Appendix – I

The details in this appendix explain some of the main concepts modelled in EnergyPlus, used to process buildings' input data for energy calculation purposes. It summarises the equations and algorithms used to process the following 20 functions:

1- Basis for the Zone and Air System Integration
2- Air System Control
3- Summary of Time Marching Solution
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6- Combined Heat and Moisture Transfer (HAMT) Model
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1- Basis for the Zone and Air System Integration:

In EnergyPlus, this process is defined as the Predictor/Corrector process. Starting with the heat balance on the zone air, solving the ordinary differential equations using a predictor-corrector approach:

$$C_z\frac{dT_z}{dt} = \sum_{i=1}^{N_{sl}} \dot{Q}_i + \sum_{i=1}^{N_{surfaces}} h_i A_i (T_{si} - T_z) + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p (T_{zi} - T_z) + \dot{m}_{inf} C_p (T_\infty - T_z) + \dot{Q}_{sys}$$

Calculating the energy stored in zone air ($C_z\frac{dT_z}{dt}$), with Cz = ρ_{air}CpCT, (ρ_{air}) being the zone air density, (Cp) is the zone air specific heat and (CT) is the sensible heat capacity multiplier, where;

$\sum_{i=1}^{N_{sl}} \dot{Q}_i$	sum of convective internal loads
$\sum_{i=1}^{N_{surfaces}} h_i A_i (T_{si} - T_z)$	convective heat transfers from the zone's surface
$\sum_{i=1}^{N_{zones}} \dot{m}_i C_p (T_{zi} - T_z)$	Heat transfers due to inter-zone air mixing
$\dot{m}_{inf} C_p (T_\infty - T_z)$	Heat transfers due to infiltration of outside air
$\dot{Q}_{sys}$	Air system output

Considering the air capacitance is zero, the system can be assumed at a steady state with:

$$-\dot{Q}_{sys} = \sum_{i=1}^{N_{sl}} \dot{Q}_i + \sum_{i=1}^{N_{surfaces}} h_i A_i (T_{si} - T_z) + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p (T_{zi} - T_z) + \dot{m}_{inf} C_p (T_\infty - T_z)$$

The system work to supply cold or hot air to the zone(s), consuming in process cooling and heating loads. This system is created to balance the tempratuer within the zone, assuming that the zone air mass flow rate supplied is equal to the sum of the air flow rates exiting the zone (through the system return air plenum), while Both air streams exit the zone at the zone mean air temperature, represented as:

$$\dot{Q}_{sys} = \dot{m}_{sys} C_p (T_{sup} - T_z)$$

Reflected on first equation above, ($C_z\frac{dT_z}{dt}$) can be represented as:

$$C_z\frac{dT_z}{dt} = \sum_{i=1}^{N_{sl}} \dot{Q}_i + \sum_{i=1}^{N_{surfaces}} h_i A_i (T_{si} - T_z) + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p (T_{zi} - T_z) + \dot{m}_{inf} C_p (T_\infty - T_z) + \dot{m}_{sys} C_p (T_{sup} - T_z)$$

Using three different solution algorithms to solve the zone air energy and moisture balance equations, EnergyPlus use 3rd Order Backward Difference and Euler method to solve the finite difference approximation, and the third being an analytical solution algorithm.

Solving a derivative as a function of time, the finite difference approximation is used as:

$$\frac{dT}{dt} = (\delta t)^{-1}({Tz}^{t} - {Tz}^{t-\delta t}) + O(\delta t)$$

Considering the cyclic nature of buildingenergy simulations, errors resulting from the many time steps are canceled over each daily cycle so there will be no net accumilation of truncation errors.

The Euler formula is to replace the derivative term in Equation 3, replacing the derivative term. By groping all the terms containing the zone mean air temptature, the other terms are considered unknown at the current time step, being lagged by one time step and collected on the other side of the equation. The resulting setup of these chnges drive a formual for updating the zone mean air tempateture:

$$Cz\frac{T_z^t - T_z^{t-\delta t}}{dt} + T_z^t\left(\sum_{i=1}^{Nsurfaces} h_i A_i + \sum_{i=1}^{Nzones} \dot{m}_i C_p + \dot{m}_{inf} C_p + \dot{m}_{sys} C_p\right) =$$

$$\sum_{i=1}^{Nsl} \dot{Q}_i^t + \dot{m}_{sys} C_p T_{supply}^t + (\sum_{i=1}^{Nsurfaces} h_i A_i T_{si} + \sum_{i=1}^{Nzones} \dot{m}_i C_p T_{zi} + \dot{m}_{inf} C_p T_\infty)^{t-\delta t}$$

By moving the lagged temparaature in the derivative approximation to the equation's right side, having the expicit appearance of the zone air temperature eliminated from one side, the energy balance equation is formulated to estimate the zone air temperatures:

$$T_z^t = \frac{\sum_{i=1}^{Nsl} \dot{Q}_i^t + \dot{m}_{sys} C_p T_{supply}^t + \left(Cz\frac{Tz}{\delta t} + \sum_{i=1}^{Nsurfaces} h_i A_i T_{si} + \sum_{i=1}^{Nzones} \dot{m}_i C_p T_{zi} + \dot{m}_{inf} C_p T_\infty\right)^{t-\delta t}}{\frac{Cz}{\delta t} + \left(\sum_{i=1}^{Nsurfaces} h_i A_i + \sum_{i=1}^{Nzones} \dot{m}_i C_p + \dot{m}_{inf} C_p + \dot{m}_{sys} C_p\right)}$$

To allow for the use of larger time steps adapting the Euler form developed above, higher order expressions for the first derivative corrispoding the the higher order trunction errors are used. It was found that the best results were obtained using the third order of the finite difference approximation:

$$\left.\frac{dTz}{dt}\right|_t \approx (\delta t)^{-1}(\frac{11}{6}T_z^t - 3T_z^{t-\delta t} + \frac{3}{2}T_z^{t-2\delta t} - \frac{1}{3}T_z^{t-3\delta t}) + O(\delta t^3)$$

Using this form of the derivative on the zone mean air temperature equation, the zone temperature update equation becomes:

$$T_z^t = \frac{\sum_{i=1}^{Nsl} \dot{Q}_i^t + \sum_{i=1}^{Nsurfaces} h_i A_i T_{si} + \sum_{i=1}^{Nzones} \dot{m}_i C_p T_{zi} + \dot{m}_{inf} C_p T_\infty + \dot{m}_{sys} C_p T_{supply}^t - (\frac{Cz}{\delta t})(-3T_z^{t-\delta t} + \frac{3}{2}T_z^{t-2\delta t} - \frac{1}{3}T_z^{t-3\delta t})}{(\frac{11}{6})\frac{Cz}{\delta t} + \sum_{i=1}^{Nsurfaces} h_i A_i + \sum_{i=1}^{Nzones} \dot{m}_i C_p + \dot{m}_{inf} C_p + \dot{m}_{sys} C_p}$$

This equation, using the third order backward difference, requires zone air temperatures at three porivious time steps, with the assumption that the time steps are at constant intervals.

