Neural surrogates for fast Grad-Shafranov equilibria reconstruction

E. Lewis[1,2], S. Pamela[1], Y. Andreopoulos[2], G. Szepesi[1]

[1] *Culham Centre for Fusion Energy, Oxfordshire, UK*

[2] *University College London, London, UK*

Introduction

Control of plasma behaviour is a key step in realising nuclear fusion as a viable energy source. Real-time control is reliant on rapid and accurate plasma simulation, the fundamental component of which is calculation of the magnetohydrodynamic (MHD) equilibirum. Reconstructing the equilibrium can provide characteristics such as stored energy, safety factor and magnetic topology in addition to acting as input for further analysis and modelling. EFIT++ is an established numerical equilibrium code [1], in use on the Joint European Torus (JET), EAST, and KSTAR tokamaks among others. While powerful, EFIT++ is computationally costly and not appropriate for real-time or large-scale applications. One such online scenario is plasma control which typically requires reconstruction times of the order of 1ms. Parallelised and approximated equilibrium codes have been developed with the aim of accelerating EFIT++'s calculation speed. rt-EFIT [2] and the GPU based P-EFIT [3] demonstrated improved performance in terms of computation time, but exhibited a trade-off in reconstruction accuracy. Surrogate models offer a solution to this problem, emulating the performance of more costly simulations at a much faster computational speed. Neural networks (NN) are a popular choice of surrogate due to their ability to accurately model high dimensional non-linear functions in a data-driven manner. They also offer a choice of loss functions that can be better oriented to the task than other hand-crafted function approximation methods.

Neural surrogates have long been in use by the fusion community for equilibrium reconstruction [4]. Many of the previous surrogates focused on predicting a set of plasma-control relevant values such as elongation, boundary position and poloidal beta. Joung et al. [4] were the first to present an EFIT++ surrogate that reconstructed the entire magnetic topology of a plasma. Additionally recent research on stellarator equilibria surrogates indicates that convolutional neural networks (CNNs) perform better than conventional feedforward neural networks (FNNs) [5].

In this paper proof-of-concept neural surrogates with the ability to rapidly reconstruct EFIT++ JET-ILW equilibria are presented. Both CNN and FNN models were developed with the goal of improving surrogate accuracy. Model inference time was below the required control threshold of 1*ms*.

