

Article **Reliability Estimation of the Compressive Concrete Strength Based on Non-Destructive Tests**

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Abstract: The uncertainty in the concrete compressive strength is one of the most challenging issues in safety checking of existing reinforced concrete (RC) buildings. The concrete compressive strength used in the assessment can highly influence the vulnerability results and thus the retrofit strategies. The need to use less expensive and less invasive in situ measurements such as the nondestructive tests should be balanced with a careful check of their structural reliability. The compressive concrete strength is characterized herein based on a large database of both in situ destructive and non-destructive results measured on the same structural members. The data are obtained from existing RC buildings mainly located in the Campania region, Southern Italy. Probabilistic linear and multilinear regression models are developed for calculating the compressive concrete strength based on non-destructive tests. Furthermore, the implementation of the concrete strength based on ultrasonic test results are investigated together with the relative measurement error through a fully probabilistic workflow. Accordingly, the relative weights of non-destructive data for calculating concrete compressive strength are estimated and compared with those recommended by the Italian national code. The results demonstrate that the effective weights of the non-destructive data are very close to the code-based recommendation.

Keywords: concrete compressive strength; existing RC structures and infrastructures; knowledge levels; destructive tests; non-destructive tests; Bayesian updating

1. Introduction

The process of the knowledge regarding the definition of structural system and the estimation of mechanical material properties are crucial steps towards the seismic evaluation of existing structures, especially in high-seismicity areas. There are different tests on structural members that can be part of this process of the knowledge. The main objective of this test campaign is to characterize the mechanical properties of materials and the potential defects in construction details. Estimation of the compressive strength of members plays a fundamental role in the seismic performance of existing RC structures/infrastructures, which is evaluated based on destructive and non-destructive tests. The most common destructive test is the core sampling, while one of the most widely used non-destructive tests is the SONREB method (performed by the combination of the results of rebound number *S* and ultrasonic pulse velocity V , $[1-3]$ $[1-3]$). The main advantage of the non-destructive test is the fact that there is no need to remove a structural sample of concrete. UNI EN 12504-2 [\[1\]](#page-16-0); UNI EN 12504-4 [\[2\]](#page-16-2) provide important instructions on implementation of these types of tests. Instead, the core sampling needs the structural sample extraction that is generally in a form of a cylinder. Then, the sample is tested in a laboratory to assess the concrete compressive strength under specific techniques ([\[3\]](#page-16-1); see also [\[4\]](#page-16-3) on how to perform this type of test). While the core destructive test is generally more reliable, it is indeed more difficult to be executed and more expensive compared to the non-destructive tests. In fact,

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in some cases, it is practically not feasible to collect enough cores according to the code provisions (NTC 2018, [\[5\]](#page-16-4)), as the normal activity cannot be interrupted; hence, the role of the non-destructive tests becomes more fundamental.

The mechanical properties of the structural concrete have been deeply studied in the past. Various methods have been presented for investigating and processing of the related results based on experimental tests. Important studies were conducted in 1980s in Europe (e.g., [\[6\]](#page-16-5)) and later in the United States (e.g., [\[7,](#page-16-6)[8\]](#page-16-7)). However, the results of these research efforts are not directly applicable to the Italian buildings stock, given the differences between the concrete constructions where these studies have been conducted. With specific reference to Italy, there is vast literature on the topic starting from 1980s (see, e.g., [\[9–](#page-16-8)[12\]](#page-16-9)). Using the ultrasonic velocity *V* and the rebound number *S*, the concrete strength is obtained with nonlinear regressions (e.g., in the RILEM standard [\[13\]](#page-16-10); see also [\[9,](#page-16-8)[10,](#page-16-11)[14\]](#page-17-0)).

More recently, other researchers have investigated the assessment of concrete strength based on probabilistic approaches. One of the most critical issues, largely discussed in the scientific community (see, e.g., $[15-18]$ $[15-18]$), is to assess correctly the uncertainty that exists in evaluating in situ test results. In fact, the reliability of the in situ concrete strength tests can depend on many factors such as the quality of the measurement, the amount of data used for the regression model, and the range of concrete strength values considered in the regression. Monti and Alessandri [\[19\]](#page-17-3) proposed a method for evaluation of material strength and calibration of the relevant confidence factors based on a Bayesian procedure. They calibrated a correlation equation to evaluate the confidence factors as a function of the number, type, and the reliability of the available in situ tests being employed and of the reliability of the available prior information. In the work of Giannini et al. [\[20\]](#page-17-4), the assessment of concrete strength was implemented combining destructive and nondestructive tests measures via Bayesian inference. Moreover, the influence of the number of coring points on the reliability of the models was studied in Trtnik et al. [\[21\]](#page-17-5) and later in Giannini et al. [\[20\]](#page-17-4). Pereira and Romao [\[22\]](#page-17-6) worked on the assessment of the concrete strength in existing buildings using as investigation technique called finite population approach to evaluate the uncertainty in the estimate of the variability of the concrete strength in a population as well as the uncertainty in the estimate of the mean value of the concrete strength. Vasanelli et al. [\[23\]](#page-17-7) estimated the in situ concrete strength considering the correlations between destructive and non-destructive tests.

This paper proposes a fully probabilistic workflow to estimate the concrete compressive strength based on both in situ destructive and non-destructive test results and to investigate the relative measurement error within a fully probabilistic framework. Moreover, it attempts to propose a methodology for calculating the relative weight of non-destructive measurements with respect to the destructive tests. This is the compilation of the workflow that was proposed in [\[24\]](#page-17-8) by the authors. The results can be directly employed to estimate the compressive strength of concrete for design/assessment purposes. To this end, a large database containing 221 pairs of core test and SONREB non-destructive test results that refer to the same structural members in various RC structures is provided herein. The buildings are mainly located in the Campania region, Southern Italy, constructed between 1930 and 1990. Hence, the proposed workflow based on the Bayesian inference and its outcomes have practical implications in (probabilistic) seismic performance assessment of existing RC construction; see, e.g., [\[25–](#page-17-9)[30\]](#page-17-10). Regression models are proposed to obtain predictive expressions for estimating the concrete compressive strength *R^c* based on the ultrasonic velocity *V* and on the rebound number *S*. In particular, expressions for logarithmic linear (ln*V*-ln*Rc*) and multilinear (ln*V*-ln*S*-ln*Rc*) regressions are derived to obtain *Rc* based on the available database. It is further explored that the regression model (ln*V*-ln*Rc*) can be used instead of the multilinear model $(lnV\text{-}lnS\text{-}lnR_c)$ without significant loss of accuracy. This is also confirmed by the significance test of the rebound index *S* in the regression. In addition, the relative weights of non-destructive tests for calculating concrete compressive strength, derived based on the proposed probabilistic workflow, are compared with the relative

weights recommended in the NTC 2018 Commentary [\[31\]](#page-17-11), which assigns 1 to destructive mental 1/3 to non-destructive tests. The results demonstrate that the non-destructive tests and 1/3 to non-destructive tests. and 1/9 to non-destructive tests. The results achievestate that the non-destructive tests effective weights are close to code provision. This result is important because it demonstrates that the non-destructive tests, implemented in terms of the ultrasonic velocity *V*, can be a reliable instrumentation to complement the destructive test (particularly where it can be a reliable instrumentation to complement the destructive test (particularly where it is not easy to execute them). ment the destructive test international to some promised the destructive test (particularly methods).

