On rate performance of *M*-ary amplitude shift keying compact ultra massive array systems for massive connectivity

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Compact ultra massive array (CUMA) is a new form of the emerging fluid antenna system where a huge number of flexible-position antennas are selected to produce the output signal. By making sure that the in-phase channels (similarly for the quadrature channels) of the desired signal at the selected antenna ports align, it builds an advantage of the desired signal over the interference. It is known that CUMA as a multiple access scheme is able to deal with hundreds of users on the same channel use, in the case of rich scattering, if binary phase shift keying is considered. It is nevertheless unclear if higher-level modulation can bring even greater network rate in this extreme massive connectivity scenario. This letter investigated this situation by presenting the average data rate expression of CUMA when *M*-ary amplitude shift keying is used, assuming a binary symmetric channel. Numerical results reveal that *M*-ary amplitude shift keying can indeed raise the rate performance considerably.

Introduction: The sixth generation (6G) mobile communications is aimed at providing TK μ extreme connectivity, meaning (i) Tbps-scale data rate, (ii) Kbps/Hz-scale spectral efficiency, and (iii) μs-scale latency [1]. The data rate target is supposed to be achievable if we are given more spectrum in the millimetre-wave and terahertz bands. The second target is extremely challenging. To obtain a spectral efficiency of 1 kbps/Hz in a single-user channel, the required received signal-to-noise ratio (SNR) is 3000 dB! This is simply inconceivable. This however could be reasonable if we are able to overlap hundreds or even thousands of users on the same channel use. The state-of-the-art technologies such as massive multiple-input multiple-output (MIMO) and non-orthogonal multiple access (NOMA) do not seem to offer such extreme massive connectivity solution. Both demand accurate channel state information (CSI) to be available at the transmitter side, and thus have scalability issues when the number of users is large. The latency target further dismisses any technology that requires huge CSI feedback and computation-intense optimization at the base station (BS) and users.

Recently, a new technology, referred to as fluid antenna system (FAS), is proposed [2, 3]. FAS represents any form of flexible-position movable and non-movable antenna systems that introduces a new degree-of-freedom in the physical layer. This is motivated by the advances in soft-material-based movable antennas [4], and radio-frequency (RF) pixel-based antennas [5–7]. Small-sized metamaterial-based switchable antennas [8], if densely packed, can also be one version of FAS as well. Antenna position flexibility has recently been studied for single [9] and multiuser communications [10, 11], presenting a new paradigm for wireless communications.

Remarkably, in [12], Wong et al. proposed a novel massive connectivity solution by adopting a compact ultra massive array (CUMA) at every user. The proposed approach builds an advantage of the desired signal over the interference by selecting two massive groups of ‘correct’ antenna ports to produce the output signal. One group focuses upon the alignment of the in-phase components of the desired signal and another for the quadrature components. The in-phase and quadrature components then combine using maximum ratio combining (MRC) to form the estimated signal, as depicted in Figure 1. The advantages of CUMA are fourfold.

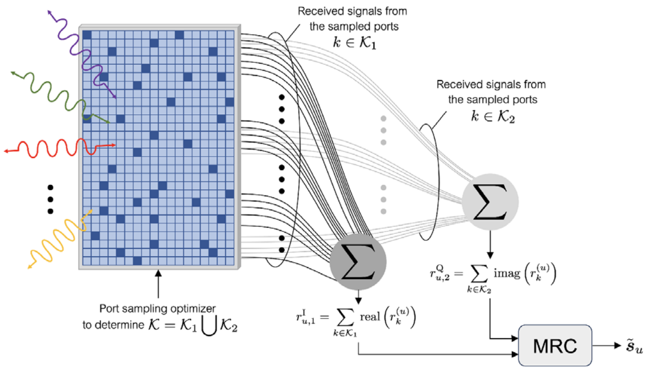

Fig. 1 *The receiver architecture of a FAS-aided user device using CUMA*

- This is a receiver-led approach, meaning that the performance of a given user does not depend upon what other users do. Accordingly, no global optimization over all the users is necessary, desirable for scalability.
- The transmitter side does not need any CSI because no precoding nor power control is adopted, contrary to what massive MIMO and NOMA require to do. This makes the scheme particularly simple.
- Each user needs only two RF chains and a single-user decoder.
- Antenna port switching only takes place once in each channel coherence time as in slow fluid antenna multiple access (FAMA) [11], which is so much more practical than fast FAMA in [10] where the switching has to be done on a symbol-by-symbol basis.

