The student as a learner, a knower, and a facilitator:

Shared epistemic agency enacted as an innovative school-based mathematics pedagogy

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I, Ijeaku Iheoma Mezue, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signed,

Abstract

This study is an inquiry into my students' participation and empowerment in all aspects of their mathematics learning that aims to explore the possibility of improving their relationship with and knowledge of mathematics. It considers the learning process in general, as well as the way it shapes the knowledge that it facilitates, and the different relationships that students have with this knowledge. This study is guided by the belief that improving my students' mathematics knowledge requires the establishment of an innovative pedagogy based on knowledge creation.

I elaborate the concept of shared epistemic agency to explain the phenomenon of students taking responsibility for the advancement of their own mathematics knowledge and that of the classroom community. The concept draws on Damşa's notion of shared epistemic agency, Scardamalia & Bereiter's discussion of knowledge building, and Nonaka's analysis of knowledge creation.

I carried out action research that applied these principles over one academic year, requiring the participants of my classroom to blend their authority with mine as they assumed the roles normally reserved for the teacher. Using six key characteristics of shared epistemic agency that I identify in the existing research, as well as the unit of analysis, I analysed the qualitative data that was collected.

I show that the shared epistemic agency that is necessary for knowledge advancement in a secondary school mathematics classroom emerged as the students participated in my innovative pedagogical environment. Moreover, I demonstrate that this agency can be reconceptualised in terms of particular kinds of

student behaviour and a particular kind of learning community. I argue that the student, in such an environment, is a competent, adaptive Participant who takes up flexible positions as a learner, a knower, and a facilitator. The classroom that developed as a democratically interactive learning community sustained the emergence of the Participant.

Impact Statement

I argue that the secondary school student is a competent individual who can take responsibility for their mathematics knowledge, and that of their learning community. In an age in which knowledge is readily available and accessible to young people, I present the notion of the Participant who can use the resources available to them, specifically through interaction with other Participants, to build and share mathematics knowledge without relying on the teacher as the epistemic authority.

This Participant is identifiable in their capacity as:

- A learner who controls their knowing and unknowing, who is productive of epistemic interactions, and who is not knowledge-less. This learner has the potential to be transformative.
- A knower with epistemic authority who is relational in their response to an unknowing, and who is interdependent with a learner.
- A facilitator with process authority who can negotiate the blending of their authority with that of other Participants to advance collective knowledge.

The new Learning Community is identifiable as a classroom community that is:

- Characterised by an interactive practice in which the Participants learn mathematics through epistemic interactions and, in which their participation positions them as learners, knowers, and facilitators.
- Productive for the creation of mathematics knowledge in a way that is demonstrably connected with the enactment of the pedagogy and its capacities for epistemic interaction.

- Democratic and egalitarian. It was presumed that all Participants are able to participate, and their participation justified this presumption.

This learning community is sustained by:

- Its definition of competence as participation in epistemic interaction.
- The formation of a common identity and a sense of belonging as a result of mutual participation, and the subsequent accountability of the students to the practice of the learning community.

Of equal benefit is the notion of the teacher as an Educator, whose purpose is to draw out from the participant their latent potential.

This study contributes to the theory and practice of mathematics education as it presents the revitalised conception of the Participant as a challenge to the efficacy of the current discourse on education in the UK. It sheds light on government policy that implicitly construes students as incapable, highlights how classroom practices such as questioning can limit students' participation, and presents an alternative to practices such as ability setting. This study presents the possibility of an alternative student, classroom, and teacher that could transform mathematics education.

The study also presents the possibility of considering the teacher as a professional capable of bringing about change in the classroom, in a manner that could be transformative to educational practices from the bottom up, heeding and reiterating the calls for reform made in both current and older research (cf. Elliott, 2011; Schon, 2008; Stenhouse, 1981).

The benefits of this research can be realised through publication in journals, discussions at educational forums and presentations at conferences; as well as, more personally, through my relationship to my own teaching practice. As a senior leader in a secondary school, I continue to challenge the presuppositions of my colleagues, urging them to consider the student as competent, and to share authority with them as partners in an educational journey.

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To God be the Glory.

1 INTRODUCTION

Socio-cultural theories of learning (e.g. Boaler & Greeno, 2000; Lave & Wenger, 1991; Rogoff et al., 1998; Wenger, 1998) suggest that the way in which mathematics learning occurs – that is – the specific practices involved in the learning process, shape and define the knowledge produced, as well as the different relationships students have with this knowledge and the uses they make of it.