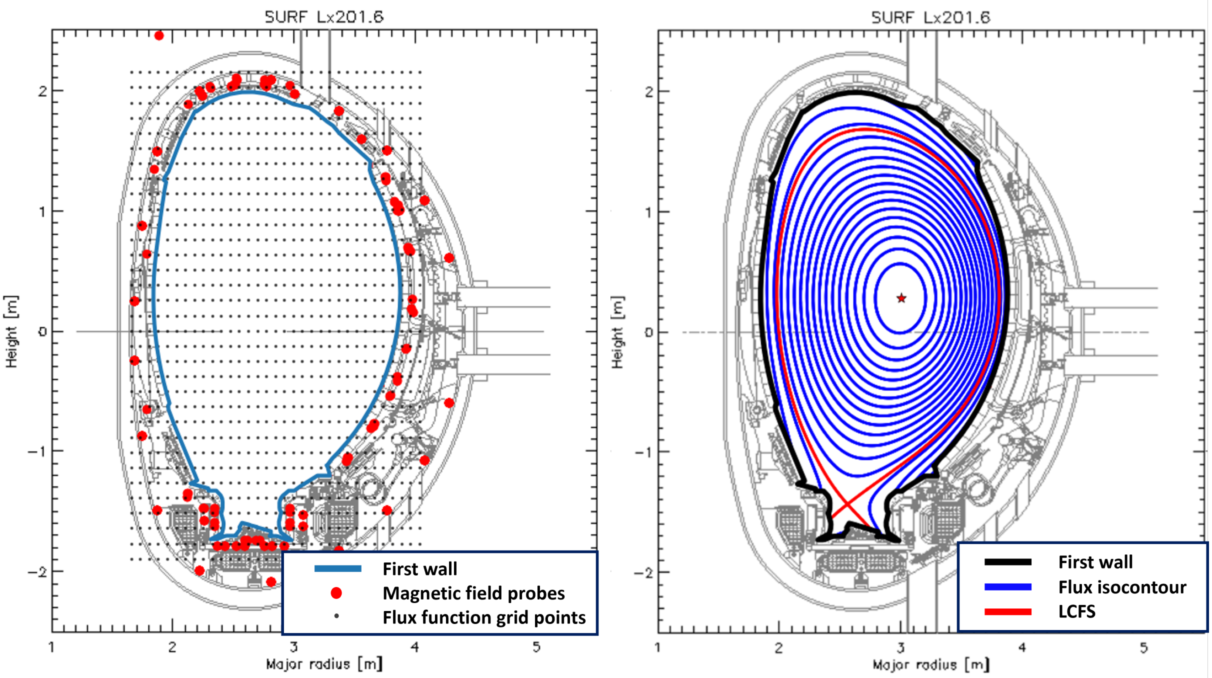

Figure 1: Poloidal cross sections of JET. Diagnostic probe locations and the magnetic topology of an equilibrium calculated by EFIT++ are displayed. 94250

Neural Surrogates for EFIT++

EFIT++ is a numerical solver which reconstructs the plasma equilibrium from tokamak magnetic diagnostics and optional additional constraints. Figure 1 details a subset of magnetic probe locations on JET, in addition to the magnetic topology of an equilibrium calculated by EFIT++. The equilibrium reconstruction requires finding the solution to the Grad-Shafranov (GS) equation, a description of an ideal two dimensional MHD plasma equilibrium with toroidal axisymmetry [6]:

$$\Delta^*\psi \equiv R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} = -\mu_0 R j_\phi = -\mu_0 R^2 p'(\psi) - \mu_0^2 f(\psi) f'(\psi) \quad (1)$$

This equation describes the plasma in terms of the poloidal flux function ψ, the toroidal current density function j_ϕ, the pressure $p(\psi)$, the magnetic permeability μ_0, and $f(\psi)$ a function related to the net poloidal current. Possible additional constraints include the magnetic field pitch angle data provided by the motional Stark Effect diagnostic and plasma pressure measurements to calculate kinetic equilibria.

Neural networks were trained as surrogates using experimental diagnostics on JET as input and equilibria calculated by EFIT++ for the same pulses as output. A range of historical pulses were sampled from 2011-2020 in order to account for the plasma facing wall upgrade JET received in 2011, significantly affecting magnetic behaviour. Measurements from a subset of sensors were used to account for variations in probe robustness and redundancy due to toroidal symmetry. The input signals were the field measured by 40 normal and tangential magnetic pickup coils, magnetic flux from 32 large single turn flux loops and plasma current from a Rogowski coil. The target output was the flux calculated by EFIT++ at points on a 33x33 grid as shown in Figure 1.

Each pulse was split into a series of time-slices corresponding to EFIT++'s temporal resolution of 0.03s. The dataset consisted of 31191 timesteps from 955 pulses. These were shuffled and divided into training, validation and test datasets with an 80:10:10 ratio. Magnetic flux output values were normalised with the maximum and minimum from the flux at the plasma boundary and at the magnetic axis respectively. Diagnostic inputs were similarly normalised with the maximum and minimum values from across all probes of the same type.

Two model architectures were used, a feedforward fully-connected network (FNN) and a convolutional neural network (CNN). Bayesian hyperparameter search was used to optimise the model architectures including the size and number of hidden layers. Both models used the Adam optimiser, dropout layers and Leaky ReLU activation functions. The FNN contained seven hidden-layers of size 100. The CNN contained two dense layers of size 1210 in addition to two transposed convolutional layers [7] of stride 5x5 and a reshaping layer, to allow the network to generate a 33x33 ψ-image from the scalar probe input values.

