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Gaussian process regression-based surrogate modelling for direct loss-based seismic design of low-rise base-isolated structures

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Abstract

Seismic base isolation has gained popularity in the last decades. As a result, many structures are now equipped with base isolation systems to offer enhanced seismic performance and meet the needs of risk-aware stakeholders. However, a robust performance-based seismic design of these types of structures is generally not carried out due to the iterative nature of common design approaches and the time/computational resources required for such iterations, which are incompatible with the preliminary design phase. Indeed, seismic risk/loss is often just assessed at the end of the design process as a final verification step. This paper offers an overview of a simplified methodology for the seismic design of low-rise structures equipped with a base isolation system to achieve a predefined level of earthquake-induced economic loss while complying with a predefined minimum level of structural reliability. The main advantage of the proposed methodology is that it requires no design iterations. The procedure is enabled by Gaussian-process-regression-based surrogate probabilistic seismic demand modelling of equivalent single degree of freedom systems (i.e., the probability distribution of peak horizontal displacements and accelerations on top of the isolation layer conditional on different ground-motion intensity levels). Combined with simplified loss models for the base isolation system and the structural and non-structural components of the superstructure, this approach allows mapping a range of structural configurations to their resulting seismic loss. A designer can then select one of the identified combinations of the strength of the superstructure and properties of the isolation system conforming with the loss target, and reliability requirements, and consequently detail the superstructure and isolation system accordingly. This paper introduces the implemented surrogate probabilistic seismic demand models and provides an overview of a tentative Direct Loss-based Design procedure for low-rise base-isolated structures.

Keywords: Seismic isolation; Direct loss-based seismic design; Gaussian process regression; Surrogate modelling

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1. Introduction and motivation

Base isolation can protect a structure and its contents from the damaging effects of earthquake-induced ground motions by introducing special isolation devices; e.g., Skinner et al. (1993). In this way, base isolation modifies the dynamic properties of the structure by elongating its natural period of vibration and can significantly reduce the floor accelerations and inter-storey drifts experienced by the superstructure; e.g., Naeim and Kelly (1999). Furthermore, it enables the dissipative structural behaviour to be concentrated in the isolation layer, thus providing convenient levels of damping. Over the years, base isolation has been used to provide structures with a superior level of seismic performance compared to traditional non-isolated designs. Base isolation has also enabled structural design solutions that meet the specific needs of risk-aware clients and risk-critical facilities (e.g., hospitals, emergency response buildings, and power-generating stations).

The Performance-based Earthquake Engineering (PBEE) approach, developed by the Pacific Earthquake Engineering Research Center (PEER), is the most appropriate framework used to provide accurate appraisals of the seismic performance of structures; e.g., SEAOC (1995), Deierlein et al. (2003), Moehle and Deierlein (2004). This allows assessing seismic risk/loss within a probabilistic framework by involving a large number of non-linear dynamic analyses of a detailed non-linear characterisation of the structure, including an inventory of structural and non-structural components. However, using such an approach to design structures that meet a specific target level of seismic loss would require an iterative, trial-and-error application of PBEE, which can be time- and resource-consuming. Hence, most isolated structures are designed following quasi-deterministic approaches, as Kazantzi & Vamvatsikos (2021) pointed out. As a result, both the economic advantage of base-isolated systems (in terms of reduction of seismic losses) and the increment of the seismic performance are usually unknown or restricted to special structures where complete iterative procedures and advanced analyses can be afforded.

To address this shortcoming, two surrogate probabilistic seismic demand models (PSDMs), representing the probability distribution of peak horizontal displacements and accelerations on top of the isolation layer conditional on different ground-motion intensity levels, are here proposed. This approach can play a valuable role in enabling computationally-cheap fragility/vulnerability model estimations and, consequently, loss/risk-oriented design. A surrogate model (or metamodel) provides a statistical approximation of a more-complex model (e.g., non-linear dynamic analysis) based on an intelligently defined input-output training database of the original model. Specifically, Gaussian-Process-regression-based surrogate modelling is proposed to predict the PSDMs of equivalent single degree of freedom, SDoF, systems representing base-isolated structures, following the procedure presented in Gentile and Galasso (2020; 2022). This enables a tentative Direct Loss-Based Design (DLBD) procedure for base-isolated systems, similar to the one proposed by Gentile and Calvi (2022) for traditional reinforced concrete structures. This direct (i.e., non-iterative) procedure can be used to optimise structural/non-structural design while assuring a required level of structural reliability and seismic-induced loss for a given site-specific seismic hazard profile.