Using the Analytical Solution algorithm as an integration approach, the objective is to aviode truncation errors caused by the three time intervals steps. This algorithm requires onlt one time step, transforming the equation as:

$$T_z^t = \left(T_z^{t-\delta t} - \frac{\sum_{i=1}^{N_{sl}} \dot{Q}_i + \sum_{i=1}^{N_{surfaces}} h_i A_i T_{si} + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p T_{zi} + \dot{m}_{inf} C_p T_\infty + \dot{m}_{sys} C_p T_{supply}}{\sum_{i=1}^{N_{surfaces}} h_i A_i + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p + \dot{m}_{inf} C_p + \dot{m}_{sys} C_p} \right) * \exp\left(-\frac{\sum_{i=1}^{N_{surfaces}} h_i A_i + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p + \dot{m}_{inf} C_p + \dot{m}_{sys} C_p}{Cz} \delta t \right) + \frac{\sum_{i=1}^{N_{sl}} \dot{Q}_i + \sum_{i=1}^{N_{surfaces}} h_i A_i T_{si} + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p T_{zi} + \dot{m}_{inf} C_p T_\infty + \dot{m}_{sys} C_p T_{supply}}{\sum_{i=1}^{N_{surfaces}} h_i A_i + \sum_{i=1}^{N_{zones}} \dot{m}_i C_p + \dot{m}_{inf} C_p + \dot{m}_{sys} C_p}$$

As the load on the zone contorls the entire simulation process, it is used as the starting point in calculating the air system demand. The simulation then accordingly defines the supply capabilites while adjusting the zone temperature if needed.

2- Air System Control:

The main function of the air system control is to sense the actual temperature within a zone, evaluate at the difference between the sampled temperature and the set (desired) temperature then sending a signal to operate the HVAC units in demand for cooling or heating. The objective is focused on maintaining the zone temperatures close to the set values. To calculate how much energy goes in to or out of a zone, the net zone load can be defined as:

$$\dot{Q}_{load} = \sum_{i=1}^{Nsl} \dot{Q}_i \sum^{\cdot} + \sum_{i=1}^{Nsurfaces} h_i A_i (T_{si} - T_z) + \sum_{i=1}^{Nzones} \dot{m}_i C_p (T_{zi} - T_z) + \dot{m}_{inf} C_p (T_\infty - T_z)$$

With multiple zones, T_z must be defined as the set zone temperatures for each zone, with the assumption that the capacity of the air system is capable of meeting the air conditioning required ($\dot{Q}_{load} = \dot{Q}_{sys}$). If the air system capacity cannot achieve the desired temperature, the air system is set to provide its maximum output to the zone. This method is named the *Predictive System Energy Balance.*

Calculating the amout of coolinh/heating supplied by the air system, corrisponding to the set zone temperatures is used through:

$$\dot{Q}_{sys} = \dot{m}_{sys} C_p \eta (T_{supply} - T_{z,desired})$$

(η) being the time step fraction between zero and one, when the air system is turned ON.

The air system defined as the Variable Air Volume (VAV) system is set to operate in a zone air temperature that can vary within limits, defined as the air damper range and the throttling range. In an Ideal system operation, when the zone air temperature rises between T_{cl} and T_{cu}, cooling is required, and the air system varies the supply air flow rate while maintaining a constant supply air temperature. When the zone air temperature drops between T_{hl} and T_{hu}, heating is required and the supply of air temperature varies while air supplied at a constant minimum flow rate. The system simulation would fall within these temperatures as long as the chosen system capacity is able to maintain the desired levels. In order to predict the air system response, the desired zone air temperature is then defined as a variable that must be calculated to determine the air system output. Using the Zone temperature updating equation,($\dot{Q}_0$)as the numerator and($\dot{Q}_{slope}$) as the denominator are used with the exclusion of the effects of zone capacitance:

$$\dot{Q}_0 = \sum_{i=1}^{Nsl} \dot{Q}_i + \sum_{i=1}^{Nsurfaces} h_i A_i T_{si} + \sum_{i=1}^{Nzones} \dot{m}_i C_p T_{zi} + \dot{m}_{inf} C_p T_\infty$$

$$\dot{Q}_{slope} = \sum_{i=1}^{Nsurfaces} h_i A_i + \sum_{i=1}^{Nzones} \dot{m}_i C_p + \dot{m}_{inf} C_p$$

When the system is operated for cooling, its Assumed that the volume flow rate varies linearly with zone air temperature. The supply air volume flow rate is normalized to the maximum flow rate, or supply air fraction. Further, to ensure sufficient fresh air is being supplied to the zone (eliminating contaminants), the minimum supply air fraction $\eta_{c,min}$ must be greater than zero:

$$\eta c = \eta c,\min + (1 - \eta c,\min)\left(\frac{Tz - Tc,lower}{Tc,upper - Tc,lower}\right) \; ; \eta_{c,min} \leq \eta_c \leq 1.0$$

On the other hand, when the system is operated for heating, it's also assumed that the heating energy output is linearly varying with the zone air temperature and normalized with respect to the maximum coil output. Yet while heating, the simulation can consider η_h to equal to zero as minimum:

$$\eta h = \left(\frac{Th,upper - Tz}{Th,upper - Th,lower}\right) ; 0 \leq \eta_h \leq 1.0$$

With these differences between the operating moods of colling and heating, two equations are needed to discribe the system using η_c and η_h. incerted in the air system output equation, the follwing expressions are used for cooling and heating respectively:

$$\dot{Q}_{sys,c} = C_p \rho (\eta_c \dot{V}max)(T_{c/c} - T_{z,pred,cool})$$

$$\dot{Q}_{sys,h} = \eta_h \dot{Q}_{h/c,max} + C_p \rho \dot{V}min \; (T_{c/c} - T_{z,pred,heat})$$

The experssions of η_c and η_h are then used to predict the zone air temprature in the cases of cooling and heating. The heating equation being valid for when the temprature is below $T_{h,upper}$; While the equation for cooling is valid for temperatures above that value.

$$T_{z,pred,heat} = \frac{\dot{Q}_{h/c,max} T_{h,upper}}{T_{h,upper} - T_{h,lower}} + \dot{Q}_0 + \frac{C_p \rho \dot{V}_{min} T_{c/c}}{\frac{Q_{\frac{h}{c},max}}{T_{h,upper} - T_{h,lower}} + C_p \rho \dot{V}_{min} + \dot{Q}_{slope}}$$

$$T_{z,pred,cool} = \frac{B1 + \sqrt{B_1^2 + B_2}}{2}$$

Where,

$$B_1 = T_{c/c} + T_{c,lower} - \frac{\eta_{c,min} - C2}{C1}$$

$$B_2 = 4\left(\frac{C_3}{C_1} + T_{c/c}\left(\frac{\eta_{c,min}}{C1} - T_{c,lower}\right)\right)$$

And,

$$C_1 = \frac{1 - \eta_{c,min}}{T_{c,upper} - T_{c,lower}} \quad , \quad C_2 = \frac{\dot{Q}_{slope}}{C_p \rho \dot{V}_{max}} \quad , \quad C_3 = \frac{\dot{Q}_0}{C_p \rho \dot{V}_{max}}$$

By calculating the predict zone air temperature for cooling and heating, the air system response is then calculated. As explained earlier when cooling, the temprature of the air supplied is constant at the system outlet. The varying volume rate is givin by:

$$\dot{V}_{supply} = \eta_c \dot{V}_{max}$$

While when heating, the air volume flow rate supplied by the system is kept at minimum while the variation in the air temperature is given by:

$$T_{supply} = T_{c/c} + \frac{\eta_h \dot{Q}_{h/c,max}}{C_p \rho \dot{V}_{min}}$$

The calculated values are used to update the zone air temperature in loops, considering the change in dialy operating parameters and environmental factors.

