Optimal Accounting Rules, Private Benefits of Control, and Efficient Liquidation

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Abstract. We study optimal accounting rules that alleviate inefficiencies caused by managerial private benefits. Accounting signals generated by the accounting rules guide the continuation decision at an interim project stage. The entrepreneur enjoys private benefits from continuation, which may induce inefficient decisions. The optimal accounting rule is characterized by a threshold, with a higher threshold representing more conservative accounting. The first-best is achieved under small private benefits. As private benefits increase, the first-best eventually is not achievable and more informative bad news is required for the manager to terminate, resulting in less conservative accounting rules. Therefore, more conservative accounting rules are associated with more efficient investment decisions.

1. Introduction

Disclosure based on accounting rules has been argued to play an important role in alleviating agency problems (Jensen and Meckling 1976, Watts and Zimmerman 1986). Properties of accounting rules, such as informativeness, timeliness, and conservative bias, have been extensively studied. Prior literature has examined settings where a financially constrained entrepreneur raises debt from outside investors to finance new projects (Gigler et al. 2009, Goex and Wagenhofer 2009, Bertomeu and Cheynel 2015, Caskey and Laux 2017). This paper studies the optimal accounting rule design in the presence of private benefits of control without restriction to specific classes of accounting signals. This flexibility allows us to elicit the qualitative features of accounting rules that are inherent to the agency problem. We find that higher agency conflicts call for less conservative accounting rules. This contrasts with what is suggested by earlier literature (Goex and Wagenhofer 2009, Gao 2013a). In addition, more conservative accounting rules are associated with but not the cause of more efficient investment decisions, as they are both endogenously caused by lower managerial private benefits.

Specifically, a financially constrained entrepreneur has exclusive access to a project and seeks financing from an outside investor. The project can be terminated at an interim stage and generates a fixed termination value. Otherwise, it continues and generates a random final cash flow. Besides the potential cash flow benefit, the entrepreneur also enjoys private benefits from continuation (Baldenius 2003, Caskey and Laux 2017). To finance the project, the entrepreneur issues debt to an outside investor.[1] The debt security is characterized by its face value, a covenant that specifies the control right of the project regarding the continuation decision and the way to split the termination value. In the interim period, a public accounting signal of the final cash flow is generated according to a rule and can be used to design the covenant. As a result, the control right can be allocated contingent on the realization of the accounting signal. For a given accounting rule, the entrepreneur chooses the face value, the termination payment to the outside investor, and the covenant to maximize the expected payoff, which consists of both the cash flow and the private benefit. The private benefit gives the entrepreneur an excessive incentive to continue the project, even if the accounting signal suggests a pessimistic prospect and thus could harm investment efficiency. Anticipating the potential distortion, a regulator designs the accounting rule to maximize investment efficiency. The objective is consistent with that of standard setters such as the Financial Accounting Standard Board.

Our main results are as follows. First, it is optimal to choose a binary accounting rule. This result is not obvious because the overall decision involves both the decision of whether to continue and which party receives the right to make such decision and so is not binary.

We establish in Proposition 1 that it is optimal to focus on the total surplus, which is determined by the ultimate binary decision of whether to continue the project. A binary accounting rule then follows.

Second, the optimal threshold depends on the magnitude of the entrepreneur's private benefit. As a benchmark, the regulator's first-best decision rule is to terminate the project if and only if the final cash flow is below the termination value, which is equivalent to setting a threshold equal to the project's termination value. We refer to this threshold as the first-best threshold. Under low agency frictions, the first-best threshold remains optimal, despite the presence of the private benefit. The entrepreneur's private benefit, when not too large, is dominated by the potential investment efficiency loss from continuing a project that should have been terminated. As a result, the efficient investment decision rule can be implemented by compensating the entrepreneur for the lost private benefit. In contrast, when the agency problem is severe, in the sense that the private benefit exceeds the potential loss from pursuing a bad project, implementing the decision rule characterized by the first-best threshold is suboptimal, and the optimal threshold falls below the first-best threshold. Therefore, agency problem results in excessive continuation.

Third, the optimal threshold falls as the agency problem becomes more severe. The reason is that the larger the private benefit, the more eager the entrepreneur is to continue. This calls for a more informative bad news signal to induce termination and thus a lower threshold.

Our paper makes several contributions to the accounting literature. First, we study the effect of mandatory disclosure on agency problems. Goex and Wagenhofer (2009) also study this question by imposing the assumption that the signals generated are either perfect or complete noise. We study this issue without imposing parametric structures on feasible securities, covenants, or accounting signals, so that the binary information structure is not a straightforward result. We show that, when the decision is binary, the optimal accounting rule can be characterized by a threshold, with a higher threshold suggesting more conservative accounting in the spirit of Gigler et al. (2009), as the higher threshold implies more informative good news but less informative bad news. Second, our paper also relates to the literature on persuasion (or more broadly, information design) prior to contracting in the presence of agency conflicts between the entrepreneur and the regulator. When the agency problem is more severe, the less conservative the optimal accounting rule should be. Our findings thus cast doubt on the conventional wisdom that more severe agency problems call for more conservative accounting. Third, our model provides a formal theory to justify the positive association between accounting conservatism and investment

efficiency, a result documented empirically (Garcia Lara et al. 2016). The positive association does not necessarily imply a causal effect of accounting conservatism on investment efficiency, as both are endogenous responses to the underlying agency problem.

The paper is organized as follows. The next section discusses related literature. Section 2 sets up the main model. Section 3 derives our main results and links the results to accounting properties such as conservatism, and Section 4 provides two extensions of the main model. Section 5 discusses implications of our results, and Section 6 concludes. The appendix contains all the proofs, and the online appendix contains some generalizations of the setting discussed in the main text.

1.1. Related Literature
Our paper relates to two streams of literature.

First, our paper relates to the analytical literature on accounting conservatism when the informativeness of accounting information is held constant and the focus is on the optimal degree of asymmetry. The literature has shown that the desirability of conservatism depends on the characteristics of the information environment and the specific setting (Guay and Verrecchia 2006, Gigler et al. 2009, Goex and Wagenhofer 2009, Caskey and Hughes 2012, Gao 2013a, Li 2013, Armstrong et al. 2016, Friedman et al. 2016, Jiang 2016, Bertomeu et al. 2017, Caskey and Laux 2017, Glover and Lin 2018).[2] We study the optimal accounting rule that includes covenant design in the presence of the agency friction that private benefits bias in favor of project continuation decisions. Gigler et al. (2009) and Guttman and Marinovic (2018) also study covenant design in different settings. They exogenously assume debt securities, whereas we allow arbitrary types of securities issued by the entrepreneur.

In this stream of literature, our paper most closely relates to Bertomeu et al. (2017) and Caskey and Laux (2017), in that these studies show a negative relation between agency frictions and accounting conservatism in the presence of ex ante earnings manipulation. These papers use classes of accounting signals in which the informativeness of the signal is fixed (Gigler et al. 2009, Li 2013). In these models, a more informative signal would always be desirable. In contrast, we investigate a setup where there are tradeoffs to having more information. Hence, we develop a complete analysis on the optimal accounting rules without restricting the informativeness of the signal. This approach allows us to study the properties of accounting rules while retaining the flexibility and generality of such rules. In this sense, our approach is similar in spirit to Bertomeu et al. (2023) but addresses a different question in a different context.

Second, our paper is related to the finance and economics literature on persuasion. Goldstein and Leitner (2018) study the optimal stress test rule in a Bayesian persuasion

setting and document that the rule is characterized by a threshold. Huang (2016) studies optimal accounting rules in a similar entrepreneur-seeking-financing setting, in which the investor can choose whether to acquire private information after the accounting disclosure. These papers do not study agency problems and optimal contracts/security design simultaneously.

2. Model

We consider a three-date model with three risk-neutral players: a regulator, an entrepreneur, and an investor. We assume that all players' discount factors equal one as assuming otherwise will not affect our results qualitatively.

2.1. Project

The entrepreneur has exclusive access to a project that requires initial investment $k > 0$ to start at date 0. At date 1, the project can either be terminated to generate a termination value $M > 0$ or continue to date 2 and generate a random final cash flow $\theta \geq 0$. In the main model, we assume that the three players share a common prior, P, about θ. We also assume that the probability density exists, denoted by p, with $\mathrm{supp}(p) = [\underline{\theta}, \overline{\theta}]$, $\underline{\theta} \geq 0$ and $\overline{\theta} > M$.

