



The contribution of the intra-firm exposures network to systemic risk[☆]



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ABSTRACT

We propose to use a systemic risk metric for an extended network which includes the inter-bank network, the banks-firms bipartite network, and the intrafirm exposures network in Uruguay. This is the first work, to the best of our knowledge, in which the intra-firm exposures network is estimated with good accuracy by using information from a firm survey. Given that the survey only includes the three most relevant debtors and creditors, we complete the full intra-firm exposures matrix by resorting to Maximum Entropy, Minimum Density and a new method which takes into account the known entries of the matrix obtained from the survey. We show that ignoring intra-firm exposures results in an important underestimation of systemic risk. Moreover, if the marginal liabilities are used as an indicator of the systemic relevance of firms, important network effects are ignored. To conclude, the paper contributes with a precise estimation of the impact of intra-firm exposures to overall systemic risk.

1. Introduction

The increasingly complex and interrelated connections in the financial system are considered one of the main sources of risk amplification and propagation of shocks. This was made evident in the worst possible way during the Global Financial Crisis (GFC) after the fall of Lehman Brothers. Since then, macroprudential policies and the interconnections in the financial system have taken central stage in the financial stability agenda.

These interconnections among financial entities have been modelled by resorting to network theory and models. Subsequently, researchers have modeled financial entities and their relationships by financial networks. Extensive literature exists on the structure of these networks and the effects of these structures on the propagation of both microeconomic and macroeconomic shocks (see Battiston and Martinez-Jaramillo, 2018; Martinez-Jaramillo et al., 2019) for an introduction.

Nevertheless, contagion through commercial indebtedness among firms or economic sectors has received less attention (Acemoglu et al., 2016), mainly due to the lack of information. However, this situation has recently changed, and now it is pos-

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sible to find works that include the real sector of the economy and its relationship with the banking system: [Poledna et al. \(2018\)](#) and [Silva et al. \(2018\)](#).

This work aims to contribute to filling this gap as well by building a commercial and financial debt network for Uruguay. We resort to a firm-level survey that included questions on the main debtors and creditors for each firm that participated in the survey (see [Baron et al., 2020](#)). Additionally, the links between the firms and the banking system are also known from the credit registry. Finally, even though the inter-bank market is small and we find low contagion through this channel, the exposures network among banks is also considered.

Evidence concerning the effect that firms' defaults may have on banks in Uruguay might be extrapolated to other countries with shallow financial sectors:

- Directly, through financial credit, the effect is quite well understood.
- Indirectly, through common asset holdings, it has not been studied due to lack of information but it is perceived to be small.
- Indirectly, through commercial credit linkages, there is less empirical evidence but the effect was explored in [Baron et al. \(2020\)](#).

In this paper, we provide an empirical quantification on the latter. For this purpose, we use information from a novel firm-level survey. The Central Bank of Uruguay conducts a survey on commercial debt among a representative sample of firms with more than 50 employees. The sample excludes firms belonging to the primary activity sector, financial intermediation, public sector, or real estate activities. Given the lack of information, connections for these sectors are inferred.

We combine the information obtained from the survey with balance sheet and credit registry data to build a commercial and financial debt network. In doing so, we also provide a series of measures of interconnectedness and the topology of the networks. Finally, we produce a systemic risk metric for the banking system and the firms with direct links to this system.

As a result, we are able to identify the most central firms in terms of commercial debt and the most central banks in the network. We also quantify the banks' exposure to credit risk originating in the firms. Most importantly, we perform a stress test exercise consisting of the propagation of a default shock to analyze the vulnerability of the network and the systemic effects of individual participants.

Nevertheless, the full structure of intra-firm exposures is unknown. The sample of firms included in the survey only report the total amount lent and borrowed and the three most important debtors and creditors. In addition, the firms also report the total number of debtors and creditors. Given this rich but partial information, we have to estimate the full matrix of intra-firm exposures. For this purpose, we resort to two well-known estimation methods: the ME ([Upper and Worms, 2004](#)) and the minimum density, (MD; [Anand et al., 2014](#)). Additionally, we developed a method that also takes into account the known entries of the intra-firm exposures matrix, which first uses the fitness model proposed in [Musmeci et al. \(2013a\)](#) and then a constrained version of the RAS algorithm to distribute the remaining amount for which the counterparties are not known.

We perform a similar analysis to [Poledna et al. \(2018\)](#), the main difference being that the intra-firm exposures network is not estimated from balance sheet data but rather by using partial network information from the firm survey. Given this additional piece of information, we are able to estimate how much the intra-firm exposures network contributes to overall systemic risk. We quantify this by using the three different estimation methods of the intra-firm exposures network.

The alternative methods to reconstruct the firm-to-firm network produce networks with different properties, which affects the estimation of risk. For instance, maximum entropy (ME) tends to create complete networks in which all entries are as homogeneous as possible while being compatible with the constraints provided by the total borrowing and lending of each individual institution. On the other hand, MD, allocates the total amount lent to and borrowed from each bank while using as few links as possible, thus producing a very sparse network that represents a lower bound in terms of connectivity.

The main objective of our work is to consider the credit risk to which banks are exposed, which arises not only from interconnections among banks but also from credit relationships between banks and firms, as well as among firms. The default of one firm can have effects on the solvency of other firms, generating a chain reaction that ultimately affects banks because of their inability to meet their bank payments. To quantify credit risk, we use DebtRank, which measures the fraction of total equity in the financial system that is potentially affected by the default of bank i or company j . In this paper, we want to address the following questions: What is the risk imposed on the financial system by commercial credit between companies? How much is the systemic risk modified, defined as the maximum loss of equity, due to the default of an agent when, in addition to the banks, the commercial credit relationships between the companies are included? Do differences exist between the banks considered more central in the network when only the inter-bank network is considered than when the intra-firm network is added? Are there differences among banks with the highest vulnerability in the network when only the inter-bank network is considered in comparison when the intra-firm network is also included? Can we identify firms in the network that are systemically important?

The last question is particularly relevant from a supervisory perspective to the extent that commercial enterprises are outside the regulatory perimeter. However, identifying which are the most important companies from the systemic point of view can guide supervisors to monitor the sectors of activity in which they are part or even, within the on-site supervision, prioritize the analysis of the credits that banks lend to these companies.

This paper offers several contributions. First, we estimate the intra-firm exposures network by resorting to three alternative methods and by doing so, we can verify the robustness of our results. Second, we estimate the contribution to systemic risk of the information contained in the intra-firm exposures network. Third, we identify systemically important firms based on the effect of their failure on banks and other firms, taking into account contagion (network) effects. Fourth, we compute the DebtRank centrality metric for both the inter-bank exposures and the extended network of exposures, including firms. Finally, we measure the importance of effective exposures from banks to firms taking into account the exposure relationships among firms.

The remaining part of the paper is organized as follows: In [Section 2](#), we place our work in the broader context of financial networks and systemic risk measurement. [Section 3](#) describes all the different data sources used in this research project. [Section 4](#) describes concepts related network analysis and the methodology. [Section 5](#) presents the results; finally, [Section 6](#) concludes.

2. Literature review

Interconnectedness has been recognized as one key elements in the amplification of the GFC. Even though interconnectedness per se is not necessarily harmful ([Martinez-Jaramillo et al., 2019](#)), it can work as an amplifier of negative shocks to the system under certain circumstances, as was the case with the GFC. Political instability leading to changes on the level of interconnectedness in the banking system could be even the case, resulting in increased levels of systemic risk ([Caceres-Santos et al., 2020](#)).

Furthermore, the effects of microeconomic and macroeconomic shocks are related to the topology of the network. [Elliott et al. \(2014\)](#) find that integration (greater dependence per agent or node) and diversification (more counterparties per agent or node) have different, non-monotonic effects on the extent of cascades.

Diversification connects the network initially, permitting cascades to travel; but as it increases further, agents or nodes are better insured against other participants' failures. Integration also faces trade-offs: increased dependence on other organizations versus less sensitivity to own investments.

One strand of theoretical and empirical literature studies contagion in the banking system or the financial system at large ([Battiston and Martinez-Jaramillo, 2018](#); [Calomiris et al., 2019](#); [Silva et al., 2016](#); [Souza et al., 2014](#)). Because the information on financial interconnections is usually not public and only partial information is available on the total debt between financial institutions (aggregate positions), a series of methodologies have been developed to complete the interconnection matrix between financial institutions.

