Achieving Least Relocation of Existing Facilities in Spatial Optimisation: A Bi-Objective Model

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Abstract

Spatial optimisation models have been widely used to support locational decision making of public service systems (e.g. hospitals, fire stations), such as selecting the optimal locations to maximise the coverage. These service systems are generally the product of long-term evolution, and there usually are existing facilities in the system. These existing facilities should not be neglected or relocated without careful consideration as they have financial or management implications. However, spatial optimisation models that account for the relocation or maintenance of existing facilities are understudied. In this study, we revisit a planning scenario where two objectives are adopted, including the minimum number of sites selected and the least relocation of existing facilities. We propose and discuss three different approaches that can achieve these two objectives. This model and the three approaches are applied to two case studies of optimising the retail stores in San Francisco and the large-scale COVID-19 vaccination network in England. The implications of this model and the efficiency of these approaches are discussed.

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1 Introduction

Spatial optimisation or facility location models are aimed at siting facilities so as to provide service to demands efficiently. A range of location models have been proposed to support varying management, planning, and decision-making contexts. In particular, the location set cover problem (LSCP) [9] has been proposed for planning applications in which the fewest facilities are to be sited so as to serve all demand within the designated service response standard. The LSCP can be written as [2]:

$$\textbf{Minimize} \quad \sum_{j=1}^{n} x_j \qquad (1)$$

Subject to:

$$\sum_{j \in N_i} x_j \geq 1 \qquad \forall i \qquad (2)$$

$$x_j \in \{0,1\} \qquad \forall j \qquad (3)$$

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Where:

$i =$	index referencing nodes of the network as demand
$j =$	index referencing nodes of the network as potential facility sites
$n =$	total number of potential sites
$S =$	maximal acceptable service distance or time standard
$d_{ij} =$	shortest distance or travel time between nodes i and j
$N_i =$	$\{j \mid d_{ij} < S\}$
$x_j =$	$\begin{cases} 1, & \text{if a facility is located at node } j \\ 0, & \text{otherwise} \end{cases}$

In applications where there are one or more existing facilities, the LSCP in its basic form as above is faced with a major problem, as it does not differentiate between sites with and without existing facilities. The scenarios where the relocation of existing facilities is concerned are common in applications. In this paper, we focus on a planning scenario where two objectives are adopted: the first one is overall efficiency, which is exactly the objective of LSCP (see Formula 1). This objective requires the least number of sites to be selected, regardless of sites with and without existing facilities. The second objective, called the least relocation of existing facilities, dictates the maximum maintenance of existing facilities (or the least relocation of existing facilities), meaning that as many existing facilities as possible should be utilised. These two objectives are not equally important and the first criterion has a higher priority than the second one.

This bi-objective problem was introduced by [8] and illustrated by a case study of optimising the locations of fire companies for the Denver fire department. This bi-objective LSCP is as follows:

$$\textbf{Minimize} \quad \sum_{j=1}^{n} x_j \qquad (4)$$

$$\textbf{Maximize} \quad \sum_{j=1}^{p} x_j \qquad (5)$$

Subject to:

$$\sum_{j \in N_i} x_j \geq 1 \qquad \forall i \qquad (6)$$

$$x_j \in 0, 1 \qquad \forall j \qquad (7)$$

In addition to the notations above, the following notations are used:

$j =$	index referencing nodes of the network as potential sites. Sites with existing facilities are indexed from 1 to p. Sites without are indexed from p+1 to n.
$p =$	total number of sites with existing facilities, $p \leq n$.

In the following, we present three approaches that are applicable to solve this problem.

The first approach is proposed in the paper as mentioned above [8], which combines the two objectives into a single one and transforms the bi-objective problem into a single-objective programming problem. More details of this approach can be found in [8].

The second approach is inspired by [7]. Originally, the author proposed a method to deal with sites with and without current facilities in spatial optimisation by keeping a specified number of current facilities in LSCP. Here, we extend this approach to solve the bi-objective LSCP. Specifically, this approach adds an additional constraint to LSCP that keeps a specified number (r) of current facilities and relocating others. By iterating all possible r values and solving a list of LSCP problems with different r, a pool of LSCP solutions with different r would be obtained, and then the LSCP solution with the maximum r would be the final solution to the bi-objective problem.

Third, this problem can be directly solved using a hierarchical or lexicographic method [6]. Specifically, Objective (4) is assigned with a higher priority than Objective (5), and these two objectives are optimised in priority order. This approach is incorporated in general-purpose mixed programming solvers like Gurobi [5]; however, it is not supported in others such as GLPK [4].

While these three approaches would derive optimal solutions to the bi-objective LSCP, the computing efficiency of these approaches are understudied. In the following section, we will compare these approaches using two case studies with different problem sizes.

2 Case studies

We present two case studies to compare the function and performance of the three approaches to the bi-objective LSCP. All processing and computation are conducted on a desktop MacOS 10.15.5, 2.7 GHz with 8 GBytes memory.

