On a Definition of Logical Consequence

Nils Kürbis
Ruhr University Bochum
University College London
University of Lodz

Abstract: Bilateralists, who accept that there are two primitive speech acts, assertion and denial, can offer an attractive definition of consequence: Y follows from X if and only if it is incoherent to assert all formulas X and to deny all formulas Y. The present paper argues that this definition has consequences many will find problematic, amongst them that truth coincides with assertibility. Philosophers who reject these consequences should therefore reject this definition of consequence.

Key words: logical consequence, assertion, denial, bilateralism

According to bilateralism in the theory of meaning, the meaning of an expression is determined by its contribution to the conditions, not only of the correct assertibility, but also of the correct deniability of sentences in which it occurs. The view originates in the work of Price (1983; 1990) and was further developed, with a particular view to its application in logic and the specifications of the meanings of the logical constants, by Smiley (1996) and Rumfitt (2000). It is an attractive position that promises to solve a puzzle inherited from Dummett's work, on which more later.[1]

Bilateralism lends itself to a striking definition of logical consequence, proposed by Restall, in terms of a notion of incoherence between assertions and denials (Restall 2005). Restall uses $[X : Y]$ to denote a *state*, where X and Y are sets of statements. There are various options for interpreting states. They might represent the statements an agent accepts and rejects, where X are the former and Y the latter, or the statements asserted and denied in a dialogue or discourse, X again those asserted, Y those denied. Some states are incoherent, for instance those in which the same formula occurs to the left and to the right of the colon. Restall defines '$X \vdash Y$' as '$[X : Y]$ is incoherent.' Given some basic assumptions, which not everyone needs to share, this consequence relation coincides with that of Gentzen's formalisation of classical logic in multiple conclusion sequent calculus.

Restall's account is adopted by Ripley. What Restall calls 'states,' Ripley calls 'positions,' and instead of coherence and incoherence, Ripley speaks of positions being within and without bounds.[2] I quote an illustrative passage in its entirety, as it also contains examples, with notation adjusted to cohere with Restall's:

> The position $[X : Y]$ asserts each thing in X and denies each thing in Y. Some positions are *in bounds*, and some are *out of bounds*. For example, the position that consists of asserting 'Melbourne is bigger than Brisbane' and 'Brisbane is bigger than Darwin' while denying 'Melbourne is bigger than Darwin' is out of bounds. It's part of what gives 'bigger' its meaning, on this view, that it is transitive. On the other hand, the position that consists

Nils Kürbis, Ruhr University Bochum, University College London, University of Lodz; nils.kurbis@filhist.uni.lodz.pl.

> of asserting 'Melbourne is bigger than Brisbane' and 'Brisbane is bigger than Darwin,' without any denials, is in bounds. It hasn't violated any of the inferential properties of the expressions contained within it.
>
> $X \vdash Y$ can now be read as the claim that the position $[X : Y]$ is out of bounds. Thus, this account gives an understanding of consequence in terms of the prior notions of constraints on assertions and denials. (Ripley 2013, 141)

Ripley agrees with Restall that it is out of bounds to put the same formula to the left and right of the colon, and if certain basic assumptions are shared, his notion of consequence coincides with Restall's.[3]

Some bilateralists may wish to follow Restall and Ripley and adopt their definition of consequence: $X \vdash Y$ iff it is incoherent or out of bounds to assert all formulas in X and deny all formulas in Y. I shall call this *AD*-consequence.

