SUPPLEMENTARY MATERIALS: On Learning the Invisible in Photoacoustic Tomography with Flat Directionally Sensitive Detector[*]

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SM1. ResCoronae-Net. Figure SM1 displays the architecture of the ResCoronae-Net.

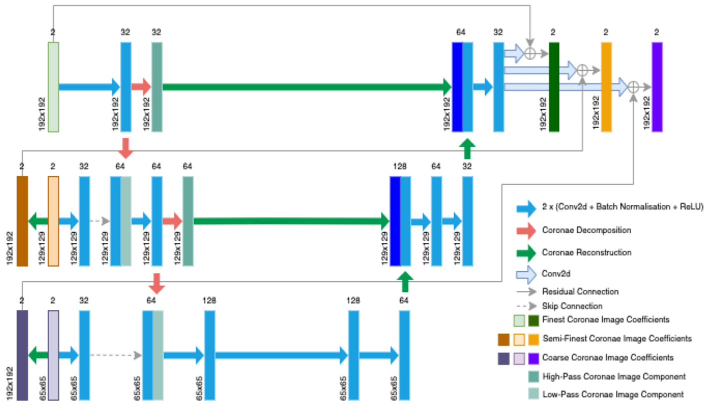

Figure SM1. *ResCoronae-Net - Architecture here shown with* 3 *scales for correcting the visible and learning the invisible Coronae coefficients in* 2*D. The input of the ResCoroneNet is scale-wise and is upsampled to the finest scale before the residual connection.*

SM2. Visualisation of Leanred Coronae coefficients. Figure SM2 (d-f) illustrates the results of learned invisible Coronae coefficients $\tilde{Q}_j^{\mathrm{L}_{\mathrm{inv}}}$ from the perfect visible Coronae coefficients (a-c) using Coronae-Net, for reference (g-i) show the perfect invisible Coronae coefficients and (j-l) are the corresponding learning errors. The error seems most pronounced at the coarse scale, though as the error magnitudes are consistent across scales, we attribute it mostly to the upsampling of the coarse to finest scale; see Figure SM2 (l).

SM3. Reference U-Net (Image Based). Figure SM3 displays the architecture of an image based reference U-Net which takes as an input the perfect visible image part $p_0^{\mathrm{P}_{\mathrm{vis}}}$ and is trained on $(p_0^{\mathrm{P}_{\mathrm{vis}}}, p_0)$ pairs with MSE loss to remove limited-view artefacts.

SM4. Reference U-Net (Coefficient Based). Figure SM4 displays the architecture of a coefficient based reference U-Net which takes as an input the (upsampled to the finest scale via zero-padding in Fourier domain) perfect visible Coronae coefficients $\tilde{Q}_j^{\mathrm{P}_{\mathrm{vis}}}$ (a.k.a. multichannel input at the top scale only, one channel per scale) and is trained on $(\tilde{Q}_j^{\mathrm{P}_{\mathrm{vis}}}, \tilde{Q}_j^{\mathrm{P}_{\mathrm{all}}})$ pairs with MSE loss to remove limited-view artefacts.

