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Can Return Forecasts Enhance International Asset Allocation? Evidence from the Sum-of-Parts Approach*

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Abstract.

We examine whether real-time return forecasts are valuable to an investor looking to allocate their portfolio across a wide selection of countries. We expand the Sum-of-Parts (SoP) method for forecasting stock returns to an international setup by adding foreign exchange returns as an additional component. We use two different methods to calculate the forecasts. The first method (Empirical Mode Decomposition) uses wavelets to frequency decompose each part into locally independent sub-signals, while the second method combines historical averages and predictive regressions. We then compare the performance of various types of portfolia under the SoP and historical average forecasts, with rebalancing taking place every period. We find that SoP forecasts deliver economic gains to an international investor over the historical average, especially when the EMD method is implemented. We further demonstrate that substantial economic gains can be generated for an international investor based in different home countries. Our results are driven by an increase in the forecast performance of each part, notably this includes foreign exchange return.

JEL Classification: sth sth

1. Introduction

This paper investigates the crucial question of whether real-time equity index return forecasts can help investors to improve their portfolio allocation internationally across countries. Generally, the return forecasting literature examines whether it is possible to beat a benchmark within that

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country and whether the forecast can help improve portfolio allocation between the domestic risk-free rate and the domestic equity index (e.g. Welch and Goyal (2008); Campbell and Thompson (2008); Ferreira and Santa-Clara (2011); Jordan et al. (2017), Jordan et al. (2014)). A separate important literature focuses on the issue of portfolio allocation internationally; this body of work tends to conduct portfolio allocation in-sample and using historical mean and variance as inputs into the decision-making problem (Solnik (1974); Solnik and Noetzlin (1982); Solnik (1993); Errunza et al. (1999)). However, an important under-researched question regards how valuable predictability is to a real-time investor who can allocate funds globally. To our knowledge this question is yet to be fully addressed. The goal of this paper is to quantify the extent to which predictability can enhance the economic value to international investors.

Our approach is to conduct out-of-sample analysis using forecasted returns in the international portfolio allocation problem. We build on the work of Ferreira and Santa-Clara (2011), who demonstrate that decomposing the equity return into separate components and then forecasting each one separately can lead to substantial improvements in forecast performance in the US. We extend their framework to a global allocation setting by introducing the change in exchange rates as an additional component; thus, all equity returns can be quoted in the same currency. This allows for cross-country portfolio allocation where returns are in the reference currency of the investor (e.g. the US dollar for a US-based investor). This enables us to address our key research question and examine how valuable return forecasts are to an agent with a global investment mandate.

We produce Sum-of-Parts forecasts by applying two different methods. The first relies on the original paper (Ferreira and Santa-Clara (2011)) and involves a combination of predictive regressions and historical averages. The second relies on frequency decomposition of the different SoP components via wavelets (Faria and Verona (2018)) which we replace with Ensemble Mode Decomposition (EMD) based on Wu and Huang (2009). We then use these real-time forecasts as expected returns in the mean-variance optimisation of an international portfolio where the investor allocates wealth to the market indices of 44 countries, and compare portfolio performance with the standard case where the forecast is the historical average (HA).

Overall, SoP model forecasts are considerably more accurate than the historical average benchmark forecast for the large majority of countries and lead to substantial improvements in forecast accuracy. Furthermore, using the model forecasts in real time increases portfolio performance and investor utility substantially relative to using the benchmark forecasts, particularly when EMD is employed. Encompassing tests reveal that, when these three forecasts are considered (EMD, original and HA Sum-of-Parts), the optimal weight on the historical average forecast is never statistically different from 0 for any country but in some cases the optimal weight on the model forecast is statistically different from 0. We demonstrate the robustness of our results using a range of portfolio specifications, sub-samples and metrics including, among others, different investor domicile, risk aversion, data frequency, country groupings, forecast error variance decompositions and rolling utility gains. The performance improvement cannot be attributed to data frequency or the level of risk aversion, the choice of home country, any pattern or specific focus in portfolio allocations across countries, the choice of benchmark or look-ahead bias; the whole exercise is out-of-sample. A series of further tests shows that our findings are due to an overall forecast improvement on all parts, where the forecast of the exchange rate growth rate plays a significant role. Our approach is thus able to considerably improve on foreign exchange (FX) forecastability. Finally, when short-selling is allowed, the portfolio allocation of a country based on the forecast seems to match the realised performance of the country's index. Portfolio weights are negative during periods of low performance and positive during periods of high performance with reasonable to high precision. These features suggest that our approach can have great practical usefulness for investors.

2. Literature review

Our approach is rooted in and extends the important work of Ferreira and Santa-Clara (2011) who demonstrate the re-assembling approach to forecasting the equity return can lead to substantial improvements in forecast accuracy. Specifically, they decompose the equity return into three components (price multiple growth, earnings growth and dividend-price ratio) and then forecast each part using appropriate predictors. They find this leads to better forecast accuracy than the historical average benchmark. They apply shrinkage to the estimates which help reduce estimation error. Further, they find that substantial gains could be made by a US investor who applied this approach in a (domestic) two-asset portfolio allocation exercise. The Sum-of-Parts approach leads to significant economic and statistical gains out-of-sample over the historical mean that range between 1.3% on a monthly and 13.4% on an annual basis. On a monthly basis, the gains can be increased to 2.6% if the forecast relies on the sum of parts obtained via a wavelet frequency decomposition of the returns time series (Faria and Verona (2018)).

In domestic settings there is no need to model the exchange rate; however, in an international setting the currency component will need to be incorporated as it creates additional gains and losses according to the changes in the FX rates across the country indices in the portfolio, since the investor valuates portfolio wealth according to the domestic currency. Conventionally, it is thought that exchange rates follow random walks (Meese and Rogoff (1983)) and therefore the numeraire might not matter for forecasting the mean; however it could still affect the standard error of the coefficient estimate potentially due to noise (Jordan et al. (2015)). The random walk of exchange rates has been challenged in recent times by studies reporting some predictive power for currency returns (Lustig and Verdelhan (2007); Ang and Chen (2010); Burnside et al. (2011); Barroso and Santa-Clara (2015); Menkhoff et al. (2012)). If this is the case, then forecasting the currency return could improve forecast power overall.

3. Methodology and Data

3.1 Expansion of the Sum-of-Parts methodology

Here, we outline the Sum-of-Parts methodology and demonstrate how it can be extended to be utilised by an investor looking to invest across multiple countries. We expand the scope from a single asset representing a stock, a portfolio or an index, to an international portfolio, with the aim to conduct a cross-country portfolio optimisation exercise and examine if Sum-of-Parts provides an advantage in such an environment. Technically, we introduce exchange rates as an additional factor and use the forecast of each country as its expected return in the optimisation of a portfolio that allocates wealth across different countries. We then examine whether Sum-of-Parts provides material gains to international investors which are exposed to currency exchange risk.

The original Sum-of-Parts approach is based on aggregating the separate forecasts of three components of stock market returns into a single return forecast. The three components are the dividend-price ratio, earnings growth and the growth of the price-earnings ratio. In Ferreira and Santa-Clara (2011), the total (gross) return of the stock market index consists of capital gains and the dividend yield. For return R, dividend D and earnings per share E,

$$1 + R_{t+1} = \frac{P_{t+1}}{P_t} + \frac{D_t}{P_t} = \frac{P_{1+1}/E_{t+1}}{P_t/E_t} \times \frac{E_{t+1}}{E_t} + \frac{D_t}{P_{t+1}} \times \frac{P_{t+1}}{P_t} \tag{1}$$

The PE ratios at times t and t+1 can be rewritten as $1 + GM_{t+1}$ where GM is the growth rate of the PE ratio. Similarly, the fraction of earnings E_{t+1}/E_t can be rewritten as $1 + GE_{t+1}$, where GE is the earnings growth rate. With a similar rewriting, the second term in (1) can be rewritten as $DP_{t+1}(1 + GM_{t+1})(1 + GE_{t+1})$ where DP is the dividend yield. Equation (1) thus becomes

$$1 + R_{t+1} = (1 + DP_{t+1})(1 + GM_{t+1})(1 + GE_{t+1}) \tag{2}$$

and taking logs leads to $r_{t+1} = dp_{t+1} + gm_{t+1} + ge_{t+1}$ where dp is the log of the dividend price ratio, ge the log of the earnings growth rate and gm the log of the price-earnings growth rate. Sum-of-Parts is markedly superior to historical mean forecasts, providing out-of-sample R^2 of 1.3% with monthly data and 13.4% with annual data.

We expand the model to an international setup by introducing a currency return component, since a change in the exchange rate of a country where portfolio wealth has been allocated with the reference currency of the investor would affect total returns. We thus include the domestic currency-to-US dollar exchange rate as a fourth component of returns. For an international investor who uses a currency different than the domestic one, the change of the spot exchange rate S between t and t+1 generates further returns and the price ratio can be written as

$$1 + R_{t+1} = \frac{P_{t+1}S_{t+1}/E_{t+1}}{P_tS_t/E_t} \times \frac{E_{t+1}}{E_t} + DP_{t+1} \times \frac{P_{t+1}S_{t+1}}{P_tS_t}$$

$$= (1 + GM_{t+1})(1 + GE_{t+1})(1 + FX_{t+1}) + DP_{t+1}(1 + GM_{t+1})(1 + GE_{t+1})(1 + FX_{t+1}) \Leftrightarrow$$

$$1 + R_{t+1} = (1 + GM_{t+1})(1 + GE_{t+1})(1 + FX_{t+1})(1 + DP_{t+1})$$

and with logs

$$r_{t+1} = fx_{t+1} + gm_{t+1} + ge_{t+1} + dp_{t+1} \tag{3}$$

where FX (fx) is the growth rate, or return, of the (log) exchange rate of the domestic currency with the US dollar, similar to Equation (2).

3.2 Forecasting methodologies

Sum-of-Parts decompositions allow separate forecasting methodologies for each part based on their individual characteristics and empirical facts. The base case SoP method uses a combination of predictive regressions and historical averages. We opt for using the last observed value of dp as the forecast for $t+1$, as per Ferreira and Santa-Clara (2011). fx and ge are forecasted as the historical average of all past available observations at time t; this is due to the fx generally being considered difficult to forecast better than a simple benchmark and that ge is affected by differing inflation rates during the early sample period. gm is forecasted via a predictive regression on the log of the price-earnings ratio as per Ferreira and Santa-Clara (2011). This is specified as $gm_{t+1} = \alpha + \beta \times log(PE_t) + \epsilon_{t+1}$. The predictive regression is conducted over a window (backlog) of the last 40 or 20 known observations up to time t, depending on the case.

However, the forecasting accuracy of Sum-of-Parts can be potentially improved by decomposing the individual components. Specifically, the predictive power of the price-earnings growth rate, one of the most important components, is low (Dai and Zhu (2020)). Faria and Verona (2018) apply wavelet decomposition and sum only some of the frequency decomposed parts, achieving significant statistical and economical gains over historical mean forecasts and a monthly out-of-sample R^2 of 2.60%. Further out-of-sample improvement could be achieved by Empirical Mode Decomposition (EMD), first introduced by Huang et al. (1998), and Ensemble Empirical Mode Decomposition (EEMD), which analyse the original time series (signal) to a small number of independent (locally orthogonal), zero-mean amplitude and frequency modulated components called Intrinsic Mode Functions (IMFs), plus any residuals. The two methods belong to the general family of wavelet methods, akin to Fourier transformations. We opt for EMD rather than EEMD because mode mixing does not appear for our low frequency dataset and therefore implementing EEMD does not lead to an improvement.

The intuition behind EMD is that it is possible to analyse a signal (time series) into separate oscillating sub-signals plus a residual by relying on local minimum and maximum points rather than focusing on distributional assumptions or time series properties. EMD is specifically designed to extract signals from non-stationary and non-linear data in a self-adaptive manner and is thus highly efficient. Each sub-signal (IMF) is extracted until a precision threshold set by the researcher is set and follows the time stamps of the original series. Their economic meaning is that they represent trend or, equivalently, memory series present in the data Faria and Verona (2018). For example if 5 oscillators plus a residual are extracted (which is often our case), the first IMF contains ”short” memory patterns, since its wave has the highest frequency, the second and third are the ”medium-term” components and the fourth and fifth contain the ”long-term” memory information. Notably, we do not set a limit to the number of IMFs but let the algorithm decide according to the precision threshold. The key property of local extraction relies on identifying local extrema according to their time stamps, determining whether an oscillator (sub-signal) crosses zero between the two extrema and separating it from the main signal in non-overlapping scale components. The signal is thus broken down into component IMFs (or intrinsic oscillatory modes with the same time stamps as the initial signal) that obey two properties: any IMF has 1) only one extremum between two subsequent zero crossings 2) a mean value of zero, which implies stationarity but does not prevent amplitude modulation or changing frequency. In other words, the algorithm treats each IMF as a sub-signal at a local level. A concise discussion of EMD and its practical features can be found in Zeiler et al. (2010) and we provide a technical description of the method below:

- Assume a time series (signal) $x(t)$ which needs to be decomposed to n IMFs $x_n(t)$ and a residual $r(t)$. Define an input signal $h(t)$ to be analysed. Initialise $h(t) = x(t), n = 1$ and the sifting step $k = 1$.
- For $h(t)$ identify local minima/ maxima, create the upper and lower envelopes $s_u(t)$, $s_d(t)$ and subtract their mean $m(t) = (s_u(t) - s_d(t))/2$ from $h(t)$.
- If $h(t) - m(t)$ does not fulfil the requirements of an IMF, then set $h(t) - m(t)$ as input signal and repeat the process (increase k by 1). This process is often called “sifting”.
- If $h(t) - m(t)$ fulfils the requirements of an IMF then store it as $x_n(t)$ and calculate $r(t) = h(t) - x_n(t)$. If $r(t)$ is not a residual then set $h(t) - x_n(t)$ as input and repeat from the second step, increasing n by 1. If $r(t)$ is a residual then the process ends. The original signal can be reconstructed as $x(t) = \sum x_n(t) + r(t)$

The stoppage criteria for sifting and for identifying residuals may vary. A residual typically contains only one extremum and is a constant or has a monotonic slope. Sifting is calibrated to stop at a threshold, e.g. if the variance of the input signal falls below a level, or according to the signal's energy ratio (the ratio of the energy of the signal at the beginning of sifting and the average envelope energy).

Dai and Zhu (2020) combine SoP and EEMD to find a monthly out-of-sample R2 above 20%. However, they use the frequency-decomposed parts that improve the stock return forecast and leave out those that reduce predictability. To avoid look-ahead bias, we combine SoP with EMD without removing any components. We select EMD over EEMD because, when EMD is applied to our low frequency dataset, mode mixing does not appear and therefore EEMD is not particularly advantageous. We apply EMD on each component series and get between 3 and 5 IMFs, plus the residuals. We then sum IMF2 with IMF3 and IMF4 with IMF5, and conduct (AR) predictive regressions on IMF1 and the two sums over a rolling window of 40 observations. We then aggregate the results of the predictive regressions with the last known residual for the corresponding time stamp of the original time series . This produces a set of forecasts for that component, and the process is applied to all four parts. The regression windows for EMD forecasts are 20 observations when all countries are present and 40 when they are introduced sequentially.

3.3 Data

The data frequency is quarterly and the sample period ranges from June 1973 to November 2018 containing end-of-quarter values. Due to differences in data availability between countries, we consider two different approaches. In the first case (all countries present), we limit our sample based on the country with the shortest time series (Poland) and use a 5-year (20 observations) window for the predictive regression. This creates a sample of 44 countries and quarterly time series of 99 observations, starting on April 1994. In the second case (sequential), new countries are introduced to the portfolio as sufficient data becomes available with a 10-year (40 observations) window for the predictive regression. There are 16 countries with full data availablility from June 1973[1], which amounts to 183 observations. Norway is introduced in January 1980, Sweden in January 1982, Italy and Malaysia in January 1986, and after that a new country is introduced approximately every six months until 1994 (Poland), where all are available. Descriptive statistics and the order of country introduction follow Table 1. All data comes from Eikon apart from the

[1] These countries are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Japan, the Netherlands, Singapore, Switzerland, the UK, the USA and South Africa. Brazil and Russia are not included in the sample due to lack of data.

risk-free rate of each country, which is proxied by the US 3-month Treasury bill available on FRED.

Table 15 in the Appendix presents two arithmetic examples where decomposing the return to its four constituents yields almost identical results to calculating the return directly. The first example uses artificial values and demonstrates perfect equality between total returns and the sum of the decomposed constituents. For a domestic investor, total return is exactly equal to the sum of ge, gp and dp. For an international investor, total return in USD is equal to the sum of ge, gp, dp and fx, and the difference between the two returns is exactly equal to the percentage change of the exchange rate. In the second example, UK values on January and February 1973 are used. There is a marginal difference of 0.0003 between total return and the sum of the three parts, but the difference between returns is again exactly equal to the return on FX. This demonstrates that measuring the constituents of returns separately and the returns themselves is virtually the same in terms of accuracy.

3.4 Optimisation of the international portfolio

The international investor allocates portfolio wealth to each country index i and a risk-free asset according to mean-standard deviation optimisation. The vector of stock market Sum-of-Parts forecasts is used as the expected returns in a typical Markowitz mean $-$ standard deviation minimization exercise with a risk-free asset where optimal weights are either restricted to be between zero and one or are unrestricted. For a vector of optimal portfolio weights w, a vector of forecasted (expected) returns $r_{fc,i}$,a risk-free rate r_f , risk aversion $\gamma = 2$, covariance matrix Σ and portfolio return r_p, the investor seeks to minimise portfolio variance

$$\min_{w} \frac{1}{\gamma} \boldsymbol{w'\Sigma_p w}$$

subject to the following constraints:

$\boldsymbol{w'1} + (1 - \boldsymbol{w'1}) = 1$

$E(r_p) = (1 - \boldsymbol{w'1})r_f + \boldsymbol{w'}r_{fc,i}$

$0 \leq w_i \leq 1$

The first constraint implies that all weights, risky and risk-free, sum to 1 while the third that weights are non-negative, and is omitted if short-selling is allowed. Solving this problem for each period leads to a vector of optimal weights $\boldsymbol{w}$ based on the Sum-of-Parts forecasts, which the investor uses in the allocation of portfolio wealth over the next period. When the next period arrives, the realised portfolio returns are observed. We compare the performance of the Sum-of-Parts approach with the standard CAPM case where the forecast is the historical average. The introduction of the third constraint changes the quadratic programming optimisation problem from allowing negative weights and thus short-selling, which has a known closed-form solution available in Pennacchi (2008), to prohibiting short-selling. In that case the problem can only be solved using numerical methods. We provide results on both cases with and without a risk-free asset. The covariance matrix is calculated on the same rolling window as the predictive regression but using monthly, instead of quarterly, returns for precision.

The solution for the optimal portfolio weights under a linear constraint without short-selling can only be found using numerical methods. We apply two methods to maximise the Sharpe ratio, the direct and the iterative approach, inorder to ensure that our results are not affected by numerical approximations. The direct method relies on turning the function of the Sharpe ratio into a quadratic expression and using a numerical algorithm to approximate the solution. Two possible candidates are the interior-point-convex and trust-region-reflective algorithms. The interior-point-convex algorithm proposes predictor-corrector steps that fall strictly within the constrains, after simplifying the problem if possible, and stops when an optimal solution has been found. The trust-region-reflective algorithm relies on the interior-reflective Newton method, which uses proposed consecutive neighbourhood regions of a function (trust regions) to gradually lower its value after a number of iterations. A similar alternative is the active-set algorithm. The iterative method relies on producing iterations of the efficient frontier in order to find the portfolio that maximises the Sharpe ratio. The consecutive interpolations gradually lead to an optimal solution, but the method is able to produce only local solutions and is relatively slower.

A point of note is that the covariance matrix of the sequential case is not guaranteed to be positive semi-definite for each step. Although there are no missing observations in the sample, the length of each return series is different. To construct the covariance matrix for series of unequal lengths, the correct statistical process is to calculate each pairwise covariance based on the data length of the shortest series. This may create numerical or precision errors and may lead to the first eigenvalue of the covariance matrix to be almost equal to zero but negative. When that issue appears, the nearest symmetric positive semi-definite covariance matrix in the Frobenius norm to an arbitrary real matrix A is shown to be $(B + H)/2$, where H is the symmetric polar factor of $B = (A + A')/2$ (Higham and Higham (1998)). In our case, the resulting differences in both the eigenvalues and the elements of the approximate matrix are miniscule. In practice, the results under the direct case are left unchanged if the issue is not treated but the results for the iterated method, which produces local solutions and can still function numerically with a non-positive semi-definite matrix, are very slightly altered. We consider this point to be of use to the interested reader, although it does not lead to a material change in our results or statistical approach.

3.5 Forecast performance and portfolio performance measures

3.5.1 Forecast performance measures

We use Theil's U to measure whether the Sum-of-Parts forecasted returns are an improvement compared to forecasts based on the historical average (HA) of all past returns. The statistic for country i is defined as

$$U_{i,T} = \sqrt{\frac{\sum_{t=1}^{T}(R_{it} - R_{it,SoP})^2}{\sum_{t=1}^{T}(R_{it} - R_{it,HA})^2}} \tag{4}$$

Where R is realised returns, R_{SoP} the Sum-of-Parts forecast and R_{HA} the historical average forecast. A Theil's U lower than 1 means that the Sum-of-Parts method performs better than the historical average, while the opposite means that HA provides a better forecast. The difference between Theil's U and 1 represents an improvement in percentage terms.