3- Summary of Time Marching Solution

Building performance calculations using EnergyPlus can simulated over many time spans a user identifies, hours, days weeks, months or years. These simulations are carried out using a time marching method based on a series of discrete bins of time referred to as time steps, recalculating the models equations at each time step.

The models used are mostly quasi-steady energy balance equations, derived to predict the conditions during each time step. A "staircase" approach is used to process the input data and boundary conditions as they are time-varying parameters. These values are calculated for a particular time step, then held constant over the entire time step. The predictions for state variables, such as temperature, are averaged over a time step, while the predictions for summed variables, such as energy use, are simple totals over the time step. Time-series then are created from the results of selected output variables at specific frequencies. The time values associated with the time-series data, or timestamps, are output at the end of the time step.

A zone is usually defined as a region of the building (can be a collection of rooms) subjected to the same type of thermal control set-up and having similar internal load profiles that can be grouped together. Different zones can interact with each other thermally through shared surfaces or by intermixing of zone air. As mentioned in the air system control section, the conditions in each zone are updated, using previously calculated values of the zone conditions. Each zone condition (temperatures) must be computed at the same simulation time and on the same time step for all zones due to heat transfer through each zone's surfaces and inter-zone mixing of air still occur; Knowing that the conditions in one zone may be changing much more rapidly than in other zones.

The initiation of a time step process starts with the use of zone temperatures calculated by the predictor-corrector formulas explained earlier. The simulation conducts a two-time-step approach, a Zone time step (specified by the user) and a System time step (limited between one minute and the value of the logged Zone time step); Updating the zone air temperature, using an adaptive time step that ensures stability. Using a System Convergence Limits object, the one-minute limit of the system time step can be increased, decreasing the simulation run times at the expense of some accuracy. Then, the maximum temperature change within any zone in the model is evaluate; If it was found that this maximum zone temperature difference is more than a preset limit of 0.3°C, the simulation shifts to take on the shorter system time step. Assuming the temperature change is linear, the number of system time steps derived by the temperatures is:

$$\left(\frac{\text{Maximum Zone Temperature Change}}{\text{MaximumZoneTemperatureDifference}\{0.3°\text{C}\}}\right) + 1.0$$

And the limit for the number of system time steps is:

$$\left(\frac{\text{Zone Time Step}}{\text{Minimum System Time Step}}\right)$$

The simulation takes on the smallest of these two values, truncates them to a whole number and calculates the system time step as:

$$\text{System Time Step} = \left(\frac{\text{Zone Time Step}}{\text{Number of System Tim eSteps}}\right)$$

4- Conduction through the Walls

Within the conduction transfer function module, the response factor equation is derived as a basic time series solution, relating the flux at a surface of an element to in infinite series of temperature histories at both sides. The heat flux (q") can be described as:

$$q''_{ko}(t) = \sum_{j=0}^{\infty} X_j T_{o,t-j\delta} - \sum_{j=0}^{\infty} Y_j T_{i,t-j\delta}$$

Where (T) is the temperature, the (i) represent the inside of the building element, (o) represent the outside of the building element, (t) represents the current time step, and X/Y are the response factors.

Due to the similarity of the higher order terms, the infinite number of terms can be replaced with the flux history terms, leading to the basic conduction transfer function (CTF) solution for the inside heat flux as:

$$q''_{ki}(t) = -Z_o T_{i,t} \sum_{j=1}^{nz} Z_j T_{i,t-j\delta} + Y_o T_{o,t} + \sum_{j=1}^{nz} Y_j T_{o,t-j\delta} + \sum_{j=1}^{nq} \Phi_j q''_{ki,t-j\delta}$$

While the outside heat flux is:

$$q''_{ko}(t) = -Y_o T_{i,t} - \sum_{j=1}^{nz} Y_j T_{i,t-j\delta} + X_o T_{o,t} + \sum_{j=1}^{nz} X_j T_{o,t-j\delta} + \sum_{j=1}^{nq} \Phi_j q''_{ko,t-j\delta}$$

Where:

X_j = Outside CTF coefficient, j = 0,1,…,nz

Y_j = Cross CTF coefficient, j = 0,1,…,nz

Z_j = Inside CTF coefficient, j = 0,1,…,nz

Φ_j = Flux CTF coefficient, j = 1,2,…,nq

T_i = Inside face temperature

T_o = Outside face temperature

q''_{ko} = Conduction heat flux on outside face

q''_{ki} = Conduction heat flux on inside face

The basic method used in for CTF calculations here is known as the State Space Method. The following linear matrix equations define the basic state space system as:

$$\frac{d[x]}{dt} = [A][x] + [B][u]$$

$$[y] = [C][x] + [D][u]$$

Where, (x) is a vector of state variables, (t) is time, (u) is a vector of inputs, (y) is the output vector, and (A, B, C, and D) are coefficient matrices. The vector of state variables (x) can be removed from the system of equations, the output vector (y) can be linked directly to the input vector (u) and time histories of the input and output vectors, applying matrix algebra.

Accordingly, the transient heat conduction equation can be solved, enforcing a finite difference grid over the various layers in the building element being analyzed. Defining the environmental temperatures (interior and exterior) as the inputs, the nodal temperatures are the state variables and the resulting heat fluxes at both surfaces are the outputs; Leading the state space representation with finite difference variables to take the form:

$$\frac{d\begin{bmatrix} T1 \\ \vdots \\ Tn \end{bmatrix}}{dt} = [A]\begin{bmatrix} T_1 \\ \vdots \\ T_n \end{bmatrix} + [B]\begin{bmatrix} T_i \\ T_o \end{bmatrix}$$

$$\begin{bmatrix} q_i'' \\ q_o'' \end{bmatrix} = [C]\begin{bmatrix} T_1 \\ \vdots \\ T_n \end{bmatrix} + [D]\begin{bmatrix} T_i \\ T_o \end{bmatrix}$$

