A Generic Circular Source Distribution for solving potential problems using Meshless Methods

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1 Abstract

Approximate solutions to the potential equation $\nabla^2 U = 0$ in a simply-connected 2-dimensional domain *D* are found using the Method of Fundamental Solutions (MFS) with sources placed on a circle. Previous research has shown that a few discrete points sources placed a large distance from *D* can give good numerical accuracy. The discrete source distribution can be random, and a circular source distribution is one which could be used for any domain. Several domains are considered, and an attempt is made to determine an optimal radius for the source distribution and number of sources required in order to give sufficient accuracy. Simple configurations for *D* give results whose accuracy depends in an ordered and predictable way on the source radius and the number of sources. With more complex domains the source radius and the number of sources have a crucial impact on accuracy.

2 Introduction

Previous research has shown that when using meshless discrete sources in the MFS, the configuration of sources relative to the domain *D* is extremely flexible. In [Mitic 2004] it was shown that, within certain limits, the source distribution can be random, and in [Mitic 2003] it was shown that sources "at infinity" (i.e., a large distance from *D* compared to the size of *D*) can produce very accurate results for the simple domain discussed in this paper. Fam's study [Fam 2002] confirmed findings on source distributions near *D*, using extensions of the MFS. All of these studies show that previous attempts to analyse meshless source distributions are insubstantial. Alwes [Alwes 2002], in an analysis of Poisson equations, considers a 'natural' radius for a circular distribution of discrete sources of "5~10 times the diameter of *D*" (without any precise definition for the term 'diameter' for a non-circular domain). The results of [Mitic 2004] and [Mitic 2003] show that a 'natural' radius is infinite, although the results of this paper indicate that for some domains ill-conditioning precludes the use of very large radii. Contrary to the findings of Poullikkis [Poullikkis 1998], sources do not have to be placed uniformly at a fixed distance from the boundary, and a minimal number of boundary elements often suffices.

The real motivation behind this study comes from [Mitic 2004]. If sources can be placed anywhere outside the domain, the idea that the same class of sources could be used for any domain is plausible. A circular source distribution has been chosen here simply because it is easy to define. Other configurations (for example, a square) should also work, although they are not considered here.

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3 Definitions and Conjectures

Given a circular source distribution, an optimal circle radius and an optimal number of sources need to be found. The domains here are all simply-connected, and I define the **nominal radius** of a domain as half of the (straight line) distance between any two points that are furthest apart. This definition does not need to be rigourous as it is needed for no more than a general guideline. Results of equivalent accuracy can be obtained from radii that vary widely.

The examples in this paper (and others) show that a circular source distribution *can* function as a generic distribution for solving the type of potential problems considered in this paper. In addition, I conjecture the following:

1. The optimal radius of the circle is 10 times the 'nominal radius' of the shape enclosed by the boundary.

2. The optimal number of sources is calculated from the number of boundary elements on curved faces and one per boundary elements on straight line faces.

3.1 Boundary Element and Circular source distribution functions

Package ***MeshlessCircle.m*** contains all the functions necessary to do the boundary element calculations in this paper. (Change the path to the actual location of the package in the import expression below to use the package.)

```
In[1]:= << "d:\\ims2004\\papers\\meshless methods\\MeshlessCircle.m"
```

In the following sections of this paper I discuss configurations of the domain *D* which provide some evidence for the conjectures above.
The top-level function in ***MeshlessCircle.m*** is *InteriorSolution*, which calculates the potential and potential gradient at an interior point of *D*. Its full prototype is:

InteriorSolution[points, sources, types, boundaryCond, interiorPoints] .

The arguments are:

points: a sequence of points that determines the boundary elements of *D*;
sources: a list of source coordinates;
boundaryCond: a list of boundary condition values, one for each boundary element;
types: a list of boundary condition types, either "Dirichlet" (potential specified)
or "Neumann" (potential gradient specified), corresponding to *boundaryCond*;
interiorPoints: a list of interior points at which to calculate the potential and potential gradient.

The source locations are generated using the function *sourcesC[r,c,n]*, which returns a list of *n* coordinates, uniformly distributed around a circle of radius *r* and centre *c*. The function *ShowSources[sources, points, pointSize]* displays the sources and the boundary (pointSize = 0.02 gives a reasonable display).

The bulk of the work in using these functions lies in generating the quantities *points* and *boundaryCond*. The other inputs are trivial, but sometimes lengthy expressions. All of them are essentially geometric, and have no bearing on the actual computation of potental and potental gradient other than to act as raw data. Therefore, all of the input expressions except for the first (section 4) for the examples that follow have been transferred to a subsidiary notebook, ***InputExpressions_Mitic.nb***. In the cases where this has been done, cross-references to and from ***InputExpressions_Mitic.nb*** are provided.

4 Linear flow in a rectangular domain

This was discussed at length in [Mitic 2004] in the context of randomly-distributed sources, and this specific case appears in [Cartwright 2001]. The domain D is rectangular with Dirichlet boundary conditions ($U = 0$ and $U = 2$) imposed on the two vertical parts of the boundary and the Neumann boundary conditions $Q = 0$ imposed on the two vertical parts of the boundary (Figure 1 below). This problem has 10 boundary elements, and represents linear heat flow in the x-direction. The results are as expected: the potential varies linearly from 0 to 2, with zero vertical flux, and constant horizontal flux whose value is $2/(0-2) = -1$. Excellent accuray is achievable using a circular source distribution, but this problem is really too simple to expose any problems with the meshless method. The radius of the circle of sources does not matter. The minimum value tried was marginally more than the nominal radius (half the diagonal, $\frac{1}{2}\sqrt{5}$) and the maximum was 'infinite' (about 100,000). The number of sources was also unimportant. As more sources are used, accuracy improves up to a point when ill-conditioning prohibits further gain in accuracy. Surprisingly, acceptable results could be obtained using only one source, but this configuration is bound to lead to trouble for more complex domains! The error for the calculated values of the potential and x- and y-components of the flux using 10 sources (as per conjecture) with a radius of 11 (about 10 times the nominal radius of the domain) was less than 0.001%. Hence the conjecture above is easily satisfied.