To determine independence of information we use the Harvey, Leybourne and Newbold (HLN) forecast encompassing test (Harvey et al. (1998)[2]. It examines two competing non-nested models and gives the optimal weight on the forecast (λ) as well as enabling the testing the null hypothesis that the optimal weight is 0. HLN is also preferable to the Diebold and Mariano (DM) test due to our relatively small sample, although both tests produce a statistic compared to a t-statistic. The test examines whether one of the forecasts encompasses all relevant information from the

[2] The Clark and West (2007) test of equal forecast accuracy for nested models has the same test statistic. However, the HLN encompassing test can be used to compare forecasts from non-nested models as well.

other. The idea is that the forecast with the worse performance may contain some information that is not fully incorporated in the better performing forecast, then a combination is preferable. However, if no such information is contained, the better performing forecast "encompasses" the worse performing one and can be used independently. The HLN encompassing test (Harvey et al. (1998)) is an evolution of the DM test (Diebold and Mariano (2002)). The DM statistic is defined as

$$DM = \frac{\bar{d}}{\sqrt{Var(\bar{d})}}$$

Where $d_t = L(e_{i,t}) - -L(e_{j,t})$, t = 1,. . . , T is a loss differential series, L(.) a loss function (e.g. mean square error) and $e_{i,t}, e_{j,t}$ are two forecast error series. Some common definitions for the loss differential d_t are $d_t = e^2_{i,t} - e^2_{j,t}$ and $d_t = |e_{i,t}| - |e_{j,t}|$. The HLN statistic modifies d_t to $d_t = (e_{i,t} - e_{j,t})e_{i,t}$ and the DM statistic as

$$HLN = T^{-1/2}\left(T + 1 - 2k + T^{-1}k(k-1)\right)^{1/2} DM \tag{5}$$

for k-step ahead forecasts and dependence between them up to lag k-1. The null hypothesis $E(d_t) = 0$ (equiv. MDM=0) is that the forecast of model i encompasses the forecast of model j. Rejecting H0 implies that forecast j stays in the forecast set.

We apply the HLN test on the collected portfolio weights for each country during the forecast period under Sum-of-Parts and Historical Average (HA). In our context, we use the test to assess country specific performance and the importance and contribution of a country to portfolio returns by examining whether a country's portfolio weight is statistically different than zero. The null hypothesis is that a country's portfolio weight is zero, i.e. nothing is invested in that country's stock index. The alternative hypothesis is that it is positive, i.e. this country generates a fraction of the portfolio's return. Specifically, the null hypothesis of the HLN test is that the SoP forecast has a weight of zero when combined with the historical average forecast. The alternative hypothesis is that the SoP forecast is not encompassed by the historical average forecast, i.e., the SoP forecast contains information above and beyond that in the historical average forecast. λ gives the optimal estimated weight on SoP and p is the test's p-value. A p-value of less than 0.10 indicates that the weight on SoP is statistically different from 0 at the 10% significance level or better when this forecast is added as a second explanatory variable to a single regression model of the historical average return. Finally, we also employ out-of-sample R^2 and mean squared error t-tests (MSE-t).

3.5.2 Portfolio performance and utility measures

We present annualised portfolio returns (R), standard deviations (SD), Sharpe ratios (Sharpe), certainty equivalents (CE) and the M2 measure of Modigliani and Modigliani (1997). Sharpe ratios are simply $E(r) - r_f/SD(r)$ for the respective portfolio. CE is also known as utility gain and is defined as:

$$CE = E(r_{SoP}) - E(r_{HA}) - (\sigma^2_{SoP} - \sigma^2_{HA}) \tag{6}$$

The second measure we use to quantify the gains to investors is the M2 measure of Modigliani and Modigliani (1997). This measure tells you how far above the actual capital market line the portfolio sits. This is very useful for comparing portfolios with different levels of risk (i.e. standard deviation). In our case this can be calculated simply as:

$$M2 = (SR_{SoP} - SR_{HA})SD_{HA} = \left(\frac{E(r_{SoP}) - r_f}{\sigma_{SoP}} - \frac{E(r_{HA}) - r_f}{\sigma_{HA}}\right)\sigma_{HA} \tag{7}$$

To examine time variation in the performance of the methods we examine two measures. Firstly, for certainty equivalent gains, we introduce focus on a Scaled Net Utility Gain (SNUG). Secondly, for and Scaled Net Cumulative Squared Error (SNCSE) as in Jordan and Vivian (2011), which belong to the family of squared error statistical measures. These measures are both looking at the performance relative to a benchmark and have the attractive feature of reaching the full sample value when the final forecast is included at time T. They also both can be interpreted as an increase (decrease) at time t indicates that the model has outperformed (underperformed) the benchmark at that point in time.

For portfolio p the Scaled Utility Gain (SUG) is defined:

$$\text{SUG} = \frac{1}{T}\sum_{1}^{t} r_{p,t} - \frac{\gamma}{2(T-1)}\sum_{1}^{t}(r_{p,t} - \bar{r}_p)^2 \qquad \text{where } \bar{r}_p = \sum_{1}^{T} r_p \tag{8}$$

The utility gain over the historical average benchmark is $SNUG = SUG_p - SUG_{HA}$, where SUG_{HA} is Equation (8) expressed for HA instead of portfolio p.

For country i:

$$\text{SNCSE} = \frac{CSE_{HA,i,t} - CSE_{SOP,i,t}}{CSE_{HA,i,T}} = \frac{\sum_{1}^{t}(r_{i,t} - \hat{r}_{HA,t})^2 - \sum_{1}^{t}(r_{i,t} - \hat{r}_{SOP,i,t})^2}{\sum_{1}^{T}(r_{i,t} - \hat{r}_{HA,i,t})^2} \tag{9}$$

4. Forecast accuracy and performance

Thus far, the forecast accuracy of Sum-of-the-Parts (SoP) and EMD methods have primarily focused on the US market. An open question is how well do these methods perform for other equity markets. In particular, firstly which method performs best internationally and secondly how well do these methods perform in emerging markets? Our empirical analysis begins by examining the forecast accuracy of the SoP method for each country in our sample denominated in US dollars. We comprehensively cover this by examining two datasets and two estimation methods. The results are affirmative. Table 2, panels (a) and (b), reports Theil's U, out-of-sample R^2 and mean squared errors for logged returns. We find that for quarterly data the SoP method performs better than the historical average for the majority of the countries. However, the most striking result is that EMD produces much better forecasts than either the original SoP approach of predictive regressions or the historical average. According to Theil's U, in the sequential case, EMD- based Sum-of-Parts outperforms the historical average in 38 countries out of 44 countries; the original SoP outperforms the historical average in 33 out of 44 countries (Table 2 a). Thus, both methods produce lower forecast errors for the vast majority of countries. However, there is a difference between the methods in terms of the magnitude of forecast gains; these are much larger for the EMD method than the original. For example, for EMD Sum-of-Parts, the greatest improvement of 28.95% is found in Pakistan (Theil's U 0.7105) whereas for the original SoP the largest gain is 2.64% in Greece. This is a consistent feature of the results as highlighted by the average improvements of 12.46% for EMD Sum-of-Parts compared to 0.96% for original SoP respectively. For the case where all countries are present throughout the sample, results are qualitatively similar. As shown in Table 2 b, EMD Sum-of-Parts performs better for 40 countries, with Pakistan improving the most in both cases (32.65% for EMD Sum-of-Parts, 6.25% for original SoP).

Out-of-sample (OOS) R^2 is vastly in favour of EMD Sum-of-Parts, with Pakistan reporting the highest OOS R^2 (54.46%) and 7 countries being above 30%, compared to a maximum OOS R^2 of 5.72% (UK) and 5 countries above 3% for original Sum-of-Parts in the sequential case. When all countries are present, the results improve further. The maximum OOS R^2 for EMD Sum-of-Parts is 56.60% (Pakistan) with 6 countries having values above 40%, while for original Sum-of-Parts the maximum is 10.66% (Pakistan) with three countries above 10%. This implies substantial gains for both forecasting methods which are, however, vastly greater when EMD is used. In the sequential case, EMD Sum-of-Parts leads to 38 countries with positive OOS R^2 compared to 25 for the original case, demonstrating an improvement in forecasting performance for the vast majority, while with all countries present the respective numbers are 40 and 35.

Forecast accuracy is tested via a one-sided t-test for mean squared errors (MSE-t test).[3] The test assesses whether the forecast error from Sum-of-Parts is smaller than the historical average for each country in the sample. In terms of statistical significance, the MSE-t tests reveal that for EMD (original) Sum-of-Parts there is statistical outperformance in 26 (5) countries at the 10% significance level for the sequential case and 30 (12) for all countries present. However, this, at least partly reflects the well-known lack of power for this test which is unfortunately an issue that has not yet been resolved in the context of non-nested models. Differences in development or geographical location do not seem to play a role.

Is it optimal to place a positive weight on the SoP forecast? Table 3, Panels (a) and (b) contain the results for the HLN encompassing tests. The results for the portfolio weights under SoP forecasts are designated by (λ_1, p_1), and for the portfolio weights under the historical average benchmark by (λ_2, p_2). For the sequential case under EMD Sum-of-Parts (Panel a), the optimal weight λ_1 is positive for all countries. Further, the optimal weight is greater than 0.5 in 38 of the 44 countries, which signifies more weight on the SoP forecast than the benchmark. The encompassing test results report that the weight on the SoP forecast is statistically different from 0 at the 10% level for all 44 countries (p_1). By contrast, for the historical average forecast, the optimal weight λ_2 is statistically significant only for 20 countries ($p_2 < 0.1$) and greater than 0.5 for 6. Three countries report negative weights, indicating that for those countries there is no value to following the historical average forecast at all. Original Sum-of-Parts for the sequential

[3] This is similar to the MSE-F tests used in Vivian and Wohar (2013) amongst others, however, MSE-F can only be applied to nested forecasts.

case still performs better than the historical average. 17 countries report statistically significant weights at the 10% level and 25 report weights above 0.5, while for the historical average there are 19 countries with weights above 0.5, 8 with statistically significant and 9 with negative weights.

The results are stronger in the case where all countries are present (Panel b), where the improvement for original Sum-of-Parts is considerable. 35 countries now report weights above 0.5, 20 are statistically significant and only 2 have negative weights, while for the historical average 9 countries are above 0.5, 7 are statistically significant and 22 have negative weights. Thus, overall, our results suggest that the SoP forecasts are greatly preferred to the historical average forecasts when EMD is used. However, for many countries the difference is not statistically significant. Consequently, how well these models perform in portfolio allocation tests will be of great interest to see how these competing approaches compare from the perspective of a real-world investor looking to allocate their portfolio across countries.

Overall, the statistical analysis of the SoP forecasts demonstrates substantial, robust improvement which makes them suitable for further analysis. We can now focus on their performance when they are used as expected returns in a mean-variance optimal portfolio.

5. Global Asset Allocation

5.1 Portfolio performance for a US-based investor

The key question of this paper is how valuable return forecasts are to an investor looking to allocate funds across different countries. The main baseline empirical results are presented in Table 4. This provides the portfolio performance and economic value to an investor based in the US (henceforth our base case, or equivalently using the US dollar in denomination) for investing in a global portfolio according to the SoP method and comparing this to the historical average benchmark. The main measures reported are annualised Certainty Equivalents, Sharpe ratios, M2, portfolio returns and standard deviations. These are presented for the 1973-2018 (sequential) and 1994-2018 (all countries present) sample periods. Portfolio weights for the SoP methods are generated under two schemes.

For the constrained case (no short-selling is allowed), it is clear that the SoP method leads to improvements in portfolio performance. For example, for the 1973-2018 sample, all SoP-based portfolios have higher returns than the HA benchmark. This leads to substantial increases in Sharpe ratios even though portfolio standard deviation also increases. The Sharpe ratio for the HA over 1973-2018 is 0.295 which increases when the original SoP method is implemented by the iterative (direct) approach to 0.490 (0.330). However, the performance of the EMD SoP method is even more striking; this leads to a Sharpe ratio increase to over 1. For the iterative (direct) approach the Sharpe ratio is 1.242 (1.093) using EMD SoP. This is a very large increase in performance.

How beneficial is this to investors? Firstly, we consider the certainty equivalent gain (CE) defined in (6) to a mean-variance investor from using the SoP method rather than the benchmark. We see here that all SoP methods lead to substantial gains for the 1973-2018 sample. Annualized gains are highest for EMD SoP at 8.80% (3.95%) for the iterative (direct) approach, which are of very large magnitude. For the original SoP method annualized gains are more modest at 1.96% (0.49%) for the iterative (direct) approach but still offering clear benefits to a mean-variance investor. The second measure we use to quantify the gains to investors is the M2 measure of Modigliani and Modigliani (1997) defined in (7). We can see that the original SoP method offers modest annualized gains according to the M2 measure of 0.11% - 0.62%. However, substantial gains of 2.51% - 2.98% are given by the EMD SoP method. Thus, the EMD SoP approach should be of great interest to a global investor looking to enhance their strategic asset allocations. For the benchmark portfolio we use the Sharpe ratio and standard deviation of the constrained case to capture the gains under low SD compared to the much more volatile All-Equity HA.

The lower section of the same table for the US presents results for the balanced panel period where all 44 countries are available throughout the sample. This is data available from 1994 with forecasts made from 1999 and the sample ending in 2018. These results are broadly similar to those which use the maximum time span of data available (1973-2018). Specifically, all SoP models generate higher Sharpe ratios than the benchmark, thus they all generate positive M2 gains and utility gains. There are two main differences. First, the original SoP models both generate clearly higher Sharpe ratios than the benchmark and thus both now have M2 gains over 0.6%. Second, the EMD SoP direct approach Sharpe ratio drops below 1, albeit remaining above 0.9 and its utility gains drops below 2%. The drop in utility gain is most likely due to the reward from reducing variance relative to the benchmark not being strong. Linked to this point, the OR SoP iterative approach has slightly higher return than the EMD SoP direct but much higher risk (variance) but they both have similar utility gains reported. The key implication of these results is that a US based investor can clearly benefit from using forecasts to guide their international investment especially when EMD SoP is employed.

5.1.1 US results - short-selling allowed

Next we look at the US unconstrained portfolia with short-selling allowed (Table 4 Panel b). These portfolia generally have higher returns and also higher risk than the portfolia which are unable to take short positions. The results are again greatly in favour of EMD SoP; over the 73-18 sample, the Sharpe ratios for both the iterative and the direct portfolios are increased compared to Panel a where short-selling is prohibited. The direct (iterative) method now produces a Sharpe ratio of 1.56 (1.33) compared to 0.22 for the original SoP and a negative ratio of -0.07 for HA. In terms of gains over the benchmark, the original SoP generate M2 gains of 3.0% and utility gains of almost 3%. EMD SoP performs much better with M2 gains of at least 14.4% and utility gains of at least 19.8%; these gains are really striking and provides compelling evidence in favour of EMD SoP.

EMD SoP performs even better over the 94-18 sample. For this balanced sample period it generates portfolio with risk similar to that of the historical average benchmark, but with much higher returns than the benchmark. Consequently, the Sharpe ratio increases to over 2 and is even close to 3 for the Direct EMD SoP portfolio. This translates into M2 and certainty equivalent gains that remain strongly positive. However, the historical average benchmark does better in this period as well generating a positive Sharpe ratio of 0.52. The benchmark actually outperforms the original SoP method over this sample. Thus, the performance of EMD SoP appears to be more robust and consistently strong compared to original SoP.

5.1.2 US results - no risk-free rate allowed

Next we consider the situation where the fund manager can only invest in risky assets. There are two reasons for this. Firstly, investment managers may face a mandate where they are required to hold all or almost all the portfolio wealth in risky assets aligned with the fund's objectives. Hence, holding a substantial weight in the risk-free rate is not feasible for them and could have serious consequences for them if they were to do so. Secondly, the constrained case with positive only weights in all assets lead to portfolio returns which were moderate albeit with high Sharpe ratio for EMD SoP. This appears to be because the portfolio allocated substantial weight to the risk-free rate on average. What would happen if investors were not able to invest in the risk-free rate? Could high Sharpe ratios still be obtained with annualised returns that are more aligned to equity market norms?

Table 4, Panel c reports the results for all-equity portfolia where short-selling is not allowed (constrained, C) and where it is allowed (unconstrained, UC). For all portfolia, we see increases in return and standard deviation in Panel c of Table 4 compared to the case where a risk-free rate was available (Panel a). Over the 73-18 sample, the historical average benchmark is more in-line with a standard all-equity portfolio with a (nominal) return of 10.78% and standard deviation of 18.53%, thereby generating a Sharpe ratio of 0.39. Both SoP methods increase return per unit of risk compared to the benchmark. The original method generates a Sharpe of 0.49. via an increase of return of 2.35% at the expense of an increase in standard deviation of 1.01% compared to the benchmark. The EMD SoP method generates a Sharpe ratio of 1.24 but due to an strikingly high annualized return of 38.50% p.a. at a standard deviation of 28.21%. Thus, it appears that the EMD SoP is very effective at identifying the countries with high projected returns and allocating wealth to them. The M2 measure is 3.0%, indicating substantial gains to an investor. The utility gain is an extremely large 23.20%, which is far higher than what is conventionally achievable. As mentioned previously, the utility gain measure does not appear to place sufficient penalty on risk compared to that actually witnessed in the market, which corresponds to the ex-post capital market line.

5.1.3 The effect of optimisation algorithm selection

There is not a definitive pattern on whether the direct or iterative method for calculating optimal portfolio weights is preferable. However, the iterative method appears to have an advantage in the absence of short-selling. Across the entire Panel a (constrained portfolia) the iterative method produces higher Sharpe Ratios, some times substantial, for both EMD SoP and OR SoP forecasts. In Panel b (unconstrained portfolia), the direct method outperforms the iterative method for EMD but there is no clear pattern for OR SoP. On the other hand, a comparison between including a risk-free asset and having an all-equity portfolio is revealing. For constrained portfolia, investing in a risk-free asset or not does not affect the Sharpe Ratio in the US - the respective values for the iterative case in Panels a and c are very similar for both samples.

5.2 Portfolio performance for different home countries

This far, it has been demonstrated that a US investor could greatly benefit from applying the (EMD) SoP approach. Can investors based in other countries also benefit from applying this method? In other words, are the empirical results sensitive to the choice of US, or any other option, as home country? To investigate this important issue, we conduct the same estimations using 8 alternative home countries. The include 4 developed markets (UK, Germany, Japan, Switzerland) and 4 emerging markets (South Africa, India, China and Chile). The results in Table 5 show that the EMD SoP method performs very well regardless of the domicile of the investor. Broadly, the results are qualitatively similar, although there is moderate variation in some of the magnitudes of the effects. In short, there is strong evidence in favour of the EMD SoP method which generates a Sharpe ratio of 0.8 or higher in almost every case.

Firstly, in terms of the HA there is large variation especially in (nominal) returns depending on the domicile of the investor as shown in Table 5 Panel A. For the 73-18 developed sample, (nominal) returns vary from 2.49% in Japan to 6.18% in the UK; however this is at least partly due to inflation and the (nominal) risk-free rate varying across countries also. The standard deviation of the benchmark only varies from 3.15% in the US to 3.84% in Switzerland. Thus, overall, there is substantial variation in the Sharpe ratio of the HA benchmark as the home country changes from 0.21 in the UK to 0.38 in Japan. Secondly, for the 4 developed markets over 73-18 the original SoP generally provides improvements in terms of performance compared to the benchmark. The notable exceptions are for Japan and Germany when the direct approach is used; for these two cases there is a reduction in Sharpe ratio and therefore M2 even though there is a positive utility gain. Thirdly and most striking, the EMD SoP consistently outperforms the other methods by a large margin. Using the direct method the EMD SoP over 73-18 has a Sharpe ratio of between 0.86 and 1.09, offering M2 gains of over 2% and Utility gains of over 3.8%. The EMD SoP iterative method performs even better; over 73-18, the Sharpe ratio is between 1.11 and 1.24, with M2 gains of over 2.5% and utility gains of over 8%. The results are broadly qualitatively similar over the 94-18 sample. However, there is more variation in performance across countries. Again, for almost all home countries the original SoP method outperforms the benchmark. In the case of China, the Sharpe ratio is negative but is less negative than for the benchmark. The EMD SoP again excels especially for the iterative method generate

Sharpe ratio of at between 0.79-1.34, M2 gains of at least 2.7% and utility gains of at least 6.7% typically. Thus, the method would have been highly beneficial to an investor looking to invest internationally regardless of their home country.

This markedly homogeneous pattern does not change in Table 6 and Table 7, which contain the all equity and unconstrained cases respectively. In Table 7, although there is greater variation with even higher values appearing, EMD Certainty equivalents are around 25% while OR SoP CE are 5-10%. However, in the 94-18 sample, the original Sum-of-Parts method is sometimes outperformed by the historical average. US, UK, Germany, Japan, South Africa, India and Chile report negative Certainty Equivalents and Sharpe Ratios that are below those of the Historical Average. Notably, EMD vastly outperforms both, with many Sharpe Ratios being close to 3. Table 6 reports the greatest divergence between EMD SoP and OR SoP. While, as noted earlier, the Constrained results are very similar to Table 7, the All Equity case (Table 6) reports staggering differences in Certainty Equivalents. All sequential EMD Certainty Equivalents range between 50 and 75%, and Sharpe Ratios of 1.5 to 2, while OR SoP reports CE of 4-8% and SR around 0.5. The 94-18 sample reports even higher respective values. EMD Certainty Equivalents are now between 75-100%, with China having a 282% value, while Sharpe ratios are often 2.5-3. OR SoP results are also increased but remain incomparable.