Boaler's research suggests that students who participate in negotiating and interrogating mathematics as they learn it are more able to use apply its principles in situations that require such practices when compared with students who learn mathematics by working through exercises from a textbook (2002a). Boaler argues that students' knowledge is applied in situations outside the classroom in a way that is dependent on the situation within the classroom, given that knowledge is co-constituted by how the learning occurred. My study, in this spirit, starts from the position that improving the mathematics knowledge of my students and their relationship with the subject requires a change in their current learning practices.

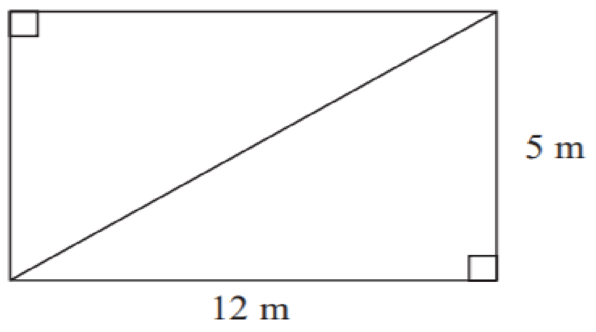
For the first fifteen years of my teaching career, my classroom pedagogy was similar to that of the mathematic teachers I had encountered, including those with whom I taught. Summarily, my role in the classroom, as the teacher, was to transfer my mathematics knowledge to the students; the students' role was to listen to me and internalise this information; this was the core of the learning process. At the start of the lesson, I would explain a topic to the students and work out a select number of examples on the board; the students then had the opportunity to ask questions. Following questions, the students would try out similar problems relevant to the topic.

At this point, I would supervise the classroom, checking students' work and answering questions that may arise; finally, a summary of our learning (or my teaching) occurred at the end of the lesson.

Due to my years of experience as a mathematics teacher, I became responsible for teaching the students who appeared to have a particular difficulty with learning mathematics. Over time I came to believe that my classroom pedagogy, combined with the nature of the students' apparent difficulties, contributed to reinforcing in the students' minds the conviction that they were not good at mathematics. The anecdotal evidence that formed the basis of my belief was what I viewed as the students' increasing reliance on my mathematics knowledge. This reliance manifested as a reluctance to take chances in the classroom, answer questions, or engage with the reasoning behind mathematical principles.

This reliance is evident when students confront a mathematics question to which the mathematical concepts to be applied are not explicit (see figure 1.1).

This rectangular frame is made from 5 straight pieces of metal.



The weight of the metal is 1.5 kg per metre.

Work out the total weight of the metal in the frame.

Figure 1.1 – Edexcel Mathematics GCSE Summer 2017 Higher Paper 1, Question 5

While my students could grasp composite mathematical concepts, such as Pythagoras' theorem, they often struggled to make sense of what is required by questions such as these. They resorted to approaches that are not mathematically rational, such as manipulating the numbers using any of the four arithmetical operations, or using formulae that relate to irrelevant information in the question, such as by calculating the area of the triangle. With a significant proportion of the reformed GCSE Mathematics Higher Paper requiring the application of mathematical knowledge in "a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions" (*National Curriculum in England*, 2021, paragraph 2), my students needed to develop this particular skill in order to achieve higher grades or even to pass mathematics at GCSE. Their learning practice needed to change.

1.1 The Context of this Research Study

Mathematics, especially at the secondary level, is an important subject both to schools and to students' own futures. Three government policies – Key Stage testing, first undertaken in 1991; the transfer of responsibility for school inspections to the Office for Standards in Education (Ofsted) in 1993; and the introduction of school performance tables under the governments' choice agenda in 1992 – drew attention to mathematics performance in schools. These policies have led school stakeholders, including local education authorities, to pressure mathematics faculties and teachers to raise their students' performance in the subject. The fall in England's position in the international ranking of students' performance in English, Mathematics and Science in the OECD PISA survey of Great Britain (Department for Education, 2010) led to the 2014 wholesale reform of the national curriculum, testing, and the performance indicators for school league tables. The government sought to emulate the more successful education systems of Finland, Singapore, and Shanghai (e.g. Department for Education & Truss, 2014). These policies and reforms continue to impact the culture and structure of secondary school mathematics departments as students' performance in mathematics becomes increasingly crucial for schools' survival.