In the case of rich scattering, CUMA as a multiple access technique can accommodate hundreds of users on the same channel use if each user has a sufficiently dense and massive FAS. Nonetheless, the results in were limited to considering the use of binary phase shift keying (BPSK). While it makes sense to consider low-rate users in massive connectivity scenarios, it is not well understood how CUMA supports higher-rate users. Motivated by this, this letter investigates the network rate performance of CUMA if users are transmitting *M*-ary amplitude shift keying (ASK). To this end, we present the rate expression of CUMA under rich scattering scenarios assuming a binary symmetric channel. Although rich scattering may not be the natural channel condition in the millimetre-wave and terahertz bands, it is possible to deploy large artificial scattering surfaces to restore the rich-scattering phenomenon even in those bands [13]. We are interested in how CUMA performs in terms of the overall rate when M increases.

The CUMA system model: We consider a downlink system where the BS has U fixed antennas, each sending an M-ary ASK signal[1] to a designated user. Hence, there are U users sharing the same channel use. Each user is equipped with a two-dimensional (2D) FAS with size of $\bar{W} = W_1\lambda \times W_2\lambda$ where λ is the wavelength. The FAS has $N = N_1 \times N_2$ flexible positions (known as ports) evenly distributed over the space, and each port represents a position, to which the antenna can be switched. To specify a given port on the FAS, we use an appropriate mapping function to convert the 2D indices (n_1, n_2) into a one-dimensional (1D) index $k = \mathrm{map}(n_1, n_2)$. In FAMA, a basic operation is to find and switch the antenna to the one where the signal-to-interference plus noise ratio (SINR) is maximized [11].

In the basic form of CUMA, user u has two real-valued received signals, one taken from the aggregate of the real parts of the received signals from the activated antenna ports in the set $\mathcal{K}_1$ and another from the ag-

[1] Evidently, it may be more interesting to consider complex modulations in CUMA. However, the interfering signals at the in-phase and quadrature components of the digital symbol are correlated, which makes the error rate analysis extremely difficult.

gregate of the imaginary parts of the signals in the set $\mathcal{K}_2$, given by [12]

$$\begin{cases} r_{u,1}^{\mathrm{I}} = \left[\sum_{k\in\mathcal{K}_1} \mathrm{real}([\mathbf{g}_{u,u}]_k)\right] s_u + \sum_{k\in\mathcal{K}_1} \mathrm{real}\left(\left[\sum_{\substack{\tilde{u}=1\\ \tilde{u}\neq u}}^{U} \mathbf{g}_{\tilde{u},u} s_{\tilde{u}}\right]_k\right), \\ r_{u,2}^{\mathrm{Q}} = \left[\sum_{k\in\mathcal{K}_2} \mathrm{imag}([\mathbf{g}_{u,u}]_k)\right] s_u + \sum_{k\in\mathcal{K}_2} \mathrm{imag}\left(\left[\sum_{\substack{\tilde{u}=1\\ \tilde{u}\neq u}}^{U} \mathbf{g}_{\tilde{u},u} s_{\tilde{u}}\right]_k\right), \end{cases} \tag{1}$$

where s_u denotes the digital symbol intended for user u with $\mathrm{E}[s_u^2] = \sigma_s^2$ for all users, the notation $[\cdot]_k$ returns the kth entry of the input vector, and $\mathbf{g}_{\tilde{u},u}$ is the channel vector from BS antenna $\tilde{u}$ (i.e. the interfering transmitter $\tilde{u}$) to user u at all the possible ports. Assuming unity average power of each channel coefficient and under rich scattering, we have

$$\rho_{k,m} \triangleq \mathrm{E}\left[[\mathbf{g}_{\tilde{u},u}]_k([\mathbf{g}_{\tilde{u},u}]_m)^*\right] = j_0\left(2\pi\sqrt{\left(\frac{(n_1-n_3)W_1}{N_1-1}\right)^2 + \left(\frac{(n_2-n_4)W_2}{N_2-1}\right)^2}\right), \tag{2}$$

where $k = \mathrm{map}(n_1, n_2)$, $m = \mathrm{map}(n_3, n_4)$ and $j_0(\cdot)$ is the zeroth-order spherical Bessel function. Note that in Equation (1), additive noise is ignored, which is justified in interference-limited scenarios (e.g. the large U case).