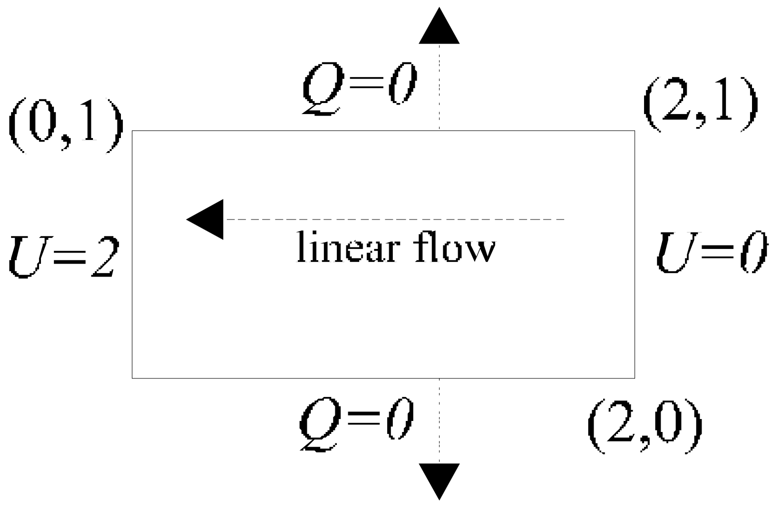
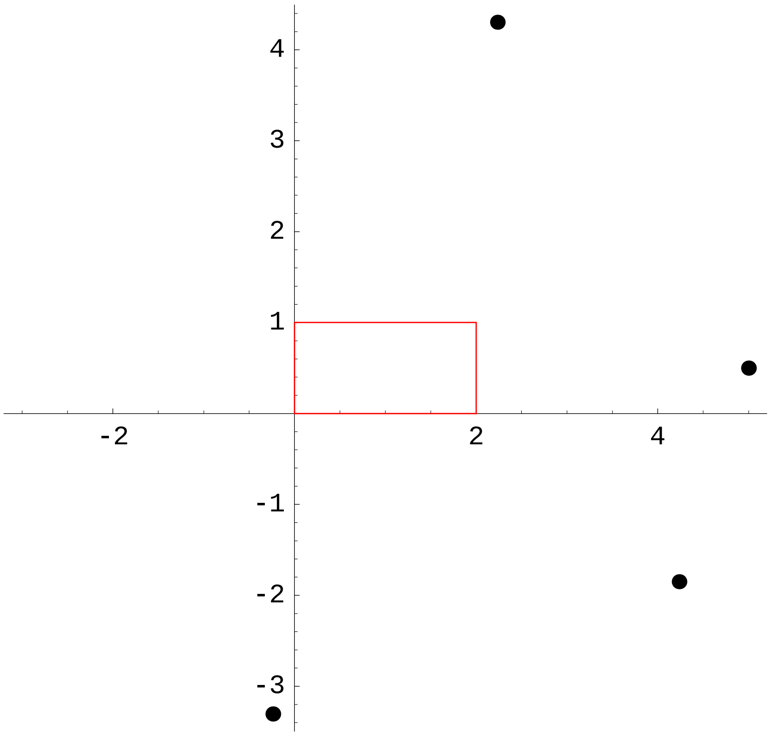


Figure 1:
Linear Flow

```
points = {{0, 0}, {.5, 0}, {1, 0}, {1.5, 0},
   {2, 0}, {2, 1}, {1.5, 1}, {1, 1}, {0.5, 1}, {0, 1}};
r = 4;  c = {1, 0.5};  pts = 10;
sources = sourcesC[r, c, pts];
types =
  {"Neumann", "Neumann", "Neumann", "Neumann",  "Dirichlet",
   "Neumann", "Neumann", "Neumann", "Neumann",  "Dirichlet"};
b = {0, 0, 0, 0, 0, 0, 0, 0, 0, 2} ;
ShowSources[sources, points,  0.02]
InteriorSolution[points, sources, types, b, {1.5, .5}]
```



- Graphics -

{0.500002, −0.999997, 3.33067×10^{-16}}

5 An Elliptical domain with Dirichlet boundary condition

This problem is taken from [Kythe 1995]. The potential at a point (x, y) on the curved part of the domain is given by $U(x, y) = x^2 + y^2$, and there is zero flux across the straight sides. (Figure 2, below). Curved surfaces (particularly concave ones) are a good test of this meshless method, and this is a relatively simple case because of the Dirichlet condition on the curved part of the boundary.

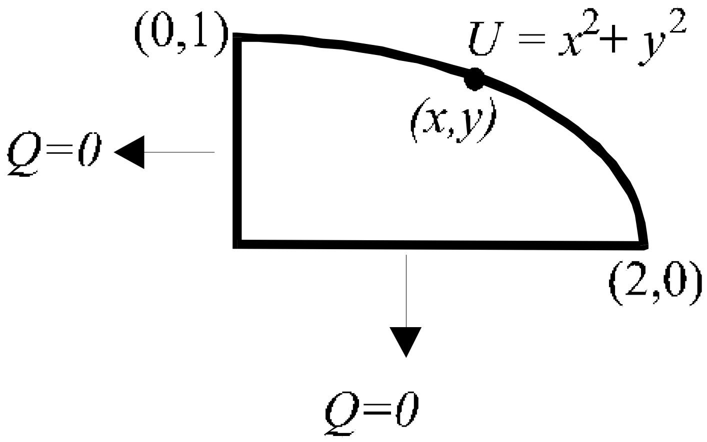
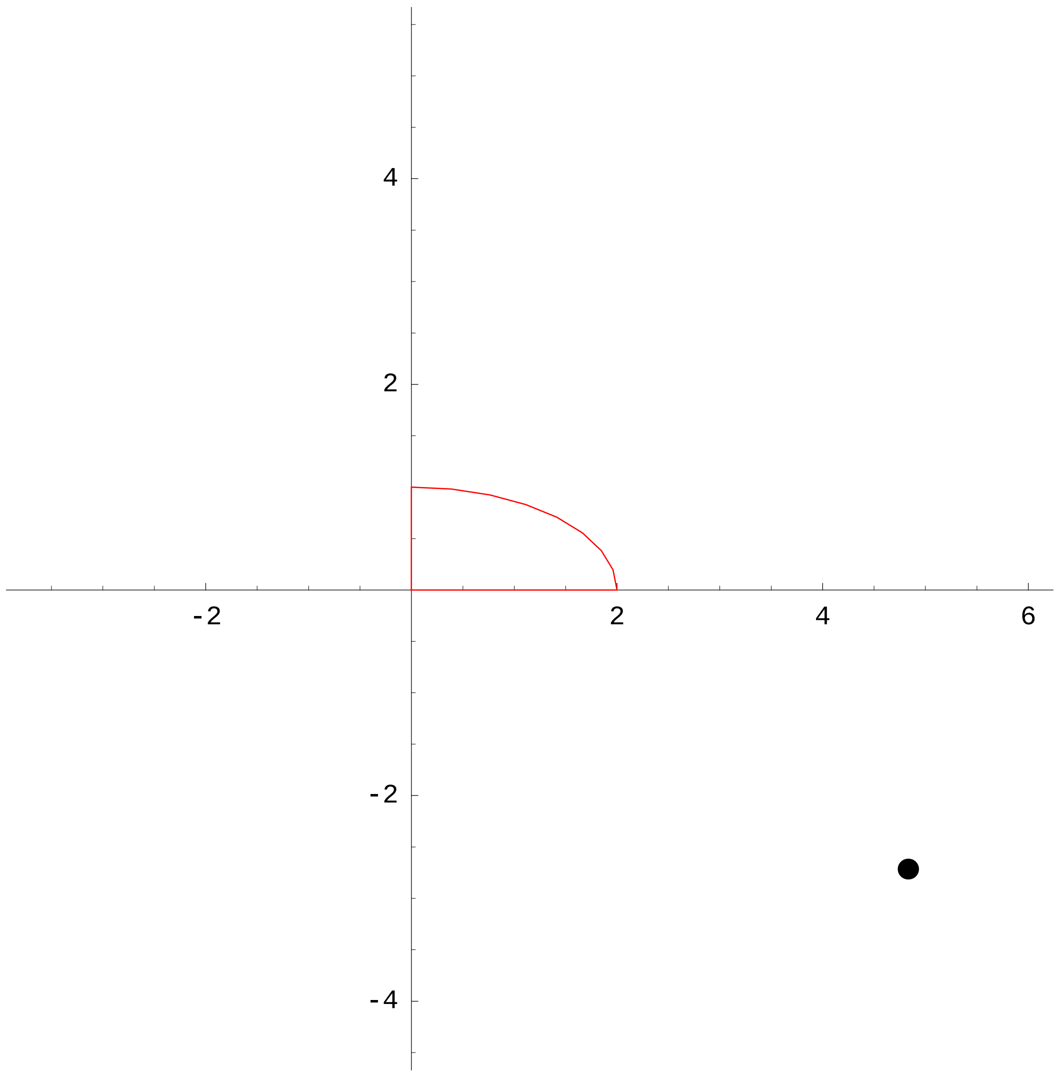


