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AQ1 Optimal Interventions in Markets with Adverse Selection†

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We study the design of interventions to stabilize financial markets plagued by adverse selection. Our contribution is to analyze the information revealed by participation decisions. Taking part in a government program carries a stigma, and outside options are mechanism dependent. We show that the efficiency of an intervention can be assessed by its impact on the market interest rate. The presence of an outside market determines the nature of optimal interventions and the choice of financial instruments (debt guarantees in our model), but it does not affect implementation costs. (*JEL* D82, D86, G01, G20, G31)

Akerlof (1970) shows how asymmetric information can create adverse selection and undermine market efficiency. Economic and legal institutions, such as auditors, underwriters, accountants, or used-car dealers, often emerge to limit adverse selection and allow markets to function. As a result, direct government interventions are usually unnecessary. If a market does collapse, however—presumably following the failure of the institutions designed to prevent the collapse in the first place—a government might want to intervene. This article asks what form these interventions should take if the goal of policy is to improve economic efficiency with minimal cost to taxpayers.

We study an economy with borrowing and investment under asymmetric information. Firms must raise capital to take advantage of profitable investment opportunities, but also have private information about the value of their existing assets. Optimal financial contracts can only partially limit adverse selection, and inefficiencies occur because the safest borrowers, facing unfairly high interest rates, drop out of the market. Competitive lenders then rationally charge a high rate to the remaining borrowers, lending and investment are inefficiently low, and there is scope for a government intervention.

We characterize cost-minimizing interventions to improve lending and investment, and we propose implementations with standard financial contracts. The key novel aspect of our analysis is the interaction between the government's intervention and the borrowing terms that firms face in the market. Potential lenders rationally

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interpret participation decisions as signals of private information. Therefore, the rate at which nonparticipants borrow depends on who participates in the program. In equilibrium, participation decisions affect outside options through signaling, while outside options influence the cost of the program through the participation constraints. This feedback distinguishes our work from the existing mechanism-design literature, in which outside options are exogenous.

Several dimensions must be considered when designing an optimal program, from the selection of types to the nature of financial contracts. Interactions between these choices and the endogenous outside options make the problem quite complex. For instance, one might conjecture that the government should try to selectively attract the good types who drop out of the private market. To do so, it might be optimal to offer different securities from the ones private lenders use. In this case, successful interventions could be cheap, or even profitable. Alternatively, if interventions are unavoidably costly, it might be important to minimize the number of participants in the program. Finally, the government might be tempted to restrict access to private markets. Our analysis clarifies these conjectures.

Our model has the following features. All firms have investment opportunities with positive net present value. A firm's privately known type θ, drawn from some interval $[\underline{\theta}, \overline{\theta}]$, indexes the quality of its legacy assets and, therefore, determines the conditional distribution of its total income y. We assume that $f(y|\theta)$ satisfies the monotone hazard rate property. In this environment, the optimal contract between firms and private investors is a debt contract. A decentralized equilibrium is characterized by a cutoff θ^D: types in $[\underline{\theta}, \theta^D]$ invest, while types above θ^D, faced with an unfairly high interest rate, forgo investment. The lack of investment by types in $[\theta^D, \overline{\theta}]$ represents the welfare loss of adverse selection. The goal of the government is to increase investment while minimizing the cost of its intervention for taxpayers.

We obtain five main results. The first result is that, regardless of how the government designs its intervention, it cannot selectively attract the best types. An equilibrium with intervention is similar to the decentralized equilibrium, but with a higher investment cutoff $\theta^T > \theta^D$. Interventions that improve investment are always costly.

The second result is that the investment level achieved by an optimal program can be assessed simply by looking at the borrowing rate outside the program. To establish this result, we show that in any equilibrium, the best type investing θ^T must weakly prefer to borrow from the market than from the government. This implies a one-to-one mapping between the investment level achieved by any program and the borrowing market rate of nonparticipants.

The third result is that the actual size of a program is, to a large extent, irrelevant. In equilibrium, the market rate is pinned down by the average quality of nonparticipants through the break-even constraint of private lenders. This average quality must be good enough to sustain private lending beyond θ^D and up to θ^T. This requirement, however, is consistent with many different participation functions. For instance, there is a unique minimal program where the government attracts all the worst types (up to some threshold), and the market then faces a truncated distribution with a better average quality than in the decentralized equilibrium. There exists, however, a continuum of other programs that achieve the same target investment θ^T, the same break-even market rate, and the same implementation cost. In some of these programs, all types below θ^T might borrow from the government with positive probability.

The fourth result is that optimal implementation requires debt-like instruments. Implementation costs are at least as high as the rents that lower types obtain if they mimic θ^T and borrow from the market. Reaching the minimum cost therefore requires that participation constraints bind simultaneously for all types. This is possible only if repayment functions are the same inside and outside the program. Since private investors use debt contracts, the government must employ debt-like securities. The government may lend directly, or it may guarantee privately issued debt. Other instruments (e.g., equity injections) are more expensive because participation constraints cannot bind for several types at once.

Our fifth result is that, even if the government could shut down the private markets, this would not lower the cost of implementation. This is because the cost of an optimal intervention is equal to the rents that all investing types obtain by mimicking type θ^T. Since θ^T is, by definition, indifferent between investing and not, its payoff is the same whether or not there is a market.

Finally, we extend our benchmark model by relaxing the assumption that investment opportunities are the same for all types. Our results continue to hold with asymmetric information about new opportunities. When we allow banks to choose the riskiness of their investments after they opt into the program, we find that moral hazard is mitigated by the endogenous response of the private interest rate and can be eliminated by indexing the terms of the government's program to that rate.

Discussion of the Literature

Our work is motivated by the history of financial crises. Calomiris and Gorton (1991) analyze the evolution of two competing views of banking panics. The "random withdrawal" theory (Diamond and Dybvig 1983; Bhattacharya and Gale 1987; Chari 1989) focuses on bank liabilities and coordination among depositors. The "asymmetric information" theory emphasizes asymmetric information about banks' assets. According to Calomiris and Gorton (1991) and Mishkin (1991), the historical evidence supports the idea that asymmetric information plays a critical role in banking crises. Several features of the financial-market collapse in fall 2008 also suggest a role for asymmetric information (Heider, Hoerova, and Holthausen 2008; Duffie 2009; Gorton 2009).[1] Governments stepped in with large-scale interventions, but there was no consensus about exactly which programs should be offered.[2] Finally, there is ample evidence that borrowing from the government (for banks), or

[1] Heider, Hoerova, and Holthausen (2008) discuss the collapse of the interbank market. Duffie (2009) discusses the OTC and repo markets. In the OTC market, the range of acceptable forms of collateral was dramatically reduced, "leaving over 80 percent of collateral in the form of cash during 2008," while the "repo financing of many forms of collateralized debt obligations and speculative-rate bonds became essentially impossible." Gorton (2009) explains how the complexity of securitized assets created asymmetric information about the size and location of risk. Investors and banks were unable to agree on prices for legacy assets or for bank equity. The classic references on financial crises (Bagehot 1873; Sprague 1910) do not discuss the role of asymmetric information explicitly.

[2] In the United States, the original Troubled Asset Relief Program (TARP) called for $700 billion to purchase illiquid assets but was transformed into a Capital Purchase Program (CPP) to invest $250 billion in US banks. As of August 2009, $307 billion of outstanding debt was issued by financial companies and guaranteed by the Federal Deposit Insurance Corporation (FDIC). The treasury also insured $306 billion of Citibank's assets, and $118 billion of Bank of America's. Soros (2009) and Stiglitz (2008) argue for equity injections; Bernanke (2009) favors asset purchases and debt guarantee; Diamond et al. (2008) view purchases and equity injection as the best alternatives; and Ausubel and Cramton (2009) argue for a careful way to "price the assets, either implicitly or explicitly."

from the IMF (for countries), carries a stigma (Peristiani 1998; Corbett and Mitchell 2000; Mitchell 2001; Furfine 2005).

Our paper builds on the rich literature that studies asymmetric information, following Akerlof (1970), Spence (1974), and Stiglitz and Weiss (1981). It is useful to relate our work to the particular branch that deals with security design. Myers and Majluf (1984) argue that debt can be used to reduce mispricing when issuers have private information. Brennan and Kraus (1987) consider various financing strategies to reduce adverse selection. Technically, we build on the contribution of Nachman and Noe (1994), who clarify the conditions under which debt is optimal in a multitype capital-raising game. DeMarzo and Duffie (1999) also discuss the optimality of debt when the security design occurs before private information is learned. Fishman and Parker (2010) analyze the externalities involved in the endogenous acquisition of private information.

Our paper is also related to the literature on government interventions to improve market outcomes. Some of the literature deals specifically with bank bailouts. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors can. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should bail out only the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to distort ex ante lending incentives. Efficient bailouts are studied by Philippon and Schnabl (2009) in the context of debt overhang, and by Farhi and Tirole (2010) in the context of collective moral hazard, while Chari and Kehoe (2009) argue that the time-inconsistency problem is more severe for the government than for private agents.

Some papers study government interventions in the presence of competitive markets. Bond and Krishnamurthy (2004) study enforcement when a defaulting borrower can be excluded only from future credit markets. Bisin and Rampini (2006) argue that market access can be a substitute for government's commitment. Golosov and Tsyvinski (2007) study the crowding-out effect of government interventions in private insurance markets.[3] Farhi, Golosov, and Tsyvinski (2009), building on Jacklin (1987), show how liquidity requirements can improve equilibrium allocations. The critical difference is that, in our paper, government intervention affects market conditions through signaling and adverse selection.