$T_1, T_2, \ldots, T_{n-1}, T_n$ are the finite difference nodal temperatuers given by (n) the number of nodes. (q_i'') and (q_o'') are the interior environmenta temperature and exterior environmental temperatures respectivlty. A simple application of this concept analysed for a one layer slab having two interior nodes and conviction at both sides led to the finite difference equation form:

$$C\frac{dT1}{dt} = hA(To - T1) + \frac{T2 - T1}{R}$$

$$C\frac{dT2}{dt} = hA(Ti - T2) + \frac{T1 - T2}{R}$$

$$q_i'' = h(Ti - T2) \qquad , \qquad q_o'' = h(T1 - To)$$

Where, $R = \frac{\ell}{kA}$, $C = \frac{\rho c p \ell A}{2}$, and (A) being the area of the layer exposed to the environmental temperatures. Plotted in a materix fomat, it ca be shown as:

$$\begin{bmatrix} \frac{dT1}{dt} \\ \frac{dT2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} - \frac{hA}{C} & \frac{1}{RC} \\ \frac{1}{RC} & -\frac{1}{RC} - \frac{hA}{C} \end{bmatrix}\begin{bmatrix} T1 \\ T2 \end{bmatrix} + \begin{bmatrix} \frac{hA}{C} & 0 \\ 0 & \frac{hA}{C} \end{bmatrix}\begin{bmatrix} To \\ Ti \end{bmatrix}$$

$$\begin{bmatrix} q_o'' \\ q_i'' \end{bmatrix} = \begin{bmatrix} 0 & -h \\ h & 0 \end{bmatrix}\begin{bmatrix} T1 \\ T2 \end{bmatrix} + \begin{bmatrix} 0 & h \\ -h & 0 \end{bmatrix}\begin{bmatrix} To \\ Ti \end{bmatrix}$$

The state space technique used here process the state space variables (nodal temperatures) to arrive at a matrix equation that gives the outputs (heat fluxes) as a function of the inputs (environmental temperatures) using matrix algebra, eliminating the need to solve for roots in the Laplace domain. Ceylan and Myers (1980) compared the results obtained by the state space method with other solution techniques and it was found that the difference was about 1%, considering the use of adequate number of nodes while computing the heat flux at the surface of a simple one-layer slab.

5- Conduction Finite Difference Solution Algorithm

Complementing the CTF solution algorithm, the conduction finite difference solution algorithm is used to simulate phase changes in materials or the variable thermal conductivity as well as for simulating zone time steps as short as one minute. Two schemes are provided, the “Crank-Nicholson” scheme and the “fully implicit” scheme. Both of the schemes share the same supporting models for material properties, data storage, solution schemes, and spatial discretization algorithms; while they differ in their fundamental heat transfer equations.

- Crank-Nicholson scheme uses an implicit finite difference scheme, coupled with an enthalpy-temperature function. It's semi-implicit and considered second-order in time, accounting for phase change energy accurately. The equation here present implicit formulation for an internal node as:

$$C_p\rho\Delta x\frac{T_i^{j+1}-T_i^j}{\Delta t}=\frac{1}{2}\left(kw\frac{T_{i+1}^{j+1}-T_i^{j+1}}{\Delta x}+kE\frac{T_{i-1}^{j+1}-T_i^{j+1}}{\Delta x}+kw\frac{T_{i+1}^{j}-T_i^{j}}{\Delta x}+kE\frac{T_{i-1}^{j}-T_i^{j}}{\Delta x}\right)$$

  Where,

  T = node temperature , C_p = specific heat of material

  ρ = density of material , Δt = calculation time step

  Δx = finite difference layer thickness (always less than construction layer thickness)

  kw = thermal conductivity for interface between i node and i+1 node

  kE = thermal conductivity for interface between i node and i-1 node

  And the sub/superscripts:

  (i) = node being modelled, (i+1) = adjacent node to interior of construction and (i-1) = adjacent node to exterior of construction

  (j) = previous time step and (j+1) = new time step

- The fully implicit scheme, based on an Adams-Moulton solution approach as a first order in time, is presented in the equation form:

$$C_p\rho\Delta x\frac{T_i^{j+1}-T_i^j}{\Delta t}=\left(kw\frac{T_{i+1}^{j+1}-T_i^{j+1}}{\Delta x}+kE\frac{T_{i-1}^{j+1}-T_i^{j+1}}{\Delta x}\right)$$

Both schemes use four types of nodes, interior surface nodes, interior nodes, material interface nodes and external surface nodes. Considering a grid for each material, consisting of a half node for each edge of the material and equal size nodes for the rest of the material.

To update a new node temperature, a Gauss-Seidell iteration scheme is opted, considering the solution is implicit. Called for each surface, 30 iterations is the limit, exiting early when the sum of all nodes temperatures between the last call and the current call changes. Typically, after three iterations the convergence is achieved with most cases of iterations being less than 10.

6- Combined Heat and Moisture Transfer Finite (HAMT) Model

The (HAMT) solution algorithm can be characterized as completely coupled, one-dimensional, finite element, heat and moisture transfer model. Its constructed to simulate the movement and storage of heat and moisture in surfaces, simultaneously from and to the internal and external environments. It simulates the effects of moisture buffering, as well as providing temperature and moisture profiles through the composite building walls.

Surfaces are structures that can be of a single layer/single material or consist of a number of layers in a combination of multiple materials. The simulation model breakdown all surfaces into its constituent materials, then split up each layer into cells through its depth, generating no more than 10 cells per material. The following equation describe the heat storage and transfer through the i^{th} cell in a surface:

$$(c_i\rho_i + c^w w_i)\Delta Vi\frac{T_i^{p+1} - T_i^p}{\Delta\tau} = \sum_j k_{ij}^w A_{ij}\frac{T_j^{p+1} - T_i^{p+1}}{x_{ij}} + \sum_j h_v \frac{\delta_{ij}}{\mu_{ij}} A_{ij}\frac{p_j^{p+1} - p_i^{p+1}}{x_{ij}}$$

Where;

T = Temperature (°C)

w = Moisture Content (kg/m^3)

k^w = Moisture dependent thermal conductivity (W/mC)

h_v = Evaporation enthalpy of water (= 2,489,000 J/kg)

δ = Vapor diffusion coefficient in air (kg/msPa)

µ = Moisture dependent vapor diffusion resistance factor

c = Specific heat capacity of dry material (J/kgC)

c^w = Specific heat capacity of water (4,180 J/kg°C@ 20°C)

P = Material Density (kg/m^3)

A = Contact Surface area (m^2)

Δvi = Cell Volume (m^3)

Δτ = Time step between calculations (s)

x = Distance between cell centers (m)

i,j = Cell indices

To calculate the temperature in a cell at the next time step, the equation can be rearranged as:

$$T_i^{p+1} = \frac{\sum_j \frac{T_j^{p+1}}{R_{ij}^h} + \sum_j h_v \frac{\delta_{ij}}{\mu_{ij}} A_{ij}\frac{p_j^{p+1} - p_i^{p+1}}{x_{ij}} + (c_i\rho_i + c^w w_i)\Delta Vi\frac{T_i^p}{\Delta\tau} + q_i^{adds}}{(c_i\rho_i + c^w w_i)\frac{\Delta Vi}{\Delta\tau} + \sum_j \frac{k_{ij}^w A_{ij}}{x_{ij}}}$$

Where q_i^{adds} represents the other sources of heat as in the thermal radiation from other surfaces. The Gauss-Seidel iteration technique is used to solve the equation, considering the i^{th} cell temperature calculation at the j^{th} cell temperature, keeping them updated as much as possible. The iteration would stop when the maximum temperature difference between two consecutive calculations in all cells gets lower than a threshold of 0.002°C.