[Anand et al. \(2018\)](#) conduct a horse race of network reconstruction methods using network data obtained from 25 different markets spanning 13 jurisdictions. They consider seven reconstruction methods and find that the best methods depend on the final purpose of the reconstructed network.

In this paper, we use three of the methods mentioned in [Anand et al. \(2018\)](#) to reconstruct the firm-to-firm network. The first is the MD method introduced by [Anand et al. \(2014\)](#). This reconstruction method minimizes the number of links necessary to distribute a given amount of loans. The second is the ME reconstruction method proposed in [Upper and Worms \(2004\)](#). Opposite to the MD reconstruction method, this method will result in a denser interconnection matrix. The third method is based on [Musmeci et al. \(2013b\)](#) fitness model, which is a novel model to reconstruct global topological properties, in particular, of the intra-firm network from known information (information from the survey and credit registry databases). This methodology uses partial information and an auxiliary non-topological property, which is interpreted as the fitness associated to each node.

Research on the effects of contagion through commercial indebtedness among firms, industries and economic sectors has received less attention (see, e.g., [Acemoglu et al., 2015](#)). However, in recent years we can find empirical and theoretical works that study the interconnection between the real sector and the financial system mainly through bipartite networks between banks and firms.

[Lux \(2014\)](#) proposes a model of a bipartite network between banks and the non-bank corporate sector and concludes that contagion due to joint exposures to counterparty risk via loans to firms is more important to contagious spread of defaults than it is to the inter-bank credit channel.

[De Masi et al. \(2009\)](#) and [De Masi and Gallegati \(2007\)](#) analyze the bank-firm network in Italy and Japan. [Poledna et al. \(2018\)](#) reconstruct and analyze the financial liability network combining the firm-bank network and the inter-bank network in the Austrian banking system. They find that all firms together create more systemic risk than the entire financial sector does.

Risk imposed by firms upon the financial system seems to be important, according to the results in the most recent empirical work. This risk is not considered when the analysis of the network is performed only within the inter-bank network.

[Aoyama et al. \(2013\)](#) analyze the bipartite credit network of the lending/borrowing relationship between banks and firms in Japan. They performed a stress test exercise, introducing distress to firm and bank nodes and evaluating the propagation through the network. More related to our work is [Baron et al. \(2020\)](#), who build a commercial and financial debt network at the sector level for Uruguay using the same data set used in this paper. They provide a series of measures of the indebtedness structure and identify the most central sectors in terms of commercial debt, as well as the most central banks in the network. They also perform a stress test consisting of the propagation of a default shock at the sector level to analyze the vulnerability of the network.

Our work includes a new layer to this analysis and considers the firm-to-firm network. To our knowledge, it is the first work that includes the banks-firms network, the inter-bank network and the intra-firm network at such a level of granularity.

3. Data

In this section, we present details of the data used to conduct the systemic risk evaluation of the financial system in Uruguay. One source of data used in this paper consists of a novel firm-level survey of 240 firms conducted by the Central Bank of Uruguay in October 2018 with information about the following:

- The total amount of commercial debts and sales credit.
- Information about the three main debtors and creditors for each firm, including their identities and the respective amounts.
- Sectoral information about firms, debtors and creditors.

The survey asked a sample of 240 Uruguayan firms questions regarding their commercial debts and credits. With this information, we can build a network of commercial debt between firms that is representative of firms with more than 50 employees. From the

survey, we not only have information on the amount of commercial debts and credits, but we are also able to identify each firm's three major debtors and creditors. We can also identify the sector of the economy to which each firm belongs, allowing us to aggregate the results at the sectoral level.

The survey does not include firms belonging to the primary activity sector, financial inter-mediation, public sector or real estate activities. Hence, the information about the connections with these sectors is incomplete, and connections are inferred from the answers of other firms that declared owing debt to or having been a creditor of some firm in these sectors.

A second database contains balance sheet information for 2015. A larger sample of the commercial credit survey, this database is representative of firms with more than 10 employees. Merging the two databases gives us balance sheet information for more than 500 firms. We use the Consumer Price Index to update balance sheet information until October 2018.¹

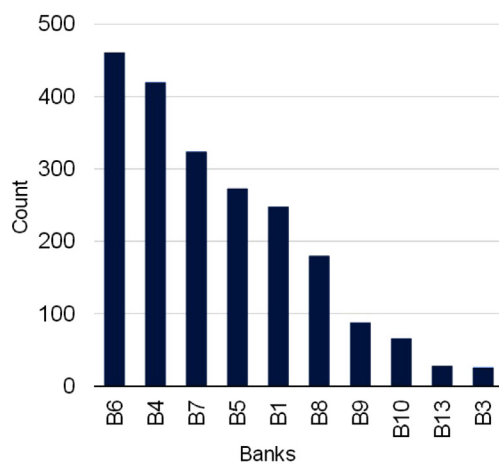
Another important data set is the Central Bank Credit Registry database containing all the loans given to firms by banks; this data set allows us to identify all the credit lent by financial institutions to companies and construct the bank-firm network.

By combining these data sets, we obtain three networks.

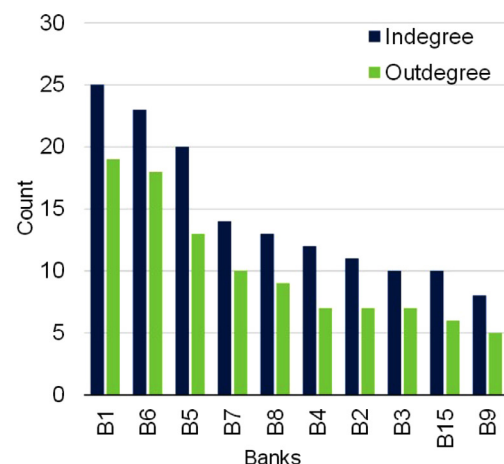
- Firm-bank network: Uruguay has 11 banks, but one of them only provides mortgage credit to families. With the data, we can build the network of credit lent by banks to firms. The total number of firms considered includes the 240 surveyed plus the three main debtors and creditors declared by those firms (Table 1). The final network has 1073 firms and 10 banks (Fig. 1a).
- Financial institutions network: We also build a network for inter-bank loans with data provided by the Superintendency of Financial Services from October 2018. In this network, we consider the loan and derivative exposures between banks and other financial institutions. The network has 26 institutions, 15 of which are other financial institutions in the Uruguayan financial system and 11 are banks. Ten of these banks lend to firms, while one bank mainly provides mortgage credit to families. According to the results presented in Table 1, 57% of the firms in the network have credit from the banking system (Fig. 1b).
- Firm-firm network: This network is constructed considering commercial lending between the 240 surveyed firms and their three major creditors and debtors. In total, there are 1073 firms in this layer.

Table 1
Banks-firms network.

Available data	
Banks	11
Other financial institutions	15
Surveyed firms	240
Surveyed firms + main creditors + main debtors	1073
Firms with bank credit	613



(a) Degree banks-firms



(b) Degree banks-financial institutions

Fig. 1. Financial institutions' degree.

4. Methodology

In this section, we describe basic concepts related network analysis. These concepts are the building blocks for the rest of the paper. We follow a similar notation used in Martinez-Jaramillo et al. (2014).

¹ We use 2015 balance sheet data because they are the latest information available.

We define a graph as an ordered pair $G = (V, E)$ where V refers to the set of vertices and E is the set of edges, which are unordered pairs (simple graph or undirected graph) from V .

A directed graph or digraph is a graph $D = (N, A)$ where N refers to the set of nodes and A is the set of arcs, which are ordered pairs from N . For directed graphs, the direction of the link is relevant.

For the above-defined mathematical objects, there are two additional objects known as the undirected weighted graph and the directed weighted graph. The important additional element is the function weight, $w : R \rightarrow E$, which assigns a number to each edge or arc, depending on whether an edge is directed (arc).

In the rest of the paper, we adopt the convention of calling networks to the weighted versions of graphs and digraphs.

The undirected networks are represented as $N = (V, E, w)$, where N refers to the network and V refers to the vertices or nodes and is a finite set different from zero. E is the unordered pairs set and w , as defined above, refers to the weight assigned to each edge or arc. (V, E) is an undirected graph.

A common mathematical representation of the networks comes in the form of a matrix.