2.1 Case study of siting stores in San Francisco

In this case, a retail chain would like to site a number of stores in San Francisco. The primary objective is to locate stores close to population centres, which are represented by 205 census tracts in this city. In this problem, we consider a set of 16 potential store sites and set the maximum service distance to access a store on the road network as 5 kilometres. The facility-demand distance matrix was derived from ArcGIS Network Analyst extension [1]. To simulate the scenario with a set of existing facilities, we randomly chose eight sites and assumed that there were existing facilities at these sites. This bi-objective problem is formulated as below: given the existing eight stores and a set of eight potential sites, at least how many sites should be selected to site the stores to cover all populations?

2.2 Case study of COVID-19 vaccination network in England

This case study aims to optimise the COVID-19 vaccination network in England. England contains 56.6 million people in 2020, which accounts for 84.3% of the UK's population. During the COVID-19 pandemic, a COVID-19 vaccination network was built and maintained to provide vaccination to residents, and this network consisted of 1,600 vaccination centres by November 2021. The locations of these vaccination centres are likely not optimised and some centres are redundant. Therefore, we formulate the location optimisation problem of the COVID-19 vaccination network as follows: given the existing 1,600 vaccination centres and a set of 21,127 potential sites (based on locations of the Point Of Interest), at least how many sites should be selected to locate the vaccination centres to cover all populations?

The demands in this problem are the populations of each Middle Layer Super Output Area (MSOA), with population-weighted centroids of MSOAs as demand points and the population of the 2011 census as weights.

2.3 Results and discussion

The results of the two case studies are presented in Table 1. Both cases verify that these three approaches derived optimal solutions with the same number of selected sites with existing facilities and sites without. In terms of computing efficiency, in the small-size San Francisco case, the three approaches solved the location problem using less than one second, demonstrating high computating efficiency. In contract, when solving the large-size COVID-19 vaccination case, Approach 3 significantly outperformed the other two approaches regarding the computing time.

Table 1 Example of test session results.

Case study (n, p)[1)]	San Francisco (8, 8)	England (21127, 1600)
Approach 1 Weighted	(4, 4, 0.1s)[2)]	(313, 107, 512m 53.9s)
Approach 2 Iterative	(4, 4, 0.2s)	(313, 107, 294m 6.5s)
Approach 3 Lexicographic	(4, 4, 0.1s)	(313,107, 54m 53.5s)

1) n and m represent the number of sites without and with existing facilities, respectively
2) the three numbers represent the number of selected sites without existing facilities, number of selected sites with existing facilities, and computing time

3 Conclusions

In this paper, we revisited a bi-objective extension of LSCP that aims to achieve two objectives simultaneously, including the minimal number of selected sites (with higher priority) and the maximal number of selected sites with existing facilities. We show that this problem can be tackled by three different approaches, using two planning cases. The results verify that these three approaches are capable of tackling this bi-objective LSCP. In terms of computing efficiency, while these approaches exhibit similar computing time in the small-size case of San Francisco, the third approach (lexicographic) shows significantly higher efficiency than the other two approaches.

This research opens up avenues for future research. First, we will attempt to analyse and understand the computational complexity of the three approaches. Second, we plan to incorporate this bi-objective LSCP into the *spopt* Python library [3], an emerging open-source project for spatial optimisation.

References

1 Huanfa Chen, Alan T. Murray, and Rui Jiang. Open-source approaches for location cover models: capabilities and efficiency. *Journal of Geographical Systems*, 23(3):361–380, April 2021. Publisher: Springer. doi:10.1007/s10109-021-00350-w.

2 Richard L. Church and Alan Murray. *Location covering models: History, applications and advancements*. Springer, New York, 2018.

3 Xin Feng, James D Gaboardi, Elijah Knaap, Sergio J Rey, and Ran Wei. Pysal/spopt. doi:10.5281/zenodo.4444156.

4 GNU Project. GLPK (GNU Linear Programming Kit), version 4.65, 2017. URL: `http://www.gnu.org/software/glpk`.

5 Inc. Gurobi Optimization. Gurobi Optimizer Reference Manual, 2016. URL: `http://www.gurobi.com`.

6 Miettinen Kaisa. *Nonlinear Multiobjective Optimization*, volume 12 of *International Series in Operations Research & Management Science*. Kluwer Academic Publishers, Boston, USA, 1999.

7 Alan T. Murray. Optimising the spatial location of urban fire stations. *Fire Safety Journal*, 62(PART A):64–71, November 2013. Publisher: Elsevier. `doi:10.1016/j.firesaf.2013.03.002`.

8 Donald R. Plane and Thomas E. Hendrick. Mathematical programming and the location of fire companies for the denver fire department. *Operations Research*, 25(4):563–578, August 1977. Publisher: INFORMS. `doi:10.1287/opre.25.4.563`.

9 Constantine Toregas, Ralph Swain, Charles ReVelle, and Lawrence Bergman. The location of emergency service facilities. *Operations Research*, 19(6):1363–1373, October 1971. Publisher: INFORMS. `doi:10.1287/opre.19.6.1363`.