2.2. Accounting Rule

Before date 0, the regulator designs an accounting rule h that generates a signal $x \in X$ at date 1, where $X \subset \mathbb{R}$ is the set of signal realizations. In particular, the accounting rule is modeled as an information structure $h : [\underline{\theta}, \overline{\theta}] \to \Delta(X)$, with $h(\cdot|\theta)$ denoting the probability measure over signal realizations, conditional on the true state being θ. We do not impose any restriction on the regulator's choice of h.

2.3. Timeline

At date 0, the entrepreneur is financially constrained. To finance the project, the entrepreneur needs to raise k from the investor by issuing a debt security (c, D, M_I), where D is the face value of the debt security, c is the covenant, and $M_I \in [0, M]$ is the payment to the investor in case of termination.[3] In particular, $c : X \to \{E, I\}$ allocates the control right of the project at date 1 according to the signal realization x. The entrepreneur obtains the control right to decide whether to continue or terminate the project if $c(x) = E$, and the investor has the control right if $c(x) = I$. If the project continues to date 2, it generates a cash flow θ, in which $\min(\theta, D)$ goes to the investor and $\max(\theta - D, 0)$ remains with the entrepreneur. If the project is terminated at date 1, the investor receives M_I, and the entrepreneur receives $M - M_I$.

2.4. Preferences

The regulator's objective is to maximize investment efficiency, measured by the expected total cash flow from this project. That is, the regulator would prefer to continue the project at date 1 if and only if the expected value of the final cash flow θ exceeds the termination value M. The entrepreneur, however, enjoys a private benefit $B \in [0, M]$, in addition to the cash flow $\max(\theta - D, 0)$, from continuing the project. The investor offers k to finance the project, provided that their expected payoff at date 0 exceeds the reservation value $\underline{v}_I > 0$.

2.5. Incentives

For a given accounting rule, subject to the investor's participation constraint, the entrepreneur chooses (c, D, M_I) to maximize the investor's expected payoff, which consists of both the cash flow benefit $\max(\theta - D, 0)$ and $M - M_I$, as well as the private benefit B. Depending on (c, D, M_I), potential conflicts of interest between the investor and the entrepreneur may arise regarding whether to continue the project at date 1. There are situations when the entrepreneur would prefer to continue due to limited liability, but the investor would prefer to terminate.[4] In addition, the private benefit gives the entrepreneur an extra incentive to continue, even if the signal suggests a pessimistic prospect. This situation generates a conflict of interest between the entrepreneur and the regulator. Anticipating the potential distortion, the regulator designs the accounting rule h to maximize investment efficiency.[5]

Intuitively, the investor's reservation value $\underline{v}_I$ captures the investor's bargaining power relative to the entrepreneur. If $\underline{v}_I$ is too large, the investor's bargaining power is so strong that the investor would ask too much from the entrepreneur, rendering it impossible to finance the project. To preclude this uninteresting case, we assume that

$$\mathbf{E}[\theta] \geq \max(\underline{v}_I, M - B). \tag{1}$$

This assumption ensures that (1) the ex ante total proceeds from the project are large enough to cover the investor's reservation value, and (2) the ex ante total proceeds are large enough to preclude the uninteresting case that the entrepreneur never wants to continue at date 1.

We next define the equilibrium of this game. Let $a : X \to \{0, 1\}$ denote a decision rule in date 1, where $a(x) = 1$ means that the project continues to date 2 upon signal realization x and $a(x) = 0$ means termination.

2.6. Equilibrium Definition

An equilibrium is a set of rules $\{(c^*, D^*, M_I^*), h^*, a^*\}$ such that[6]

(i) For any $x \in X$,

$$a^*(x) \in \arg \max_{a \in \{0,1\}} a \cdot (\mathbf{E}^h[\min(D,\theta)|x] - M_I) \text{ if } c(x) = I,$$

and

$$a^*(x) \in \arg \max_{a \in \{0,1\}} a \cdot [\mathbf{E}^h[\max(\theta - D, 0)|x]$$
$$+ B - (M - M_I)] \text{ if } c(x) = E,$$

where $\mathbf{E}^h[\cdot]$ denotes the expectation operator under accounting rule h.

(ii) Given the accounting rule h, $\{(c, D^*, M_I^*), a^*\}$ maximizes

$$\mathbf{E}[a(x) \cdot (\mathbf{E}^h[\max(\theta - D, 0)|x] + B)$$
$$+ (1 - a(x)) \cdot (M - M_I)] \qquad (2)$$

subject to $M_I \in [0, M]$, Condition (i), and

$$v_I^h(D, M_I, a) = \mathbf{E}[a(x) \cdot \mathbf{E}^h[\min(\theta, D)|x]$$
$$+ (1 - a(x)) \cdot M_I] \geq \underline{v}_I,$$

where $v_I^h(D, M_I, a)$ is the investor's expected payoff, given D, M_I and the decision rule a.

(iii) The accounting rule h^* maximizes

$$\mathbf{E}[a(x) \cdot \mathbf{E}^h[\theta|x] + (1 - a(x)) \cdot M] \qquad (3)$$

subject to Conditions (i) and (ii).

In this definition, Condition (i) is an incentive compatibility (IC) constraint. It requires that the project continues/terminates at date 1 if it is optimal for the party with the control right to do so. Condition (ii) states that the entrepreneur chooses (c, D, M_I) to maximize the expected payoff given by (2), subject to the IC constraint and the investor's participation constraint. Condition (iii) states that, anticipating the entrepreneur's choice of (c, D, M_I), the regulator proposes an accounting rule to maximize the investment efficiency as given by (3).

3. Analysis and Results

The equilibrium is solved through backward induction. We first solve for the entrepreneur's optimal response to any given accounting rule chosen by the regulator, followed by solving for the regulator's optimal accounting rule given the entrepreneur's best response.

For any accounting rule h, let $a_h : X \to \{0, 1\}$ be defined by

$$a_h(x) = \begin{cases} 1 \text{ if } \mathbf{E}^h(\theta|x) + B > M \\ 0 \text{ otherwise.} \end{cases} \qquad (4)$$

Then a_h is the decision rule that maximizes the total surplus from the entrepreneur's perspective (i.e., including the private benefit B) under the accounting rule h. Let

DM^h denote the set of all pairs of (D, M_I) that make the investor just break even when a_h is implemented. That is,

$$DM^h = \{(D, M_I) \in \mathbb{R}_+ \times [0, M] : v_I^h(D, M_I, a_h) = \underline{v}_I\}.$$

We show in the appendix that DM^h is not empty. Intuitively, because the expected payoff from the project when a_h is implemented is higher than the entrepreneur's reservation payoff, there will be some security that makes the investor at least break even.

The following proposition shows that, for any given accounting rule, it is always optimal for the entrepreneur to implement a_h with some element of DM^h by a proper design of the covenant.

Proposition 1. *For any given accounting rule h, (i) for any pair $(D, M_I) \in DM^h$, there exists a covenant c such that (D, M_I, c) implements a_h; (ii) any $(D, M_I) \in DM^h$ together with its covenant in (i) is optimal for the entrepreneur.*[7]

In principle, the accounting rule design involves a binary allocation of control rights and a binary decision (i.e., continue or terminate) associated with each such allocation, so that the design of the accounting rule involves four actions. Proposition 1 shows that it is sufficient to consider a binary action problem in designing the accounting rule. That is, for any accounting rule h, the entrepreneur is always able and willing to implement the (binary) decision rule a_h. This result stems from the public nature of the signal. Because public signals do not cause information asymmetry between the entrepreneur and the investor, there always exists a way to allocate the total surplus without sacrificing the efficiency of the decision. As a result, the regulator's problem becomes

$$\max_h \; \mathbf{E}[a_h(x) \cdot \mathbf{E}^h(\theta|x) + (1 - a_h(x)) \cdot M]. \qquad (5)$$

Because the decision is binary, it is sufficient to consider binary (action-based) signal realizations, with one inducing termination and the other inducing continuation. Without loss of generality, let $X = \{0, 1\}$, with $x = 0$ (1) referring to termination (continuation). Then the accounting rule is characterized by a function $h^*(\theta) = \Pr(x = 1|\theta)$. The regulator then chooses $h^*(\theta)$ to maximize investment efficiency (5), taking into account the entrepreneur's response characterized in Proposition 1. The results are summarized in Proposition 2.