Figure 2:
Elliptical Domain with Dirichlet Boundary Condition

The numerical accuracy obtained is, on the whole, insensitive to changes in the number of sources and the source radius. Sufficient accuracy may be obtained, as in the reactangular domain, by using 8 sources (0.8 times the number of boundary elements) with a radius of 10 (about 10 times the nominal radius of the domain, $\frac{1}{2}\sqrt{5}$ again). The 'infinite radius' calculation also gives correct results, despite increased ill-conditioning. The results for the potential and flux (x- and y-components) at the interior point (1.0, 0.5) with radius 10000 (8 sources) are {1.02718, 0.60582, −0.30291}, which agree closely with results from much smaller radii. In this case it does not seem worth using too large a radius in order to minimise any inaccuracy due to ill-conditioning.

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 5.*
The numerical output is the calculated potential and flux (x- and y-components) at the interior point (1.0, 0.5).




```
Out[6]=  - Graphics -

Out[11]=  {1.02717, 0.605895, -0.30289}
```

6 A Triangular domain with a boundary condition discontinuity

Cartwright [Cartwright 2001] presents a problem in which there is a discontinuity in the boundary conditions at the corners of the domain. This problem has a Neumann boundary condition defined on the hypotenuse of a triangle and Dirichlet boundary conditions on the other two sides (Figure 3, below). This necessitates additional complications for traditional BEM (boundary element method) calculations, but not for meshless methods. The method proposed in this paper still gives correct results. The nominal radius of the domain may be taken as half the length of the hypotenuse of the triangle, $\frac{1}{2}\sqrt{2}$. With the radius of the circle of sources set to 10 times the nominal radius of the domain and using a number of sources equal to the number of boundary elements, results for the potential and flux at an interior point are accurate to within 0.5%. The calculation is very insensitive both to the number of sources used and to the radius of the source circle. The case where the latter only just encloses the domain is accurate for a radius of 1.3 or more. The results are not accurate for a smaller radius than 1.3. It appears that there is no upper bound for the radius, so the "sources at infinity" concept of [Mitic 2004] applies. At least 6 sources are required to solve this problem. The "solutions" obtained using 5 sources are accurate to within 2%, but are completely wrong with fewer sources. As for the radius, there appears that there is no upper bound for the number of sources, subject to ill-conditioning limiting accuracy at some stage. 100 sources works. I have used a constant element BEM to check my results for this problem, and conclude that Cartwright's results are wrong!

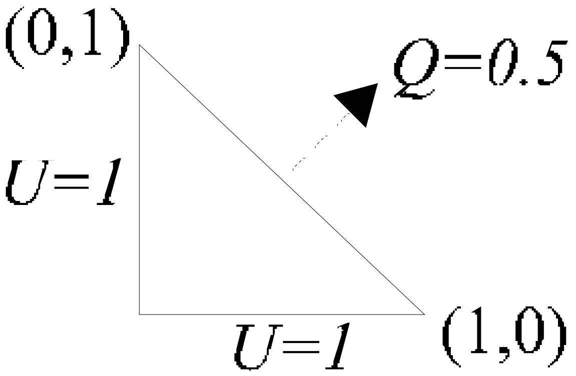


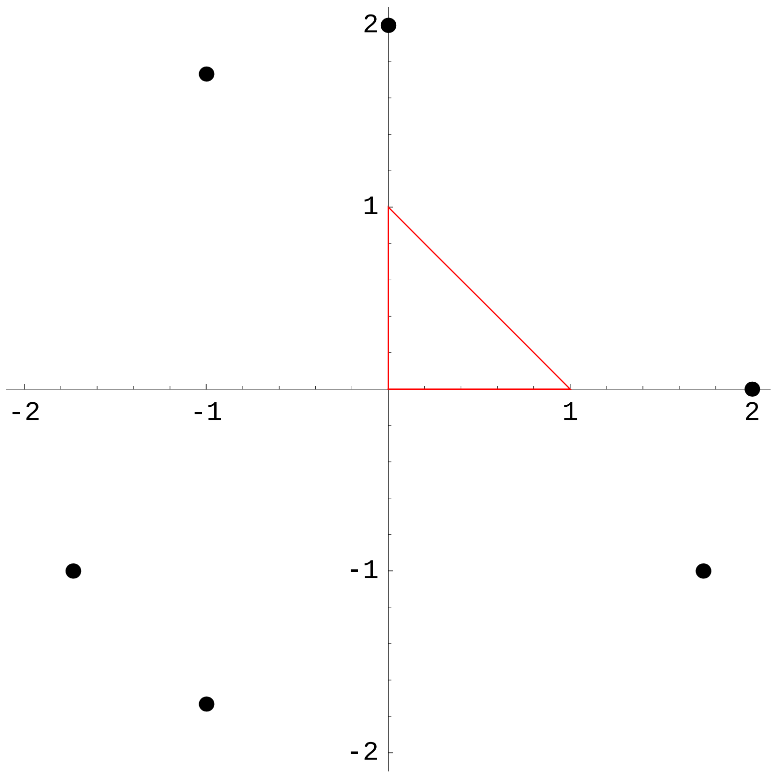
Figure 3:

Triangular Domain

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 6.*

The numerical output is the calculated potential and flux (*x*- and *y*-components) at the interior point (0.25,0.25).