7- Outside Surface Heat Balance

Accounting for the heat balance at the outside surfaces, four parameters are considered to achieve equilibrium. The absorbed direct and diffuse solar radiation heat flux ($q''_{\alpha sol}$), the air and surroundings thermal radiation flux exchange (q''_{LWR}) and the convective flux exchange with the outside air (q''_{conv}) balancing the conduction heat flux into the wall/surface (q''_{ko}):

$$q''_{\alpha sol} + q''_{LWR} + q''_{conv} - q''_{ko} = 0$$

The absorbed direct and diffuse solar radiation heat flux ($q''_{\alpha sol}$) is a short-wave radiation factor, influenced by localized parameters such as: the selected location characteristics, surface facing angle, surface facing tilt, surface face material properties and weather conditions. While the air and surroundings thermal radiation flux exchange (q''_{LWR}) is a long-wave radiation factor, calculated by identifying the surface absorptivity, surface/sky/ground temperatures and the corresponding sky/ground view factors. To simplify the calculations, few building loads calculations' assumptions were considered such as: surfaces are at a uniform temperature, each surface emits or reflects diffusely and even distribution of energy flux output across the surface. These assumptions were found to be frequently used in most critical engineering applications.

8- Inside Heat Balance

The internal heat balance mainly accounts for the inside faces of zone surfaces. The heat balance is modeled within the conduction the building elements, the convection through air, the absorption/reflectance through short-waves (from the solar radiation through windows and/or emittance from internal sources such as lights) and long-wave radiant interchange (absorption and emittance of other zone surfaces, equipment, and people defined as low temperature radiation sources):

$$q''_{LWX} + q''_{SW} + q''_{LWS} + q''_{ki} + q''_{sol} + q''_{conv} = 0$$

Where, (q''_{LWX}) is the net long-wave radiant exchange flux between zone surfaces, (q''_{SW}) is the net short-wave radiation flux to surface from lights, (q''_{LWS}) is the long-wave radiation flux from equipment in zone, (q''_{ki}) is the conduction flux through walls, (q''_{sol}) is the transmitted solar radiation flux absorbed at surfaces and (q''_{conv}) being the convective heat flux to the zone air.

9- Climate Calculations

Weather characteristics are important inputs to any energy modeling software. Latitude, longitude and elevation are the essential to define key parameters the barometric pressure, solar position and the sky radiance. Weather details are also required, hourly or sub-hourly input data featuring the yearly weather conditions (average temperatures, relative humidity, wind characteristics, precipitation levels, daylighting potential… etc.) are needed to calculate the key energy consuming systems, such as cooling/heating loads and lighting requirements. EnergyPlus provides an access to select the region, where the simulation will take on the climate data as an input. The calculations consider the latest weather files linked to the software database in carrying out the simulation. It allows the user to analyze their buildings' yearly performance as well as for limited/selected time intervals.

To simulate a 24 hour run of how a building behave under specific climatic conditions, EnergyPlus uses a range multiplier profile to represent the period's variance in temperature, creating an air temperature at each time-step out of the maximum dry-bulb temperature (measured at an exposed space to the air, shielded from radiation and moisture), the entered daily range and the multiplier values as:

$$T_{current} = T_{max} - (T_{range} \times T_{Multiplier})$$

Where,

$T_{current}$ = Air temperature at the current hour of day

T_{max} = Input maximum dry-bulb temperature

T_{range} = Input daily temperature range

$T_{Multiplier}$ = Range multiplier defined based on typical conditions of diurnal temperatures

As for the horizontal infrared radiation intensity, its calculated (in W/m^2) using:

$$\mathrm{Horizontal\_IR} = \mathrm{Sky}_{\mathrm{emissivity}} \times 5.6697\mathrm{e}^{-8} \times \mathrm{Temperature}^{4}_{\mathrm{drybulb}}$$

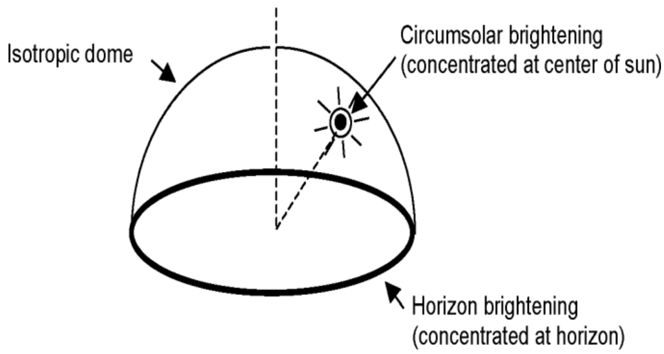
Where, the constant $5.6697\mathrm{e}^{-8}$ is the Stefan-Boltzmann constant and the $\mathrm{Sky}_{\mathrm{emissivity}}$ calculated by:

$$\mathrm{Sky}_{\mathrm{emissivity}} = \left(0.787 + 0.764 \times \ln\left(\frac{\mathrm{Temperature}_{\mathrm{dewpoint}}}{273}\right)\right) \times (1 + 0.0224\mathrm{N} - 0.0035\mathrm{N}^2 + 0.00028\mathrm{N}^3)$$

Where $\mathrm{Temperature}_{\mathrm{dewpoint}}$ is the dew-point temperature (K) and the N being the opaque sky cover (zero when the sky is clear).

10- Sky Radiance Model

A key parameter influencing the buildings' energy performance is the interaction of the buildings' external surfaces with the sky radiance. To calculate the diffuse sky irradiance on a surface, EnergyPlus accounts for the surface orientation, the sky radiance distribution and the effects of shadowing from surrounding surfaces (such as overhangs or window sheds). Three distributions govern the sky radiance distribution, the Isotropic distribution covering the entire sky dome, the Circumsolar brightening distribution centered at the position of the sun and a Horizontal brightening distribution; This was based on an empirical model relaying on real skies radiance measurements Perez et al., 1990.



Sky clearness and brightness are the two main factors defining the proportions of these distributions, measured from the sun position and the solar measurements extracted from the selected weather files. In the simple case of no shadowing, the sky diffuse irradiance can be expressed as:

$$\mathrm{I}_{\mathrm{sky}} = \mathrm{I}_{\mathrm{horizon}} + \mathrm{I}_{\mathrm{dome}} + \mathrm{I}_{\mathrm{circumsolar}}$$

And,

$$\mathrm{I}_{\mathrm{horizon}} = \mathrm{I}_{\mathrm{h}} F_2 \sin S \quad , \quad \mathrm{I}_{\mathrm{dome}} = \mathrm{I}_{\mathrm{h}}(1 - F_1)(1 + \cos S)/2 \quad , \quad \mathrm{I}_{\mathrm{circumsolar}} = \mathrm{I}_{\mathrm{h}} F_1 a/b$$

Where,

I_{h} = horizontal solar irradiance (W/m2)

F_1 = circumsolar brightening coefficient

F_2 = horizon brightening coefficient

S = surface tilt (radians)

a = maximum of (0, cos α) , α = incidence angle of sun on the surface (radians)

b = maximum of (0.087, cos Z) , Z = solar zenith angle (radians)

provided that F_1 and F_2 are dependents on the relative optical air mass, the extraterrestrial irradiance (having an average annual value of 1353 W/m^2) and the direct normal solar irradiance.

