932, 2023.

[16] G. L. Stüber, *Principles of mobile communication*. Springer, 2001, vol. 2.

[17] R. B. Nelsen, *An introduction to copulas*. Springer, 2006.

[18] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*. Academic, 7th ed., 2007.

[19] F. R. Ghadi *et al.*, "Fluid antenna-assisted dirty multiple access channels over composite fading," *IEEE Commun. Lett.*, vol. 28, no. 2, pp. 382–386, Feb. 2024.

[20] H.-C. Yang, S. Choi and M.-S. Alouini, "Ultra-reliable low-latency transmission of small data over fading channels: A data-oriented analysis," *IEEE Commun. Lett.*, vol. 24, no. 3, pp. 515–519, Mar. 2020.

[21] E. W. Weisstein, "Euler-Mascheroni constant," [Online] Available: https://mathworld.wolfram.com/, 2002.

[22] Z. Wang and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun*, vol. 51, no. 8, pp. 1389–1398, Aug. 2003.

[23] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge university press, 2005.

[24] S. Liu, J. Cheng, and N. C. Beaulieu, "Asymptotic error analysis of diversity schemes on arbitrarily correlated Rayleigh channels," *IEEE Trans. Commun.*, vol. 58, no. 5, pp. 1351–1355, May 2010.