The corresponding Constant Element BEM solutions are respectively {1.01468, 0.198341, 0.198341}.



1.04419, 0.176777, 0.176777

---

7 A Circular Cylinder heat transfer problem

This problem [Kythe 1995] presents a stringent test of the hypotheses proposed in this paper. The problem models heat flow through the walls of a pipe, maintaining a constant temperature on the inner and outer faces of the pipe. The diagram below (Figure 4) shows the cross-section through one quarter of the pipe, with the boundary condition $Q = 0$ on the straight faces to ensure that the flow is entirely radial. Concentrating on calculating the value of U at an interior point, an analytic solution is available. With inner temperature T_{in} and outer temperature T_{out}, and corresponding radii r_{in} and r_{out}, the temperature T_r at radius r is $T_r = T_{\text{in}} + (T_{\text{in}} - T_{\text{out}}) \frac{\text{Log}(\frac{r}{r_{\text{in}}})}{\text{Log}(\frac{r_{\text{in}}}{r_{\text{out}}})}$. The correct temparature at radius 20 is therefore 621.44. In this case the nominal radius may be taken as $\frac{\sqrt{30^2+30^2}}{2} \sim 21$. The calculation below shows the result of the "circle radius ~ ten times nominal radius with number of sources ~ 0.8 times the number of boundary elements" combination. The calculated U value is accurate to within 3%. Unfortunately it is hard to improve on this figure.

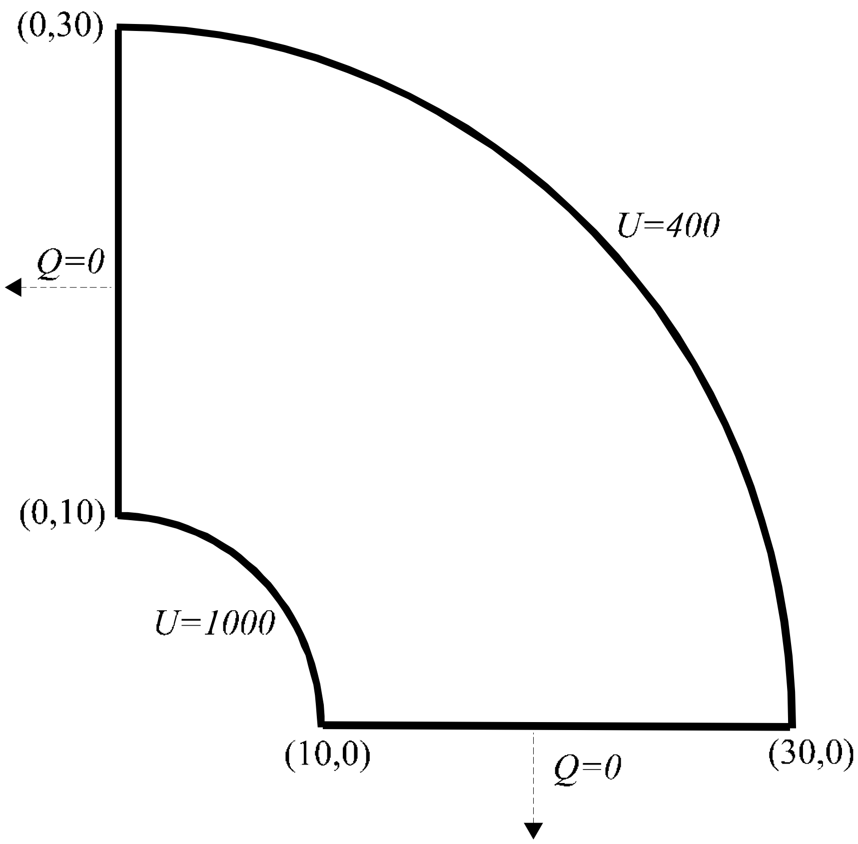
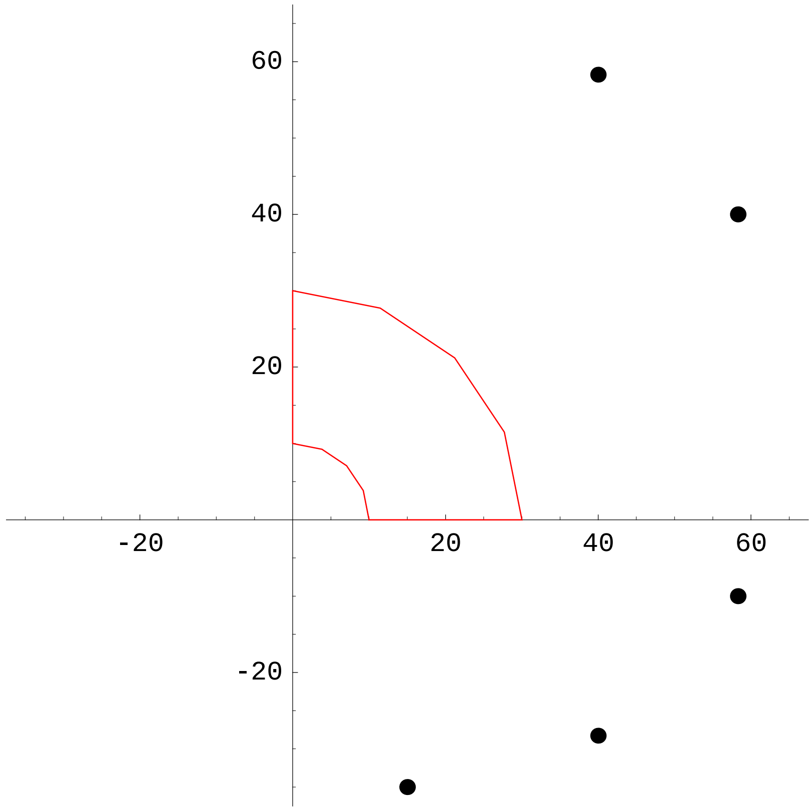


Figure 4:

Circular Domain : Heat Transfer

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 7.*
The numerical output is the calculated potential and flux (*x*-and *y*-components) at the interior point (14.1421, 14.1421), which is the mid-point of a circle of radius 20 in Figure 4. The potential should be constant on this line.