The most closely related papers are Minelli and Modica (2009) and Tirole (2010). Minelli and Modica (2009), building on Stiglitz and Weiss (1981), model the intervention as a sequential game between the government and a monopolistic lender. Like us, Tirole (2010) emphasizes the role of endogenous outside options. Our models assume different frictions that prevent the efficient financing of new projects. Tirole (2010) assumes moral hazard in addition to adverse selection, while we follow Myers and Majluf (1984) and assume that returns of old and new projects are fungible. Some results are, nonetheless, similar. For example, Tirole (2010) also finds that the government cannot selectively attract good types, and that it does not

[3] There is also an extensive literature on how government interventions can improve risk sharing. For excellent surveys, see Kocherlakota (2006), Golosov, Tsyvinski, and Werning (2007); and Kocherlakota (2009).

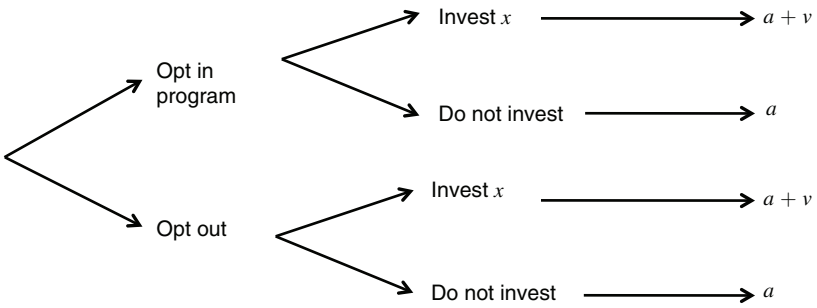

FIGURE 1. TIMING

benefit from shutting down private markets. Another difference between our work and both Minelli and Modica (2009) and Tirole (2010) is that we allow for continuous payoffs (as opposed to binary ones). This allows us to discuss security design.

We present our model in Section I. In Section II, we characterize its decentralized equilibria. We formally describe the mechanism-design problem in Section III. In Section IV, we characterize lower bounds on the costs of government interventions. Those bounds can actually be achieved by simple, common interventions, as we show in Section V. Section VI discusses extensions, and we close the paper with some final remarks in Section VII.

I. The Model

Our model has three dates, $t = 0, 1, 2$, and a continuum of firms with preexisting "legacy" assets. The firms start with private information regarding the quality of their legacy assets and receive the opportunity to make a new investment at time 1. The government can offer various programs at time 0, and firms can borrow and lend in a competitive market at time 1. We assume that all agents are risk neutral and we normalize the risk-free rate to zero. The timing of the model is depicted in Figure 1.

Initial Assets and Cash Balance.—Firms start with cash and legacy assets, and no preexisting liabilities. Cash is liquid and can be kept, invested, or lent at time 1. Let c_t denote the cash holdings at the beginning of period t. All firms start with c_0 in cash, but c_1 can differ from c_0 if the government injects cash in the firms at time 0. Cash holdings cannot be negative: $c_t \geq 0$ for all t.

The book value of legacy assets, A, is known, but some assets may be impaired, and the eventual payoff at time 2 is the random variable $a \in [0, A]$. Firms privately know their type θ, which determines the conditional distribution of the value of legacy assets $f_a(a \mid \theta)$. Types are drawn from a compact set $\Theta \subset [\underline{\theta}, \overline{\theta}]$ with cumulative distribution $H(\theta)$.

Investment and Borrowing.—Firms receive investment opportunities at time 1. Investment requires the fixed amount x and delivers a random payoff v at time 2. Firms can borrow at time 1 in a competitive market. After learning its type θ, a firm offers a contract (l, y^l) to the competitive investors, where l is the amount raised from investors at time 1, and y^l is the schedule of repayments to investors at time 2.

Without government intervention, the funding gap of the firms is $l_0 \equiv x - c_0$. The government can reduce the funding gap with cash injections, denoted by m. In this case, $c_1 = c_0 + m$, and the firm only needs to borrow $l = l_0 - m$. We use the generic notation l for the amount actually raised from private investors. In period 2, the cash balance of the firm is $c_2(i) = c_1 + l - x \cdot i$, and its total income is

$$\tau_2(i) = c_2(i) + a + v \cdot i, \tag{1}$$

where $i \in \{0, 1\}$ is a dummy for the decision to invest at time 1. Total firm income at time 2 depends on the realization of the two random variables a and v. This total income will be split among initial owners, new lenders, and (potentially) the government.

Assumptions.—For simplicity, we assume in our benchmark model that all firms receive the same investment opportunities. The random payoff v is distributed on $[0, V]$ according to the density function $f_v(v)$. Let $\overline{v} \equiv E[v]$ be the expected value of v. To make the problem interesting, we assume that new projects have positive NPV and that firms need to borrow in order to invest: $\overline{v} > x > c_0$. We further assume that contracts can be written only on the total income of the firm at time 2:

ASSUMPTION A1: *The only observable outcome is total income* τ_2 *defined in equation* (1).

Under Assumption 1, repayment schedules can be contingent on total income τ_2 but not on a and v separately. If a firm does not invest, it keeps $c_2 = c_1$, and its total income at time 2 is $a + c_1$. If it invests, it ends up with $c_2 = 0$ and total income $a + v$.[4] We define total income conditional on investment as: $y \equiv a + v$. The distribution of y, which is the convolution of f_a and f_v, is denoted by f. Since f_a depends on θ, so does f. Let Y denote the support of y. We assume that $f(y \mid \theta)$ satisfies the strict monotone hazard rate property:

ASSUMPTION A2: *For all* $(y, \theta) \in Y \times \Theta$, $f(y \mid \theta) > 0$, *and* $f(y \mid \theta)/(1 - F(y \mid \theta))$ *is decreasing in* θ.

Total income is used to repay the loans taken at time 1 according to a schedule y^l. When the government intervenes, the firm also might need to repay the government, according to a schedule y^g. Our last assumption is to impose a monotonicity condition on the repayment schedules.

ASSUMPTION A3: *The repayment schedules* y^l *and* y^g *of private lenders and of the government are nondecreasing in* τ_2.

Let us briefly discuss the main features of our model. We introduce a binary investment technology to simplify the strategy space of firms, but it is easy to extend

[4] It will become clear that the government never finds it optimal to inject cash into the banks beyond $m = l_0$.

the model to partial investment, for instance, by having $i \in \{0, 1/2, 1\}$. In order to stay close to the workhorse model of Myers and Majluf (1984), we initially assume that all firms receive the same investment opportunities. In this context, A1 makes private information relevant by preventing the parties from contracting directly on v (by spinning off the new investment, for example). As an extension, we introduce private information on v in Section VI. Assumption A2 defines a natural ranking among types regarding the quality of their legacy assets, from the worst type $\underline{\theta}$ to the best type $\overline{\theta}$. The strict inequalities in A2 are not crucial, but they simplify some of the proofs. We allow for a general set of types Θ because, while much intuition can be obtained with just two types, the implementation results are somewhat special for the two-type case, as we explain in Section V. Assumption A3 has been standard in the literature on financial contracting since Innes (1990) and Nachman and Noe (1994). It renders optimal contracts more realistic by effectively smoothing sharp discontinuities in repayments, and it can be formally justified by the possibility of hidden trades.[5]

II. Equilibria without Interventions

Because the credit market is competitive and investors are risk neutral, in any candidate equilibrium, the expected repayments to the lenders must be at least the size of the loan

$$E\left[y^l \mid \mathcal{I}\right] \geq l, \tag{2}$$

where $\mathcal{I}$ denotes the information set of the private lenders at the time they make the loan. Under symmetric information, investment decisions would have been independent of the quality of legacy assets, and all firms would invest since $\overline{v} \geq x$. The symmetric-information allocation is an equilibrium under asymmetric information when firms can issue risk-free debt. By contrast, adverse selection can occur when new investments are risky and when there is significant downside risk on legacy assets.

Contracting Game.—The contracting game is potentially complex because the kind of security a firm offers might signal its type. Under assumptions A1 to A3, however, it is a standard result that all firms that invest offer the same security, and this security is a debt contract.[6] The intuition is that bad types want to mimic good types, while good types seek to separate from bad types. Contracts with high repayments for low income realizations are relatively more attractive for good types than

[5] The justification is that if repayments were to decrease with income, the borrower could secretly add cash to the bank's balance sheet by borrowing from a third party, obtaining the lower repayment, immediately repaying the third party, and obtaining strictly higher returns. See Sections 3.6 and 6.6 in Tirole (2006) for further discussion.

[6] There are several ways to obtain this result. One is to let each bank offer one contract and solve the issuance/signaling game. Nachman and Noe (1994) show in Theorem 5 that the unique equilibrium is pooling on the same debt contract as long as the distribution of payoffs can be ranked by hazard rate dominance (A2). Another way to obtain the result is to follow Myerson (1983) and let banks offer a menu of securities in a first stage (see, also, Maskin and Tirole 1992). The inscrutability principle then ensures that no signaling occurs during the contract-proposal phase, and we can focus on one incentive-compatible menu. Standard design arguments can then be used to show that, under A1–A3, the best menu specifies the same debt contract for all types that invest.

for bad types. This is the core idea of Myers and Majluf (1984) in a model with two types, extended by Nachman and Noe (1994) to an arbitrary set of types.

Equilibria without Government Intervention.—We have explained above that all investing firms offer the same debt contract. Our next step is to characterize the set of firms that actually invest. Let r be the (gross) interest rate at which firms borrow. We can define the expected repayment function for type θ as

$$\rho(\theta, rl) \equiv \int_Y \min(y; rl)\, f(y \mid \theta)\, dy. \tag{3}$$

For a given θ, the function $\rho(\theta, rl)$ is increasing in the face value rl. Under symmetric information, the fair interest rate r_θ^* on a loan l to a firm with type θ is implicitly given by $l \equiv \rho(\theta, r_\theta^* l)$. Since $\rho(\theta, rl)$ is increasing in θ, the fair rate is decreasing in θ for any given l. With private information, however, the interest rate cannot depend explicitly on θ, and better types end up facing an unfair rate. This is the source of adverse selection.