Proposition 2. *Suppose $\underline{\theta} \leq M - B$. The optimal accounting rule h^* is characterized by a threshold $\widehat{\theta} \in (M - B, M]$, such that*

$$h^*(\theta) = \begin{cases} 1 \text{ if } \theta > \widehat{\theta} \\ 0 \text{ if } \theta \leq \widehat{\theta}. \end{cases}$$

In particular, if $B \leq \mathbf{E}(M - \theta|\theta \leq M)$, then $\widehat{\theta} = M$. If $B > \mathbf{E}(M - \theta|\theta \leq M)$, then $\widehat{\theta} \in (M - B, M)$ and is uniquely

determined by

$$\mathbf{E}[\theta|\theta \le \widehat{\theta}] = M - B. \qquad (6)$$

Moreover, the optimal decision rule is $a^*(x) = x$.

This proposition has an immediate corollary showing that full disclosure is not optimal, which we formally state here.

Corollary 1. *Full disclosure is suboptimal for the regulator.*

It is straightforward to see that the first best threshold is reached when $\underline{\theta} > M - B$. When the prior is sufficiently good, the optimal accounting rule features no information disclosure and the project is always continued. This is why Proposition 2 focuses on the less extreme case of $\underline{\theta} \le M - B$.

In Proposition 2, the regulator's efficient outcome is to continue if and only if $\theta > M$. If B is sufficiently small, setting $\widehat{\theta} = M$ induces the first-best outcome, as the entrepreneur can be compensated for the forgone private benefit. If B is sufficiently large, it is too costly to compensate for the forgone private benefit. As a compromise, the regulator has to reduce $\widehat{\theta}$ until

$$\mathbf{E}[\theta|\theta \le \widehat{\theta}] = M - B,$$

when the inefficiency from continuation is sufficiently large to overcome the forgone private benefit. This makes the low signal (i.e., $x = 0$) just pessimistic enough so the entrepreneur terminates the project. As a result, the regulator reduces $\widehat{\theta}$ to make the low signal just informative enough to convince the entrepreneur to terminate.

We now explain the intuition of the corollary. Intuitively, when B is sufficiently large, the distortion induced by the entrepreneur's excessive willingness to continue is so severe that the regulator's efficient outcome is not achievable, and the threshold has to be set at some $\widehat{\theta} \in (M - B, M)$, resulting in full disclosure being suboptimal for the regulator. Full disclosure induces a continuation/termination threshold $M - B$, which is below the optimal threshold $\widehat{\theta}$ given by Proposition 2, leading to inefficient continuation for $\theta \in (M - B, \widehat{\theta}]$. This corollary also differentiates our paper from many previous papers on the optimal degree of conservatism (Gigler et al. 2009, Bertomeu et al. 2017, Caskey and Laux 2017) when a parametric functional form is imposed on the accounting signals, as otherwise full disclosure is optimal.

According to Gigler et al. (2009), accounting conservatism affects not only the incidence of good news and bad news but also their information content. In particular, more conservative accounting makes good news less likely to arrive but more informative once it arrives. In our setting, higher threshold of $\widehat{\theta}$ implies exactly more informative good news and less informative bad news. Therefore, higher $\widehat{\theta}$ corresponds to more conservative

Figure 1. (Color online) Our Conservatism Definition
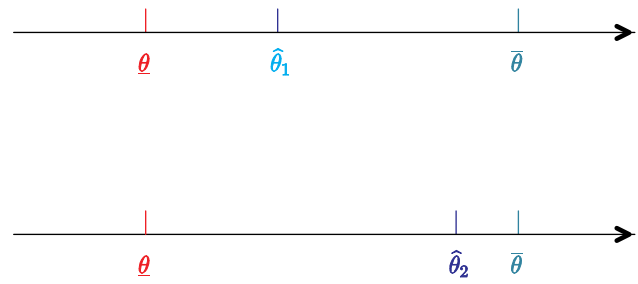
accounting, as formalized by the following definition. We provide a graphical illustration in Figure 1 with two thresholds $\widehat{\theta}_1$ and $\widehat{\theta}_2$ such that $\widehat{\theta}_1 < \widehat{\theta}_2$. Upon observing $x = 1$, the investor is more confident that θ is high when the threshold is $\widehat{\theta}_2$, as the interval $(\widehat{\theta}_2, \overline{\theta}]$ is smaller than $(\widehat{\theta}_1, \overline{\theta}]$. Meanwhile, the informativeness of $x = 0$ decreases because, upon observing $x = 0$, the investor is less confident that θ is low when the threshold is $\widehat{\theta}_2$ as the interval $[\underline{\theta}, \widehat{\theta}_2]$ is larger than $[\underline{\theta}, \widehat{\theta}_1]$. Thus, accounting is more conservative when $\widehat{\theta} = \widehat{\theta}_2$. This leads us to introduce the following definition of the accounting rule in our setting being relatively more or less conservative.

Definition 1. The accounting rule is more conservative or less liberal for higher $\widehat{\theta}$.

We now explore the comparative statics with respect to $\widehat{\theta}$, which characterizes the optimal accounting rule and allows us to connect the results on the optimal accounting rule with accounting conservatism. Because most of the results in this section, except Proposition 7, are immediate corollaries of Proposition 2, the proofs are omitted. From Proposition 2, the threshold $\widehat{\theta}$ depends on the magnitude of B. We therefore state the following conditions regarding B subsequently used in our comparative statics.

Corollary 2. *When $B \le \mathbf{E}(M - \theta|\theta \le M)$ is satisfied, the threshold $\widehat{\theta}$ increases in the liquidation value M. When $B > \mathbf{E}(M - \theta|\theta \le M)$, the threshold $\widehat{\theta}$ decreases in private benefit B and increases in the liquidation value M.*

This result is immediate from Proposition 2. When the private benefit is small, $\widehat{\theta} = M$ and the regulator's efficient outcome is achieved. When B is large, the regulator has to reduce $\widehat{\theta}$ so that the bad news is just informative enough to justify termination. The larger B is, the higher private benefit the entrepreneur enjoys from continuation, calling for a more pessimistic bad news to convince the entrepreneur to terminate, resulting in a lower $\widehat{\theta}$. Similarly, higher M makes termination more attractive, so a less pessimistic bad news is sufficient to convince the entrepreneur to terminate, that is, a higher $\widehat{\theta}$.

Corollary 2 implies that accounting is more conservative (or less liberal) if M is larger. It also implies that accounting is more liberal (or less conservative) when

M is smaller or B is larger. If we interpret larger B as higher agency cost, this result contrasts with the view that higher agency cost demands accounting to be more conservative (Watts 2003a). Higher agency cost still results in lower investment efficiency; as in our case, larger B results in lower $\widehat{\theta}$, generating more overinvestment. However, higher agency cost is associated with less, not more, conservative accounting. In other words, if the regulator chooses more conservative accounting when the private benefit becomes larger, the investment efficiency will be even lower, making this choice suboptimal.

In addition, when B is sufficiently large, $\widehat{\theta} < M$ implies overinvestment because projects with $\theta \in (\widehat{\theta}, M)$ should be terminated but are continued. Higher B results in lower $\widehat{\theta}$, implying more overinvestment and thus lower investment efficiency. Meanwhile, lower $\widehat{\theta}$ implies more liberal accounting. We thus document a positive association between lower investment efficiency and more liberal accounting, or, equivalently, higher investment efficiency and more conservative accounting, summarized in the following corollary. The proof is again omitted, as it directly follows Proposition 2.[8]

Corollary 3. *When $B > \mathbf{E}(M - \theta | \theta \leq M)$, more conservative accounting is associated with less overinvestment thus higher investment efficiency.*

4. Extensions
4.1. Moral Hazard at the Security Issuance Stage
This section introduces moral hazard into the main model and allows the entrepreneur to propose any security from

$$S = \{s : [\underline{\theta}, \overline{\theta}] \rightarrow [0, \overline{\theta}] | s(\theta) \in [0, \theta] \text{ for all } \theta\}.$$

As in the security design literature, we impose the dual monotonicity constraint on the entrepreneur's choice of securities. That is, both the payment to the investor, $s(\theta)$, and the residual cash flow to the entrepreneur, $\theta - s(\theta)$, are nondecreasing in θ. We show that the optimal security is debt, thus providing a microfoundation of the debt financing used by our main model.

