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NONLINEAR ENGINEERING AND ROBOTICS

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Enhanced Adaptive Control over Robotic Systems via Generalized Momentum Dynamic Extensions

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Adaptive control and parameter estimation have been widely employed in robotics to deal with parametric uncertainty. However, these techniques may suffer from parameter drift, dependence on acceleration estimates and conservative requirements for system excitation. To overcome these limitations, composite adaptation laws can be used. In this paper, we propose an enhanced composite adaptive control approach for robotic systems that exploits the acceleration-free momentum dynamics and regressor extensions to offer faster parameter and tracking convergence while relaxing excitation conditions and providing a clear physical interpretation. The effectiveness of the proposed approach is validated through experimental evaluation on a 3-DoF robotic leg.

Keywords: adaptive control, parameter estimation, motion control

1. Introduction

Model-based approaches have been widely used in the development of robotic systems to formulate provably stable algorithms for manipulation, locomotion, and force rendering. However,

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these algorithms highly depend on the accuracy of parameter estimates, such as the inertia tensor, mass, and friction coefficients. In real-life scenarios, these parameters may not be accurately known in advance or change over time due to various factors. To address this, researchers have explored adaptive control and online parameter estimation techniques that update controller parameters in real-time based on system feedback.

The field of adaptive control and estimation has been extensively studied for several decades, with significant progress made by the robotics community in the 1980s and 90s. One of the most notable contributions was the passivity-based controller proposed by Slotine and Li [17]. However, this celebrated controller in general does not yield parameter convergence, which results in numerical drift, sensitivity to noise, and dependence on persistency of excitation (PE). To overcome these issues, researchers combined the tracking-based adaptive law with online parameter estimation techniques, proposing the composite adaptive control framework [11, 16]. Although composite adaptation significantly improves parameter error dynamics, the convergence still depends upon PE. One way to combat this is to increase the amount of data present in the underlying model through regression extension, which was first proposed by Lion [15] and Kreisselmeier [13]. This idea has been exploited in various scenarios, leading to the framework of concurrent learning [23, 31]. The attractive properties of such estimators are that the norm of the parameter error is monotonically decreasing with convergence supported by a condition strictly weaker than PE, namely, that of square integrability of eigenvalues of extended regressor matrices [14, 29, 30]. Thanks to the above properties, these estimators have been used in safety-critical controllers [10, 33].

The aforementioned estimation techniques rely on the assumption that some of the system outputs can be represented linearly with respect to the set of constant parameters. In robotics, this linear representation is known as the regressor form [18] and is the standard in identification [24]. However, as most models of robotic systems are given by second-order differential equations, the underlying linear representation is dependent upon accelerations, which are usually subject to significant noise. To overcome this limitation, several techniques have been proposed, such as filtering based on integration by parts [25] or exploitation of linearly parameterized physical quantities as mechanical power and energy [20, 21]. However, the resulting scalar power or energy regressors may require more data due to the "mixing" of the different components of the dynamics. Meanwhile, alternative acceleration-free models have been effectively exploited in the context of disturbance observers [6, 7]. Relying on the filtered model of generalized momentum, this technique exploits the Hamiltonian view of dynamics, facilitating the transformation of second-order dynamics into pairs of first-order governing equations. However, to the best of our knowledge, this technique has not yet been studied in the context of adaptation.

In this paper, we propose a composite adaptation approach that effectively combines momentum dynamics with filtered regressor extension. This approach allows for faster parameter and tracking convergence while relaxing the excitation conditions, as demonstrated through analytical analysis and experimental evaluation.

The remainder of this paper is organized as follows. In the next section, we provide a short background on models of robotic systems, passivity-based adaptive control and its composite modification. In Section 3, the momentum-based extended regressors are proposed and integrated into the framework of composite adaptation, whose convergence is proven via the Lyapunov direct method. In Section 4, we provide an experimental evaluation of the proposed adaptation law on a practical 3-DoF robotic manipulator (a leg of a commercial quadruped robot) and compare the performance of the proposed controller with a conventional scheme. Finally, we present a relevant discussion, conclusions, and outline avenues for future work.

2. Linear parameterization and adaptive control

In this section, we will briefly review the basic concepts of dynamics, linear parameterization, conventional passivity-based adaptive control, and composite adaptation, which will facilitate the understanding of the proposed scheme.

2.1. Dynamics and linear parameterization

The dynamics of a robotic system can be described by the equations of motion which relate the forces acting on the system to the second-order derivatives of coordinates $\mathbf{q} \in \mathbb{R}^n$:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u}. \tag{2.1}$$

Here, $\mathbf{u} \in \mathbb{R}^n$ represents a vector of control inputs, $\mathbf{M} \in \mathbb{R}^{n\times n}$ is a positive definite symmetric inertia matrix, $\mathbf{C} \in \mathbb{R}^{n\times n}$ is a matrix of Coriolis and centrifugal terms, and $\mathbf{g} \in \mathbb{R}^n$ describes the effect of gravity and other position-dependent (potential) forces.

The above equation of motion is typically obtained using the Lagrangian formalism:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = u_i, \quad i = 1,\, 2,\, \ldots,\, n, \tag{2.2}$$

where $\mathcal{L}(\mathbf{q},\, \dot{\mathbf{q}},\, t) \triangleq \mathcal{K}(\mathbf{q},\, \dot{\mathbf{q}}) - U(\mathbf{q})$ is the Lagrangian, which is equal to the difference of kinetic and potential energy of the system.

