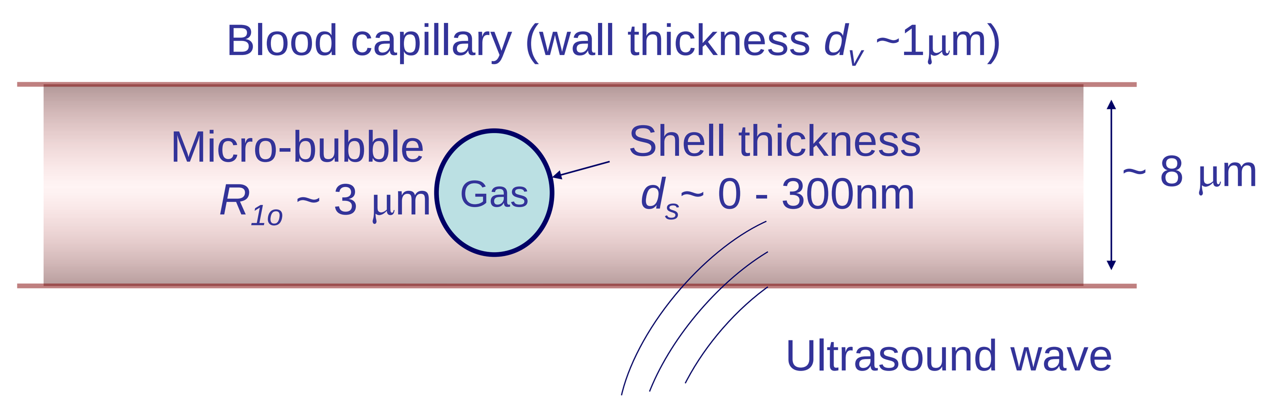
MODELLING OF THE DYNAMICS OF A COATED MICROBUBBLE CONFINED IN A BLOOD VESSEL

Sergey Martynov, Eleanor Stride and Nader Saffari
Department of Mechanical Engineering, UCL, Torrington Place, London, WC1E 7JE, UK

UCL

Objectives

- To develop the numerical model for the dynamics of a bubble, encapsulated in a shell, and located in a narrow elastic vessel filled with a viscous fluid
- To study the effects of bubble coating and vessel stiffness, on the bubble-vessel deformations, radiated pressure, amplitude and frequency spectra of oscillations



Assumptions and Governing Equations

Fluid

- an incompressible homogenous Newtonian liquid with properties similar to water

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho_l \frac{\partial \mathbf{u}}{\partial t} + \rho_l (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla\cdot\left(-p\mathbf{I} + \mu_l\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)\right) = 0$$

Bubble shell

- a finite-thickness linear elastic structure, undergoing small deformations

$$\rho \frac{\partial^2 \xi}{\partial t^2} - \nabla\cdot\sigma = 0$$

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right), \quad \varepsilon = \frac{1}{2}\left(\frac{\partial \xi_i}{\partial x_j} + \frac{\partial \xi_j}{\partial x_i} + \frac{\partial \xi_l}{\partial x_k}\frac{\partial \xi_k}{\partial x_j}\right)$$

Gas inside the bubble

$$p_g = p_{g,eq}\left(\frac{V_{eq}}{V}\right)^{\gamma}$$

Vessel wall

- a long elastic membrane-type tube of a thickness d_v ~1μm, radius R_v ~4-100μm, and elastic modulus E_v ~ 0.1-10 MPa

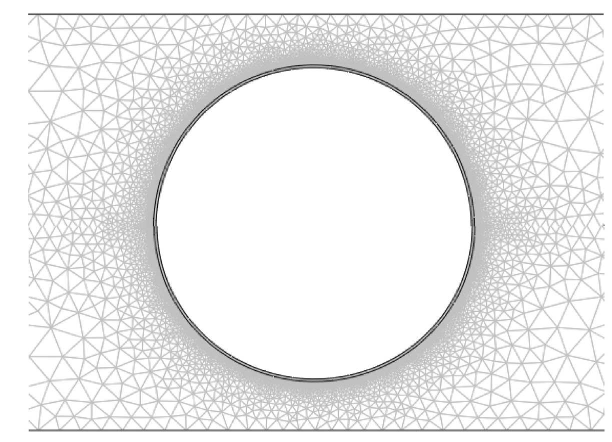
$$\rho_v d_v \ddot{R}_v = p_i - p_a - \frac{(1-\nu^2)E_v d_v}{R_v^2}\left(R_v - R_{vo}\right)$$

Ultrasound wave:

$$p_{external} = p_o + p_{ac}\sin(\omega t)$$

Solution Method

The governing equations subject to boundary conditions are solved using the FEM package COMSOL MultiPhysics v. 3.4. The fluid-structure interaction is calculated using the ALE method on the non-uniform moving mesh.