1.1.1 Mathematics – An Important Secondary School Subject

In England, all 15-to-16-year-olds sit the General Certificate of Secondary Education (GCSE) at the end of their compulsory secondary school education. Children start secondary school having sat standardised English Reading and Mathematics tests at age 11, the end of primary school. The difference between the two test scores (the progress measure), statistically calculated by the Department for Education, is used to determine students' progress in mathematics in secondary schools.

In their bid to quantify what is happening in schools and give parents more information and power regarding the choice of schools, successful governments in England have introduced and continuously improved test-based school accountability measures (Leckie & Goldstein, 2019). In 2016, As part of this measure, the government introduced the floor target "Progress 8" (DfE, 2019), replacing the previous floor target that judged a given school on the percentage of its students who achieved five C+ GCSE grades, including English and Mathematics. This new target measured individual students' progress in eight subjects from primary school national tests to their GCSE examinations. A school's Progress 8 score is the average of their students' scores presented with a 95% confidence interval. While this floor target is calculated based on students' performance across eight subjects, English and Mathematics are double-counted. A Progress 8 score ranges from -1.0 to +1.0; a score below -0.5 indicates failure to achieve the minimum standard expected by the government, and a score of +0.5 or above indicates that the students in the school are progressing above the expected level. These measures further contribute to the emphasis placed on students' performance in the particular subject of mathematics that informs teaching practices in schools and mathematics classrooms.

These accountability measures rank schools on students' attainment and progress in GCSE examinations. Before the COVID-19 pandemic began in early 2020, these rankings, in the form of "league tables", were published in the national press, and validated a school's reputation both in the local and national contexts. As school funding follows pupils, these accountability measures and accompanying league tables are a form of "consequential accountability" (Hanushek and Raymond, 2005), which assigns consequences to institutions that fail to meet expectations. Parental choice determines pupil intake; schools that produce positive results, therefore, become oversubscribed, while "failing" schools struggle to meet their intake quota. Poor performance also triggers an inspection by Ofsted that results in an Outstanding, Good, or Requiring Improvement rating; all schools are required to make public the full documentation of these inspections. Publicity acts as a further aspect of the control and policing of school performance, and legitimises government policy.

For students, these qualifications act as a threshold for accessing post-16 education and employment. Secondary school mathematics and its study has historically conferred positive status on students who perform well in it. It is a gatekeeper to entry into elite universities (P. Davies & Ercolani, 2019) and a higher earning power leading to economic stability (Levine & Zimmerman, 1995).

1.1.2 The Secondary School

This study took place in an inner-city London state school that enjoys a measure of popularity in the local community. The school has a cohort of 1300 mixed-gender, culturally-diverse pupils ranging in age from 11 to 16 years old. The school has enjoyed increasingly strong examination results, with a Progress 8 score of 0.3. In its last school inspection, Ofsted graded the school as Outstanding. The mathematics faculty is in a block of twelve classrooms, one of which is an ICT suite. My classroom, where the research took place, is the ICT suite. The mathematics faculty designed the Year 10 curriculum map with the intention of having students progress through the GCSE mathematics content over two years; thus, they would typically complete the program by the end of the Year 10, having started at the beginning of Year 9.

This study focuses on one mathematics class of students who were, at the beginning of the research, just commencing Year 10 studies. I chose to focus on this class as, at the time, I only taught one Year 10 class and one Year 11 class; I did not select the latter class for two reasons. Firstly, Year 11 is a shortened year, as the GCSE examinations start in May, and the students are no longer in lessons; secondly, Year 11 was planned by the mathematics faculty as a revision year, and the students would have already completed the curriculum content. I wanted the study to take place over a sufficient amount of time, and involve students learning new content.

The students in this study were 14 to 15 years old and in their fourth year of secondary education (that is Year 10), having commenced a programme of study that culminates in a series of external examinations across May and June the following academic year. The students were loosely assigned to mathematics class groups based on assumptions about their mathematical abilities, as is conventional (Boaler, 2014). This perception was based on students' performance in the internal mathematics examinations that took place at the end of the previous academic year; the mathematics faculty considered my class as of a lower-middle ability.

1.1.3 Myself: The Mathematics Teacher

From the beginning of my teaching journey, I have been aware of the potential of alternative approaches to secondary school education, having spent my formative years educated in another continent. As a Postgraduate Certificate of Education (PGCE) trainee at the UCL Institute of Education (IOE) in London, I was acutely aware of the fact that the form of UK education system was not universal, and this contributed to my initial endorsement of alternative pedagogy. Attending to these and other personal motivations and assumptions is a significant aspect of becoming a reflexive qualitative researcher; it is crucial to be faithful to the influence of my positionality on the research process and findings. Jane Miller referred to telling one's story as part of the research process, as the "autobiography of the question" (Miller, 1995, p. 23); she argued that it is a powerful validation of our experiences and their potential for rethinking teaching. To this end, in reflecting on the journey that led me to this study, I highlight two further motivations that changed my thinking as a mathematics teacher and made me consider adapting my pedagogical methods in the classroom.

