A case study of an expert in computational thinking in the context of mathematics education research

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We conducted semi-structured interviews with three experienced mathematics education researchers with great expertise in the design and use of digital technologies, including programming skills, to investigate their views and perceptions on computational thinking (CT) and its impact on mathematical learning. In this paper we report on our findings from one of them, Mark, and we suggest ways for adapting the very recent Mathematical Digital Competency (MDC) framework to encompass CT practices and dispositions. Our aim is to offer insights into how CT is perceived and understood by him, by prompting him to reflect on his own CT practices and competencies. We offer suggestions for an MDC framework for mathematics teacher educators that encompasses CT.

Keywords: Computational thinking, mathematics education researchers, mathematical digital competency, digital technologies.

Introduction

Computational Thinking (CT) has made its appearance in the mathematics education of the digital era over a decade ago and ever since then, the upsurge of interest in its influence/impact on mathematical teaching and learning is evident. Looking back at PME44, when Inprasitha (2021) announced the theme of the conference being on "Mathematics Education in the 4th Industrial Revolution", CT was characterised as an essential competency for a digital society. The relationship between CT and mathematics has been of particular interest. Indeed, some see CT as offering the potential to transform school mathematics (e.g., Perez, 2018).

Teacher education will be critical in enabling mathematics teachers to realise the potential of CT to transform mathematics. Yet, to date, educational literature on CT, or computational competency or the "new digital age competency" as sometimes is referred to (e.g., Grover & Pea, 2013), has mainly focused on students' CT. Undoubtedly, to promote effective CT teaching (Weintrop et al., 2016), one should focus on teacher education and professional development as argued by Lee et al. (2020). To our knowledge, there is not any research that investigates mathematics teacher educators' (MTEs) expertise in CT.

We address this gap by investigating MTEs' CT and their computational practices in order to better articulate the knowledge and beliefs required by mathematics teacher educators. To do this, we consider the "telling case" (Mitchell, 1984) of Mark, an experienced 'mathematics education with technology' researcher and teacher educator, Mark, who has extensive knowledge of Programming, Artificial Intelligence (AI) and Machine Learning (ML) from an over 15-years active design-based research background on educational technologies in mathematics education. We present initial findings from an exploratory study in order to consider ways for adapting the Mathematical Digital

Competency (MDC) framework (Geraniou & Jankvist, 2019) to encompass mathematics teacher educators' CT practices and dispositions.

Computational thinking and mathematics education

Cuny, Snyder and Wing (2010) defined CT as "*the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent*". The consensus of research (e.g., Shute et al., 2017) is that, whilst there are practices in common, CT is a distinct and separate discipline to mathematics, and that CT is less about the use of technology and computers and more about the concepts, practices and processes involved. However, CT involves practices that are also required in mathematics, such as "decomposition, abstraction, algorithm design, debugging, iteration, and generalization" (Li et al., 2020, p.156). There has been some research regarding the teaching of CT both in general (see Grover & Pea's, 2013) and specifically in mathematics and other STEM subjects (see, e.g., Lee et al., 2020). Our recent work (Geraniou & Hodgen, 2022) indicated that unlike the teachers in Sands et al.'s (2018), neither of the two MTEs we interviewed viewed CT narrowly as synonymous with doing mathematics nor simply using digital tools to do mathematics. However, both appeared to have had limited opportunities to articulate the relationship between CT and mathematics and, as a result, viewed CT as closely tied to computers and other digital tools. This outcome, together with anecdotal data from our own past experiences and work, suggest that there is variation in the way CT is perceived and potentially used by teachers and teacher educators. This argument goes against Perez's (2018) claim that the practices identified in his review represent a consensus in mathematics education. All these findings suggest the need to investigate further mathematics education researchers' (as well as mathematics teachers') perspectives on what CT is and assess their CT practices, offering support towards enriching their mathematics teaching practices.