2. Methodology 2. Methodology

This section presents the methodology used in this paper (see also the flowchart in This section presents the methodology used in this paper (see also the flowchart in Figure [1\)](#page-2-0). Section [2.1](#page-2-1) proposes the first step of the procedure that consists of converting *f*_{core} into its equivalent in situ cylindrical f_c and cube R_c values. Section [2.2](#page-3-0) employs the data in order to obtain the probabilistic logarithmic regression models ($ln V$ - $ln R_c$) and ($ln V$ - $ln S$ ln*Rc*). Finally, Section 2.3 introduces the Bayesian workflow for quantifying the relative ln*Rc*). Finally, Section [2.3](#page-4-0) introduces the Bayesian workflow for quantifying the relative error in non-destructive (ultrasonic) test measurements, denoted as *Rultr*, and calculating error in non-destructive (ultrasonic) test measurements, denoted as *Rultr*, and calculating their relative weights with respect to destructive (core) test results *Rc*. The probabilistic their relative weights with respect to destructive (core) test results *Rc*. The probabilistic workflow consists of the following steps: (i) calculation on the conditional probability workflow consists of the following steps: (i) calculation on the conditional probability model for predicting R_{ultr} given the core strength R_c ; (ii) characterization of the uncertainty in the concrete strength considering both destructive and non-destructive test results; (iii) estimation of the relative weights associated with each ultrasonic non-destructive test data. data.

Figure 1. Flowchart of the proposed procedure in this manuscript. **Figure 1.** Flowchart of the proposed procedure in this manuscript.

2.1. Conversion of f_{core} into the Equivalent In Situ Cylindrical f_c and Cube R_c Values

nevertheless, the strength measured on a core specimen is different from the actual in nevertheless, the strength measured on a core test are given in NTC 2018 [\[5\]](#page-16-4) and in its situ strength. The specifications to use core test are given in NTC 2018 [5] and in its strength. The specifications to use core test are given in NTC 2018 [5] and in its Commen-Commentary [\[31\]](#page-17-11). The code refers to the Italian council for public works recommendations tary [31]. The code refers to the Italian council for public works recommendations (CSLP (CSLP guidelines, [\[32\]](#page-17-12)) and to the UNI EN 13791 [\[33\]](#page-17-13). In particular, the CSLP guidelines [\[32\]](#page-17-12) indicate that the estimate of the resistance of the cast-in-place concrete, based on the core specimen extraction, should be modified by implementing corrective coefficients. Different formulations from the literature proposed factors to quantify these corrections. Core test is the most reliable method to evaluate the concrete compressive strength;

In this work, a relationship proposed in $[12,34]$ $[12,34]$ to convert the concrete strength from the core specimen, denoted as f_{core} , into the equivalent cylindrical strength f_c is used. The main factors considered in this relationship to estimate the real concrete strength are the size and geometry of the cores, presence of reinforcing bars, and the effect of drilling damage: size and geometry of the cores, presence of α

$$
f_c = (C_{H/D} \cdot C_{dia} \cdot C_a \cdot C_d) \cdot f_{core},
$$
\n(1)

where

• *C_{H/D}* is the correction for height/diameter ratio H/D of the specimen; $C_{H/D} = 2/(1.50$ $+ D/H$).

- C_{dia} is the correction for the diameter of the core; C_{dia} = [1.06, 1.00, 0.98] for *D* = [50, 100, 150] mm, respectively. Linear interpolation can be used for the diameters included between the indicated intervals; thus, the general linear equation can be $C_{dia} = 1 +$ 0.0012 × (100 − *D*) when *D* ≤ 100 mm and $C_{dia} = 1 - 0.0004 \times (D - 100)$ when *D* > 100 mm.
- C_a is the correction for the presence of reinforcing bars; $C_a = 1.0$ for the absence of bars, and varying between 1.03 for small-diameter bars (φ10) and 1.13 for large-diameter bars (φ20).
- *C_d* is the correction for damage due to drilling; $C_d = 1.20$ for $f_{core} < 20$ MPa, and 1.10 for $f_{core} \geq 20$ MPa.

Regarding the correction coefficient C_d , it is important to note that the lower the original concrete quality, the larger the drilling damage (see [\[12\]](#page-16-9)). Alternatively, it is important to note that there is another formulation for the assessment of in situ concrete strength, proposed from Reluis-DPC Guidelines [\[35\]](#page-17-15). The functional form to convert *fcore* to *f^c* is very similar to Equation (1), including (a) a geometry correction factor based on *H*/*D* with different expressions (depending on the humidity condition for the core specimen) but similar values compared to $C_{H/D}$; (b) the diameter correction factor with the same expression as that of *Cdia*; (3) the correction factor for damages due to drilling (equal to the constant value 1.06); (d) the factor taking into account the humidity condition for the core specimen (which was not considered in Equation (1)). There is no coefficient in Reluis-DPC Guidelines [\[35\]](#page-17-15) related to the presence of reinforcing bars (i.e., similar to *Ca* in Equation (1)). In this study, we employ Equation (1) instead of the Reluis-DPC expression due to the lack of data about the conservation of the specimen after the extraction (related to the inclusion of the humidity corrections used in (a) and (d) of the Reluis-DPC expression denoted above).

Once the cylinder strength *fc* is evaluated, a factor of 0.83 is suggested in NTC 2018 [\[5\]](#page-16-4) to convert f_c to the cube strength R_c for normal concrete strength as follows:

$$
R_c = f_c / 0.83. \tag{2}
$$

It should be noted that the above conversion is for standard samples with $H/D = 2$. This conversion is useful because many literature regression predictions that correlate *V* and *S* to the concrete strength use the cube concrete strength. Many research endeavors in Italy re dedicated to *fc*-*R^c* mapping (see, e.g., [\[10](#page-16-11)[,36](#page-17-16)[,37\]](#page-17-17)). The conversion presented in Equation (2) is also proposed by Masi [\[11\]](#page-16-12) and ACI 214.4-R03 [\[38\]](#page-17-18). Given the linear relation in Equation (2), the standard deviation and coefficient of determination (see Section [2.2\)](#page-3-0) of the logarithmic linear regression are the same for *f^c* and *Rc*.