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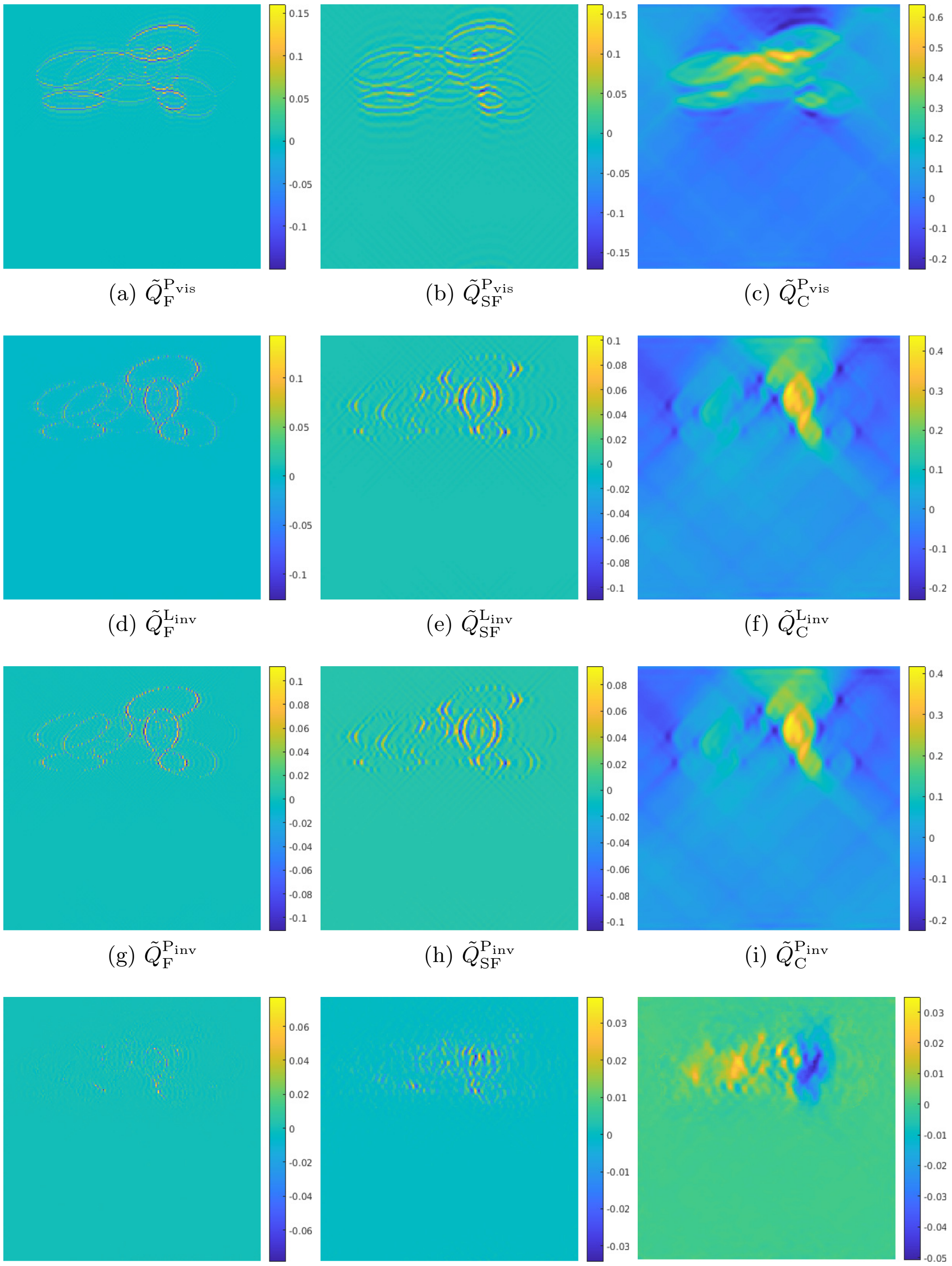

(a) $\tilde{Q}_{\mathrm{F}}^{\mathrm{P}_{\mathrm{vis}}}$ (b) $\tilde{Q}_{\mathrm{SF}}^{\mathrm{P}_{\mathrm{vis}}}$ (c) $\tilde{Q}_{\mathrm{C}}^{\mathrm{P}_{\mathrm{vis}}}$

(d) $\tilde{Q}_{\mathrm{F}}^{\mathrm{L}_{\mathrm{inv}}}$ (e) $\tilde{Q}_{\mathrm{SF}}^{\mathrm{L}_{\mathrm{inv}}}$ (f) $\tilde{Q}_{\mathrm{C}}^{\mathrm{L}_{\mathrm{inv}}}$

(g) $\tilde{Q}_{\mathrm{F}}^{\mathrm{P}_{\mathrm{inv}}}$ (h) $\tilde{Q}_{\mathrm{SF}}^{\mathrm{P}_{\mathrm{inv}}}$ (i) $\tilde{Q}_{\mathrm{C}}^{\mathrm{P}_{\mathrm{inv}}}$