The definition has many merits. One in particular is worth underlining. The left and right hand sides of $\vdash$ are treated in exactly the same fashion: in each case, something is done to all formulas in them. The definition thereby evades objections often levelled against multiple conclusions when they are given their ordinary explanation, where something is done to *all* formulas to the left of $\vdash$ (asserted, assumed to be true, assigned the truth value True) and the same is done to *some* formulas on its right. It is often objected that we have no practice of reasoning in which such multiple conclusions are drawn or that such multiple conclusions cannot be understood without a prior understanding of disjunction, which means that they presuppose what is to be explained, if the aim is to specify the meanings of the logical constants in inferential terms.[4] Few object to the mere listing of formulas to the left of $\vdash$ in single conclusion calculus. In *AD*-consequence, the right hand side of $\vdash$ is treated equally to its left, and so the objection that multiple conclusions presuppose an understanding of disjunction has no more target. Some work may still be needed to convince the opponents that proponents of *AD*-consequence can answer the objection that there is no practice of arguments that draw multiple conclusions.[5] However, the symmetry in the treatments of the antecedent and consequent provides leverage for an answer that is not available in unilateral multiple conclusion logic. Even if there was no practice of arguments with multiple conclusions, with *AD*-consequence, an extension of this practice to such arguments would require no more conceptual resources than are already available to anyone capable of engaging in the practice with multiple premises. None of this is decisive and all of it would require spelling out. But be that as it may, the prospects are there, and it appears as if *AD*-consequence, if certain basic assumptions about what is incoherent or out of bounds are accepted, permits an inferentialist[6] justification of classical logic, which, arguably, remains elusive within a framework that adopts single conclusion sequent calculus or natural deduction.[7]

My aim here is not to extoll the virtues of *AD*-consequence. I rather wish to discuss what I consider to be a fundamental problem with it. It commits its proponents to consequences which do not appear to be sufficiently appreciated. At least I know of no discussion where they are explicitly endorsed. These consequences may even show the account to be incoherent. The main observation on which my argument turns has been made before by Rumfitt in two footnotes.[8] The contribution of the present paper is to draw attention to Rumfitt's observation, which should be more widely known and calls for a response from proponents of *AD*-consequence. Furthermore, I spell it out in a little more detail and draw further consequences for the relation between truth and assertibility in the context of a bilateralist theory of meaning. Restall and Ripley are keen to point out that different philosophers may consider different states or positions to be incoherent or out of bounds: this is a question of a wider philosophical outlook. And indeed, for some philosophers the consequences I am going to discuss may not be problematic at all. In two cases I will indicate who might be amongst them. But I will also give examples of philosophers to whom these consequences are unacceptable, and some of them are bilateralists who explicitly deny them.

(a) It is incoherent to assert a proposition and deny that it is assertible. Let's use Rumfitt's method of representing assertions and denials by yes-no questions and their answers. Consider N who asserts and denies the following:

N: Is there any tea left? Yes.

Also N: Is it assertible that there is any tea left? No.

This is incoherent: the second speech act denies that the first speech act could have been performed. Clearly this generalises: for any proposition A, it is incoherent to assert A and deny that A is assertible. Hence according to AD-consequence, every proposition entails its own assertibility.[9] Few philosophers will accept this consequence.

Analogously, it is out of bounds to assert 'There is no hippopotamus in the room' and deny 'It is deniable that there is a hippopotamus in the room.' Hence according to AD-consequence, every negated proposition entails its own deniability, and consequently, any false proposition does, if falsity entails truth of negation. But to bring out the problem, it suffices to focus on assertibility.

(b) It is out of bounds to assert a proposition and to deny that it has been asserted. The reasons are similar to those why case (a) is incoherent: it is self-defeating to deny that an assertion was made that has in fact been made. But then according to AD-consequence, every proposition entails that it has been asserted, i.e., A entails that it has been asserted that A. This will go too far even for many of the staunchest anti-realists.

(c) It is out of bounds for me to assert 'There is tea left' and deny that I have asserted this. If I do this, I have put myself firmly out of bounds of respectability. Take your favourite example of a liar at the helm of government to appreciate the problem. Someone who asserts A and denies that he asserted it is lying or deceiving or misleading. According to AD-consequence, it therefore follows that A entails that I have asserted that A. Few will be happy with such a conclusion.

Maybe cases (b) and (c) may be dismissed as frivolous. Maybe they are merely instances of some version of Moore's Paradox that are otherwise to be dealt with or it is a problem with liars, not logic. I am not convinced by this myself. Moore's Paradox, whatever its solution, turns on a non-logical notion and as such is not a logical paradox. It involves belief, and resources relating to that notion are available to dispel it. Maybe its solution is to be found in pragmatics. According to AD-consequence, however, assertion and denial are notions at the foundations of logic, so the difficulties posed by cases (b) and (c) are also located there. They do appear to show an incoherence that is rather more worrying than Moore's Paradox. Be that as it may, I shall not pursue these cases any further. Let me just remark that from case (c) it appears that what may be an unforeseen consequence of AD-consequence is a version Protagoras' measure doctrine, announced at the opening of his *Truth*: 'Man is the measure of all things, of those that are, that they are, and of those that are not, that they are not.'[10] It is generally interpreted, following Plato's account in the *Theaetetus*, as expressing a kind of relativism: any truth is only truth for me or truth for you. Here it appears that truth is only that which I asserted, and if you run through a similar argument yourself, you'll see that truth is only that which you asserted. Thus it appears that for Protagoras and his sophist colleagues, (c) is not out of bounds. For Socrates and Theaetetus it is: the measure doctrine is self-defeating (*Theaetetus* 171a–c).[11]