The following sections present the development and validation of the proposed surrogate PSDMs based on Gaussian Process (GP) regressions, enabling the proposed DLBD, which is also briefly described considering its strengths and limitations.

2. Surrogated Probabilistic Seismic Demand Models

Two GP regressions are used to surrogate the parameters of the PSDMs of SDoF systems representing base-isolated structures (Figure 1). Specifically, the surrogate models maps the SDoF input parameters $\boldsymbol{X} = \{f_y, t_1, h_{iso}, Hyst\}$ (i.e., yield strength of the isolation system normalised by the total weight of the structure f_y; pre-yield period of the isolation system t_1; post-yield to pre-yield stiffness ratio of the isolation system h_{iso}; and the hysteresis model $Hyst$; Section 2.2) to the output PSDMs parameters $\boldsymbol{Y}_{1,2} = \{a_{1,2}, \sigma_{1,2}\}$ (slope a, and log-normal standard deviation σ of the inelastic segment of bi-linear PSDMs in terms of the specific engineering demand parameter conditional on the selected intensity measure, where the subscripts refer to each of the PSDMs; Section 2.1). The GP regressions are trained on the results of a database of cloud-based non-linear time history analysis NLTHA (section 2.3) of representative SDoF systems (section 2.2).

2.1. Considered PSDMs

A PSDM describes the probability distribution of a considered engineering demand parameter (EPD) conditional to the ground-motion intensity measure (IM). In this study, two separate PSDMs are implemented. The first describes the displacement ductility demand, defined as the ratio between the displacement on top of the isolation system and its yield displacement (μ), whereas the second represents the acceleration demand normalised by the yield acceleration at the top of the isolation system (α).