(j) $\tilde{Q}_{\mathrm{F}}^{\mathrm{L}_{\mathrm{inv}}} - \tilde{Q}_{\mathrm{F}}^{\mathrm{P}_{\mathrm{inv}}}$ (k) $\tilde{Q}_{\mathrm{SF}}^{\mathrm{L}_{\mathrm{inv}}} - \tilde{Q}_{\mathrm{SF}}^{\mathrm{P}_{\mathrm{inv}}}$ (l) $\tilde{Q}_{\mathrm{C}}^{\mathrm{L}_{\mathrm{inv}}} - \tilde{Q}_{\mathrm{C}}^{\mathrm{P}_{\mathrm{inv}}}$

Figure SM2. *Ellipses - Visualisation of upsampled Coronae coefficients at each scale* $j \in \{C, SF, F\}$ *(*$\theta_{max} = \pi/4$*): (a-c) perfect visible Coronae coefficients* $\tilde{Q}_j^{P_{vis}}$*, (d-f) learned invisible Coronae coefficients* $\tilde{Q}_j^{L_{inv}}$*, (g-i) perfect invisible Coronae coefficients* $\tilde{Q}_j^{P_{inv}}$*, and (j-l) the corresponding learning errors* $\tilde{Q}_j^{L_{inv}} - \tilde{Q}_j^{P_{inv}}$*.*

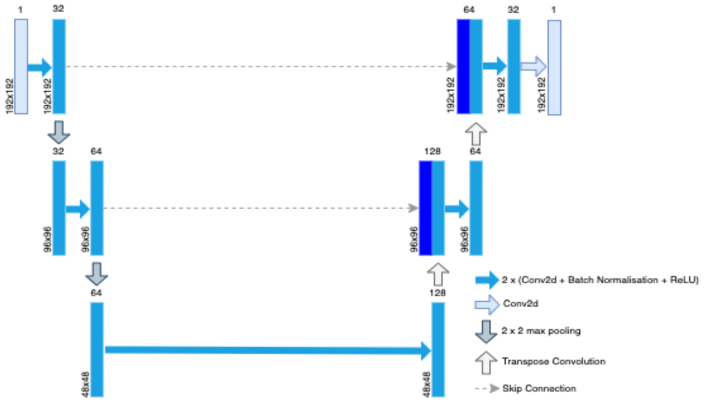

Figure SM3. 3 *scales U-Net (image based) architecture used in learned post-processing. The input is the visible component of the image and the network returns the prediction of the complete (visible and invisible) image. Additionally, max-pooling and* 2*D deconvolution are used as the downsampling and upsampling in the network, respectively*

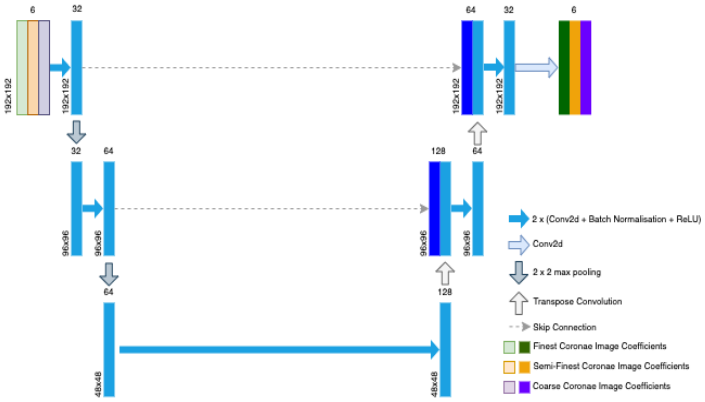

Figure SM4. 3 *scales U-Net (coefficient based) architecture used in learned post-processing. The input are the concatenated visible and invisible (upsampled) Coronae coefficients from each scale and the output are the visible and invisible (upsampled) Coronae coefficients. Additionally, max-pooling and* 2*D deconvolution are used as the downsampling and upsampling in the network, respectively.*

SM5. Reference ResU-Nets (Image Based). Figure SM5 displays the architecture of the image based reference ResU-Net which takes as an input the imperfect visible image part $p_0^{\mathrm{I_{vis}}}$ and is trained on $(p_0^{\mathrm{I_{vis}}}, p_0)$ pairs with MSE loss to remove artefacts due to limited angle and imperfect ℓ_1 reconstruction of the visible part of the image.