Out[19]= {601.521, −20.4227, −20.4227}

Correct results depend on the number of sources. Table 1 shows the variation of the calculated potential at radius 20 for a varying number of sources. Too many sources gives increasingly inaccurate results. A particular problem is that the results are totally inaccurate if the number of sources is approximately equal to the number of boundary elements (16 in this case). Using 16 sources gives $U(20) \sim 506$ and using 15 sources gives $U(20) \sim 1226$. Using roughly 0.8 times the number of boundary elements (12 or 13) for the number of sources gives tolerable accuracy for U, but not for Q. We are more successful using about 0.5 or 0.6 times the number of boundary elements for the number of sources. No explanation is apparent yet for this alarming variation in results!

Out[76]//DisplayForm=

Sources	U (20)	% error
4	635.69	2.3
5	628.21	1.1
6	631.19	1.6
7	622.34	0.1
8	605.38	2.6
10	606.07	2.5
12	599.59	3.5
20	589.34	5.2
30	604.65	2.7
40	604.65	2.7
50	604.65	2.7
100	604.65	2.7

Table 1:

Heat Flow : Variation of Potential with Number of Sources

The following results (Table 2) are obtained by varying the radius of the circle on which the sources lie, using 12 sources. Clearly, increasing the radius improves the result, but only up to a point. The result for "infinite radius" is surprisingly less accurate than for some smaller radii, presumably due to increased ill-conditioning.

Radius	U (20)	% error
40	599.59	3.5
100	603.55	2.9
200	603.98	2.8
500	604.10	2.8
1000	604.11	2.8
5000	605.78	2.5
10000	605.78	2.5
50000	614.17	1.2
100000	635.87	2.3

Table 2:

Heat Flow : Variation of Potential with Radius

One further interesting experiment is to investigate the stability of the results with an "infinite" source radius, and varying the number of sources. Many problems then disappear. The problem is much less sensitive to the number of sources used. The results for the sources at radius 100000 (i.e. "infinity") with 16 sources (i.e., one per boundary element) is typical: $U \sim 635.6$, $Q\text{x} \sim Q\text{y} \sim -20.02$. The same accuracy is achievable with a range of sources from 5 to 100.

8 Torsion of an Ellipse

This problem is taken from [Brebbia 1989], and concerns torsion of an ellipse. The configuration is as shown in Figure 5, with a Neumann condition defined by *Q1(x,y)* on the elliptical boundary, where $Q1(x, y) = \frac{75\,x\,y}{\sqrt{25\,x^2+10000\,y^2}}$. The nominal radius is $\sqrt{10^2+5^2} \sim 5.6$.

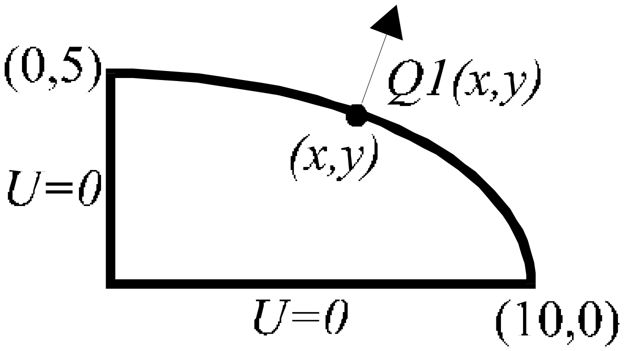
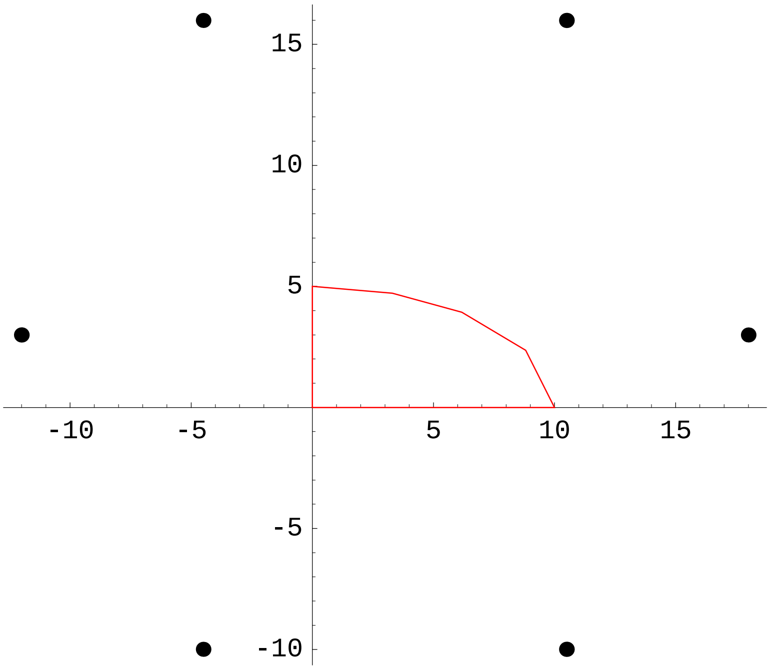


Figure 5:

Torsion of an Ellipse

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 8.*
The numerical output is the calculated potential and flux (*x*- and *y*-components) at the interior point (2, 2). The corresponding solutions from [Brebbia 1989] are given after the *Mathematica* outputs.



`Out[37]=` {2.53575, 1.24581, 1.23106}

Despite the complication of boundary condition on the curved part of the boundary (for which we need to apply *Q1* at points on that part of the boundary), this problem is stable with respect to changes in both the source radius and the number of sources. Results agree well with Brebbia and Dominguez's values (potential - 2.40, x-flux ~ 1.22 and y-flux ~ 1.22). For example, the "infinite radius with number of sources = number of boundary elements" configuration gives $U \sim 2.461$, $Qx \sim 1.212$ and $Qy \sim 1.230$. Changing the radius and number of sources makes little difference to the overall results.

9 A Motz problem

This Motz problem presents particular difficulties for traditional BEM techniques because of discontinuities in the boundary conditions. Figure 6 shows a Motz configuration. The problem is solved by several techniques in [Fam 2002]. The nominal radius of the boundary is $\sqrt{14^2 + 7^2} \sim 8$.

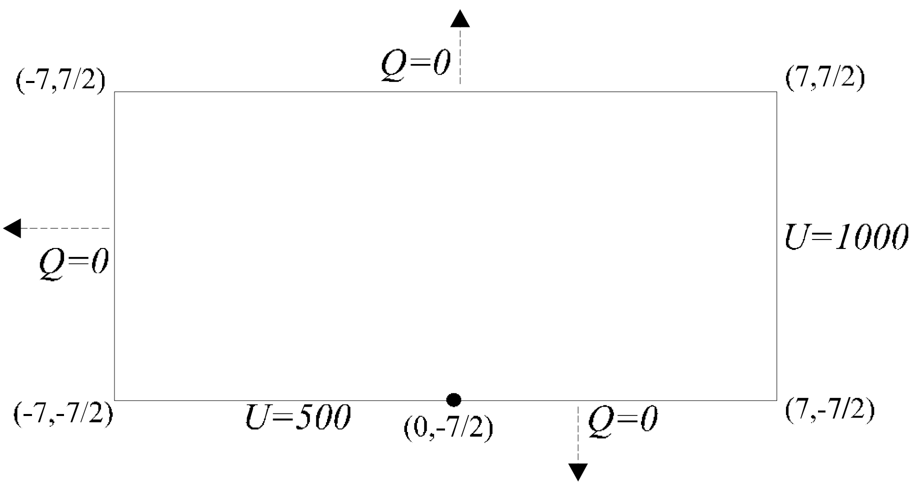


Figure 6:

Motz problem

The parameters of the computed solution are:

- 1 boundary element per boundary condition (5 in all);
- 5 sources;
- solutions calculated for interior points $(x, -7/2) - x$ in [1.4, 7].

