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Scaling in City Size Distributions

From the late 19th century to the middle of the last, many contributions to the emergence of city science were made by scholars in Germany. Walter Isard (1956) in his book *Location and Space Economy* summarised these under the banner of location theory where the focus was on how the size and spatial distribution of economic activities in cities and regions could be explained as a function of their agglomeration economies, their density and the relative dependence on one another through transportation costs. The work of economists and geographers such as Alfred Weber, Christaller and Losch amongst others set the scene for the development of this new science – regional science as it came to be called – and this dominated the development of urban modelling which after some volatile early attempts, is slowly becoming more widely accepted.

In one small corner of this world in location theory, there was a focus on deriving empirical laws pertaining to the size and spatial distribution of economic activities. In 1949, Zipf presented some of these relationships in his book *Human Behavior and Principle of Least Effort* which popularised the 'rank size rule' or law as it was sometimes called. This law related to city sizes as well as many other size distributions such as income (which in turn had been formalised by Vilfredo Pareto in the late 19th century). If we rank n cities from the largest to the smallest as $p_1, p_2, \dots, p_n$, and if we then multiply each city size by its rank r, then this would result in a dramatic regularity which could be written as $p_r r = K$ where K is a constant of proportionality and the relationship is that of a rectangular hyperbola. In fact, the relationship can be written in different ways and perhaps in a more characteristic form, it is $p_r = Kr^{-1}$ or an inverse power law. In terms of statistical physics, this is a particularly simple scaling law which can be further generalised with the exponent on rank varying for different types of relationship between size, and frequency (which is rank by any other name).

This is one of many such scaling laws that emerge from applications of basic mechanics to human phenomena such as the analogy between Newton's second law of motion – gravitation – and the interaction between human populations that now constitutes the core of what has come to be called 'social physics'. In fact Krugman (1996) argues that the rank size relationship with respect to city size is in fact one of the few iron laws in economics, saying: "This simple regularity is puzzling; even more puzzling is the fact that it has apparently remained true for at least the past century. Standard models of urban systems offer no explanation of the power law".

In this context, it is clear that much of this thinking is not particularly new. As Denise Pumain has pointed out many times, much of it was anticipated almost as soon as Newton articulated his laws in the late 17th century. Central place theory which encapsulates much of this physics of scaling with direct applicability to geographic hierarchies and hinterlands was explored in the early 19th century by Reynaud (1841)(see Robic, 1982; Cottineau and Morphet, 2016). There were attempts at applying gravitational theory throughout the 19th century and there are even suggestions that a unifying theory of city size might be possible in the work of Weber (1899, 1967). However in terms of city size, the first contribution that we are aware of was published in 1913 by Felix Auerbach in a paper entitled 'Das Gesetz der

Bevolkerungskonzentration' which translates as 'The Law of Population Concentration'. As we do not think that the paper has been translated before, we consider it so important to development of ideas about scaling and city size, we publish Antonio Ciccone's translation here and it follows as the first paper of this issue. Moreover, Rybski and Ciccone's (2022) commentary on the paper provides essential background to the evolution of ideas in this field and we refer readers to this background and wider context.

In Ciccone's translation that follows, Auerbach focuses on developing and interpreting city size in terms of overall population size and urban density. In fact he begins by showing that the basic relation $p_{\%}r = K$ is fairly precise for the distribution of the largest 94 towns in Germany from the Population Census in 1910. With some volatility for a handful of the largest cities which is also very clear from many data sets for city size which have been explored over the last 100 years or more, predicted $K^{'}$ settles down very quickly and by the time the 20th rank is reached, the relation varies by no more than 2-3% for the rest of the data. If $K^{'}$ is very large, the population is likely to be concentrated in a few towns while if it is small, the population of towns is quite decentralised. This is a measure that does not seem to have been used before and it marks the fact that Auerbach spends much more effort in figuring out how the towns in the distribution relate to one another than most scholars who have explored the rank size rule have done hitherto. If we want to compare different systems, we still need to note that the average concentration $K^{'}$ depends on both the number and size of cities in the distribution where the number is always uncertain due the fact that a minimum size of cities is often chosen arbitrarily as there is no natural lower bound.

The second measure divides the value of $K^{'}$ for the lowest city size $p_{\$}$ (which we assume is approximately stable over most of the range) by the total population in the system $P_{\$} = \sum_{\%(!}^{\$} p_{\%}$, that is $S_{\$} = p_{\$}n/P_{\$}$. When $S_{\$}$ is large, it is likely that the populations are large and close to one another whereas when it is small, this means there are many small populations. But the interpretation is ambiguous. More is needed and only in the last paragraph does Auerbach speculate on the rank size rule, and it took Lotka in 1925 to make the connection to such scaling completely clear. Nevertheless, it was Auerbach who led the way as Rybski and Ciccone (2022) point out in their own interpretation.

I welcome this translation as I think our field has now reached the point where some of the early origins need to be laid bare. As readers can judge from reading Ciccone's translation below, Auerbach's work not only presented an early version of power law scaling for city sizes but also began to explore how cities might relate to one another in terms of their sizes. There is little on this in the voluminous literature on rank size since his pioneering paper but I consider that his work shines a light on aspects of city size that we could well explore further. There is the question of minimum and maximum sizes of cities, and the increasingly important question of what is happening to empirical laws such as these as cities grow every bigger, as their boundaries become ever more ambiguous, and as they begin to fuse.

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References

Cottineau C, Morphet R (2016) *Jean Reynaud: Seer Of Space*. University College London: CASA Working Papers Series, Paper 205.

Isard W (1956) *Location and Space Economy: A General Theory Relating to Industrial Location, Market Areas, Land Use, Trade, and Urban Structure*. Cambridge MA: The MIT Press.

Krugman P (1996) Confronting the mystery of urban hierarchy. *Journal of the Japanese and the International Economies 10*: 399—418.

Lotka A J (1925) *Elements of Physical Biology*, Baltimore MD: Williams and Wilkins Company.

Reynaud J (1841) Villes, *Encyclopédie Nouvelle*, Paris, éd. par C. Gosselin, t. VIII, 670-687.

Robic M C (1982) Cent ans avant Christaller... une théorie des lieux centraux. *L'Espace Géographique* XI :5-12.

Rybski D, Ciccone A (2022) Auerbach, Lotka, Zipf – Pioneers of power-law city-size distributions. Preprint. 14412 Potsdam, Germany: Potsdam Institute for Climate Impact Research.

Weber A F (1899, 1967) *The Growth of Cities in the Nineteenth Century: A Study in Statistics*. New York: Macmillan and Company (1899) and Ithaca NY: Cornell University Press.

Zipf G K (1949) *Human Behavior and the Principle of Least Effort*. Cambridge MA: Addison-Wesley.