1.1.3.1 Getting Expectations Wrong

In January 2008, certain events caused me to rethink the traditional pedagogical approach to mathematics that I was implementing, and particularly to question its assumptions regarding the role of the learner. It started with a student who took her GCSE mathematics examination twenty months early, in November 2007, at the start of Year 10.

Kaome (real name withheld) was one of the students in what was then my Year 10 mathematics class. My class was a "border-line" class; mathematics teachers use this term to refer to groups of students whom the faculty considers to have difficulty learning mathematics, but who, with academic support, could achieve a pass grade C in the GCSE examination at the end of their secondary education (June 2009 for Kaome). Achieving a grade C was of great importance to schools, given the presiding government's accountability measures, which were based on the percentage of students who achieved a grade C and above in subjects (see section 1.1.1). Kaome's academic profile hitherto was based on her performance in the primary school Mathematics Standardised Assessment Tests (Year 6 SATs) that positioned her on entry to secondary school as of “average ability” having achieved the national expected level (Gibbs, 2011) and her performance in the Year 9 SATs that positioned her as of “border-line ability” having achieved the national expected level (Department for Children, Schools and Families, 2009)]. Based on our school’s internal statistics, 52% of students who achieved a level 5 in their Year 9 SATs achieved a C grade [or above] at the GCSE level   Kaome achieved a level 5 in her Year 9 SATs.

Being “border-line”, Kaome was availed of only a limited field of mathematical concepts; she was perceived by the pedagogical authorities as lacking the cognitive capacities required to engage with higher-level concepts. Moreover, the teaching procedures even prevented Kaome and other “border-line” students from being able to explore such concepts on their own. For instance, when covering the topic of linear equations such as 2x + 7 = 15, students in “border-line” classes were only exposed to methods informed by “what is happening to x?”-style flow diagrams such as that shown in figure 1.2 below.

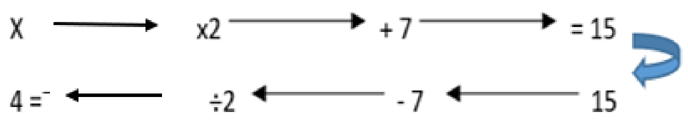


Figure 1.2 – Solving Linear Equations in Border-line GCSE Mathematics Classes

This method cannot be applied to equations such as 2x + 7 = 3x + 11, which have variables on both sides of the equal sign.

In January 2008, Kaome achieved a grade B in GCSE Mathematics. Her parents had sent her to a Saturday school in preparation for the November 2007 GCSE examinations. In personal communications, her mother informed me that the Saturday school had expected Kaome to achieve an A grade; thus, their view was that she had underachieved, while we (myself and the mathematics faculty) believed that she had over-achieved. It came down to a difference in expectations.

After achieving a B grade, Kaome moved to the higher-ability mathematics class; the faculty no longer considered her to be a “border-line” student, but now assumed that she was capable of reckoning with more advanced material. Due to a subsequent change in self-perception, she herself behaved like such a student who achieves A grades in both GCSE Mathematics and Statistics. Three years later, she went on to study medicine at university.

As a mathematics teacher, I had judged Kaome wrongly; I had relied on statistical information to limit my expectations of my students, including Kaome. In doing so, I justified to myself the restriction of the mathematics learning that I made accessible to them. As a consequence of my experience with Kaome, I decided to change this approach.

1.1.3.2 Students Taking Responsibility for their Mathematics Knowledge

As Head of Faculty, in November 2009 I decided to give all students in Year 11 the opportunity to enter their GCSE examinations eight months early. As a result of this decision, the school achieved its best GCSE Mathematics results to date, with 84% of the cohort achieving a grade C or better by the end of Year 11. In the following year, the mathematics faculty allowed all students in any secondary-level year group to enter GCSE Mathematics at a time of their choosing within the broader timeline of secondary study. Expectations for achievement became the responsibility of the individual students themselves; expectations became an index of students' beliefs about themselves and their own agency, and were no longer limited by teachers or based on past examination performance.