Case (a) is not so easily dismissed. Many philosophers deny that every proposition entails its own assertibility, and they have good reasons to do so. Amongst them are philosophers of a realist bent. They will not accept the inference: 'Every even number is the sum of two primes,' therefore 'It is assertible that every even number is the sum of two primes.' Many philosophers agree that

there are truths that are not assertible: assertibility is tied to epistemic notions, such as evidence, justification, warrant, and those philosophers think that there are truths for which we neither had nor have nor ever will have a warrant. These are not just the realists. Some anti-realistically inclined philosophers, are amongst them, too.[12]

Let □ stand for 'It is assertible that,' and for ease of exposition, let's avail ourselves of a conditional that satisfies the deduction theorem.[13] 'Assertibility' here means 'correct assertibility,' a notion according to which only truths are assertible. Plain *A* represents the truth of *A*. We therefore have $\Box A \to A$. □ so interpreted should not be the vacuous modality, according to which *A* is logically equivalent to $\Box A$, if there is a distinction between truth and correct assertibility: $A \to \Box A$ should fail. A consequence of *AD*-consequence, however, is that truth implies correct assertibility, as it is incoherent to assert *A* and deny $\Box A$, i.e., that it is correctly assertible that *A*. $A \to \Box A$ follows, and consequently truth coincides with correct assertibility. *AD*-consequence, therefore, is not one for philosophers who wish to resist a collapse of truth into correct assertibility.

One of Rumfitt's footnotes contains two potential avenues for responses by proponents of *AD*-consequence.[14] Rumfitt observes that they require notions of incoherence and rejection sufficiently strong to avoid versions of Moore's paradox. Rumfitt proposes that incoherence must be logical incoherence and rejection rejection as false. The former avenue, however, is explicitly ruled out by Ripley, who underlines that 'there is nothing interestingly logical about the notion of incoherence in play here. The notion that we can find playing a role in our conversational practices is a material one. . . . [It] does not reduce to a prior logical inconsistency, or anything like it' (Ripley 2017, 310).[15] The incoherence of positions is a material notion of incoherence, similar to the one between '*a* is red all over' and '*a* is green all over.' For Restall, it is important that the definition of consequence (and the derivation of the structural features of sequents from it) is '*independent* of the logical vocabulary used in the statements that are themselves asserted and denied' (Restall 2013, 83), and thus presumably also independent of a logical notion of incoherence. Restall's and Ripley's intention is that the meanings of the logical connectives are defined, in inferentialist fashion, by rules of inference governing them in the resulting sequent calculus, with its sequents explained in terms of assertion, denial and material incoherence. Logical incoherence is then a special case of being out of bounds and definable once the logical connectives have been defined: an evident option is that a position is logically incoherent if it commits to the assertions of *A* and of ¬*A*. The second avenue of escape is also not congenial to their inferentialist outlook (cf. Restall 2009, 245): the notion of falsity is to be explained in terms of the notion of rejection, and so a notion of rejection as false is not available at the initial stages of theorising at which the definition of consequence is situated: if it was, one might as well define consequence in terms of truth and falsity.