In particular, after designing the contract but before the disclosure of the accounting signal, the entrepreneur can make an effort choice $l \in \{0, 1\}$, where $l = 1$ means exerting effort and $l = 0$ means no effort. The effort is unobservable to the investor and the regulator. Exerting effort improves the distribution of the cash flow θ. However, the entrepreneur incurs disutility z from exerting effort. In addition, we assume that the entrepreneur has a more optimistic belief about the distribution of θ conditional on exerting effort, relative to the investors and the regulator. The accounting literature has used the overoptimistic assumption, for example, Laux and Stocken (2012) and Infuehr and Laux (2022), whereas Gervais (2010) provides a review of the analytical literature of

managerial overoptimism in finance. This assumption is introduced for a technical reason: Our main results still hold without this assumption (i.e., let $\tilde{P}_1 = P_1$), except possibly for a knife-edge case of the parameter values. To formalize this extension, we introduce the following notations. If $l = 1$, from the entrepreneur's perspective, the cumulative distribution function (CDF) of cash flow θ is P_1; from the investors' and the regulator's perspective, the cash flow θ follows CDF $\tilde{P}_1$. Accordingly, let p_1 and $\tilde{p}_1$ denote the respective probability density functions (PDFs). If $l = 0$, all the players believe that cash flow θ follows CDF P_0. Let p_0 denote the PDF of P_0. We formalize the setting by the assumptions as follows.

Assumption 3. *The probability density functions p_0, p_1, and $\tilde{p}_1$ have full support on $[\underline{\theta}, \overline{\theta}]$; $\frac{p_1(\theta)}{p_0(\theta)}$, $\frac{\tilde{p}_1(\theta)}{p_0(\theta)}$, and $\frac{p_1(\theta)}{\tilde{p}_1(\theta)}$ strictly increase in θ, that is, the strict motonelikelihood ratio property (MLRP) holds.*

The full support assumption is a regularity condition for the sake of simplicity. The MLRP assumptions that $\frac{p_1(\theta)}{p_0(\theta)}$ and $\frac{\tilde{p}_1(\theta)}{p_0(\theta)}$ strictly increase in θ imply that P_1 and $\tilde{P}_1$ first order stochastically dominate (FOSD) P_0. That is, exerting effort improves the cash flow from all players' perspective. The MLRP assumptions that $\frac{p_1(\theta)}{\tilde{p}_1(\theta)}$ strictly increases in θ implies that P_1 FOSDs $\tilde{P}_1$, meaning that the entrepreneur is more optimistic about the improvement, relative to the investors and the regulator.

We make a final assumption so that the agency friction is nontrivial.

Assumption 4. *θ and M cannot be sufficiently large, i.e., $\int_{\underline{\theta}}^{\overline{\theta}} \max(\theta, M) p_0(\theta) d\theta < \underline{v}_I$.*

This assumption ensures that the project can be financed only if the entrepreneur exerts effort, as the investor cannot break even absent the entrepreneur's effort. This makes the moral hazard problem not trivial. The next proposition shows that the optimal accounting rule is qualitatively the same as that characterized in the main setting.

Proposition 3. *Any optimal security that finances the project must be a debt security, and the optimal accounting rule is still characterized by Proposition 2 with the expectations taken under distribution P_1.*

The optimal security is debt because its residual cash flow gives the entrepreneur the highest incentive to exert effort. Because the optimal accounting rule in our main model is derived with debt contracts and the optimal security is debt in this extension, it is straightforward that the optimal accounting rule characterized by Proposition 2 remains optimal in this extension. The only difference is that the expectations are now taken under distribution P_1, that is, conditional on the effort being exerted from the entrepreneur's perspective.

4.2. Constraining the Informativeness of Accounting Signals

In the main setting, we follow Gigler et al. (2009) and use the relative informativeness of good news versus bad news to measure conservatism. Their study also controls for the total amount of information while comparing the informativeness of good news and bad news. To focus our analysis on accounting rule design, we do not put any constraint on the informativeness of the accounting signal in the main model. This section discusses what the results would be when the informativeness of accounting signals cannot exceed some upper bound.

The accounting signal is informative in the sense that it reduces the receivers' uncertainty about the cash flow θ. As axiomatized in Shannon's information theory, uncertainty is measured by entropy, and a signal conveys information because the receipt of it reduces the entropy about the random state. In this extension, we follow the rational inattention literature (Sims 2003) and adopt entropy reduction (i.e., mutual information) to measure the informativeness of the accounting signal.[9] We impose an upper bound on the entropy reduction of accounting signals to control for the informativeness. In particular, given the prior distribution P, the ex ante entropy of θ is

$$H_{prior} = -\int_{\underline{\theta}}^{\overline{\theta}} p(\theta) \ln p(\theta) d\theta.$$

The accounting signal indicates whether θ is above or below a threshold $\widehat{\theta}$. Thus, the ex post entropy, that is, the expected entropy after observing the accounting signal, following Cover and Thomas (2006), is[10]

$$H_{post} = -\int_{\underline{\theta}}^{\overline{\theta}} p(\theta) \ln p(\theta) d\theta + P(\widehat{\theta}) \ln P(\widehat{\theta})$$
$$+ [1 - P(\widehat{\theta})]\ln[1 - P(\widehat{\theta})]. \quad (6)$$

Therefore, given an upper bound Λ, the informativeness constraint is

$$I(\widehat{\theta}) = H_{prior} - H_{post} = -P(\widehat{\theta}) \ln P(\widehat{\theta})$$
$$- [1 - P(\widehat{\theta})]\ln[1 - P(\widehat{\theta})] \le \Lambda,$$

where $I(\widehat{\theta})$ is the entropy reduction (i.e., mutual information) when the threshold is $\widehat{\theta}$. Note that $I(\widehat{\theta}) = 0$ for $\widehat{\theta} \in \{\underline{\theta}, \overline{\theta}\}$. This is intuitive because in this case the accounting signal does not reveal any information about θ. When $\widehat{\theta}$ increases from $\underline{\theta}$ to $\overline{\theta}$, $I(\widehat{\theta})$ first increases and then decreases, achieving its maximum $\ln 2$ when $P(\widehat{\theta}) = 1/2$. Hence, in the case of $\Lambda < \ln 2$, equation $I(\theta) = \Lambda$ has two solutions, denoted by θ_1 and θ_2. Without loss of generality, let $\theta_1 \le \theta_2$. Therefore, only accounting rules with threshold in $[\underline{\theta}, \theta_1] \cup [\theta_2, \overline{\theta}]$ are feasible. In this case, a corner solution may be obtained, as illustrated in the following proposition.

Proposition 4. *Let $\widehat{\theta}_{constraint}$ denote the threshold of the optimal accounting rule under the informational constraint, and θ^* be determined by $\mathbf{E}[\theta|\theta \le \theta^*] = M - B$.[11] Then,*

$$\widehat{\theta}_{constraint}$$
$$= \begin{cases} \min(M,\theta^*) & \begin{array}{l} \text{if } \Lambda \ge \ln 2, \text{ or if } \Lambda < \ln 2 \\ \text{and } \min(M,\theta^*) \in [\underline{\theta}, \theta_1] \cup [\theta_2, \overline{\theta}]; \end{array} \\ \theta_2 & \begin{array}{l} \text{if } \Lambda < \ln 2, M \in (\theta_1, \theta_2), \\ \mathbf{E}(\theta|\theta \le \theta_2) \le M - B \\ \text{and } \mathbf{E}(\theta|\theta_1 \le \theta \le \theta_2) < M; \end{array} \\ \theta_1 \text{ or } \theta_2 & \begin{array}{l} \text{if } \Lambda < \ln 2, M \in (\theta_1, \theta_2), \\ \mathbf{E}(\theta|\theta \le \theta_2) \le M - B \\ \text{and } \mathbf{E}(\theta|\theta_1 \le \theta \le \theta_2) = M; \end{array} \\ \theta_1 & \text{otherwise}. \end{cases}$$
$$(7)$$

Intuitively, when $\Lambda \ge \ln 2$, the informativeness constraint is slack, and the optimal threshold characterized by Proposition 2, which is $\min(M, \theta^*)$, remains optimal. When $\Lambda < \ln 2$, $\min(M, \theta^*)$ is still optimal if it belongs to $[\underline{\theta}, \theta_1] \cup [\theta_2, \overline{\theta}]$, because the informativeness constraint remains slack.

When $\min(M, \theta^*)$ falls in (θ_1, θ_2), the informativeness constraint becomes binding because it is only feasible to choose a threshold in $[\underline{\theta}, \theta_1] \cup [\theta_2, \overline{\theta}]$. To make sense of the accounting rule, the threshold should induce liquidation upon the low signal and continuation upon the high signal. Any threshold in $[\underline{\theta}, \theta_1]$ serves this purpose because $\theta_1 < \theta^*$, and so does any threshold in $[\theta_2, \theta^*]$ if $\theta^* \ge \theta_2$ (i.e., $\mathbf{E}(\theta|\theta \le \theta_2) \le M - B$). However, any threshold $\theta' \in [\underline{\theta}, \theta_1]$ is dominated by θ_1, because for all $\theta \in [\theta', \theta_1]$, liquidation leads to M while continuation generates θ, which is smaller than M.