We focus on linearly parameterized systems, which have kinetic and potential energies expressed as linear functions $\boldsymbol{\varphi}$ and unknown but constant parameters $\boldsymbol{\theta} \in \mathbb{R}^l$:

$$\mathcal{K} = \boldsymbol{\varphi}_{\mathcal{K}}^T(\mathbf{q},\, \dot{\mathbf{q}})\boldsymbol{\theta}, \quad \mathcal{U} = \boldsymbol{\varphi}_{\mathcal{U}}^T(\mathbf{q})\boldsymbol{\theta}. \tag{2.3}$$

Here, $\boldsymbol{\varphi}_{\mathcal{K}}(\mathbf{q},\, \dot{\mathbf{q}}) \in \mathbb{R}^l$ and $\boldsymbol{\varphi}_{\mathcal{U}}(\mathbf{q}) \in \mathbb{R}^l$ are *kinetic and potential energy regressors* expressed in generalized coordinates.

Continuing with the Euler – Lagrange equations, one can obtain the dynamics $i = 1,\, \ldots,\, n$ in a linearly parameterized form:

$$\boldsymbol{\Phi}(\mathbf{q},\, \dot{\mathbf{q}},\, \ddot{\mathbf{q}})\boldsymbol{\theta} = \mathbf{M}(\mathbf{q},\, \boldsymbol{\theta})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\, \dot{\mathbf{q}},\, \boldsymbol{\theta})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q},\, \boldsymbol{\theta}) = \mathbf{u}, \tag{2.4}$$

where the *dynamics regressor* $\boldsymbol{\Phi} \in \mathbb{R}^{n\times \ell}$ consists of rows calculated as

$$\boldsymbol{\Phi}_i = \left[\sum_{j=1}^{n} \frac{\partial^2 \boldsymbol{\varphi}_{\mathcal{K}}}{\partial \dot{q}_i\, \partial \dot{q}_j}\ddot{q}_j + \sum_{j=1}^{n} \frac{\partial^2 \boldsymbol{\varphi}_{\mathcal{K}}}{\partial \dot{q}_i\, \partial q_j}\dot{q}_j - \frac{\partial \boldsymbol{\varphi}_{\mathcal{K}}}{\partial q_i} + \frac{\partial \boldsymbol{\varphi}_{\mathcal{U}}}{\partial q_i}\right]. \tag{2.5}$$

This representation is widely used in robotics to develop various control schemes. One such scheme is the widely used passivity-based adaptive controller, originally proposed by Slotine and Li [17].

2.2. Passivity-based adaptive control

The passivity-based adaptive controller is given by the following expressions:

$$\mathbf{u} = \mathbf{Y}_c(\mathbf{q},\, \dot{\mathbf{q}},\, \dot{\mathbf{q}}_s,\, \ddot{\mathbf{q}}_s)\widehat{\boldsymbol{\theta}} - \mathbf{K}\mathbf{s}, \tag{2.6}$$

$$\dot{\widehat{\boldsymbol{\theta}}} = -\gamma_c \mathbf{Y}_c^T(t)\mathbf{s}. \tag{2.7}$$

Here, $\mathbf{s} = \dot{\widetilde{\mathbf{q}}} + \lambda\widetilde{\mathbf{q}}$ is the so-called sliding surface with $\lambda > 0$ regulating the convergence of the trajectories. The tracking error between the actual $\mathbf{q}$ and the desired trajectory $\mathbf{q}_d$ is defined via $\widetilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$, and $\dot{\mathbf{q}}_s = \dot{\mathbf{q}}_d - \lambda\widetilde{\mathbf{q}}$ is referred to as the "virtual velocity". The "controller regressor" $\mathbf{Y}_c(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s)$ is calculated in a way that allows for the linear parameterization of the virtual dynamics $\mathbf{Y}_c(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s)\widehat{\boldsymbol{\theta}} = \widehat{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}}_s + \widehat{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_s + \widehat{\mathbf{g}}(\mathbf{q})$, where $\gamma_c > 0$ is the adaptation parameter (similar to a gradient descent step) that regulates the convergence of the parameter estimates, and $\mathbf{K} \succ 0$ regulates the speed of convergence to the sliding surface.

The convergence is proven via Lyapunov's direct method for the following Lyapunov candidate function:

$$V(t) = \frac{1}{2}\left(\mathbf{s}^T\mathbf{M}(\mathbf{q})\mathbf{s} + \gamma_c^{-1}\widetilde{\boldsymbol{\theta}}^T\widetilde{\boldsymbol{\theta}}\right), \tag{2.8}$$

where $\widetilde{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ is referred to as the *parameter estimation error*.

Taking the time derivative of the equation above and substituting the controller and adaptation law dynamics yields:

$$\dot{V}(t) = \left(\mathbf{s}^T\mathbf{Y}_c + \gamma_c^{-1}\dot{\widehat{\boldsymbol{\theta}}}^T\right)\widetilde{\boldsymbol{\theta}} - \mathbf{s}^T\mathbf{K}\mathbf{s} = -\mathbf{s}^T\mathbf{K}\mathbf{s} \leqslant 0. \tag{2.9}$$

Invoking Barballat's lemma implies that convergence to the surface $\mathbf{s} = \mathbf{0}$ is achieved as $t \to \infty$, which in turn ensures that $\widetilde{\mathbf{q}}$ and $\dot{\widetilde{\mathbf{q}}}$ tend to $\mathbf{0}$ as well.