Validation Cases

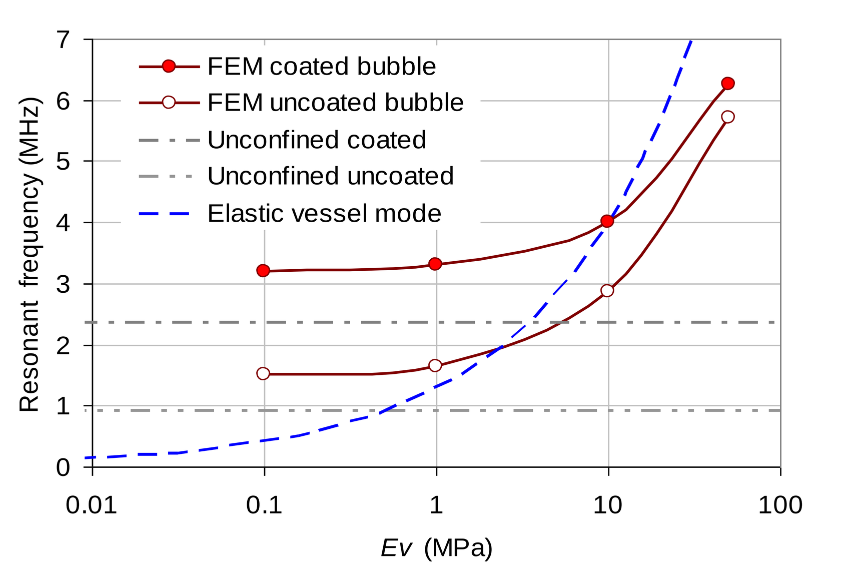
- Dynamics of a spherical (unconfined) coated bubble (Hoff, et al, 2000);
- Resonance frequency of oscillations of uncoated bubble confined in:
  - a rigid vessel (Oguz and Proseperetti, 1998), and
  - an elastic vessel (Qin and Ferarra, 2007);
- Free non-spherical oscillations of uncoated bubbles (Hao and Prosperetti, 1999);
- Static buckling of a hollow spherical shell (Koiter, 1969)

Results of studies of the effects of bubble encapsulation and confinement

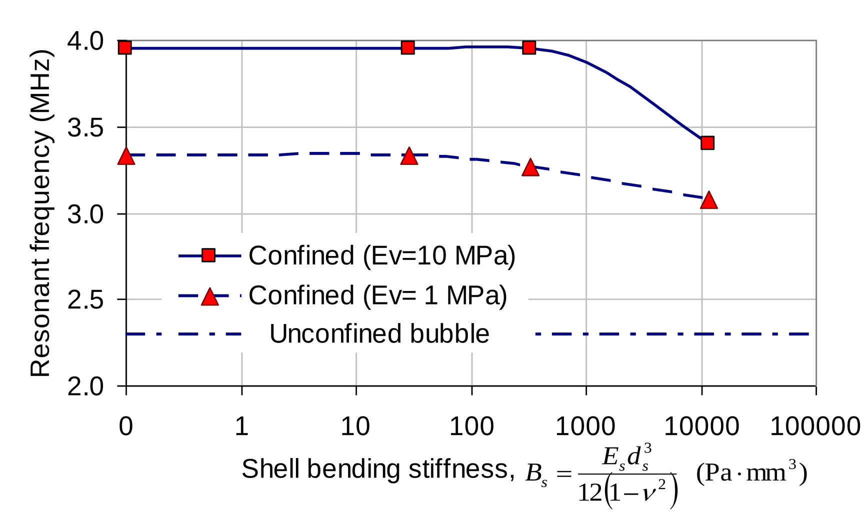
The results of analysis of free and forced oscillations are shown for bubbles with the internal radius R_{1o} = 3μm and shell thickness d_s = 0 – 300 nm, placed in vessels with radius R_v = 4μm, length L_v = 200 μm, wall thickness d_v = 1μm, and elastic modulus E_v = 0.1 - 50 MPa

Resonance frequency of confined bubbles

The effect of the vessel stiffness E_v on the resonance frequency of coated bubbles (d_s = 50 nm, E_s = 23.6 MPa) and uncoated bubbles (σ=0.0643 Pa m)