The quality of SoP forecasts is verified in Table 8, which contains Theil's U statistics for all the economies that are considered as home countries. The values for minimum, maximum and average improvement and the number of countries where the SoP forecast outperforms the historical average for EMD SoP are very similar to those in Table 2. For the original method, the averages are similar for most cases but for the 94-18 sample China, the UK and Germany report improvement for slightly fewer countries compared to the US. Apart from acting as a successful robustness test, our results show that the Sum-of-Parts method is applicable to countries outside the US. Although investors located in both developed and developing economies experience economic and performance gains, it is notable that non-Western home countries perform comparably, if not better, to their Western counterparts. This observation becomes more apparent across tables 5,6 and 7, where CE and SR values are quite similar and often outperform the US, the UK, Germany, Japan and Switzerland.

6. Further analysis and robustness checks

This section provides additional analysis on a range of pertinent issues. It serves to examine how our main results are impacted by adjusting the way our allocation exercise is implemented. We address the following questions here. Firstly, we consider investor preferences and what impact there is from allowing the investor to be conservative than we have modelled so far. Secondly, we examine whether including only one type of market substantially the main implications from our study. Thirdly, we provide insights into which countries have substantial allocations to them and when; this enables us to assess how widespread allocations across countries are and an indication of how much the portfolio adjusts. Fourthly, we provide further evidence on whether there is time variation in the effects we consider. Fifthly, we examine an alternative frequency of data. Finally we consider how much each component contributes to variation in the return.

6.1 Risk aversion

Thus far, we have focused on a single level of risk aversion, i.e. $\gamma = 2$. What if investors are actually more risk averse than we have considered thus far? To address this issue, Table 9 conducts the same exercise but with risk aversion $\gamma = 3$ and $\gamma = 5$ for the illustrative case of the US-based investor for the constrained and unconstrained portfolia. Naturally the portfolio returns and standard deviations are reduced with a higher level of risk aversion. This is because the weights in risky assets are reduced since risk aversion appears in the denominator of the portfolio allocation formula. Most importantly, however, the Sharpe ratios for EMD SoP are consistently substantially higher than the historical average benchmark and in almost all cases above 1. Consequently, large gains are feasible. According to M2 these are all above 1% and even greater when short-selling is allowed. Utility gains for EMD SoP with the iterative method are at least 5% when $\gamma = 3$ and at least 3.26% when $\gamma = 5$. Thus, although these do reduce somewhat as risk aversion increases they still remain substantial. Thus, our main findings are remarkably robust to adjusting the level of risk aversion. For the original SoP method the results also are similar as the level of risk aversion is increased. Generally it does improve over the HA benchmark. However, the improvements are of more moderate magnitudes. For the unconstrained case where short selling is allowed, however, it underperforms for the 94-18 sample.

6.2 Developed only and emerging market only portfolios

In our analysis we have allowed the investor to select across whichever countries are available. However, over our sample period there is an expansion of countries especially from emerging markets. It also tends to be the case that emerging markets experience greater fluctuations in value than developed ones but that developed indices tend to be more highly correlated with each other. This has lead to mutual funds being established with a developed market only remit or an emerging market only remit. A natural question is therefore, how our proposed methods perform in these differing settings. Are our main results robust if we confine the investor to only investing in one of these groups of countries?

We calculate the same portfolio performance measures as earlier after dividing the countries into developed markets (DM) and emerging markets (EM)[4]. For brevity, Table 13 only reports the results with the US dollar as base currency, but the measures are remarkably robust to the choice of home country. Several main points emerge. Firstly, it is very clear from Table 13 that the SoP methods consistently outperform the HA benchmark regardless of whether investment is directed to developed markets or to emerging markets. Secondly, the EMD SoP performs best. Thirdly, generally we see that emerging market only investment leads to strongly higher returns, higher risk and generally higher Sharpe ratios than developed market only investment. The only exception is where short-selling is allowed and this leads to a Sharpe ratios which is similar to the emerging market. The gains appear to be higher when investing in emerging markets only, which are driven by the higher returns generated by these portfolia. Finally, it appears that the performance of the emerging markets only portfolio over 94-18 is similar if not slightly better than the all country present case (Table 4), but that the developed market only performance is weaker than this. This highlights the importance of including emerging markets into strategic asset allocations; an international investor who excludes emerging markets would on average gain substantially less than one who embraces them.

[4] According to MSCI, DMs include Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Japan, Netherlands, Singapore, Switzerland, the UK, the USA, Norway, Sweden, Italy, Spain, New Zealand, Finland, Portugal, we also add Luxembourg, and Israel (24 countries), while EMs include South Africa, Malaysia, Thailand, Sri Lanka, S. Korea, Taiwan, Philipinnes, Chile, India, Greece, Turkey, Mexico, Hungary, Pakistan, Cyprus, Colombia, China, Czech Republic, Peru and Poland. We generally follow MSCI but add Luxembourg to DMs and Sri Lanka, Pakistan and Cyprus to EMs for completeness. The order here refers to the ordering of the countries in the sample.

6.3 Portfolio weights and heat maps

The encompassing tests showed the EMD Sum-of-Parts to produce statistically significant portfolio weights. It is, however, important to explore further whether there are any patterns or characteristic features in the different samples we have explored, which would signify the presence of trends. This becomes very important when short-selling is allowed.

Table 12 Panels G7 and E7 provide statistics on portfolio weights for a US based investor into the major developed (G7) and emerging (E7) economies. Interestingly, the mean and median weights are clearly positive for US and France but clearly negative for Germany, Italy and Japan. The standard deviations of the weights are also substantial suggesting there is a lot of adjustment by the EMD approach over time. This is also apparent from the quantile information. For most of the G7 there are large differences between the lower quartile and upper quartile weights. For the US 25% Q is -34.39% and 75% Q is 32.46%. This is typical for other G7 countries except for Japan and Italy which have narrower ranges of weights; Japan's, for example, go from 25% Q is -13.64% and 75% Q is 5.95%. From Panel E7, it is clear that the weights in emerging markets generally have a narrower range than their developed counterparts. The standard deviations of the weights are all lower than for the G7 countries. Further, for Korea and Turkey the weights range is especially narrow from about -2.7% for the lower quartile to about +3.5% for the upper quartile. The mean and median for all the E7 markets is close to 0. These lower weights in general is likely due to these countries returns in generally have higher variation, which leads to lower weights when the portfolio allocation formula is applied.

Figures 2 and 3 present the results for a US-based investor for different samples and portfolia. The results, again, proved to be robust to different home countries, risk aversion and other factors, so we focus on the most comprehensive case of the US dollar with short-selling for the full sample (44 countries), DMs (24 countries) and EMs (20 countries) for the sequential (73-18) and all countries present (94-18) cases with and without (all equity) a risk-free asset.

Apart from a general tendency to invest more in large markets rather than emerging economies when all 44 countries are present, there does not seem to be a discernible pattern. The allocations, both positive, and negative are greater in DMs than EMs and become even greater when a risk-free asset is not present. This tendency can be explained by the larger volatilities in emerging markets, which lead to lower respective weights in favour of developed markets. However, a closer examination between the DM and EM samples does not show any particular country bias. Investment positions are spread all across the period and countries when volatilities are comparable.

An important observation is the ability of the EMD method to follow observed patterns in financial markets. Figure 3, Panel B presents two characteristic cases that demonstrate a remarkably the weights for the USA and Greece taken from the 94-18 DM and DE sub-samples respectively. The dot com bubble, the 2008-2009 financial crisis and the bull market of the last decade appear quite prominently, which shows a clear adaptation to market trends. The match is even better for Greece. The four periods of large short-selling in blue and purple correspond to the 2008 financial crisis, the country's bailouts in 2009-2010, the 2012-13 debt crisis as well as the political crisis, elections and referendum period in 2014-2016, while the 2013-14 period in red matches the significant increase of the country's stock market index at that time.

Table 12, where descriptive statistics and quantiles are reported for selected cases, completes the picture of the composition of EMD SoP portfolia when short-selling is allowed. The statistics are calculated on the 94-18 period with (ACP) and without (all equity) a risk-free asset as sample averages and averages-of-averages for each of the 9 home countries, the 24 developed markets (DMs), the 20 emerging markets (EMs), the 7 largest developed markets (G7) and the 7 largest emerging markets present in our sample (E7).

6.4 Time Variation in Performance.

Thus far, we have seen very strong performance from the EMD SoP measure and that this is robust across two sample periods. However, how consistent is this performance over time, especially at a more granular level? To examine this question we look at the performance of the utility gain measure of time and the forecasting performance over time. Specifically we consider a Rolling Utility Gain (RUG) measure for the former and the Scaled Net Cumulative Squared Error (CSE) (Jordan and Vivian (2011)) for the later. They belong to the family of squared error statistical measures and refer to portfolios (Equation 7) and countries i (Equation 6).

In terms of the rolling utility gain, we see from Figure 4 that there is consistent outperformance of the benchmark by EMD SoP. EMD (constrained) demonstrates a much steadier, smoother and scaled net utility gain (SNUG) of up to 6% compared to the original SoP which generates gains of between 0.6% and 1.2%. With short-selling allowed, the EMD gains are up to 20% while original Sum-of-Parts fluctuates much more for practically zero end-of-period gains. Moreover, the trajectory of the rolling gains are strongly upwards indicating that throughout the sample the EMD approach is outperforming the benchmark. There are no obvious periods of underperformance for EMD, while in contrast, the original SoP appears to underperform including during the financial crisis period.

In terms of forecast performance, we report the average and median of the measure across the sample of countries. This is reported in the Scaled Net CSE graphs (Figure 5). The results suggest that there is consistent outperformance of the benchmark by the EMD SoP method. The trend in the graphs is clearly upward. The only period of substantial underperformance occurs around the time of the Global Financial Crisis (2007-2009). However, an upward trend espeically in terms of median is apparent again soon afterwards. For the original SoP method forecast performance is much more mixed and time-varying. Performance is good in the early 2000s but weak during the Financial Crisis (2007-2009). The magnitudes of the gains and losses are also much smaller.

The findings again demonstrate a clear advantage both in terms of utility and in terms of forecast accuracy for a USD-based investor that uses EMD forecasts over both the historical average and the original method.

6.5 Data frequency

The analysis of this paper thus far has been on quarterly data. The seminal paper of Ferreira and Santa-Clara (2011) report an out-of-sample R^2 improvement of 13.4% with annual data and 1.3% with monthly data. It is natural to check the robustness of our results to an alternative data frequency. With annual data there would be insufficient observations for appropriate analysis to be conducted given that data is only available since 1973 at best. Thus, we examine using monthly data, even though we anticipate for it to be more challenging to capture reversion in earnings multiples at this frequency. with monthly data and see if the qualitative patterns we identify change. Table 14 contains the collected results on the US Theil's for monthly data (Sharpe ratios, Certainty Equivalents, M2, returns, standard deviations) and is directly comparable to Table 4. There is not a significant difference for EMD Sum-of-Parts. 37 countries beat the historical average, with a maximum improvement of 17.6% and a mean improvement of 8.91%. The results are weaker for original Sum-of-Parts, where only 15 countries perform better than the historical average in the 73-18 period.

How are the portfolio allocation results impacted by the switch to monthly frequency data? We consistently find the CE and SR values are comparable to the earlier quarterly results presented in Table 4. This is the case for both the sequential and the all countries present portfolia. In almost all cases the Sharpe ratios remain above 1 with substantial utility gains feasible. Thus, data frequency does not materially affect portfolio performance for EMD SoP. For original SoP, where only 15 countries perform better than the historical average in the 73-18 period. However, Sharpe ratios and Certainty Equivalents are higher than those under HA forecasts, implying that an improvement even in a small cluster of countries can be beneficial. As earlier, the all-equity portfolio performs best.

In some ways it is surprising that the EMD SoP method continues to work so well with monthly frequency data. This is because the valuation multiple growth is perceived to be an important driver of the overall predictability of the SoP approach. However, reversion in valuation ratios is very gradual and thus should be more difficult to detect in monthly data than in quarterly. This naturally leads to the question of how much does each component contribute to the overall predictability of the forecast[5].

6.6 The contribution of each part to forecast performance

An important issue and robustness check is to determine what drives the considerable improvement in performance, particularly when Sum-of-Parts is combined with EMD. We are particularly interested in the contribution of the foreign exchange growth rate to the total forecast, since this is our technical extension. A high contribution would provide strong evidence of our method's ability to better forecast and utilise changes in the exchange rate, a feat that has remained challenging. We adopt three approaches and focus on the sequential case, since it provides the most data. The first is to conduct a set of linear regressions with the total Sum-of-Parts

[5] For the concerned reader, we annualise quarterly and monthly Sharpe ratios by multiplying by $\sqrt{4}$ and $\sqrt{12}$ respectively. According to Lo (2002), this is appropriate only for iid returns and can lead to significant discrepancies. A series of AR(1) estimations showed very low autocorrelations in realised portfolio returns, which makes our approach reasonably suitable. At any rate, the monthly and quarterly Sharpe ratios can be easily inferred.

forecast as dependent variable and all parts but one as independent variables. Since a regression with all four parts present would provide an R^2 of 1, "leaving one [part] out" and calculating $1 - R^2$ for that regression (akin to Shapley regressions) would show how important that part is to the total forecast. The second approach is to calculate Theil's U for each part in each country using the part's historical average as benchmark. This intends to show how much the forecast of each component improves. The third is estimating a 4-dimensional VAR model with the total Sum-of-Parts forecast and three out of four parts forecasts, and then examine their Forecast Error Variance Decompositions (FEVD). The Generalized FEVD represents the contribution to the forecast error variance of equation-wise shocks to the response variables in the model. When the SoP forecast is shocked by one standard devation, the contribution of forecast error of the specified part settles at (converges to) the long-run value. Similarly, when a part is shocked by one standard deviation, the contribution of forecast error of the SoP forecast settles at the long-run value. To avoid singularity in the covariance matrix of the VAR (since the total forecast is a linear combination of all other variables), we omit DP because is the part with the lowest contribution according to our earlier findings. We also opt for Generalised instead of Orthogonalised FEVD due to the large sensitivity of the latter to the ordering of variables in the VAR, which makes interpretation difficult given the large number of sub-cases. The drawback of Generalised FEVDs not summing to 1 is small.

Table 10, Panels (a) and (b) contain our results for the "leave-one-out" regressions for EMD and original SoP in the sequential and all countries present cases. Each column reports $1 - R^2$ for the regression where that part is missing. It is evident that the part with the lowest contribution is the dividend yield, which motivates us to omit it from the VAR later. The part with the highest contribution is GM, with a country average of 0.66 in the sequential case and 25-30% in the original method, which agrees with Ferreira and Santa-Clara (2011). The respective contributions of GM are 45% and 8-15%, while FX contributes by 15% and 4-6%. Table 11, Panels (a) and (b) corroborate our findings for the parts' Theil's U. The part with the highest improvement is DP, while the other parts improve by 20-25%. These findings show that FX plays a significant but not central role to the quality of the total forecast.

We estimate the VAR for every country over 100 periods and collect each FEVD. Figure 1, Panels (a) and (b) report the average FEVD of each part forecast if there is a shock on the Sum-of-Parts forecast. Similarly, Panels (c) and (d) report the average FEVD of the Sum-of-Parts forecast if there is a shock in the respective forecast of a part. In both cases, the results are the same and are not significantly affected by the number of lags. The GM forecast converges to roughly 0.35, the GE forecast to 0.13 and the FX forecast to 0.2. Although the most important part is again GM, now FX is clearly the second most important forecast component. To conclude, the main driver of our results seems to be not only an overall improvement in forecast performance for each part but the method's ability to utilise the foreign exchange growth rate effectively.

7. Conclusion

This paper primarily investigates whether forecasting of international stock returns is beneficial to an investor with a global mandate. Firstly, we demonstrate that the sum of the parts method can be easily augmented to suit an international investment setting. Specifically, returns are decomposed into four components rather than three, with the foreign exchange rate return added to earnings growth, the dividend yield and the change in price-earnings ratio. Secondly, we examine whether stock returns can be forecasted in each of the 44 countries in our sample. We find in general that the sum of the parts method can lead to improved forecasts especially when empirical mode decomposition (EMD) is used. In that case, the vast majority of portfolio weights is statistically different than zero, while in both cases the investor's home country does not seem to play a role. Thus, important gains can be achieved by non-US dollar investors. Thirdly, we examine our key question of whether return forecasts can be used in real-time portfolio allocation by an investor with a global remit and whether this improves performance over using the historical average benchmark. We demonstrate that substantial gains are possible both in terms of the economic value and in terms of portfolio performance metrics and the individual forecasting of each component provides a substantial improvement in the performance of an international portfolio under mean-variance optimisation. Specifically, EMD Sum-of-Parts forecasting performs much better than the historical average forecast. We report Sharpe Ratios consistently above 1 in many cases under EMD, a considerable improvement under the original Sum-of-Parts method. Certainty equivalents and M2 provide similar implications regarding model performance, with certainty equivalents consistently above 20% above 100% in some cases. Our results are not driven by a single component; in fact EMD has the ability to improve the forecast performance of each part. Specifically, the FX growth rate is the second most important component of the aggregate Sum-of-Parts forecast; this highights that FX movements are potentially important to an international investor and that the proposed approach can help improve forecasts of it. Our main conclusion is that by using the EMD Sum-of-Parts approach that substantial gains are feasible to a global investor regardless of which country they are domiciled in.

Tables and Figures

	Mean	SD	Min	Max	Obs
Stock return	0.0210	0.1418	-0.9808	0.9333	6223
FX return	-0.0041	0.0539	-0.5891	0.2082	6223
DP ratio	0.0077	0.0049	0	0.0952	6223
Earnings growth rate	0.0184	0.1631	-2.4258	2.7988	6223
PE growth rate	0.0010	0.2041	-2.7629	2.4036	6223
PE ratio	2.6694	0.4443	-0.5108	4.7095	6223

(a) Descriptive statistics

Australia	TOTMKAU	Sri Lanka	TOTMKCY
Austria	TOTMKOE	South Korea	TOTMKKO
Belgium	TOTMKBG	New Zealand	TOTMKNZ
Canada	TOTMKCN	Finland	TOTMKFN
Denmark	TOTMKDK	Taiwan	TOTMKTA
France	TOTMKFR	Philippines	TOTMKPH
Germany	TOTMKBD	Chile	TOTMKCL
Hong Kong	TOTMKHK	India	TOTMKIN
Ireland	TOTMKIR	Greece	TOTMKGR
Japan	TOTMKJP	Portugal	TOTMKPT
Netherland	TOTMKNL	Turkey	TOTMKTK
Singapore	TOTMKSG	Mexico	TOTMKMX
Switzerland	TOTMKSW	Hungary	TOTMKHN
UK	TOTMKUK	Luxembourg	TOTMKLX
USA	TOTMKUS	Pakistan	TOTMKPK
S. Africa	TOTMKSA	Cyprus	TOTMKCP
Norway	TOTMKNW	Israel	TOTMKIS
Sweden	TOTMKSD	Colombia	TOTMKCB
Italy	TOTMKIT	China	TOTMKCH
Malaysia	TOTMKMY	Czech Republic	TOTMKCZ
Thailand	TOTMKTH	Peru	TOTMKPE
Spain	TOTMKES	Poland	TOTMKPO

(b) Country Datastream/ Eikon RICs

Table 1: Descriptive statistics and data codes

Note: All values are reported in logs with quarterly frequency between June 1973 and November 2018 (where available). Country observations range between 183 for the whole period and 99 for Poland, which has the shortest sample. The codes refer to the price index in local currency.