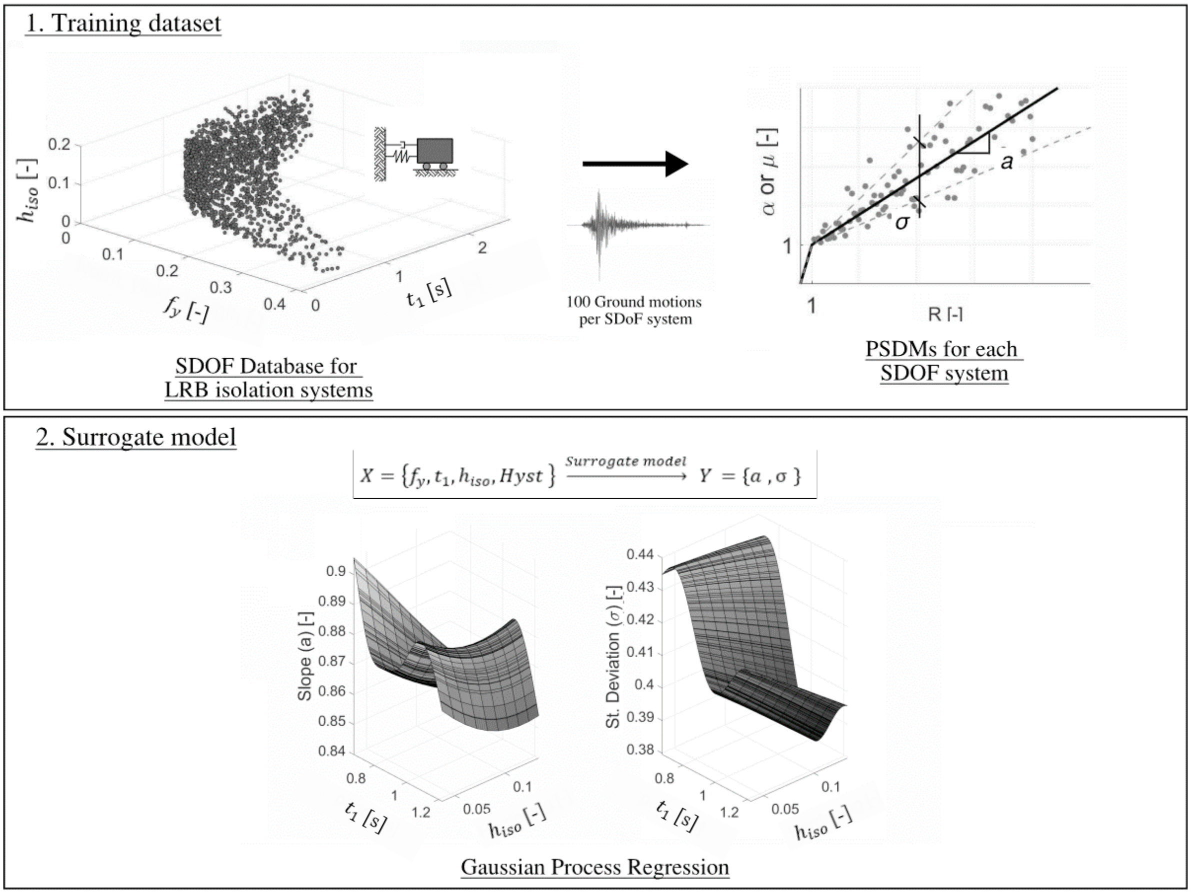

Figure 1. Training dataset and surrogate model. f_y: yield strength of the isolation system normalised by the total weight of the structure; t_1: pre-yield period of the isolation system; h_{iso}: post-yield to pre-yield stiffness ratio of the isolation system; LRB: Lead-rubber bearings, α: acceleration on top of the isolation system divided by f_y; μ: displacement ductility of the isolation system; R: Pseudo-spectral acceleration at t_1 divided by f_y; $Hyst$: hysteresis model; a: slope for the inelastic segment of the PSDM; σ: logarithmic standard deviation of the PSDM.

Both are conditioned on $R = SA(t_1)/f_y$, where $SA(t_1)$ is the pseudo-spectral acceleration at the pre-yield period of the isolation system and f_y is the yield strength of the isolation system normalised by the total weight of the structure. In this way, the model is represented in a normalised space where the yield point corresponds to the coordinate (1,1).

A bi-linear model is implemented for each EDP, as shown in Eq 1. and Eq 2. A bi-linear PSDM is considered an adequate simplified representation of the behaviour of the structure; e.g., O'Reilly and Monteiro (2019). The first segment of the model describes the deterministic behaviour of the isolation system in the elastic range, while the second segment represents the inelastic behaviour described by the median value of the EDP (characterised by the slope a) and its associated dispersion represented by the logarithmic standard deviation $\sigma_{\ln(\mu-1)|R-1}$. This simplified model is characterised by homoscedasticity and normal residuals in the logarithmic space (i.e., ε is a standard Normal variable). The selection of R is convenient because it allows representing both models in the normalised space and to easily separate the two segments.

$$\begin{cases} \mu = R \\ \ln(\mu - 1) = \ln(a_1) + \ln(R - 1) + \varepsilon\sigma_{\ln(\mu-1)|R-1,1} \end{cases} \quad (1)$$

$$\begin{cases} \alpha = R \\ \ln(\alpha - 1) = \ln(a_2) + \ln(R - 1) + \varepsilon\sigma_{\ln(\alpha-1)|R-1,2} \end{cases} \quad (2)$$

2.2. SDoF database

A database of SDoF systems is defined to encompass a wide range of design possibilities representing feasible design configurations for different types of isolation systems (e.g., Lead Rubber Bearings, LRBs; High Damping Rubber Bearings; Friction Pendulum Systems). The considered database includes 2,000 SDoF systems for each isolation system type. To do so, the mapping variables $\boldsymbol{X} = \{f_y, t_1, h_{iso}, Hyst\}$ are derived based on the detailing parameters of each isolation system typology. Note that the hysteresis rule ($Hyst$) is a categorical variable, meaning that for each type of isolation system, the parameters controlling the shape of the hysteresis curve are constant among all the SDoF systems of that isolation type category. This study focuses specifically on LRB isolation systems.

In this case, three parameters define LRBs: height of the rubber, $height_{rubber}$; rubber-to-lead cross-section area ratio, A_{ratio}; and total weight of the structure divided by the bearing area of the isolators, σ_{eq}. Those variables and the mechanical properties of the isolator materials (yield stress, fy_{lead}; and shear modulus of the lead plugs, G_{lead}, and shear modulus of the rubber, G_{rubber}) are sampled with plain Monte-Carlo sampling using the distributions shown in Table 1. The mapping variables (f_y , t_1, h_{iso}) are then computed from the sampled values of the random variables by following the general theory of LRBs, e.g., Naeim and Kelly (1999).