In the mathematical literature, there are two ways to describe an adjacency matrix. The first is defined through an ordered list of arcs (i, j) . For an undirected graph, the adjacency matrix is defined as

$$A_{ij} = \begin{cases} 1 & \text{if } \{i, j\} \in E \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

We could take directions into account and define the following two matrices A_{ij}^+ and A_{ij}^- which can be useful for performing computations related to in and out degrees.

$$A_{ij}^+ = \begin{cases} 1 & \text{if } (i, j) \in E, \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$A_{ij}^- = \begin{cases} 1 & \text{if } (j, i) \in E, \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

By neighbor of a node i , we refer to the existence of an edge that connects the nodes. In particular, from the adjacency matrix we define the neighbors of a node i . For an undirected graph $G = (V, E)$, the vertex $j \in V$ is the neighbor of the node $i \in V$, if an edge that connects them.

$$N(i) = \{j \in V : a_{ij} = 1\}$$

A weighted matrix represents a weighted network. In the financial context, the weight of the arcs in the directed networks represent, for instance, money flows, exposures, correlations, and transaction values.

We define an external weight matrix W^+ from a directed network $R = (N, A, w)$ as follows:

$$W_{ij}^+ = \begin{cases} w_{ij} & \text{if } (i, j) \in A, \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

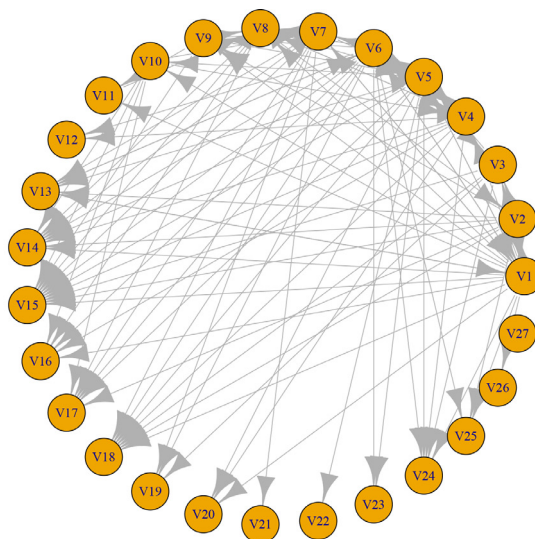
The internal weight matrix W^- is defined as follows:

$$W_{ij}^- = \begin{cases} w_{ij} & \text{if } (j, i) \in A, \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

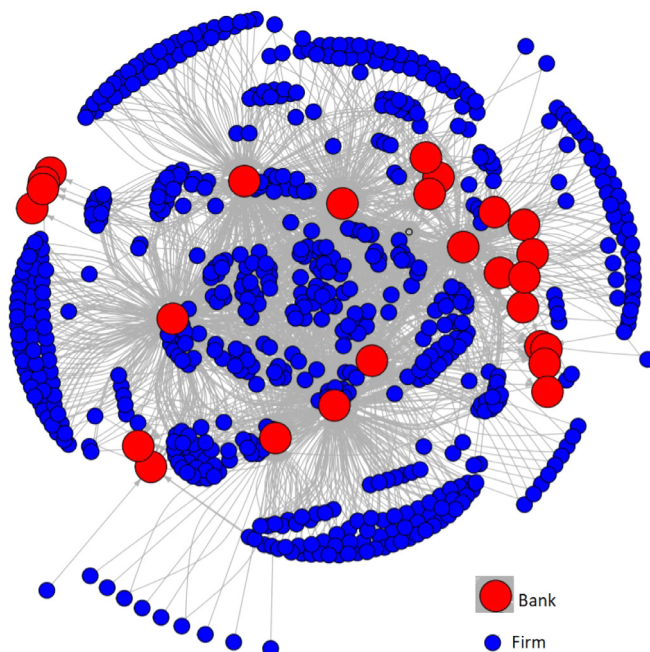
The interpretation of money flow used in financial networks depends on the matrix to which money belongs. For instance, to calculate losses from contagion in an exposures network, (i, j) , the matrix W^+ refers to the amount of money that institution i is exposed to by institution j ; in other words, the amount of money that j owes to i . The interpretation for W^- is the same but in the other direction.

We can define $W = W^+ + W^-$ as the total weight matrix (i.e., the connection between both institutions).

Fig. 2a shows the network representation of the inter-bank exposures network, whereas in Fig. 2b we plot the bank-firm exposures of the Uruguayan financial system. We use credit registry data that allows us to identify all the credit lent by financial institutions to firms. This bipartite network represents banks' credit portfolios, where bank nodes are colored red and firms blue. In the center, most of the firms are highly interconnected with red nodes (banks). The banks in the top left-hand side of Fig. 2b have few connections with firms, whereas the firms in the right-hand side show overlapping portfolios.



(a) Inter-bank exposures



(b) Bank-firm exposures

Fig. 2. Exposures networks.

4.1. Network metrics

To characterize the network and identify the nodes (banks) that are more central, we use conventional measures of centrality. [Borgatti \(2006\)](#) develop a framework for the measurement of centrality by assessing the nodes involved in the walk structure of a network. They establish four key dimensions: type of node, type of walk, property of walk and choice of summary measure. The authors define a walk from node u to node v as “a sequence of adjacent nodes that begins with u and ends with v ”.

Because our interest is in the bank credit risk exposure, we compare the differences in the centrality measures when considering the bank-to-bank network and the inferred firm and bank network.

Silva et al. (2016) divide the structural measures into three categories: local, quasi-local and global. Local measures refer to and describe only the node to which these measures belong. They do not take into account either neighboring nodes or the rest of the nodes in the network. Quasi-local measures refer only to the node to which these measures belong, but they take into account the characteristics of each node's neighbors as additional information. These measures are relevant to identifying the nodes to which a given node connects and provide useful information that complements the information provided by local measures. Global measures, in turn, use global information from a network and are useful in identifying the characteristics of the network as a system, not only the characteristics of individual nodes.

4.1.1. Centrality

Centrality measures are an important tool for ranking nodes according to their relevance in the network. The larger the centrality measure, the greater the importance a node has in a network (Martinez-Jaramillo et al., 2014). Some examples of centrality measures we use in this analysis are degree, strength and DebtRank.

The degree centrality of a node is defined as the number of edges attached to it. The greater the degree centrality, the larger the number of institutions affected. Degree centrality is defined as the degree of each node:

$$C_d(i) = d_i \quad (6)$$

In-degree centrality only considers the edges that extend to node i . Out-degree centrality, on the other hand, only considers the edges that originate in node i .

The strength centrality refers to the sum of its interbank assets and liabilities. Inner strength is the sum of its interbank assets, and the outer strength is the sum of its interbank liabilities. This information is useful in determining which bank is lending (borrowing) the most in the network.

$$C_s(i) = s_i \quad (7)$$

A ranking of banks differs if we consider the inter-bank market or the financial credit between banks and firms. Moreover, we investigate banks and firms' centrality by adding to these two layers a third one that considers the firm-to-firm commercial debt layer. Then, our results are referred to as a multi-layer network.

4.2. DebtRank

Battiston et al. (2012) introduced DebtRank as a method to compute the systemic importance of financial institutions based on their position within a network of inter-bank exposures.

The algorithm assumes that when an institution suffers some losses, they are propagated to its creditors because of credit quality deterioration, and it can therefore account for the propagation of shocks before the default of financial institutions.

Here, we consider the approach to computing the DebtRank as proposed in Bardoscia et al. (2015). Given a network W of inter-bank exposures, with W_{ij} the exposure of institution i towards institution j , and the equity of institution i denoted by E_i and the relative loss of equity of bank i denoted by h_i , the algorithm reduces to the calculation of the fixed point of the following map:

$$h_i(t) = \min \left\{ 1, \sum_j \frac{W_{ij}}{E_i} h_j(t-1) + h_i(1) \right\}, \quad (8)$$

where $h_i(0) = 0$, and $h_i(1)$ is the relative loss of i associated with an exogenous shock. Eq. (8) implements the idea that the loss of bank i is due to the exogenous shock plus a contribution that comes from exposures to institution i .

DebtRank can be used to compute the impact and vulnerability metrics for each institution in the network. The overall impact of institution i is the loss that an exogenous shock corresponding to the default of i would cause to the system. It is important to note that when we measure the impact we subtract the losses due to the exogenous shock, so that the *impact* only refers to losses due to network effects. Conversely, the vulnerability of institution i is its average loss if the exogenous shock causes the default of another institution.

4.3. Beyond inter-bank exposures

In the network literature on financial contagion and systemic risk, inter-bank exposures have been the main object of study. Less work has been published on bank-to-firm networks and even less on firm-to-firm networks. There are a few exceptions to these extensions: Lux (2014), Lux and Luu (2019), Marotta et al. (2015), Silva et al. (2018), Poledna et al. (2018) and Baron et al. (2020). Nevertheless, only in Poledna et al. (2018) is the firm-to-firm network estimated from aggregated balance sheet data, and in Baron et al. (2020), intra-firm exposure is estimated at the sectoral level.