What became immediately noticeable to myself and my fellow faculty members was the change that took place in students' participation in their learning once they had decided to sit their GCSE examinations. The students took responsibility for what they did not know and sought to know; they became more tenacious and creative in their desire for knowledge, and supported each other's learning. Over the next four years, the faculty achieved figures ranging between 79% and 84% of students achieving a grade C or above. More students referred to themselves as "good at maths", and, upon receiving their results in the January of the academic year, it became common for students to register to take the next set of GCSE examinations to achieve a better grade. In September 2013, however, the government began to penalise schools for early entry examinations, and our faculty stopped offering this opportunity to students.

1.1.3.3 Questioning the Taken-for-Granted

Having observed how early entry for GCSE examination challenged the taken-for-granted relationships between assessment procedures and student performance, I challenged myself to look further beyond my current thinking. Part of the learner discourse that I had initially internalised tended to link certain coordinates, such as presumed ability, ethnicity, gender, and economic profile to students' mathematics achievement (cf Boaler et al., 2011). Subsequent independent research led me to discover that, beyond what I had seen as fact or simply assumed, other factors such as students' perceptions of gender and ability can impede progress, especially during group work (Pozzi et al., 1993). I observed that the differential performance of ethnic minority groups is partly explained by other factors such as their attendance of lower-performing schools (Kingdon & Cassen, 2010); and that the teacher's attitude towards characteristics such as ethnicity (positive or prejudicial) can have a significant impact on students' participation and achievement in mathematics learning (Boaler et al., 2011). The literature confirmed what I had come to realise: that my perception of my students influenced my behaviour towards them, and, therefore, their experiences in my classroom. I decided to attempt to bring my actions in line with my expectations of the students.

I started by changing how I expected the students in my classroom to learn mathematics; I sought to restore the motivation I noticed in my students when they were able to take responsibility for the timing of their entry into GCSE Mathematics. Above all, I wanted them to make more decisions about what they wanted to learn and how they learnt it. Two years before the commencement of my doctoral research, I began pursuing this aim by giving the students in my mathematics classes the opportunity to choose the sequence in which we would learn the topics in the curriculum; I also gave them new responsibilities, asking each to prepare a mathematics topic and teach it to their peers, with the hope of bolstering their confidence in their abilities. While I sincerely believe that they had a positive effect on my students' participation, given their informal nature, I could not effectively analyse the impact of these new measures. This research study was undertaken in order to discover, with analytical clarity, the most effective means for improving the conditions of my students' engagement with mathematics.

Thus, this study aims to empower students to actively participate in all aspects of their mathematics learning in order to improve their relationship with the subject and their grasp of it. Most concretely, I am concerned to discover more effective forms of pedagogy that encourage students to apply their knowledge rationally to solve problems in the secondary school mathematics classroom; and, ultimately, that will improve their performance in GCSE examinations. To this end, I explore how "shared epistemic agency" is developed and sustained in mathematics classrooms. Shared epistemic agency, discussed in full in chapter 4, is the central concept that I have developed and used to describe and analyse students' participation in learning environments for the creation of knowledge. I propose that students with this type of agency are actively engaged in their learning, taking responsibility for what they know and do not know and acting to further their own and their peers' knowledge; if this agency is able to be sustained over a period of time, I hold that it is a powerful facilitator of the advancement of the collective knowledge of all the students in the classroom.

1.1.4 The Thesis Outline.

I have organised this study across six further chapters. In chapter 2, in which I develop my theoretical framework, I review literature on the key concepts of agency, social learning theories, pedagogy and the existing constructs of knowledge building, knowledge creation and shared epistemic agency that inform the design of the study.

This review leads me to conclude that my elaborated idea of shared epistemic agency, which embodies the six essential characteristics of the ideal learner that I have extracted from the extant literature on education theory, was the particular kind of agency required to improve the participation of the students in my classroom. An innovative pedagogy that could support the development of this agency was also needed. Students with shared epistemic agency: intentionally act to resolve a mathematics unknowing, they seek to extend their knowledge, they explicate knowledge to each other, they take control of the learning process and as a result, they create knowledge new to them. The review revealed characterisations of shared epistemic agency in short-term classroom projects, outside a high-stakes assessment system, but these were important differences to my classroom setting. The following research questions then emanated

1. What are the indicators of shared epistemic agency in the mathematics classroom?
2. What sustains the emergence of shared epistemic agency in the mathematics classroom?

Chapter 3 presents the qualitative action-research methodology I employed throughout the study, and the specific research design that it informed, which was developed to fit my particular aims. In this chapter, I explain the innovative pedagogy that is at the heart of this study, address the ethical issues in relation to the intervention and explain the methods of data collection.