Table 1. SDoF database definition: assumed distributions for the LRB dataset.

Random Variable	$height_{rubber}$ [m]	A_{ratio} [-]	$\sigma_{eq.}$ [MPa]	fy_{lead} [MPa]	G_{lead} [MPa]	G_{rubber} [MPa]
Assumed Distribution	$\sim U(0.15, 1.00)$	$\sim U(3, 25)$	$\sim U(6, 25)$	$\sim U(10, 13)$	$\sim N(130, 5)$	$\sim N(1, 0.1)$

2.3. Seismic response analyses

For each SDoF system in the database, 100 ground motion records are used to perform cloud-based NLTHA. The ground motions are selected from the SIMBAD database (Selected Input Motions for displacement-Based Assessment and Design); Smerzini et al. (2014). These recorded ground motions are characterised by moment magnitudes in the range of 5-7.3, source-to-station distances smaller than 35km and peak ground acceleration values in the range 0.29g -1.77g. The aim is to consider a broad range of strong ground motions so that the resulting surrogate model can be flexible enough to accommodate various design conditions. Although the response of isolated structures can be influenced by site-specific conditions (e.g., soft soils), such distinctions are not considered in building the database. However, users can re-fit the surrogate model by considering any set of ground motions (by filtering the existing results or by running NLTHA of un-considered records).

The ground motion records are scaled so that a non-linear response is achieved ($\mu > 1.0$ and $\alpha > 1.0$) since the elastic range of the PSDMs is automatically defined (see Section 2.1). To do so, each ground motion is selected randomly and is linearly scaled in amplitude using a scaling factor. The scaling factor is computed by selecting a random ductility value (100 equally spaced values between 1 and 20) and following the equal displacement rule ($\mu \simeq R$) as a reasonable approximation. Clearly, the resulting NLTHA will not result exactly in the assumed ductility value since the equal displacement rule only applies on average to such non-linear analyses. Finally, the cloud analysis results for each SDoF system in the database are used to fit the PDSMs as per Section 2.4.

2.4. Training of the surrogate model

GPs are statistical distributions over functions, entirely defined by their mean and covariance functions; e.g., Rasmussen and Williams (2006). A GP regression involves conditioning a prior GP (in a Bayesian framework) to an input-output training dataset (in this case, $\boldsymbol{X}$ and $\boldsymbol{Y}$ defined in Section 2). GP regressions are particularly convenient to fit a model to a dataset of observations because they are non-parametric statistical models, which are therefore not constrained to any specific functional form. The user only needs to define the typology of the covariance function to provide a predictive statistical model; e.g., Rasmussen and Williams (2006). For this work, a squared exponential covariance function is used since it can model the expected smoothness of the input-output map (i.e., a small perturbation of the input SDoF parameters causes a small variation of the PSDM parameters). The hyperparameters of the covariance are calibrated using a maximum likelihood approach and a quasi-Newton optimisation method; e.g., Gentile and Galasso (2020). The assumptions adopted for the training process are consistent with those implemented by Gentile and Galasso (2022), where they are extensively explained.

3. Validation of the Gaussian Process Regression

The trained surrogate models are subjected to several tests to assess the effectiveness of their predictions. This section shows the validation results related to the surrogate models for the LRB isolation systems. To assess the predictions within the dataset, the normalised root mean squared error (NRMSE) is calculated according to Eq. 3, where s_i represents the predicted outputs (e.g., the PSDM parameters) and m_i the modelled outputs for the *i*-th dataset input vector. The NRMSE values for the slope of the PSDMs correspond to 2.7% and 2.5%, respectively, for the ductility-based and acceleration-based PSDMs. The NRMSE values for the logarithmic standard deviation are equal to 6.6% and 5.9%, respectively.

$$NRMSE = \frac{mean(\sqrt{(s_i - m_i)^2})}{mean(m_i)} \tag{3}$$

To assess the predictive power of the surrogate models for unseen data (i.e., generalisation outside of the training dataset), 10-fold cross-validation is performed. To do so, the dataset is first randomly divided into ten equally-sized subsets. Then, ten more GP regressions are fitted for each surrogate model by leaving out one subset at a time and using the remaining nine subsets for the training. The excluded subset is used as a testing benchmark for each GP regression to compute the in-fold predicted-vs-modelled errors. Finally, the in-fold NRMSE is calculated by aggregating the predicted-vs-modelled errors of the ten GP regressions.