Poledna et al. (2018) characterize the different exposures in a liability matrix that links the banking system with the real economy, represented by the firms that borrow from the banking system. In their representation of the Austrian economy, the bipartite network (at least the one with measurable links to the banking system) has two types of nodes: banks B and firms F , $|B| = b$, $|F| = f$, $n = b + f$. There are links between banks (inter-bank), links between banks and firms (firms' deposits in banks and banks' credits to firms), and links between firms (intra-firm). Mathematically, such pattern of interactions can be represented by a matrix with the following block structure:

$$W_{n \times n} = \begin{pmatrix} BB_{b \times b} & BF_{b \times f} \\ FB_{f \times b} & FF_{f \times f} \end{pmatrix}, \quad (9)$$

where BB is the inter-bank exposures matrix, BF is the bank-firms loans matrix, FB is the firms' deposits in banks and FF is the intra-firm exposures matrix.

As previously mentioned, the relations in BB have been extensively studied in the literature about inter-bank contagion, and the BF block has also been studied as a bipartite network (see, e.g., [Ramadiah et al., 2020](#)). The FB block is harder to study empirically and, to the best of our knowledge, no studies use this data, with the exception of [Silva et al. \(2018\)](#). Finally, [Silva et al. \(2018\)](#) and [Poledna et al. \(2018\)](#) also study the FF block; however, they estimate it from payment system data and from aggregated balance sheet data, respectively. Another exception that uses the same data upon which we base our empirical analysis is [Baron et al. \(2020\)](#), who perform the analysis at the sectoral level. The contribution of this paper is then to estimate the FB block at the firm level by resorting to survey data.

In the following, we perform a systemic risk analysis of the whole system represented by the matrix $W_{n \times n}$. To this end, we use an extension of the DebtRank algorithm that also accounts for the bank-firm and firm-firm interactions. The algorithm is the same as the one described in the previous section:

$$h_i(t) = \min \left\{ 1, \sum_j \frac{(W_{n \times n})_{ij}}{E_i} h_j(t-1) + h_i(1) \right\}, \quad (10)$$

where we replaced W with $W_{n \times n}$, and where the index i runs over banks and firms, instead of only over banks.

4.4. Reconstruction of firm-to-firm network

We consider alternative methods to reconstruct the firm-to-firm network: ME ([Upper and Worms, 2004](#)), MD ([Anand et al., 2014](#)), and a fitness model ([Caldarelli et al., 2002](#); [Park and Newman, 2004](#); [Squartini and Garlaschelli, 2011](#)).

We also use a combination of a fitness model and ME. The fitness model can be used to compute probabilities for missing links to exist in the network. These probabilities are computed such that the number of creditors and debtors of each individual institution is equal, on average, to the one observed empirically. The linking probabilities can then be used to produce adjacency matrices that correspond to plausible network structures compatible with the number of counterparties of each institution. As with the ME case, the RAS algorithm can then be used to assign weights to the existing links. This method generates networks with a connectivity degree that is intermediate between those of the ME and MD. These and other methods are well documented in [Anand et al. \(2018\)](#).

We now provide more details about how we reconstruct the firm-to-firm network, taking into account all the information available from the survey. We have a system of N firms. For each firm i , we know the total amount a_i of loans to other firms, the total amount ℓ_i of money borrowed from other firms, the number of creditors k_i^{out} and the number of debtors k_i^{in} (the convention we use is that a link goes from the borrower to the lender). We also know the identity of a subset of creditors and debtors. We denote these subsets, respectively, by v_i^{out} and v_i^{in} .

Given the information specified above, we have an incomplete matrix of inter-firm exposures that we need to fill by satisfying the constraint on the total in and out degree and in and out strength of each node (in and out strengths are a_i and ℓ_i respectively). To achieve this goal, we proceed with a two-step method: First, we reconstruct a binary adjacency matrix that satisfies (on average) the constraint on in and out degree using a fitness model. Second, we assign weights to the links using the RAS method.

4.4.1. Binary adjacency matrix via the fitness model

According to the fitness model the probability that a link from node i to node j exists is given by [Park and Newman \(2004\)](#); [Squartini and Garlaschelli \(2011\)](#):

$$p_{i \rightarrow j} = \frac{x_i^{\text{out}} x_j^{\text{in}}}{1 + x_i^{\text{out}} x_j^{\text{in}}}, \quad (11)$$

where the set of variables x_i^{out} and x_i^{in} , called fitness, have to be computed in such a way that the constraints on the in and out degrees are satisfied; that is, by solving the following set of equations:

$$k_i^{\text{out}} - |v_i^{\text{out}}| = \sum_{j \notin v_i^{\text{out}}} \frac{x_i^{\text{out}} x_j^{\text{in}}}{1 + x_i^{\text{out}} x_j^{\text{in}}} \quad \forall i \in \{1 \dots N\} \quad (12)$$

$$k_i^{\text{in}} - |v_i^{\text{in}}| = \sum_{j \notin v_i^{\text{in}}} \frac{x_i^{\text{in}} x_j^{\text{out}}}{1 + x_i^{\text{in}} x_j^{\text{out}}} \quad \forall i \in \{1 \dots N\}, \quad (13)$$

where $|v_i^{\text{in}}|$ and $|v_i^{\text{out}}|$ represent the number of elements in sets v_i^{in} and v_i^{out} (i.e., the numbers of creditors and debtors for which we know the exposures).

Once we have solved the above set of equations to determine the values of the x_i 's, we can generate an instance of a binary adjacency matrix by drawing each link $i \rightarrow j$ with probability $p_{i \rightarrow j}$.

4.4.2. Assigning weights via the RAS algorithm

Once we have determined which links are present in the network, we have to assign weights to those links. We know the weight of the links from the data (i.e., those in the sets v_i^{in} and v_i^{out}) and thus we assign them the known weights. To the other links, we assign weights through the following iterative procedure (n denotes the iteration):

$$W_{i \rightarrow j}^{(2n)} = \frac{W_{i \rightarrow j}^{(2n-1)}}{\sum_{j \notin v_i^{\text{out}}} W_{i \rightarrow j}^{(2n-1)}} \left(\ell_i - \sum_{j \in v_i^{\text{out}}} W_{i \rightarrow j} \right) \quad (14)$$

$$W_{j \rightarrow i}^{(2n+1)} = \frac{W_{j \rightarrow i}^{(2n)}}{\sum_{j \notin v_i^{\text{in}}} W_{j \rightarrow i}^{(2n)}} \left(a_i - \sum_{j \in v_i^{\text{in}}} W_{j \rightarrow i} \right) \quad (15)$$

The idea of the above iterations is that in even (odd) steps, we re-scale the unknown weights such that the sum of the elements in each row (column) is equal to the total amount of debt (credit). Clearly, when we enforce the sum over the rows, the one over the columns will be off, and vice versa. The idea is to iterate the equations until we reach a given precision.

4.5. Building a network of effective exposures of banks towards firms

Let us denote by V_{ai} the exposure of bank a to firm i . In our case, this is associated with a loan from the bank to the firm. However, in the presence of credit relationships between firms, a bank can be exposed to firms to which it did not directly lend because if bank a lends to firm i and not to firm j , but firm i lends to firm j , the inability of firm j to pay its debt to firm i may affect bank a . In Fig. 3, bank a is only exposed to firm 1 because firm 1 is not linked, through commercial credit, to any other firm. In the right figure, bank a is directly linked to firm 1 and indirectly exposed to firm 3 and firm 4 because they owe debt to firm 1.

Rather than trying to construct a micro-founded model of how these types of shocks propagate in the network of firms, we consider the existence of effective exposures of banks to firms.