Teacher knowledge, competencies, and computational thinking

Recent research into teacher knowledge and instructional quality has shown that a key aspect of teacher knowledge is not just the knowledge itself, but also the enaction of this knowledge (Tabach, 2021), or the extent to which pedagogic strategies and tasks are cognitively challenging for students. König et al. (2021) refer to this enaction of knowledge as cognitive activation and conceive of it as a central aspect of teacher competencies (König et al., 2021). Tabach (2021) discusses this shift from knowledge to competencies, as inspired by Niss and Højgaard's (2019) view on what it means to be mathematically competent as articulated in the Danish KOM mathematical competencies framework. "By focusing on mathematical competence rather than on mathematical subject matter as the integrating factor of mathematics across all its manifestations, we have chosen to focus on the exercise of mathematics, i.e., the enactment of mathematical activities and processes" (Niss & Højgaard, 2019, p. 12). Perez's (2018) review of CT in mathematics education indicates a ~~similar~~ shift towards competencies by highlighting the practices and dispositions involved in CT. He highlights a range of practices, including elements, such as "developing algorithms and automations" as well as composite practices, such as "efficient and effective combinations of resources, testing and debugging" (p. 428).

The Danish KOM mathematical competencies framework was presented in 2011 by Niss and Højgaard, to represent the mathematical competencies possessed by students, but teachers too. In a more recent publication, Niss and Højgaard (2019) defined mathematical competence as comprising "knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role" (Niss & Højgaard, 2011, p. 49). Building upon this framework, Geraniou and Jankvist (2019) proposed that students' having Mathematical Digital Competency (MDC) involves the following three elements:

- "[MDC1]: *Being able to engage in a techno-mathematical discourse*. In particular, this involves aspects of the artefact-instrument duality in the sense that instrumentation has taken place and thereby initiated the process of becoming techno-mathematically fluent.
- [MDC2]: *Being aware of which digital tools to apply within different mathematical situations and context, and being aware of the different tools' capabilities and limitations*. In particular, this involves aspects of the instrumentation–instrumentalisation duality.
- [MDC3]: *Being able to use digital technology reflectively in problem solving and when learning mathematics*. This involves being aware and taking advantage of digital tools serving both pragmatic and epistemic purposes, and in particular, aspects of the scheme-technique duality, both in relation to one's predicative and operative form of knowledge" (p. 43).

We also agree with Krumsvik and Jones's (2013) characterisation regarding teacher's digital competence that involves two dimensions, that of the competence to use technology for personal use and additionally that of the competence to use technology in pedagogical settings. This idea that has been conceptualised by Chick and Beswick (2018) as *meta pedagogical content knowledge* (meta-PCK) of MTEs. Extending these theoretical ideas, we suggest that mathematics educators' expertise may involve a further conceptualisation or in other words a meta-MDC, where for example, they engage in a meta-discourse about their own practice and the capabilities and limitations of the particular tools supporting their practice and how these relate to more general aspects of CT.

All the above made us consider that there is a clear link between CT practices and MDC. We should also look into the composite CT practices as it is particularly challenging for educators to become competent at a meta-level in combining these various elements. So, our research question is: *In what ways can mathematics educators conceptualise CT in relation to MDC?*

Design and methods

We carried out an exploratory study with three MTEs, who have extensive experience with research in the use of digital technologies for mathematical teaching and learning. We believed that identifying MTEs' beliefs would shed light onto what CT is, what CT practices are, what the relationship between CT and mathematical thinking is, how CT practices can be promoted among mathematics teachers, why CT practices are useful (or not) and what they offer to mathematics education. We interviewed those three MTEs independently. Our initial findings based on our discussions with two of the three MTEs, were discussed in a recent publication (Geraniou & Hodgen, 2022). In this paper, we will focus on our discussions with the remaining MTE, Mark.

We carried out a 60-minute interview with Mark and we present a vignette of our discussions. The interview consisted of two parts. In the first part, Mark had to present and reflect on a mathematical activity involving CT, using the Think-Aloud protocol (Güss, 2018). We asked him to reflect on (a) the programming aspects, (b) mathematical definitions, (c) the structure of the mathematical and the tool's language, and (d) the algorithms. Given that CT is a relatively new area of interest, we wanted a task that would enable Mark to articulate various aspects of CT practices. Hence, we asked Mark to bring along a problem he was familiar with. This has an advantage of generating a range of ideas in a relatively short space of time, but has some limitations in terms of comparing the MTEs' beliefs. In the second part, we asked Mark for his own definition of CT using Cuny, Snyder and Wing's (2010) definition and discussed the CT practices presented in Perez's (2018) paper.