The key for CUMA is to establish an advantage of the desired signal over the interference in Equation (1) and this is done by carefully choosing the sets, $\mathcal{K}_1$ and $\mathcal{K}_2$. To do so, the simplest form of CUMA selects $\mathcal{K}_1$ such that

$$\mathrm{sign}(\mathrm{real}([\mathbf{g}_{u,u}]_k)) > 0, \forall k \in \mathcal{K}_1. \tag{3}$$

The set $\mathcal{K}_2$ is chosen in a similar fashion. More sophisticated criteria are possible [12]. Nevertheless, for mathematical tractability, Equation (3) is assumed. In so doing, the sets ensure that the aggregation in Equation (1) makes the desired signal stronger while the interfering signals combine randomly, and could be averaged out in a manner similar to massive MIMO.

With the two signals, $r_{u,1}^{\mathrm{I}}$ and $r_{u,2}^{\mathrm{Q}}$, MRC is adopted to combine them to produce the estimate for detection of s_u. In this letter, s_u is chosen from the set of M-ary ASK alphabets and error-correcting codes are not used.

Main result: Here, we present the main result which quantifies the network rate of CUMA in the case of M-ary ASK and rich scattering assuming a binary symmetric channel. The result is given in the following theorem.

Theorem 1. *The average network rate of* M*-ary ASK CUMA is given by*

$$R = U \log_2 M\left[1 + p_e \log_2(p_e) + (1-p_e)\log_2(1-p_e)\right], \tag{4}$$

where p_e *is the bit-error-rate (BER) given by*

$$p_e = \frac{2(M-1)}{M\log_2 M}\int_0^\infty Q\left(\sqrt{\frac{6}{M^2-1}\left(\frac{z}{\sigma_2^2}\right)}\right) f_Z(z)dz, \tag{5}$$

where $Q(\cdot)$ *is the Gaussian-Q function,*

$$\sigma_2^2 = \frac{1}{4}\left(N + \sum_{m=2}^{N}\sum_{k=1}^{m-1}\rho_{k,m}\right), \tag{6}$$

and

$$f_Z(z) = \int_0^z f_{Z_1}(x) f_{Z_1}(z-x)dx, \tag{7}$$

in which $f_{Z_1}(z)$ *is given by Equation (8) (see top of this page) where* $I \triangleq U-1$, $\Gamma(\cdot)$ *is the gamma function,* $\mathcal{M}_{a,b}(t)$ *is the Whittaker* M *function given by [14, Section 9.220 on p. 1024], and*

Table 1. Simulation parameters

		f (GHz)	
		6	26
Size, $\bar{W}$		$3\lambda \times 1.6\lambda$	$13\lambda \times 7\lambda$
Number of ports $N_1 \times N_2$	I	7×3	27×13
	II	31×3	131×13
	III	61×3	261×13

The size of the 2D FAS at each user is 15 cm × 8 cm.
Case I is the non-compact case with minimum spacing of 0.5λ.
Case II is the compact case with minimum spacing of 0.1λ.
Case III is the very compact case with minimum spacing of 0.05λ.

$$f_{Z_1}(z) = \frac{\Gamma\left(\frac{I+1}{2}\right)}{\Gamma\left(\frac{I}{2}\right)\Gamma\left(\frac{I}{2}\right)}\left(\frac{1}{2^{\frac{I}{2}}}\right)\left(\frac{N}{2}\sqrt{\frac{1}{\pi}}\right)^{-\frac{1}{2}} z^{-\frac{3}{4}} e^{-\frac{1}{4\sigma_1^2}\left(\frac{N}{2}\sqrt{\frac{1}{\pi}}\right)^2\left(\frac{2\sigma_1^2+z}{\sigma_1^2+z}\right)} \left(\frac{2}{1+\frac{z}{\sigma_1^2}}\right)^{\frac{2I+1}{4}} \mathcal{M}_{-\frac{2I+1}{4},-\frac{1}{4}}\left(\frac{\left(\frac{N}{2}\sqrt{\frac{1}{\pi}}\right)^2 z}{2\sigma_1^2(\sigma_1^2+z)}\right), \tag{8}$$

$$\sigma_1^2 = \frac{N}{4}\left(1-\frac{1}{\pi}\right) + 2\sum_{m=2}^{N}\sum_{k=1}^{m-1}\mathrm{cov}(X_k^+, X_m^+). \tag{9}$$

where $X_k^+ = \max\{0, \mathrm{real}([\mathbf{g}_{u,u}]_k)\}$ *and* $\mathrm{cov}(X_k^+, X_m^+)$ *is given by*

$$\mathrm{cov}(X_k^+, X_m^+) = \frac{(1-\rho_{k,m}^2)^{\frac{3}{2}}}{4\pi} - \frac{1}{4\pi} + \frac{\rho_{k,m}}{2\sqrt{\pi}}\mathcal{W}\left(-\sqrt{\frac{2\rho_{k,m}^2}{1-\rho_{k,m}^2}}, 1, \frac{1}{2}\right), \tag{10}$$

where $\mathcal{W}(a, b, c)$ *is a special integral of the Q function [15, (2)].*