These can be computed as follows: Let W_{kj} denote the amount lent by firm j to firm k , and D_k denote the total debt of firm k (this includes debt to firms as well as to banks). We can say that firm j owns a fraction, $\frac{W_{kj}}{D_k}$, of k 's debt. If firm k lent an amount W_{ik} to firm i , k owns a fraction, $\frac{W_{ik}}{D_i}$, of i 's debt. This means that firm j effectively owns a fraction, $\frac{W_{ik} W_{kj}}{D_i D_k}$, of k 's debt. This intuition can be extended to all types of paths in the network, so that we can say that bank a is effectively exposed to firm i by an amount equal to the following:

$$\tilde{V}_{ai} = V_{ai} + \sum_j \frac{V_{aj}}{D_j} \Pi_{ij} D_i + \sum_{jk} \frac{V_{aj}}{D_j} \Pi_{ik} \Pi_{kj} D_i + \dots \quad (16)$$

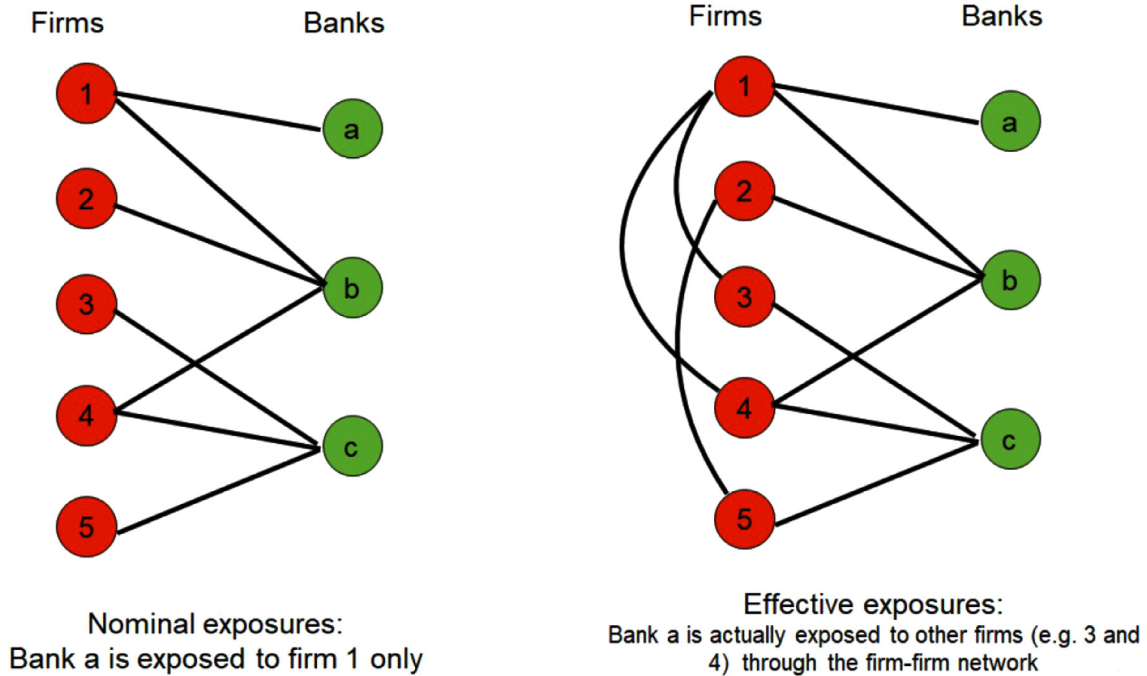


Fig. 3. Effective exposures.

$$= \sum_j [(1 - \Pi)^{-1}]_{ij} \frac{V_{aj}}{D_j} D_i, \quad (17)$$

where $\Pi_{ij} = W_{ij}/D_i$.

The intuition of Eq. (16) is the following: The effective exposure of a to i is equal to a sum that accounts for all possible paths in the network that connect a to i . The first term of the right-hand side is the direct exposure of a to i . In the second term, the factor D_i is the loss associated with the default of firm i . A fraction Π_{ij} of this loss is passed to firm j , which in turn passes a fraction V_{aj}/D_j to bank a . The sum over j accounts for all possible paths of length 2 from a to i in the network. In the third term of the right-hand side, the loss is passed from i to k , then from k to j and finally from j to a , so that paths of length 3 are considered, and so on.

The underlying assumption is that if a firm defaults, then its creditors (linearly) propagate some loss to their creditors, and so on. In practice, the propagation could stop if some creditors absorb the loss without passing it on further. The exposures calculated following Eq. (16) are therefore only an upper bound to effective exposures, whereas nominal exposures are a lower bound.

5. Results

The first result is the successful application of the DebtRank algorithm to the inter-bank exposures network. This network consists of two layers: the unsecured lending layer and the derivatives layer. The unsecured lending layer considers inter-bank credit without collateral, whereas the derivatives layer includes exchange rate swaps among banks.

Fig. 4 presents the systemic risk profile for the banking system in Uruguay. Neglecting the derivatives layer can underestimate banks' systemic importance in the network. It is both important and useful to consider both layers of exposures to estimate systemic risk and to obtain a profile of the banking system in order to compute the contribution of each type of exposure for each individual bank to systemic risk. As an example, DebtRank obtained for the failure of bank 4 when considering only the unsecured credit exposure is about three times lower than when we consider both layers (unsecured credit exposure and derivative) together. Considering only the unsecured lending exposure, bank 10 has a lower DebtRank than banks 1, 5 and 6, but the risk imposed by bank 10 is higher than the risk imposed by those banks when we consider both layers simultaneously.

The second set of results involves the bank-firms bipartite and the intra-firm exposures networks. Similar to our method for the inter-bank exposures, we compute the DebtRank for the full exposures network for all the three reconstruction methods. In this way, we can find not only systemically important banks, but also systemically important firms.

Fig. 5a and b show the difference between effective and nominal exposures to point out the size of the effective exposures due to indirect links. In particular, Fig. 5a shows the effective exposures effect of the inter-bank network. Each entity has three bars of effective exposures (one bar for each reconstruction method). The effect is similar for each methodology, but for some entities, the effective exposure due to indirect links is slightly larger for the constrained RAS methodology (e.g., entities 5 and 10). Fig. 5b shows the difference between effective and nominal exposures' effect according to each methodology for the intra-firm network, mainly

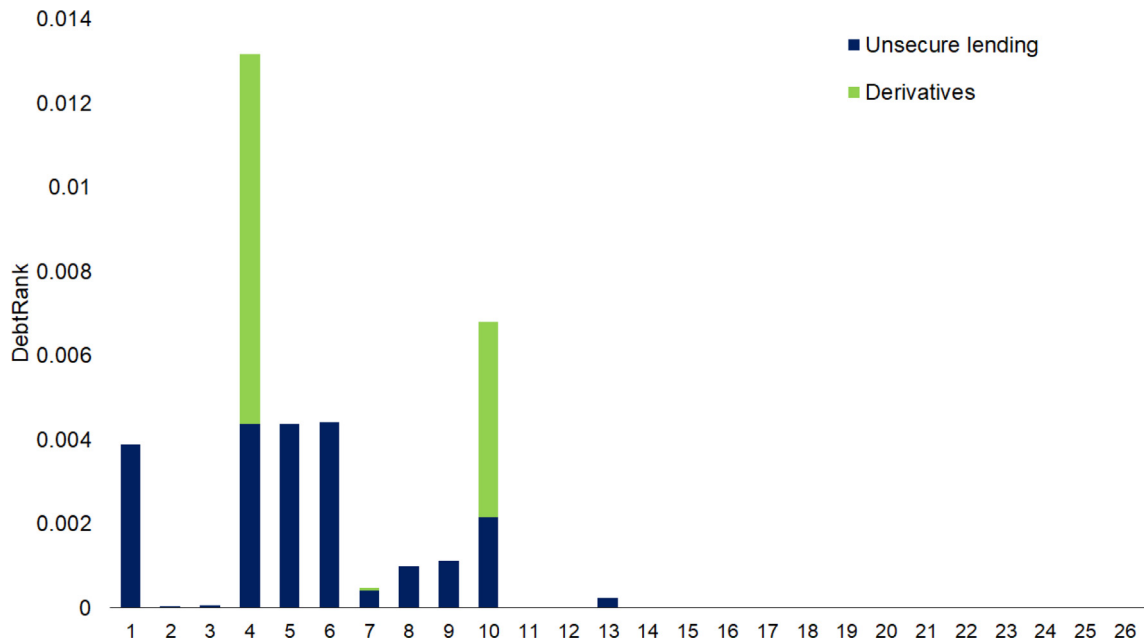
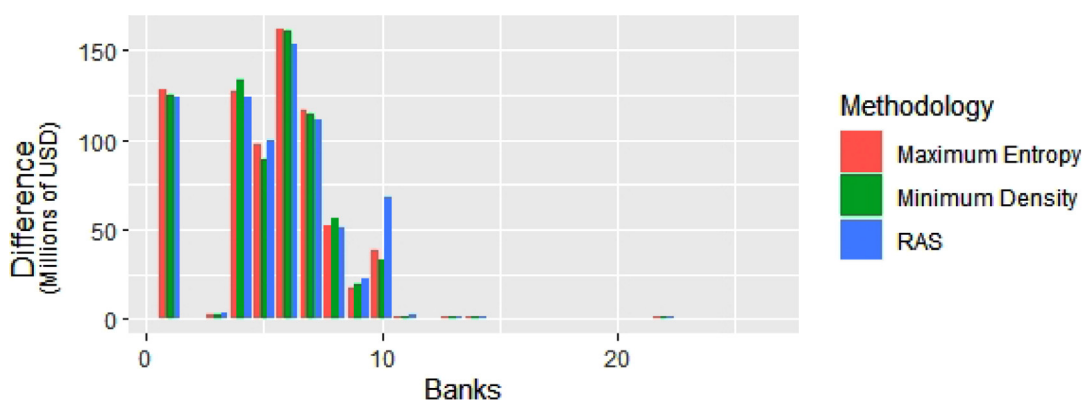
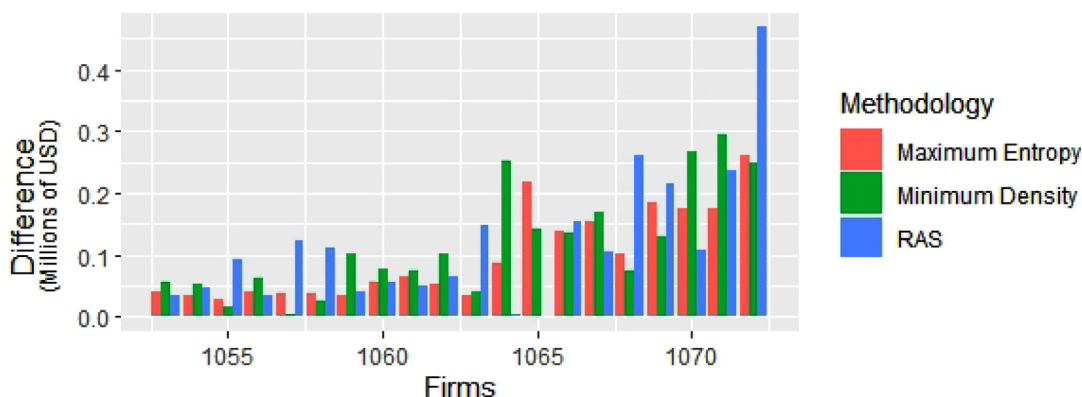