Without government intervention, firms need to borrow $l = l_0 = x - c_0$. Given a market rate r, a type θ wants to invest if and only if $E[a \mid \theta] + \overline{v} - \rho(\theta, rl_0) \geq E[a \mid \theta] + c_0$. This investment condition is equivalent to[7]

$$\overline{v} - x \geq \rho(\theta, rl_0) - l_0. \tag{4}$$

The term $\rho(\theta, rl_0) - l_0$ measures the informational rents paid by the firm. The rents are zero when the rate is fair. The information cost is positive when $r > r_\theta^*$ and negative (a subsidy) when $r < r_\theta^*$. When informational rents are too large, firms might decide not to invest.

Since the right-hand side of equation (4) is increasing in θ, if θ wants to invest at rate r, any type below θ also wants to invest at that same rate. The set of investing types is, therefore, $[\underline{\theta}, \hat{\theta}]$, and the marginal type $\hat{\theta}$ is defined by

$$\overline{v} - x \equiv \rho(\hat{\theta}, rl_0) - l_0. \tag{5}$$

The borrowing rate depends on the market's perception about the mix of firms that invest. Let $h(\cdot \mid 1)$ describe the market's beliefs about the type of firms that borrow to invest. Investors' beliefs must be consistent with Bayes' rule: $H(\theta \mid 1) = H(\theta)/H(\hat{\theta})$ if $\theta \in [\underline{\theta}, \hat{\theta}]$, and 0 otherwise. Finally, the rate r must satisfy the zero-profit condition for private investors:

$$l_0 = \int_{\underline{\theta}}^{\hat{\theta}} \rho(\theta, rl_0)\, dH(\theta \mid 1). \tag{6}$$

[7] We use the conventional assumption that, when indifferent, banks choose to invest.

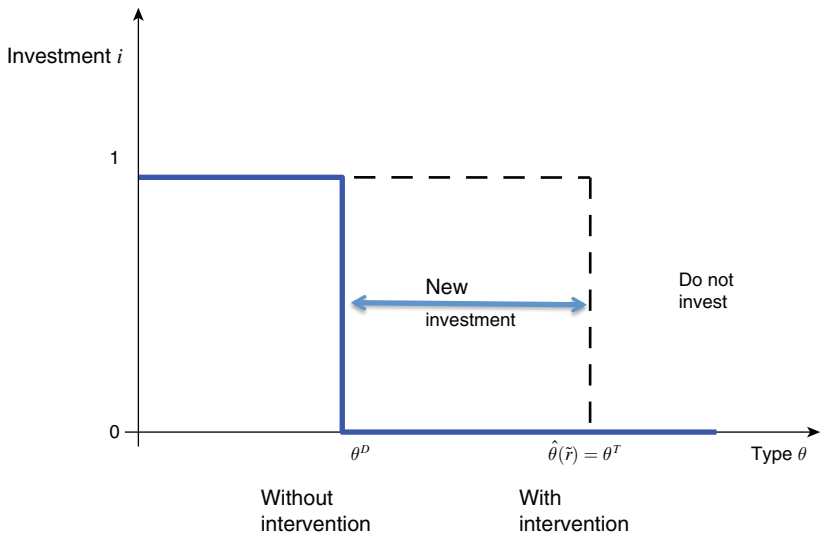

FIGURE 2. EQUILIBRIUM INVESTMENT

PROPOSITION 1: *The efficient outcome is sustainable without government intervention if and only if there is a borrowing rate r such that* (4) *and* (6) *hold for* $\hat{\theta} = \overline{\theta}$. *All other equilibria have* $\hat{\theta}$ *and are inefficient.*

The intuition for Proposition 1 is as follows. The potential for adverse selection exists because the investment condition (4) is more likely to hold for worse types than for better types.[8] Multiple equilibria are possible because of the endogenous response of the interest rate.[9] Let r^D denote the lowest interest rate that can be supported without government intervention, and let θ^D be the corresponding threshold. The best decentralized equilibrium (r^D, θ^D) depends on c_0 and on the prior distribution of types.

In the remainder of the paper, we examine cases in which the efficient outcome is not sustainable as a decentralized equilibrium—i.e., $\theta^D < \overline{\theta}$. It is clear that higher cash levels increase θ^D and improve economic efficiency. Therefore, governments might seek to inject liquidity into the firms. The equilibrium level of investment, before and after the government's intervention, is depicted in Figure 2. Our goal is to design the most cost-effective interventions that achieve a given level of investment. We do so formally in the next sections.

III. Mechanism Design with a Competitive Fringe

In this section, we present the government's objective and describe the mechanism-design problem. Without intervention, the best equilibrium is (r^D, θ^D) described

[8] If the scale of investment were a choice variable, the separating equilibrium would involve good banks scaling down to signal their types. With our technological assumption, they scale down to zero. The only important point is that in both cases the equilibrium can be inefficient.

[9] Multiplicity arises for the same reasons as in signaling games.

above. The government's goal is to find the cheapest possible way to implement any given level of investment.[10] We denote the cost of a government program by Ψ. While the government's objective is straightforward, the mechanism-design problem is nonstandard because we assume that private markets remain open. The market rate for nonparticipating firms, then, depends on the mechanism the government uses because participation decisions convey information about private types. This interrelationship does not exist in standard mechanism design, where outside options are independent from the mechanism.[11] We refer to our model as mechanism design with a "competitive fringe."

Government's Strategy.—A government program $\mathcal{P}$ is a menu of contracts. The revelation principle applies and we can, without loss of generality, consider programs with one contract per type.[12] A contract specifies the cash m injected at time 1 and the schedule of payments y^g received by the government at time 2. A generic program, therefore, takes the form: $\mathcal{P} = \{m_\theta, y^g_\theta\}_{\theta \in \Theta}$. Under A3, y^g_θ is increasing in y for all $\theta \in \Theta$.[13]

Firms' Strategy.—A firm's strategy consists of a participation decision and an investment decision as a function of its type. At time 0, after the announcement of the government's program, each firm chooses a contract in $\mathcal{P}$ or opts out by choosing $\mathcal{O}$. We allow firms to randomize their participation decisions. At time 1, given its type and its realized participation decision, each firm decides whether or not to invest:

$$i : \Theta \times \{\mathcal{P} \cup \mathcal{O}\} \to \{0,1\}.$$

The choice of a government contract in $\mathcal{P}$ is observed by the market and induces a private lending contract (l_θ, y^l_θ). If a firm of type θ chooses a contract $\{m_{\theta'}, y^g_{\theta'}\}$ designed for θ', its cash becomes $c_1 = c_0 + m_{\theta'}$. If it invests, it must then borrow $l_{\theta'} = x - c_0 - m_{\theta'}$ from the private market, and its expected payoff is

$$V(\theta, \theta', 1) = \int_0^\infty \left(y - y^l_{\theta'}(y) - y^g_{\theta'}(y)\right) f(y \mid \theta)\, dy. \tag{7}$$

If it does not invest, its expected payoff is $V(\theta, \theta', 0) = E[a \mid \theta] + c_0 + m_{\theta'} - \int_0^\infty y^g_{\theta'}(y) f(y \mid \theta)\, dy$. If the firm opts out of the government program, it has the

[10] In a general equilibrium model, one could—after the cost minimization—solve for the optimal level of investment. We do not study this second stage here. Rather, we characterize the cost-minimizing intervention for any particular level of investment the government might want to implement.

[11] In common agency problems, the interrelationship of the design problems is more complex since both principals that offer contracts have bargaining power. In our paper, the principal's (the government's) mechanism induces a competitive market's response.

[12] In our model firms are ex ante identical, so the government offers one menu. Otherwise the government should condition on observable characteristics, such as size, or leverage, and our results would apply after this conditioning.

[13] This covers any program based on equity payoffs (common stock, preferred stock, warrants, etc.), as well as all types of direct lending and debt-guarantee programs. The case of asset purchases can be analyzed by allowing y^g_θ to depend on a. We discuss this extension in Section VI.

option to borrow in the private market at an interest rate $\tilde{r}$. The outside option of a type θ firm is, therefore,

$$\tilde{V}(\theta, \tilde{r}) \;=\; E[a \,|\, \theta] \;+\; \max\{c_0, \overline{v} \;-\; \rho(\theta, \tilde{r}\, l_0)\}. \tag{8}$$

Competitive Fringe.—Regardless of whether a firm opts in or out, the interest rate at which it borrows must satisfy the break-even condition of competitive lenders in equation (2). The information set $\mathcal{I}$ contains the equilibrium strategies of the firms—the participation and investment mappings—and the observed choices—the particular contract in $\mathcal{P}$ and the decision to demand a loan of size l.[14] The loan is $l_0 = x - c_0$ for firms opting out, and $l_0 - m_\theta$ for firms opting in and choosing the contract designed for θ.

Equilibrium Conditions.—Fix a government intervention and market rate $\tilde{r}$ for nonparticipating firms. Let $\Theta_{\mathcal{P},1}$ denote the set of types that participate and invest, and let $\Theta_{\mathcal{P},0}$ denote the types that participate but do not invest. Define $\Theta_{\mathcal{P}} = \Theta_{\mathcal{P},1} \cup \Theta_{\mathcal{P},0}$. Similarly, we can define $\Theta_{\mathcal{O},1}$ (respectively $\Theta_{\mathcal{O},0}$) to be the corresponding nonparticipating sets of types, and $\Theta_{\mathcal{O}} = \Theta_{\mathcal{O},1} \cup \Theta_{\mathcal{O},0}$. In order to have an equilibrium, we must have

- For $i \in \{0,1\}$ and $\theta \in \Theta_{\mathcal{P},i}$, $V(\theta, \theta, i) \geq \max\big(V(\theta, \theta', j), \tilde{V}(\theta, \tilde{r})\big)$, for $j \in \{0,1\}$ and $\theta' \in \Theta_{\mathcal{P}}$.
- For all $\theta \in \Theta_{\mathcal{O}}$, $\tilde{V}(\theta, \tilde{r}) \geq V(\theta, \theta', j)$ for $j \in \{0,1\}$ and $\theta' \in \Theta_{\mathcal{P}}$, and $\theta \in \Theta_{\mathcal{O},1} \Leftrightarrow \overline{v} - x \geq \rho(\theta\,, \tilde{r}\, l_0) - l_0$.