	TU	OOS R^2	MSE-t	p MSE-t	TU	OOS R^2	MSE-t	p MSE-t
	SEQ OR SoP				SEQ EMD			
Australia	0.9997	1.52%	0.5290	0.2980	0.8822	25.97%	1.5530	0.0600
Austria	0.9978	1.04%	0.4580	0.3230	0.8925	17.97%	0.9770	0.1640
Belgium	0.9883	1.86%	0.6010	0.2740	0.9795	9.13%	0.5840	0.2800
Canada	0.9999	1.24%	0.4120	0.3400	0.8730	25.15%	1.9020	0.0290
Denmark	1.0086	-1.22%	-0.3090	0.6210	0.9835	8.59%	0.4650	0.3210
France	0.9936	2.19%	0.8800	0.1890	0.8456	28.50%	2.3510	0.0090
Germany	0.9926	1.72%	0.7220	0.2350	0.8933	20.37%	1.4950	0.0670
Hong Kong	0.9844	3.45%	1.4480	0.0740	0.8762	24.29%	1.6530	0.0490
Ireland	0.9979	2.19%	0.6400	0.2610	0.8961	24.11%	1.5190	0.0640
Japan	1.0006	1.74%	0.4540	0.3250	0.8254	31.42%	2.3670	0.0090
Netherlands	1.0027	0.89%	0.2480	0.4020	0.8826	26.22%	1.3930	0.0820
Singapore	0.9941	0.98%	0.3130	0.3770	0.9850	4.92%	0.2850	0.3880
Switzerland	1.0265	-3.69%	-0.9390	0.8260	0.8445	28.97%	2.4110	0.0080
UK	0.9765	5.72%	1.9890	0.0230	0.8683	27.12%	1.9190	0.0270
US	0.9894	2.94%	1.0080	0.1570	0.9101	18.33%	1.1730	0.1200
South Africa	0.9882	4.44%	1.4410	0.0750	0.8441	30.00%	3.1100	0.0010
Norway	1.0150	-2.15%	-0.6360	0.7380	1.0028	2.53%	0.1460	0.4420
Sweden	0.9952	-0.30%	-0.1240	0.5490	1.0024	-4.42%	-0.2200	0.5870
Italy	0.9938	-0.41%	-0.1150	0.5460	0.9189	16.25%	1.0910	0.1380
Malaysia	0.9897	-2.45%	-1.1060	0.8660	0.8996	21.37%	1.1620	0.1230
Thailand	0.9854	0.10%	0.0290	0.4880	1.3717	-7.62%	-0.2690	0.6060
Spain	0.9985	-1.13%	-0.4740	0.6820	0.8045	34.28%	2.5510	0.0050
Sri Lanka	0.9942	0.78%	0.2760	0.3910	0.8618	25.64%	1.7460	0.0400
South Korea	0.9973	-0.32%	-0.1090	0.5430	0.8146	24.20%	1.4120	0.0790
New Zealand	1.0291	-4.97%	-1.8120	0.9650	0.8446	31.78%	2.6580	0.0040
Finland	0.9988	0.10%	0.0520	0.4790	0.8761	21.97%	1.3100	0.0950
Taiwan	0.9804	-1.75%	-0.3410	0.6340	0.8654	20.72%	1.2340	0.1090
Philippines	0.9924	1.40%	0.5740	0.2830	0.8588	28.23%	1.9860	0.0240
Chile	0.9864	-0.56%	-0.1380	0.5550	0.8049	32.37%	2.0690	0.0190
India	0.9779	2.57%	1.6420	0.0500	0.7775	39.70%	3.4310	0.0000
Greece	0.9736	-2.54%	-0.9000	0.8160	0.7468	44.82%	3.1190	0.0010
Portugal	0.9990	-0.15%	-0.0540	0.5220	0.8877	21.45%	1.2920	0.0980
Turkey	0.9789	2.39%	0.7240	0.2350	0.9047	22.12%	1.4500	0.0740
Mexico	0.9961	-2.05%	-0.7340	0.7680	0.8844	22.79%	1.5480	0.0610
Hungary	0.9925	0.91%	0.2700	0.3940	0.8814	29.20%	1.7020	0.0440
Luxembourg	1.0188	-5.87%	-0.8800	0.8110	1.6211	-120.10%	-3.1380	0.9990
Pakistan	0.9740	5.13%	1.8750	0.0300	0.7105	54.46%	2.3280	0.0100
Cyprus	0.9998	-5.36%	-1.0650	0.8570	0.9879	-1.18%	-0.0480	0.5190
Israel	0.9830	3.55%	0.9080	0.1820	1.1010	-17.61%	-0.6970	0.7570
Colombia	0.9929	1.79%	0.9420	0.1730	0.9031	11.39%	0.5920	0.2770
China	1.0114	-1.24%	-0.5450	0.7070	0.9115	15.86%	1.0690	0.1430
Czech	1.0009	-0.77%	-0.1720	0.5680	0.8881	19.61%	1.6410	0.0500
Peru	1.0092	0.73%	0.1700	0.4330	1.0480	-10.39%	-0.4200	0.6630
Poland	1.0125	-3.89%	-0.8610	0.8050	0.9512	13.13%	0.6490	0.2580

(a) Sequential case, 73-18

	TU	OOS R^2	MSE-t	p MSE-t	TU	OOS R^2	MSE-t	p MSE-t
	SEQ OR SoP				SEQ EMD			
Australia	0.9825	2.86%	1.6130	0.0530	0.8159	39.38%	2.0210	0.0220
Austria	0.9903	0.73%	0.2730	0.3920	0.9339	19.08%	0.7210	0.2360
Belgium	0.9895	1.33%	0.3820	0.3510	0.9702	14.32%	0.6290	0.2650
Canada	0.9852	2.03%	0.9230	0.1780	0.7969	40.31%	1.9850	0.0240
Denmark	0.9840	2.94%	1.2600	0.1040	0.9030	25.08%	1.0080	0.1570
France	0.9847	2.92%	1.2480	0.1060	0.8770	25.34%	1.7640	0.0390
Germany	0.9921	0.60%	0.2130	0.4160	0.8186	33.52%	2.2890	0.0110
Hong Kong	0.9950	1.65%	0.5320	0.2970	0.8383	30.92%	1.7370	0.0410
Ireland	0.9865	2.77%	0.8150	0.2070	0.9187	23.63%	1.0370	0.1500
Japan	1.1057	-22.56%	-2.3050	0.9890	0.8289	28.83%	1.6760	0.0470
Netherlands	0.9944	1.07%	0.2830	0.3890	0.8566	32.79%	1.4960	0.0670
Singapore	0.9818	3.09%	0.9360	0.1750	0.9452	10.69%	0.4880	0.3130
Switzerland	1.0050	-1.40%	-0.5120	0.6960	0.9017	21.93%	1.3740	0.0850
UK	0.9533	8.57%	3.3070	0.0000	0.8023	40.31%	2.0570	0.0200
US	0.9446	10.24%	2.7630	0.0030	0.8617	28.77%	1.4300	0.0760

(To be continued)

	TU	OOS R^2	MSE-t	p MSE-t	TU	OOS R^2	MSE-t	p MSE-t
South Africa	0.9697	5.54%	X	0.0350	0.8472	29.80%	2.4080	0.0080
Norway	0.9978	0.40%	0.2040	0.4190	0.9464	14.75%	0.6710	0.2510
Sweden	0.9894	1.07%	0.4250	0.3350	0.7920	39.27%	2.5210	0.0060
Italy	0.9928	0.28%	0.0910	0.4640	0.8876	24.69%	1.6750	0.0470
Malaysia	0.9852	3.16%	0.9680	0.1660	0.8793	21.57%	1.0290	0.1520
Thailand	0.9727	6.21%	1.8430	0.0330	1.6883	-35.47%	-0.9850	0.8380
Spain	0.9883	2.11%	0.8720	0.1920	0.7747	41.69%	3.2660	0.0010
Sri Lanka	0.9813	3.51%	0.6730	0.2500	0.8250	31.62%	1.8700	0.0310
South Korea	1.0005	1.63%	0.3530	0.3620	0.7757	37.80%	2.3680	0.0090
New Zealand	1.0255	-4.77%	-1.5320	0.9370	0.8495	30.70%	2.0930	0.0180
Finland	0.9741	7.89%	1.3690	0.0850	0.8200	34.84%	2.0170	0.0220
Taiwan	0.9773	3.71%	1.0170	0.1550	0.8246	28.10%	1.5990	0.0550
Philippines	0.9775	4.19%	1.3730	0.0850	0.8172	33.43%	2.1810	0.0150
Chile	0.9933	1.65%	0.9120	0.1810	0.7903	38.08%	2.2250	0.0130
India	0.9682	5.70%	1.9600	0.0250	0.7358	46.75%	4.1550	0.0000
Greece	0.9780	4.59%	1.4060	0.0800	0.7384	51.24%	3.6660	0.0000
Portugal	0.9897	1.65%	0.7720	0.2200	0.8460	30.56%	1.9160	0.0280
Turkey	0.9630	10.44%	2.3770	0.0090	0.8654	34.22%	2.2540	0.0120
Mexico	1.0090	-0.19%	-0.0490	0.5200	0.8613	27.91%	1.5630	0.0590
Hungary	0.9787	3.43%	1.6660	0.0480	0.8720	32.53%	1.7740	0.0380
Luxembourg	1.0158	-3.82%	-1.0300	0.8480	1.4379	-71.84%	-2.4410	0.9930
Pakistan	0.9375	10.66%	2.2880	0.0110	0.6735	56.60%	2.5870	0.0050
Cyprus	1.0141	0.08%	0.0140	0.4940	1.0863	-12.03%	-0.5470	0.7080
Israel	1.0112	-2.04%	-0.2940	0.6160	0.9325	10.61%	0.5400	0.2950
Colombia	1.0319	-5.72%	-1.1580	0.8770	0.9278	12.44%	0.8080	0.2100
China	0.9847	3.63%	0.7460	0.2280	0.8249	26.47%	1.2920	0.0980
Czech	0.9878	1.14%	0.5050	0.3070	0.7683	37.99%	2.3860	0.0090
Peru	1.2148	-46.62%	-2.4570	0.9930	1.0429	-9.16%	-0.4200	0.6630
Poland	1.0086	-1.23%	-0.4710	0.6810	0.9940	4.69%	0.2120	0.4160

(b) All countries present case, 94-18

Table 2: US Theil U, Out-of-sample R^2 and one-sided MSE tests for Original and EMD Sum-of-Parts, 73-18 (Panel a) and 94-18 (Panel b)

	λ_1	p_1	λ_2	p_2	λ_1	p_1	λ_2	p_2
	OR SoP				EMD SoP			
Australia	0.7510	0.0620	0.2490	0.2980	0.7950	0.0010	0.2060	0.1240
Austria	0.7460	0.0880	0.2550	0.3180	0.6560	0.0000	0.3440	0.0200
Belgium	0.7440	0.0400	0.2570	0.2630	0.5750	0.0000	0.4250	0.0030
Canada	0.6660	0.0540	0.3340	0.2040	0.8340	0.0000	0.1660	0.1810
Denmark	0.4070	0.0890	0.5930	0.0310	0.5700	0.0010	0.4300	0.0030
France	0.8440	0.0200	0.1560	0.3420	0.8130	0.0000	0.1870	0.0560
Germany	0.8430	0.0440	0.1570	0.3700	0.6840	0.0000	0.3160	0.0070
Hong Kong	1.4870	0.0180	-0.4870	0.7650	0.7840	0.0000	0.2170	0.0860
Ireland	0.6640	0.0070	0.3360	0.0920	0.7160	0.0000	0.2840	0.0240
Japan	0.6450	0.0240	0.3550	0.1370	0.7840	0.0000	0.2160	0.0330
Netherlands	0.5820	0.0490	0.4180	0.1000	0.7430	0.0000	0.2570	0.0750
Singapore	0.7120	0.1560	0.2880	0.3320	0.5500	0.0020	0.4500	0.0090
Switzerland	0.2310	0.2090	0.7690	0.0050	0.8600	0.0000	0.1400	0.1510
UK	1.2610	0.0010	-0.2610	0.7570	0.7930	0.0000	0.2070	0.0570
US	0.8940	0.0140	0.1060	0.3920	0.7390	0.0000	0.2610	0.1230
South Africa	1.0260	0.0030	-0.0260	0.5290	0.9380	0.0000	0.0620	0.3240
Norway	0.2680	0.2380	0.7320	0.0230	0.5270	0.0050	0.4730	0.0070
Sweden	0.3690	0.3650	0.6310	0.2770	0.4680	0.0000	0.5320	0.0050
Italy	0.3980	0.3320	0.6020	0.2470	0.6670	0.0010	0.3330	0.0150
Malaysia	-0.8940	0.7650	1.8940	0.0760	0.9040	0.0140	0.0960	0.3800
Thailand	0.5220	0.2460	0.4780	0.2730	0.4710	0.0070	0.5290	0.0120
Spain	-0.3250	0.5730	1.3250	0.2240	0.8700	0.0000	0.1300	0.1520
Sri Lanka	0.7600	0.2180	0.2400	0.3990	0.8080	0.0000	0.1920	0.1610
South Korea	0.3830	0.3650	0.6170	0.2830	0.7900	0.0020	0.2100	0.1470
New Zealand	-0.8950	0.8800	1.8950	0.0100	0.8620	0.0000	0.1380	0.1180
Finland	0.5520	0.2910	0.4480	0.3290	0.8010	0.0040	0.1990	0.1790
Taiwan	0.3010	0.3090	0.6990	0.1150	0.7660	0.0070	0.2340	0.0870
Philippines	1.0490	0.1420	-0.0490	0.5210	0.8450	0.0000	0.1550	0.1960
Chile	0.4140	0.2520	0.5860	0.1800	0.8360	0.0000	0.1640	0.1410
India	2.8980	0.0280	-1.8980	0.9020	1.0390	0.0000	-0.0390	0.6230
Greece	-0.5420	0.6820	1.5420	0.1020	1.0190	0.0000	-0.0190	0.5520
Portugal	0.4640	0.2510	0.5360	0.2080	0.8030	0.0010	0.1970	0.2150
Turkey	1.2100	0.1150	-0.2110	0.5840	0.8950	0.0110	0.1060	0.3120
Mexico	-0.4530	0.6380	1.4530	0.1410	1.0070	0.0040	-0.0070	0.5090
Hungary	0.7560	0.2220	0.2440	0.3990	0.9770	0.0030	0.0230	0.4620
Luxembourg	-0.1600	0.5830	1.1600	0.0650	0.2010	0.0270	0.7990	0.0000
Pakistan	2.6050	0.0170	-1.6050	0.9290	0.9480	0.0000	0.0520	0.3740
Cyprus	0.0110	0.4910	0.9890	0.0180	0.4950	0.0000	0.5050	0.0020
Israel	1.1970	0.0670	-0.1970	0.6020	0.3900	0.0280	0.6100	0.0020
Colombia	2.6670	0.1290	-1.6670	0.7630	0.6550	0.0130	0.3450	0.1110
China	-0.1670	0.5530	1.1670	0.1730	0.7250	0.0040	0.2750	0.1180
Czech	0.3340	0.3670	0.6660	0.2470	0.8850	0.0010	0.1150	0.3150
Peru	0.6060	0.1700	0.3950	0.2730	0.3940	0.0580	0.6060	0.0210
Poland	-1.0150	0.7200	2.0150	0.1370	0.7160	0.0210	0.2840	0.2100

(a) Sequential case, 73-18

	λ_1	p_1	λ_2	p_2	λ_1	p_1	λ_2	p_2
	OR SoP				EMD SoP			
Australia	2.8940	3.10%	-1.8940	0.9000	0.9150	0.30%	0.0850	0.2430
Austria	0.6660	14.20%	0.3340	0.2960	0.6570	0.60%	0.3430	0.0650
Belgium	0.7090	11.10%	0.2910	0.2950	0.5920	0.10%	0.4080	0.0060
Canada	1.6430	10.20%	-0.6430	0.7010	1.0580	0.30%	-0.0580	0.6050
Denmark	1.3540	2.70%	-0.3540	0.6980	0.6970	0.60%	0.3030	0.0310
France	1.3830	3.20%	-0.3830	0.7090	0.7600	0.10%	0.2400	0.0310
Germany	0.7920	28.40%	0.2080	0.4410	0.8760	0.00%	0.1240	0.2050
Hong Kong	0.9110	13.00%	0.0890	0.4540	0.7330	0.10%	0.2670	0.0120
Ireland	0.7560	1.10%	0.2440	0.2360	0.6930	0.40%	0.3070	0.0630
Japan	0.1620	13.30%	0.8380	0.0000	0.7450	0.00%	0.2550	0.0620
Netherlands	0.7040	14.80%	0.2960	0.3550	0.7940	0.20%	0.2060	0.1210
Singapore	1.2950	7.30%	-0.2950	0.6380	0.5800	1.40%	0.4200	0.0010
Switzerland	0.0210	49.10%	0.9790	0.1510	0.7500	0.10%	0.2510	0.0610
UK	4.1200	0.00%	-3.1200	0.9980	0.8640	0.30%	0.1370	0.1580
US	2.9610	0.10%	-1.9610	0.9880	0.8020	0.10%	0.1980	0.1690

(To be continued)

	λ_1	p_1	λ_2	p_2	λ_1	p_1	λ_2	p_2
South Africa	1.9460	1.00%	-0.9460	0.8840	0.8550	0.00%	0.1450	0.1410
Norway	0.6530	20.10%	0.3470	0.3190	0.6860	1.70%	0.3140	0.1210
Sweden	0.9880	20.10%	0.0130	0.4960	0.9100	0.00%	0.0900	0.2590
Italy	0.5960	29.70%	0.4040	0.3480	0.7480	0.00%	0.2520	0.0410
Malaysia	2.0170	10.60%	-1.0170	0.7420	0.7930	1.20%	0.2080	0.2110
Thailand	1.6980	0.70%	-0.6980	0.8600	0.3980	1.90%	0.6020	0.0490
Spain	1.1860	7.10%	-0.1860	0.5930	0.8730	0.00%	0.1270	0.1130
Sri Lanka	0.9780	9.40%	0.0220	0.4880	0.8600	0.00%	0.1400	0.2230
South Korea	0.7910	17.50%	0.2090	0.4010	0.9430	0.00%	0.0570	0.3730
New Zealand	-0.7130	81.90%	1.7130	0.0200	0.8290	0.00%	0.1710	0.0830
Finland	1.3100	1.90%	-0.3100	0.7040	0.9130	0.10%	0.0870	0.3070
Taiwan	1.6150	8.00%	-0.6150	0.7160	0.8280	0.40%	0.1720	0.1270
Philippines	1.4430	2.40%	-0.4430	0.7450	0.8260	0.00%	0.1740	0.1060
Chile	1.7250	10.50%	-0.7250	0.7040	0.8920	0.00%	0.1080	0.2280
India	2.0610	0.70%	-1.0610	0.9090	1.0520	0.00%	-0.0520	0.7040
Greece	1.5990	2.10%	-0.5990	0.7670	0.9930	0.00%	0.0070	0.4760
Portugal	1.4190	12.30%	-0.4190	0.6380	0.8250	0.00%	0.1750	0.1600
Turkey	1.9760	0.20%	-0.9760	0.9470	0.9830	0.10%	0.0170	0.4580
Mexico	0.4760	17.20%	0.5240	0.1440	0.8660	0.20%	0.1340	0.2910
Hungary	1.5450	1.20%	-0.5450	0.8090	0.9090	0.20%	0.0910	0.3130
Luxembourg	-0.0840	55.70%	1.0840	0.0290	0.2730	0.50%	0.7270	0.0000
Pakistan	2.7210	0.60%	-1.7210	0.9690	0.9240	0.00%	0.0760	0.2650
Cyprus	0.5040	3.40%	0.4960	0.0360	0.4490	0.00%	0.5510	0.0000
Israel	0.3840	17.10%	0.6160	0.0630	0.5590	0.10%	0.4410	0.0000
Colombia	0.0250	47.60%	0.9750	0.0120	0.6230	0.10%	0.3770	0.0220
China	1.1780	10.80%	-0.1780	0.5790	0.8050	0.60%	0.1950	0.1600
Czech	0.9030	13.50%	0.0970	0.4520	0.8990	0.00%	0.1010	0.2360
Peru	0.0960	26.80%	0.9040	0.0000	0.4070	3.60%	0.5930	0.0120
Poland	0.0290	48.90%	0.9710	0.1670	0.5430	0.80%	0.4570	0.0250

(b) All countries present case, 94-18

Table 3: US HLN Encompassing tests for Original and EMD Sum-of-Parts forecasts, 73-18 (Panel a) and 94-18 (Panel b)

	CE	R	SD	Sharpe	M2
73-18	Panel (a) - Constrained portfolio				
HA		4.51%	3.15%	0.2945	
OR Dir	0.49%	5.12%	4.66%	0.3301	0.11%
OR Iter	1.60%	6.47%	6.81%	0.4241	0.41%
EMD Dir	3.95%	8.57%	4.56%	1.0925	2.51%
EMD Iter	10.64%	16.35%	11.42%	1.1178	2.59%
94-18					
HA		3.32%	4.04%	0.4005	
OR Dir	0.65%	3.95%	3.75%	0.5976	0.80%
OR Iter	1.90%	5.46%	6.34%	0.5924	0.78%
EMD Dir	1.72%	5.00%	3.55%	0.9286	2.13%
EMD Iter	7.80%	11.70%	8.60%	1.1627	3.08%
73-18	Panel (b) - Unconstrained portfolio				
HA		2.88%	10.39%	-0.0670	
OR Dir	2.99%	5.73%	9.64%	0.2227	-0.23%
OR Iter	2.98%	5.37%	7.63%	0.2345	-0.19%
EMD Dir	24.91%	29.47%	16.59%	1.5603	16.91%
EMD Iter	20.35%	24.75%	16.12%	1.3135	14.35%
94-18					
HA		6.47%	9.12%	0.5231	
OR Dir	-2.78%	3.48%	7.87%	0.2263	-2.71%
OR Iter	-2.43%	3.60%	6.26%	0.3032	-2.00%
EMD Dir	20.47%	26.86%	8.64%	2.9101	21.77%
EMD Iter	16.57%	23.19%	9.89%	2.1724	15.04%
73-18	Panel (c) - All Equity portfolio				
HA C		10.78%	18.53%	0.3886	
OR C	1.97%	13.13%	19.54%	0.4888	0.61%
EMD C	23.20%	38.50%	28.21%	1.2378	2.97%
94-18					
HA C		9.61%	21.83%	0.3623	
OR C	3.97%	12.02%	17.87%	0.5770	0.71%
EMD C	21.45%	31.63%	23.07%	1.2969	3.62%

Table 4: Results for the USA

Note: Annualised certainty equivalents (CE), returns (R), standard deviations (SD), Sharpe Ratios and M2 for a US-based investor with USD as the home currency. The 73-18 sample refers to the sequential case, where a country is introduced to the portfolio when enough data becomes available. The 94-18 sample refers to the all countries present case, where all 44 countries are present from the start of the period. The constrained portfolio (C) prohibits short-selling, the unconstrained portfolio (UC) allows short-selling and the All Equity (AE) portfolio does not have a risk-free asset.