Besides, even if these avenues of escape succeeded in excluding cases (b) and (c), it is not evident that either would exclude case (a). Given the notions at the foundations of bilateral logic, assertion and denial, at first glance at least it looks as if it should be logically incoherent to assert that there is tea left and to deny that this is assertible: one contradicts the other. This also seems to be the case if it is asserted that there is tea left and rejected as false that this is assertible. It even looks as if an appeal to a notion of material incoherence makes things worst. According to Restall and Ripley, and a large number of philosophers agree, our practices of assertion and denial are governed by norms of coherence. These norms are violated if it is asserted that *A* and denied that *A* is assertible. One is incompatible with the other. This may not be quite the same notion of incompatibility as that which holds between '*a* is red all over' and '*a* is green all over,' but nothing in what has been said establishes this. This appearance may well arise because the latter is a metaphysical incompatibility, and as such perceived as very firm, while the former incompatibility arises from our practices, and thus may seem less firm, as our practices may well be different. There is a serious question, however, whether such a line is attractive to inferentialists, who accept that at least some of our practices are very firm indeed. After all, what is at stake here is logic. The commitments in-

curred by asserting that there is tea left and denying that this is assertible (on one popular account, possession of a warrant that there is tea left and the denial that such a warrant is available) strike me as incoherent in a way that is not far from the way in which asserting that something is red all over as well as green all over is incoherent. In any case, the notion of incoherence at the heart of *AD*-consequence is left rather underdetermined; and, I think, deliberately so, to leave space for different ways of spelling it out and different ways of formalising logics on its basis. Even if we have here indicated an avenue of escape to the proponents of *AD*-consequence, we can note that to provide a more detailed account of incoherence and incompatibility is a challenge posed by the present paper, and the burden of proof is on them to provide one.

Throughout Dummett's writings, although the equation of truth with correct assertibility is strongly suggested, there are passages showing him to shy away from this conclusion. He voices doubt about the equation and seems to try to resist it. At various points Dummett argues that the distinction between truth and correct assertibility may be drawn by looking closely at the antecedents of conditionals (e.g., Dummett 1993b, 193). In later years, however, Dummett cheerfully admits that truth and correct assertibility coincide: 'The truth of a sentence should be equated with its being objectively correct to assert it' (Dummett 2002, 294). So while for the later Dummett, the consequence that truth collapses into correct assertibility would be welcome, for the earlier Dummett this is debatable.

For at least two prominent bilateralists, the coincidence of truth and assertibility that results from *AD*-consequence is not acceptable. Indeed, according to Price it is one of the main selling points of bilateralism that it allows us to draw a distinction between truth and assertibility. *A* and 'It is assertible that *A*' share their assertibility conditions. But according to Price, they do not share their deniability conditions, and this, argues Price, allows bilateralists to distinguish their meanings. The bilateralist proposal 'makes it easy to draw this distinction. For the relevant notion of equivalence of meaning is now that of having the same denial conditions, as well as the same assertion conditions; whereas it may be deniable that "S" is assertible, without it also being deniable that S' (Price 1983, 167). The distinction between truth and assertibility can be drawn because, although the deniability conditions of *A* are also deniability conditions of 'It is assertible that *A*,' the converse does not hold, and the deniability conditions of 'It is assertible that *A*' may not include the deniability conditions of *A*. Thus Price rejects the inference from *A* to 'It is assertible that *A*.'

Rumfitt concurs that the bilateral account of meaning, but maybe not Dummett's, permits to block the inference of 'It is assertible that *A*' from *A* and thus to prevent truth from collapsing into assertibility, while at the same time largely respecting Dummettian restrictions on the determination of meaning by use. According to Rumfitt, an attempt to keep hold of the distinction between truth and assertibility that our practice clearly draws, and to formulate principles on which to found a theory of meaning that respects that practice rather than must propose to revise it, almost invariably leads to bilateralism (Rumfitt 2002, 319–320). Although Rumfitt accepts that if it is assertible that *A*, then *A*, and that if it is deniable that *A*, then ¬*A*, he denies 'that we are in general entitled to assert the converse claim (a) that if sentence *A* is true, then it is correct to assert it. Similarly, [he denies] that we are in general entitled to assert (b) that if *A* is false, then it is correct to deny it' (Rumfitt 2002, 314). Consequently, any bilateralist who follows Price and Rumfitt must reject *AD*-consequence.

Classical logic is generally taken to be the logic of realists who accept that there are truths that are unassertible. Even anti-realistically inclined philosophers like Rumfitt agree: this is the reason he prefers classical logic over intuitionist logic, and why he recommends the bilateral account of meaning as one on the basis of which a smooth account of how this is possible can be given. Let me stress again that what has been presented here may not present a problem for Restall and Ripley: maybe they are amongst the philosophers who accept that truth implies assertibility, and they may be in good company. But we've at least got the rather curious fact that an admittedly attractive and on the face of it non-committal definition of logical consequence rather immediately yields a strong anti-realists thesis.