These conditions summarize the incentive, investment and participation constraints for both participating and nonparticipating firms.

Private lenders must expect to break even, and their beliefs must be consistent with the equilibrium behavior of the firms. For instance, if $H_{\mathcal{O},1}$ denotes the market's perception about the distribution of firm types that choose to invest alone, the outside rate $\tilde{r}$ must satisfy

$$l_0 \;=\; \int_{\Theta_{\mathcal{O},1}} \rho(\theta, \tilde{r}\, l_0)\, dH_{\mathcal{O},1}(\theta). \tag{9}$$

Similar conditions must hold for firms that opt in and borrow $l_\theta = x - c_0 - m_\theta$. Finally, the resource constraint $y^l(y) + y^g(y) \leq y$ must hold for all contracts. In what follows, we, without loss of generality, restrict our attention to interventions where the government receives junior claims—i.e., where new lenders are paid first according to $y^l_\theta(y) = \min(y, r_\theta l_\theta)$.[15]

[14] In other words, we assume that the private market sees not only whether a firm accepts public money, but also under what conditions. Then, if in an equilibrium the government program is fully separating, there is no asymmetric information left in the market. Despite this possibility, we show that programs in which all participating banks pool at the same option are optimal.

[15] To see this, imagine a program where the government has some senior claims $\tilde{y}^{gs}_\theta$ and some junior claims $\tilde{y}^{gj}_\theta$. The optimal private debt contract is $\tilde{y}^l_\theta = \min(y - y^{gs}_\theta, \tilde{r}_\theta l_\theta)$, and from the resource constraint we have $y^{gs}_\theta \leq y$ and $\tilde{y}^{gj}_\theta = 0$ for all $y < \tilde{r}_\theta l_\theta$. Now consider an alternative program where all government claims are junior. The private contract is $y^l_\theta = \min(y, r_\theta l_\theta)$. Define $y^g_\theta = \min(y - y^l_\theta, \tilde{r}_\theta l_\theta) + (\tilde{y}^{gs}_\theta + \tilde{y}^{gj}_\theta)\, 1_{y > \tilde{r}_\theta l_\theta}$. This contract gives exactly the same payoff function to the bank, so it leaves all participation, incentive, and investment constraints

To sum up, the design problem is complex because the participation decision is influenced by the nonparticipation payoffs that depend on the market reaction, which is, in turn, endogenous to the mechanism. In Section IV, we characterize the set of feasible interventions and derive lower bounds on their costs. In Section V, we show that these bounds are actually achieved by realistic and commonly used interventions.

IV. Cost-Minimizing Interventions

In this section, we study feasible interventions and derive lower bounds for their costs. We analyze interventions where the competitive fringe is active—i.e., where some firms invest without the government's assistance. When the competitive fringe is active, the marker rate $\tilde{r}$ is pinned down by equation (9). The case where the competitive fringe is completely inactive—i.e., when $\Theta_{\mathcal{O},1} = \varnothing$—is discussed in Section V.

A. *Feasible Interventions*

Any firm opting out of the program can borrow in the market at rate $\tilde{r}$. This rate defines a marginal type $\hat{\theta}(\tilde{r})$ for which condition (4) holds with equality. The government knows that the outside option of any type below $\hat{\theta}$ is to invest, while the outside option of any type above $\hat{\theta}$ is to do nothing. We will establish that the government cannot selectively attract better types by showing that in all feasible interventions, the types that invest with the help of the government are worse than $\hat{\theta}$.

We first introduce some notation and establish an important building block of our analysis in Lemma 1. Firms that participate and invest receive income $y - y^l_\theta - y^g_\theta$. The difference between inside and outside payoffs conditional on investment in both cases is then: $\gamma_\theta(y) \equiv \min(y, \tilde{r} l_0) - \min(y, r_\theta l_\theta) - y^g_\theta(y)$. From the participation constraint $V(\theta, \theta, 1) \geq \tilde{V}(\theta, \tilde{r})$ of investing types, we then obtain

$$E[\gamma_\theta(y) \,|\, \theta] \;\geq\; 0 \text{ for all } \theta \in \Theta_{\mathcal{P},1}. \tag{10}$$

The following Lemma establishes a Single Crossing Property that plays a central role in our analysis.

LEMMA 1 *Single Crossing Property*: *If* $E[\gamma_\theta(y) \,|\, \theta] \geq 0$ *for some* $\theta \in \Theta$, *then* $E[\gamma_\theta(y) \,|\, \theta'] \geq 0$ *for all* $\theta' < \theta$. *If, in addition,* $\gamma_\theta(y) \neq 0$ *for some* $y \in Y$, *then* $E[\gamma_\theta(y) \,|\, \theta'] > 0$.

PROOF:

See Appendix.

Lemma 1 says that if a type prefers the strategy "opt-in-and-invest" to the strategy "invest-alone," then all types below it have the same preference. In other words, irrespective of the way the government intervenes (so long as repayments to the

unchanged. Finally, since $l_\theta = E[\tilde{y}^l_\theta] = E[y^l_\theta]$, it follows that $E[y^g_\theta] = E[\tilde{y}^{gs}_\theta + \tilde{y}^{gj}_\theta]$, and the cost to the government is unchanged.

government increase in the firms' total income at time 2), if some type prefers to invest with the help of the government, rather than investing alone, all worse types want to do the same. This result is driven by the same forces that cause the original market failure—namely, that good types expect to repay *relatively* more than bad types. An important subtlety, however, is that what matters is the difference in expected payments inside or outside the program, both of which are increasing in θ. An additional complication is that the function $\gamma_\theta(y)$ is not monotonic. It is typically positive for low values of y and then decreasing. The key is that it can switch sign only once, for some income realization $\hat{y}$. This explains why first-order stochastic dominance is not enough, and why we need conditional stochastic dominance (or hazard-rate dominance) in A2. We can then apply stochastic dominance conditional on income being more (or less) than $\hat{y}$, and obtain our result. The interesting point is that A2 is also the necessary and sufficient condition for debt to be optimal in the capital-raising game under asymmetric information (Nachman and Noe 1994).

Lemma 1 has important implications. First, as long as some firms invest without the government's help, no type above $\hat{\theta}(\tilde{r})$ invests.

PROPOSITION 2: *When the competitive fringe is active*, *the set of investing firms is contained in the interval* $[\underline{\theta}, \hat{\theta}(\tilde{r})]$.

PROOF:

For nonparticipating types ($\theta \in \Theta_{\mathcal{O}}$), the proposition follows from the definition of $\hat{\theta}(\tilde{r})$. For participating types, we argue by contradiction. Suppose that there is a participating type θ strictly above $\hat{\theta}(\tilde{r})$ that invests—that is, $\theta \in \Theta_{\mathcal{P},1}$. Because $\theta > \hat{\theta}(\tilde{r})$, we have that $E[a\,|\,\theta] + c_0 > E[a\,|\,\theta] + \overline{v} - \rho(\theta, \tilde{r}\, l_0)$. Moreover, since $\theta \in \Theta_{\mathcal{P},1}$, we have that $E[a\,|\,\theta] + \overline{v} - \rho(\theta, r_\theta l_\theta) - E[y_\theta^g(y)\,|\,\theta] \geq E[a\,|\,\theta] + c_0$. Together, these two inequalities imply that $E[\gamma_\theta(y)\,|\,\theta] > 0$. From Lemma 1, we then know that $E[\gamma_{\theta'}(y)\,|\,\theta] > 0$ for all types $\theta' < \theta$. Therefore, $\left[\underline{\theta}, \hat{\theta}(\tilde{r})\right] \subset \Theta_{\mathcal{P},1}$ and $\Theta_{\mathcal{O},1} \subset \left[\hat{\theta}(\tilde{r}), \overline{\theta}\right]$, but this contradicts the definition of $\hat{\theta}(\tilde{r})$, which says that types above $\hat{\theta}(\tilde{r})$ do not invest at the market rate $\tilde{r}$.

Proposition 2 shows that no type above $\hat{\theta}(\tilde{r})$ will invest, and it is reminiscent of the standard adverse-selection unraveling result. In other words, the government cannot eliminate adverse selection. It can only limit the unraveling.

The key point to understand is that, in equilibrium, the government cannot attract a type above $\hat{\theta}(\tilde{r})$ and make this type invest. The reason is that, as shown in Lemma 1, all types worse than $\hat{\theta}(\tilde{r})$ would choose to mimic this type rather than go to the market. But, then, private lenders would anticipate lending only to types above $\hat{\theta}(\tilde{r})$ and would rationally charge a low rate, inducing investment beyond $\hat{\theta}(\tilde{r})$, who, therefore, could not be the marginal type. Hence, irrespective of the intervention, the best investing type is still willing to be financed by the market.

Proposition 2 also implies that we can, without loss of generality, focus on programs where only types below $\hat{\theta}$ participate in the government program. To see why, imagine that a type $\theta' > \hat{\theta}(\tilde{r})$ participates. We know that this type does not invest and must get at least its outside option. But, then, the government can simply have this type drop out (by charging an infinitesimal fee, for instance). This does not affect the outside market rate because θ' does not invest. Therefore, we have the following corollary to Proposition 2.

COROLLARY 1: *Without loss of generality, only types below $\hat{\theta}(\tilde{r})$ participate in the government's program:* $\Theta_{\mathcal{P}} \subset \left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$.

Another important implication of Lemma 1 is that the government cannot design a program that attracts only good firms and induces them to invest. This suggests that all interventions that increase investment will be costly. This is, indeed, what we show next.

B. *Minimum Cost of Intervention*

In this section, we obtain a lower bound for the cost of interventions based on the minimal rents necessary to implement a given level of investment.