The in-fold NRMSE values for the slope of the acceleration and ductility PSDMs are 2.8% and 2.6% for the ductility-based and acceleration-based PSDMs, respectively. In contrast, the in-fold NRMSE for the logarithmic standard deviation is 6.7% and 6.1%, in the usual order (see Figure 2). Therefore, given the uncertainties commonly involved in the seismic performance assessment and risk models, the error introduced by using the provided GP regressions is deemed acceptable.

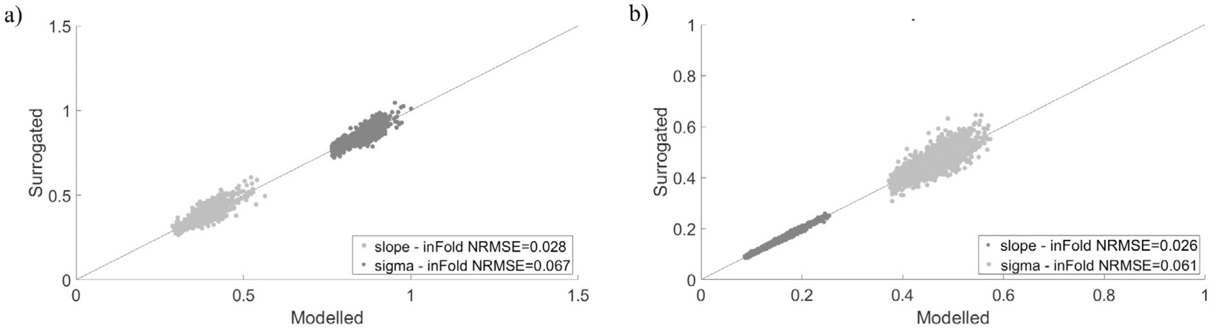

Figure 2. Surrogated (GP regression) versus modelled (SDoF cloud analysis) points; (a) ductility PSDM (b) acceleration PSDM

4. Tentative DLBD procedure for base-isolated structures

4.1. Overview

A brief description of a tentative DLBD procedure for base-isolated structures is presented in this section. Further work is underway to complete the formulation and validate it using a comprehensive set of case-study structures.

The procedure allows, practically without iterations, setting an economic loss target and identifying a combination of *design parameters* of the isolation system (f_y, t_1, h_{iso}) and superstructure (yield force of the superstructure normalised by the total weight of the structure, $f_{y_{SS}}$) consistent with that target. A predefined minimum level of structural reliability of the isolation system is also imposed. A designer and/or a client can select the desired target for the expected annual loss of the structure (this includes isolation system, superstructure and its contents), EAL_{target} and a maximum mean annual frequency of exceedance (MAFE) for the near-collapse damage state of the isolation system, $\lambda_{NC,\text{lim}}$. An additional design requirement is set for the MAFE of the yield damage state of the superstructure, $\lambda_{yield_{SS}}$, to assure an elastic behaviour of the superstructure. The design requirements are summarised in Eq. 4.

$$EAL = EAL_{target};\ \lambda_{NC} < \lambda_{NC,lim};\ \lambda_{yield_{SS}} < \lambda_{yield_{SS},lim} \tag{4}$$

The surrogate PSDMs described above are at the base of a simplified reliability- and loss-assessment module that, in turn, enables the non-iterative feature of the proposed DLBD. This module is fully automated and can run on a large set of potential design solutions (i.e., a *set of seed structures*) at a remarkably low computational cost. The seed structures that comply with the design requirements (Eq. 4) are selected from the seed space and referred to as *design candidates*. Finally, a *design solution* is arbitrarily chosen among the candidates based on user/client preferences. Structural detailing, strictly not part of DLBD, is finally provided for posterior detailing of the final structure, Figure 3.

