HAR Inference: Recommendations for Practice

Rejoinder

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We are lucky to have such distinguished discussants and for them to provide such thoughtful comments and suggestions. We agree with much of what they say, and in this brief rejoinder we focus on a few common threads in their remarks.

First and most importantly, our aim has been to suggest a new convention for HAR inference that is informed by and builds on the massive body of research on HAR inference over the past twenty years. This research – which builds on fundamental contributions by our discussants – has been deep and has yielded important insights, yet it is largely ignored in empirical practice. It is time for that to change. The proposed rules embody those insights and turn them into concrete recommendations. It is true that the truncation parameters are unnecessarily large in some of our designs, but not in all of them, and if a single rule that is not data dependent is to be recommended, it should control size in situations with moderate dependence, not just low dependence.

Second, as West points out, in practice HAR inference might be used for testing individual coefficients, confidence intervals, and testing joint hypotheses, all in the same regression. Given the better power, and comparable size, of the NW test than EWC for testing multiple hypotheses, in our view this gives an edge to using NW with fixed-b asymptotics and our proposed rule.

Third, we neither pretend nor hope to have the last word on this topic. Indeed a number of the suggestions by the discussants are meritorious, and we have left some stones unturned. Müller’s suggestion that the EWC estimator have degrees of freedom v capped at 20 makes sense; at that point, the power loss is minimal compared to the oracle estimator, but capping v controls the rejection rate under the null in a very large data set. We did not look at VARHAC estimators, but they should be on the table too, although deriving a lag length rule would require developing fixed-*b* theory for VARHAC. We have intentionally avoided data-dependent rules (an idea raised or implicit in the comments by Müller, Sun, and West), but for sophisticated practitioners data dependence might be warranted; in any event, data-dependent rules could be incorporated into standard software, making them easy to use. And there is clearly more to do concerning understanding whether it is desirable to impose the null for testing; here, Vogelsang's comments make good progress beyond what is in our paper. Researchers should continue to investigate these and other methods. In this regard, we hope that our new data-based DFM design will provide a useful testbed for other researchers, and we encourage them to use it (the core software is compact and is available online). We think the design is sufficiently rich that it unlikely that it can be "gamed" to produce tests that work well in this design but not in reality or in other settings with moderate dependence.

Fourth, we have followed LLS in approaching the truncation parameter choice through the lens of trading off the null rejection rate and size-adjusted power. As both West and Sun point out, a different approach is to start with a tradeoff between Type I and Type II errors, as done by Sun, Phillips, and Jin (2008). One can argue the intellectual merits of the two approaches but, in keeping with the practical motivation of this paper, we instead point out that the two approaches have similar implications. In particular, by minimizing the linear loss function proposed in the comments by Sun – i.e., $b^*_{SPJ} = \arg\min_b \left(w/(1+w)\right)e_{\mathrm{I}}(b) + \left(1/(1+w)\right)e_{\mathrm{II}}(b,\delta^2)$, where w is the ratio of the weight on the Type I error $e_{\mathrm{I}}(b)$ relative to the weight on the type II error $e_{\mathrm{II}}(b,\delta^2)$ and where the test uses fixed-*b* critical values – we obtain the optimal SPJ rule,

$$b^*_{SPJ} = b_{0,SPJ}T^{-q/(1+q)}, \text{ where } b_{0,SPJ} = \left[\frac{2q\left(wG'_m(\chi^\alpha_m) - G'_{m,\delta^2}(\chi^\alpha_m)\right)k^{(q)}(0)\omega^{(q)}}{\delta^2 G'_{m+2,\delta^2}(\chi^\alpha_m)\int k^2}\right]^{1/(1+q)}, \quad (1)$$

where terms are as defined in the text of our paper.[1] This expression applies for positive low-frequency correlation, i.e., $\omega^{(1)}$, $\omega^{(2)} > 0$, which is the relevant case if the truncation parameter choice is not data-dependent.

The key point is that the rate in (1) is the same as in our Equation (22) for the size/size-adjusted power tradeoff; the only difference is the expression for the constant term. In particular, it can be seen that there exists some choice of w such that the sequence in (1) is equivalent to the sequence provided by the rule provided by our Equation (22). Assuming the local alternative δ is the worst-case alternative from a size-adjusted-power standpoint (as in our paper), one obtains that

$$b_{0,SPJ} = b_0 \kappa_q^{-1} \left(w - \frac{G'_{m,\delta^2}(\chi_m^\alpha)}{G'_m(\chi_m^\alpha)} \right)^{1/(1+q)} q^{1/(1+q)}, \tag{2}$$

where b_0 is the optimal constant for our rule given in our Equation (22) and κ_q is defined following (22). Solving (2) for $b_{0,SPJ} = b_0$ yields the relation between κ and w for which the two approaches yield the same rules.

When (2) is evaluated for the NW and EWC/EWP kernels, 5% tests, an AR(1) with coefficient $\rho = 0.7$ (what we use for our rules), and $w = 9$ (which corresponds to our weight of $\kappa = 0.9$ on the squared size distortion and is within the range of values considered by Sun (2014a)), the Sun, Phillips, and Jin (2008) rule (1) implies,

$$S_{SPJ}^* = 1.9T^{1/2} \quad \text{(NW Type I/II optimal)} \tag{3}$$

$$\nu_{SPJ}^* = 0.3T^{2/3} \quad \text{(EWC/EWP Type I/II optimal)} \tag{4}$$

[1] We follow Sun's comment and refer to this as the SPJ rule, however this rule follows from the setup in Sun's comment but not strictly from Sun, Phillips, and Jin (2008): Sun's comment uses fixed-*b* critical values (as do we) whereas the SPJ optimal rule uses a polynomial correction to standard normal critical values.

These are close to our proposed rules (respectively, $1.3T^{1/2}$ and $0.4T^{2/3}$). Thus, rules quite similar to ours, with identical asymptotic order, also emerge from the Type I/II approach.

We thank again the discussants for their efforts and insightful comments.