Table 3, below, shows the results for U at the same points as were used in [Fam 2002] using a small radius and an "infinite" radius. For comparison, Fam's results using the Whiteman BEM are reproduced. It is hard to draw conclusions from these results other than to note that the meshless results are consistently lower than the Whiteman BEM results. Fam presents several methods of solution for this problem, and the meshless results given here are comparable with all of Fam's results. There is no guarantee that the Whiteman BEM produces "correct" results, although I suspect that they are more reliable than some others. If so, a small radius produces more accurate results a large radius.

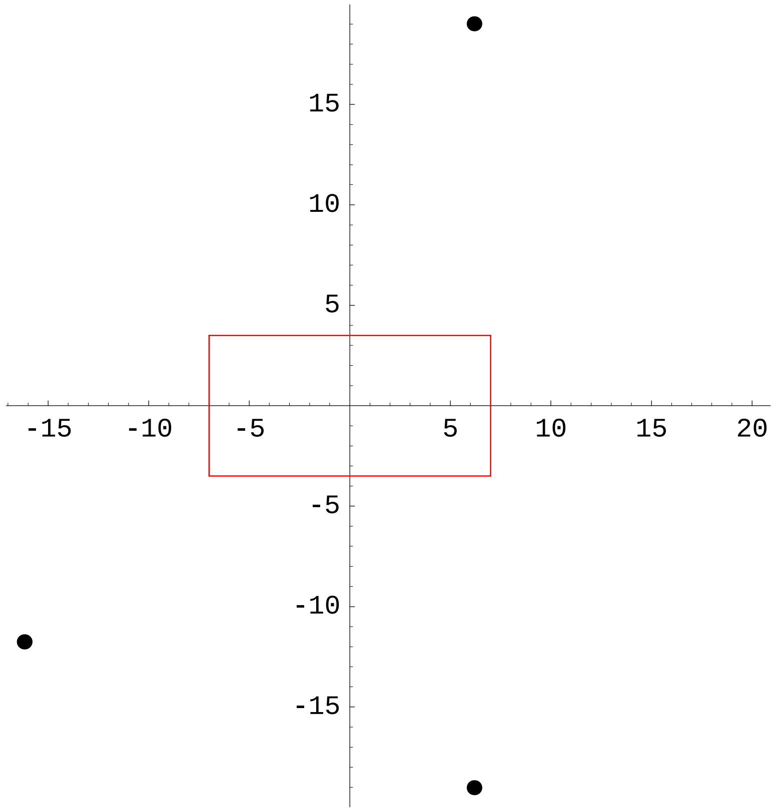
x	U (circular point sources) (radius = 10)	U (circular point sources) (radius infinite)	U (Whiteman BEM)
7	1030.3	1113.6	1000
5.6	957.4	1008.2	930
4.2	882.8	910	860
2.8	807.8	819.1	780
1.4	733.6	735.5	690

Table 3:

Motz problem : potentials at (x, –7 / 2)

```
InteriorPoints = {{1.4, -7 / 2}, {2.8, -7 / 2},
   {4.2, -7 / 2}, {5.6, -7 / 2}, {7.0, -7 / 2}};
```

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 9.*
The numerical output is a vector of calculated potentials at the set of interior points $(x, -7/2)$, where x is as in Table 3. Table 3 also gives the calculated Whiteman BEM potentials from [Fam 2002]. This is their simplest computation, but cannot be taken as an 'accurate' solution.



{757.336, 830.907, 906.641, 985.721, 1069.41}

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10 Fluid Flow past a circular cylinder

Here we consider Zhang's fluid flow problem in [Zhang 2001]. This is a standard configuration of fluid flow past a circular cylinder (the circular arc part of the domain) and is shown in Figure 7. Domains such as this have caused problems in the past, and I think this is due to the concave part of the boundary. In Figure 7, U is the stream function. There are 'natural' boundary conditions on the boundary segments OA, AB and BC, and the boundary conditions on CD and DO are calculated from the analytical solution, $U(x, y) = y\left(1 - \frac{1}{y^2+(x-4)^2}\right)$ with y=2 and x=0 respectively.
The nominal radius of the domain is $\sqrt{4^2 + 2^2} \sim 2.2$.

The first output below is the calculated potential at (0,0). The second output below is a vector of calculated potentials at a sequence of points on the line OC, as discussed in Section 10.1.