Similarly, any threshold $\theta' \in [\theta_2, \theta^*]$ is dominated by θ_2. Recall that we are considering the case $\theta^* \ge \theta_2$, so that $\min(M, \theta^*) \in (\theta_1, \theta_2)$ implies $M \in (\theta_1, \theta_2)$. Hence, for all $\theta \in [\theta_2, \theta']$, the outcome from continuation, θ, exceeds M, the outcome from liquidation. It is thus sufficient for the regulator to focus on two candidate thresholds, namely, θ_1 and θ_2. To compare θ_1 and θ_2, θ_2 is better only when liquidation is more favorable than continuation for the event (θ_1, θ_2), that is, $\mathbf{E}(\theta|\theta_1 \le \theta \le \theta_2) < M$. This gives rise to the condition for the optimality of θ_2 in Equation (7). Otherwise, it is optimal to choose θ_1.

In case of binding informativeness constraint, it is worth noting that the optimal threshold is purely determined by the constraint and is independent of B and thus the underlying agency problem. In practice, firms encounter diverse information environments depending on their business characteristics and industry. According to this extension, we would see that the optimal accounting rule for firms operating in stringent information environments are mainly driven by the informativeness

constraint rather than agency issues. This offers some additional implications, which we discuss in Section 5.

5. Implications

Our results provide several empirical implications, some of which have not been tested and some of which provides alternative interpretations of some findings of the empirical literature. Please see Table 1 for a summary.

First, Corollary 2 shows that accounting becomes more conservative for larger liquidation value (*M*). To the extent that *M* measures the tangibility of firms' assets, our model predicts that conservatism is positively associated with the tangibility of firms' assets consistent with the empirical literature (Roychowdhury and Watts 2007). Incorporating measurement of tangible assets is an interesting extension to further explore this implication.

Second, Corollary 2 also shows that, when the agency problem is sufficiently severe (i.e., private benefit *B* is sufficiently large), higher private benefits result in less conservative accounting. To the extent that, in reality, the private benefit is usually so large that first-best is not achievable, our results throw some caution to the claim that accounting conservatism alleviates agency problems. For example, LaFond and Roychowdhury (2008) document a negative association between accounting conservatism and agency cost proxied by managerial ownership. One possible reason for this discrepancy could be that managerial ownership reflects more than managerial private benefit.

Third, Corollary 3 shows that more conservative accounting is associated with higher investment efficiency. Although this implication is consistent with empirical findings (Ahmed and Duellman 2011, Garcia Lara et al. 2016), the interpretation is different. For example, Garcia Lara et al. (2016) find that more conservative accounting, by resolving debt-equity conflicts, reduces overinvestment and leads to higher investment efficiency. Our model, however, only predicts an *association*, as both higher investment efficiency and more conservative accounting are caused by low managerial private benefits. Specifically, our model implies an optimal level of conservatism adopted by firms and that a higher level of conservatism reduces investment efficiency. We thus provide an alternative explanation for the findings in the empirical literature and illustrate the importance of differentiating between *causal* relations

and *associations.* In addition, Proposition 4 shows that such association may be weaker or even nonexistent for firms when the information generating process is very costly (e.g., firms with complicated business models and complicated transactions).

Fourth, we are able to show that accounting should be more conservative if there is more downside risk. This is consistent with accounting rules in practice. For example, extant accounting rules require that most intangible investments (e.g., research and development expenditures) be directly subtracted in arriving at earnings, but tangible investments can be accounted for as assets and thus not subtracted as expenses. This implies that accounting earnings are more conservative for intangible investments than for tangible ones, as low earnings numbers are less informative for intangible investments. However, intangible investments typically have higher downside risk, due to, for example, a higher failure rate for developing novel products and/or technology. Our results thus provide a justification for these rules. Explicitly modelling the uncertainty of the intangible investments' value when selling is an interesting extension to further explore this implication.

Finally, according to Proposition 4, we know that the empirical implications discussed above are more likely to be observed when firms' earnings quality is sufficiently high (i.e., $\Lambda \geq \ln 2$). When firms' earnings quality is not sufficiently high (i.e., $\Lambda < \ln 2$), and when firms' assets are sufficiently tangible (i.e., when *M* is sufficiently large), then firms' accounting properties only depend on firms' earnings quality and becomes more conservative when earnings quality goes down (as the optimal threshold is θ_2 and we can show that θ_2 decreases in Λ). Similarly, when firms' earnings quality is not sufficiently high (i.e., $\Lambda < \ln 2$), and when firms' assets are sufficiently intangible (i.e., when *M* is sufficiently small), then firms' accounting properties only depend on firms' earnings quality and becomes less conservative when earnings quality goes down (as the optimal threshold is θ_1 and we can show that θ_1 increases in Λ).

6. Conclusion

We propose a model where the accounting system provides a signal to both an entrepreneur and an outside investor about whether to continue or terminate a project. The entrepreneur enjoys a private benefit from

Table 1. Stays as Comparative Statics

Exogenous variables\endogenous constructs	Accounting conservatism	Investment efficiency
Liquidation value	+	+ When private benefit is large
Private benefit	−	− When private benefit is large
More downside risk	+	Ambiguous

continuation, which may distort the investment decision. We study the properties of optimal accounting rules that maximize investment efficiency from the regulator's perspective. The main takeaway is that the higher the private benefit, the less conservative (or more liberal) the accounting. We also document a positive association, instead of a causal relationship, between accounting conservatism and investment efficiency. We show that they are both caused by low managerial private benefit and is not caused by each other.

Future work can extend our main model in several directions. First, our main model focuses on managerial private benefits. However, other informational asymmetries may also determine preferences over reporting systems; for example, managers may be privately informed about the profitability of their project. Studying the optimal accounting rules with different agency frictions will help us better understand how accounting rules evolve to alleviate various agency problems and lead to richer empirical predictions; for example, how does signaling private information interact with optimal accounting rule design to affect the properties of accounting rules (see Jiang and Yang (2017) for an example)?

Second, we model optimal accounting rules in the Bayesian persuasion framework, which implies ex ante commitment and thus no manipulation by the entrepreneur. In addition, the accounting rule is modelled in a reduced form. Future research can relax both of these assumptions and explore whether conservatism improves economic efficiency, in the presence of entrepreneurial manipulation and explicit modelling of accounting measurements using, for example, a two-step measurement approach (Gao 2013a, b).

Acknowledgments

Appendix
A.1. Proof of Proposition 2
Proof. Let μ denote the entrepreneur's posterior mean of θ. From Proposition 6, the entrepreneur is indifferent between termination and continuation when $\mu = M - B$. According to the tie-breaking rule, the entrepreneur chooses termination when $\mu = M - B$. Then the regulator's expected payoff is

$$V(\mu) = \begin{cases} \mu & \text{if } \mu > M - B \\ M & \text{if } \mu \le M - B. \end{cases}$$

According to the "posterior approach" of Bayesian persuasion (Kamenica and Genzkow 2011), because V has only two linear

segments, it is sufficient and optimal to have two posteriors with each linear segment containing a posterior mean. Because of the linearity of the expected payoff, we can combine the two posteriors to form a new posterior and still preserve the same average posterior mean. Thus, let $\mu_1 > M - B$ and $\mu_0 \le M - B$. This binary information structure can be represented by a function m, that is, $m(\theta) \equiv \Pr(x = 1 | \theta)$. Now we derive the optimal m.

The regulator's problem is[12]

$$\max_m \int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) p(\theta) d\theta + M \left[1 - \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) p(\theta) d\theta \right]$$

s.t.

$$\int_{\underline{\theta}}^{\overline{\theta}} \theta[1 - m(\theta)] p(\theta) d\theta \le (M - B) \int_{\underline{\theta}}^{\overline{\theta}} [1 - m(\theta)] p(\theta) d\theta, \quad \text{(A.1)}$$

and

$$\int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) p(\theta) d\theta \ge (M - B) \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) p(\theta) d\theta, \quad \text{(A.2)}$$

where Inequalities (A.1) and (A.2) correspond to explicitly writing out $\mathbf{E}[\theta | x = 0] \le M - B$ and $\mathbf{E}[\theta | x = 1] \ge M - B$, respectively. The Lagrangian can be written as

$$L = \int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) p(\theta) d\theta + M \left[1 - \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) p(\theta) d\theta \right]$$

$$+ \lambda_1 \int_{\underline{\theta}}^{\overline{\theta}} [M - B - \theta](1 - m(\theta)) p(\theta) d\theta$$

$$+ \lambda_2 \int_{\underline{\theta}}^{\overline{\theta}} [\theta - (M - B)] m(\theta) p(\theta) d\theta,$$

where λ_1 and λ_2 are the Lagrangian multipliers of Inequalities (A.1) and (A.2), respectively. We now discuss two cases, corresponding to when $\lambda_1 = 0$ and $\lambda_2 = 0$, respectively.