However, it is important to note that the adaptive controller may not necessarily result in precise estimation of the unknown parameters, generating instead the values that ensure only $\mathbf{s} \to 0$. Convergence of parameter estimates is supported by the concept of *persistence of excitation* (PE), which requires that the bounded signal $\mathbf{Y}(t) \in \mathbb{R}^{n\times\ell}$ satisfies the following:

$$\int\limits_t^{t+T} \mathbf{Y}^T(\tau)\mathbf{Y}(\tau)\,d\tau \succeq c\mathbf{I} \tag{2.10}$$

for some $T > 0$, $c > 0$ and all $t > 0$.

However, this condition is rarely satisfied in practice, leading to numerical drift of parameter estimates and potential instability [12, 29]. This has motivated researchers to enhance this class of adaptive controllers, with one of the most effective techniques being the *composite adaptation*.

2.3. Prediction error and composite adaptation

The concept of composite adaptation encompasses the integration of a "prediction regression" within a linearly parameterized model:

$$\widetilde{\mathbf{y}}(t) = \widehat{\mathbf{y}}(t) - \mathbf{y}(t) = \mathbf{Y}(t)\widehat{\boldsymbol{\theta}} - \mathbf{Y}(t)\boldsymbol{\theta} = \mathbf{Y}(t)\widetilde{\boldsymbol{\theta}}. \tag{2.11}$$

Here, $\widehat{\mathbf{y}}$ represents the predicted system outputs, while the discrepancy between the prediction $\widehat{\mathbf{y}}$ and the measured output $\mathbf{y}$ is referred to as the "prediction error", denoted by $\widetilde{\mathbf{y}}$.

It is important to note that the same parameters $\boldsymbol{\theta}$ are utilized to parameterize the robot in the conventional adaptation scheme (2.7). However, the regressor matrices Y and Y_c differ. The signals $\mathbf{y}(t)$ and $\mathbf{Y}(t)$ are highly dependent on the application and system properties. They typically represent a combination of measurable terms that describe various physical phenomena expressed in a linearly parameterizable form, with some common types briefly discussed at the beginning of the next section. Importantly, these signals are not contingent upon the specific implementation of the controller but rather rely on the assumption that the control input is known or measurable. In the subsequent section, we will delve further into constructing one of the realizations of $\mathbf{y}(t)$ and $\mathbf{Y}(t)$ using dynamically extended generalized momentum regression. In the meantime, let us continue with a brief overview of composite adaptation.

Regarding the specific linear parameterization, the adaptation law is modified by incorporating the prediction error term with a gain parameter $\gamma_p > 0$, resulting in:

$$\dot{\widehat{\boldsymbol{\theta}}} = -\gamma_c \mathbf{Y}_c^T \mathbf{s} - \gamma_p \mathbf{Y}^T \widetilde{\mathbf{y}}. \tag{2.12}$$

Substituting this adaptive law into the derivative of the Lyapunov candidate (2.9) yields:

$$\dot{V}(t) = -\mathbf{s}^T \mathbf{K}\mathbf{s} - \frac{\gamma_p}{\gamma_c} \widetilde{\boldsymbol{\theta}}^T \mathbf{Y}^T \mathbf{Y} \widetilde{\boldsymbol{\theta}} \leqslant 0. \tag{2.13}$$

The dynamics of the parameter estimates are governed by the following law:

$$\dot{\widetilde{\boldsymbol{\theta}}} = -\gamma_p \mathbf{Y}^T(t)\mathbf{Y}(t)\widetilde{\boldsymbol{\theta}} - \gamma_c \mathbf{Y}_c^T(t)\mathbf{s}(t). \tag{2.14}$$

This adaptation law ensures time-variant first-order error dynamics with a converging input $\mathbf{z}(t) = \mathbf{Y}_c^T(t)\mathbf{s}(t)$. Consequently, parameter estimates exhibit smoother and faster convergence than in the case of the conventional adaptive control thanks to the "feedback term" $-\gamma \mathbf{Y}^T(t)\mathbf{Y}(t) \preceq 0$. The conventional adaptation algorithm is restored by setting the composite gain $\gamma_p = 0$, thus indicating that composite adaptation only enhances convergence and smoothness of estimate transients. However, using the composite adaptive law raises two issues that need to be addressed, namely: choosing an accurate, noise-free linear parameterization $\mathbf{y} = \mathbf{Y}\boldsymbol{\theta}$, and secondly, reducing excitation requirements to improve parameter convergence properties in the absence of PE. These will be studied in the next section.

3. Momentum extensions and enhanced composite adaptation

Composite adaptive control has shown promise in improving parameter estimates and trajectory tracking, but its implementation requires an additional linear parameterization. However, selecting an appropriate linear model for a general robotic system is ambiguous. The naive approach of using the full linear representation of dynamics (2.11) is challenging because it relies on accelerations $\ddot{\mathbf{q}}$. Although several filtering techniques have been proposed in the literature [12, 25, 30], they are computationally expensive for complex systems, since integration by parts of (2.1) is required. Another alternative is linear parameterization of energy or power [20, 21], but these regressors demand more excitation and further constrain the already restrictive PE conditions, which are seldom satisfied in practice.