Mark's vignette

A mathematical activity

Mark presented an activity that he had recently created at the request of a teacher. The activity was designed to enable students to investigate the modelling of an infectious disease such as COVID. It was designed in Scratch using a simple model of the effect of different factors (movement, handwashing, the transmission rate, and healthcare capacity) on infection and death rates. The environment allowed students to change these factors to explore their effects. The activity, as seen by students, is illustrated in Figure 1. The movement of people, represented as coloured sprites, was modelled as a random process and students could use a slider to alter the level of movement, thus reflecting the effect of social distancing restrictions. When the sprites 'meet', the likelihood of infection was again modelled randomly and students could use a slider to alter the level of handwashing, thus reflecting the effect of hygiene measures. The likelihood of recovery is affected by the healthcare capacity, which can be altered either by students or the teacher. Sprites are in one of four states: susceptible to the disease (yellow), infected (red), recovered and assumed immune (blue) or dead (black). The graph in the bottom left-hand corner shows the level of infection over time against the health care capacity.

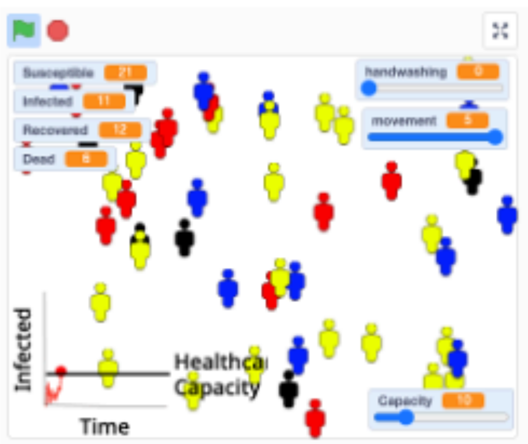

Figure 1: The Scratch activity as explored by students

Mark's definition of computational thinking

Mark saw a strong overlap between mathematics and CT with the notion of variable being key to modelling in both disciplines. Nevertheless, despite this common interest, he considered the two disciplines as distinct. He defined CT as follows:

> Mark: [It is] many things … the simple answer is ... being able to think … with a specific programming language or specific tools … in a way that allows you to address a particular problem … [And] being able to develop something in a way that is... general or abstract, which could be configurable. So going back to the example here, I want to give the people the possibility to ... change the parameters so [they can] understand the model in principle.

So, for Mark, CT involves not only practices, such as understanding coding and algorithms, and dispositions, such as thinking like a programme, but also purposes, such as developing general solutions to problems.

Mark's computational thinking in light of MDC and pedagogy

Unsurprisingly, given his expertise and background, Mark demonstrated considerable facility with CT and we found evidence for many of Perez's (2018) CT practices and dispositions. More significantly in terms of our interest in MTE's knowledge of CT, he also demonstrated a consideration of the cognitive activation directed at teachers and pedagogy that appear to align well with MDC (Geraniou & Jankvist, 2019).

MDC1. Being able to engage in a techno-mathematical discourse: Mark engaged in a techno-mathematical discourse about CT. He reflected not only on the cognitive activation of tasks, but also on *how* and *why* to cognitively activate tasks **(**König et al., 2021). Mark was careful to distinguish the "*relatively unexplored' and 'half-baked" activity* microworld environment (using a term from the literature on microworlds - e.g., Kynigos, 2007) from the actual task that a teacher would set students which might be how to reduce the transmission or to "*fix"* some "*broken*" aspect of the activity. This could enable teachers to "*expose students to this idea that there's a variable that is [between] 0 and 1 and it has an impact".* He noted that he would "*flag the potential … to ask this kind of 'what if' questions".* Indeed, for Mark, it is the point at which CT and mathematics "meet" where the problem becomes pedagogically productive:

> Mark: A lot of the code is just setting up things and ... not so important. ... Where maths meets computation somehow is here, because, this is a code for the person that moves around. So from Scratch, you have these ... sprites ... that move around and [we] define the movement ... [as] random … The other thing ... [is] when they touch the edge ... [they] bounce back which … simulates a small school or a city or whatever you want.