Corollary 1 argues that, when the competitive fringe is active, without loss, participating types, $\Theta_{\mathcal{P}}$, are a subset of $[\underline{\theta}, \hat{\theta}(\tilde{r})]$. Lemma 1 and the analysis of decentralized equilibria in Section II imply that the most attractive type to mimic for types in $\left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$ is the best type investing, $\hat{\theta}(\tilde{r})$. Moreover, Proposition 2 tells us that when the competitive fringe is active, the best type investing $\hat{\theta}(\tilde{r})$ invests without government help. Then, total rents paid to the firms with types in $\left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$ must be at least equal to the ones enjoyed from mimicking $\hat{\theta}(\tilde{r})$, which are equal to $\int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} \left(l_0 - \rho(\theta, \tilde{r} l_0)\right) dH(\theta)$. This last point says that if a type below $\hat{\theta}(\tilde{r})$ mimics it, the rents will be equal to the ones that that type would obtain by investing alone in the market; hence, participation constraints are tight. The tricky issue is that, possibly, only some types in $\left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$ participate, and they could be doing so randomly. But (9) tells us that the market breaks even for nonparticipating firms, thus all rents enjoyed by firms are paid by the government. Since rents are independent of the level of investment below type $\hat{\theta}(\tilde{r})$, an optimal program must induce investment for all types in $\left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$. The equilibrium level of investment, before and after the government's intervention, is depicted in Figure 2 above.

Based on these two observations, we obtain Theorem 1, which describes the properties of feasible and optimal programs:

THEOREM 1: *When the competitive fringe is active, feasible programs are characterized by an investment cutoff $\hat{\theta}$ and an associated market rate $\tilde{r}$. The cost of a feasible program cannot be less than the informational rents at rate $\tilde{r}$.*

$$\Psi^*(\tilde{r}) = \int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} \left(l_0 - \rho(\theta, \tilde{r} l_0)\right) dH(\theta). \tag{11}$$

A program reaches the lower bound $\Psi^*(\tilde{r})$ if and only if all types below $\hat{\theta}$ invest and $\gamma_\theta(\cdot)$ is identically zero for all types above the lowest type.

PROOF:

We have already argued that given an investment cutoff $\hat{\theta}$, the minimal cost is given by (11). From equation (10), we know that $E[\gamma_\theta(y) \mid \theta] \geq 0$ for all participating types. From Lemma 1, we further know that if $\gamma_\theta(y) \neq 0$ for some $y \in Y$ and some type $\theta \in \Theta_{\mathcal{P}}$, then $E[\gamma_\theta(y) \mid \theta'] > 0$ for all $\theta < \theta'$, implying a cost $\Psi > \Psi^*(\tilde{r})$.

For the cost to be $\Psi^*(\tilde{r})$, we must, therefore, have $\gamma_\theta(y) = 0$ for all $y \in Y$ and all $\theta \in \Theta_{\mathcal{P},1}/\{\underline{\theta}\}$.

Our analysis shows that cost minimization requires two things: making all types below $\hat{\theta}(\tilde{r})$ invest (this maximizes the pie to be split), and making all participation constraints bind (this minimizes the rents to the firms). The lower bound of costs cannot be less than the informational rents at rate $\tilde{r}$ and is strictly positive for all interventions that increase investment—i.e., for all interventions that implement $\hat{\theta}(\tilde{r}) > \theta^D$, since θ^D is the highest type for which the break-even constraint (6) holds. To go beyond θ^D, the government is forced to pay information rents. Theorem 1 tells us that the number and types of participating firms matters only through $\tilde{r}$. Hence, we have the following corollary:

COROLLARY 2: *For a given outside rate $\tilde{r}$, the minimum cost does not depend on the participation in the program.*

The intuition of this result is as follows: The market rate pins down the investment threshold (cf. Proposition 2), which, in turn, pins down the unavoidable rents that the government needs to pay. There are many participation regimes that would lead to the same rate. The crucial feature is that the relative proportion of nonparticipating and investing types is such that it leads to the same break-even interest rate for the market.

More formally, let $p(\theta) \in [0,1]$ be the probability of participation in the program for any type $\theta \in [\underline{\theta}, \hat{\theta}(\tilde{r})]$. Corollary 2 tells us that the actual participation rate of various types is irrelevant, so long as it leads to the same market rate. The equilibrium participation function p must be such that lenders break even on types opting out, when the rate is $\tilde{r}$—that is:

$$l_0 = \int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} \left[\int_Y \min(y, \tilde{r} l_0) f(y \mid \theta)\, dy \right] \frac{\big(1 - p(\theta)\big)\, dH(\theta)}{\int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} \big(1 - p(s)\big)\, dH(s)}. \tag{12}$$

There are many participation functions p that induce the same $\tilde{r}$, as Figure 3 illustrates.

The minimal-sized program, depicted in Figure 3A, attracts the worst types—that is, $p(\theta) = 1_{\theta < \theta^p}$ for all types below some cutoff θ^p, and zero otherwise. The cutoff θ^P must be such that (12) holds at $\tilde{r}$. At the other extreme, we can have $p(\theta) \to 1$ for all $\theta \in [\underline{\theta}, \hat{\theta}(\tilde{r})]$ and $\Theta_{\mathcal{O},1} \to \varnothing$.[16] Any size between the minimal size and the limit where all investing types participate delivers the same exact cost and level of investment. So, any size between $H(\theta^p)$ and $H\big(\hat{\theta}(\tilde{r})\big)$ can be an equilibrium.

One way to grasp this result is the following. Start from the minimal intervention with size $H(\theta^p)$. Consider the types left out to invest alone—that is, types between θ^p and $\hat{\theta}(\tilde{r})$. Now take a random sample of these types. By definition, the expected repayment for this random sample is exactly equal to the loan. This has two implications. First, removing the sample does not change the market rate $\tilde{r}$.

[16] For instance, start from $p(\theta) = 1_{\theta < \theta^p}$ and let p increase uniformly for all types in $[\underline{\theta}, \hat{\theta}(\tilde{r})]$.

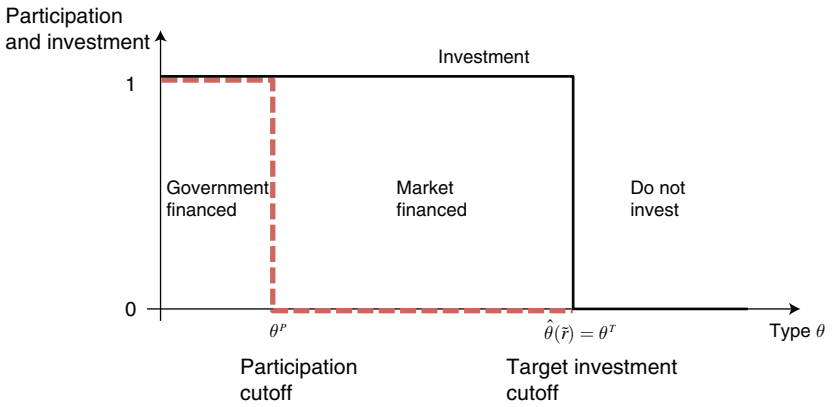

FIGURE 3A. MINIMAL PROGRAM

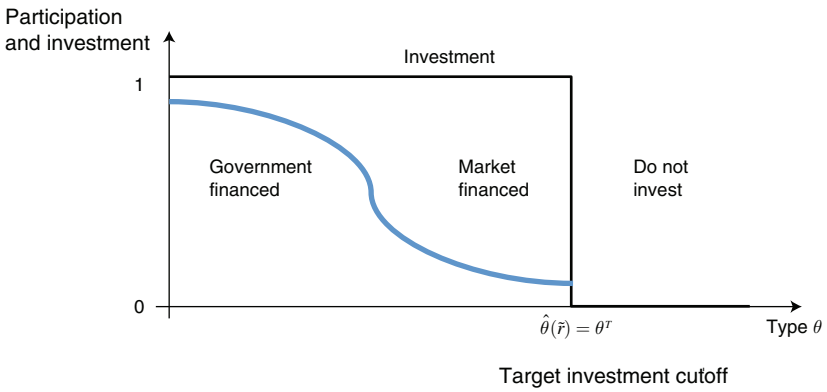

FIGURE 3B. LARGER PROGRAM

Second, adding the sample to the program does not change the expected cost for the government. Therefore, any random sample can be taken, and any size between $H(\theta^p)$ and $H\big(\hat{\theta}(\tilde{r})\big)$ can be obtained. Actually, the range of possible participation is even greater than is suggested by this explanation: One can have a program where some types below θ^p do not participate, and participation schedules can obviously be discontinuous and contain holes. All these programs would appear different, since participation would vary greatly, but their cost and investment level would be exactly the same.

The second important implication of Theorem 1 concerns the schedule of payments received by the government. The requirement that $\gamma_\theta(\cdot)$ be identically zero restricts the shape of the payoff functions that the government should offer, as explained in the following corollary.

COROLLARY 3: *Any feasible program that achieves investment* $[\underline{\theta}, \hat{\theta}(\tilde{r})]$ *at minimum cost* $\Psi^*(\tilde{r})$ *must be such that, for all* $y \in Y$ *and all* $\theta \in \Theta_{\mathcal{P},1}/\{\underline{\theta}\}$

$$y_\theta^g(y) = \min(y, \tilde{r} l_0) - \min(y, r_\theta l_\theta). \tag{13}$$

The intuition of this result is as follows: Theorem 1 tells us that reaching the lower bound on cost requires that, for all income realizations, the difference between repayments in and out of the program are the same. (This way, the government makes sure that participation constraints bind for all types at the same time.) This implies that the shape of repayments in the program must be the same as that of repayments outside the program. Since the market employs debt contracts, in order to match the shape of repayments, the government must employ debt-like instruments.

Theorem 1 tells us what the government can hope to achieve, but the derivation of the lower bound takes into account only the participation constraints of the firms, ignoring the investment and incentive constraints. In the next section, we show how to design interventions that reach the lower bound and establish that (13) is achieved by debt-like instruments.