Fig. 4. Banking system's risk profile.



(a) Banks' effective exposures



(b) Firms' effective exposures

Fig. 5. Effective exposures.

the 20 firms with the larger effective exposure. For the intra-firm network, it is not so clear whether one of the methodologies has a larger effective exposure. For instance, some firms have larger effective exposures for the RAS methodology, whereas for other firms, the ME contains the larger effect.

The survey information provides data for 1187 observations (Fig. 6a), which presents a sparse matrix. Applying the RAS algorithm to complete the matrix from the known information (Fig. 6b) increased the non-zero elements to 14,999 observations. This means that the RAS algorithm provides more useful information for vulnerability and impact analysis.

In Fig. 6c, we use the ME methodology, which shows a plentiful matrix with 430,487 observations. On the other hand, with the MD methodology, the number of observations decreases to 1184—almost the same number of observations from the survey (Fig. 6d). In Fig. 7, we show the intersection between the survey observations (blue) and MD methodology (red). Notably, the intersection between the MD methodology (red) and the survey observations (blue) is minimal, we find four intersections between the two matrices (red circles, Fig. 7). The MD methodology does not resemble the original matrix; in fact, only three arcs are present in both structures. The constrained RAS algorithm takes into account all of the information available from the survey to reconstruct the firm-to-firm network and respects the original structure of the information.

We consider the information about intra-firm exposures to analyze the changes in vulnerability among both banks and firms. We break down the systemic importance of banks and firms into their vulnerability regarding shocks suffered by other banks and firms. We show the top 20 banks and top 20 firms by the different methods (Fig. 8a and b).

For more detail, we plot the vulnerability for all banks and all firms for each method. In the base case, we found that at least one bank was importantly vulnerable when we included intra-firm exposure information. Its vulnerability reaches 90%, meaning it has an important effect on the inter-bank network. Notably, this is a small, non-systemic bank with a very small proportion of total credit. Concerning firms, we find that in some cases, their vulnerability goes from 0.1% to 0.8% (see Appendix, Fig. 11a).

The different methods of reconstructing the firm-to-firm credit network have different effects on the quantification of vulnerability, directly affecting the effective exposures. As expected, a sparse network obtained with the MD methodology results in lower quantification of the vulnerability than that of a denser intra-firm network, like the one obtained with the ME algorithm. We can consider both metrics as lower and upper bounds of the vulnerability quantification, while the RAS algorithm provides an interme-

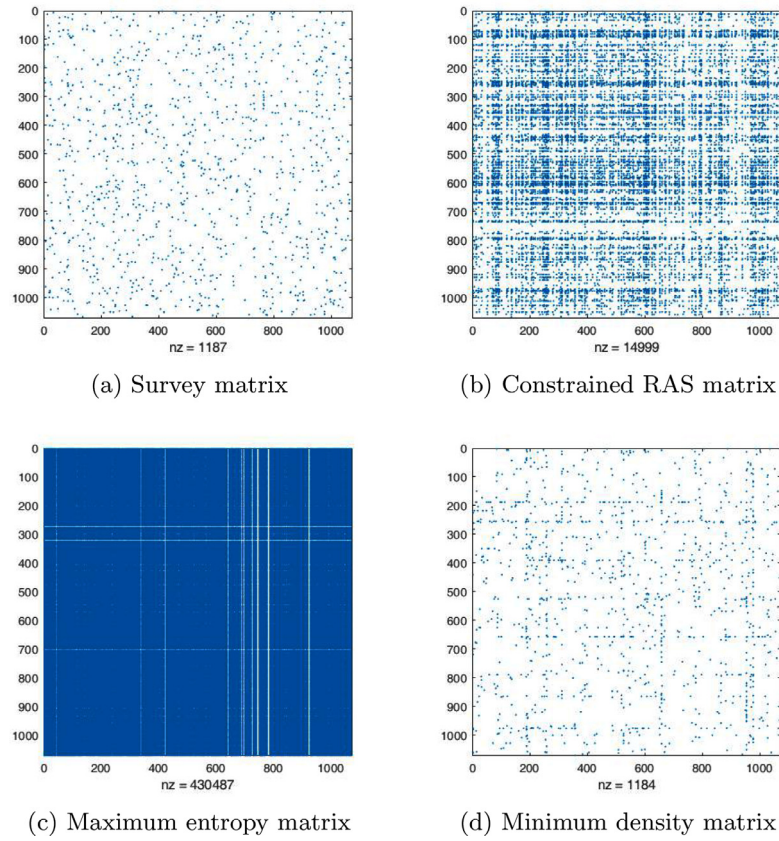


Fig. 6. Matrix of exposures-estimation methods.

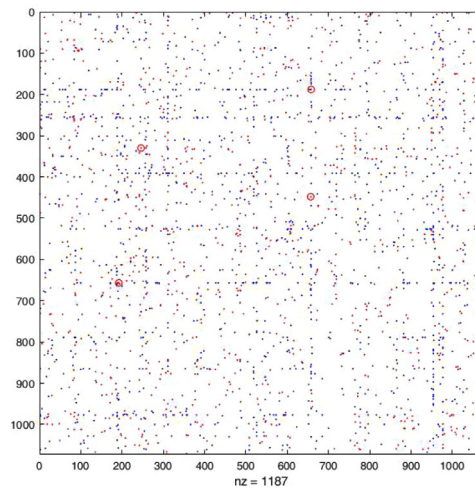
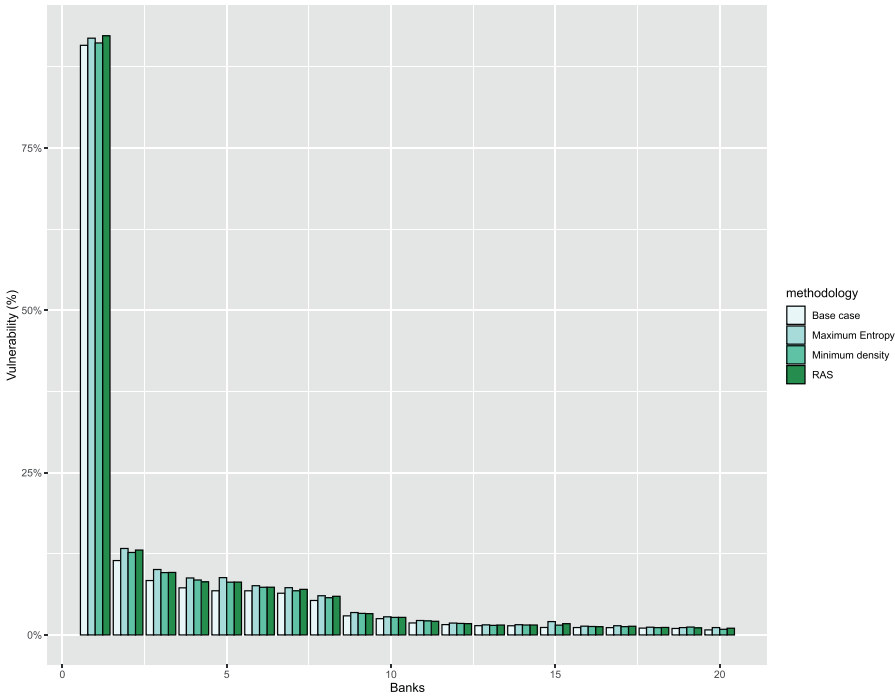