2.2. Probabilistic Logarithmic Linear and Multilinear Regressions

Linear least squares fitting is a procedure for finding the best fitting line to a given dataset by minimizing the sum of the squares of the residuals of the data from the line. The regression can also be regarded as a probabilistic model to define the conditional probability distribution of dependent variable with respect to independent variable. This probabilistic model has many applications in the structural engineering literature (see, e.g., [\[39](#page-17-19)[,40\]](#page-18-0)). Herein, this probabilistic model is implemented to obtain the lognormal probability distribution of $\ln R_c$ | *V*. This is equivalent to fit a power-law curve to the R_c -*V* response in the arithmetic scale that gives the conditional median of *Rc* for a certain velocity *V* value, denoted as $\eta_{R_c|V}$, as follows:

$$
\ln \eta_{R_c|V} = \ln a + b \cdot \ln V = \ln \left(a \cdot V^b \right), \quad \eta_{R_c|V} = a \cdot V^b
$$

$$
\beta_{R_c|V} = \sqrt{\sum_{i=1}^n \left(\ln \frac{R_{ci}}{a \cdot V_i^b} \right)^2 / (n-2)}
$$
 (3)

where ln*a* and *b* are the logarithmic linear regression parameters; $\beta_{R_c|V}$ is the logarithmic standard deviation of regression (i.e., the standard error of regression); {*Vⁱ* , *Rci*}, *i* = 1:*n*, are the *n* in situ core test data. The conditional logarithmic standard deviation of the regression follows the *homoscedasticity* assumption. One of the most well-known measures indicating how well the sample regression line fits the data is called the *coefficient of determination* or the *R*-square (R^2) defined as

$$
R^{2} = 1 - \frac{\sum_{i=1}^{n} \left(\ln \frac{R_{ci}}{a \cdot V_{i}^{b}}\right)^{2}}{\sum_{i=1}^{n} \left(\ln R_{c,i} - \overline{\ln R_{c}}\right)^{2}}, \quad \frac{\sum_{i=1}^{n} \ln R_{c,i}}{\ln R_{c}} = \frac{\sum_{i=1}^{n} \ln R_{c,i}}{n}.
$$
 (4)

R² varies between zero and one and offers a variance measure reduction provided by means of the regression. The perfect fit has $R^2 \approx 1$ ($R^2 \approx 0$ shows a poor fit). It should be noted that each building can have its own characterization of non-destructive parameters versus the concrete strength, based on the data related only to that building. Nevertheless, in the absence of building-specific calibration between destructive and non-destructive data, the regression prediction, albeit imperfect, is useful.

The probabilistic logarithmic multilinear regression model is also employed herein, where the ln*S* and ln*V* are used as independent predictor variables, as follows:

$$
\ln \eta_{R_c|V,S} = \ln a + b \cdot \ln S + c \cdot \ln V = \ln \left(a \cdot S^b \cdot V^c \right), \quad \eta_{R_c|V,S} = a \cdot S^b \cdot V^c
$$

$$
\beta_{R_c|V,S} = \sqrt{\sum_{i=1}^n \left(\ln \frac{R_{ci}}{a \cdot S^b \cdot V^c} \right)^2 / (n-3)}.
$$
 (5)

2.3. Bayesian Workflow for Quantifying the Relative Error in Non-Destructive Test Measurements and Calculating Their Relative Weights with Respect to Destructive Test Results

A preliminary version of this Bayesian workflow was presented first in [\[24\]](#page-17-8). Herein, we describe the complete methodology in a stepwise manner.

2.3.1. The Conditional Probability Model for Predicting *Rultr* Given the Core Strength *R^c*

The procedure relies on a probabilistic regression model in the logarithmic scale of ultrasonic resistance R_{ultr} versus destructive test resistance R_c measured at the same location. This regression model does not imply the inclusion of rebound parameter *S*. Later, in Section [3,](#page-6-0) it is demonstrated that *S* is not significant within a multilinear regression model shown in Equation (5). This helps in characterizing the conditional probability model for predicting *Rultr* given *R^c* based on a linear regression model between ln*Rultr* and ln*R^c* as follows:

$$
\ln \eta_{R_{ultr}|R_c} = \ln a_f + b_f \cdot \ln R_c = \ln \left(a_f \cdot R_c^{b_f} \right)
$$

\n
$$
\beta_{R_{ultr}|R_c} = \sqrt{\sum_{i=1}^{n} \left(\ln R_{ultr,i} - \ln \eta_{R_{ultr}|R_c,i} \right)^2 \over n-2} = \sqrt{\sum_{i=1}^{n} \left(\ln \left(a \cdot V_i^b \right) - \ln \eta_{R_{ultr}|R_c,i} \right)^2} \tag{6}
$$

where ln a_f and b_f are the parameters of the linear regression; $\eta_{R_{ultr}|R_c}$ is the conditional median of *Rultr* given *Rc*; *βRultr*|*R^c* is the logarithmic standard deviation (dispersion) of *Rultr* given *Rc*. It should be noted that an early version of this relationship (a proportional one) was proposed in [\[24\]](#page-17-8), which evolved into a power–law relationship in the arithmetic scale.

2.3.2. Characterizing the Uncertainty in the Concrete Strength Considering Both Destructive and Non-Destructive Test Results