In the chapter 4, which concerns my analytical methods, I present an original unit of analysis: an Episode of shared epistemic agency that exemplifies the objects of interest; that is, the interplay of the six characteristics mentioned above. An Episode is a snapshot of students' purposeful interactions to resolve an unknowing, hence produce knowledge new to the students. Focusing the analysis on Episodes thus allows me to select relevant moments from the hours of data.

In chapter 5, I present the findings from these Episodes. To facilitate answering the research questions and meet the aims of the study, in the first section of this chapter, I used my analysis of episodes to present a more detailed description of how the characteristics of shared epistemic agency manifested in the classroom as the students enacted the innovative pedagogy. In the second section, I elaborate on what was unique about students' epistemic interaction and I present findings that highlight how student positionings and authority impacted on the way they advanced mathematics knowledge in the classroom.

Subsequently, chapter 6 contains a discussion of the two themes that emanate from these findings in responds to the research questions. I critique the idea of shared epistemic agency as an encapsulation of the six characteristics and I propose a more holistic view of the construct. The chapter also puts forward a conceptualisation of the student and the mathematics classroom that emerged from the study and it reflects on the action research process. This reflection focusses on my role as a participants and the innovative pedagogy.

In the seventh and final chapter, I outline the contributions this study makes to the field of mathematics education, and I present a challenge to current educational policy and classroom practice. In my contribution to theory, I present my extension to the existing construct of shared epistemic agency and I indicate the extent to which this study has fulfilled its aims of participation and empowerment. My final contribution as a teacher researcher identifies the value of action research as a meta-methodology. I note the limitations of the research study and end with a call for teachers to become researchers in a bid to improve the profession.

2 THEORETICAL FRAMEWORK

This chapter, organised into five sections, reviews the key concepts and constructs that inform this study's design. The first section addresses the concept of student agency, which includes the narrower concept of epistemic agency, and outlines the learning theories pertinent to its forms. The second section focuses on social theories of learning, in particular addressing the knowledge-creation metaphor of learning and Wenger's (1998) communities of practice. The third section considers the conventional pedagogy that this study seeks to transform in order to achieve its aims, and discusses a picture of authority that is useful for describing pedagogy in general and teachers' and students' participation in it. The fourth section lays out the twin theories of knowledge building and shared epistemic agency that underpin this study. In contrast, the fifth section examines other studies that have worked on transforming pedagogies, especially within the context of mathematics education, supporting my claim that what these studies lack is a focus on an innovative pedagogy such as I am developing that supports everyday practice in the mathematics classroom.

2.1 Agency

This section discusses three approaches to agency that have informed the approach that I develop and utilise in this study. They are:

- Bandura's individualistic and calculative perspective on human agency as the "capacity of individual human beings to make choices and to act on these choices in a way that makes a difference in their lives" (Martin, 2004, p. 135).

This perspective opposed the tradition of behaviourism that viewed human behaviour as determined mechanistically by environmental stimuli.

- Emirbeyer and Mische's "situated agency" (Emirbayer & Mische, 1998, p. 963)), which, drawing on the work of influential 20^{th}-century social philosophers George Herbert Mead, Hans Joas, and John Dewey's, views of agency as a rational and evaluative capacity. In their view, individuals (actors) can respond to changing environments by continually reconstructing their view of the past as they attempt to understand the conditionings of the emergent present, and use this subsequent understanding as the basis upon which to shape and control their future responses. The inherited conception of a "deliberative attitude" (Mead, 1932, p. 76) represents actors as able to actively constitute their environment by selectively controlling their responses to emergent situations and structural factors such as race, culture, gender, and poverty that otherwise constrain their agency.
- Scardamalia's epistemic agency, which identifies the academic sphere as a locus of the knowledge-building practice of learning, and which connects this practice with the general capacity of the human being (Bereiter & Scardamalia, 1998). Epistemic agency "refers to the amount of individual or collective control people have over the whole range of components of knowledge building" (Scardamalia & Bereiter, 2006, p. 106). The word "epistemic" itself, from Ancient Greek *epìstamai* ("to know"), means "relating to knowledge and knowing".

These three approaches to agency are all underpinned by an attention to the social and relational qualities of agency, though the first two have slightly different backgrounds and assumptions from each other. While this study draws on ideas from both Bandura and Emirbayar and Mische's theories, Scardamalia's work on epistemic agency is the primary influence.