V. Implementation

We now study feasible interventions that reach the lower bound $\Psi^*(\tilde{r})$. Corollary 3 shows that financial instruments that do not have the payoff structure of equation (13) cannot achieve the lowest cost. This suggests that the government should intervene with debt-like instruments. This is, indeed, what we show.

Recall from Section II that the best decentralized equilibrium is characterized by $\theta^D < \overline{\theta}$. We can, then, think of the implementation as follows. The government chooses a target for aggregate investment $\theta^T \in (\theta^D, \overline{\theta}]$.[17] Through equation (5), this is equivalent to choosing a target R^T for the market rate. For now, we take $\hat{\theta}$ and R^T as given, and we study the minimum cost of implementing $\hat{\theta} = \theta^T$. Implementation determines the structure of payoffs and implies an allocation of types to the sets $\Theta_{\mathcal{P},1}$ and $\Theta_{\mathcal{O},1}$.

As explained in Corollary 2, the actual participation rate of various types is irrelevant so long as it leads to the same market rate. Given a participation function consistent with $\tilde{r} = R^T$ in the competitive fringe, the government still has several choices regarding the size of its loans (lend l_0 or less) and the security design (direct lending versus debt guarantees).

For simplicity in the following discussion, we consider a participation function $p(\theta) = 1_{\theta<\theta^p}$, depicted in Figure 3a. Then, θ^p is uniquely pinned down by the break-even constraint $l_0 = \left(1/\left(H(\theta^T) - H(\theta^p)\right)\right) \int_{\theta^p}^{\theta^T} \rho(\theta, R^T l_0)\, dH(\theta)$. We start with the simplest program: direct lending.

PROPOSITION 3: *With an active competitive fringe, direct lending by the government of l_0 at rate R^T uniquely implements the desired investment at the minimum cost.*

PROOF:

In order to achieve the investment target θ^T, the interest rate R^T is such that $\hat{\theta}(R^T) = \theta^T$ in equation (5). Note that $1 < R^T < r^D$ since $\theta^T \in (\theta^D, \overline{\theta}]$. The program corresponds to $m_\theta = l_0$, $l_\theta = 0$, and $y_\theta^g(y) = \min(y, R^T l_0)$ for all types. The incentive and investment constraints are clearly satisfied for all participating firms. Firms in

[17] This is equivalent to a target $xH(\theta^T)$ for investment spending, or a target $(\overline{v} - x)H(\theta^T)$ for value added.

$\Theta_{\mathcal{O},0}$ do not want to participate without investing since $R^T > 1$. We cannot have $\tilde{r} < R^T$ since all firms would then drop out, and the market rate would be $\tilde{r} = r^D > R^T$. We cannot have $\tilde{r} > R^T$; otherwise, $\Theta_{\mathcal{O},1} = \varnothing$, which contradicts the fact that the competitive fringe is active. Hence, we must have $\tilde{r} = R^T$. Then, $\gamma_\theta(y) = 0$ and $\Psi = \Psi^*(R^T)$.

The direct-lending program implements the desired level of investment and achieves the minimum cost, which are the only outcomes the government cares about. The key points are that (i) at R^T, target investment θ^T is achieved; (ii) the incentive constraints are trivially satisfied since the government program offers one option for all types; and (iii) given that $\tilde{r} = R^T$ is the market rate, participation constraints hold with equality for all participating types. With other programs, such as equity injections, the participation constraint binds only for the best participating type, while all others receive rents.[18]

We now discuss equivalent implementations varying by the size of the loan and the security design:

Size of Loans: Consider a program where the government lends $m < l_0$ at a rate $R < R^T$. Participating types $[\underline{\theta}, \theta^p]$ must now borrow $l^u = l_0 - m$ on the market at a rate r that satisfies the zero-profit condition of the lenders:

$$l^u \;=\; l_0 \;-\; m \;=\; \frac{1}{H(\theta^p)} \int_{\underline{\theta}}^{\theta^p} \rho(\theta, r\, l^u)\, dH(\theta). \tag{14}$$

Given θ^p, equation (14) defines a schedule $r\,l^u$ strictly decreasing in m. Finally, the face value of the government loan Rm must satisfy the condition $\gamma = 0$—i.e., $Rm + rl^u(m) = R^T l_0$. We cannot have $R < 1$; otherwise, some firms would take the cash without investing. The maximal program (analyzed in Proposition 3) corresponds to $m = l_0$, $rl^u = 0$, and $R = R^T$. The minimal lending program $m^{\min}$ is defined as the unique solution to $rl^u(m) + m - R^T l_0 = 0$;[19] that is,

$$rl^u(m^{\min}) \;+\; m^{\min} \;-\; R^T l_0 \;=\; 0. \tag{15}$$

Any outcome that can be implemented by lending l_0 at rate R^T can also be implemented by a continuum of programs with $m \in (m^{\min}, l_0)$ and $R \in (1, R^T]$.[20]

[18] The only exception is when only the worst type participates—i.e., $\Theta_{\mathcal{P}} = \{\underline{\theta}\}$. With an atom-less distribution, such a program cannot increase investment, but if $\Pr(\theta = \underline{\theta}) > 0$, the government might choose a program where $\Theta_{\mathcal{P}} = \{\underline{\theta}\}$. Then, optimality requires that $E[\gamma_{\underline{\theta}}(y) \,|\, \underline{\theta}] = 0$, and this can be achieved in various ways. For instance, the government might offer cash m against a share α of equity returns. Opting into the program reveals the type to the lenders, and we must have $E[\min(y, r_{\underline{\theta}} l_{\underline{\theta}}) \,|\, \underline{\theta}] = l_0 - m$. The condition $E[\gamma_{\underline{\theta}}(y) \,|\, \underline{\theta}] = 0$ simply implies $(1-\alpha)m - \alpha E[y \,|\, \underline{\theta}] = \rho(\underline{\theta}, \tilde{r}\, l_0) - l_0$. The parameters (m, α) pin down the generosity of the program and, therefore, in equilibrium, the outside rate $\tilde{r}$. The more bad types that opt in, the lower is $\tilde{r}$, and the more costly the program becomes.

[19] The solution exists because the function is continuous, negative at $m = l_0$ since $rl^u(l_0) = 0$ and $R^T > 1$, and positive at $m = 0$ since $r\,l^u(l_0) \geq R^T l_0$. The solution is unique because $\frac{\partial rl^u}{\partial m} < -1$. To see why, notice that (14) implies $\frac{\partial rl^u}{\partial m} = -\frac{H(\theta^p)}{\int_{\underline{\theta}}^{\theta^p} (\partial \rho(\theta, rl^u)/\partial rl^u)\, H(\theta)}$, and (3) implies $\frac{\partial \rho(\theta, rl^u)}{\partial rl^u} = 1 - F(rl^u \,|\, \theta) < 1$.

[20] The only potential issue is unicity. The endogeneity of the borrowing rate r could lead to multiple equilibria for some distribution H. This problem can be ruled out if we allow coordination on the best feasible outcome, or if we impose enough concavity on H (log-concavity is often used in mechanism design for this purpose). Note,

Security Design: The government also has a choice of which debt instrument to use. In particular, direct-lending and debt-guarantee programs are equivalent. Instead of lending directly, the government can guarantee offers to guarantee new debt issuance up to an amount S (the size of the secured loan) for a fee ϕ paid upfront: $m = -\phi S$. Private lenders accept an interest rate of 1 on the guaranteed debt, so the budget constraint at time 1 becomes

$$x = c_0 + (1 - \phi)S + l^u,$$

where l^u is the unsecured loan. The government has to make payments in case of default—that is, whenever $a + v < S$.

The debt guarantee and direct lending programs are equivalent when $R = (1 - \phi)^{-1}$ and $m = (1 - \phi)S$. In practice, central firms use direct lending, while governments seem to favor debt guarantees. A reason might be that, while equivalent in market-value terms, the programs differ in accounting terms since debt guarantees are contingent liabilities and do not appear as increases in public debt.

Implementation without Competitive Fringe.—We now ask what happens if the government has the power to shut down the market. We obtain the following result:

PROPOSITION 4: *The cost of implementing $\theta^T > \theta^D$ remains the same, $\Psi^*(R^T)$, even if the government shuts down private markets. The minimum cost $\Psi^*(R^T)$ is achieved by direct government lending of l_0 at rate R^T.*

PROOF:

Notice, first, that it is still true (and, in fact, easier to show) that, regardless of the type of government program, only types below a threshold invest. The difference between participation and nonparticipation payoffs conditional on investing in the program is

$$\overline{v} - c_0 - \int_Y [\min(y, r_\theta l_\theta) + y_\theta^g(y)] f(y \mid \theta)\, dy.$$

The last term is increasing in θ. This immediately implies that the investment and participation set is $[\underline{\theta}, \theta^T]$ and that θ^T is the most attractive type to mimic. The type θ^T is indifferent between investing and not investing, so its payoff is $E[a \mid \theta^T] + c_0$. Let $y_{\theta^T}^g(y)$ denote the repayment function of type θ^T, and redefine $\gamma_\theta(y) \equiv y_{\theta^T}^g(y) - y_\theta^g(y)$. Incentive compatibility implies that $\gamma_\theta^g(y) \geq 0$ for all θ. Then, for the reasons explained in the proof of Theorem 1, a lower bound on cost is achieved when $\gamma_\theta^g(y) = 0$ for all y and all $\theta > \underline{\theta}$. This can be achieved by direct lending of l_0 at rate R^T.

This result is not obvious for two reasons: monopoly power and stigma. If the government is the sole lender, it has monopoly power and can set any lending rate

however, that this multiplicity does not arise from the intervention itself, since it is also present without intervention, as discussed in Section II. It is different from the multiplicity created by menus, which occurs for any distribution, as discussed below.