	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
73-18			Switzerland					UK		
HA		3.38%	3.84%	0.2527			6.18%	3.62%	0.2076	
OR Dir	0.88%	4.40%	5.34%	0.3727	0.46%	0.88%	7.14%	4.60%	0.3726	0.60%
OR Iter	1.03%	4.87%	7.77%	0.3161	0.24%	0.91%	7.32%	6.09%	0.3113	0.38%
EMD Dir	4.47%	8.14%	6.52%	0.8778	2.40%	4.01%	10.39%	5.74%	0.8644	2.38%
EMD Iter	7.54%	11.09%	8.56%	1.0814	3.12%	10.86%	18.02%	10.54%	1.1947	3.57%
94-18										
HA		2.10%	5.21%	0.2653			3.94%	5.41%	0.2644	
OR Dir	0.33%	2.35%	4.25%	0.3818	0.61%	0.96%	4.76%	3.93%	0.5728	1.67%
OR Iter	3.15%	5.56%	7.58%	0.6383	1.94%	2.38%	6.37%	5.87%	0.6579	2.13%
EMD Dir	3.05%	5.04%	3.99%	1.0821	4.25%	3.40%	7.18%	3.76%	1.2448	5.31%
EMD Iter	6.74%	9.35%	8.80%	0.9805	3.72%	9.55%	14.00%	8.96%	1.2824	5.51%
73-18			Germany					Japan		
HA		4.48%	3.69%	0.2485			2.49%	3.19%	0.3812	
OR Dir	0.20%	4.90%	5.91%	0.2252	-0.09%	0.14%	2.81%	5.33%	0.2882	-0.30%
OR Iter	2.30%	7.31%	8.18%	0.4580	0.77%	1.59%	4.76%	8.81%	0.3948	0.04%
EMD Dir	3.81%	8.43%	5.18%	0.9393	2.55%	3.82%	6.48%	5.18%	1.0043	1.99%
EMD Iter	9.20%	14.51%	9.79%	1.1175	3.20%	10.42%	13.97%	10.77%	1.1787	2.54%
94-18										
HA		2.98%	3.94%	0.2892			2.11%	4.34%	0.4620	
OR Dir	0.16%	3.11%	3.71%	0.3442	0.22%	-0.22%	1.88%	4.20%	0.4230	-0.17%
OR Iter	1.36%	4.53%	5.89%	0.4574	0.66%	0.50%	2.81%	6.27%	0.4320	-0.13%
EMD Dir	3.12%	6.08%	3.72%	1.1402	3.35%	2.47%	4.66%	5.19%	0.8780	1.81%
EMD Iter	7.54%	11.09%	8.56%	1.0814	3.12%	8.10%	11.02%	9.98%	1.0940	2.74%
73-18			S. Africa							
HA		11.70%	3.50%	0.2519						
OR Dir	0.59%	12.50%	5.79%	0.2908	0.14%					
OR Iter	1.80%	14.05%	8.18%	0.3948	0.50%					
EMD Dir	4.83%	16.69%	5.34%	1.0999	2.96%					
EMD Iter	10.47%	23.11%	10.30%	1.1929	3.29%					
94-18								India		
HA		9.07%	4.22%	0.2825			7.86%	4.55%	0.2292	
OR Dir	0.07%	9.16%	4.51%	0.2850	0.01%	0.58%	8.36%	3.62%	0.4271	0.90%
OR Iter	1.62%	10.94%	6.54%	0.4674	0.78%	1.77%	9.66%	4.92%	0.5778	1.58%
EMD Dir	2.69%	11.70%	3.50%	1.0929	3.42%	3.35%	11.13%	3.60%	1.1974	4.40%
EMD Iter	8.36%	18.00%	8.68%	1.1661	3.73%	8.43%	16.62%	7.29%	1.3437	5.07%
94-18			Chile					China		
HA		4.79%	4.29%	0.2084			1.85%	6.63%	0.023	
OR Dir	0.99%	5.67%	2.68%	0.6614	1.94%	0.55%	2.29%	5.69%	0.103	0.54%
OR Iter	1.78%	6.67%	5.23%	0.5294	1.38%	-1.25%	1.43%	11.25%	-0.024	0.31%
EMD Dir	3.31%	8.02%	3.19%	1.2923	4.65%	1.99%	3.99%	7.66%	0.2986	1.83%
EMD Iter	8.07%	13.23%	7.43%	1.2564	4.50%	9.50%	12.53%	12.71%	0.852	5.50%

Table 5: Constrained case for selected home countries

Note: Annualised certainty equivalents (CE), returns (R), standard deviations (SD), Sharpe Ratios and M2 for Switzerland, the UK, Germany, Japan, S. Africa, India, Chile and China as home currencies. The 73-18 sample refers to the sequential case, where a country is introduced to the portfolio when enough data becomes available. The 94-18 sample refers to the all countries present case, where all 44 countries are present from the start of the period. The constrained portfolio (C) prohibits short-selling, the unconstrained portfolio (UC) allows short-selling and the All Equity (AE) portfolio does not have a risk-free asset. The All Equity portfolia are based on iterative weights.

	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
73-18			Switzerland					UK		
HA C		8.12%	20.49%	0.2786			10.40%	16.20%	0.3073	
OR C	1.30%	10.91%	23.84%	0.3564	0.40%	2.25%	13.14%	17.66%	0.4370	0.83%
EMD C	24.99%	37.15%	28.70%	1.2106	3.68%	24.79%	39.65%	26.61%	1.2864	3.90%
94-18										
HA C		8.41%	28.95%	0.2655			9.56%	21.17%	0.3332	
OR C	6.47%	10.81%	20.77%	0.4857	1.15%	4.84%	12.71%	16.72%	0.6105	1.87%
EMD C	24.54%	33.90%	30.54%	1.0862	4.28%	23.88%	34.36%	23.23%	1.3710	5.99%
73-18			Germany					Japan		
HA C		8.95%	18.51%	0.2908			8.30%	20.31%	0.3457	
OR C	2.68%	12.77%	21.38%	0.4305	0.67%	2.79%	12.22%	22.94%	0.4772	0.31%
EMD C	23.59%	36.35%	26.89%	1.2190	3.58%	25.49%	37.87%	28.64%	1.2776	2.86%
94-18										
HA C		7.59%	21.14%	0.2719			9.25%	25.44%	0.3596	
OR C	4.37%	10.56%	17.55%	0.4972	0.82%	3.54%	10.45%	20.32%	0.5093	0.20%
EMD C	22.43%	32.14%	25.67%	1.1801	3.51%	23.93%	34.18%	27.35%	1.2463	3.40%
73-18			S. Africa							
HA C			16.83%	19.48%	0.3084					
OR C	2.50%	21.02%	23.44%	0.4354	0.64%					
EMD C	26.28%	48.15%	29.73%	1.2558	3.51%					
94-18								India		
HA C	12.08%	20.75%	0.2028				10.23%	19.69%	0.1730	
OR C	5.14%	16.24%	18.21%	0.4590	0.74%	4.95%	13.99%	16.38%	0.4374	0.95%
EMD C	25.98%	38.92%	22.73%	1.3659	4.57%	23.92%	34.81%	21.31%	1.3134	4.93%
94-18			Chile					China		
HA C		9.33%	17.29%	0.3139			-3.30%	38.51%	-0.1299	
OR C	4.54%	12.97%	14.48%	0.6267	1.80%	9.89%	2.10%	32.16%	0.0123	-0.07%
EMD C	23.43%	33.74%	19.93%	1.4969	5.53%	38.15%	32.66%	35.56%	0.8707	5.62%

Table 6: All Equity case for selected home countries

Note: Annualised certainty equivalents (CE), returns (R), standard deviations (SD), Sharpe Ratios and M2 for Switzerland, the UK, Germany, Japan, S. Africa, India, Chile and China as home currencies. The 73-18 sample refers to the sequential case, where a country is introduced to the portfolio when enough data becomes available. The 94-18 sample refers to the all countries present case, where all 44 countries are present from the start of the period. The constrained portfolio (C) prohibits short-selling, the unconstrained portfolio (UC) allows short-selling and the All Equity (AE) portfolio does not have a risk-free asset. The All Equity portfolia are based on iterative weights.

	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
73-18			Switzerland					UK		
HA		-0.11%	12.09%	-0.2086			3.56%	11.47%	-0.1623	
OR Dir	5.15%	4.80%	11.06%	0.2160	5.13%	5.35%	8.63%	10.14%	0.3159	5.49%
OR Iter	6.41%	5.60%	8.75%	0.3642	6.93%	5.02%	8.13%	9.30%	0.2914	5.20%
EMD Dir	25.19%	25.72%	14.52%	1.6056	21.94%	26.58%	31.19%	15.38%	1.6756	21.08%
EMD Iter	19.16%	19.25%	12.90%	1.3057	18.31%	21.57%	25.70%	13.71%	1.4793	18.83%
94-18										
HA		1.07%	10.95%	0.0322			6.41%	9.27%	0.4207	
OR Dir	2.48%	3.26%	9.51%	0.2672	2.57%	-1.31%	4.83%	7.67%	0.3028	-1.09%
OR Iter	3.50%	3.96%	7.64%	0.4243	4.29%	-0.69%	5.28%	6.48%	0.4281	0.07%
EMD Dir	29.15%	30.36%	11.57%	2.5620	27.69%	20.93%	27.31%	9.16%	2.7085	21.21%
EMD Iter	23.95%	25.08%	11.17%	2.1810	23.52%	15.60%	22.17%	10.14%	1.9389	14.08%
73-18			Germany					Japan		
HA		-0.67%	11.34%	-0.3740			0.68%	13.17%	-0.0457	
OR Dir	7.17%	6.38%	10.80%	0.2603	7.19%	5.50%	6.22%	13.33%	0.3706	5.48%
OR Iter	7.12%	6.02%	9.27%	0.2641	7.24%	5.00%	5.23%	11.33%	0.3485	5.19%
EMD Dir	26.16%	26.10%	13.77%	1.6364	22.80%	24.65%	15.29%	13.45%	1.6124	21.84%
EMD Iter	9.20%	14.51%	9.79%	1.1175	20.74%	19.62%	20.37%	13.45%	1.4200	19.30%
94-18										
HA		5.88%	8.61%	0.4689			3.47%	8.96%	0.3756	
OR Dir	-1.77%	3.98%	7.86%	0.2732	-1.69%	-0.90%	2.36%	7.72%	0.2922	-0.75%
OR Iter	-1.75%	3.82%	6.62%	0.2992	-1.46%	-1.05%	2.01%	6.29%	0.3025	-0.65%
EMD Dir	21.01%	26.97%	9.08%	2.7691	19.81%	19.95%	23.25%	7.96%	2.9062	22.66%
EMD Iter	14.66%	20.66%	9.31%	2.0211	13.37%	13.67%	17.05%	8.50%	1.9948	14.50%
73-18			S. Africa							
HA		9.96%	10.73%	-0.0794						
OR Dir	4.64%	14.45%	9.99%	0.3638	4.76%					
OR Iter	4.87%	14.37%	8.29%	0.4286	5.45%					
EMD Dir	28.50%	39.57%	15.01%	1.9153	21.41%					
EMD Iter	23.19%	34.20%	14.80%	1.5801	17.81%					
94-18								India		
HA		11.34%	9.56%	0.3627			9.71%	9.44%	0.3058	
OR Dir	-2.15%	8.91%	7.90%	0.1302	-2.22%	-1.25%	8.19%	7.92%	0.1732	-1.25%
OR Iter	-2.03%	8.86%	6.78%	0.1451	-2.08%	-1.08%	8.17%	6.65%	0.2036	-0.97%
EMD Dir	18.59%	29.74%	8.51%	2.5703	21.11%	20.79%	30.36%	8.66%	2.7177	22.77%
EMD Iter	13.46%	24.62%	8.56%	1.9557	15.23%	16.78%	26.42%	9.07%	2.1616	17.52%
94-18			Chile					China		
HA		7.59%	8.76%	0.4220			-3.92%	18.19%	-0.3092	
OR Dir	-1.33%	6.09%	7.70%	0.2842	-1.21%	10.50%	5.10%	13.52%	0.2512	10.19%
OR Iter	-2.40%	4.87%	6.65%	0.1456	-2.42%	11.10%	5.16%	11.34%	0.3046	11.16%
EMD Dir	19.28%	26.77%	8.14%	2.8090	20.90%	36.13%	29.97%	10.33%	2.7368	55.40%
EMD Iter	12.33%	19.82%	8.13%	1.9572	13.44%	34.86%	29.62%	14.09%	1.9808	34.86%

Table 7: Unconstrained case for selected home countries

Note: Annualised certainty equivalents (CE), returns (R), standard deviations (SD), Sharpe Ratios and M2 for Switzerland, the UK, Germany, Japan, S. Africa, India, Chile and China as home currencies. The 73-18 sample refers to the sequential case, where a country is introduced to the portfolio when enough data becomes available. The 94-18 sample refers to the all countries present case, where all 44 countries are present from the start of the period. The constrained portfolio (C) prohibits short-selling, the unconstrained portfolio (UC) allows short-selling and the All Equity (AE) portfolio does not have a risk-free asset. The All Equity portfolia are based on iterative weights.

	Countries above HA	Median TU	Max Improv.	Avg Improv.	Countries above HA	Median TU	Max Improv.	Avg Improv.
		USA				UK		
EMD Seq	38	0.8861	0.2895	0.1246	35	0.9073	0.2755	0.1174
EMD ACP	40	0.8530	0.3265	0.1515	39	0.8677	0.2945	0.1399
OR Seq	33	0.9947	0.0264	0.0096	29	0.9950	0.0386	0.0127
OR ACP	33	0.9881	0.0625	0.0197	27	0.9927	0.0601	0.0201
		Germany				Japan		
EMD Seq	40	0.8981	0.2596	0.1118	41	0.8966	0.2733	0.1165
EMD ACP	39	0.8482	0.2818	0.1534	41	0.8526	0.3088	0.1504
OR Seq	27	0.9966	0.0356	0.0120	23	0.9984	0.0229	0.0096
OR ACP	28	0.9901	0.0555	0.0203	34	0.9904	0.0479	0.0172
		Switzerland				S. Africa		
EMD Seq	40	0.8923	0.2696	0.1155	38	0.8891	0.2386	0.1188
EMD ACP	39	0.8515	0.2956	0.1558	39	0.8571	0.2699	0.1501
OR Seq	28	0.9951	0.0312	0.0131	29	0.9954	0.0332	0.0132
OR all	32	0.9884	0.0556	0.0193	32	0.9920	0.0498	0.0169
		Chile				India		
EMD ACP	39	0.8777	0.3048	0.1322	38	0.8713	0.2965	0.1333
OR ACP	24	0.9979	0.0688	0.0214	34	0.9873	0.0677	0.0219
		China						
EMD ACP	41	0.7166	0.3891	0.2691				
OR ACP	39	0.9685	0.0892	0.0372				

Table 8: Theil's U statistics for different home countries

Note: Seq denotes the sequential case (73-18) and ACP the all countries present case (94-18).

73-18	CE	R	SD	SR	M2	73-18	CE	R	SD	SR	M2
		$\gamma = 3$, Constrained portfolio						$\gamma = 5$, Constrained portfolio			
HA		4.20%	2.31%	0.2675		HA		3.95%	1.75%	0.2119	
OR Dir	0.36%	4.61%	3.28%	0.3137	0.11%	OR Dir	0.23%	4.20%	2.28%	0.2717	0.10%
OR Iter	1.15%	5.50%	4.63%	0.4161	0.34%	OR Iter	0.73%	4.73%	2.96%	0.3905	0.31%
EMD Dir	2.65%	6.90%	3.24%	1.0268	1.75%	EMD Dir	1.60%	5.57%	2.26%	0.8824	1.17%
EMD Iter	7.35%	12.09%	7.77%	1.0957	1.91%	EMD Iter	4.53%	8.69%	4.89%	1.0447	1.45%
94-18						94-18					
HA		2.78%	2.79%	0.3863		HA		2.35%	1.85%	0.3508	
OR Dir	0.43%	3.20%	2.61%	0.5733	0.52%	OR Dir	0.25%	2.60%	1.75%	0.5141	0.30%
OR Iter	1.32%	4.21%	4.31%	0.5811	0.54%	OR Iter	0.81%	3.21%	2.72%	0.5525	0.37%
EMD Dir	1.14%	3.90%	2.47%	0.8917	1.41%	EMD Dir	0.68%	3.02%	1.66%	0.7975	0.82%
EMD Iter	5.33%	8.37%	5.79%	1.1504	2.13%	EMD Iter	3.26%	5.70%	3.58%	1.1178	1.42%
		$\gamma = 3$, Unconstrained portfolio						$\gamma = 5$, Unconstrained portfolio			
HA		3.12%	6.97%	-0.0666		HA		3.30%	4.28%	-0.0650	
OR Dir	1.95%	5.01%	6.53%	0.2191	1.99%	OR Dir	1.15%	4.44%	4.10%	0.2094	1.18%
OR Iter	1.87%	4.77%	5.21%	0.2289	2.06%	OR Iter	1.07%	4.29%	3.34%	0.2141	1.20%
EMD Dir	16.94%	20.84%	11.25%	1.5347	11.16%	EMD Dir	10.33%	13.94%	7.00%	1.4800	6.62%
EMD Iter	13.87%	17.69%	10.92%	1.2923	9.47%	EMD Iter	8.47%	12.05%	6.79%	1.2465	5.62%
94-18						94-18					
HA		4.88%	6.16%	0.5165		HA		3.61%	3.81%	0.5005	
OR Dir	-1.89%	2.89%	5.22%	0.2274	-1.78%	OR Dir	-1.15%	2.42%	3.13%	0.2271	-1.04%
OR Iter	-2.42%	2.26%	4.11%	0.1350	-2.35%	OR Iter	-1.49%	2.04%	2.50%	0.1329	-1.40%
EMD Dir	13.63%	18.47%	5.79%	2.8957	14.65%	EMD Dir	8.17%	11.77%	3.54%	2.8438	8.93%
EMD Iter	11.08%	16.03%	6.63%	2.1620	10.13%	EMD Iter	6.67%	10.30%	4.04%	2.1281	6.21%

Table 9: US EMD portfolio performance metrics for risk aversion $\gamma = 3, 5$

Note: Results for the sequential (73-18) and all countries present (94-18) cases with EMD Sum-of-Parts, for constrained (no short-selling) and unconstrained (short-selling allowed) portfolia. The results for the all-equity portfolio are identical to Table 4, Panel (c) and are thus ommited.

	DP	FX	GE	GM	DP	FX	GE	GM
Missing IV	SEQ EMD				SEQ OR SoP			
Australia	0.0040	0.2228	0.2877	0.6942	0.0266	0.0500	0.0393	0.2223
Austria	0.0163	0.1012	0.6009	0.3993	0.0099	0.0152	0.1365	0.4164
Belgium	0.0197	0.1809	0.5880	0.7390	0.0274	0.0166	0.0352	0.2039
Canada	0.0094	0.0859	0.5487	0.6115	0.0182	0.0262	0.0693	0.5432
Denmark	0.1202	0.2312	0.8202	0.8894	0.0013	0.0060	0.0331	0.2227
France	0.0034	0.1708	0.4419	0.6107	0.0119	0.0413	0.0768	0.2636
Germany	0.0031	0.1575	0.5845	0.5955	0.0054	0.0228	0.0295	0.3930
Hong Kong	0.0051	0.0144	0.4065	0.9001	0.0093	0.0036	0.0458	0.3368
Ireland	0.0356	0.1179	0.6306	0.8755	0.0194	0.0247	0.0527	0.3390
Japan	0.0138	0.1845	0.3783	0.7424	0.0013	0.0089	0.0158	0.2701
Netherlands	0.0516	0.2316	0.7090	0.7682	0.0204	0.0156	0.0189	0.2155
Singapore	0.0073	0.0335	0.4392	0.8341	0.0069	0.0026	0.0199	0.5585
Switzerland	0.0023	0.2954	0.4165	0.5397	0.0248	0.0347	0.0867	0.7220
UK	0.0044	0.1769	0.3772	0.7319	0.0368	0.0744	0.1665	0.5837
US	0.0030	0.0008	0.3694	0.7763	0.0468	0.0000	0.0256	0.3126
South Africa	0.0039	0.3310	0.2710	0.5799	0.0224	0.0365	0.0507	0.1993
Norway	0.0173	0.0924	0.7286	0.6618	0.0112	0.0091	0.0227	0.3873
Sweden	0.0122	0.0870	0.5339	0.4641	0.0109	0.0134	0.0623	0.2289
Italy	0.0066	0.1152	0.3608	0.7393	0.0468	0.0179	0.1901	0.2177
Malaysia	0.0139	0.3198	0.2742	0.6792	0.0089	0.0419	0.0601	0.2251
Thailand	0.0887	0.0899	0.4068	0.8089	0.0078	0.0059	0.0390	0.2935
Spain	0.0057	0.1232	0.2399	0.7592	0.0479	0.0397	0.1243	0.1126
Sri Lanka	0.0068	0.0321	0.1669	0.7778	0.1141	0.0193	0.0260	0.2479
South Korea	0.0156	0.1194	0.2667	0.5745	0.0021	0.0101	0.0270	0.1934
New Zealand	0.0154	0.2357	0.4568	0.3853	0.0214	0.0514	0.0691	0.3170
Finland	0.0512	0.1368	0.5082	0.8526	0.0179	0.0179	0.0587	0.2978
Taiwan	0.0633	0.1098	0.5635	0.8650	0.0299	0.0030	0.0995	0.4362
Philippines	0.0129	0.0585	0.4071	0.6838	0.0089	0.0577	0.0696	0.2907
Chile	0.0042	0.1376	0.1777	0.3528	0.0136	0.1214	0.0472	0.2269
India	0.0015	0.0502	0.1073	0.5670	0.0064	0.0922	0.0886	0.2427
Greece	0.1942	0.2917	0.8287	0.5057	0.0267	0.0320	0.3496	0.6778
Portugal	0.0164	0.1798	0.5473	0.6298	0.0376	0.0228	0.0203	0.1187
Turkey	0.0097	0.1997	0.2092	0.3524	0.0015	0.0714	0.0482	0.2444
Mexico	0.0028	0.1921	0.5051	0.3719	0.0038	0.0735	0.1470	0.4269
Hungary	0.1964	0.4373	0.5384	0.5195	0.0086	0.0462	0.0739	0.3446
Luxembourg	0.2380	0.2461	0.7125	0.8940	0.0139	0.0120	0.0677	0.1503
Pakistan	0.0052	0.0260	0.2192	0.5511	0.0306	0.0141	0.0642	0.3175
Cyprus	0.2584	0.3331	0.7518	0.6609	0.0287	0.0014	0.1034	0.1354
Israel	0.1431	0.2167	0.3999	0.7652	0.0139	0.0155	0.0701	0.1419
Colombia	0.0048	0.1876	0.5251	0.4695	0.0763	0.5059	0.2626	0.3299
China	0.0106	0.0069	0.3928	0.8904	0.0084	0.0000	0.0421	0.0441
Czech	0.0138	0.1357	0.4637	0.7681	0.0193	0.0084	0.0334	0.0911
Peru	0.2216	0.2382	0.6475	0.8799	0.0142	0.0132	0.0581	0.4270
Poland	0.0052	0.0715	0.2774	0.3165	0.0170	0.0228	0.0993	0.1609