Alternatively, generalized momentum models have shown success in acceleration-free disturbance observers [6, 7]. As we will further show, these momentum models, coupled with regressor extensions [29], enhance the quality of composite adaptive control.

3.1. Extended momentum regressors

To build the linear prediction model, we use the notion of generalized momentum $\mathbf{p} \in \mathbb{R}^n$ and its linear parameterization given by

$$\mathbf{p} = \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} = \mathbf{M}(\mathbf{q})\dot{\mathbf{q}} = \frac{\partial \boldsymbol{\varphi}_K^T}{\partial \dot{\mathbf{q}}} \boldsymbol{\theta} = \boldsymbol{\Phi}_{\mathbf{p}}(\mathbf{q}, \dot{\mathbf{q}})\boldsymbol{\theta}, \tag{3.1}$$

where $\boldsymbol{\Phi}_{\mathbf{p}} \in \mathbb{R}^{n\times l}$ is the *generalized momentum regressor.*

Taking the time derivative of the Euler – Lagrange equations yields:

$$\dot{\mathbf{p}} = \frac{\partial \mathcal{L}}{\partial \mathbf{q}} + \mathbf{u} = \left[\frac{\partial \varphi_{\mathcal{K}}}{\partial \mathbf{q}} - \frac{\partial \varphi_{\mathcal{U}}}{\partial \mathbf{q}}\right]^T \boldsymbol{\theta} + \mathbf{u}. \tag{3.2}$$

We can now define the matrix $\boldsymbol{\Phi}_{\mathbf{h}}(\mathbf{q}, \dot{\mathbf{q}}) = \left[\frac{\partial \varphi_{\mathcal{K}}}{\partial \mathbf{q}} - \frac{\partial \varphi_{\mathcal{U}}}{\partial \mathbf{q}}\right]^T$ to write:

$$\left[\frac{d}{dt}\boldsymbol{\Phi}_{\mathbf{p}}(\mathbf{q}, \dot{\mathbf{q}}) - \boldsymbol{\Phi}_{\mathbf{h}}(\mathbf{q}, \dot{\mathbf{q}})\right]\boldsymbol{\theta} = \mathbf{u}. \tag{3.3}$$

Now, similarly to the design of disturbance observers [6] we define the dynamics of prediction error via momentum residual:

$$\dot{\tilde{\mathbf{y}}} = \dot{\tilde{\mathbf{p}}} = \dot{\mathbf{p}} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} - \mathbf{u} = \left[\frac{d}{dt}\boldsymbol{\Phi}_{\mathbf{p}} - \boldsymbol{\Phi}_{\mathbf{h}}\right]\boldsymbol{\theta} - \mathbf{u}. \tag{3.4}$$

Next, we proceed with using the above in the context of filtered dynamic regressor extension — a particular technique that will alleviate the acceleration requirements and increase the dimension of regression.

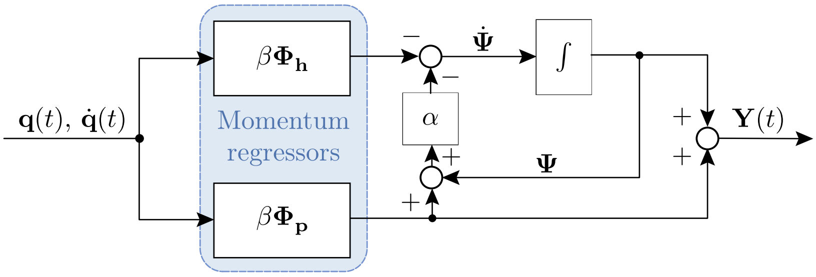

Fig. 1. A schematic overview of regressor computation based on the momentum models

3.2. Acceleration-free dynamical extensions

The concept of extensions is a valuable tool for constructing high-dimensional regression models as follows:

$$\mathbf{Y}_e = col\left\{\mathbf{Y}_1^T, \ldots, \mathbf{Y}_r^T\right\}, \quad \mathbf{y}_e = col\{\mathbf{y}_1, \ldots, \mathbf{y}_r\}, \tag{3.5}$$

where $\mathbf{Y}_e \in \mathbb{R}^{nr\times l}$ and $\mathbf{y}_e \in \mathbb{R}^{nr}$ are the extended regressors and predictions, respectively, $r \in \mathbb{N}$ is referred to as the *order of extension*, while the $col\{x_1, \ldots, x_n\}$ stacks up its vector arguments x_k into a column vector $\mathbf{x} = [x_1, \ldots, x_n]$.

Various methods such as time delays or LTI filters can be used to construct the extended regressor [29]. For the proposed momentum-based models, integration can provide an acceleration-free family of regressors, but often leads to errors and drift. Alternatively, filtering techniques can be used to address these issues [30], with the stability of such a scheme investigated recently in [28].