The procedure for Bayesian updating of the probability distribution for concrete strength is outlined herein. D defines the set of available test data consisting of the core test data (destructive, denoted as D_{core}) and ultrasonic test data (non-destructive, denoted as D_{ultr}); thus, $D = \{D_{\text{core}}, D_{\text{ultr}}\}$. By employing the Bayes theorem, the posterior (updated) joint distribution of the median *η* and logarithmic standard deviation *β* of the concrete strength given D, $P(\eta, \beta | D)$, can be expressed as follows:

$$
P(\eta, \beta | \mathbf{D}) = c^{-1} P(\mathbf{D} | \eta, \beta) P(\eta, \beta), \tag{7}
$$

where $P(D|\eta, \beta)$ is the likelihood of our data D; $P(\eta, \beta)$ is the prior joint distribution of the concrete strength parameters; c^{-1} is the normalizing constant. Assuming that, prior to implementing D, random variables *η* and *β* are independent, Equation (7) can be rewritten as

$$
P(\eta, \beta | D) = c^{-1} P(D_{\text{core}} | \eta, \beta) P(D_{\text{ultr}} | D_{\text{core}}, \eta, \beta) P(\eta) P(\beta),
$$
\n(8)

where $P(D_{core}|\eta, \beta)$ and $P(D_{ultr}|D_{core}, \eta, \beta)$ are the likelihoods of the two sets, D_{core} and D_{ultr} , respectively. We note that the likelihood of the ultrasonic test is conditioned on D_{core} data. Assuming independence between core test measurements (which is not true in general), the likelihood of observing the core test measurements can be written as

$$
P(\text{D}_{\text{core}}|\eta,\beta) = \prod_{i} P(R_{c,i}|\eta,\beta) = \prod_{i} \left(\frac{1}{\beta R_{c,i}} \phi \left(\frac{\ln \left(\frac{R_{c,i}}{\eta} \right)}{\beta} \right) \right),\tag{9}
$$

where $\phi(\cdot)$ is the standard normal probability density function; $D_{\text{core}} = \{R_{c,i}, i = 1:n\}$. On the other hand, the likelihood $P(D_{\text{ultr}}|D_{\text{core}}, \eta, \beta)$ can be written by employing the total probability theorem:

$$
P(\mathbf{D}_{\mathrm{ultr}}|\mathbf{D}_{\mathrm{core}},\eta,\beta) = \prod_{k} P(R_{\mathrm{ultr},k}|\mathbf{D}_{\mathrm{core}},\eta,\beta) = \prod_{k} \Big(\sum_{i} P(R_{\mathrm{ultr},k}|\mathbf{R}_{c,i}) P(\mathbf{R}_{c,i}|\eta,\beta)\Big),\tag{10}
$$

where $D_{ultr} = {R_{ultr,k}, k = 1:n}$. In Equation (10), the probability of observing the *k*th ultrasonic data $P(R_{ultr,k}|D_{core}, \eta, \beta)$ can be expanded with respect to the vector of the core values $\{R_{c,i},$ $i = 1:n$. We note that the conditioning on D_{core} manifests itself in the conditional probability *P*(*Rultr*,*^k* |*Rc*,*i*), which is a lognormal distribution with the median and logarithmic standard deviation derived from the probabilistic regression model in Equation (6). Hence, it can be expressed as

$$
P(R_{ultr,k}|R_{c,i}) = \frac{1}{\beta_{R_{ultr}|R_c} R_{ultr,k}} \phi \left(\frac{\ln(R_{ultr,k}/(a_f R_{c,i}^{b_f}))}{\beta_{R_{ultr}|R_c}} \right). \tag{11}
$$

Substituting Equation (11) into Equation (10), we obtain

$$
P(\text{D}_{\text{ultr}}|\text{D}_{\text{core}}, \eta, \beta) = \prod_{k} \left(\sum_{i} \frac{1}{\beta_{R_{\text{ultr}}|R_{c}} R_{\text{ultr},k}} \phi \left(\frac{\ln \left(\frac{R_{\text{ultr},k}}{(a_{f} R_{c,i}^{\ b_{f}})} \right)}{\beta_{R_{\text{ultr}}|R_{c}}} \right) \cdot \frac{1}{\beta R_{c,i}} \phi \left(\frac{\ln \left(\frac{R_{c,i}}{\eta} \right)}{\beta} \right) \right).
$$
(12)

Substituting Equation (9) and Equation (12) into Equation (8), the updated joint probability distribution *P*(*η*, *β* |D) can be estimated.

The mechanical property of interest herein is the average of concrete strength across the whole data (i.e., a constant value of concrete strength is used for structural analysis, performance assessment, etc.). This means that we are specifically interested in the probability distribution for concrete median strength value. The updated marginal distribution of the median *η*, *P*(*η* |D), can be calculated directly from the updated joint probability distribution by integrating *P*(*η*, *β* |D) over the domain of dispersion parameter *β*. This marginal probability *P*(*η* |D) of *η* takes into account both destructive and non-destructive test results. The maximum likelihood of the median *η*, denoted herein as *R*ˆ *c*, can directly

be estimated from of the joint distribution $P(\eta, \beta | D)$. The coefficient of variation of the median *η*, denoted as COV*R^c* , is estimated as follows:

$$
COV_{R_c} = \frac{\sqrt{E(\eta^2) - E(\eta)^2}}{E(\eta)}, E(\eta) = \int_{\Omega_\eta} \eta \cdot p(\eta | \mathbf{D}) d\eta, E(\eta^2) = \int_{\Omega_\eta} \eta^2 \cdot p(\eta | \mathbf{D}) d\eta. \tag{13}
$$

As a result, parameters \hat{R}_c and COV_{R_c} can directly be used as the median and logarithmic standard deviation of an equivalent lognormal probability density function for concrete strength.

2.3.3. Estimating the Relative Weights Associated with Each Ultrasonic Non-Destructive Test Data

In order to obtain an estimate of the relative weight of the *k*th ultrasonic test given the measurements of the core test D_{core}, we start with the probability of observing the *k*th ultrasonic data given the maximum likelihood estimates of the joint distribution $P(\eta, \beta | D)$ that can be shown as $P(R_{ultr,k}|\eta_{ML} = \hat{R}_c, \beta_{ML})$ (we note that we drop the conditioning on D_{core} since it is already embedded in the estimated maximum likelihood parameters $η_{ML}$ and β_{ML}). With reference to Equation (10), this probability can be written as

$$
P(R_{ultr,k}|\eta_{ML}, \beta_{ML}) = \sum_{i} P(R_{ultr,k}|R_{c,i}) P(R_{c,i}|\eta_{ML}, \beta_{ML})
$$

$$
= \sum_{i} \frac{1}{\beta_{R_{ultr}|R_{c}} R_{ultr,k}} \phi\left(\frac{\ln(R_{ultr,k}/(a_{f}R_{c,i}^{b_{f}}))}{\beta_{R_{ultr}|R_{c}}}\right) \cdot \frac{1}{\beta_{ML}R_{c,i}} \phi\left(\frac{\ln(R_{c,i}/\eta_{ML})}{\beta_{ML}}\right)
$$
(14)

Equation (14) can be interpreted as the "exact" probability content of $P(R_{ultr,k}|\eta_{ML} = \hat{R}_c, \beta_{ML})$ (in the sense of if we want to account for the relative measurement error explicitly). The goal herein is to obtain a weight to be applied to a nondestructive data in order for it to obtain the same reliability of the destructive test. The equivalent weight of a given ultrasonic test can then be seen as the likelihood of observing the ultrasonic test results, calculated based on the conditional probability estimated over all observed core tests. Thus, Equation (14) can be seen as the weight of the *k*th ultrasonic test.