SM6. Reference ResU-Nets (Coefficient Based). Figure SM6 displays the architecture of the coefficient based reference ResU-Net which takes as an input the (upsampled to the finest scale via zero-padding in Fourier domain) imperfect visible Coronae coefficients $\tilde{Q}_j^{\mathrm{I_{vis}}}$ and is trained on $(\tilde{Q}_j^{\mathrm{I_{vis}}}, \tilde{Q}_j^{\mathrm{P_{all}}})$ pairs with MSE loss to remove artefacts due to limited angle and imperfect visible Coronae coefficients computed from ℓ_1 reconstruction.

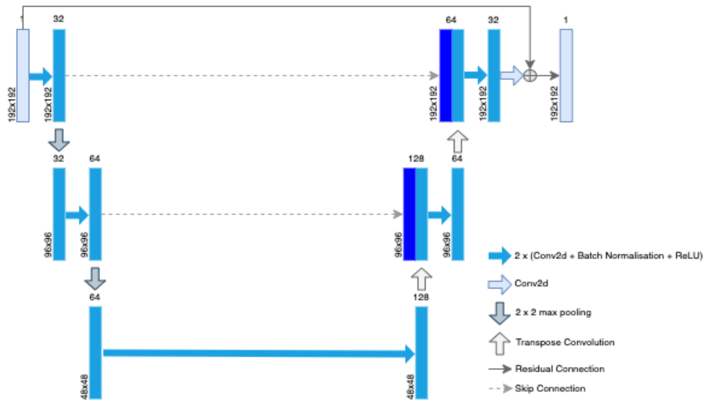

Figure SM5. 3 *scales ResU-Net (image based) architecture used in learned post-processing. The network is based on the U-Net in Figure* SM3 *with additional skip connection.*

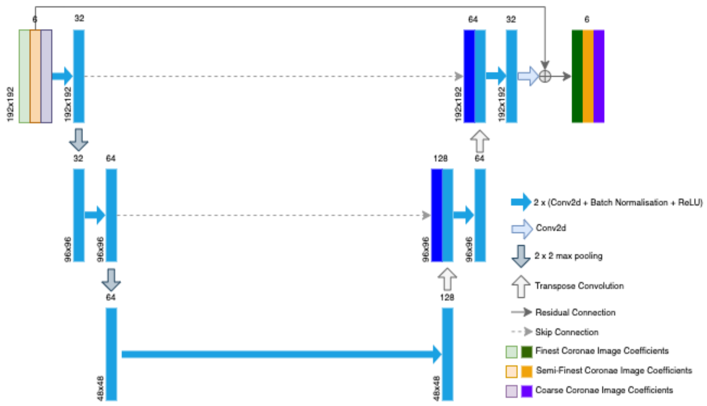

Figure SM6. 3 *scales ResU-Net (coefficient based) architecture used in learned post-processing. The network is based on the U-Net in Figure* SM4 *with additional skip connection.*

SM7. VR-FISTA.

A version of **FISTA** adapted to (**VR**) is summarized in Algorithm **VR-FISTA**.

VR-FISTA Fast Iterative Shrinkage Thresholding Algorithm

Initialisation: $y^1 = \mathbf{0}$, $f^1 = \mathbf{1}$, $\Lambda = \mathrm{diag}(2^{j-2})$, $j \geq j_0$, $\alpha^1 = 1$, $\mu = \frac{1}{L}$, $\tau > 0$, $\eta > 0$ and $K_{\max}$

$k := 1$

Repeat

$\tilde{z}^k = y^k - \mu \breve{\Psi} \hat{\mathbf{A}}_\angle^\dagger (\hat{\mathbf{A}}_\angle \breve{\Psi}^\dagger y^k - \hat{g}_\angle)$

$f^{k+1} = \arg\min_f \mu\tau ||\Lambda f||_1 + \frac{1}{2}||f - \tilde{z}^k||_2^2$

$\alpha^{k+1} = (1 + \sqrt{1 + 4(\alpha^k)^2})/2$

$y^{k+1} = f^{k+1} + \frac{\alpha^k - 1}{\alpha^{k+1}}(f^{k+1} - f^k)$

$k = k + 1$

Until $||f^k - f^{k-1}||_2 / ||f^{k-1}||_2 < \eta$ or $k > K_{\max}$