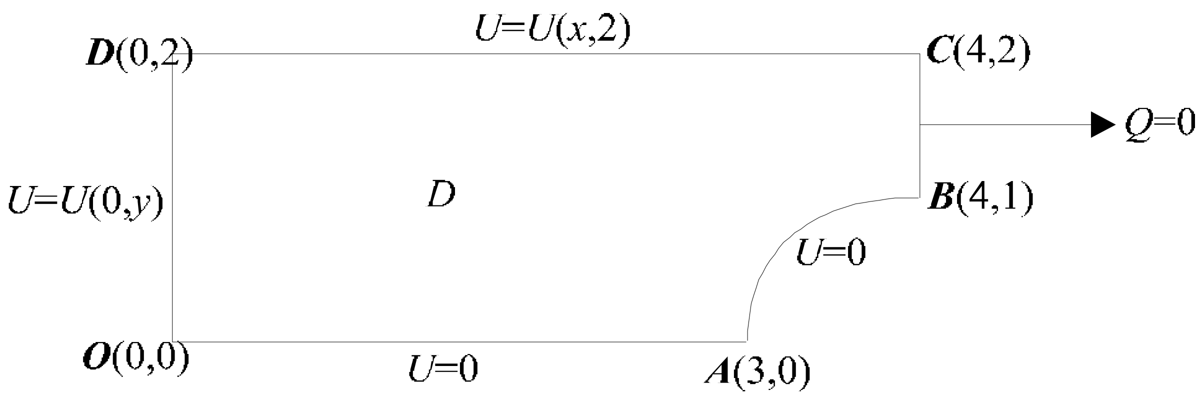
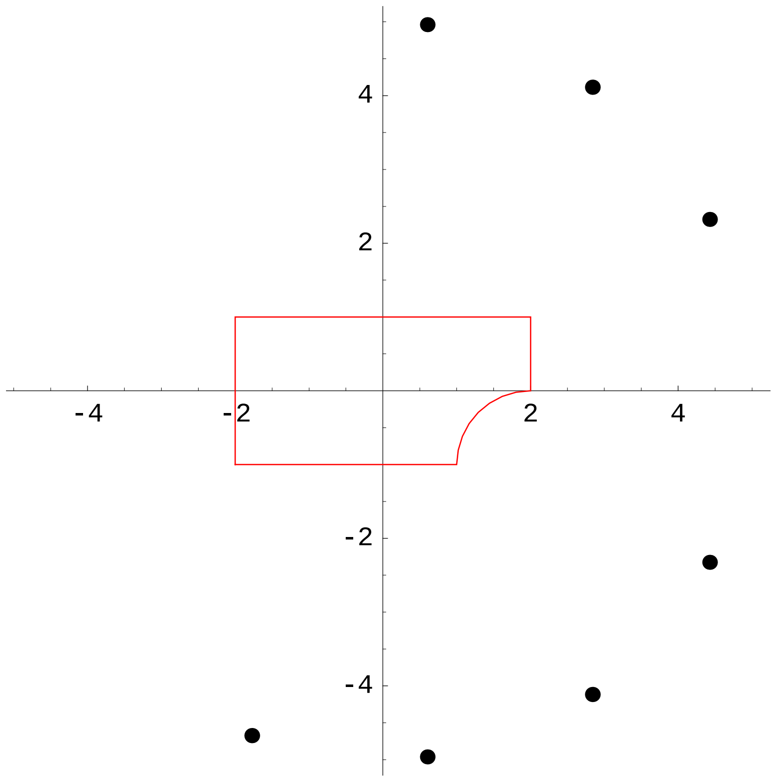


Figure 7:

Fluid Flow past a CircularCylinder

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 10.*
The numerical output is a vector of calculated potentials at a set of 19 interior points, equally spaced on the line *OC* in Figure 7.



Out[164]=

{0.0789327, 0.189132, 0.28342, 0.36843, 0.448432, 0.525529, 0.600084, 0.671316, 0.73796, 0.798939, 0.853938, 0.90381, 0.950725, 0.997986, 1.04944, 1.10839, 1.17602, 1.24918, 1.31757}

For comparison, the corresponding exact solutions are:

```
Out[45]= {0.0930796, 0.184615, 0.274249, 0.361538, 0.445946, 0.526829,
         0.603448, 0.675, 0.740708, 0.8, 0.852809, 0.9, 0.943836,
         0.988235, 1.03846, 1.1, 1.17692, 1.27059, 1.37945}
```

10.1 Comparison of calculated potentials on a diagonal with exact values

For radius $r = 10$, the calculated potentials agree closely with the exact results except near the boundary $x = -2$. For larger radii (e.g. $r = 50$, which is about 20 times the nominal dimension of the region, the maximum error is about 3%. Increasing the radius further produces more error, particularly near the vertical boundaries of the region.

In contrast to the number of nodes used by Zhang [Zhang 2001] to discretise the boundary (ranging from 26 to 104), I solved this problem using only 20 boundary elements and as few as 12 sources. The discretisation was very crude but it sufficed: 3 boundary elements on OA, 8 on AB, 1 on BC, 4 on CD and 2 on DO. The following results show the calculated potentials at the same 19 interior points as Zhang. The parameters in this comparison are: 13 sources (one for each boundary element on the straight line sides and one for each boundary element on the convex sides and none for concave sides, making 13+0+0), with source radius 22 (10 times the nominal radius). The percentage errors recorded below are considerably worse if the source radius exceeds 20 times the nominal radius. This problem is less sensitive to the number of sources, but the number conjectured appears to be optimal.

```
potExact = Map[uExact[#[[1]], #[[2]]] &, intPoints] // N;
perCentErrors = 100 (potExact - calcPotentials) / potExact
```

```
{-8.02602, -6.7199, -2.5103, 0.203486, 1.4191, 1.63932, 1.33876,
 0.853842, 0.370855, -0.0507658, -0.435211, -0.831093, -1.24257,
 -1.56965, -1.59213, -1.02109, 0.420308, 2.98827, 7.04065}
```

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11 Saint Venant torsion

This torsion problem, originally from [Canas 1997] and analysed further in [Fam 2003] provides a very stringent test of the conditions that are necessary to obtain good accuracy. The original problem contains only Neumann boundary conditions. The discussion in [Mitic 2004] showed that a meshless BE method involving no Dirichlet boundary conditions could not converge, let alone to a correct numerical value. Hence, in order to solve this problem, one boundary point (point B in Figure 8, below) is "anchored" by imposing an arbitrary Dirichlet boundary condition $U = 0$. In order to do this, a very small straight line portion of the boundary from (1, 0.01) to (1, −0.01) is extracted, and the curved portion CBA is distorted slightly to fit with the inserted straight line portion. This is the same boundary condition that Fam imposes, and its physical interpretation is to stipulate that there should be no rigid body motion of the domain.