NOTES

1. This paper was written in response to a lecture by Graham Priest and subsequent discussion. Thank you also to Bernhard Weiss for discussions of truth and assertibilty. Referees for *Thought* made valuable suggestions for improvement.

2. Restall applies his definition to the analysis of issues in the philosophy of logic (Restall 2009, Restall 2013) and also speaks of positions and characterises the incoherent positions as being out of bounds or as involving clashes.

3. As pointed out by a referee, it should be noted that Ripley and Restall do not share these basic assumptions—Ripley rejects the transitivity of consequence, while Restall accepts it. This incompatibility between their respective accounts is of no consequence to the present paper.

4. See, for instance, Dummett 1993a, 186–187. Steinberger develops a Dummettian position and criticises *AD*-consequence from that perspective (Steinberger 2011). Humberstone shows how to translate a single conclusion bilateral logic into multiple conclusion unilateral logic (Humberstone 2000), thereby showing the close connection between these two approaches.

5. Thank you to a referee for pointing this out.

6. A referee has questioned whether this really is an inferentialist account of consequence. Inference has a direction in the step from premises to conclusion, and valid consequences bring commitments to their conclusions, given acceptance of their premises. Neither features in *AD*-consequence. For present purposes it suffices that some philosophers who consider themselves inferentialists are attracted to *AD*-consequence, and I shall pass this question on to them.

7. The literature ensuing from Dummett's work on the theory of meaning and the justification of deduction should suffice as evidence for this claim. Arguments for classical logic based on single conclusion bilateral logic or natural deduction are also inconclusive. Rumfitt argues that such a logic must be classical (Rumfitt 2000, Rumfitt 2002). Kürbis formalises an intuitionist bilateral logic which he maintains satisfies Rumfitt's requirements on satisfactory bilateral logics (Kürbis 2016). Wansing proposes that the logic of bilateralism is a constructive logic with strong negation (Wansing 2017), Nelson's logic N4 (Nelson 1949). This logic receives an equivalent formulation by omitting the coordination principles Non-Contradiction and Reductio for assertion and denial from Rumfitt's classical bilateral logic, an observation first made by Gibbard (2002). Taking assertion and denial to be mutually exclusive speaks for a bilateral version of intuitionist logic with strong negation, N3, which keeps Non-Contradiction while omitting Reductio: this option is very natural indeed, but to my knowledge it has so far not had any champions. For the terminology and semantic characterisations of N4 and N3, see Wansing 2001; for their proof-theoretic characterisation in unilateral natural deduction, see Prawitz 1965, Appendix B, §2. It has been pointed out by a referee that Rumfitt may be interpreted as making only a weaker claim than the one I attribute to him here: that the bilateral framework permits classical negation to be governed by harmonious rules, not that a bilateral logic must be classical. Notice however that a stronger reading is also possible, and it is this one that I take to be pertinent in the question concerning the justification of deduction: Rumfitt characterises bilateral intuitionist negation rules as 'anomalous' (Rumfitt 2000, 805f.) in a way comparable to how Dummett characterises unilateral classical negation rules. From this perspective, just as Dummett gives an argument for intuitionist and against classical logic, Rumfitt aims to give an argument for classical and against intuitionist logic. For further argument, see Kürbis 2016.

8. Rumfitt 2008, 80 and Rumfitt 2015, 51. See note 9 for further details.

9. Rumfitt's first example is: 'accepting that there will never be sufficient reason for accepting or rejecting "There is a god," while rejecting that very same statement, is self-defeating. But "There is a god" is in no sense a consequence of "There will never be sufficient grounds for accepting or rejecting 'There is a god'"' (Rumfitt 2008, 80). His second example relates to the dispute between classicists and intuitionists. Rumfitt points out that according to *AD*-consequence, 'logicians in both schools will accept [$A \lor \neg A$] as valid,' because [$\emptyset : A \lor \neg A$] is incoherent also for the intuitionist: intuitionists agree with classicists that it is incoherent to deny $A \lor \neg A$, that is to assert its negation' (Rumfitt 2015, 51). Earlier in this footnote Rumfitt comments that 'care must be taken to gloss "incoherent" [as used in *AD*-consequence] if this reading [of $X : Y$] is not to generate versions of Moore's paradox; we need "incoherent" to mean something like "logically contradictory" and "reject" to mean "reject as false" rather than as impolite, or ungrounded.' More on this later.