To realize the extension filtering-based momentum framework, we apply a collection of first-order filters with transfer functions $H_i(s) = \frac{\beta_i}{\alpha_i + s}$ to (3.3). The resulting extended regressors $\mathbf{Y}_i$ and predictions $\mathbf{y}_i$ are given by

$$\dot{\mathbf{Y}}_i + \alpha_i\mathbf{Y}_i = \beta_i[\dot{\boldsymbol{\Phi}}_{\mathbf{p}}(t) - \boldsymbol{\Phi}_{\mathbf{h}}(t)], \tag{3.6}$$

$$\dot{\mathbf{y}}_i + \alpha_i\mathbf{y}_i = \beta_i\mathbf{u}(t), \tag{3.7}$$

where $\alpha_i > 0$, $\beta_i > 0$ are time constants and gains of the filters, respectively. The solution of the above yields the extended regression of (3.5) and ensures that the accumulation of errors is avoided thanks to the exponential forgetting of initial conditions.

It is worth noting that the use of the filtering momentum residual makes it possible to avoid the differentiation of the momentum regressor $\frac{d}{dt}\mathbf{\Phi}_\mathbf{p}$. To achieve this, we introduce the *residual momentum regressor* $\mathbf{\Psi}_i(t) = \mathbf{Y}_i(t) - \beta\mathbf{\Phi}_\mathbf{p}(t)$ and form a simple LTI ODE that takes the position and speed-dependent momentum parameterization as input and produces a regressor as output (Fig. 1):

$$\dot{\mathbf{\Psi}}_i + \alpha_i \mathbf{\Psi}_i = -\beta_i[\alpha\mathbf{\Phi}_\mathbf{p}(t) + \mathbf{\Phi}_\mathbf{h}(t)]. \tag{3.8}$$

Solving the above with $\mathbf{\Psi}_i(t_0) = -\beta\mathbf{\Phi}_\mathbf{p}(t_0)$ yields a solution equivalent to (3.6). Importantly, all calculations are based on the position and velocity $\mathbf{q}$, $\dot{\mathbf{q}}$ which we assume to be measurable, and no acceleration $\ddot{\mathbf{q}}$ is required. Moreover, the output is propagated through first-order stable filters, which further reduces the effect of noise.

We will now move on to discuss the proposed enhancement over adaptive controllers that leverages the idea of generalized momentum and regressor extensions.

3.3. Momentum enhanced composite adaptive controller

In this work, we propose an enhancement for composite adaptive controllers that leverages the generalized momentum and regressor extensions. Specifically, we enhance the composite version of the passivity-based adaptive controller by replacing the constant gain sliding feedback $\mathbf{Ks}$ with a varying adaptive momentum-like term $\mathbf{M}(\mathbf{q}, \widehat{\boldsymbol{\theta}})\mathbf{s} = \lambda\mathbf{\Phi}_\mathbf{p}(\mathbf{q}, \mathbf{s})\widehat{\boldsymbol{\theta}}$, and consequently use the extensions (3.6), (3.7) in the composite adaptive law. The resulting controller is given by

$$\mathbf{u} = [\mathbf{Y}_c(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s) - \lambda\mathbf{\Phi}_\mathbf{p}(\mathbf{q}, \mathbf{s})]\widehat{\boldsymbol{\theta}}, \tag{3.9}$$

$$\dot{\widehat{\boldsymbol{\theta}}} = -\gamma_c[\mathbf{Y}_c - \lambda\mathbf{\Phi}_\mathbf{p}(\mathbf{q}, \mathbf{s})]^T\mathbf{s} - \gamma_p\sum_{i=1}^{r}\mathbf{Y}_i^T\widetilde{\mathbf{y}}_i. \tag{3.10}$$

The stability and convergence analysis of the proposed scheme follows a Lyapunov candidate (2.8) whose derivative is

$$\dot{V}(t) \leqslant -\lambda\mathbf{s}^T\mathbf{M}(\mathbf{q})\mathbf{s} - \gamma_p\sigma_{\min}(t)\widetilde{\boldsymbol{\theta}}^T\widetilde{\boldsymbol{\theta}}, \tag{3.11}$$

where $\sigma_{\min}$ is the minimal singular value of extended regressor $\mathbf{Y}_e$.

Application of the Grönwall–Bellman inequality yields the following implication:

$$V(t) \leqslant e^{-2\lambda t}V(0) + \int\limits_0^t e^{-2\lambda(t-\tau)}\varepsilon(t)\|\widetilde{\boldsymbol{\theta}}(\tau)\|^2\,d\tau, \tag{3.12}$$

where $\varepsilon(t) = \lambda\gamma_c^{-1} - \gamma_p\sigma_{\min}(t)$.

The expression above, together with the results presented in [28, 29], suggests that the proposed controller and adaptive law exhibits several distinguishing properties:

- global asymptotic convergence of $\|\mathbf{s}\|$ and $\|\widetilde{\boldsymbol{\theta}}\|$ is guaranteed, provided that $\sigma_{\min} \notin L^2$;
- global exponential convergence of $\|\mathbf{s}\|$ and $\|\widetilde{\boldsymbol{\theta}}\|$ is ensured if $\sigma_{\min}(t)$ is uniformly strictly positive or PE;
- once convergence to the sliding surface is achieved, the dynamics of parameter error norm $\|\widetilde{\boldsymbol{\theta}}\|$ is monotonically decreasing, i.e.: $\|\widetilde{\boldsymbol{\theta}}(t_2)\| \leqslant \|\widetilde{\boldsymbol{\theta}}(t_1)\|$, $\forall t_2$, t_1 such that $t_2 > t_1 > 0$.