(a) Sequential case, 73-18

	DP	FX	GE	GM	DP	FX	GE	GM
Missing IV	SEQ EMD				SEQ OR SoP			
Australia	0.0035	0.2675	0.2137	0.3761	0.0182	0.1171	0.1079	0.1513
Austria	0.0297	0.1125	0.4701	0.4352	0.0197	0.0924	0.3795	0.2488
Belgium	0.0319	0.1171	0.5658	0.7488	0.0462	0.0177	0.0878	0.1506
Canada	0.0078	0.0846	0.3318	0.3067	0.0055	0.0660	0.1873	0.1472
Denmark	0.0309	0.1473	0.6690	0.7946	0.0063	0.0838	0.1141	0.3456
France	0.0048	0.1403	0.3745	0.6714	0.0106	0.0623	0.1071	0.1321
Germany	0.0051	0.1394	0.4566	0.7792	0.0070	0.0634	0.0935	0.1643
Hong Kong	0.0071	0.0072	0.4952	0.8612	0.0069	0.0000	0.1093	0.1938
Ireland	0.0507	0.1397	0.5870	0.6382	0.0056	0.0122	0.0942	0.2841
Japan	0.0406	0.2092	0.5379	0.8862	0.0004	0.0023	0.0176	0.3581
Netherlands	0.0630	0.1914	0.4492	0.6638	0.0132	0.0261	0.0959	0.1971
Singapore	0.0150	0.0396	0.3657	0.7516	0.0078	0.0171	0.1054	0.2768
Switzerland	0.0030	0.2085	0.7269	0.4128	0.0130	0.0371	0.1370	0.3269
UK	0.0052	0.1640	0.3379	0.5162	0.0185	0.0435	0.1046	0.5088
US	0.0016	0.0013	0.4937	0.7080	0.0064	0.0000	0.1962	0.4896

(To be continued)

	DP	FX	GE	GM	DP	FX	GE	GM
South Africa	0.0035	0.3732	0.2314	0.2968	0.0090	0.1261	0.0375	0.0790
Norway	0.0446	0.1518	0.7490	0.6435	0.0184	0.0405	0.0604	0.1539
Sweden	0.0118	0.0923	0.6479	0.7283	0.0185	0.0541	0.4611	0.3760
Italy	0.0143	0.1413	0.4490	0.7510	0.0385	0.0474	0.3828	0.2530
Malaysia	0.0195	0.2058	0.3836	0.8355	0.0124	0.0516	0.0316	0.1221
Thailand	0.0178	0.0188	0.2617	0.7016	0.0048	0.0060	0.0464	0.0981
Spain	0.0067	0.1285	0.3057	0.7852	0.0439	0.0869	0.2799	0.2351
Sri Lanka	0.0084	0.0357	0.1844	0.7517	0.1204	0.0371	0.0884	0.3313
South Korea	0.0092	0.1022	0.3489	0.7214	0.0021	0.0660	0.0758	0.1707
New Zealand	0.0205	0.2879	0.4502	0.3911	0.0167	0.1034	0.1285	0.1590
Finland	0.0883	0.1670	0.5085	0.8728	0.0177	0.0236	0.1372	0.3941
Taiwan	0.0739	0.1139	0.6016	0.9314	0.0168	0.0150	0.1174	0.1543
Philippines	0.0144	0.0622	0.3269	0.6273	0.0060	0.0324	0.0735	0.2463
Chile	0.0032	0.1483	0.1805	0.3541	0.0214	0.1204	0.1129	0.1555
India	0.0017	0.0439	0.0976	0.5386	0.0045	0.0678	0.1768	0.2041
Greece	0.1869	0.2681	0.8315	0.4969	0.0173	0.0320	0.3518	0.8219
Portugal	0.0221	0.1755	0.5759	0.6650	0.0517	0.0959	0.4057	0.2506
Turkey	0.0582	0.3919	0.5236	0.4917	0.0041	0.2036	0.3061	0.2082
Mexico	0.0026	0.1196	0.4752	0.2802	0.0025	0.2515	0.1070	0.1746
Hungary	0.2184	0.4114	0.6505	0.6612	0.0084	0.0728	0.1893	0.3976
Luxembourg	0.2371	0.2426	0.7478	0.9361	0.0130	0.0193	0.1420	0.2004
Pakistan	0.0047	0.0275	0.2457	0.7275	0.0451	0.0791	0.1452	0.3044
Cyprus	0.6807	0.5441	0.7747	0.9394	0.0620	0.0021	0.1440	0.1177
Israel	0.1984	0.2696	0.5657	0.7715	0.0051	0.0047	0.1538	0.0861
Colombia	0.0408	0.1887	0.4773	0.6504	0.0083	0.1194	0.1780	0.3679
China	0.0125	0.0086	0.3968	0.9030	0.0108	0.0000	0.1035	0.3183
Czech	0.0101	0.1281	0.4369	0.7606	0.0186	0.0047	0.0199	0.1165
Peru	0.4237	0.4178	0.6984	0.9144	0.0064	0.0020	0.0624	0.4034
Poland	0.1418	0.1852	0.3673	0.5860	0.0117	0.0739	0.1036	0.1130

(b) All countries present case, 94-18

Table 10: $1 - R^2$ of Shapley regressions (leave-one-out) for the sequential case, 73-18 (Panel a) and 94-18 (Panel b)

Note: $1 - R^2$ of regressions where the returns forecast is the dependent variables and all the parts forecasts apart from the one denoted are the independent variables. $1 - R^2$ measures the relative importance of the part that has been left out to the total forecast in terms of explanatory power.

	DP	FX	GE	GM	DP	FX	GE	GM
Missing IV	SEQ EMD				SEQ OR SoP			
Australia	0.3687	0.7775	0.6247	0.8437	0.4582	1.0025	0.9967	0.9904
Austria	0.3828	0.8138	0.8094	0.8884	0.4646	0.9988	1.0059	0.9758
Belgium	0.3142	0.8050	0.7676	0.8315	0.3986	1.0017	0.9988	0.9780
Canada	0.2756	0.8332	0.6707	0.7991	0.1982	1.0004	1.0024	0.9816
Denmark	0.2627	0.8030	0.7516	0.7970	0.3118	1.0020	0.9933	0.9808
France	0.2561	0.8008	0.7649	0.8657	0.3052	1.0030	0.9946	0.9822
Germany	0.2327	0.8177	0.8037	0.8832	0.3035	0.9990	1.0027	0.9754
Hong Kong	0.3240	0.6546	0.6066	0.8133	0.4673	1.0071	0.9953	0.9731
Ireland	0.1850	0.8006	0.7586	0.8181	0.2204	1.0036	0.9980	0.9945
Japan	0.1578	0.8308	0.6964	0.7633	0.1990	0.9990	1.0031	0.9958
Netherlands	0.2123	0.8075	0.7820	0.7860	0.2666	0.9995	0.9995	0.9875
Singapore	0.3644	0.8326	0.6971	0.8560	0.4456	0.9995	0.9989	0.9693
Switzerland	0.2099	0.8421	0.8235	0.9147	0.2741	0.9978	1.0008	0.9934
UK	0.1891	0.8159	0.6900	0.8606	0.2659	1.0024	0.9959	0.9871
US	0.1183	0.9901	0.6328	0.8830	0.1370		0.9990	0.9908
South Africa	0.1917	0.7819	0.6387	0.8385	0.2260	1.0020	0.9928	0.9967
Norway	0.3091	0.8391	0.8970	0.8943	0.4440	1.0012	1.0020	0.9826
Sweden	0.4189	0.7923	0.8215	0.8600	0.5036	1.0000	0.9965	0.9814
Italy	0.3829	0.7844	0.7627	0.8572	0.4725	0.9999	0.9942	0.9804
Malaysia	0.4596	0.8071	0.7193	0.7853	0.5664	1.0031	0.9959	0.9748
Thailand	0.4434	0.6676	0.8770	0.6794	0.6392	1.0037	0.9980	0.9787
Spain	0.2554	0.7848	0.7085	0.7884	0.3362	1.0011	0.9975	0.9854
Sri Lanka	0.3858	0.7516	0.8949	0.8194	0.5003	1.0017	1.0022	0.9822
South Korea	0.6062	0.9460	0.8534	0.8043	0.8139	1.0121	1.0053	0.9764
New Zealand	0.4903	0.8139	0.8403	0.8033	0.6024	1.0017	1.0048	0.9771
Finland	0.3184	0.7835	0.7804	0.7297	0.3943	1.0029	0.9944	0.9885
Taiwan	0.3027	0.7881	0.8896	0.7587	0.3680	1.0012	0.9962	0.9749
Philippines	0.2879	0.7216	0.8165	0.8784	0.3462	1.0103	0.9980	0.9794
Chile	0.3981	0.7981	0.9111	0.7391	0.5188	1.0024	0.9994	0.9443
India	0.5017	0.7108	0.7798	0.8361	0.5987	1.0049	0.9696	0.9711
Greece	0.5080	0.7710	0.8415	0.9313	0.6763	1.0060	0.9979	0.9807
Portugal	0.3322	0.7733	0.8446	0.8527	0.4166	1.0027	1.0021	0.9279
Turkey	0.1953	0.6668	0.7498	0.9566	0.2438	1.0269	0.9169	0.9497
Mexico	0.5626	0.7700	0.7253	0.9336	0.7220	1.0099	0.9481	0.9751
Hungary	0.5849	0.7359	0.8564	0.7487	0.7583	1.0035	1.0065	0.9863
Luxembourg	0.3645	0.7640	0.8614	0.8246	0.4768	1.0022	0.9995	0.9906
Pakistan	0.4995	0.6986	0.7747	0.8141	0.6705	1.0033	0.9959	0.9755
Cyprus	0.2612	0.7859	0.7136	0.8246	0.3744	1.0004	0.8806	0.9239
Israel	0.4936	0.8140	0.8423	0.6865	0.6167	1.0028	0.9906	0.9935
Colombia	0.3775	0.8142	0.8288	0.9736	0.5281	1.0067	0.9686	0.9862
China	0.2793	0.9907	0.9043	0.8701	0.3907	0.9907	1.0035	0.9895
Czech	0.3511	0.8841	0.8312	0.8256	0.4026	0.9991	0.9859	0.9858
Peru	0.5891	0.6961	0.8986	0.8521	0.6652	1.0016	1.0101	0.9944
Poland	0.3231	0.9148	0.7654	0.8292	0.3731	1.0015	1.0068	0.9472

(a) Sequential case, 73-18

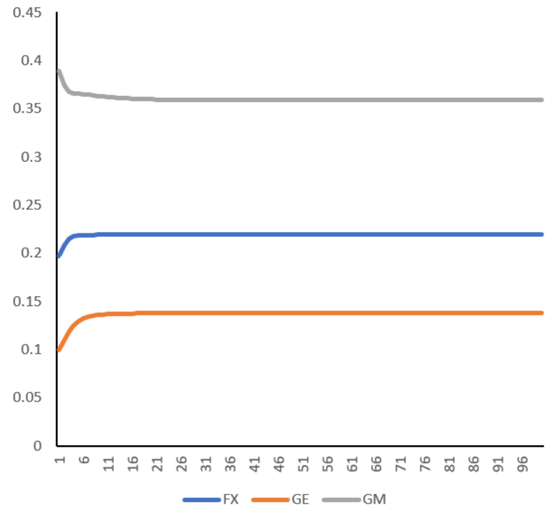
	DP	FX	GE	GM	DP	FX	GE	GM
Missing IV	SEQ EMD				SEQ OR SoP			
Australia	0.3264	0.7396	0.6214	0.8478	0.4611	1.0009	0.9981	0.9639
Austria	0.3482	0.7517	0.7749	0.7694	0.4780	1.0009	0.9999	0.9708
Belgium	0.4061	0.7520	0.7336	0.7791	0.5196	1.0009	1.0006	0.9810
Canada	0.2602	0.8237	0.6376	0.7820	0.3500	0.9995	0.9925	0.9663
Denmark	0.4644	0.7497	0.8063	0.8359	0.6121	1.0005	1.0005	0.9745
France	0.4000	0.7537	0.6757	0.8009	0.5426	1.0007	0.9981	0.9688
Germany	0.3091	0.7525	0.7896	0.7533	0.4334	1.0009	0.9988	0.9703
Hong Kong	0.4892	0.7962	0.6022	0.7359	0.6485	1.0000	1.0035	0.9702
Ireland	0.3640	0.7476	0.6663	0.7778	0.4544	1.0005	0.9963	0.9882
Japan	0.2079	0.7988	0.6987	0.7325	0.2605	1.0016	1.0062	1.0135
Netherlands	0.4377	0.7516	0.7619	0.7354	0.5470	1.0009	1.0014	0.9591
Singapore	0.3955	0.8279	0.6244	0.7540	0.4938	1.0018	1.0012	0.9710
Switzerland	0.2055	0.8084	0.7786	0.8650	0.2718	1.0018	0.9968	0.9701
UK	0.3567	0.7599	0.6671	0.7858	0.5289	0.9986	0.9987	0.9616
US	0.2972	0.9900	0.5925	0.8514	0.4217		0.9995	0.9584

(To be continued)

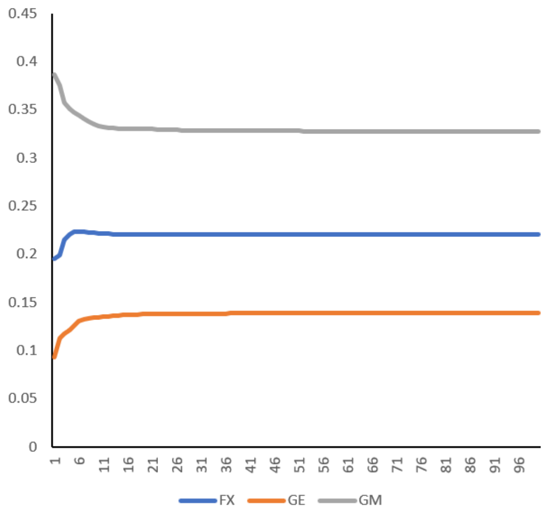
	DP	FX	GE	GM	DP	FX	GE	GM
South Africa	0.4169	0.7251	0.5835	0.7963	0.5444	1.0042	0.9928	0.9747
Norway	0.3158	0.7874	0.7791	0.8438	0.4643	0.9991	0.9994	0.9805
Sweden	0.3364	0.7831	0.8117	0.7964	0.4334	0.9997	0.9899	0.9688
Italy	0.3586	0.7539	0.7311	0.8168	0.4707	1.0009	0.9952	0.9727
Malaysia	0.3802	1.1758	0.6635	0.6879	0.4737	1.0246	0.9993	0.9696
Thailand	0.3694	0.7416	0.9164	0.6630	0.4450	1.0213	0.9106	0.9693
Spain	0.2507	0.7532	0.6686	0.7664	0.3434	1.0011	0.9957	0.9667
Sri Lanka	0.3737	0.7257	0.8832	0.8007	0.5446	1.0011	1.0015	0.9718
South Korea	0.5719	0.7854	0.8448	0.7942	0.7340	1.0148	1.0036	0.9628
New Zealand	0.4716	0.7723	0.8170	0.7927	0.6175	1.0015	1.0046	0.9749
Finland	0.3161	0.7552	0.7124	0.6953	0.4072	1.0002	0.9952	0.9682
Taiwan	0.3160	0.7436	0.8523	0.7215	0.4055	1.0032	0.9997	0.9709
Philippines	0.2855	0.7367	0.7679	0.8566	0.3576	1.0102	1.0007	0.9656
Chile	0.4926	0.7803	0.8626	0.7356	0.6723	1.0009	1.0001	0.9715
India	0.5392	0.6994	0.7627	0.7863	0.6499	1.0004	0.9820	0.9677
Greece	0.4625	0.7626	0.8109	0.9189	0.6372	1.0020	1.0002	0.9716
Portugal	0.3201	0.7515	0.7129	0.8603	0.4152	1.0008	0.9957	0.9718
Turkey	0.3208	0.6634	0.6664	0.9601	0.4378	1.0206	0.9672	0.9621
Mexico	0.5164	0.7066	0.8205	0.9540	0.6784	1.0195	0.9956	0.9675
Hungary	0.5528	0.7220	0.8410	0.7206	0.7443	1.0030	1.0030	0.9802
Luxembourg	0.3539	0.7520	0.8110	0.7832	0.5074	1.0009	1.0006	0.9828
Pakistan	0.4068	0.6672	0.7050	0.7264	0.6680	1.0028	0.9916	0.9509
Cyprus	0.3803	0.7553	0.7788	0.9040	0.5043	1.0006	0.9722	0.9920
Israel	0.3315	0.7727	0.8131	0.6549	0.4509	1.0026	0.9729	0.9880
Colombia	0.3764	0.7939	0.8289	0.7987	0.5436	1.0056	1.0010	0.9828
China	0.3734	0.9878	0.9368	0.8476	0.5306	0.9878	1.0006	0.9765
Czech	0.4196	0.8188	0.8475	0.7722	0.5323	1.0007	1.0124	0.9795
Peru	0.3883	0.6681	0.8557	0.8539	0.5122	1.0023	0.9726	1.0004
Poland	0.3086	0.8504	0.8074	0.8127	0.3828	1.0022	0.9992	0.9809

(b) All countries present case, 94-18

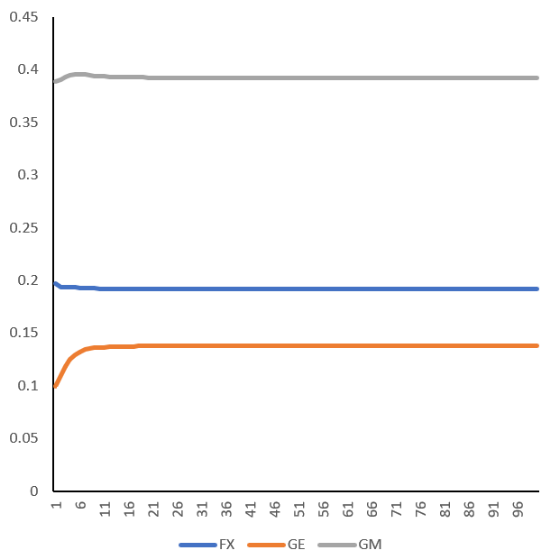
Table 11: Theil U of the different parts with each part's historical average as benchmark, 73-18 (Panel a) and 94-18 (Panel b)



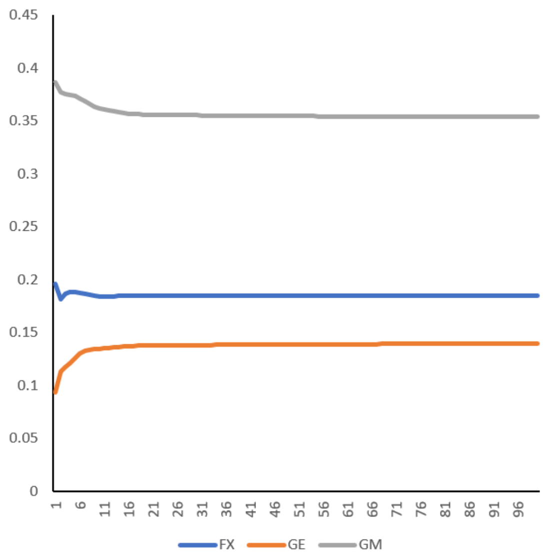
(a) 1 lag, shock on forcSoP



(b) 2 lags, shock on forcSoP



(c) 1 lag, shock on part



(d) 2 lags, shock on part

Fig. 1: Average Generalised Forecast Error Variance Decompositions (FEVD) with one and two lags in the VAR model

Note: Generalised FEVDs of the FX, GE and GM forecasts in a 4-dimensional VAR together with the total SoP forecast (forcSoP). When the SoP forecast is shocked, the contribution of forecast error of the specified part converges to the long-run value (Panels a,b). When the part is shocked, the contribution of forecast error on the total forecast converges to the long run value (Panels c, d). To allow for estimation, the least important part (DP) is dropped.