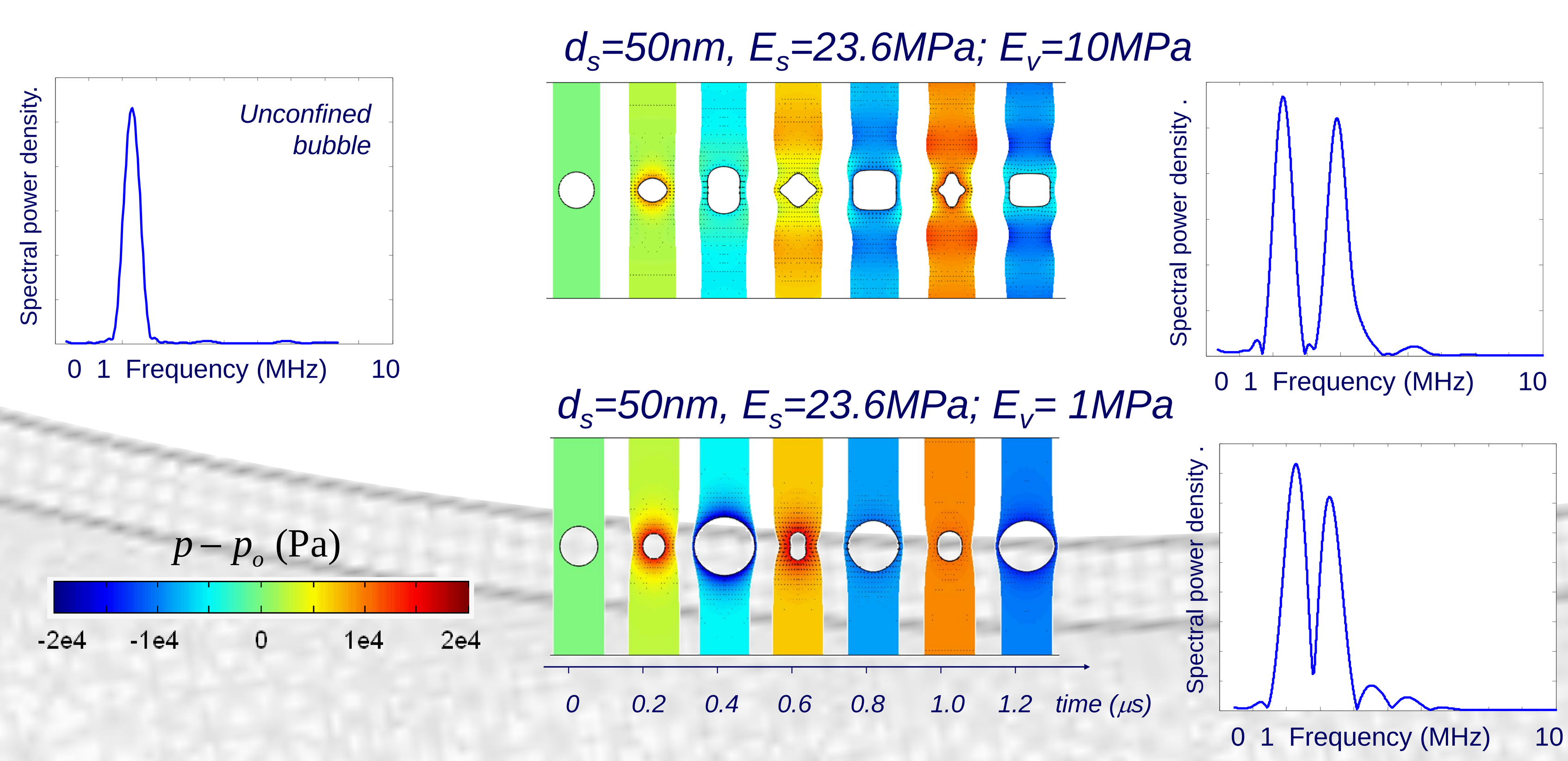


The effect of the bubble shell bending stiffness B_s on the resonance frequency of bubbles with radius R_{1o}=3μm confined in vessels with R_v=4μm and E_v= 1MPa and 10 MPa