Proof. The proof follows from what appeared in [12]. The main step here is to obtain the bit-energy to the noise power density ratio, $\frac{E_b}{N_0}$, from the resulting output signal, which gives

$$\frac{E_b}{N_0} = \frac{1}{\log_2 M}\left(\frac{z}{\sigma_2^2}\right), \tag{11}$$

in which the ratio, $\frac{z}{\sigma_2^2}$, is the signal-to-interference ratio (SIR) and it has been shown in [12, Theorem 4] that the overall interference behaves like a Gaussian noise. With Equation (11) and the BER expression for M-ary ASK in [16, p. 194], we obtain Equation (5). Finally, we use the rate expression of a binary symmetric channel with p_e in Equation (5) and multiply it by the number of bits per symbol, $\log_2 M$, and the number of users, U, to complete the proof. □

The main insight derived from Theorem 1 is that it seems possible to increase the network rate, R, by increasing the order of modulation, M. This point will be confirmed by the numerical results in the next section.

Numerical results: In this section, numerical results using Equation (4) in Theorem 1 are provided. We will examine the rate performance at 6 GHz (the 5G Mid-band) and 26 GHz (the millimetre-wave band). As explained before, rich scattering is still assumed in both bands because it is possible to create the scattering conditions utilizing large reflective surfaces [13]. The size of FAS at each user is set to be 15 cm × 8 cm, the size of a typical handset. The performance of CUMA rightly depends on the number of available ports on the FAS, $N = N_1 \times N_2$, which can be interpreted as the level of compactness or port density since the area is fixed. To investigate this, three cases of compactness are considered and the detailed settings are given in Table 1. Moreover, the limiting factor here is the interference as the only noise source. Also, the parameter, M, controls the modulation.

The rate results for 6 GHz are provided in Figure 2 while we look into the case of 26 GHz in Figure 3. The great news, already reported in [12], is that as the number of users increases, the network rate keeps rising, meaning that CUMA is a massive connectivity solution with its

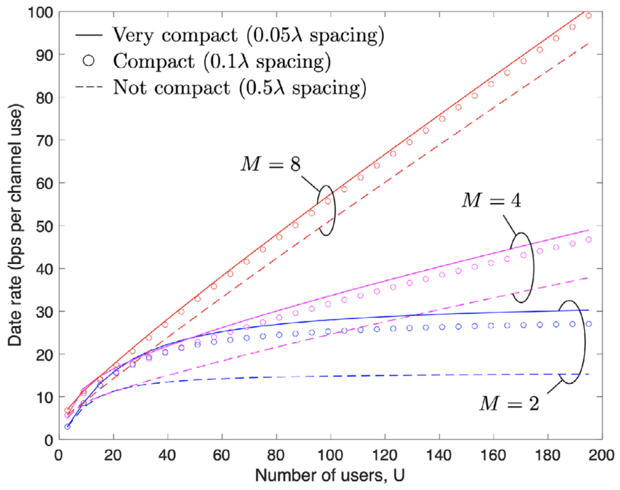

Fig. 2 *The network rate performance of CUMA at* 6 GHz

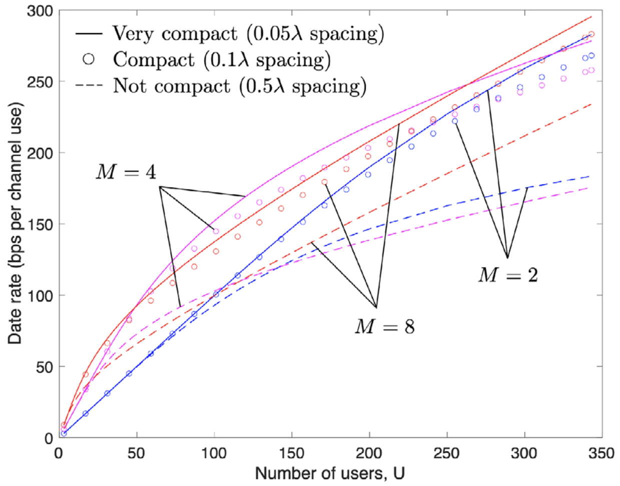

Fig. 3 *The network rate performance of CUMA at* 26 GHz

complexity unrelated to the number of users in the network. On the other hand, compactness always delivers better rate performance. Evidently, the rate performance is highly dependent on the size of FAS at each user; therefore, CUMA at a higher frequency band performs better than that at lower frequency, as can be seen by comparing the results in Figures 2 and 3.[2]