To consider the shadowing of surfaces, the irradiation distribution values are multiplied with a ratio that takes into account the irradiation influence of shadowing objects, having the sky diffuse irradiance represented as:

$$I'_{sky} = R_{horizon} I_{horizon} + R_{dome} I_{dome} + R_{circumsolar} I_{circumsolar}$$

EnergyPlus allow for two settings, a simple sky diffuse model with $R_{horizon}$ and R_{dome} calculated once for each surface, as they are independent of sun position, or a detailed sky diffuse model, calculating $R_{horizon}$ and R_{dome} at every time-step for each surface.

By dividing the horizon line into 24 equal intervals, considering the unobstructed irradiance on the surface and the sunlit fraction from radiation at each interval, the horizon irradiation ratio $R_{horizon}$ is factored. While the isotropic sky dome ratio R_{dome} factors 6 altitude intervals to the horizontal 24 intervals, giving a total of 144 points accounting for the sky dome grid. With the assumption that the circumsolar region is concentrated at the solar disk, the circumsolar ratio $R_{circumsolar}$ is calculated by factoring the irradiance from circumsolar region with obstructions divided by the irradiance from circumsolar without obstructions.

11- Daylighting Calculations

Daylighting effect on the energy consumption is reflected with the consideration of daylight availability, the site conditions, windows' glare/solar-gain qualities and he designed lighting control systems. The simulation factors in three main steps, calculating the daylight factors (ratios of the interior illuminance to the exterior horizontal illuminance), applying the daylighting calculations at each heat-balance time step as long as the sun is up and balancing the electric lighting loads, based on the lighting energy needed to make up the difference between the daylighting illuminance level and the design illuminance. EnergyPlus daylight calculation take into account four sky types, a clear, clear turbid, intermediate, and overcast. The calculations are simulated hourly based on the sun-path/sun-positions several times a year whereas based on the selected geographical location.

The daylight factors are classified as interior illuminance factors, window luminance factors and windows background luminance factors, calculated on hourly intervals based on the sun position and the sun paths at the specific simulation span of days defined by the user. The calculations start with evaluating the exterior horizontal daylight illuminance (from the sky and sun). Then, calculating the interior illuminance and window background luminance for each window, considering the design shading inputs. The ratio of the exterior horizontal illuminance to the windows luminance effect is the daylight factor.

Considering the source of the daylight, the sun or the sky, the daylight factors are calculated accordingly. Originating from their sources (sky/sun) or reflected, the light through windows reaching the zone or reflected by interior surfaces are considered. Taking on the sky condition (clear, clear turbid, intermediate, and overcast) and zone geometry, the daylight factors are calculated.

$$d_{sky} = \frac{\text{Illuminance at reference point due to sky−related light}}{\text{Exterior horizontal illuminance due to light from the sky}}, \; d_{sky} = \frac{\text{Illuminance at reference point due to sun−related light}}{\text{Exterior horizontal illuminance due to light from the sun}}$$

$$w_{sky} = \frac{\text{Average window luminance due to sky-related light}}{\text{Exterior horizontal illuminance due to light from the sky}}, \; w_{sky} = \frac{\text{Average window luminance due to sun-related light}}{\text{Exterior horizontal illuminance due to light from the sun}}$$

$$b_{sky} = \frac{\text{Window background luminance due to sky-related light}}{\text{Exterior horizontal illuminance due to light from the sky}}, \; b_{sky} = \frac{\text{Window background luminance due to sun-related light}}{\text{Exterior horizontal illuminance due to light from the sun}}$$

For a day-lit zone, with N windows, these six daylight factors are calculated for each combination of reference point, window, sky-condition/sun-position and shading characteristics:

$$\begin{bmatrix} \text{Ref point 1} \\ \text{Ref point 2} \end{bmatrix} \begin{bmatrix} \text{Window 1} \\ \text{Window 2} \\ \dots \\ \text{Window N} \end{bmatrix} \begin{bmatrix} \text{Clear sky, first sun} - \text{up hour} \\ \text{Clear/turbid sky, first sun} - \text{up hour} \\ \text{Intermediate sky, first sun} - \text{up hour} \\ \text{Overcast sky, first sun} - \text{up hour} \\ \dots \\ \text{Clear sky, last sun} - \text{up hour} \\ \text{Clear/turbid sky, last sun} - \text{up hour} \\ \text{Intermediate sky, last sun} - \text{up hour} \\ \text{Overcase sky, last sun} - \text{up hour} \end{bmatrix} \begin{bmatrix} \text{Unshaded window} \\ \text{Shaded window} \end{bmatrix}$$

12- Time-Step Daylighting Calculation

The solar irradiance data out of the weather file is the foundation for the daylighting calculations. The exterior horizontal illuminance from the sun and the sky are logged at each time step within a zone having one or two daylighting reference points specified. At each reference point, the interior illuminance for each window is calculated by interpolating the daylight illuminance factors at the timely sun position. These daylighting factors are used to calculate net illuminance and glare due to all windows at each zone, leading for the assessment of lighting control required and determining the electrical loads required to meet the illuminance standard specified by the user.

The electrical power required to balance the illuminance requirements is highly dependent on the selected/applied lighting system. EnergyPlus calculates a factorial electric lighting output (f_L) needed to meet the illuminance set-point at a reference point by:

$$f_L(i_L) = \max\left[0, \frac{I_{set}(i_L) - I_{tot}(i_L)}{I_{set}(i_L)}\right]$$

Where, (I_{set}) is the illuminance set-point and (I_{tot}) is the daylight illuminance at the reference point (i). Assuming that the electric lights at full power produce an illuminance equal to I_{set} at the reference point. Calculating the fractional lighting input power corresponding to (f_L) then can be calculated as per the user's lighting control system selection.

13- Window Calculation Module

Windows' properties have a significant role in buildings' energy performance. To understand their contribution, first, the components of a window are to be explained. Typically, windows can be composed of multiple transparent/semitransparent layers. The main layer is the glazing, layered with a gap in between that is filled with air or other gases designed to deliver a specific function. The material selection is incorporated in the calculations based on their thermal and optical qualities. These layers are fixed with a frame, and based on the user's design, windows can be fitted with dividers and internal/external shading system.

The optical properties of the glazing system selected control the amount of solar radiation penetrating a zone throw windows. The solar transmittance, reflectance and absorption properties of each layer in the window profile contribute in the resultant lighting and zone heat balance calculations. To calculate the Reflectance (R), the following equation is used:

$$R = \frac{r + (1-r)^2 r\tau^2}{1 - r^2\tau^2}$$

Where, the transmittance (τ) is calculated as $\tau = \frac{[(1-r)^4 + 4r^2T^2]^{1/2} - (1-r)^2}{2r^2T}$, $r = \left(\frac{n-1}{n+1}\right)^2$, (T) is the transmittance at normal incidence and (n) is the material's index of refraction.