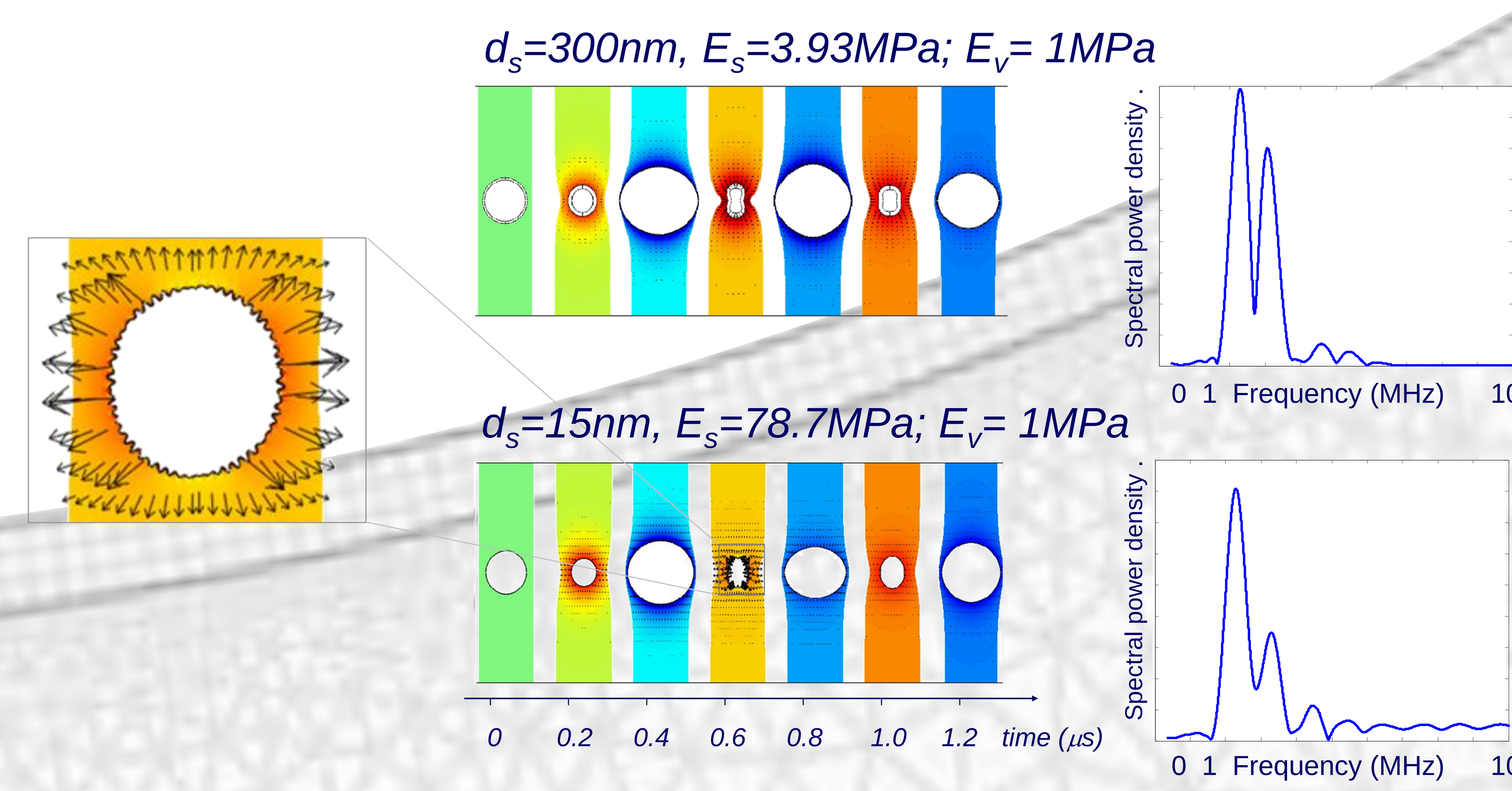


Forced bubble oscillations (f = 2.3MHz, p_{ac} = 10kPa) – shape deformations and radiated pressure frequency spectra*

The effect of the vessel wall stiffness E_v on the forced oscillations of coated bubbles (d_s = 50 nm, E_s = 23.6 MPa)



The effect of the bubble shell bending stiffness B_s on the forced oscillations of bubbles with radius R_{1o}=3μm confined in vessels with R_v=4μm and E_v=1 MPa



* The bubble and vessel deformations are shown magnified by 10 times

Conclusions

The resonance frequency of a bubble is shown to be strongly affected by the vessel wall stiffness, and the membrane (bulk) stiffness of the bubble coating, while the influence of the coating bending stiffness is relatively small. The resonance frequency is shown to increase with the vessel wall stiffness.

The wall deformation pattern is mainly influenced by the vessel wall stiffness and liquid viscosity; the amplitude of wall deformations and wall stresses depends on the membrane stiffness of the bubble coating.

The development of a buckling instability in the bubble shell is governed by the shell bending stiffness, and also depends on the pressure field in the liquid around the bubble, which is strongly affected by the vessel and bubble stiffness, and the vessel diameter.

Acknowledgements

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