2.1.1 Human Agency

This section starts with the work of Albert Bandura – the *locus classicus* of a discussion of agency to which a considerable majority of researchers in the social sciences have referred since its initial dissemination. In a gesture that helped to make him one of the most influential psychologists in modern history, he challenged the then-predominant behaviourist perspective, positing his "Social Cognitive Theory" of learning and development. Bandura dealt with human behaviour and agency in terms of a triadic framework of reciprocity among environmental variables, behaviours, and personal factors such as cognition (Bandura, 1999, p. 156) He later extended this theory to address how people seek to exercise control over their lives by means of the self-regulation of their actions and thoughts (Bandura, 1986). He claimed that much of human behaviour is performed not only to accommodate the preferences of others, but is also "motivated and regulated by internal standards and self-evaluative reactions to [one's] own actions" (Bandura, 1986, p. 20). Moreover, he argues for construing agency as emergent and interactive, claiming that thoughts emerge from neurological processes initiated and sustained by social interactions. From this socio-cognitive perspective, he identifies four moments of human agency that determine the influence of thought on human actions: intentionality (distinguished from the 'intentionality' that is discussed by earlier psychologists

Brentano (Fréchette, 2013) and Husserl (Husserl et al., 2019)) , and which continues to be used as a term in cognitive science and philosophy of mind), forethought, self-reactiveness, and self-reflectiveness (Bandura, 2001). Agency, in the first place, can be understood as a characteristic of whosoever carries out their actions intentionally; people are agentic if their actions are intentional. Intentions themselves are understood as the proactive commitment to bringing about a desired outcome. Furthermore, successful outcomes that are brought about accidentally, even with intention, are not viewed as agentic, given the separation of intention from the decisive action or event. On the other hand, a successfully intentional action may confer agency on a person even if it does not succeed in bringing about the desired outcome. The critical feature of individual agency is the power to generate actions for a given purpose, regardless of whether the outcome of such actions is of benefit or not, or whether it produces the intended consequences. Student A asking an adjacent student, B, for help with a mathematics question is evidence of a student's intention to solve a mathematics problem. The agency emerges in the activity of asking for help, and is present regardless of the outcome of the request – that is, whether or not help is eventually received or whether such help in fact leads to a correct solution.

Forethought extends agency temporally beyond the present moment of intentionality, connecting it with forward-directed planning (Bandura, 2001, p. 7). People anticipate future consequences of their actions and select current actions to bring about future success. An anticipated future success cannot be a source of current motivation and action (i.e. an intention) since it does not exist. However, when individuals represent the consequences of their intended actions cognitively in the present, they become a source of present self-guidance, motivation, and behavioural regulation in anticipation of a projected goal and future outcome. Individuals exercise agency by acting to shape the present to meet a desired future. In this sense, they transcend the constraints of the present. Following through with the previous example, student A asks the questions of student B because they feel that student B's response would help them solve the mathematics problem, a goal which it is in their interest to achieve. The decision to ask the question requires a degree of forward-planning. In Bandura's terms, forethought is the capacity of student A to be motivated to persevere with seeking to answer the question, as student A can imagine the future benefits that will accrue if they can solve the mathematics problem (p. 7)

Self-reactiveness as a feature of human agency is the ability of the individual to motivate and self-regulate themselves to execute intended actions for a desired outcome. It includes all the sub-functions of self-regulation that link thought to action, such as self-monitoring, self-guidance, and self-correction. Self-reactiveness is an important element for the achievement of intended actions. Thus, in our example, student A is not only a planner and a forethinker; they can also change how they behave in order to encourage student B to give them the answer to their question or to answer further questions. This could involve such strategies as, for example, not giving in to frustration if student B is too slow to respond.

Having solved the problem with the help of student B, student A can also look back and decide on whether their course of action was the right one. This attests to Bandura's final feature of agency, *self-reflectiveness*: the capacity to understand and be aware of one's thoughts and actions and to evaluate their adequacy. In this metacognitive activity, individuals judge the validity of their predictions against the anticipated outcome of their actions. They consider external effects, such as the impact of other people's actions, established practices and beliefs, and the anticipated impact of these factors on their future success. People's beliefs in their capacity to exercise control over their own functioning and over environmental events constitute the final frontier of human agency (p. 10). People act because they believe they can produce effects with their actions. The strength of one's belief in this ability correlates positively with the effort invested in actions.