To evaluate the transmittance (T) through the window's multi-layer system, the recursion relations between the transmission of the different layers has to be solved. Assuming a window system with (i) to (j) layers, the front (R^f) and back (R^b)reflectance and the absorption of layer (j), (A_j) are taken into account. With (N) being the number of layers, each variable is a function of wave length, calculated as:

$$T_{i,j} = \frac{T_{i,j-1}T_{j,j}}{1 - R^f_{j,j}R^b_{j-1,i}}$$

$$R^f_{i,j} = R^f_{i,j-1} + \frac{T^2_{i,j-1}R^f_{j,j}}{1 - R^f_{j,j}R^b_{j-1,i}} \quad , \quad R^b_{j,i} = R^b_{j,j} + \frac{T^2_{j,j}R^b_{j-1,i}}{1 - R^b_{j-1,i}R^f_{j,j}}$$

$$A^f_j = \frac{T_{1,j-1}\left(1 - T_{j,j} - R^f_{j,j}\right)}{1 - R^f_{j,N}R^b_{j-1,1}} + \frac{T_{1,j}R^f_{j+1,N}\left(1 - T_{j,j} - R^b_{j,j}\right)}{1 - R^f_{j,N}R^b_{j-1,1}}$$

To calculate the spectral average (P_s), the glazing system properties above are integrated over the wavelength, leading to a spectral-average solar property of:

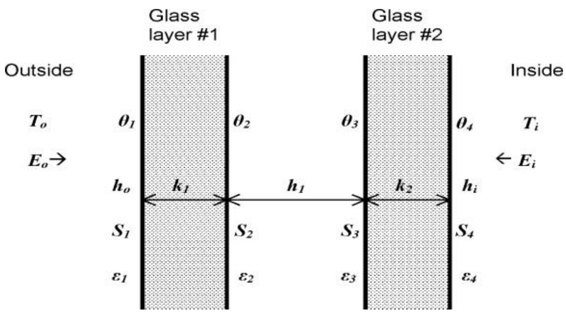
$$Ps = \frac{\int P(\lambda)E_s(\lambda)d\lambda}{\int E_s(\lambda)d\lambda}$$

$E_s(\lambda)$, being the solar spectral irradiance function.

The spectral calculations are used as an input to be correlated with the gazing system's angular performance with 10-degree increments. The layers' spectral and angle properties are calculated, by linear interpolation, for each incident angel to then calculate the entire glazing system's performance. Six polynomial curves fitted with six coefficients, a cosine incident angle function, are generated from each angle; These curves are used to simulate the optical properties at each time-step.

14- Window Heat Balance Calculation

To solve the heat balance equation for windows, the glass face's temperatures are calculated at every time step on each face. Any window system would be having two faces exposed to inner and outer environments, requiring two set of equations to be solved. Within the simple case of a two glazing layer window system, the variables can be displayed as in this figure:



To simplify the calculations, few assumptions were made such as the glazing layer thickness is assumed to be thin as if the heat storage within the layer could be neglected. Second, the heat flow is one dimensional and perpendicular to the glazing face. Third, and as in most glass products, the gazing layers are considered opaque to infrared waves. Finally, due to the nature of glass conductivity being high, the glazing faces are assumed to be isothermal.

In the simple case of double glazing, the four representations below are used to solve the heat balance problem in contribution to windows system:

$$E_o\varepsilon_1 - \varepsilon_1\sigma\theta_1^4 + k_1(\theta_2 - \theta_1) + h_o(T_o - \theta_1) + S_1 = 0$$

$$k_1\,(\theta_1 - \theta_2) + h_1\,(\theta_3 - \theta_2) + \sigma\,\frac{\varepsilon_2\varepsilon_3}{1-(1-\varepsilon_2)(1-\varepsilon_3)}\,(\theta_3^4 - \theta_2^4) + S_2 = 0$$

$$h_1\,(\theta_2 - \theta_3) + k_2\,(\theta_4 - \theta_3) + \sigma\,\frac{\varepsilon_2\varepsilon_3}{1-(1-\varepsilon_2)(1-\varepsilon_3)}\,(\theta_2^4 - \theta_3^4) + S_3 = 0$$

$$E_i\varepsilon_4 - \varepsilon_4\sigma\theta_4^4 + k_2(\theta_3 - \theta_4) + h_i(T_i - \theta_4) + S_4 = 0$$

Where, (ε) is the emissivity of a face, (k) is the conductance of a glass layer, (h) is the air convective conductance, (T) is the Temperature of indoor/outdoor air, (E) is the long-wave radiation incident on the window, (θ) is the temperature of faces and (S) is the short-wave and long-wave radiation from zone internal sources absorbed by a face.

To solve the above, the equations are first linearized by defining ($h_{r,i}$) as ($\varepsilon_i\sigma\theta_i^3$) and rewriting the equations in a matrix form as $A\theta = B$. the first time-step take on the values of θ_i as if the layers are a simple reinforces concrete network. The internal facing surface convection coefficient h_i is then re-evaluated with θ_{2N} and using θ_i to evaluate the radiative conductance's $h_{r,i}$. The matrix $\theta = A^{-1}B$ is then solved using Lower-Upper decomposition, leading to a new θ_i: $\theta_i \rightarrow (\theta_i + \theta_{prev,i})/2$. Then the cycle starts solving for a new θ_i until $\Delta\theta_i$ becomes less than the tolerance value, defined as:

$$\frac{1}{2N}\sum_{i=1}^{2N}|\Delta\theta i| < 0.02K$$

This θ_{2N} takes place in the zone heat balance solution of the previously explained section of Outdoor/Exterior Convection heat balance as well as in the calculation for the occupant thermal comfort solution.

15- Infiltration

To model the air infiltration within a zone, the user shall expect some amount of uncertainty. Many assumptions are considered in modelling the zone air behaviour being mixed with input air of different characteristics as the assumption of immediate mixing of zone air. Three models are used in EnergyPlus to simulate the infiltration process, the Design Flow Rate model, the Effective Leakage Area model and the Flow Coefficient model.

A. Infiltration Design Flow Rate (DFR) Model:

Infiltration of air can be explained as the mixing of air within a zone by an external air, mostly through doors and windows being opened. Other source of infiltration can be through cracks around windows and/or through some building elements. The user's input of the design flow rate is controlled by temperature difference and wind speed. The infiltration simulated with the DFR model is calculated using:

$$\text{Infiltration} = (I_{design})\,(F_{schedule})\,[A + B|(T_{zone} - T_{odb})| + C\,(\text{WindSpeed}) + D\,(\text{Windspeed}^2)]$$

The values of the coefficients are circumstantial. Depending on the infiltration potential, an ideal process would be to customize these coefficients accordingly. EnergyPlus uses a set of defaults that result in a constant volume flow of infiltration under all conditions.

B. Infiltration by Effective Leakage Area (ELA) Model

Also referred to as "basic model", based on the work of Sherman and Grimsrud (1980), this model calculates the infiltration by:

$$\text{Infiltration} = (F_{\text{Schedule}}) \frac{A_L}{1000} \sqrt{C_s \, \Delta T + C_w \, (\text{Windspeed})^2}$$

(A_L) here represents the effective air leakage area in cm^2, that corresponds to a 4 Pa pressure differential. (C_s) is the stack-induced infiltration coefficient in $(L/s)^2/(cm^4 \cdot K)$, while (C_w) is the wind-induced infiltration coefficient in $(L/s)^2/(cm^4 \cdot (m/s)^2)$. As for ($\Delta T$), it's the absolute temperature difference between the zone air and the outdoor air.