ACP	US	UK	Switz'nd	S. Africa	Japan	India	Germany	China	Chile	
Min	-0.5246	-0.5307	-0.5257	-0.5141	-0.5157	-0.5216	-0.5346	-0.4774	-0.5154	
Max	0.5289	0.5346	0.5264	0.5266	0.5306	0.5297	0.5387	0.4780	0.5126	
Mean	0.0045	0.0040	0.0033	0.0019	0.0029	0.0035	0.0042	0.0005	0.0034	
Median	0.0081	0.0024	0.0051	-0.0021	-0.0007	0.0047	0.0039	-0.0021	0.0029	
SD	0.2261	0.2258	0.2191	0.2196	0.2221	0.2237	0.2241	0.2010	0.2210	
5% Q	-0.3559	-0.3581	-0.3510	-0.3456	-0.3540	-0.3593	-0.3597	-0.3226	-0.3555	
25% Q	-0.1526	-0.1462	-0.1458	-0.1476	-0.1465	-0.1497	-0.1492	-0.1308	-0.1453	
50% Q	0.0081	0.0024	0.0051	-0.0021	-0.0007	0.0047	0.0039	-0.0021	0.0029	
75% Q	0.1526	0.1496	0.1460	0.1428	0.1491	0.1530	0.1441	0.1348	0.1541	
95% Q	0.3726	0.3708	0.3643	0.3645	0.3603	0.3637	0.3725	0.3261	0.3587	
Long w's	49.42%	49.63%	49.19%	48.96%	49.31%	49.40%	49.14%	49.60%	49.25%	
All Equity	US	UK	Switz'nd	S. Africa	Japan	India	Germany	China	Chile	
Min	-0.7461	-0.7526	-0.7781	-0.7734	-0.7708	-0.7485	-0.7839	-0.9319	-0.7637	
Max	0.7582	0.8047	0.8137	0.8257	0.8379	0.7760	0.7962	0.9543	0.7988	
Mean	0.0227	0.0227	0.0227	0.0227	0.0227	0.0227	0.0227	0.0227	0.0227	
Median	0.0367	0.0317	0.0264	0.0207	0.0302	0.0260	0.0301	0.0091	0.0308	
SD	0.3599	0.3632	0.3877	0.3936	0.3993	0.3599	0.3742	0.5976	0.3699	
5% Q	-0.5535	-0.5583	-0.6043	-0.5923	-0.6132	-0.5592	-0.5804	-0.8281	-0.5747	
25% Q	-0.2437	-0.2344	-0.2584	-0.2720	-0.2761	-0.2326	-0.2388	-0.4945	-0.2461	
50% Q	0.0367	0.0317	0.0264	0.0207	0.0302	0.0260	0.0301	0.0091	0.0308	
75% Q	0.2778	0.2819	0.3166	0.3185	0.3227	0.2863	0.2840	0.5455	0.2897	
95% Q	0.5757	0.5920	0.6179	0.6309	0.6241	0.5757	0.6070	0.8685	0.5882	
Long w's	51.75%	50.72%	49.97%	50.63%	50.20%	51.81%	50.32%	50.63%	50.43%	
Full samples	ACP	AE	DMs ACP	EMs ACP	G7 ACP	E7 ACP	DMs AE	EMs AE	G7 AE	E7 AE
Min	-0.9733	-1.0000	-0.9768	-0.9274	-0.9733	-0.6011	-1.0000	-1.0000	-1.0000	-0.8086
Max	0.9771	1.0000	0.9602	0.9391	0.9771	0.3235	1.0000	1.0000	1.0000	0.6782
Mean	0.0045	0.0227	0.0058	0.0161	0.0022	0.0048	0.0417	0.0500	0.0665	-0.0024
Med	-0.0023	0.0115	-0.0053	0.0027	-0.0269	0.0023	0.0241	0.0356	0.0323	-0.0039
SD	0.2732	0.4222	0.3351	0.3161	0.4303	0.0943	0.5671	0.4683	0.6027	0.1709
5% Q	-0.4594	-0.7578	-0.5627	-0.5132	-0.7457	-0.1333	-0.5627	-0.7924	-1.0000	-0.2605
25% Q	-0.1067	-0.1836	-0.1676	-0.1639	-0.2372	-0.0500	-0.1676	-0.2325	-0.3395	-0.0949
50% Q	-0.0023	0.0115	-0.0053	0.0027	-0.0269	0.0023	-0.0053	0.0356	0.0323	-0.0039
75% Q	0.1102	0.2111	0.0865	0.0938	0.2665	0.0510	0.0865	0.1696	0.5180	0.0840
95% Q	0.4822	0.8903	0.3138	0.2773	0.7921	0.1778	0.3138	0.4823	1.0000	0.3069
Long w's	49.42%	51.75%	49.16%	42.30%	46.84%	50.99%	49.16%	45.09%	52.44%	48.46%
G7	Canada	France	Germany	Japan	UK	US	Italy			
Min	-0.9070	-0.9733	-0.9314	-0.6207	-0.9682	-0.9709	-0.7167			
Max	0.9060	0.9756	0.9238	0.5498	0.9771	0.9411	0.4409			
Mean	0.0054	0.1069	-0.1316	-0.0441	0.0064	0.1321	-0.0599			
Median	-0.0218	0.2058	-0.0781	-0.0409	0.0214	0.2520	-0.0761			
SD	0.4231	0.5339	0.4235	0.1780	0.4880	0.5471	0.1786			
5% Q	-0.6212	-0.8044	-0.8448	-0.3366	-0.8051	-0.7565	-0.3113			
25% Q	-0.3480	-0.2814	-0.4321	-0.1364	-0.3750	-0.3439	-0.1546			
50% Q	-0.0218	0.2058	-0.0781	-0.0409	0.0214	0.2520	-0.0761			
75% Q	0.3025	0.4441	0.1672	0.0595	0.3246	0.6230	0.0150			
95% Q	0.6810	0.9524	0.4512	0.2721	0.9069	0.8776	0.2171			
Long w's	49.37%	54.43%	40.51%	37.97%	51.90%	62.03%	31.65%			
E7	S. Africa	Korea	Taiwan	India	Turkey	Mexico	China			
Min	-0.6011	-0.1847	-0.2064	-0.1096	-0.1242	-0.2019	-0.1921			
Max	0.3235	0.1523	0.1918	0.2469	0.1041	0.1776	0.2973			
Mean	0.0230	0.0055	-0.0008	0.0246	-0.0004	-0.0260	0.0075			
Median	0.0205	0.0062	-0.0076	0.0263	-0.0044	-0.0243	0.0053			
SD	0.1651	0.0553	0.0748	0.0819	0.0465	0.0821	0.0949			
5% Q	-0.2109	-0.0817	-0.1141	-0.0943	-0.0690	-0.1572	-0.1438			
25% Q	-0.0748	-0.0246	-0.0469	-0.0366	-0.0275	-0.0854	-0.0568			
50% Q	0.0205	0.0062	-0.0076	0.0263	-0.0044	-0.0243	0.0053			
75% Q	0.1413	0.0356	0.0340	0.0713	0.0310	0.0289	0.0637			
95% Q	0.2592	0.0967	0.1369	0.1983	0.0771	0.1210	0.1489			
Long w's	53.16%	58.23%	49.37%	60.76%	44.30%	36.71%	54.43%			

Table 12: Portfolio weights statistics and quantiles (Q) with short-selling

Note: "ACP" and "All-Equity" report averages across countries of the respective statistic (e.g. average of country averages) and of the percentage of positive weights (long w's) for the all countries present (ACP) and all-equity (AE) cases with short-selling and EMD forecasting. "Full Samples" reports the statistics of the entire respective sample (i.e. not averages) and the same cases as above. ACP and AE refer to the 44-country full sample, DMs to the 24 developed markets' sample, EMs to the 20 emerging markets sample, G7 to USA, UK, Canada, Japan, France, Germany and Italy, while E7 to the 7 largest emerging markets in our sample (S. Africa, China, Taiwan, Korea, India, Turkey, Mexico). The G7 and E7 country values are from the ACP case that includes a risk-free asset.

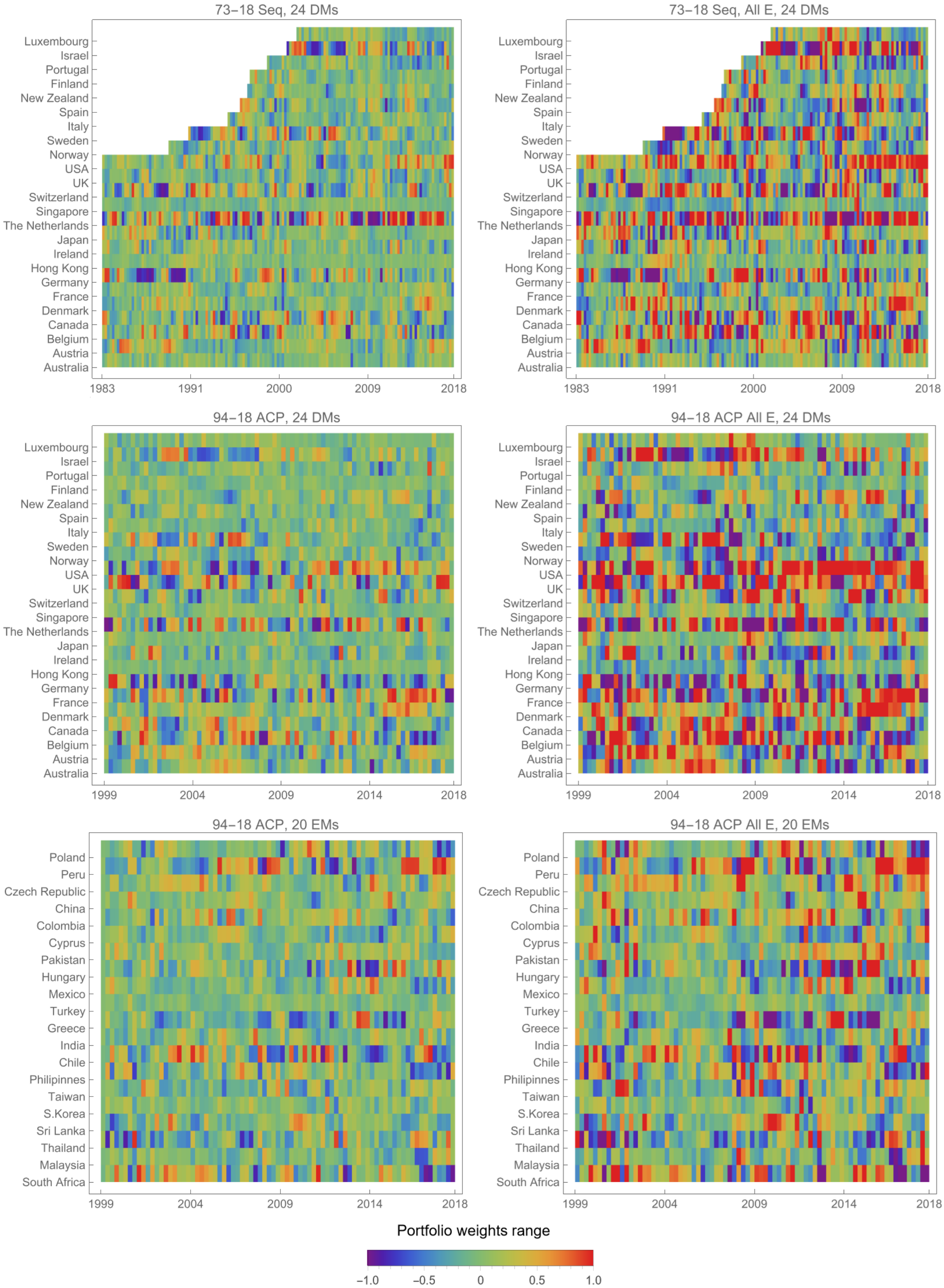
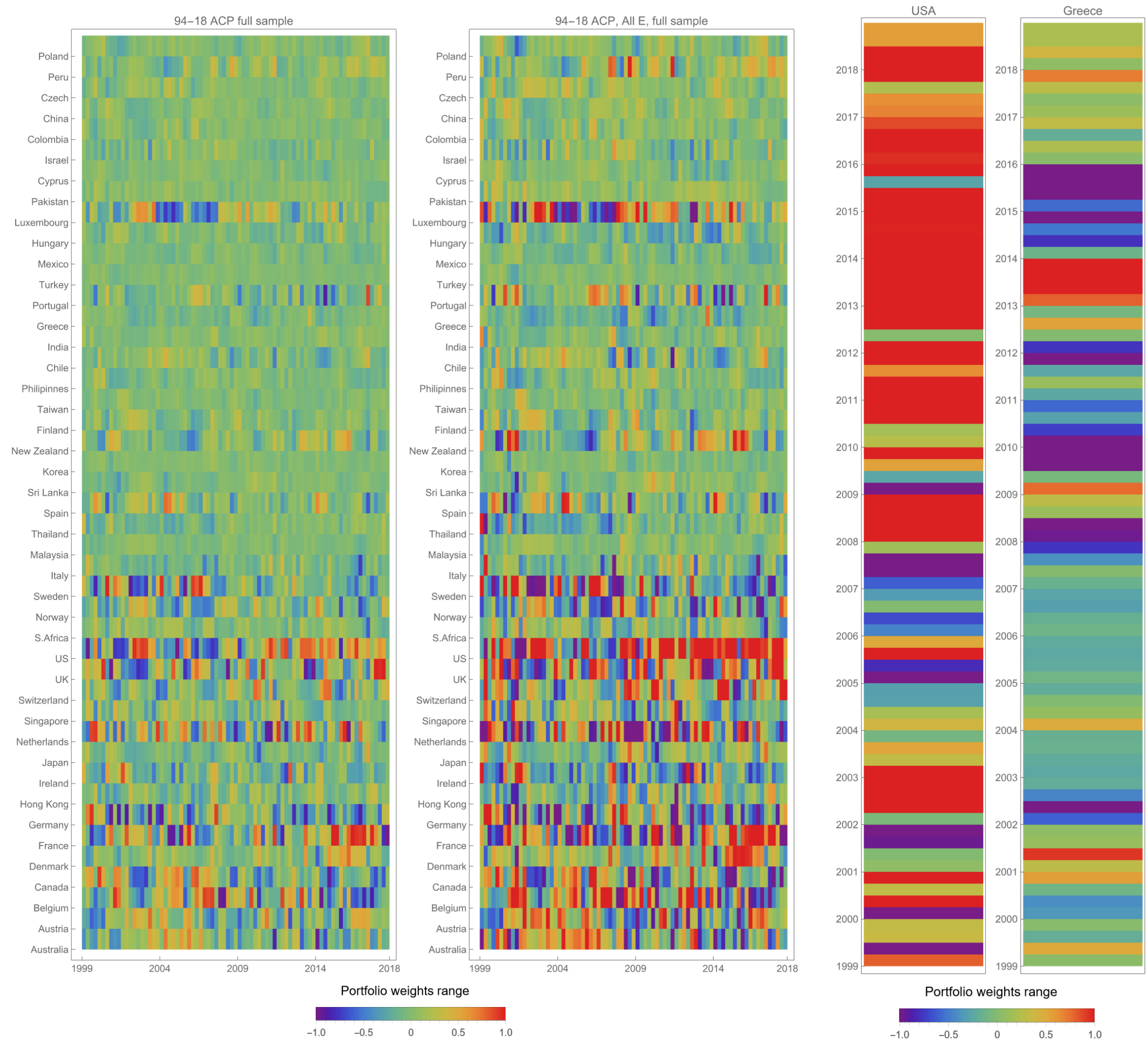


Fig. 2: Heat maps for a US-based investor.

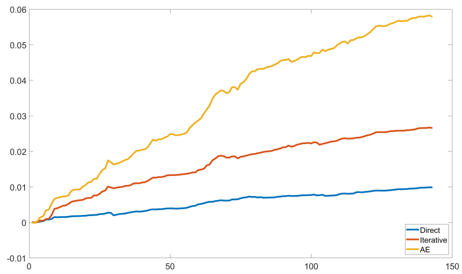


(a) 44 Countries, Unconstrained weights, 94-18

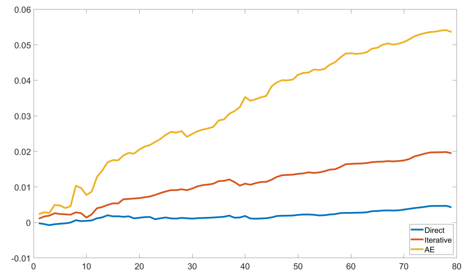
(b) All Equity, 94-18

Fig. 3: Heat maps for a US-based investor (continued)

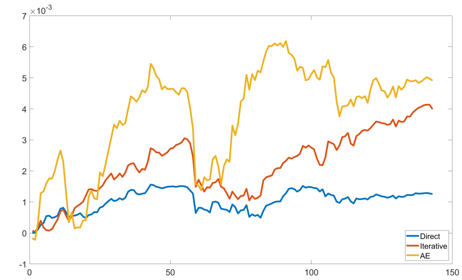
Note: Panel (a) reports the portfolio weights under short-selling with and without (AE) a risk-free asset when all countries are present (1994-2018). Panel (b) reports the All-Equity weights with short-selling for USA (DM sub-sample) and Greece (EM sub-sample).



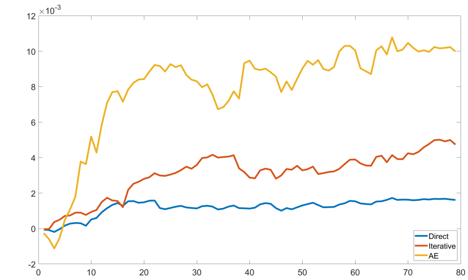
(a) EMD Sequential, 73-18



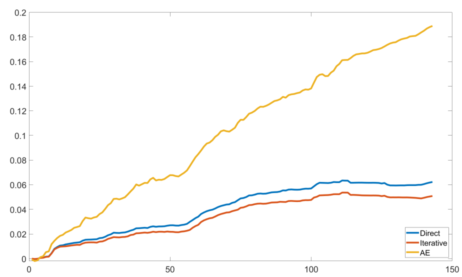
(b) EMD ACP, 94-18



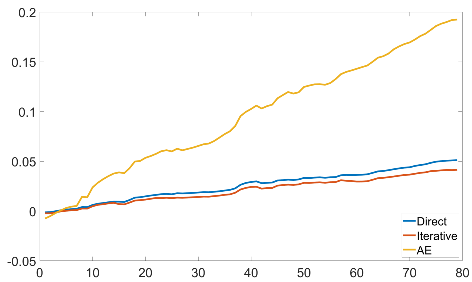
(c) OR Sequential, 73-18



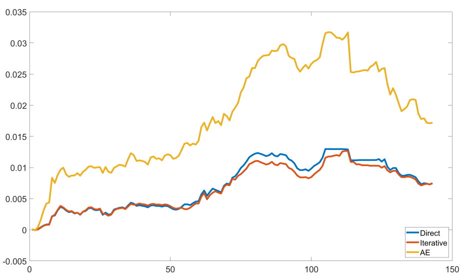
(d) OR SoP ACP, 94-18



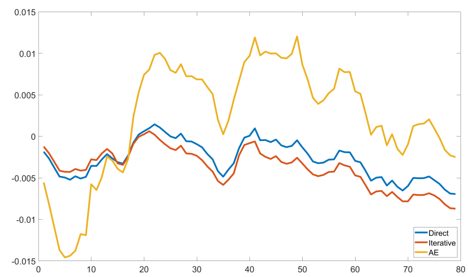
(e) EMD Sequential, 73-18, SS



(f) EMD ACP, 94-18, SS



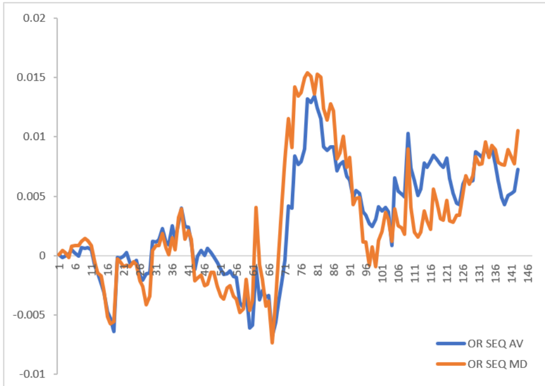
(g) OR Sequential, 73-18, SS



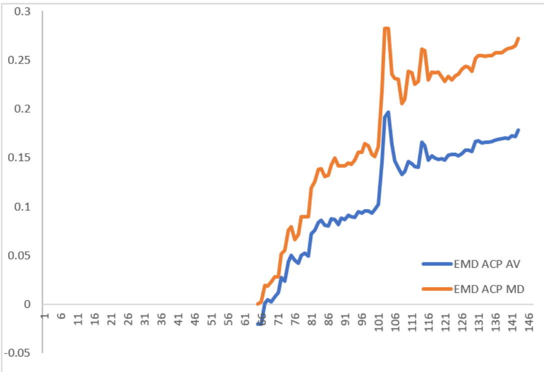
(h) OR SoP ACP, 94-18, SS

Fig. 4: Rolling utility gains for different cases, US-based investor.

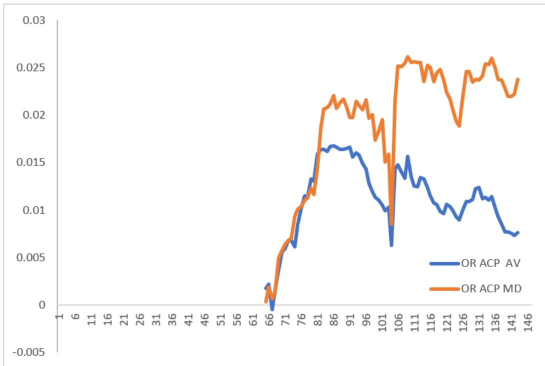
(a) EMD Sequential



(b) Original SoP Sequential



(c) EMD All Countries Present



(d) Original SoP All Countries Present

Fig. 5: Cumulative Squared Errors

Note: Average (AV) and median (MD) CSE across countries for different cases.