3. Application

3.1. Brief Overview of the In Situ Tests

As underlined in CSLP guidelines [\[32\]](#page-17-12), the choice of the number of destructive and nondestructive tests on concrete specimens is among the most challenging issues during field investigation and should follow precise recommendations. The reliability of the concrete strength depends on the number of concrete cores extracted as well as non-destructive measurements carried out. Rebound index and ultrasonic (non-destructive) test can be carried out quickly, which are less expensive, and cause the least damage to the surfaces of the structures. However, the estimation of the resistance is not straightforward as it requires complex calibrations. Core drilling is the reference method for calibrating non-destructive measurements. In fact, estimating concrete strength by coring, except for the recourse to some corrective coefficients, does not require a real correlation for the interpretation of data. However, it causes localized and moderate damages to the structure, is obviously slower and more expensive in execution. Finally, it is important to note that between the non-destructive tests, rebound index test is less expensive compared to the ultrasonic test; however, this work shows (see Section [3.3\)](#page-9-0) that rebound index test has a weak relation with the concrete core strength, while the ultrasonic test indicates a much better correlation.

3.1.1. Destructive Tests

Concrete strength can more reliably be estimated by core test; however, it is important to follow all the prescriptions from the codes in extracting and analyzing the core specimen. The procedures for removal, the process on the extracted samples to obtain the specimens,

and the relative compression test methods are described in UNI EN 12504-1 ("*Removal from concrete in structures—Carrots—Removal, examination and compression test",* [\[4\]](#page-16-3)), UNI EN 12390-1 ("*Testing hardened concrete—Part 1: Shape, dimensions and other requirements for specimens and moulds*", [\[41\]](#page-18-1)), UNI EN 12390-2 ("*Testing hardened concrete—Part 2: Making and curing specimens for strength tests*", [\[42\]](#page-18-2)), and UNI EN 12390-3 ("*Testing hardened concrete— Part 3: Compressive strength of test specimens*", [\[43\]](#page-18-3)). It is important to respect some main recommendations (as highlighted also in the CSLP guidelines, [\[32\]](#page-17-12)):

- The diameter of the extracted carrots should be at least three times greater than the maximum diameter of the aggregates.
- Carrots intended for resistance assessment should not contain reinforcing steel bars.
- Carrots with defects should be evaluated carefully and separately.
- The height/diameter ratio (slenderness) of the specimens should, if possible, be equal to one or two.
- Before breaking, the core samples should be aired out for at least 24 h.

3.1.2. Non-Destructive Tests

The code recommendations for the ultrasonic and rebound index tests and for their combination can be found in Reluis Guidelines [\[35\]](#page-17-15) and CSLP Guidelines [\[32\]](#page-17-12), UNI EN 12504 [\[1,](#page-16-0)[2\]](#page-16-2). The ultrasonic test technique is based on the measurement of the ultrasound propagation velocity, i.e., the propagation of longitudinal elastic waves inside the concrete. It takes into account the global mechanical properties of the material, since the speed of wave propagating within a homogeneous material depends on material density, elastic modulus, and Poisson's coefficient. As a first approximation, the propagation speed is proportional to the square root of the elastic modulus and inversely proportional to the square root of the density. The compressive strength is estimated based on the ultrasound transmission velocity, assuming the validity of a proportional relationship between compressive strength and elastic modulus, using experimental correlations.

The propagation velocity of elastic waves is influenced by various factors, including the humidity content, the composition of the concrete mixture, and the degree of maturation. In determination of velocity, possible presence of steel reinforcement bars and any possible macroscopic defects should be considered. The estimated average speed of the ultrasonic waves, *V*, should be controlled, i.e., measurements that lead to transmission speeds higher than 4800 m/s or less than 2500 m/s should be further verified by considering the following indicative values (CSLP Guidelines [\[32\]](#page-17-12)):

- For concrete of poor quality: *V* < 3000 m/s;
- For medium-quality concrete: $3000 \text{ m/s} \le V \le 4000 \text{ m/s}$;
- For concrete of good quality: *V* > 4000 m/s.

The rebound index method uses the sclerometer to measure the elastic energy absorbed by the concrete after an impact. This energy can be put in relation to concrete stiffness and strength by empirical relationships. The relation between rebound index and concrete strength can be influenced by numerous other factors, including the humidity conditions of the concrete on the surface (a wet surface leads to a lower rebound index); the presence of a carbonated surface layer (increases the rebound index); a surface texture (a rough surface generally provides a lower rebound index); the orientation of the instrument; the age of the concrete; the size and type of the aggregates. Since only the concrete near the point of impact strongly influences the rebound index values, the test methodology is sensible for local conditions.

The combined method consists of the application of two survey methods, the rebound index plus the measurement of ultrasound propagation speed, which is known as the SONREB method for assessing the strength of concrete. The available results from two different methods makes it possible to estimate the resistance through several correlations. The validity of the SONREB method is derived from the compensation of the inaccuracies of the two non-destructive methods used. In fact, it has been noted that the humidity content

underestimates the sclerometric index measurement, and overestimates the ultrasonic velocity. Moreover, as the age of the concrete increases, the sclerometric index increases while the ultrasonic speed decreases.

3.2. The Database

The database used in this study consists of 221 test results. The data are obtained from 20 different structures. The buildings have been built in the past century between the 1930s to the 1990s, mainly in the Campania region, Southern Italy (with some exceptions related, however, to other Italian regions). The importance classes for buildings are mainly related to Class II (ordinary buildings with normal occupancy) and Class III (important buildings with high occupancy), defined in NTC 2018 [\[5\]](#page-16-4). The tests are implemented on RC beams, columns, and walls. Table [1](#page-8-0) summarizes the data. The information about the buildings is just related to their construction period (CP), and to ensure the privacy, more specific information cannot be shared. The following issues are addressed:

* BN = Building Number; CP = Construction Period.

- \triangleright For all the 221 data points, concrete strength from destructive core tests, ultrasonic velocity *V* and rebound number *S* from non-destructive tests are available. For each test, they are measured on the same structural member.
- \triangleright For BN 8, CP and type of structural member is not available.
- \triangleright For BNs 7, 12 and 20, CPs are not available.
- ➢ BN 18 and 19 belong to the same building but refer to two different periods (in particular, the lower floors have been built in 60s, while the last floor has been built in 80s as superelevation).
- \geq The number of the tests corresponding to the same building is always less than or equal to 30. For each BN, the average and the standard deviation values of the test data are shown in the last two columns.
- \triangleright A significant part of the data is collected based on personal communications with different professional engineers. The data associated with three buildings, BNs 3, 8 and 21, are obtained based on the tests available in the literature.