Case 1. Suppose that $\mathbf{E}[\theta] \le M - B$. Because $\mathbf{E}[\theta] = \Pr(x = 1) \mathbf{E}[\theta | x = 1] + \Pr(x = 0) \mathbf{E}[\theta | x = 0]$, if $\mathbf{E}[\theta | x = 1] \ge M - B$, we must have $\mathbf{E}[\theta | x = 0] \le M - B$. Thus, Inequality (A.2) implies Inequality (A.1), and we can drop Inequality (A.1) from the optimization problem, that is, $\lambda_1 = 0$, and thus,

$$L = (1 + \lambda_2) \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta - M + \frac{\lambda}{1 + \lambda} B \right] m(\theta) p(\theta) d\theta + M.$$

Because L is increasing in m when $\theta - M + \frac{\lambda}{1+\lambda} B > 0$ and decreasing in m when $\theta - M + \frac{\lambda}{1+\lambda} B \le 0$, to maximize L, the optimal $m(\theta)$ has to be

$$m(\theta) = \begin{cases} 1 & \text{if } \theta > M - \frac{\lambda}{1 + \lambda} B \equiv \widehat{\theta} \\ 0 & \text{if } \theta \le \widehat{\theta}. \end{cases}$$

If $\lambda > 0$, $\widehat{\theta} = M - \frac{\lambda}{1+\lambda}B > M - B$ as $\frac{\lambda}{1+\lambda} < 1$. Thus, $\mathbf{E}[\theta|x=1] > \mathbf{E}[\theta|\theta > M - B] > M - B$. This implies that Inequality (A.2) is not binding, and thus $\lambda = 0$, which is a contradiction. We therefore have $\lambda = 0$ and $\widehat{\theta} = M$.

Case 2. Suppose that $\mathbf{E}[\theta] > M - B$. Again $\mathbf{E}[\theta] = \Pr(x=1)\mathbf{E}[\theta|x=1] + \Pr(x=0)\mathbf{E}[\theta|x=0]$. Thus, if $\mathbf{E}[\theta|x=0] \leq M - B$, we must have $\mathbf{E}[\theta|x=1] > M - B$. This implies that Inequality (A.1) implies Inequality (A.2) and that (A.2) can be dropped from the optimization problem, that is, $\lambda_2 = 0$, and thus,

$$L = (1+\lambda) \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta - M + \frac{\lambda}{1+\lambda}B \right] m(\theta)p(\theta)d\theta$$

$$+ M + \lambda \int_{\underline{\theta}}^{\overline{\theta}} (M - B - \theta)p(\theta)d\theta.$$

Both the second and third terms of the Lagrangian do not depend on $m(\theta)$. Thus, the optimization of the first term leads to

$$m(\theta) = \begin{cases} 1 \text{ if } \theta > \widehat{\theta} = M - \dfrac{\lambda}{1+\lambda}B \\ 0 \text{ if } \theta \leq \widehat{\theta}. \end{cases}$$

We now discuss two subcases.

Case 2.1. If $\mathbf{E}[\theta|\theta \leq M] \leq M - B$, then, following similar logic as case 1, we have $\lambda = 0$ and $\widehat{\theta} = M$.

Case 2.2. If $\mathbf{E}[\theta|\theta \leq M] > M - B$, then there exists a unique $\widehat{\theta} \in (M - B, M)$ such that $\mathbf{E}[\theta|\theta \leq \widehat{\theta}] = M - B$. This implies that Equation (A.1) binds and $\lambda > 0$.

Note that $\mathbf{E}[\theta] \leq M - B$ implies that $\mathbf{E}[\theta|\theta \leq M] \leq M - B$. Thus, we can combine Case 1 and Case 2.1 and state our results as follows: $\widehat{\theta} = M$ when $\mathbf{E}[\theta|\theta \leq M] \leq M - B$ and $\widehat{\theta}$ is defined by $\mathbf{E}[\theta|\theta \leq \widehat{\theta}] = M - B$ when $\mathbf{E}[\theta|\theta \leq M] > M - B$. $\square$

A.2. Proof of Proposition 3

Proof. Note that

$$\int_{\underline{\theta}}^{\overline{\theta}} \max(\theta, M)p_0(\theta)d\theta < \underline{v}_I$$

implies that the investor can break even only if the entrepreneur exerts effort. Then the optimal security $s(\theta)$ that finances the project solves the following optimization problem:

$$\max_{s(\cdot)} \int_{\underline{\theta}}^{\hat{\theta}} (M - M_I)p_1(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} [\theta - s(\theta) + B]p_1(\theta)d\theta,$$

subject to

$$\int_{\underline{\theta}}^{\hat{\theta}} M_I \tilde{p}_1(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} s(\theta)\tilde{p}_1(\theta)d\theta \geq \underline{v}_I, \tag{IR}$$

and

$$\int_{\underline{\theta}}^{\hat{\theta}} (M - M_I)[p_1(\theta) - p_0(\theta)]d\theta$$

$$+ \int_{\hat{\theta}}^{\overline{\theta}} [\theta - s(\theta) + B][p_1(\theta) - p_0(\theta)]d\theta \geq z. \tag{IC}$$

The optimal security $s(\theta)$ has to promise the investor at least the reservation utility (i.e., the individual rationality constraint (IR)) and ensure that the entrepreneur exerts high effort (i.e., the incentive compatibility constraint (IC)).

We rewrite $s(\theta)$ as

$$s(\theta) = \int_{\underline{\theta}}^{\theta} n(\theta')d\theta' + s(\underline{\theta}),$$

where $n(\theta')$ is the slope of $s(\theta)$. Then the dual-monotonicity assumption amounts to $n(\theta) \in [0,1]$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. We now express the objective function and the constraints in $n(\theta)$. Note that

$$\int_{\hat{\theta}}^{\overline{\theta}} s(\theta)p_1(\theta)d\theta$$

$$= \int_{\hat{\theta}}^{\overline{\theta}} \left[s(\widehat{\theta}) + \int_{\hat{\theta}}^{\theta} n(\theta')d\theta' \right] p_1(\theta)d\theta$$

$$= [1 - P_1(\widehat{\theta})]s(\widehat{\theta}) + \int_{\hat{\theta}}^{\overline{\theta}} \int_{\theta'}^{\overline{\theta}} p_1(\theta)d\theta \cdot n(\theta')d\theta'$$

$$= [1 - P_1(\widehat{\theta})]s(\widehat{\theta}) + \int_{\hat{\theta}}^{\overline{\theta}} [1 - P_1(\theta')]n(\theta')d\theta'$$

$$= [1 - P_1(\widehat{\theta})]s(\widehat{\theta}) + \int_{\hat{\theta}}^{\overline{\theta}} [1 - P_1(\theta)]n(\theta)d\theta.$$

Similarly, we obtain

$$\int_{\hat{\theta}}^{\overline{\theta}} [\theta - s(\theta)]p_1(\theta)d\theta$$

$$= [1 - P_1(\widehat{\theta})][\widehat{\theta} - s(\widehat{\theta})] + \int_{\hat{\theta}}^{\overline{\theta}} [1 - P_1(\theta)][1 - n(\theta)]d\theta,$$

and

$$\int_{\hat{\theta}}^{\overline{\theta}} [\theta - s(\theta)][p_1(\theta) - p_0(\theta)]d\theta$$

$$= [P_0(\widehat{\theta}) - P_1(\widehat{\theta})][\widehat{\theta} - s(\widehat{\theta})]$$

$$+ \int_{\hat{\theta}}^{\overline{\theta}} [P_0(\theta) - P_1(\theta)][1 - n(\theta)]d\theta.$$

We can rewrite the entrepreneur's optimization problem as

$$\max_{n(\cdot), s(\hat{\theta})} (M - M_I)P_1(\widehat{\theta}) + [1 - P_1(\widehat{\theta})][B + \widehat{\theta} - s(\widehat{\theta})]$$

$$+ \int_{\hat{\theta}}^{\overline{\theta}} [1 - P_1(\theta)][1 - n(\theta)]d\theta,$$

subject to

$$M_I \tilde{P}_1(\widehat{\theta}) + [1 - \tilde{P}_1(\widehat{\theta})]s(\widehat{\theta}) + \int_{\hat{\theta}}^{\overline{\theta}} [1 - \tilde{P}_1(\theta)]n(\theta)d\theta \geq \underline{v}_I \tag{IR'}$$

and

$$[P_0(\widehat{\theta}) - P_1(\widehat{\theta})][B - (M - M_I) + \widehat{\theta} - s(\widehat{\theta})]$$
$$+ \int_{\widehat{\theta}}^{\overline{\theta}} [P_0(\theta) - P_1(\theta)][1 - n(\theta)]d\theta \geq z. \qquad \text{(IC}')$$