	Developed markets					Emerging markets				
			US Constrained							
73-18	CE	R	SD	Sharpe	M2					
HA		4.45%	2.59%	0.3356						
OR Dir	0.76%	5.50%	3.71%	0.5189	0.48%					
OR Iter	2.47%	7.50%	6.48%	0.6054	0.70%					
EMD Dir	3.39%	8.04%	5.12%	0.8707	1.39%					
EMD Iter	7.64%	12.76%	8.56%	1.0725	1.91%					
			US Unconstrained							
HA		5.92%	9.49%	0.2471						
OR Dir	3.01%	7.34%	10.75%	0.3496	0.97%					
OR Iter	3.10%	7.06%	8.86%	0.3930	1.38%					
EMD Dir	20.57%	27.40%	13.42%	1.7746	14.49%					
EMD Iter	15.57%	21.83%	11.16%	1.6353	13.17%					
			US Constrained					US Constrained		
94-18	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
HA		2.85%	4.37%	0.2618			3.44%	5.75%	0.302	
OR Dir	0.44%	3.22%	3.54%	0.4291	0.73%	0.65%	3.95%	3.75%	0.598	1.70%
OR Iter	2.04%	5.10%	6.42%	0.5295	1.17%	1.90%	5.46%	6.34%	0.592	1.67%
EMD Dir	1.76%	4.68%	5.05%	0.5879	1.42%	4.17%	7.67%	6.20%	0.962	3.80%
EMD Iter	5.44%	8.69%	7.67%	0.9103	2.83%	11.35%	15.69%	11.09%	1.262	5.52%
			US Unconstrained					US Unconstrained		
HA		4.24%	8.34%	0.3042			2.74%	16.88%	0.062	
OR Dir	0.37%	4.38%	6.81%	0.3933	0.74%	0.65%	3.95%	3.75%	0.598	9.05%
OR Iter	-0.01%	3.93%	6.23%	0.3570	0.44%	1.90%	5.46%	6.34%	0.592	8.96%
EMD Dir	14.62%	18.58%	6.46%	2.6110	19.24%	41.94%	43.57%	13.19%	3.175	52.57%
EMD Iter	10.87%	14.75%	5.79%	2.2511	16.24%	35.17%	37.84%	16.65%	2.170	35.60%
			All Equity case					All Equity case		
73-18	CE	R	SD	Sharpe	M2					
HA C		12.05%	17.85%	0.4744	0.36%					
OR C	1.66%	13.51%	18.40%	0.5399	0.53%					
EMD C	16.22%	32.79%	27.75%	1.0523	1.86%					
94-18						CE	R	SD	Sharpe	M2
HA C		7.04%	20.04%	0.2663	0.02%		10.51%	28.53%	0.3088	0.04%
OR C	4.48%	10.81%	18.18%	0.5009	1.04%	3.97%	12.02%	17.87%	0.5770	1.58%
EMD C	13.36%	20.96%	21.38%	0.9006	2.79%	31.56%	42.81%	29.80%	1.3795	6.20%

Table 13: Results for emerging and developed markets, EMD Sum-of-Parts

Note: As earlier, the 73-18 sample refers to the the Sequential case and the 94-18 sample to the All Countries Present case. Developed Markets include Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Japan, Netherlands, Singapore, Switzerland, the UK, the USA, Norway, Sweden, Italy, Spain, New Zealand, Finland, Portugal, Luxembourg and Israel (24 countries), while EMs include South Africa, Malaysia, Thailand, Sri Lanka, S. Korea, Taiwan, Philippines, Chile, India, Greece, Turkey, Mexico, Hungary, Pakistan, Cyprus, Colombia, China, Czech Republic, Peru and Poland.

	Constrained portfolio					Unconstrained portfolio				
73-18	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
HA		4.54%	2.84%	0.3277			3.02%	10.36%	-0.0329	
OR Dir	0.58%	5.22%	4.20%	0.3835	0.16%	4.91%	8.09%	11.11%	0.4031	4.52%
OR Iter	2.32%	7.21%	6.55%	0.5505	0.63%	2.79%	5.57%	9.16%	0.2141	2.56%
EMD Dir	7.66%	12.44%	5.65%	1.5632	3.51%	41.75%	45.60%	13.80%	3.0428	31.86%
EMD Iter	13.98%	19.10%	8.13%	1.9059	4.48%	31.15%	34.43%	11.57%	2.6645	27.94%
94-18										
HA		3.88%	4.58%	0.4704			3.87%	4.58%	0.4711	
OR Dir	0.69%	4.55%	4.44%	0.6372	0.76%	1.52%	6.21%	10.19%	0.4417	-0.13%
OR Iter	1.57%	5.55%	5.56%	0.6875	0.99%	0.10%	4.51%	8.62%	0.3241	-0.67%
EMD Dir	6.80%	10.71%	4.92%	1.8262	6.20%	48.38%	53.48%	11.99%	4.3173	17.60%
EMD Iter	11.68%	15.75%	6.37%	2.2015	7.92%	36.71%	41.49%	10.61%	3.7489	15.00%

All-Equity

	Constrained portfolio				
73-18	CE	R	SD	Sharpe	M2
HA		3.47%	9.32%	0.4218	0.27%
OR	2.62%	4.73%	11.20%	0.5458	0.62%
EMD	35.82%	16.22%	12.95%	2.0084	4.77%
94-18					
HA		3.16%	10.58%	0.4234	-0.22%
OR	4.70%	4.44%	9.13%	0.7335	1.20%
EMD	33.25%	14.40%	11.29%	2.1219	7.56%

(a) Portfolio performance metrics

	EMD Seq	EMD ACP	OR Seq	OR ACP		EMD Seq	EMD ACP	OR Seq	OR ACP
Australia	0.8239	0.8210	1.0046	0.9949	Sri Lanka	0.9022	0.8089	0.9994	0.9930
Austria	0.8398	0.7600	1.0062	0.9960	Korea	0.9253	0.8838	0.9980	0.9986
Belgium	0.9247	0.9173	0.9991	0.9959	New Zealand	0.9467	0.9159	1.0205	1.0216
Canada	0.8491	0.8480	1.0066	0.9916	Finland	0.9874	0.9414	1.0069	0.9941
Denmark	1.1445	0.8994	1.0174	0.9969	Taiwan	0.9652	0.8932	1.0006	0.9956
France	0.8521	0.8198	1.0019	0.9898	Philippines	0.9818	0.9231	0.9984	0.9894
Germany	0.9104	0.8851	1.0026	1.0066	Chile	0.9272	0.8624	1.0035	0.9983
Hong Kong	0.8945	0.8634	0.9924	0.9981	India	0.9009	0.7961	0.9948	0.9840
Ireland	0.9241	0.9551	1.0137	1.0009	Greece	1.0473	1.0705	0.9926	0.9804
Japan	0.8707	0.8319	1.0062	1.1746	Portugal	0.9021	0.8436	1.0003	0.9972
Netherlands	0.8961	0.8576	1.0051	0.9957	Turkey	0.9375	0.8439	0.9953	0.9803
Singapore	0.8734	0.8269	0.9964	0.9886	Mexico	0.9534	0.8442	1.0071	1.0121
Switzerland	0.8807	0.8591	1.0179	1.0088	Hungary	1.0318	1.0217	1.0020	0.9924
UK	0.8733	0.8307	0.9936	0.9771	Luxembourg	1.0418	1.0199	1.0214	1.0283
US	0.8572	0.8148	0.9972	0.9692	Pakistan	0.9539	0.8905	0.9955	0.9684
South Africa	0.8569	0.8479	0.9964	0.9898	Cyprus	1.0951	1.1129	1.0104	1.0415
Norway	0.9611	0.9024	1.0121	1.0054	Israel	1.0287	1.1975	1.0056	1.0298
Sweden	0.9689	0.9206	1.0006	1.0008	Colombia	0.8845	0.8553	1.0048	1.0537
Italy	0.9347	0.8594	0.9987	0.9940	China	0.9535	0.8665	1.0025	1.0014
Malaysia	0.8263	0.7882	1.0010	1.0016	Czech	0.9299	0.8382	1.0038	1.0031
Thailand	0.9803	1.0313	0.9991	0.9886	Peru	1.3099	1.6768	1.0295	1.4175
Spain	0.8997	0.8540	1.0003	0.9938	Poland	0.9524	0.8863	1.0034	1.0054

(b) Theil's U

Table 14: Results for a US-based investor using monthly data

Appendix

Additional Graphs

	Example		UK Jan-Feb 1973	
	t	t+1	t	t+1
Index	100	110	310.74	305.20
Dividend		5		3.63
Earnings	8	10	11.969	13.0184
USD spot FX	1.5	1.75	0.8514	0.8879
PE ratio	12.5	11	18.1	16.3
	Values	Logs	Values	Logs
Return (local)	1.15	0.1398	0.9822	-0.0180
PE growth rate	0.88	-0.1278	0.9006	-0.1047
Dividend yield	1.0455	0.0445	1.0030	0.0030
Earnings growth	1.25	0.2231	1.0877	0.0840
Sum-of-Parts		0.1398		-0.0177
Index ret USD	1.3417	0.2939	1.0243	0.0240
FX ret	1.1667	0.1542	1.0429	0.0420
USD ret - FX ret		0.1398		-0.0180
R-GE-GM-DP-FX		0		0.000307

Table 15: Sum-of-Parts numerical and real data examples.

Note: The index in the numerical example does not include dividends. In the real data example, the index is the total return index which includes dividends, and the dividend value is the annualized value.

	VaR (m)	ES (m)	VaR (ann)	ES (ann)	Skew	Kurt
73-18			Constrained portfolio			
HA	-0.94%	-1.71%	-11.25%	-20.56%	-0.6542	3.3949
OR Dir	-1.59%	-2.54%	-19.06%	-30.53%	-0.6538	3.4832
OR Iter	-2.48%	-3.74%	-29.71%	-44.92%	0.2170	3.5713
EMD Dir	-1.27%	-2.70%	-15.29%	-32.42%	0.1838	4.4318
EMD Iter	-1.71%	-2.65%	-20.56%	-31.80%	1.4650	6.8285
94-18						
HA	-1.70%	-2.52%	-20.44%	-30.24%	0.5842	2.7134
OR Dir	-1.47%	-2.89%	-17.66%	-34.66%	-1.0207	8.2601
OR Iter	-2.44%	-3.44%	-29.32%	-41.29%	-0.1561	2.0236
EMD Dir	-1.03%	-2.19%	-12.41%	-26.30%	0.2989	2.8545
EMD Iter	-1.45%	-2.35%	-17.34%	-28.20%	0.4283	1.2190
73-18			Unconstrained portfolio			
HA	-4.88%	-7.07%	-58.51%	-84.80%	-0.4546	1.3923
OR Dir	-4.40%	-6.30%	-52.77%	-75.54%	-0.1197	1.1288
OR Iter	-4.09%	-5.75%	-49.12%	-68.97%	-0.1861	1.7996
EMD Dir	-1.99%	-3.72%	-23.91%	-44.59%	0.5451	1.1247
EMD Iter	-1.14%	-2.81%	-13.67%	-33.73%	1.0103	2.3648
94-18						
HA	-1.70%	-3.87%	-20.44%	-46.44%	0.5854	2.7136
OR Dir	-4.25%	-5.26%	-50.97%	-63.17%	0.1460	-0.0602
OR Iter	-3.70%	-4.88%	-44.41%	-58.59%	0.1902	0.6556
EMD Dir	-0.60%	-1.96%	-7.24%	-23.56%	0.6347	0.9039
EMD Iter	-0.25%	-1.36%	-3.05%	-16.30%	0.8951	1.3622

Table 16: Non-parametric Value-at-Risk and Expected Shortfall for monthly data

Note: Value-at-Risk (VaR) and Expected Shortfall (ES) for monthly data. Var is the value at the 5th percentile of returns while ES the average of all returns below VaR.

	Constrained portfolio					Unconstrained portfolio				
73-18	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
HA		4.51%	3.15%	0.2945			2.88%	10.39%	-0.0670	
OR Dir	0.49%	5.12%	4.66%	0.3301	0.11%	2.99%	5.73%	9.64%	0.2227	3.01%
OR Iter	1.60%	6.47%	6.81%	0.4241	0.41%	2.91%	5.34%	7.91%	0.2223	3.01%
EMD Dir	3.96%	8.62%	4.98%	1.0107	2.26%	25.33%	29.19%	14.34%	1.7861	19.26%
EMD Iter	11.07%	16.31%	9.13%	1.3950	3.47%	22.69%	26.61%	14.56%	1.5818	17.14%
94-18										
HA		3.32%	4.04%	0.4005			6.47%	9.12%	0.5231	
OR Dir	0.65%	3.95%	3.75%	0.5976	0.80%	-2.78%	3.48%	7.87%	0.2263	-2.71%
OR Iter	1.90%	5.46%	6.34%	0.5924	0.78%	-3.49%	2.53%	6.16%	0.1349	-3.54%
EMD Dir	1.62%	4.84%	2.51%	1.2499	3.43%	18.45%	24.61%	7.20%	3.1815	24.24%
EMD Iter	10.20%	13.89%	7.31%	1.6685	5.13%	16.16%	22.64%	9.18%	2.2812	16.03%

All-Equity

	Constrained portfolio					Unconstrained portfolio				
73-18	CE	R	SD	Sharpe	M2	CE	R	SD	Sharpe	M2
HA		10.78%	18.53%	0.3886	0.30%		10.39%	21.45%	0.3177	
OR	1.97%	13.13%	19.54%	0.4888	0.61%	6.90%	21.19%	29.15%	0.6041	6.97%
EMD	27.87%	40.87%	23.78%	1.5682	4.01%	88.03%	113.32%	44.14%	2.4858	26.53%
94-18										
HA		9.61%	21.83%	0.3623	-0.15%		14.11%	23.70%	0.5234	
OR	3.97%	12.02%	17.87%	0.5770	0.71%	-1.01%	12.49%	22.37%	0.4821	-0.37%
EMD	28.64%	37.34%	19.63%	1.8153	5.72%	77.79%	92.14%	24.21%	3.7353	29.29%

(a) Portfolio performance metrics

	EMD Seq	EMD ACP	OR Seq	OR ACP		EMD Seq	EMD ACP	OR Seq	OR ACP
Australia	0.8603	0.8836	0.9997	0.9825	Sri Lanka	0.8310	0.8641	0.9942	0.9813
Austria	0.9642	0.9707	0.9978	0.9903	Korea	0.8290	0.9643	0.9973	1.0005
Belgium	0.9143	0.8875	0.9883	0.9895	New Zealand	0.8845	0.9027	1.0291	1.0255
Canada	0.9077	0.9472	0.9999	0.9852	Finland	0.9444	0.9535	0.9988	0.9741
Denmark	1.0407	0.9627	1.0086	0.9840	Taiwan	1.0693	1.0100	0.9804	0.9773
France	0.7827	0.7845	0.9936	0.9847	Philipinnes	0.7213	0.7739	0.9924	0.9775
Germany	0.9431	0.8680	0.9926	0.9921	Chile	0.7603	0.7449	0.9864	0.9933
Hong Kong	0.9091	0.9987	0.9844	0.9950	India	0.8627	0.7840	0.9779	0.9682
Ireland	1.0528	1.1093	0.9979	0.9865	Greece	0.9141	0.7741	0.9736	0.9780
Japan	0.8761	0.9500	1.0006	1.1057	Portugal	0.8390	0.8756	0.9990	0.9897
Netherlands	0.8980	0.9978	1.0027	0.9944	Turkey	0.9371	0.8193	0.9789	0.9630
Singapore	0.9693	0.9587	0.9941	0.9818	Mexico	0.8306	0.8422	0.9961	1.0090
Switzerland	0.9373	0.9299	1.0265	1.0050	Hungary	0.9401	0.8508	0.9925	0.9787
UK	0.8997	0.9027	0.9765	0.9533	Luxembourg	1.5776	1.0555	1.0188	1.0158
US	0.9736	0.9038	0.9894	0.9446	Pakistan	0.8581	0.8511	0.9740	0.9375
South Africa	0.8085	0.8343	0.9882	0.9697	Cyprus	1.0621	1.2062	0.9998	1.0141
Norway	1.0067	0.9344	1.0150	0.9978	Israel	1.4070	1.3920	0.9830	1.0112
Sweden	1.0444	0.9088	0.9952	0.9894	Colombia	0.7508	0.7780	0.9929	1.0319
Italy	0.9269	0.8704	0.9938	0.9928	China	0.8766	0.9272	1.0114	0.9847
Malaysia	0.9389	1.0462	0.9897	0.9852	Czech	0.8828	0.8687	1.0009	0.9878
Thailand	1.1330	1.3346	0.9854	0.9727	Peru	1.0971	1.1206	1.0092	1.2148
Spain	0.7412	0.7420	0.9985	0.9883	Poland	0.8247	1.0195	1.0125	1.0086

(b) Theil's U

Table 17: Look-ahead bias check - Results for a US-based investor

References

Ang, A. and Chen, J. (2010). Yield curve predictors of foreign exchange returns. In *AFA 2011 Denver Meetings Paper*.

Barroso, P. and Santa-Clara, P. (2015). Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120.

Burnside, C., Eichenbaum, M., and Rebelo, S. (2011). Carry trade and momentum in currency markets. *Annual Review of Financial Economics*, 3(1):511–535.

Campbell, J. Y. and Thompson, S. B. (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *The Review of Financial Studies*, 21(4):1509–1531.

Clark, T. E. and West, K. D. (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of econometrics*, 138(1):291–311.

Dai, Z. and Zhu, H. (2020). Forecasting stock market returns by combining sum-of-the-parts and ensemble empirical mode decomposition. *Applied Economics*, 52(21):2309–2323.

Diebold, F. X. and Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & economic statistics*, 20(1):134–144.

Errunza, V., Hogan, K., and Hung, M.-W. (1999). Can the gains from international diversification be achieved without trading abroad? *The Journal of Finance*, 54(6):2075–2107.

Faria, G. and Verona, F. (2018). Forecasting stock market returns by summing the frequency-decomposed parts. *Journal of Empirical Finance*, 45:228–242.

Ferreira, M. A. and Santa-Clara, P. (2011). Forecasting stock market returns: The sum of the parts is more than the whole. *Journal of Financial Economics*, 100(3):514–537.

Harvey, D. I., Leybourne, S. J., and Newbold, P. (1998). Tests for forecast encompassing. *Journal of Business & Economic Statistics*, 16(2):254–259.

Higham, D. J. and Higham, N. J. (1998). Structured backward error and condition of generalized eigenvalue problems. *SIAM Journal on Matrix Analysis and Applications*, 20(2):493–512.

Huang, N. E., Shen, Z., Long, S. R., Wu, M. C., Shih, H. H., Zheng, Q., Yen, N.-C., Tung, C. C., and Liu, H. H. (1998). The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis. *Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences*, 454(1971):903–995.

Jordan, S. J. and Vivian, A. (2011). Forecasting stock returns internationally: can fundamental-price models beat the historical average? In *IFABS 2011 Conference (Rome) Paper*.

Jordan, S. J., Vivian, A., and Wohar, M. E. (2014). Sticky prices or economically-linked economies: The case of forecasting the chinese stock market. *Journal of International Money and Finance*, 41:95–109.

Jordan, S. J., Vivian, A., and Wohar, M. E. (2015). Location, location, location: currency effects and return predictability? *Applied Economics*, 47(18):1883–1898.

Jordan, S. J., Vivian, A., and Wohar, M. E. (2017). Forecasting market returns: bagging or combining? *International Journal of Forecasting*, 33(1):102–120.

Lo, A. W. (2002). The statistics of sharpe ratios. *Financial analysts journal*, 58(4):36–52.

Lustig, H. and Verdelhan, A. (2007). The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review*, 97(1):89–117.
Meese, R. A. and Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of international economics*, 14(1-2):3–24.
Menkhoff, L., Sarno, L., Schmeling, M., and Schrimpf, A. (2012). Carry trades and global foreign exchange volatility. *The Journal of Finance*, 67(2):681–718.
Modigliani, F. and Modigliani, L. (1997). Risk-adjusted performance. *Journal of portfolio management*, 23(2):45–54.
Pennacchi, G. G. (2008). *Theory of asset pricing*. Pearson/Addison-Wesley Boston.
Solnik, B. (1993). The performance of international asset allocation strategies using conditioning information. *Journal of Empirical Finance*, 1(1):33–55.
Solnik, B. and Noetzlin, B. (1982). Optimal international asset allocation. *The Journal of Portfolio Management*, 9(1):11–21.
Solnik, B. H. (1974). Why not diversify internationally rather than domestically? *Financial analysts journal*, 30(4):48–54.
Vivian, A. and Wohar, M. E. (2013). The output gap and stock returns: Do cyclical fluctuations predict portfolio returns? *International Review of Financial Analysis*, 26:40–50.
Welch, I. and Goyal, A. (2008). A comprehensive look at the empirical performance of equity premium prediction. *The Review of Financial Studies*, 21(4):1455–1508.
Wu, Z. and Huang, N. E. (2009). Ensemble empirical mode decomposition: a noise-assisted data analysis method. *Advances in adaptive data analysis*, 1(01):1–41.
Zeiler, A., Faltermeier, R., Keck, I. R., Tomé, A. M., Puntonet, C. G., and Lang, E. W. (2010). Empirical mode decomposition-an introduction. In *The 2010 international joint conference on neural networks (IJCNN)*, pages 1–8. IEEE.