10. Here quoted after Burnyeat 2012a, 4.

11. See Burnyeat for discussion (Burnyeat 2012b). It is, incidentally, also out of bounds for someone to deny that you asserted that *A*, if you asserted that *A*, so *A* entails that you asserted that *A*. This is reminiscent of the kind of sophisms peddled by Euthydemus and Dionysodorus, only that they claim that everyone knows everything (if they know one thing) (*Euthydemus* 293b–294a). If we accept that knowledge provides the norm of assertion, then asserting *A* and denying that I know that *A* violates this norm and is thus out of bounds. So according to *AD*-consequence, from *A* it follows that I know that *A*, and I am all-knowing, as the sophists would have it. McCabe (2019) shows the relevance of the *Euthydemus* to contemporary philosophy of logic and language. Leigh (2019) provides an overview of McCabe's paper and commentaries on it, amongst which Kürbis 2019, an attempt to draw parallels between Plato's portrayal of the views of the sophists and recent views.

12. Rumfitt provides extensive discussion, independently of bilateralism (Rumfitt 2015).

13. A little care is needed in formalising modal logic so that the deduction theorem holds. For natural deduction, Prawitz's systems or adaptations thereof will do (Prawitz 1965, chap. VI); for sequent calculus, see (Indrzejczak 2020, chap. 3).

14. As pointed out by a referee, to whom many thanks for the suggestion to add the following discussion.

15. In this paper, Ripley discusses Rumfitt's footnote in a footnote. Following the example of my predecessors, I will here add another. Ripley complains that Rumfitt misunderstands Restall. It seems to me rather that Ripley misunderstands Rumfitt. Ripley writes: 'There is nothing at all incoherent (pace verificationists) about supposing that there is a god, but that there will never be sufficient grounds for accepting or rejecting "There is a god." But coherence (in the relevant sense) is the same under supposition and not; so there is nothing incoherent (in the relevant sense) about the actual pattern of acceptances and rejections Rumfitt points to' (Ripley 2017, 312). According to Rumfitt, the following position is out of bounds: N: 'Will there ever be sufficient grounds for accepting or rejecting "There is a god"? No.' Also N: 'Is there a god? No.' The latter denial commits N to the possession of sufficient grounds for rejecting 'There is a god,' a possibility ruled out by the former denial. Rumfitt's point is that this has to do with features that 'govern the practice of making assertions and denials' (Restall 2013, 83). Nothing in what Ripley says affects this point.

REFERENCES

Burnyeat, M. 2012a. "Protagoras and Self-Refutation in Later Greek Philosophy," in *Explorations in Ancient and Modern Philosophy*, vol. 1. Cambridge: Cambridge University Press.

Burnyeat, M. 2012b. "Protagoras and Self-Refutation in Plato's *Theaetetus*," in *Explorations in Ancient and Modern Philosophy*, vol. 1. Cambridge: Cambridge University Press.

Dummett, M. 1993a. *The Logical Basis of Metaphysics*. Cambridge, MA: Harvard University Press.

Dummett, M. 1993b. "The Source of the Concept of Truth," in *The Seas of Language*, 188–201. Oxford: Clarendon Press. https://doi.org/10.1093/0198236212.003.0008

Dummett, M. 2002. "'Yes,' 'No' and 'Can't Say,'" *Mind* 111 (442): 289–295. https://doi.org/10.1093/mind/111.442.289

Gibbard, P. 2002. "Price and Rumfitt on Rejective Negation and Classical Logic," *Mind* 111 (442): 297–304. https://doi.org/10.1093/mind/111.442.297

Humberstone, L. 2000. "The Revival of Rejective Negation," *Journal of Philosophical Logic* 29: 331–381. https://doi.org/10.1023/A:1004747920321