A closer observation of the results in the figures reveals that the value of M plays a role in affecting the rate performance of CUMA. It is however not trivial to figure out if a larger value of M is desirable when the number of users is not so large. In the extreme massive connectivity case, i.e. very large U, by contrast, the results in Figure 2 do show a clearer picture that a larger value of M greatly enhances the network rate. It is worth noting that although $M = 8$, i.e. 3 bits/symbol, achieves the highest rate among the cases, the delivered data is less than 1 bit per user per channel use. In fact, it can be seen that with $U = 200$ users using 8-ASK at 6 GHz, each user on average delivers 0.5 bits per channel use, much better than 0.15 bits per channel use if 2-ASK is used. With less users, there are obvious crossovers when different M values are considered, meaning that sometimes a smaller M can benefit and achieve a better overall rate.

Now, let us turn our attention to the 26 GHz case in Figure 3. The results illustrate a less clear situation as far as the impact of M is concerned. We believe that if the number of users, U, is in the thousands, then we should observe a higher network rate delivered if M is larger, just like the case of 6 GHz. Unfortunately, we are unable to obtain the results with such a large number of users. In fact, for $U > 350$, it exceeds our computing capability to calculate Equation (8). Basically, the perplexing situation at 6 GHz when $U < 20$ occurs here when $U < 350$ in the case of 26 GHz. The crossovers suggest that it should be difficult to generalize how the network rate would behave if M changes. Despite this, it is clear to see that both $M = 4$ and $M = 8$ outperform the case of $M = 2$, showing clear benefits of using higher-order modulation. For $50 \leq U \leq 250$, the results indicate that $M = 4$ appears to obtain a higher rate, while $M = 8$ does best if $U < 50$. Finally, it is useful to point out that at 26 GHz with $U = 350$ users, CUMA is able to deliver 0.86 bits per user per channel use using 8-ASK.

Finally, to put these results in context of existing solutions, it should be noted that both massive MIMO and NOMA are not expected to achieve similar results to CUMA under practical conditions. The reason is that even for slow fading it would be nearly impossible to find hundreds of users in the same timeframe in which their channels are seemingly static. This will eliminate any chance of precoding and power control being effective, not to mention the enormous complexity of the required optimization with such large number of users and the huge overhead for CSI acquisition.

Practical limitations and future research directions: Before we conclude this letter, it is worth pointing out that the results reported here rely on the assumptions that mutual coupling does not exist for CUMA and that rich scattering always occurs regardless of the operating frequency. These assumptions are indeed unrealistic. Future research should look to examine CUMA under realistic mutual coupling conditions. Also, the approach of exploiting artificial scattering surfaces in [13] should be explored to see how close rich scattering conditions can be restored to benefit CUMA.

Conclusion: This letter investigated the network rate performance of the emerging CUMA system for M-ary ASK communications. CUMA is a new technique of using FAS for massive multiple access that has the ability to accommodate hundreds or more users on the same channel use without the need of precoding and interference cancellation. Our results illustrated that using a higher-order M-ary ASK modulation in CUMA could enhance the network rate greatly, especially in extreme massive connectivity.

Author contributions: **Kai—Kit Wong**: Conceptualization; formal analysis; investigation; methodology; writing—original draft; writing—review and editing. **Chan—Byoung Chae**: Investigation; methodology; writing—review and editing. **Yangyang Zhang**: Conceptualization; writing—review and editing.

Acknowledgements: The work of Wong is supported by the Engineering and Physical Sciences Research Council (EPSRC) under grant EP/W026813/1 while the work of Chae is supported by the Institute of Information and Communication Technology Promotion (IITP) grant funded by the Ministry of Science and ICT (MSIT), Korea (2021-0-02208, 2022R1A5A1027646).

Conflict of interest statement: The authors declare no conflicts of interest.

Data availability statement: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Received: *7 November 2023* Accepted: *23 December 2023*
doi: 10.1049/ell2.13077

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[2]Readers should be reminded that this is only true if the rich scattering conditions in the channel are maintained regardless of the frequency.

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