The properties described above share similarities with those of dynamically extended parameter estimators [28, 29] and composite/concurrent adaptation controllers [16, 31, 34]. However, the proposed scheme is applicable to a general type of robotic systems described by (2.1). Although a detailed analysis of these properties is beyond the scope of this paper due to space limitations, we provide some insights into their origins.

The asymptotic convergence of $\mathbf{s}$ is ensured by conventional controllers, and hence we do not elaborate on it further. On the other hand, the convergence of parameter estimates is guaranteed if the integral term in (3.12) can be made to approach zero as $t \to \infty$. Invoking the mean value theorem shows that this can be achieved if $\sigma_{\min} \notin L^2$, which is weaker than persistent excitation on $\mathbf{Y}$. Exponential convergence, however, relies on the fact that, if $\sigma_{\min}(t)$ is either PE or uniformly positive definite, one can choose γ_p such that $\lambda < \gamma_c\gamma_p\sigma_{\min}(t)$, making the integral part uniformly negative. This, in turn, causes $V(t)$ in (3.12) to be upper-bounded by an exponential curve $e^{-2\lambda t}V(0)$. The monotonic decrease after convergence to the sliding surface is demonstrated by setting $\mathbf{s} = 0$ in the definition of the Lyapunov candidate (2.8) and its derivative (3.11). This results in $\frac{d}{dt}|\widetilde{\boldsymbol{\theta}}| \leqslant \sqrt{\gamma_c\gamma_p\sigma_{\min}(t)}|\widetilde{\boldsymbol{\theta}}|$ and, since $\sqrt{\gamma_c\gamma_p\sigma_{\min}(t)} \geqslant 0$, $|\widetilde{\boldsymbol{\theta}}(t)|$ monotonically decreases.

It is worth noting that these properties cannot be satisfied by conventional adaptive laws, as they require the prediction regressor to have full rank at least on some intervals, and thus $\sigma_{\min}(t) \neq 0$. This is only possible with an extended regression of order $nr \geqslant l$.

Having discussed the properties of the proposed scheme, we now proceed with the implementation and experimental evaluation on a practical robotic system.

4. Experimental evaluation

This section evaluates the performance of the proposed enhanced adaptive controller, as given by equations (3.9) and (3.10), and compares it with that of a conventional adaptive controller, given by equations (2.6) and (2.7).

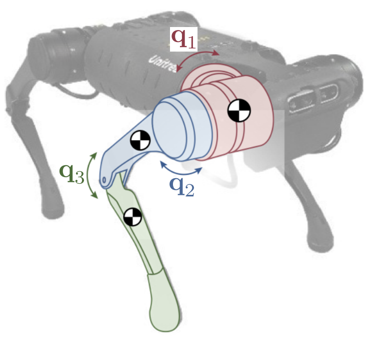

Fig. 2. Photo of A1 robot with an overlaid schematic representation of the leg

The experimental framework for this investigation centered on a single limb from the Unitree A1 quadruped, as delineated in Figure 2. The leg is actuated by three identical motors with a maximum torque of 33.5 Nm and a rotation speed of 21 rad/sec. Each motor is equipped with 12-bit encoders and torque (current) sensing capabilities. The robot is equipped with a versatile API that allows real-time control over torques at a rate of 500–1000 Hz through Ethernet communication. It is imperative to highlight that the study was conducted using the

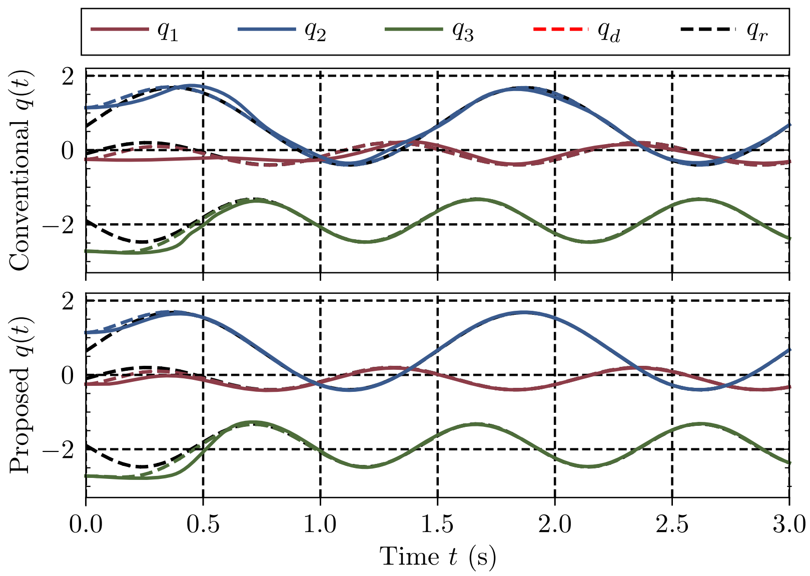

Fig. 3. Comparison of the trajectory tracking behavior of the conventional (top) and proposed (bottom) adaptive controllers

leg exclusively as a three-degree-of-freedom (3-DOF) manipulator, with the quadruped's base firmly fixed, thereby concentrating solely on the dynamics of the robotic manipulator.