(or choose any mechanism) without the fear that firms will drop out and borrow from the market. However, given that the equilibrium feasible investment and participation set is still $[\underline{\theta}, \theta^T]$, the government cannot avoid rewarding rents to all types $[\underline{\theta}, \theta^T]$ who would otherwise mimic θ^T. At an optimal intervention, regardless of whether there is a private market, this type is indifferent between investing and not investing, so it always earns $E[a \mid \theta^T] + c_0$. Hence, the rents that the government must pay to induce investment $[\underline{\theta}, \theta^T]$ are independent from the existence of a market. We conclude that the competitive fringe does not impose extra costs when the government uses an optimal implementation.[21]

We summarize our implementation results in the following Theorem.

THEOREM 2: *Direct-lending and debt-guarantee programs uniquely implement any investment set of the form* $[\underline{\theta}, \hat{\theta}(\tilde{r})]$ *at minimum cost* $\Psi^*(\tilde{r})$. *The cost of implementation does not change even if the government has the power to shut down private markets.*

We conclude this section with a brief discussion of menus, arguing that they are less robust than simple programs. Notice that the payoff structure of equation (13) applies to all interventions. Even with menus, the government must use debt-like instruments. Proposition 3 describes an optimal implementation using one debt contract. In the Appendix, we describe interventions with menus of debt contracts. While (nontrivial) menus can implement an optimal outcome, they can never do so uniquely. Alongside the equilibrium where each type chooses the correct contract, there is always an equilibrium where all the types pool on the contract designed for the worst type. Any nontrivial menu can, thus, overshoot its target and end up costing more than intended. This cannot happen with simple programs since all participating types *already* pool on the unique contract offered by the government. In this sense, simple programs are more robust than programs with menus.

VI. Extensions

In this section, we provide three important extensions to our main results. The first extension is to consider asymmetric information with respect to new investment opportunities. The second is to analyze asset purchases. The third is to consider the consequences of moral hazard in addition to adverse selection.

A. *Asymmetric Information about New Loans*

We have assumed so far that the distribution of v is independent of the firm's type. Let us now relax this assumption, while maintaining assumptions A2 and A3. We replace A1 by

ASSUMPTION A1′: $E[v \mid \underline{\theta}] > x$ *and* $E[v \mid \theta]$ *is increasing in* θ.

[21] The case of inactive fringe also arises when the market rate for nonparticipating firms is too high for investment outside the program. Our results apply to the case of an inactive fringe regardless of what causes the inactivity.

Our results hold under A1′. Firms continue to offer debt contracts, and expected repayments are still given by $\rho(\theta, rl)$. The investment condition, however, becomes $E[v \mid \theta] - x > \rho(\theta, rl) - l$ for type θ. The significant difference is that it might potentially hold for good firms and not for bad firms. This does not, however, change the nature of the decentralized equilibrium: There is still some threshold below which types invest. Proposition 1 still holds, and the results in Sections III to V are unaffected.[22]

B. *Asset Purchases*

Our benchmark model follows Myers and Majluf (1984) in assuming that cash flows are fungible. Assumption A1 rules out contracts written directly on a. We can dispense with this assumption in two ways: by assuming asymmetric information with respect to v, as explained above,[23] or by allowing asset purchases, but not spin-offs. In other words, we can allow contracts that are increasing in *both* a and y.[24] One such contract is the purchase of Z units of face value of the legacy assets. If p is the purchase prize, the net payoffs are $aZ/A - pZ$. All our results hold in this setup. We can show that firms continue to offer debt contracts and that optimal interventions still use debt-guarantee or direct-lending programs. Moreover, we can show that, among suboptimal interventions, asset purchases do strictly worse than equity injections.

AQ2

C. *Moral Hazard*

Our benchmark model takes investment opportunities as exogenous. In practice, however, firms can partially control the riskiness of their new loans. To understand how endogenous risk-taking affects our results, we introduce a new project with random payoff v'. This project also costs x but is riskier (in the sense of second-order stochastic dominance) and has a lower expected value: $E[v'] < E[v]$. To emphasize moral hazard created by government interventions, we assume that market participants can detect the choice of v', but this choice is not contractible by the government.[25]

ASSUMPTION A4: *The choice of project v' is observed by private lenders but cannot be controlled by the government.*

[22] Note that A1′ implies that all types still have positive NPV projects. If instead $E[v \mid \underline{\theta}] < x$, the complication is that Proposition 1 need not hold. Essentially, adverse selection worsens and total market breakdowns are possible (equilibria where decentralized investment is zero). Interventions can become, at the same time, more desirable and more expensive (since the government finances some negative NPV projects). As long as A2 holds, however, debt contracts are still optimal, and, based on the discussion in Section V, we conjecture that optimal interventions still involve simple debt-guarantee programs or direct lending.

[23] A particularly simple case to analyze is when investment simply scales up existing operations. We can capture this with $v = \alpha a$ for some constant α. In this case, we can allow contracts to be written on either a or v.

[24] This rules out a contract on $v = y - a$, which is effectively a spin-off. Details can be found in an earlier version of the paper and are available upon request.

[25] This is so either because the government has an inferior detection technology and does not observe v', or because this choice cannot be verified as in the incomplete contract literature. The point of A4 is to study inefficiencies *created* by government interventions. If v' is not observable by private investors, risk-shifting occurs with or without government intervention.

Under A4, we can derive three important results (the proofs are in the Appendix). First, there is no risk-shifting without government intervention, and Proposition 1 is unchanged. The intuition is that choosing v' is doubly costly. It increases the borrowing rate because of greater objective risk (a direct effect), but it also sends a negative signal about the firm's type. This indirect effect occurs because good types dislike high rates relatively more than bad types.[26]

The second result is that government interventions are more likely to induce moral hazard when government loans are larger. Risk-taking might occur because the government lends at a rate that is independent of the project chosen. The risk of moral hazard is maximized by the program (l_0, R^T) and minimized by the program $(m^{\min}, 1)$ described in Section V.

The third point is that, even if moral hazard occurs with minimal lending, the government can prevent risk shifting by making its lending rate contingent on the rate charged by private lenders. The government lends $m^{\min}$, defined by equation (15) at rate $R = 1$ as long as the private lenders' rate r does not exceed the break-even rate consistent with no risk shifting, which we define as r^*. Otherwise, the government charges the same rate as the private lenders. Any risk-shifting deviation would bring the firm back to the case without intervention, where we have already seen that risk shifting does not take place. Hence, deviations never occur and risk-shifting is eliminated.

Formally, define r^* as the solution to

$$l_0 - m^{\min} = E[\rho(\theta, m^{\min} + r^* l^u) \mid \theta \in [\underline{\theta}, \theta^p]. \tag{16}$$

We then have the following proposition:

PROPOSITION 5: *Suppose that the government lends $m^{\min}$ at rate $R = 1$ as long as $r \leq r^*$ and threatens to charge the private rate if $r > r^*$. Then, there is no risk shifting.*

We conclude that moral hazard has important implications for the design of interventions. It breaks the equivalence results of Section V between programs with large and small loans. To prevent moral hazard, it is necessary to observe a private lending rate, so the government should not be the sole lender. With small-loan interventions, the government can prevent risk shifting by observing private rates even if the government itself has no direct monitoring technology. In practice, it is important to choose a private rate that the borrower cannot manipulate (i.e., the rate on a large corporate bond issuance). The same results apply to debt guarantees: the guarantee should not cover the entire loan.

[26] In fact, it is easy to see that good types would be willing sacrifice NPV in exchange for *safer* projects because such antirisk shifting would function as a costly signaling device. Good banks would become too conservative in their lending policies during a crisis in order to signal their types. We note that this would make debt guarantees more appealing, since guarantees would lean against the conservatism bias.

VII. Conclusion

Since we have already summarized our findings in the introduction, we conclude with a short discussion of how our theoretical results relate to actual interventions—in particular, those used during the recent financial crisis. We first note that the presence of a stigma attached to government programs is an important issue. Policymakers understand that the acceptance of government assistance could be an admission of financial weakness and that this can result in insufficient uptake. In the fall of 2008, the US Treasury was concerned about the market reaction if the best banks decided to opt out of its recapitalization program. The second important dimension is the choice of financial instruments. In our model, cost minimization requires debt instruments (either direct lending or debt guarantees). In practice, debt guarantees and direct lending are widely used, but equity injections are also prevalent, and asset buybacks are sometimes used, as well. While a detailed discussion is clearly beyond the scope of this paper, we conjecture the following general result: The optimal choice of security is directly related to the nature of the financial friction and the associated market failure. We illustrate this conjecture with two recent papers. Philippon and Schnabl (2009) study optimal bailouts when investment is limited by debt overhang. In that context, the key friction is lack of capital, and equity injections dominate debt guarantees. Tirole (2010) studies a model where the pledgeable income of new projects is limited by moral hazard, and legacy assets need to be sold to raise capital. In that context, the optimal intervention uses assets purchases. Our conjecture is therefore that the optimal instrument for a particular intervention should resemble the contracts used by private-market participants. Dealing with debt overhang requires equity; unfreezing the debt market requires debt guarantees; and jump starting the securitization market requires asset purchases. A general proof of this conjecture is a project for future research.

