Introduction to the Digital Experiences in Mathematics Education (DEME)
Special Issue on
"Supporting transitions within, across and beyond digital experiences for the teaching and learning of mathematics"

Guest Editors
Eirini Geraniou (e.geraniou@ucl.ac.uk)
Manolis Mavrikis (m.mavrikis@ucl.ac.uk)
Carla Finesilver (carla.finesilver@kcl.ac.uk)
Anna Baccaglini-Frank (anna.baccaglinifrank@unipi.it)

In recent decades there has been a considerable quantity of research devoted to the potential impact of digital resources in education, including how such resources might successfully be integrated into the mathematics classroom (e.g., Drijvers et al, 2010; Forgasz, 2006; Ruthven, 2014). Both the research and teaching community recognise by now that technology has the potential to provide "students with appropriate 'vehicles" for developing "Mathematical Ways of Thinking" (Papert, 1983) and that computer-based representations provide "various forms of records produced, with many different types of linkages, and with accompanying pedagogical approaches" (Goldin and Kaput, 1996 p. 417). While some of these ideas have been explored in the literature (e.g., Geraniou & Mavrikis, 2015; Gurtner J.-L.,1992; Hölzl, 1996), the idea of 'bridging' or 'transitioning' between dynamic digital resources and non-digital static resources so far has not been a major focus of research in mathematics education. For this special issue, we were therefore interested in seeing how students (and subsequently teachers) conceptualise mathematics as they make transitions when using digital resources, involving different mathematical representations and different semiotic systems that may be using 'new' representations of mathematical concepts. Whether we consider digital resources, such as graphing software, computer algebra systems or dynamic geometry environments as 'vehicles for mathematical ways of thinking', or as a means for providing an increasing variety of visual, dynamic and linked computer-based representations that can support the learning of mathematics, 'transitioning' between such resources becomes crucial particularly when considering the flexibility, variety and multimodality of digital environments in combination with all non digital forms of learning.

For this special issue, we invited papers that focused specifically on such transitions, highlighting in particular three types of transition related to digital resources, *within*, *beyond* and *across*:

1) transitions between mathematical experiences and representations of mathematical objects within a particular digital resource ('*transitions within*'), i.e., what mathematical knowledge is used by students during their interactions with a digital resource, how they make transitions between different mathematical representations of the same mathematical object or concept within a digital resource, and how the mathematical representations in the digital resource support mathematical learning;
2) transitions from a digital resource to non-digital resources or vice versa ('*transitions beyond*'), i.e., what mathematical knowledge is gained by students from their interactions with a digital resource and how this can then be used outside the digital resource (such as when doing "paper-and-pencil" mathematics, undertaking traditional exam papers) in formal and/or informal settings, and how this transition is

supported (or not) by the design and implementation of both the digital and non-digital resources;
3) transitions across digital resources (‘*transitions across*’), i.e., what mathematical knowledge is gained from a digital resource and then used in subsequent interactions with a different digital resource, which could involve, e.g. different representations of the same mathematical concepts.

We received a high number of proposals and accepted thirteen papers, collected in the two volumes dedicated to this Special Issue. The papers focus on a variety of types of transitions, involving many different digital resources: GeoGebra (Bach; Bretscher; Lisarelli; Miragliotta), STEP - an online assessment of mathematical reasoning platform (Yerushalmy et al.), an online undergraduate learning-support system (Kontorovich & Locke), a 3D turtle geometry microworld (Kynigos et al.), Techtivities – digital activities involving video animations and interactive sets of Cartesian axes (Johnson et al.), the GGBot - a 3D printed drawing robot running on the SNAP! visual programming language (Del Zozzo & Santi), Excel (Maracci et al.), Touchtimes – a multi-touch iPad application (Bakos), virtual manipulatives (Rich), and a simulation with table and graphs (Panorkou et al.).

Most authors chose to focus on *transitions within* and/or *transitions beyond*. Lisarelli introduces a task specifically designed to work on a 'transition beyond' that involves moving from the graph of a function in a dynamic geometry environment (DGE) to the Cartesian graph of the same function in a paper-and-pencil environment. Miragliotta focuses on the process of geometric prediction and analyses a ‘transition beyond’ in terms of possible relationships between predictions, developed during the resolution of geometrical tasks within a paper-and-pencil environment, and subsequent explorations within a DGE. Bach tackles ‘transitions within’, analysing the opportunity offered by DGEs to perform transitions within the environment itself, particularly when dragging. Maracci, Pocalana and Carlino investigate how the use of artefacts, the process of construction and emergence of resources nurture each other, as they analyse the complex transitions that can occur around the production and sharing of signs when digital artefacts and resources are available. Kynigos and Karavako capture mathematical meanings evolving through a flow of ‘transitions within’ different representational contexts of a digital resource and then beyond these to digital, artistic and pure mathematical representations of ideas around music and dance. Finally, Rich discusses ‘transitions within’ the representations embedded in virtual manipulatives, and potentially also in making further transitions beyond other representations.