4.1. Regressor models

To implement the conventional and proposed adaptive schemes, we first built a simplified model of the manipulator by assuming that the unknown parameters of each body are given by the following vector:

$$\boldsymbol{\theta}_i = \left[m_i x_{c_i},\, m_i y_{c_i},\, m_i z_{c_i},\, m_i,\, I_{xx_i},\, I_{yy_i},\, I_{zz_i}\right]^T \in \mathbb{R}^7,$$

where m_i is the mass of the body and $\mathbf{r}_{c_i} = \left[x_{c_i},\, y_{c_i},\, z_{c_i}\right]^T$ is the location of its center of mass (COM) with respect to the body frame. Note that we assume the COM to be initially unknown and located not in the center of the frame. I_{xx_i}, I_{yy_i}, I_{zz_i} are the diagonal components of the inertia tensor.

The potential energy regressors are given by

$$\phi_{\mathcal{U}_i} = \left[\mathbf{g}^T \mathbf{R}_i(\mathbf{q}) \;\; \mathbf{g}^T \mathbf{r}_i(\mathbf{q}) \;\; \mathbf{0}_{1\times 3}\right]^T,$$

where $\mathbf{r}_i \in \mathbb{R}^3$ is the Cartesian position of the origin of the local frame connected to the ith body, $\mathbf{R}_i \in \mathrm{SO}(3)$ is the rotation matrix describing the orientation of the body with respect to the world frame, and $\mathbf{g} = [0,\, 0,\, 9.81]^T$ is the spatial gravity acceleration.

The kinetic energy regressors of each body are given by

$$\phi_{\mathcal{K}_i} = \begin{bmatrix} -\omega_{y_i} v_{z_i} + \omega_{z_i} v_{y_i} \\ \omega_{x_i} v_{z_i} - \omega_{z_i} v_{x_i} \\ -\omega_{x_i} v_{y_i} + \omega_{y_i} v_{x_i} \\ \frac{1}{2} v_{x_i}^2 + \frac{1}{2} v_{y_i}^2 + \frac{1}{2} v_{z_i}^2 \\ \frac{1}{2}\omega_{x_i}^2 \\ \frac{1}{2}\omega_{y_i}^2 \\ \frac{1}{2}\omega_{z_i}^2 \end{bmatrix}$$

where $\mathbf{v}_i(\mathbf{q}, \dot{\mathbf{q}})$ and $\boldsymbol{\omega}_i(\mathbf{q}, \dot{\mathbf{q}})$ are the linear and angular velocities of the ith body, respectively, expressed in the inertial frame.

The kinematic quantities were computed as functions of joint coordinates and velocities using the pinocchio C++ library [36]. The complete dynamics, as given by equation (2.5), and momentum regressors were obtained via fast analytical derivatives with respect to $\mathbf{q}$ and $\dot{\mathbf{q}}$. Thanks to the efficient implementation of the pinocchio library, all necessary computations were completed within 1 ms, ensuring a 1 kHz sampling rate for the practical implementation of the controller.

The momentum regressors were extended with order $r = 8$, while gains and filter coefficients were uniformly distributed in the intervals $\beta_i \in [6, 10]$, $\alpha_i \in [8, 12]$, respectively. The resulting LTI systems, given by equations (3.7) and (3.8), were discretized exactly to provide an accurate recursive implementation without solving underlying ODEs numerically.

To obtain the relative parameters, which is important for conditioning the adaptation problem, we scaled the regressors with nominal parameters provided by the manufacturer as $\mathbf{Y}\,\mathrm{diag}\{\boldsymbol{\theta}_0\}$. We also identified a set of basic lumped parameters $\widehat{\boldsymbol{\theta}}_0$ and their values via singular-value decomposition over sampled regressors and well-known least squares-based techniques reported in [18, 24]. Additionally, we identified the parameters of a simple viscous and dry friction model that was compensated for via a feed-forward term.

4.2. Tuning and comparison

Both controllers were given a task to track desired trajectories given by a critically damped reference model with a natural frequency $\omega > 0$:

$$\ddot{\mathbf{q}}_d + 2\omega(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}_r) + \omega^2(\mathbf{q}_d - \mathbf{q}_r) = \ddot{\mathbf{q}}_r(t).$$

To reduce the computational burden, we discretize underlying ODEs of the reference model similarly to the LTI filters used in extensions. The reference trajectory is given by a harmonic signal:

$$\mathbf{q}_{r_i} = A_{0_i} + A_i \sin 2\pi\nu_i t,$$

where the amplitudes are chosen to account for the joint limits as $A_0 = [-0.1,\ 0.64,\ -1]$ and $A = [0.3,\ 1,\ -0.57]$, while the frequencies are given by $\nu = [0.95,\ 0.66,\ 1.05]$.

To ensure a fair comparison between the passivity-based and proposed controllers, both were tuned with the same adaptation $\gamma_c = 0.5$ and sliding gain $\lambda = 7$, respectively. The feedback gains in the conventional controller represented the average scaled nominal inertia along the desired