Indrzejczak, A. 2020. *Trees and Sequents. An Introduction to the Theory and Applications of Propositional Sequent Calculi.* Basel: Birkhaüser. https://doi.org/10.1007/978-3-030-57145-0

Kürbis, N. 2016. "Some Comments on Ian Rumfitt's Bilateralism," *Journal of Philosophical Logic* 45: 623–644. https://doi.org/10.1007/s10992-016-9395-9

Kürbis, N. 2019. "The Importance of Being Erroneous," *Australasian Philosophical Review* 3(2): 155–166. https://doi.org/10.1080/24740500.2020.1716661

Leigh, F. 2019. "Says Who? Modes of Speaking in the *Euthydemus*," *Australasian Philosophical Review* 3(2): 123–130. https://doi.org/10.1080/24740500.2020.1716658

McCabe, M. 2019. "First Chop Your *Logos* . . . : Socrates and the Sophists on Language, Logic and Development," *Australasian Philosophical Review* 3(2): 131–150. https://doi.org/10.1080/24740500.2020.1716659

Nelson, D. 1949. "Constructible Falsity," *Journal of Symbolic Logic* 14: 16–26. https://doi.org/10.2307/2268973

Plato. 1997a. "Euthydemos," in *Complete Works*, ed. J. M. Cooper. Indianapolis: Hackett.

Plato. 1997b. "Theaetetus," in *Complete Works*, ed. J. M. Cooper. Indianapolis: Hackett.

Prawitz, D. 1965. *Natural Deduction: A Proof-Theoretical Study*. Stockholm: Almqvist and Wiksell.

Price, H. 1983. "Sense, Assertion, Dummett and Denial," *Mind* 92: 161–173. https://doi.org/10.1093/mind/XCII.366.161

Price, H. 1990. "Why 'Not'?" *Mind* 99: 221–238. https://doi.org/10.1093/mind/XCIX.394.221

Restall, G. 2005. "Multiple Conclusions," in *Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress*, ed. P. Hájek, L. Valdés-Villanueva, and D. Westerståhl, 189–205. London: King's College Publications.

Restall, G. 2009. "Truth Values and Proof Theory," *Studia Logica* 92: 241–264. https://doi.org/10.1007/s11225-009-9197-y

Restall, G. 2013. "Assertion, Denial and Non-Classical Theories," in *Paraconsistency: Logic and Applications*, ed. K. Tanaka, F. Berto, E. Mares, and F. Paoli, 81–99. Dordrecht: Springer. https://doi.org/10.1007/978-94-007-4438-7_6

Ripley, D. 2013. "Paradoxes and Failures of Cut," *Australasian Journal of Philosophy* 91(1): 139–164. https://doi.org/10.1080/00048402.2011.630010

Ripley, D. 2017. "Bilateralism, Coherence, Warrant," in *Act-Based Conceptions of Propositional Content: Contemporary and Historical Perspectives*, ed. F. Moltmann and M. Textor. Oxford: Oxford University Press.

Rumfitt, I. 2000. "'Yes' and 'No,'" *Mind* 109: 781–823. https://doi.org/10.1093/mind/109.436.781

Rumfitt, I. 2002. "Unilateralism Disarmed: A Reply to Dummett and Gibbard," *Mind* 111: 305–312. https://doi.org/10.1093/mind/111.442.305

Rumfitt, I. 2008. "Knowledge by Deduction," *Grazer Philosophische Studien* 77: 61–84. https://doi.org/10.1163/18756735-90000844

Rumfitt, I. 2015. *The Boundary Stones of Thought: An Essay in the Philosophy of Logic*. Oxford: Clarendon Press. https://doi.org/10.1093/acprof:oso/9780198733638.001.0001

Smiley, T. 1996. "Rejection," *Analysis* 56: 1–9. https://doi.org/10.1093/analys/56.1.1

Steinberger, F. 2011. "Why Conclusions Should Remain Single," *Journal of Philosophical Logic* 40(3): 333–355. https://doi.org/10.1007/s10992-010-9153-3

Wansing, H. 2001. "Negation," in *The Blackwell Guide to Philosophical Logic*, ed. L. Goble. Oxford: Blackwell.

Wansing, H. 2017. "A More General General Proof Theory," *Journal of Applied Logic* 25: 23–46. https://doi.org/10.1016/j.jal.2017.01.002