Results and Discussion

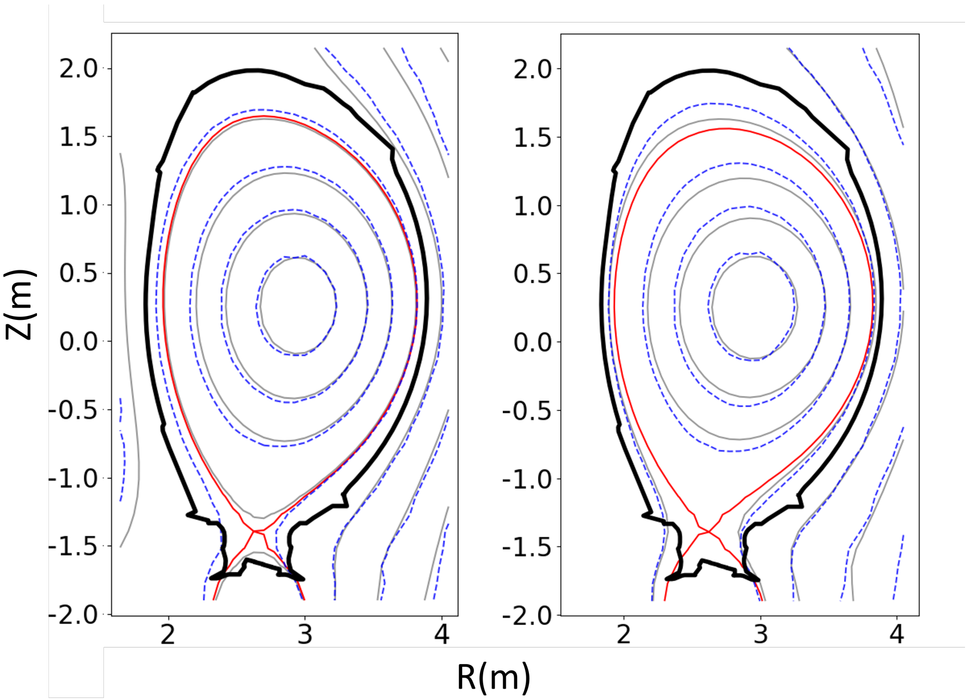

Figure 2: JET-ILW equilibria reconstructed by the FNN. Isocontours predicted by surrogate (blue) compared to EFIT++ ground-truth (grey). First wall (black) and LCFS (red) indicated. Pulse #94250.

Model inference times for a single equilibrium reconstruction were on average roughly $500\mu s$

(benchmarked on an Intel 4 core i7 processor PC), below the 1*ms* limit required for real-time plasma-control. By visual inspection of flux isocontours in Figure 2 there is very good agreement between the FNN surrogate and the original EFIT++ simulation. The mean squared error (MSE) reported by the FNN model on an unseen test set was 0.0023, in terms relative to the ψ-values this is a mean average percentage error (MAPE) of 9%. The MSE of the CNN was 0.0255 and MAPE 34%. This is contrary to the expectation that the CNN would perform better due to its superior ability to extract and predict features from grid-data and capture spatial relationships between the values in the ψ-output. This is likely due to the more extensive hyperparameter tuning applied to the FNN model, further optimisation of the CNN and a consideration of the number of trainable weights in each model is necessary before a valid comparison is possible.

Summary and Further Work

Neural surrogates with the ability to rapidly reconstruct JET-ILW plasma equilibria were trained. The models reproduce the poloidal flux function ψ(R,Z) calculated by EFIT++, an equilibrium solver used on JET. Two model architectures FNN and CNN were tested. Model inference time is of the order of microseconds, demonstrating the viability of surrogate EFIT++ models for real-time control scenarios.

In addition to optimisation of the CNN, future work will involve modifying the surrogate loss function to incorporate terms relating to the Grad-Shafranov equation, thereby introducing a physical constraint to the network as in Joung et al [4]. These derivatives will be acquired through automatic differentiation of the neural network. Finally surrogates of more sophisticated EFIT++ simulations with added constraints such as the motional stark effect and force and pressure profiles will be investigated.

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

References

[1] L. L. Lao et al. Nuclear Fusion **25**, 1611-1622 (1985)
[2] J. R. Ferron et al. Nuclear Fusion **38**, 1055 (1998)
[3] X. N. Yue et al. Plasma Physics and Controlled Fusion **55**, (2013)
[4] S. Joung et al. Nuclear Fusion **60**, (2020)
[5] A. Merlo et al. Nuclear Fusion **61**, (2021)
[6] J. Wesson and D. J. Campbell, Tokamaks 3rd ed, 118 (2004)
[7] A. Dosovitskiy et al. IEEE Transactions on Pattern Analysis and Machine Intelligence **39**, (2017)