C. Infiltration by Flow Coefficient (FC) Model

Referred to as the "enhanced model", the FC model is founded on the work of Walker and Wilson (1998). It models the infiltration value using:

$$\text{Infiltration} = (F_{\text{Schedule}}) \sqrt{(c \, C_s \, \Delta T^n)^2 + (c \, C_w \, (s * \text{Windspeed})^{2n})^2}$$

Where (c) is the flow coefficient in $m^3/(s \cdot Pa^n)$, (n) is the pressure exponent and (s) is the shelter factor.

16- Ventilation

Ventilation in building is the process of purposeful flow of air from the outdoor environment and directly into a controlled thermal zone in order to provide some amount of non-mechanical cooling. The ventilation model used in EnergyPlus is controlled by the set schedule, the temperature difference between the inside/outside environment and the wind speed. The Zone Ventilation Design Flow model takes on the environmental conditions to formulate the ventilation flow rate, taking into account the number of ventilation points in a zone, having the flow rate of each summed up when they are more than one. The basic equation used to calculate the ventilation using this model is:

$$\text{Ventilation} = (V_{\text{design}}) \, (F_{\text{schedule}}) \lfloor A + B \, |T_{\text{zone}} - T_{\text{odb}}| + C \, (\text{WindSpeed}) + D \, (\text{WindSpeed}^2) \rfloor$$

Ideally, a detailed analysis of the ventilation design is required to determine a custom set of coefficients (A, B, C and D). However, to simplify the model, EnergyPlus defaults are 1,0,0,0 which gives a constant volume flow of ventilation under all conditions.

17- Air Exchange

In modeling the air exchange, air mixing between the zones is accounted for by evaluating the energy and mass balance for the receiving zones, a result of the temperature difference and constant air mixing. The process is treated as a convective gain, while the masses considered are air, water vapor and CO_2. To calculated the energy added to receiving zone air by mixing mass flow $\dot{Q}_{\text{MixingFlowToReceivingZone}}$ (W), the following equation is used:

$$\dot{Q}_{\text{MixingFlowToReceivingZone}} = \rho_{\text{Avg}} C_{p,\text{Avg}} \dot{V}_{\text{Air}} (T_{\text{SourceZone}} - T_{\text{ReceivingZone}})$$

Where,

$C_{p,Avg}$ is the average specific heat of air within the two zones (J/kg.K), ρ_{Avg} is the average density of air within the two zones (kg/s), $\dot{V}_{Air}$ is the volume rate of air flow defined by the user (m³/s) and T for the Temperatures. The energy loss from the source zone simply uses the same equation while changing the temperature difference to $(T_{ReceivingZone} - T_{SourceZone})$.

18- Zone Internal Gains

The sources of internal heat gains within a zone are generally a result of lighting systems, equipment setup (based on the building's function) and the people occupying the zone. The total heat gain is an accumulation of convective, radiant and latent gains out of the various possible sources. The convective gains are considered instantaneous heat gains to the zone air. While the radian gains are absorbed/released from surfaces within the zone confirming toward surfaces heat balance. As for the latent gains, they are attributed to ventilation and HVAC systems. With EnergyPlus, the user gets to define the systems within a zone, their setup (in quantity and distribution) and their schedule of operation/application.

People as heat gains sources are contributing to the zone heat balance calculations in consideration of the metabolic rate a human body generate. A combination of heat emitted through the body surface and the respiratory tract. A consideration is given to the metabolic rate based on gender and age, with a metabolic rate of females and children factored by 0.85 and 0.75 respectively of that of a male.

19- Set-point Managers

In order to calculate the updated conditions of a zone, each time-step take one the current conditions (calculated at the previous time-step) and subject it to the controlled set of limitations the user defines as in HVAC temperature set-points. At each time-step the model refers to the input schedule set-points, that are usually seasonal, to manage the HVAC system. The model compares each specific set-point to that defined in the user's schedule to account for the operations required to have them as equals. To meet the cooling and heating loads, the set-points manager operated using the following equation:

$$T_{set} = T_z + \frac{\dot{Q}_z}{C_{p,air}\ \dot{m}_z}$$

Where T_{set} is the set-point temperature, T_z is the control zone temperature, $\dot{Q}_z$ is the zone load (operating as cooling for when its less than zero and heating if greater than zero), $C_{p,air}$ is the specific heat of air and $\dot{m}_z$ being the zone supply air mass flow rate.

20- Occupant Thermal Comfort

Thermal comfort is an input parameter the user defines based on the building's function. It's an interpretation of the thermal, physiological and psychological responses of people toward the surrounding environment. The model was created to empirically determine the building's occupants' thermal response, using parameters such as the activity levels, work efficiency and the desired air velocity, logged in the model along with their schedules. As it being considered in the heat gains calculations, the occupants metabolic rate is a variable influencing the thermal comport conditions. The other case-sensitive variable would be the resistance of clothing. The metabolic rate is measured as a rate of the internal heat production rate of an occupant (H) over the occupant's body surface area A_{Du} , calculated as:

$$A_{Du} = 0.202\ (\text{weight})^{0.425}\ (\text{height})^{0.725}$$

In EnergyPlus, an average person height and weight are used, 1.73 m by 70 kg are resulting in a 1.8 m^2 surface area is used. While, the metabolic rate is measured in mets, where 1 met = 58.2 W/ m^2.

The environmental variables influencing the thermal comfort conditions are defined by the zone's air temperature, mean radiant temperature, relative air velocity and the water vapor pressure in ambient air. Whereas the physiological variables are associated with the skin temperature, body-core internal temperature, sweat rate, skin wetness and the thermal conductance between the body-core and the skin.

Most mathematical models identify the thermal comfort with a seven (Hot, Warm, Slightly warm, neutral, Slightly cool, Cool and Cold) or nine (Very hot, Hot, Warm, Slightly warm, neutral, Slightly cool, Cool, Cold and Very cold) sensation scales. The Fanger Comfort Model, the Pierce Two-Node Model and the KSU Two-Node Model are the most notable models concerning the thermal comfort; all fundamentally aim to achieve energy heat balance to an occupant, using energy (heat) exchange laws to measure the thermal sensation.

All the modes of energy loss from body L (in W/m^2) can be calculated by summing:

$$\mathrm{L} = \mathrm{Q_{res}} + \mathrm{Q_{dry}} + \mathrm{E_{sk}}$$

Where, $\mathrm{Q_{dry}}$ is the sensible heat flow from the skin, $\mathrm{Q_{res}}$ is the the rate of respiratory heat loss and $\mathrm{E_{sk}}$ is the Total evaporative heat loss from skin.

The Fanger model calculates the Predicted Mean Vote (PMV) thermal sensation scale, quantifying how the energy loss (L) deviates from the metabolic rate (M), as PMV is equivalent to the difference in Internal Heat Production (H) and body energy loss multiplied by a thermal sensation transmittance coefficient:

$$\mathrm{PMV} = (0.303\ \mathrm{e}^{-0.036\mathrm{M}} + 0.028)(\mathrm{H} - \mathrm{L})$$