Denote the Lagrangian multipliers for (IR′) and (IC′) as λ and μ, respectively. Then the Lagrangian is

$$L = (M - M_I)P_1(\widehat{\theta}) + [1 - P_1(\widehat{\theta})][B + \widehat{\theta} - s(\widehat{\theta})]$$
$$+ \int_{\widehat{\theta}}^{\overline{\theta}} [1 - P_1(\theta)][1 - n(\theta)]d\theta$$
$$+ \lambda \left[M_I \tilde{P}_1(\widehat{\theta}) + [1 - \tilde{P}_1(\widehat{\theta})]s(\widehat{\theta}) \right.$$
$$\left. + \int_{\widehat{\theta}}^{\overline{\theta}} [1 - \tilde{P}_1(\theta)]n(\theta)d\theta - \underline{v}_I \right]$$
$$+ \mu \left([P_0(\widehat{\theta}) - P_1(\widehat{\theta})][B - (M - M_I) + \widehat{\theta} - s(\widehat{\theta})] \right.$$
$$\left. + \int_{\widehat{\theta}}^{\overline{\theta}} [P_0(\theta) - P_1(\theta)][1 - n(\theta)]d\theta - z \right).$$

The derivatives of L with respect to $n(\theta)$ and $s(\widehat{\theta})$ are

$$\frac{\partial L}{\partial n(\theta)} = \lambda[1 - \tilde{P}_1(\theta)] - [1 - P_1(\theta)] - \mu[P_0(\theta) - P_1(\theta)],$$

and

$$\frac{\partial L}{\partial s(\widehat{\theta})} = \lambda[1 - \tilde{P}_1(\widehat{\theta})] - [1 - P_1(\widehat{\theta})] - \mu[P_0(\widehat{\theta}) - P_1(\widehat{\theta})].$$

Strict MLRP conditions imply that $\tilde{P}_1(\theta) - P_1(\theta) > 0$ and $P_0(\theta) - P_1(\theta) > 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$. As the Lagrangian multiplier for (IC′), $\mu \geq 0$. Then, if $\lambda \leq 1$, we have $\frac{\partial L}{\partial s(\widehat{\theta})} < 0$ and $\frac{\partial L}{\partial n(\theta)} < 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$ and thus $s(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, violating the (IR) constraint. Hence, it must be the case that $\lambda > 1$. Define

$$f(\theta) = \lambda[1 - \tilde{P}_1(\theta)] - [1 - P_1(\theta)] - \mu[P_0(\theta) - P_1(\theta)],$$

so $\frac{\partial L}{\partial n(\theta)} = f(\theta)$ and $\frac{\partial L}{\partial s(\widehat{\theta})} = f(\widehat{\theta})$. Note that

$$f(\underline{\theta}) = \lambda - 1 > 0,$$

and

$$f(\overline{\theta}) = 0.$$

In addition, MLRP implies monotone hazard rate, that is, for all $\theta \in [\underline{\theta}, \overline{\theta}]$,

$$\frac{p_0(\theta)}{1 - P_0(\theta)} > \frac{p_1(\theta)}{1 - P_1(\theta)},$$

or, equivalently,

$$\frac{d}{d\theta} \ln \left(\frac{1 - P_0(\theta)}{1 - P_1(\theta)} \right) < 0.$$

Hence, $\frac{1 - P_0(\theta)}{1 - P_1(\theta)}$ strictly decreases in θ. For the same argument, $\frac{1 - \tilde{P}_1(\theta)}{1 - P_1(\theta)}$ also strictly decreases in θ. Because $f(\theta)$ can be rewritten

as

$$f(\theta) = (\lambda - 1) - [1 - P_1(\theta)]$$
$$\left[\lambda \left(1 - \frac{1 - \tilde{P}_1(\theta)}{1 - P_1(\theta)} \right) + \mu \left(1 - \frac{1 - P_0(\theta)}{1 - P_1(\theta)} \right) \right],$$

it is also strictly decreasing in θ. Hence, $f(\theta)$ is either strictly positive on $[\underline{\theta}, \overline{\theta})$, or cross zero from above at a unique point $\theta_0 \in (\underline{\theta}, \overline{\theta})$. In the former case, $s(\widehat{\theta}) = \widehat{\theta}$ and $n(\theta) = 1$ for $\theta > \widehat{\theta}$, resulting in $s(\theta) = \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, which is a debt security with face value equal to $\overline{\theta}$. Now we consider the latter case as follows.

If $\theta_0 < \widehat{\theta}$, then $f(\theta) < 0$ on $[\widehat{\theta}, \overline{\theta}]$, implying that $s(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, a violation of the IR constraint.

If $\theta_0 = \widehat{\theta}$, then $f(\widehat{\theta}) = 0$ and $f(\theta) < 0$ on $(\widehat{\theta}, \overline{\theta}]$. In this case, $n(\theta) = 0$ for $\theta > \widehat{\theta}$, resulting in $s(\theta) = s(\widehat{\theta}) \in (0, \widehat{\theta}]$ for all $\theta > \widehat{\theta}$. (Note that $s(\widehat{\theta})$ has to be positive as otherwise $s(\theta) = 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, which again violates the IR constraint.) This amounts to issuing $\frac{s(\widehat{\theta})}{\widehat{\theta}}$ fraction of a debt security with face value $\widehat{\theta}$.

If $\theta_0 > \widehat{\theta}$, then $f(\widehat{\theta}) > 0$ and $s(\widehat{\theta}) = \widehat{\theta}$. In addition,

$$n(\theta) = \begin{cases} 1 \text{ if } \theta \leq \theta_0 \\ 0 \text{ if } \theta > \theta_0, \end{cases}$$

that is,

$$s(\theta) = \begin{cases} \theta \text{ if } \theta \leq \theta_0 \\ \theta_0 \text{ if } \theta > \theta_0, \end{cases}$$

implying that $s(\theta) = \min(\theta, \theta_0)$, which is a debt security.

Therefore, we have shown that, for any given binary information structure characterized by a cutoff $\widehat{\theta}$, the optimal security that finances the project is always debt. We next show that, if $B \leq \mathbf{E}_1(M - \theta | \theta \leq M)$, then $\widehat{\theta} = M$; and if $B > \mathbf{E}_1(M - \theta | \theta \leq M)$, then $\widehat{\theta} \in (M - B, M)$ and is uniquely characterized by $\mathbf{E}_1[\tilde{\theta} | \theta \leq \widehat{\theta}] = M - B$, where $\mathbf{E}_1(\cdot)$ denotes the expectation under probability distribution P_1. The proof is similar to that of Proposition 2.

The regulator's problem is

$$\max_m \int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) \tilde{p}_1(\theta) d\theta + M \left[1 - \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) \tilde{p}_1(\theta) d\theta \right],$$

s.t.

$$\int_{\underline{\theta}}^{\overline{\theta}} \theta[1 - m(\theta)]p_1(\theta)d\theta \leq (M - B) \int_{\underline{\theta}}^{\overline{\theta}} [1 - m(\theta)]p_1(\theta)d\theta,$$

$$\text{(A.3)}$$

and

$$\int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) p_1(\theta) d\theta \geq (M - B) \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) p_1(\theta) d\theta, \qquad \text{(A.4)}$$

where Inequalities (A.3) and (A.4) correspond to $\mathbf{E}_1[\theta | x = 0] \leq M - B$ and $\mathbf{E}_1[\theta | x = 1] \geq M - B$, respectively. We now discuss two cases.