Figur[e 2](#page-9-1) shows the three test typologies, the Core test, the Rebound index test, and the Ultrasonic test, performed on BN 2. the Ultrasonic test, performed on BN 2. Eigure 2 shows the three test type logies the Care test the Reb

Figure 2. The three tests performed on BN2 (see Table [1\)](#page-8-0): (a) Core test; (b) Rebound index test; (c) IIItrasonic test Ultrasonic test. (**c**) Ultrasonic test.

3.3. Results and Comparisons

3.3.1. Regression Predictive Equations **in the 221 data pairs in the**

3.3.1. Regression Predictive Equations
Figure 3 illustrates the logarithmic linear regression performed on the 221 data pairs $\{(V_i, R_{c,i}), i = 1:n = 221\}$. The data points associated with each building are shown with distinct colors (the amount of data for each building is shown in Table 1). Fi[gu](#page-8-0)re 3 also
shows the regression line i.e. the median u_{max} (see Section 2.2), with dark grey solid shows the regression line, i.e., the median $\eta_{R_c|V}$ (see Section 2.2), wi[th d](#page-3-0)ark grey solid line and the model parameters a, b, and $\beta_{R_c|V}$ as well as R^2 are reported in the figure (see Equation (3)).

Figure 3. The logarithmic linear regression between *Rc* and ultrasonic velocity *V*.

Figures [4](#page-10-0) and [5](#page-10-1) show the scatter plots for test data $\{(S_i, V_i, R_{c,i}), i = 1:n = 221\}$ on a natural logarithmic scale. In Figure [4,](#page-10-0) the regression model is derived for {(*Vⁱ* , *Rc*,*ⁱ*), *S* = 39.7, $i = 1:n = 221$ } where the constant rebound number $S = 39.7$ corresponds to the mean value extracted from the data. On the same page, Figure 5 illustrates the regression model $\{({V}_i,$ $R_{c,i}$), $V = 3445 \text{ m/s}$, $i = 1:n = 221$ } where the constant ultrasonic velocity $V = 3445 \text{ m/s}$ is the

mean of data. Figures 4 a[nd](#page-10-1) 5 also show the regression prediction model (i.e., regression li[ne](#page-10-0) in blue solid line and the estimated model parameters *a*, *b*, *c* and $\beta_{R_c|V,S}$; see Equation (5)). Figure 6 illustrates the three-dimensional (3D) representation of the multilinear logarith[mi](#page-11-0)c
magnetic and islaminable that P_1 has a similar small than with the allowance and with V_1 regression, which reveals that *Rc* has a significant correlation with the ultrasonic velocity *V*; $T_{\rm c}$ has a significant correlation with the attractive velocity \dot{V} ,
however, there is low statistical correlation with the rebound index *S*. To this end, a test of significance for the rebound index S is c[ondu](#page-13-0)cted in Section 3.3.3 in order to check the feasibility of this parameter within the multilinear regression.

Figure 4. The multilinear logarithmic regression with $S = 39.7$.

Figure 5. The multilinear logarithmic regression with $V = 3445$ m/s.

Figure 6. Multilinear logarithmic regression in the 3D space. **Figure 6.** Multilinear logarithmic regression in the 3D space.

Based on the collected database, the following two probabilistic logarithmic regres-Based on the collected database, the following two probabilistic logarithmic regression sion models (linear, Equation (15), and multilinear, Equation (16)) in the arithmetic scale models (linear, Equation (15), and multilinear, Equation (16)) in the arithmetic scale are proposed herein:

$$
\eta_{R_c|V} = \left(1.21 \times 10^{-5}\right) \cdot V^{1.79}; \beta_{R_c|V} = 0.23,
$$
\n(15)

$$
\eta_{R_c|V,S} = \left(1.06 \times 10^{-5}\right) \cdot S^{0.07} \cdot V^{1.78}; \beta_{R_c|V,S} = 0.23. \tag{16}
$$

3.3.2. Comparison with Literature

Figure 7 illustrates com[par](#page-12-0)isons of the proposed multilinear regression model with regression models in the literature. The functional forms of the existing models in the literature are as follows:

RILEM (1993) :
$$
R_c = (7.7 \times 10^{-11}) \cdot S^{1.40} \cdot V^{2.60}
$$
, (17)

Gasparik (1992) :
$$
R_c = (6.69 \times 10^{-8}) \cdot S^{1.246} \cdot V^{1.85}
$$
, (18)

Di Leo and Pascale (1994) :
$$
R_c = (1.0 \times 10^{-9}) \cdot S^{1.058} \cdot V^{2.446}
$$
, (19)

Del Monte et al. (2004) :
$$
R_c = (3.7 \times 10^{-7}) \cdot S^{1.13} \cdot V^{1.69}
$$
. (20)

 $RN₂$ **RN** $\frac{BN}{BN}$ $a=7.7*10^{\text{-}11}$ $a = 6.69*10^{8}$ BN 4
BN 5
BN 6
BN 7
BN 8
BN 9 $BN:$ $BN \theta$
 $BN 7$ $b = 1.40$ $b = 1.25$ à 2.60 $BN 8$ $= 1.85$ **BNC** BN 10
BN 10
BN 11 $\bar{8}$ R [MPa] COO **BN10** [MPa] BN 11
BN 11
BN 12 $= 0.23$ R^2 $= 0.39$ **BN12** BN 13
BN 14 **BN** 13 BN 14
BN 15
BN 15
BN 16
BN 17 00000000 COOOOOO **BN15** BN 16
BN 17 **BN18** BN 18
BN 18
BN 19 BN 19
BN 20 **BN 20 BN 21** $BN₂₁$ **RILEM Gaspari** Propose $Propos$ 2000 3000 4000 3000 4000 2000 model model Ultrasonic velocity, V [m/s] Ultrasonic velocity, V [m/s] (**a**) (**b**) $\frac{BN}{BN}$ BN 2 $BN₃$ $a = 3.7*10^{-7}$ BN $a = 1.0*10^{-9}$ $\frac{BN}{BN}$ BN 4 \overline{RN} BN 5
BN 6
BN 7
BN 8 $b = 1.13$ $b = 1.06$ $BN 6$ $BN7$ BN 8
BN 8
BN 9 $= 1.69$ $\overline{3}$ 2.45 $BN₉$ BN 10
BN 11
BN 12
BN 13 **BN10** $[MPa] \centering% \includegraphics[width=1.8\columnwidth]{figures/fig_0a}% \label{fig:1a}% \includegraphics[width=1.8\columnwidth]{figures/fig_0b}% \label{fig:1b}% \includegraphics[width=1.8\columnwidth]{figures/fig_0b}% \label{fig:1b}% \includegraphics[width=1.8\columnwidth]{figures/fig_0b}% \label{fig:1b}% \includegraphics[width=1.8\columnwidth]{figures/fig_0b}% \label{fig:1b}% \includegraphics[width=1.8\columnwidth]{figures/fig_0b}% \label{fig:1b}% \includegraphics[width=1.8\columnwidth]{figures/fig_0b}% \label{fig:1b}% \includegraphics[width=1.8\columnwidth]{figures/fig$ R_{ρ} [MPa] BN 10
BN 11
BN 12
BN 13 $= 0.31$ 0.35 $\overline{1}$ \sim 00000000 **BN14** 000000 **BN 14 BN 15** BN 14
BN 15
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BN 20
BN 21
DelMo 8 Di Lec Propose Propo model 400 3000 model Ultrasonic velocity, V [m/s] Ultrasonic velocity, V [m/s] (**c**) (**d**)