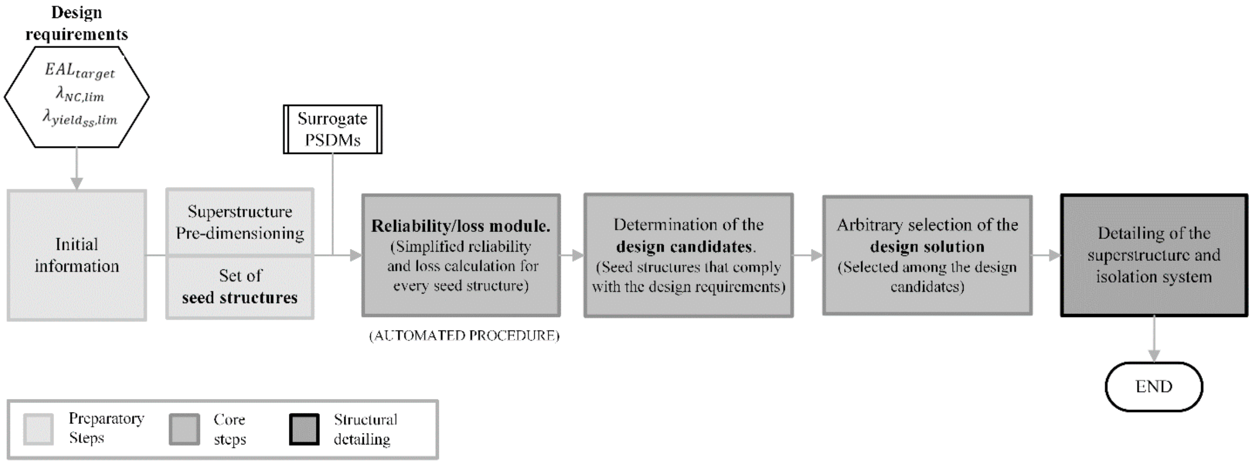

Figure 3. Schematic representation of the tentative DLBD procedure.

4.2. Loss- and reliability-assessment module

For the reliability assessment, the isolation system near-collapse fragility curve, $F_{NC}(IM)$, is first estimated assuming a log-normal distribution and using the ductility-based surrogate PSDM (see Eq. 1). Then, by adopting a site-specific hazard curve (describing the MAFE of $SA(t_1)$, λ_{IM}), the λ_{NC} is computed according to Eq. 5, where $F_{DS}(IM)$ corresponds to the fragility curve of the specific damage state DS and $\frac{d\lambda_{IM}}{dIM}$ corresponds to the derivative of λ_{IM} with respect to IM. An analogous procedure is followed to compute the $\lambda_{yield_{SS}}$.

$$\lambda_{DS} = \int_0^{\infty} F_{DS}(IM) \left| \frac{d\lambda_{IM}}{dIM} \right| dIM \tag{5}$$

The loss assessment first involves computing the fragility curves of the four subsystems within the structure: isolation system, superstructure, drift-sensitive non-structural components (NSCD) and acceleration-sensitive non-structural components (NSCA). This is done for a set of subcomponent-specific damage states (DSs), as described in Section 4.3. The fragility curves for the superstructure, NSCD and NSCA are computed using the acceleration-based surrogate PSDM (Eq. 2) and approximate relations expressing the acceleration profile along the height of the superstructure; e.g., FEMA (2012). The fragility curves for the isolation system are computed using the ductility-based surrogate PSDM (Eq.1). The vulnerability curves for each subsystem (n) are computed according to Eq .6 and represent the expected (mean) economic loss for a given IM level normalised by the total reconstruction cost, or loss ratio, $LR_n(IM)$. This requires using damage-to-loss ratios for each DS, DLR_{DSi}, representing the mean LR constrained on the realisation of the *i*-th DS. Given the site-specific hazard curve, the EAL for each subsystem is then computed according to Eq.7 and the overall EAL is calculated by aggregating all the EAL_n, Eq. 8.

$$LR_n = \sum_{i=1}^{\#DS+1} (F_{DS_{i-1}}(IM) - F_{DS_i}(IM))\, DLR_{DSi} \tag{6}$$

$$EAL_n = \int_0^{\infty} LR_n(IM) \left| \frac{d\lambda_{IM}}{dIM} \right| dIM \tag{7}$$

$$EAL = \sum_{n=1}^{4} EAL_n \tag{8}$$

4.3. Main steps of the procedure

The main steps for the DDBD procedure can be divided into three phases (Figure 3). The preparatory steps involve the initial design decisions, a pre-dimensioning of the superstructure and the definition of the set of seed structures. The core steps involve the reliability and loss calculations for all the seeds, the identification of the design candidates, and the final selection of the design solution. The third phase is added for completeness, and it involves detailing the isolation system layout, the isolation devices and the superstructure. Any design methodology can be used for detailing since this phase is essentially not part of DLBD. A summary of the main steps for each phase is shown below.