Transitions *across* are the focus of only two papers, and in different ways. Bakos includes analysis of ‘transitions across’ the two different microworlds in TouchTimes, and the different multiplicative models portrayed by each of them. Moreover, she identifies internal shifts in thinking that the teachers experience personally and professionally while undergoing double instrumental genesis. Panorkou, York and Germia examine the “messiness” of transitions between related artefacts. They look at how students’ covariational reasoning is shaped and reorganized through transitions between artefacts and how the artefacts’ synergy (a construct explored in the previous Special Issue of this journal) provides a constructive space for students to shape and reorganise their reasoning.

Our call was designed to allow for researchers’ different interpretations of the three types of transition, and there was also an interesting variety in this aspect of the submissions. For example, Kontorovich and Locke explore the role of students' transitions within a learning-support module in relation to their learning of mathematics, which they describe through the lens of commognition in terms of "re-routinization" – a process of repeated development of conventional routines to be implemented in already familiar mathematical tasks. Johnson, Olson, Tsinnajinnie and Bechtold look at "boundary transitions" within, across, and beyond a set of digital resources. For them, "boundary transitions" are brokering moves between communities: they involve boundary objects, which may be tangible (e.g., digital resources) or intangible (e.g., the notion of function) and can link different instructional items (transitions beyond), different forms of the same instructional item (transitions between), or parts of a single instructional item (transitions within)." Del Zozzo and Santi introduce transitions between domains of activity as "domestications of the eye" for the learning of mathematics with the GGBot, defining a 'transition between' as a transformation of sensuous cognition that occurs when shifting between domains over time. Yerushalmy, Olsher, Harel and Chazan explicitly consider "transitions in thinking" to describe changes in the range of graphs students create to exemplify a story, as their source-path-goal schema evolves. Specifically, they explore how STEP supports the transition from thinking about 'distance' as only total distance travelled to a coordinated view that also includes distance from a starting location. Bretscher uses the TPACK framework to analyse teachers' knowledge arising in four interviews, and defines transitions as events that provide potential opportunities for students to create situated abstractions by generalizing or abstracting mathematical meaning from specific contexts.

Overall, the papers in this Special Issue help us gain insight into a variety of questions we posed in our call, such as:

- How do the different mathematical representations within digital resources impact the three types of transition?
- How can 'bridging' within, beyond and across digital resources be scaffolded both by design of the digital resources and by teachers' actions?
- What is the impact of the 'dragging' feature of dynamic digital resources on students' learning when different transitions are involved?
- Which mathematical and digital competencies of students are elicited by transitions within, beyond and across different digital resources?
- What theoretical frameworks can help us understand better the three types of transition?

Although further research is definitely needed – as suggested by open questions proposed in many of the papers – we hope readers of this section appreciate our attempt to create a reference volume foregrounding some evolving pathways for research in mathematics education on how transitions within, beyond and across digital resources foster the development of mathematical knowledge, on how the design of digital resources and in particular the mathematical representations in those digital resources may support or hinder these transitions, and particularly on the need to better understand the role of educators in supporting such transitions. Finally, we wish to kindly thank the contributing authors and all the reviewers, without whom this Special Issue would have never seen the light.

References

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics, 75*(2), 213-234.

Forgasz, H. (2006). Factors that encourage or inhibit computer use for secondary mathematics teaching, *Journal of Computers in Mathematics and Science Teaching, 25*(1), 77-93.

Geraniou, E., & Mavrikis, M. (2015). Building Bridges to Algebra through a Constructionist Learning Environment. *Constructivist Foundations, 10* (3), 321-330.

Goldin, G. A., & Kaput, J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 397–430). Mahwah, NJ: Erlbaum.

Gurtner J.-L. (1992). *Between Logo and mathematics: A road of tunnels and bridges*. In: Hoyles C. & Noss R. (eds.) Learning mathematics and Logo. MIT Press, Cambridge MA: 247– 268.

Hölzl, R. (1996). How does 'dragging' affect the learning of geometry? *International Journal of Computers for Mathematical Learning, 1,* 169–187. https://doi.org/10.1007/BF00571077

Papert, S. (1983). *Mindstorms: children, computers, and powerful ideas*. Basic Books.

Ruthven, K. (2014). Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. In Clark-Wilson, A., Robutti, O. & Sinclair, N. (Eds), *The mathematics teacher in the digital era: An international perspective on technology focused professional development*, (pp.373-393). Dordrecht: Springer.