mainly in the Campania region.

Figure 7. (a) RILEM 1993 [\[13\]](#page-16-10) regression shown with constant $S = 39.7$; (b) Gasparik 1992 [\[14\]](#page-17-0) regression shown with constant $S = 39.7$; (**c**) Di Leo and Pascale 1994 [\[9\]](#page-16-8) regression shown with $\frac{6}{10}$ constant *S* = 39.7; (**d**) Del Monte et al., 2004 [\[10\]](#page-16-11) regression shown with constant *S* = 39.7.

Figure 7 shows the scatter plots of test data = { $(S_i, V_i, R_{c,i})$, $i = 1:n = 221$ } for a constant value of $S = 39.7$ (the average value for the dataset). The coefficients *a*, *b* and *c* of these regressions, shown in the figures, are taken based on Equations (17)–(20) [\[9,](#page-16-8)[10,](#page-16-11)[13,](#page-16-10)[14\]](#page-17-0). Finally, *R*² is also calculat[ed](#page-12-0) in the arithmetic scale and shown in Figure 7:

$$
R^{2} = 1 - \frac{\sum_{i=1}^{n} \left[R_{c,i} - \left(a \cdot S_{i}{}^{b} \cdot V_{i}{}^{c} \right) \right]^{2}}{\sum_{i=1}^{n} \left(R_{c,i} - \overline{R_{c}} \right)^{2}}.
$$
 (21)

rithmic residuals sets, one is that of *R^c* given V based on the original regression between It should be noted that although R^2 is calculated based on Equation (21), it is not a least squares error estimate. That is, the reported R^2 is simply representing the goodness of fit of a prescribed regression model (herein, alternative regressions with constant rebound fied the presented. The slope of the slope of the slope of the slope of the regression line is a random varianumber *S* are presented).

The comparison between the literature regression models (blue solid line) and the results is regression models (blue solid line) and the results is results in the results of the results of the results of the results of the proposed multilinear regression model (black dashed line) is also proposed in Figure [7.](#page-12-0)
Line and the celtain of the certain It can be noted that the proposed multilinear regression model is mainly governed by the parameter *V*, while the coefficient for the *S* parameter is very small. To this end, a significance test on the parameter *S* in the regression is performed in the next section.

The value of R^2 of the proposed formulation is higher with respect to the other literature predictions. This is expected because the presented regression is the logarithmic linear least squares fit to the same data. Nevertheless, in absence of building-specific

calibration between destructive and non-destructive data, the regression prediction is useful, mainly in the Campania region.

3.3.3. Significance Test for the Rebound Index *S* in the Regression

A one-side *p*-value test is implemented [\[47\]](#page-18-7) to test the significance of the rebound index *S* in the regression. The test is performed on the potential trend among two logarithmic residuals sets, one is that of *Rc* given V based on the original regression between *Rc* and V, and one is that of *S* (considered as a potential independent variable) on V. The slope of the regression line to be zero (test of hypothesis, [\[47\]](#page-18-7)) helps to derive the significance of the trend among the two logarithmic residuals. The slope significance is quantified through the *p*-value, assuming that the slope of the regression line is a random variable described by Student's *t*-distribution [\[47\]](#page-18-7). The hypothesis is rejected, and the results are statistically significant if the *p*-value is smaller than a certain (small) value, e.g., 0.01. Figure 8 show[s a](#page-13-1) p-value that is quite high; thus, *S* is not statistically significant as a second independent regression variable. Moreover, the small R² value of the logarithmic residual-residual plot confirms this issue. The high *p*-value for the second predictor variable (S) and similar R^2 between the multi-variable (V and S) and mono-variable (only V) regressions offer another confirmation to use only V. Therefore, it is recommended to use the logarithmic linear regression based on ultrasonic velocity V as the only independent variable.

Figure 8. Logarithmic residual–residual plot normalized with respect to V. **Figure 8.** Logarithmic residual–residual plot normalized with respect to V.

and Non-Destructive Tests and Based on Both Destructive Strength Based on Both Destructive Tests and Based on B 3.3.4. The Uncertainty Characterization of Concrete Strength Based on Both Destructive

This section presents the uncertainty characterization of concrete strength based on both destructive and non-destructive tests within the Bayesian workflow, as presented in Section 2.3.2. The procedure relies on a probabilistic logarithmic linear regression model built to predict the ultrasonic resistance R_{ultr} as a function of the core resistance R_c measured at the same structural element (see Section [2.3.1,](#page-4-2) Equation (6)). Figure [9](#page-14-0) shows the probabilistic regression model. The probabilistic regression model.

probabilistic regression model.

Figure 9. The probabilistic logarithmic linear regression model of R_{ultr} given R_c .