Hence, Mark was able to engage in a techno-mathematical and computational discourse ***at** a meta-pedagogic level*, as indicated by how he distinguishes the key moment in coding with Scratch.

MDC2. Being aware of […] digital tools […] and their capabilities and limitations: It was striking that Mark reflected on how his practice was embedded in the programming environment and thus his knowledge was *distributed* across the programming language (Helliwell & Chorney, 2021; Hodgen, 2011):

Mark: Scratch ... is a different way of thinking. ... [The] programming language becomes an object to think with. ... had someone asked me to do this part with the transmission rate, ... I'm not sure I would have done it this way. … But it's an interesting notion that you pick a number, and then you compare it to the transmission rate.

Here, Mark demonstrates meta-level thinking in his consideration of how Scratch structures one's thinking in particular ways.

He went on to reflect on the constraints of Scratch in terms of "cutting corners" and distinguished this from the simplification involved in constructing models in general:

Mark: I'm using [cutting corners] also in a computational way, ... because of Scratch. … It's me thinking of the limitations of Scratch. ... Obviously in any modelling you have to simplify … [I] was being critical of Scratch … that's why I said I would cut corners.

This demonstrates an awareness not only of the tools of CT but also of how teachers think, and act, pedagogically with these tools and the benefits and limitations of these.

MDC3. Being able to use digital technology reflectively in problem solving and when learning mathematics: He reflected on modelling computationally and mathematically. In particular, he noted that "*the actual models [of transmission] have differential equations in them*" and are hence beyond much of school mathematics. However, he considered that pedagogic models in CT classrooms do not need to be "authentic", but should rather be "meaningful":

Mark: Obviously, if you wanted to have a proper model, it would be mathematically very complex and so this [model] is targeted to early secondary. ... So, it's very simple, the code, to be able to achieve this and it doesn't reflect obviously a proper COVID mathematical model. But I think that's actually what makes it kind of useful.

So, in Mark's view, the pedagogic task is to model modelling in order that the CT model is "*close enough*" to key aspects of the "*real, more complex*" model "*because ... it doesn't happen always that when you are close to that person you get the virus, which is close to reality. ... [I]t happens based on a transmission rate*." This shows not only an awareness of computational modelling as a pedagogical exercise but also at a meta-level what is key in supporting students' interpretation of the mathematical model.

Conclusion

Our study indicated that Mark was very skilled in CT and mathematics and offered insightful comments about CT in relation to mathematical modelling in the Scratch environment, subsequently revealing his own MDC. Reflecting upon our past work (Geraniou & Hodgen, 2022), we remind our readers that we had highlighted the need to articulate the nature of Computational Thinking Pedagogical Content Knowledge (CTPCK). In this paper, however, the data from Mark led us to a different avenue to knowledge, that of competencies. We argue that possessing CT is a competency and in fact a mathematical digital competency, based on the definitions shared by Geraniou and Jankvist (2019). In more detail, we suggested some adaptations to the three elements of MDC regarding students to encompass an MDC framework for mathematics educators that considers CT practices:

- [MDC1:] *Being able to engage in a techno-mathematical* ***and computational*** *discourse* ***at a meta-pedagogic level****.*
- [MDC2:] *Being aware of which digital tools to apply within different mathematical situations and context, and being aware of the different tools' capabilities and limitations*, ***so as to think, and act, pedagogically with these tools, while considering the benefits and limitations of these****.*
- [MDC3:] *Being able to use digital technology reflectively in problem solving and when learning mathematics,* ***considering and applying computational modelling as a pedagogic enterprise.***

What distinguishes Mark's vignette is his ability to reflect about the nature of CT framed in a pedagogical manner, indicating how he enacted his own mathematical knowledge and computational thinking as several integrated competencies, in our case MDCs.

Our future work entails the wider empirical investigation of this framework for MTEs' competencies with regards to CT practices. We want to identify the CT elements mathematics teacher educators and mathematics teachers possess and those CT elements they should acquire to enrich their mathematics teaching practice. We conclude by posing a challenge to our readers: Is CT better conceived as CTPCK or MDC?

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