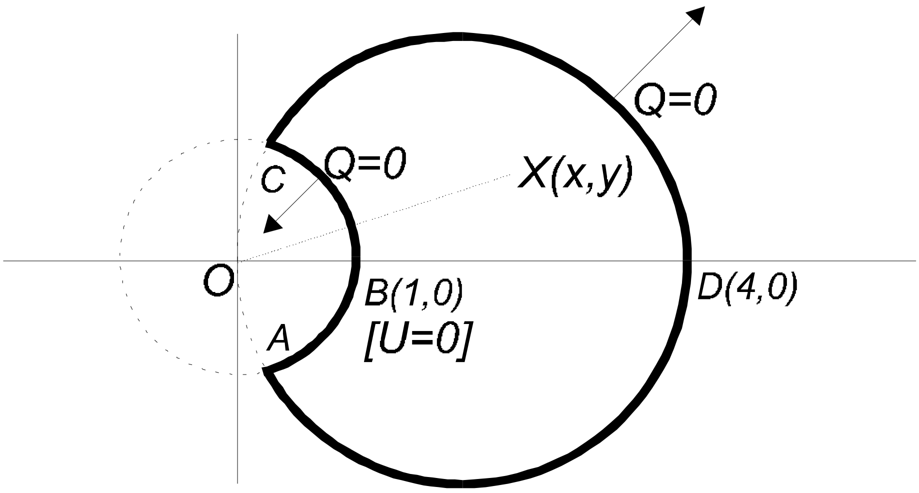
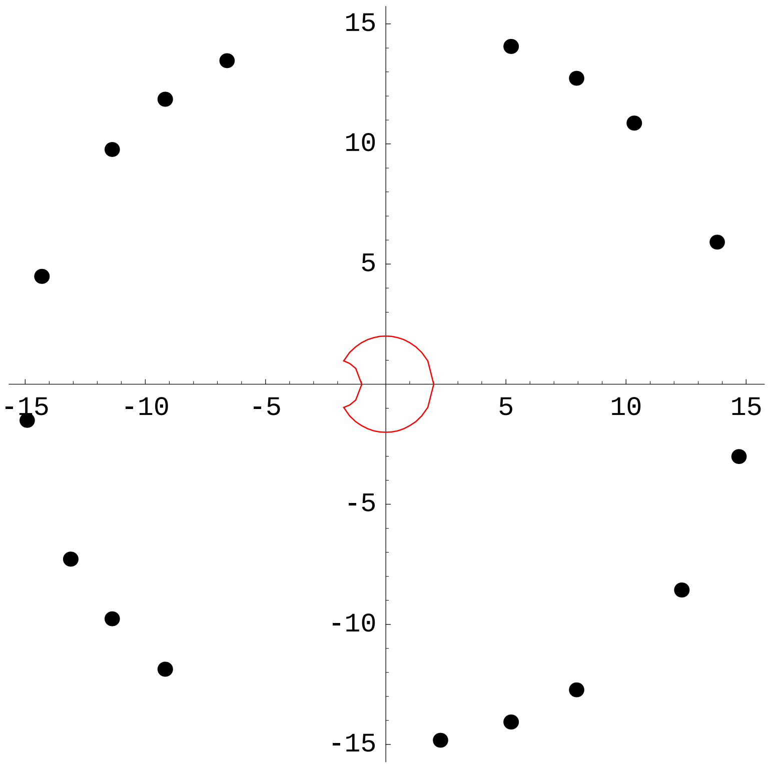


Figure 8:

Saint Venant Torsion

The exact solution for the potential at a point *X*(*x*,*y*) in the doamin is given in [Canas 1997] as *U(x,y)* = 2 sinθ (r + 1/r) where θ is the angle *BOX* in Figure 8 and r is the distance *OX*. For four test points {1,1}, {2,1}, {3,1} and {3,0} the corresponding exact solutions for the potential are 3.0, 2.4, 2.2 and 0. The *Mathematica* code uses two functions to define the geometry of the two circles concerned: Y for the larger circle CDA and Ys for the smaller circle ABC. The discretisation of the boundary comprises 37 boundary elements, with 6 on the concave part ABC, the one straight line element at B and the other 30 on CDA.

The inputs for this problem may be found in ***InputExpressions_Mitic.nb****, section 11.*
The outputs are:



Out[124]=

{3.03989, 2.45171, 2.34563, 0.000106812}

Table 4 shows how the solutions at the test points vary as the radius of the source circle increases (in multiples of the nominal radius NR = 2)

Multiple of NR	U(–1, 1)	U(0, 1)	U(1, 1)	U(1, 0)
2	2.87939	2.33771	2.13252	0
5	2.81563	2.22689	1.86201	.0000026
10	3.03307	2.40686	2.22598	–.003098
20	2.98682	2.38379	2.21362	–.004883
25	2.86670	2.31885	2.15112	–.00146484
30	2.60928	2.20499	2.10248	0
50	2.28738	2.04742	1.99670	–.0011597
1000	1.72013	1.77242	1.81926	.00035095

Table 4:

Saint Venant : Variation of calculated potential with Radius

Accuracy deteriorates rapidly as the sources radius increases, and the "infinite" radius case is clearly inapplicable. A possible explanation is the dominance of Neumann boundary conditions in this problem.

Table 5 shows how the calculated solution for the potental varies with the number of sources used for a given source radius, 20 = 10 NR. The results appear to be stable over a large range for the number of sources. There is no point in using too many sources as the time taken to do the computations increases with the number of sources, and ill-conditioning can affect the result. Using other radii for the source circle does not present such a clear view. For example, using a source radius of 10 (5 times the nominal radius) gives accurate results for 20-30 sources, but very inaccurate results (typically with errors of 20% or more) for other numbers.

Sources	U (−1, 1)	U (0, 1)	U (1, 1)	U (1, 0)
10	1.71881	1.77184	1.81905	0
15	2.35566	2.08144	2.01926	0
20	2.86701	2.32014	2.15302	.0000012
25	3.03535	2.40887	2.22655	−.0003443
30	3.04002	2.41376	2.23283	.0037384
40	3.03688	2.41066	2.22977	.0007057
50	3.03684	2.41060	2.22970	.0006561
60	3.03662	2.41039	2.22948	.0004387
80	3.03669	2.41049	2.22959	.0005169
100	3.03619	2.40996	2.22913	.00002098
150	3.03682	2.41059	2.22968	.0006628
200	3.03623	2.41002	2.22912	.000092506
500	3.03573	2.40947	2.22859	−.00048137

Table 5:

Saint Venant : Variation of calculated potential with Number of Sources

These investigations show that, for this problem, the radius of the source circle is the more important factor for achieving good accuracy. This is also true of the circular cylinder heat transfer problem, but the optimal radii are rather different (10 times the nominal radius in the former case and infinite in the latter). Discrepancies such as this make it hard to specify a generic pattern for the source circle.

12 Conclusion

The aim of this study was to find a generic circular source determination for solving the BVP $\nabla^2 U = 0$ with a wide variety of domains and boundary conditions defined on those domains. The parameters determinable are the number of sources and the source radius. The conjecture of this paper came about by trying many variations on these input parameters for all of the cases considered here. The result below are the only ones that can be applied in all cases.

Optimal source circle radius = 10*Nominal radius of the domain.

Optimal number of sources = One for each boundary element on straight line portions of the domain + one for each boundary element on convex portions of the domain (with none for concave portions of the domain).

Ideally I would have liked an "infinite source radius" to have worked for all cases, but this failed totally for the Zheng fluid flow problem and the Saint Venant torsion problem. In all other cases it provided an ideal situation: use a very large radius, in which case the number of sources is relatively unimportant and a convenient (but not the only one!) value to choose is the total number of boundary elements. No reason is yet apparent for the different behaviour in the case of the Zheng and Saint Venant problems. Choosing the optimal source radius to be ten times the domain's nominal radius works, but the number of sources is then critical. The circular cylinder heat flow problem shows that the solution can then be extremely ill-conditioned with respect to the number of sources used. One must be cautious in accepting the conclusions in the bold in this section. In particular, the formula for the optimal number of sources causes some concern. It fits all the cases discussed here, but there is no guarantee that it will fit others. However, some of the cases here are extreme, and will, I hope, provide a good sample for determining the parameters of the source circle.

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