Following the Bayesian updating procedure proposed in Section [2.3.2,](#page-4-1) the marginal distribution of the median of the concrete strength η , denoted as $P(\eta|D)$, can be calculated directly from the joint distribution $P(\eta, \beta|D)$ in Equation (8). The errors in both destructive and non-destructive tests are incorporated within this distribution. It can be directly employed for uncertainty propagation within the framework of performance-based design
 $\frac{1}{2}$ and assessment of structures. Figure [10](#page-14-1) illustrates the posterior distribution $P(\eta|D)$ with a thick black line. The prior probability distribution of the median η , denoted as $P(\eta)$ in Equation (8) , is assumed to have the distribution proposed by Verderame et al. [\[48\]](#page-18-8), which is based on the typical values of the post-World War II constructions in Italy. The prior distribution is shown in Figure 10 and \sim distribution is a lognormal distribution with the median equal to 16.5/0.83 MPa and a COV
cause to 0.15. This prior distribution is shown in Figure 10 with a thin groy line. Moreover, equal to 0.15. This prior distribution is shown in Figure [10](#page-14-1) with a thin grey line. Moreover, equal to 0.15. This prior distribution is shown in Figure [10](#page-14-1) while a thin grey line. Moreover we assigned a non-informative prior to $P(\beta)$ in Equation (8). Figure 10 also shows the maximum likelihood estimate of the median, denoted as *R*ˆ *c*, in Section [2.3.2,](#page-4-1) with a red maximum intermode estimate of the median, denoted as K_c , in Section 2.3.2, which a red dotted line. The maximum likelihood is estimated from of the maximum likelihood of the $\frac{d}{dt}$ intervals the maximum intermode is estimated from or the maximum intermode of the position *P*(*η*, *β*|*D*). Moreover, the coefficient of variation (COV) of the median *η*, denoted as COV_{R_c} , is calculated through Equation (13), and shown in this figure. distribution of the median of the concrete strength *η*, denoted as (|), can be calculated Following the Bayesian updating procedure proposed in Section 2.3.2, the marger we assigned a non-mormanive prior to $I(p)$ in Equation (o). Tigure to also show

Figure 10. The posterior and prior distribution of the median of the concrete strength R_c considering both core and ultrasonic test results. both core and ultrasonic test results.

3.3.5. Non-Destructive Data Relative Weights for the Concrete Compressive Strength 3.3.5. Non-Destructive Data Relative Weights for the Concrete Compressive Strength $\frac{1}{2}$ Estimation

on the results of the destructive tests. Generally, these weights are directly employed for calculating the weighted average of the concrete compressive strength for existing for calculating the weighted average of the concrete compressive strength for existing This section proposes the results of the non-destructive data relative weights based structures. With reference to NTC (2018) [\[5\]](#page-16-4), the concrete compressive strength of specimens extracted from columns and beams at each floor should be weighted by assigning weights 1 and 1/3 to destructive and non-destructive results, respectively.

, *ul t k r R*

Figure [11](#page-15-0) presents the histogram and statistics of the relative weights of 221 ultrasonic rigure 11 presents the mistogram and statistics of the relative weights or 221 ultrasonic
pieces of data, calculated based on Equation (14). The calculation of the statistics for the relative weights is based on the first two moments of the set of weights. In particular, the expected value is calculated as follows: where E(*w2*) can be calculated as follows:

$$
E_w = \frac{\sum_{k} w_k \cdot R_{ultr,k}}{\sum_{k} R_{ultr,k}},
$$
\n(22)

 $\overline{}$, $\overline{}$

where E_w is the expected value of the weights. Moreover, the standard deviation on the set of the weights can be calculated as follows: $\frac{1}{2}$ as the expected value of the weights, redectively. The standard deviation on

$$
\sigma_w = \sqrt{\mathrm{E}(w^2) - (\mathrm{E}_w)^2},\tag{23}
$$

where $E(w^2)$ can be calculated as follows: as 0.14 and 0.42. The code weight *w* = 1/3 is very close to the estimated mean weight. where $E(u')$ can be calculated as follows.

$$
E\left(w^2\right) = \frac{\sum_{k} w_k^2 \cdot R_{ults,k}}{\sum_{k} R_{ults,k}}
$$
(24)

Figure 11. Histogram and statistics of the 221 relative weights of the ultrasonic test data calculated based on Equation (14) and NTC 2018-based weight $w = 1/3$.

The E_w and $\mathrm{E}_w \pm \sigma_w$ values calculated from Equations (22) and (23) are reported in Figure [11](#page-15-0) as blue solid and dash dotted lines, respectively. The red line indicating the code-based weight of $w = 1/3$ is also shown in Figure [11.](#page-15-0) The expected value of the weights calculated according to Equation (24) is $E_w = 0.28$, and the confidence band of one standard deviation around the expected value, calculated according to Equation (23), is estimated as 0.14 and 0.42. The code weight $w = 1/3$ is very close to the estimated mean weight.

As a result, the mean of the non-destructive test effective weights is very close to the code-base recommended values. This result is important because it demonstrates how the non-destructive tests, implemented in terms of the ultrasonic velocity *V*, can be reliably used within the concrete strength calculation to complement the destructive test, in particular where it is not easy to execute them.

4. Conclusions

This paper proposes a Bayesian workflow to characterize the uncertainty in the concrete compressive strength based on both in situ destructive and non-destructive data. It further attempts to estimate the relative weight of non-destructive measurements with reference to the destructive tests in order to estimate the compressive strength for design/assessment purposes. A database containing core tests and SONREB non-destructive

test data, performed on the same structural members of various RC structures, is assembled in this work. The buildings are mainly located in the Campania region, Southern Italy, constructed between 1930s and 1990s. Probabilistic logarithmic linear and multilinear regression models are implemented to obtain predictive expressions for estimating the concrete compressive strength, *Rc*, based on the ultrasonic velocity, *V*, and on the rebound number, *S*. The implementation of the concrete strength based on non-destructive test results is successively considered by estimating the relative measurement error with a probabilistic methodology. The Bayesian inference framework presented herein manages to quantify the relative error of the non-destructive tests with respect to the destructive core tests. The main findings of the work are herein summarized:

- The logarithmic linear regression model (ln*V*-ln*Rc*) can be adopted instead of the logarithmic multilinear model (ln*V*-ln*S*-ln*Rc*) without significant loss of accuracy. This is also confirmed from the significance test of the rebound index *S* in the regression.
- It is good to have an own regression for each specific building among destructive and non-destructive tests. However, it can be noted that the proposed multilinear regression model has a good fit with respect to the considered data that allows its usage for similar data.
- The relative weights of non-destructive tests for calculating concrete compressive strength are derived based on the proposed probabilistic workflow. The mean of the weights is very close to the relative weight recommended in the NTC 2018 Commentary, which assigns a value of 1/3 to non-destructive tests.

Author Contributions: Conceptualization, All; methodology, All; formal analysis, A.M. and H.E.; writing—original draft preparation, A.M. and H.E.; writing—review and editing, All; supervision, F.J. and A.P. All authors have read and agreed to the published version of the manuscript.

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