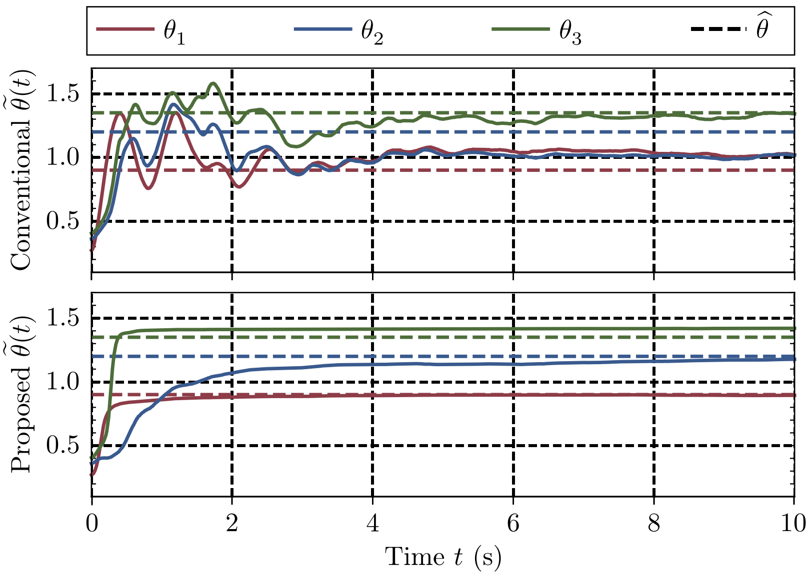

Fig. 4. The parameter convergence under conventional (top) and proposed (bottom) adaptive laws

trajectory, $\mathbf{K} = \lambda\overline{\mathbf{M}}$. The gain for the composite adaptation was chosen as $\gamma_p = 0.5$, and the reference model was tuned with a natural frequency of $\omega = 8$ rad/s.

The experimental joint trajectories for both controllers are depicted in Fig. 3. The proposed controller outperformed the conventional, with a transient time of around 0.5 s, which is roughly 3 times faster than the passivity-based control. Sample temporal plots of dynamical parameter estimates (masses) are depicted in Fig. 4, with the dashed lines representing the identified parameters $\widehat{\boldsymbol{\theta}}_0$ (ground truth). The proposed adaptation law showed superior performance in terms of convergence rate, accuracy of estimates, and their smoothness, thanks to the power of composite adaptation provided by extended regression. The estimates rapidly converged to the region near the ground truth under the proposed adaptation, while the conventional estimator introduced visible oscillations and static errors. Increasing the learning gain γ_c in the conventional controller yielded parameter drift and instability. However, it should be noted that there are still small estimation errors in the proposed algorithm, which can be attributed to unaccounted dynamics, such as stick-slip friction and neglected off-diagonal inertia terms.

The overall performance of the controllers as summarized by the norms of $|\widetilde{\mathbf{q}}|$ and $|\widetilde{\boldsymbol{\theta}}|$ is demonstrated in Fig. 5. Note how the proposed law ensures faster convergence for both parameters and tracking errors, with a monotonic, exponential-like behavior in the case of errors. It is worth mentioning that the proposed controller is also easier to tune, only requiring positive values for λ, γ_c, and γ_p, all of which can be chosen to be much greater than the coefficients of the conventional controller, without compromising stability and noise rejection.

5. Conclusion

In this paper, we propose a modification to composite adaptive control for robotic systems. The proposed framework utilizes dynamically extended momentum dynamics similar to those used in designing disturbance observers. We analyze the conditions for exponential and asymptotic stability and demonstrate that the proposed modification offers faster parameter and

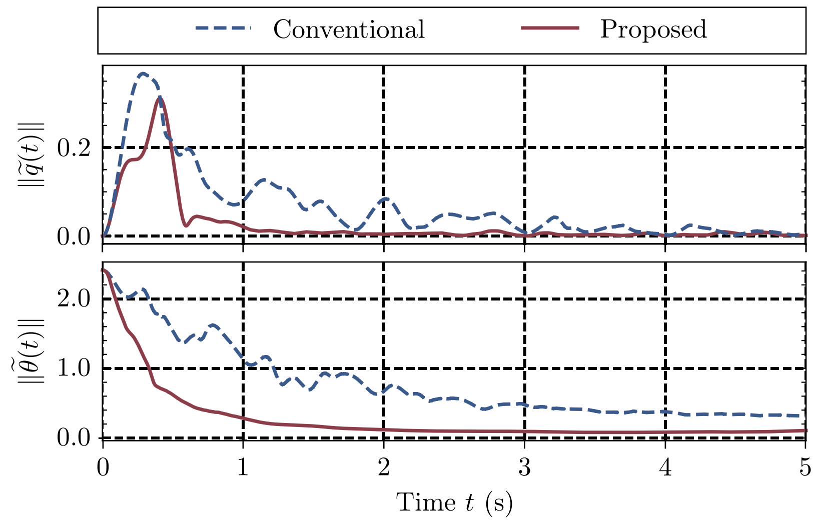

Fig. 5. The summarized tracking (top) and parameter estimation (bottom) behavior under proposed and conventional controllers

tracking convergence, while also relaxing excitation requirements. These properties are similar to those of dynamically extended parameter estimators and composite/concurrent adaptation controllers, but can now be applied to general cases of robotic systems. The effectiveness of the proposed approach is demonstrated through experimental evaluation on a 3-DoF robotic leg, and the results demonstrate significant improvement over the conventional scheme, with 3 times faster convergence of tracking errors and an exponential-like convergence of parameter estimates.

In the future, we aim to enhance convergence behavior by using advanced mixing techniques. We also plan to account for parametric constraints via natural adaptive laws and validate the proposed techniques on various practical systems, including legged robots and quadrotor UAVs.

Conflict of interest

The authors declare that they have no conflict of interest.

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