Case 1. $\mathbf{E}_1(\theta) \leq M - B$. In this case, Inequality (A.4) implies Inequality (A.3), and thus the Lagrangian can be written as

$$L = \int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) \tilde{p}_1(\theta) d\theta + M \left[1 - \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) \tilde{p}_1(\theta) d\theta \right]$$

$$+ \lambda \int_{\underline{\theta}}^{\overline{\theta}} [\theta - (M - B)] m(\theta) p_1(\theta) d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \left[1 + \lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)} \right] \left[\theta - M + \frac{\lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)}}{1 + \lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)}} B \right]$$

$$m(\theta) \tilde{p}_1(\theta) d\theta + M,$$

where λ is the Lagrangian multiplier. Because L is increasing in m when $\theta - M + \frac{\lambda \frac{p_1(\theta)}{\tilde{p}_1(\theta)}}{1 + \lambda \frac{p_1(\theta)}{\tilde{p}_1(\theta)}} B > 0$ and decreasing in m when $\theta - M + \frac{\lambda \frac{p_1(\theta)}{\tilde{p}_1(\theta)}}{1 + \lambda \frac{p_1(\theta)}{\tilde{p}_1(\theta)}} B \leq 0$, to maximize L, the optimal $m(\theta)$ has to be

$$m(\theta) = \begin{cases} 1 & \text{if } \theta - M + \frac{\lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)}}{1 + \lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)}} B > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Because $\frac{p_1(\theta)}{\tilde{p}_1(\theta)}$ is increasing in θ, there is a unique $\widehat{\theta}$ such that $\theta - M + \frac{\lambda \frac{p_1(\theta)}{\tilde{p}_1(\theta)}}{1 + \lambda \frac{p_1(\theta)}{\tilde{p}_1(\theta)}} B = 0$. Hence, $m(\theta) = 1_{\{\theta > \hat{\theta}\}}$. If $\lambda > 0$, $\widehat{\theta} = M - \frac{\lambda \frac{p_1(\hat{\theta})}{\tilde{p}_1(\hat{\theta})}}{1 + \lambda \frac{p_1(\hat{\theta})}{\tilde{p}_1(\hat{\theta})}} B > M - B$. Thus, $\mathbf{E}_1[\theta | x = 1] > \mathbf{E}_1[\theta | \theta > M - B] > M - B$. This implies that Inequality (A.4) is not binding, and thus $\lambda = 0$, which is a contradiction. We therefore have $\lambda = 0$ and $\widehat{\theta} = M$.

Case 2. $\mathbf{E}_1(\theta) > M - B$. In this case, Inequality (A.3) implies Inequality (A.4), and thus the Lagrangian can be written as

$$L = \int_{\underline{\theta}}^{\overline{\theta}} \theta m(\theta) \tilde{p}_1(\theta) d\theta + M \left[1 - \int_{\underline{\theta}}^{\overline{\theta}} m(\theta) \tilde{p}_1(\theta) d\theta \right]$$

$$+ \lambda \int_{\underline{\theta}}^{\overline{\theta}} [M - B - \theta](1 - m(\theta)) p_1(\theta) d\theta$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \left[1 + \lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)} \right] \left[\theta - M + \frac{\lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)}}{1 + \lambda \cdot \frac{p_1(\theta)}{\tilde{p}_1(\theta)}} B \right]$$

$$m(\theta) \tilde{p}_1(\theta) d\theta + M + \lambda \int_{\underline{\theta}}^{\overline{\theta}} (M - B - \theta) p_1(\theta) d\theta,$$

where again λ is the Lagrangian multiplier. Neither the second nor the third term of the Lagrangian depends on $m(\theta)$.

Thus, the same argument as in Case 1 leads to $m(\theta) = 1_{\{\theta > \hat{\theta}\}}$, where $\widehat{\theta}$ is uniquely pinned down by $\widehat{\theta} - M + \frac{\lambda \frac{p_1(\hat{\theta})}{\tilde{p}_1(\hat{\theta})}}{1 + \lambda \frac{p_1(\hat{\theta})}{\tilde{p}_1(\hat{\theta})}} B = 0$.

We now discuss two subcases.

Case 2.1. If $\mathbf{E}_1[\theta | \theta \leq M] \leq M - B$, then, following the similar logic in Case 1, we have $\lambda = 0$ and $\widehat{\theta} = M$.

Case 2.2. If $\mathbf{E}_1[\theta | \theta \leq M] > M - B$, then there exists a unique $\widehat{\theta} \in (M - B, M)$ such that $\mathbf{E}_1[\theta | \theta \leq \widehat{\theta}] = M - B$. This implies that Inequality (A.3) binds and $\lambda > 0$.

Note that $\mathbf{E}_1[\theta] \leq M - B$ implies that $\mathbf{E}_1[\theta | \theta \leq M] \leq M - B$. Thus, we can combine Case 1 and Case 2.1 and state our results as follows: $\widehat{\theta} = M$ when $\mathbf{E}_1[\theta | \theta \leq M] \leq M - B$, and $\widehat{\theta}$ is defined by $\mathbf{E}_1[\theta | \theta \leq \widehat{\theta}] = M - B$ when $\mathbf{E}_1[\theta | \theta \leq M] > M - B$. This completes the proof. □

A.3. Proof of Proposition 4

Proof. The proof is no more than the discussion right after the proposition and is thus omitted.

Endnotes

[1] In the online appendix, we show that our results are robust to a variety of security designs.

[2] For example, Caskey and Hughes (2012) find that, in the presence of an asset-substitution problem and when debt securities are issued, the optimal accounting rule should be fair value based, subject to conservative adjustment; in Goex and Wagenhofer (2009), the optimal accounting rule is conditionally conservative in the presence of a moral hazard problem and debt security; in Bertomeu and Cheynel (2015), it is asymmetric when there are frictions in credit markets; and in Jiang and Yang (2017), it features a lower bound and a sufficient statistic in the presence of asymmetric information and equity security.

[3] We restrict our attention to debt financing in the main model to focus our analysis on accounting rule design. We show in the appendix that our results are robust to more general security designs, when debt is not the (uniquely) optimal security. We also microfound debt financing by introducing informational frictions in Section 4.1.

[4] This is different from Goex and Wagenhofer (2009) in the sense that we assume excessive continuation whereas they assume excessive termination.

[5] Here, the regulator's incentive can also be justified by interpreting B as some redistributive rent extraction that the entrepreneur takes (from someone other than the investors; there are certainly other parties in the firm) but that has no social value in the aggregate.

[6] We assume the tie-breaking rule that, whenever a party is indifferent between continuation and termination, the party chooses termination.

[7] We prove a stronger result in the appendix, showing that it is optimal for the entrepreneur to implement the decision rule a_h, regardless of the type of the securities, so long as the security payoff is increasing with respect to the underlying state. In Section 4.1, however, we introduce informational frictions to pin down the security design, showing that the optimal security must be debt, providing a micro-foundation of the debt financing used in the main model.

[8] In the online appendix, we provide a sufficient and necessary condition to characterize the impact of more general (nonparametric) belief changes on $\widehat{\theta}$. Specifically, to capture a belief change, we perturb the density of the prior, $p(\theta)$, by $\alpha \cdot \Delta p(\theta)$, where α is the magnitude of the perturbation. We are able to show that the marginal effect of how $\widehat{\theta}$

varies under the perturbed belief $p + \alpha \cdot \Delta p$, that is, $\frac{\partial \hat{\theta}}{\partial \alpha}\big|_{\alpha=0} > 0$ if $\int_{\underline{\theta}}^{\hat{\theta}} \triangle p(\theta)(M - B - \theta) d\theta > 0$. This implies that $\widehat{\theta}$ increases when there is more downside risk in the prior distribution of θ, that is, when more density $\triangle p(\theta)$ is applied to lower values of θ.

[9] Accounting applications include Jiang and Yang (2017) and Lu (2022).

[10] The low (high) accounting signal arrives with probability $P(\widehat{\theta})$ $(1 - P(\widehat{\theta}))$, and $\frac{p(\theta)}{P(\widehat{\theta})}\left(\frac{p(\theta)}{1-P(\widehat{\theta})}\right)$ is the posterior density of θ conditional on the arrival of the low (high) signal. Then, the expected entropy of the posterios is

$$H_{post} = -P(\hat{\theta}) \int_{\underline{\theta}}^{\hat{\theta}} \frac{p(\theta)}{P(\hat{\theta})} \ln \frac{p(\theta)}{P(\hat{\theta})} d\theta$$
$$- [1 - P(\hat{\theta})] \int_{\hat{\theta}}^{\overline{\theta}} \frac{p(\theta)}{1 - P(\hat{\theta})} \ln \frac{p(\theta)}{1 - P(\hat{\theta})} d\theta$$
$$= - \int_{\underline{\theta}}^{\overline{\theta}} p(\theta) \ln p(\theta) d\theta + P(\hat{\theta}) \ln P(\hat{\theta}) + [1 - P(\hat{\theta})] \ln[1 - P(\hat{\theta})].$$

[11] The existence and uniqueness of θ^* is guaranteed by our assumptions regarding the prior distribution of θ.

[12] This is the same Lagrangian method proposed in Bertomeu and Cheynel (2015).

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