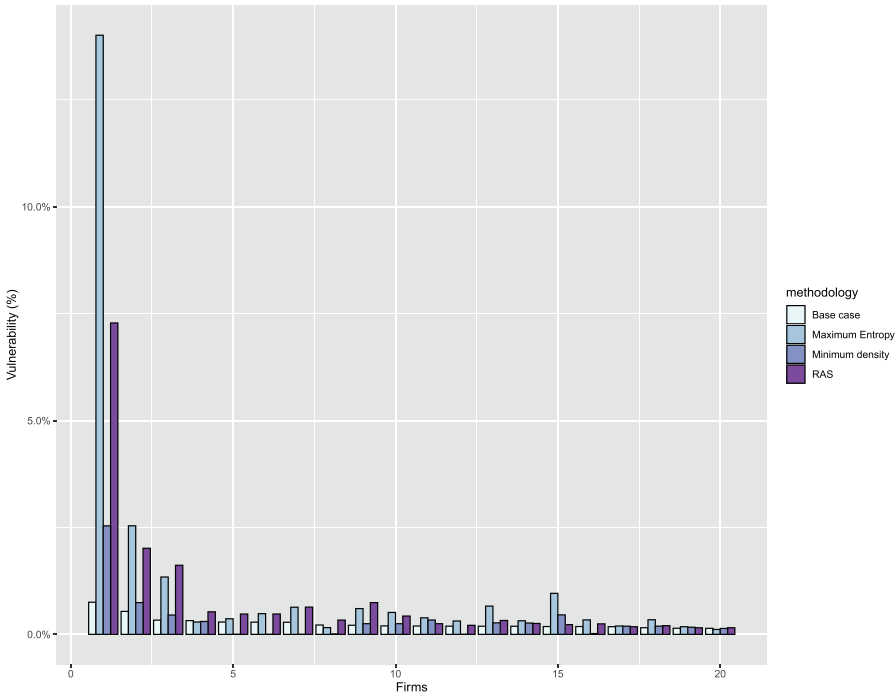
Fig. 7. Intersection survey and minimum density matrix.

diate result. For instance, the RAS algorithm presents a slight increase in the vulnerability of some banks. The same applies to firms, whose vulnerability goes from 1% to 7%. At the aggregate level, the vulnerability of firms when considering intra-firm exposures is around 21% higher as compared to the base case (see Appendix, Fig. 11b). The 21% results from computing the aggregated sum of the differences for each firm between the intra-firm network from the RAS methodology and the base case.

The analysis using the ME method shows more firms with higher vulnerability than in the base case and when using the RAS methodology. The vulnerability of firms goes from 1% to 14%, and the aggregate vulnerability for firms, including intra-firm exposures, increases by around 37% relative to the base case (see Appendix, Fig. 11c).



(a) Banks



(b) Firms

Fig. 8. Vulnerability with different methodologies.
Losses as percentages of Tier 1.

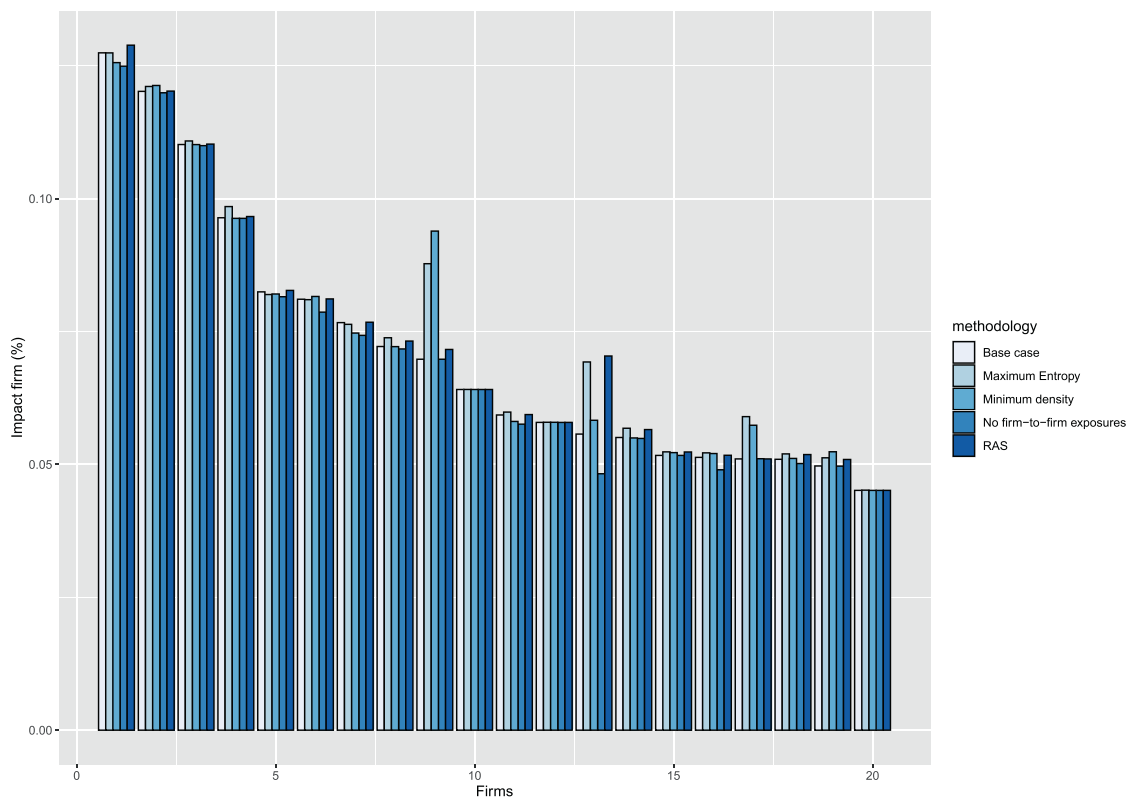


Fig. 9. Firm impact with different methodologies.

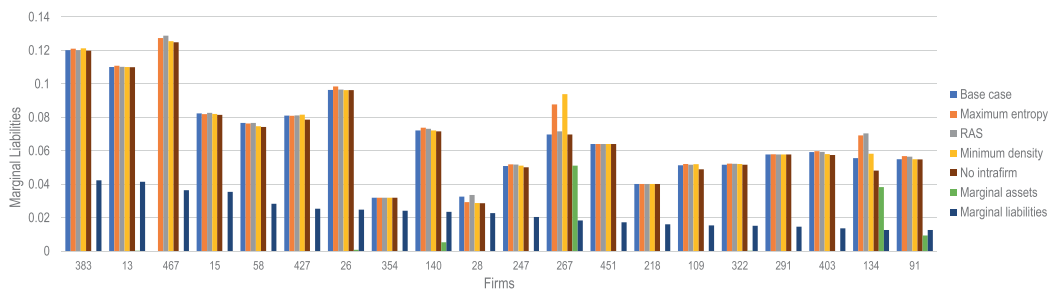
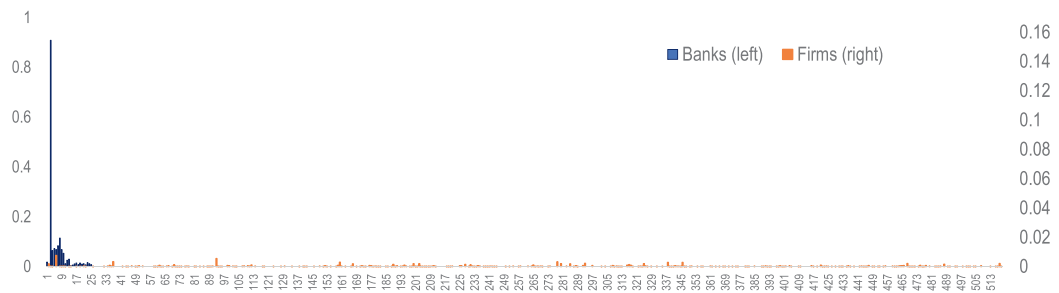


Fig. 10. Ranking by marginal liabilities.