Preparatory steps:

- Selection of EAL_{target} , $\lambda_{NC,lim}$ and $\lambda_{yield_{SS},lim}$.
- Selection of the isolation system type (e.g., LRB, high damping rubber bearings, friction pendulum system).
- Definition of a set of damage states relevant for each subsystem. The designer can specify these damage states based on the type of isolation system, the characteristics of the structure, the inventory of non-structural components and the client's specific requirements. For example, the damage states can be set as inspection, replacement and near-collapse damage states for the isolation system. Slight, moderate and extensive damage states for the superstructure and slight, moderate, extensive and complete damage states for the NSCD and NSCA. The definition of these damage states can be taken from any relevant guideline or standard, e.g., HAZUS guideline, FEMA (2020).
- Definition of DLRs for each subsystem and each DS.
- Selection of the lateral-load resisting system and material for the superstructure (e.g., reinforced concrete wall).
- Pre-dimensioning of the superstructure. This involves the definition of the basic geometric properties and seismic mass of the superstructure and the isolation base. This can be based on gravity design.
- Computation of the yield displacement of the superstructure following direct displacement-based design principles; Priestly et al. (2007).
- Definition of the set of seed structures in terms of ranges for the isolation system properties and the strength of the superstructure to be considered. The values for the properties of the isolation system can be calculated from the parameters specific to the selected system; see Section 2.2.

Core steps:

- Computation of the EAL, λ_{NC} and $\lambda_{yield_{SS}}$ for each seed structure using the simplified loss- and reliability-assessment module.
- Determination of the design candidates by selecting the seed structures that comply with the design requirements (Eq. 4).
- Selection of the final design solution from the design candidates. This decision can be based on any desired consideration (e.g., economic considerations, facility to manufacture the isolation devices, easiness of implementing the design solution).

Structural detailing:

- Detailing the isolation system and the superstructure in such a way that the design parameters of the design solution are met. This includes the selection of the isolation devices layout, the detailing of the isolation devices, the detail of the superstructure to reach the desired yield strength and the detailing of the foundation system.

5. Conclusions and limitations

This paper presented the formulation, calibration and validation of two surrogate probabilistic seismic demand models (PSDMs) based on Gaussian Process (GP) regressions. A database of 2000 cloud-based non-linear time-time history analyses was used to calibrate the PSDM surrogate models for lead rubber bearing isolation systems. In addition, a 10-fold cross-validation was performed, showing adequate prediction capacity of the adopted GP regressions.

The proposed surrogate PSDMs enabled the proposal of a direct loss-based design (DLBD) of base-isolated structures. This procedure allows designing structures that would achieve a given economic loss target for a given site-specific hazard profile while complying with a predefined minimum level of structural reliability. A general overview of the tentative DLBD procedure for base-isolated structures was also presented. Several remarks about this work can be given:

- Surrogate models based on GP regressions represent an appealing alternative to generate predictions of PSDMs. An advantage of this type of model is that it is non-parametric and requires no previous knowledge of the functional form of the input-output mapping.
- The proposed surrogate PSDMs have been proven effective and efficient in overcoming the high computational cost required to compute analytical fragility curves, deemed incompatible with the preliminary design phase.
- The validation errors of the surrogate PSDMs lie within acceptable ranges, especially considering the uncertainties and approximations generally affecting seismic risk analyses.
- The complete description, applicability and validation of the proposed DLBD methodology will be developed in future work.

The proposed tentative DLBD is currently affected by limitations that the authors are currently addressing:

- The SDoF models can only capture the first mode response of the combined isolation and superstructure system. This implies that the maximum acceleration and displacement response of the superstructure are assumed to happen at the same instant as the maximum response of the isolation layer. Thus, the procedure will lose its effectiveness for structures where higher modes are important.
- Since the probabilistic seismic performance analysis considers a complete range of intensity measure levels, and the dynamic properties of isolated structures depend on the effective stiffness of the isolation system, the relative stiffness of the superstructure needs to be high enough with respect to the effective stiffness of the isolation system, even at low-intensity measure levels, such that the second dynamic mode of the isolated structure is not significant in the response.
- In highly damped isolated systems, the coupling of modal shapes can generate a high floor acceleration response, e.g., Skinner et al. (1993); the implemented models cannot capture this effect.
- This procedure is only applicable to structures with regular superstructures. The torsional response of the superstructure is not yet included in the methodology.

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