Appendix

PROOF OF LEMMA 1:

The proof focuses on the shape of the function $\gamma_\theta(y) \equiv \min(y, \tilde{r} l_0) - \min(y, r_\theta l_\theta) - y^g_\theta(y)$. We first show that γ_θ is weakly positive for low values of y, and then decreasing. Recall that y^g_θ is increasing in y, and the resource constraint imposes $y^g \leq y - \min(y, r_\theta l_\theta)$. For $y \leq \min(\tilde{r} l_0, r_\theta l_\theta)$, we have $y = \min(y, r_\theta l_\theta)$ and, therefore, $\gamma_\theta = -y^g \geq 0$. If $\tilde{r} l_0 < r_\theta l_\theta$, then γ is decreasing for all $y > \tilde{r} l_0$. If $\tilde{r} l_0 > r_\theta l_\theta$, the relevant case in later analysis, then for $y \in [r_\theta l_\theta, \tilde{r} l_0]$, we have $\gamma_\theta = y - r_\theta l_\theta - y^g_\theta(y)$. Since $y^g \leq y - r_\theta l_\theta$, this means $\gamma_\theta \geq 0$. For $y > \tilde{r} l_0$, we have $\gamma_\theta(y) = \tilde{r} l_0 - r_\theta l_\theta - y^g_\theta(y)$ decreasing in y since y^g_θ is increasing. We conclude that, in all cases, γ_θ is weakly positive for low values of y, and then decreasing. There are two possibilities: either $\gamma_\theta(y) \geq 0$ for all y, or there exists a $\hat{y}$ such that $\gamma_\theta(y) < 0$ for all $y > \hat{y}$. If $\gamma_\theta(y) \geq 0$, then the lemma (with both weak and strong inequalities) follows directly from the first part of A2. In the second case, $E[\gamma_\theta(y) \mid \theta] \geq 0$ implies that

$$\int_0^{\hat{y}} \gamma_\theta(y) f(y \mid \theta)\, dy \;\geq\; \int_{\hat{y}}^{\infty} -\gamma_\theta(y) f(y \mid \theta)\, dy. \tag{A1}$$

Since $\gamma_\theta(y) < 0$ for all $y > \hat{y}$, both sides are strictly positive. Consider $\theta' < \theta$.

For $y > \hat{y}$, we know that γ_θ is negative and decreasing, implying that $-\gamma_\theta$ is positive and increasing. The monotone hazard rate property implies conditional expectation dominance (see Nachman and Noe 1994) that gives us $\int_{\hat{y}}^{\infty} -\gamma_\theta(y) \frac{f(y|\theta)}{1-F(\hat{y}|\theta)}\, dy > \int_{\hat{y}}^{\infty} -\gamma_\theta(y) \frac{f(y|\theta')}{1-F(\hat{y}|\theta')}\, dy$ or

$$\text{(A2)} \qquad \int_{\hat{y}}^{\infty} -\gamma_\theta(y) f(y|\theta)\, dy \;>\; \frac{1 \;-\; F(\hat{y}|\theta)}{1 \;-\; F(\hat{y}|\theta')} \int_{\hat{y}}^{\infty} -\gamma_\theta(y) f(y|\theta')\, dy.$$

The monotone hazard rate property implies that $\frac{1-F(y|\theta)}{1-F(y|\theta')}$ is increasing in y, which, together with the definition of MHR, implies that $f(y|\theta) \leq f(y|\theta') \frac{1-F(\hat{y}|\theta)}{1-F(\hat{y}|\theta')}$. Then, for all $y < \hat{y}$, we have

$$\text{(A3)} \qquad \int_{0}^{\hat{y}} \gamma_\theta(y) f(y|\theta)\, dy \;<\; \frac{1 \;-\; F(\hat{y}|\theta)}{1 \;-\; F(\hat{y}|\theta')} \int_{0}^{\hat{y}} \gamma_\theta(y) f(y|\theta')\, dy.$$

Combining (A2) with (A3), we finally obtain

$$\frac{1 \;-\; F(\hat{y}|\theta)}{1 \;-\; F(\hat{y}|\theta')} \int_{0}^{\hat{y}} \gamma_\theta(y) f(y|\theta')\, dy \;>\; \frac{1 \;-\; F(\hat{y}|\theta)}{1 \;-\; F(\hat{y}|\theta')} \int_{\hat{y}}^{\infty} -\gamma_\theta(y) f(y|\theta')\, dy,$$

and, therefore, $E[\gamma_\theta(y)\,|\,\theta'] = \int_0^{\infty} \gamma_\theta(y) f(y|\theta')\, dy > 0$.

Menus

Suppose that the government offers a menu of contracts (m_θ, R_θ) where m_θ is the loan and R_θ is the interest rate. Type θ then borrows $l_\theta = l_0 - m_\theta$ from the market, and since the choice of the contract reveals the type, the zero profit condition type by type implies

$$\text{(B1)} \qquad \rho(\theta, r_\theta l_\theta) \;=\; l_0 \;-\; m_\theta.$$

In order for the menu to be optimal, we need $\gamma_\theta(y) = 0$, or, equivalently,

$$\text{(B2)} \qquad R_\theta m_\theta \;+\; r_\theta l_\theta \;=\; R^T l_0.$$

The menu is feasible if and only if (B1) and (B2) are satisfied and $R_\theta \geq 1$ for all θ. There are obviously several ways to design a menu. Since $\frac{\partial \rho(\theta, r_\theta l_\theta)}{\partial r_\theta l_\theta} = 1 - F(r_\theta l_\theta|\theta)$, the menu must solve the differential system

$$\frac{\partial \rho}{\partial \theta}\, d\theta \;+\; \big(1 \;-\; F(r_\theta l_\theta|\theta)\big)\, d(r_\theta l_\theta) \;+\; dm_\theta \;=\; 0,$$

$$d(R_\theta m_\theta) \;+\; d(r_\theta l_\theta) \;=\; 0.$$

For concreteness, we study $\Theta_{\mathcal{P}} = [\underline{\theta}, \theta^p]$ and $R_\theta = 1$ for all types in $\Theta_{\mathcal{P}}$. Then, the schedule is pinned down by the differential equation

$$F(r_\theta l_\theta | \theta)\, dm_\theta \;=\; -\frac{\partial \rho}{\partial \theta}\, d\theta \text{ for all } \theta \in \Theta_{\mathcal{P}},$$

and the initial condition $\rho(\theta^p, R^T l_0 - m_{\theta^p}) = l_0 - m_{\theta^p}$. Government loans decrease with θ and compensate the types for revealing their private information. This menu is clearly feasible and reaches the lower bound for cost $\Psi^*(R^T)$.

Menus, however, are susceptible to multiple equilibria. To see why, imagine that all types in $\Theta_{\mathcal{P}}$ pool on the contract designed for the worst type $\underline{\theta}$. Let $\overline{r}$ be the corresponding break-even rate. Clearly, we must have $\overline{r} < r_{\underline{\theta}}$. Therefore, $R_{\underline{\theta}} m_{\underline{\theta}} + \overline{r}\, l_{\underline{\theta}} < R_{\underline{\theta}} m_{\underline{\theta}} + r_{\underline{\theta}} l_{\underline{\theta}} = R^T l_0$. This deviation is profitable for all types and is incompatible with θ^T remaining the marginal type. The equilibrium will then be one of a unique contract of lending $m_{\underline{\theta}}$ at rate $R_{\underline{\theta}}$, but with a strictly higher marginal type $\hat{\theta} > \theta^T$ and a strictly higher cost.

Moral Hazard

We prove three claims: (i) There is no risk shifting without interventions; (ii) smaller government loans create less moral hazard; and (iii) there exists a simple contingent program that removes moral hazard.

For (i), consider an equilibrium with risk shifting. Let r be the borrowing rate for v and $r' > r$ for v'. Type θ chooses to risk shift if and only if

$$\overline{v} \;-\; \rho(\theta, r l_0) \;>\; \overline{v}' \;-\; \rho'(\theta, r' l_0).$$

It is clear that if a particular type θ wants to risk shift, a worse type would also want to risk shift.[27] Hence, the set of risk-shifting types is $\Theta' = [\underline{\theta}, \theta']$ for some θ'. We first show that $\Theta' = \varnothing$ when there is no intervention.

LEMMA 2: *The decentralized equilibrium without intervention is unaffected by the availability of project v'.*

PROOF:

Consider the highest type θ' that chooses v'. For type θ', we have $\rho(\theta', r' l_0) > l_0$ since θ' pools with lower types. This type can strictly benefit by choosing v because v is safer and because it would pool with better types.

For (ii), suppose that the government lends l_0 at R^T. Then, the condition for risk-shifting to occur is

$$\text{(C1)} \qquad \rho(\theta, R^T l_0) \;-\; \rho'(\theta, R^T l_0) \;>\; \overline{v} \;-\; \overline{v}'.$$

[27] This always holds when banks can offer menus of borrowing and projects and the inscrutability principle holds, as in Myerson (1983). If there is signaling at the proposal stage, we need a refinement to rule out unreasonable equilibria where risk taking happens to be a good signal.

The firm faces no penalty for risk-shifting, apart from the NPV loss. If condition (C1) holds for $\underline{\theta}$, then some risk-shifting does occur in the program with large loans. When $m < l_0$, on the other hand, we need to look for a cutoff θ' such that

$$l_0 - m = E[\rho'(\theta, Rm + r'l^u) \,|\, \theta \in [\underline{\theta}, \theta']],$$

and

$$l_0 - m = E[\rho(\theta, Rm + rl^u) \,|\, \theta \in [\underline{\theta}, \theta^p]].$$

The condition for risk-shifting is the indifference of the marginal type: $\rho(\theta', Rm + rl^u) - \rho'(\theta, Rm + r'l^u) = \overline{v} - \overline{v}'$. This is clearly stronger than condition (C1) because of the risk-sensitive rate and the adverse signaling effect. The government needs to maximize the amount borrowed on the market to minimize risk-shifting incentives. Within the class of interventions studied in the paper, it does so by lending $m = m^{\min}$ defined in (15).

For (iii), consider the implementation with lending $m^{\min}$ at rate $R = 1$. Define r^* as the solution to

$$l_0 - m^{\min} = E[\rho(\theta, m^{\min} + r^* l^u) \,|\, \theta \in [\underline{\theta}, \theta^p]].$$

Suppose that the government offers to lend $m^{\min}$ at rate $R = 1$ as long as $r \leq r^*$, and at a rate equal to the private rate for this firm ($R = r$) otherwise. If a firm risk-shifts, its interest rate will strictly exceed r^* and it will be punished by losing the government subsidy. From the above discussion, it is clear that no firm will ever risk-shift.

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