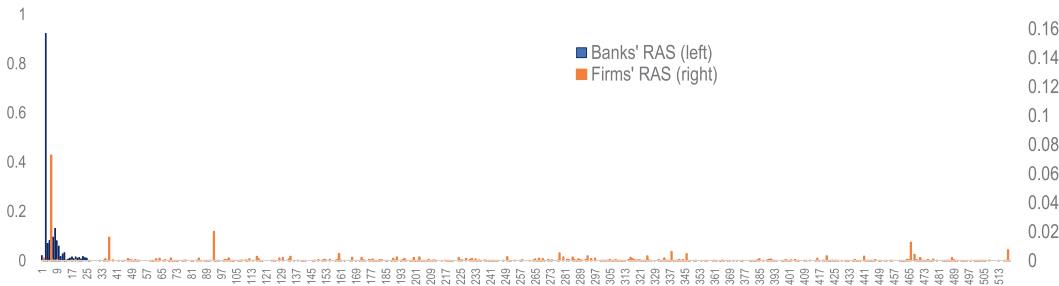
On the other hand, the MD approach shows lower levels of vulnerability for firms, from 0.1% to 1.0%, in contrast with the ME methodology and the RAS algorithm. Aggregate vulnerability for firms, when we consider intra-firm exposures, is around 11% higher as compared to the base case (see Appendix, Fig. 11d).

In the next section, we analyze the changes in firm impact due to the inclusion of intra-firm exposures. We quantify the aggregate firm impact using the different methodologies. As mentioned above, the overall impact of an institution i is the relative equity loss that an exogenous shock corresponding to the default of i would cause to the system. In Fig. 9, we plot the five cases: no firm-to-firm exposure (only DebtRank information), the base case, ME, MD, and constrained RAS. We order the firms by impact and compare their behavior when using different methodologies. For better visualization and comparison, we only plot the top 20 entities.

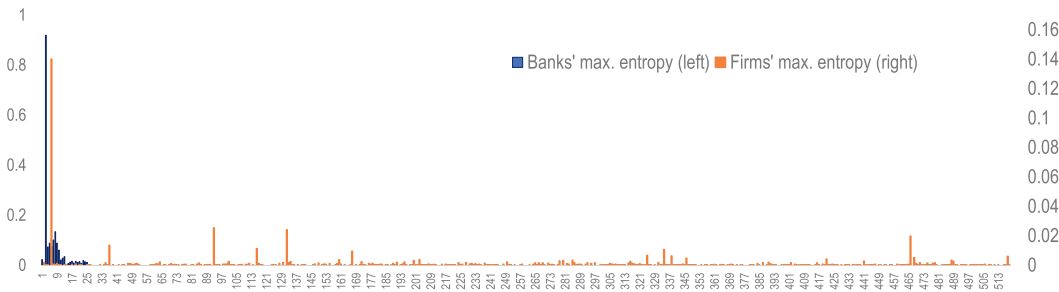
If the firm's impact does not include intra-firm exposure information, we find that some firms have larger contributions to the overall system, such as entities that show an impact of around 10% to 12%. Then, we compute the aggregate sum of the difference between including and not including the intra-firm exposures for each methodology (base case, ME, MD, and constrained RAS) to quantify the aggregated firm impact of the shocks received due to other firms defaulting. For the base case, the intra-firm exposures increase the aggregated firm impact by around 18%. In contrast, intra-firm exposures with the ME method show around an 84% increase of the aggregate impact, as measured by the aggregate losses. Intra-firm exposures with the MD methodology increase the aggregate firm impact by around 62%. Finally, with the constrained RAS methodology, the intra-firm exposures increase the aggregate firm impact by around 63%.



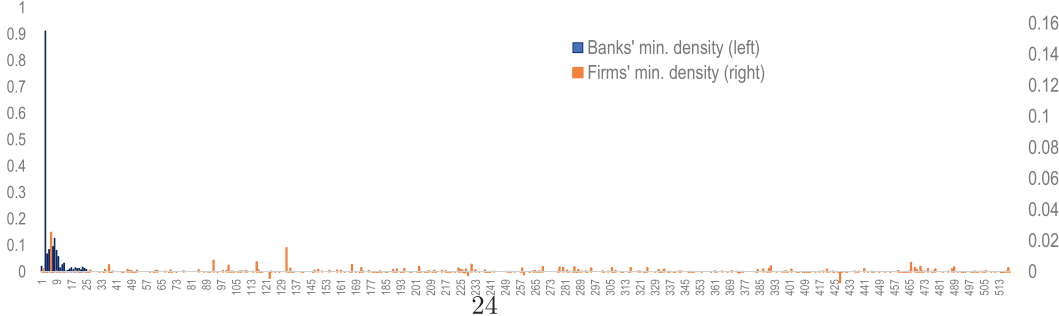
(a) Base case



(b) RAS



(c) Maximum entropy



(d) Minimum density

Fig. 11. Vulnerability Losses as percentages of Tier 1.

Table 2
Top 10 ranking table by different methodologies.

Survey	Max. entropy	RAS	Min. density	Base case	Marg. assets	Marg. liab.
467	467	467	467	467	267	383
383	383	383	383	383	140	13
13	13	13	13	13	26	467
26	26	26	26	26	13	15
15	267	15	267	15	427	58
427	15	427	15	427	383	427
58	427	58	427	58	467	26
140	58	140	58	140	15	140
267	140	267	140	267	58	267
451	451	451	451	451	451	451

In Fig. 10, we order vulnerability, including intra-firm exposure information, by marginal liabilities. For each entity, we have the information for the three methodologies (RAS, ME, and MD), marginal assets, marginal liabilities, information on vulnerability without intra-firm exposures (0), and DebtRank (1). Importantly, we order the ranking by marginal liabilities because this is how contagion propagates. In the constrained RAS and DebtRank methodologies, most of the entities are not ordered in the same way as when entities are ranked by marginal liabilities. For instance, the third firm can affect around 13% of the equity in a firm network.

In Table 2 we illustrate the differences in the top 10 entities by method. We take as the pivot the survey information, and we found that the main differences occur when we order the entities by marginal assets and marginal liabilities, and with ME and MD. For the first 10 entities, the RAS and survey information have the same order of entities.

6. Conclusions and further work

This work has many interesting takeaways, but the most important is that ignoring the intra-firm exposures leads to systemic risk being underestimated. Moreover, the most important vulnerability among Uruguayan banks to financial contagion comes from the real sector of the economy, as opposed to the well-studied interbank exposures.

In the first part of our work, we compute the DebtRank for banks considering only direct inter-bank credit and derivatives exposures. The results showed a small impact from such direct exposures. Then, by using information from the credit registry, we built the bipartite banks-firms network to study the impact of firm failures on the banking system, considering network effects from the interbank exposures network. However, the most novel part of this work is the estimation of the intrafirm exposures network and its contribution to the systemic risk faced by the banking system.

We estimated the intrafirm exposures network using three methods and documented the differences; then, we estimated the contribution of the information contained in the intrafirm exposures network to systemic risk; finally, we were able to identify systemically important firms based on their impact on banks and other firms, taking into account contagion (network) effects. In order to achieve these goals, we computed the DebtRank centrality metric for both the interbank network and for an extended network of exposures, including firms.

Before concluding, we would like to highlight two side products of this work: (i) the comparison of three methods of reconstructing firm-firm exposures networks in terms of systemic risk (the ME, the MD, and a constrained version that uses the RAS algorithm) and (ii) the computation of effective exposures, which show that banks are exposed among themselves beyond their direct credit lines given to firms through firm-firm lending relationships.

Declaration of Competing Interest

Authors declare that they have no conflict of interest.

Appendix A. Additional results

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