Bandura's social cognitive theory also recognises the necessity of collective agency in the precipitation of positive effects; indeed, it is clearly the case that individuals work with others to bring about what they cannot accomplish independently. A key ingredient of collective agency is the belief, mutually held by the individuals that make up a group, in their collective power to bring about the desired results; Bandura refers to this as the "belief of collective efficacy" , noting that it consists in the group members' knowledge, intentions, skills, and the "interactive, coordinated, and synergistic flexibles of their transactions" (p. 14), which together determine the group's attainments.

Although Bandura's view of human agency is interactive and relational, it still emphasises the capacities of the individual, even as it recognises collective agency. This individualist view, though proffered in the distinctive context of modern psychology, can be traced back to the conception of agency as personal autonomy leading to individual empowerment and emancipation that was articulated by Immanuel Kant in the 18th century (Biesta & Tedder, 2006, p. 4). An emphasis on the empowerment of the individual student, who can follow a course of action to meet a desired outcome, and persevere and reflect on the achievement of the outcome for future purposes, is relevant to this study. However, students in mainstream education do not learn in isolation, and this study would be limited if it did not progress beyond the individual perspective alone. Schools are institutions with social structures such as rules and regulations, traditional teaching practices, curriculum maps, and school-wide assessments. Educational policy that includes, for instance, the GCSE curriculum also has bearing on the agency of students. Both social structures and educational policies impact students and their agency in emergent classroom situations. They impose competing views of how students should engage with learning and constrain the actions they may want to take to produce an outcome, or cause students to re-evaluate their thoughts, habits, and beliefs about consequential outcomes. Since this study seeks to challenge the received views of students as passive and constrained, it requires as a framework a conception of human agency that follows Bandura's – in other words, one which considers the subject to be emergent, dynamic, and interactive – while also mitigating the individualist emphasis of the latter's theories, in order to account for the distributed nature of the social and policy-led pressures that weigh on the students' agency.

2.1.2 Situated Agency

In order to do justice to this interplay, I turn to Emirbayer and Mische's sociological conception of agency as *situated* (Emirbayer & Mische, 1998, p. 963). As noted above, Emirbayer and Mische drew on the work George Herbert Mead; they were also influenced by Hans Joas, and John Dewey's work, situating them within the tradition known as American Pragmatism. Pragmatism rejects the mind-matter and rational-normative dichotomies, offering a theory of knowledge that takes as its point of departure the interactions and transactions that take place in nature – itself understood as "a moving whole of interacting parts" (Dewey in Biesta, 2014, p. 36). On a pragmatic view, the experiences of living organisms cannot be separated in thought from their implication in an environment; organisms interactively adapt to their living circumstances, and are constituted by their attunement to ever-changing environmental conditions. Emirbayer and Mische (1998) characterise their approach as 'relational pragmatics', due, on the one hand, to their allegiance with contemporary and classical pragmatism, and, on the other, to their conception of agency as intrinsically relational and social (p. 973). Their view of agency focuses on actors and their engagement (and disengagement) with the different contexts and environments that constitute their flexible yet structured social universes.

Emirbayer and Mische argue that a conception of agency should neither be limited to considerations of the individual pursuit of interests and needs (as in the Kantian tradition), nor to a view of human actions as totally constrained within cultural and structural contexts (as, for example, in structuralist anthropology (p. 974). Thus, they seek to reconceptualise agency in order to account for the historical and temporal nature of human experience, and to demonstrate how this temporality interacts with structural contexts informed by the past and oriented towards the present and the future. In their view, human actions, through an interplay of habit, imagination, and judgement, reproduce and can also transform the contextual determinations to which they respond. Individuals can orient themselves towards the past, present, or future at any point in time, and change their orientation as they see fit. Applied to our case, the consequence is that students can and do change their relationships with each other and with their contexts.

According to Emirbayer and Mische, agency has three dimensions. In the first dimension, the *iterational* element*,* individuals can change the dogmatic schemes of action that have developed over time in a society (p. 976). Their agency lies in the capacity for selecting, deciding, locating, and recognising which actions to change, or else contemplating whether to reproduce existing schemas of experience, activities, expectations of others, or situations developed in the past. In other words, it involves participants knowing what to do with existing knowledge and practices. The second dimension is the *projective* element, on which agency is conceptualised as the ability of individuals to reconfigure their current actions to create a desired future. This dimension, that draws parallels to Bandura's forethought and self-reactiveness, is the creative-reconstructive dimension of agency, where existing cultural practices do not constrain agents' actions, but rather, constitute challenges to which they can respond. They are able to invent new thoughts and actions to bring about a desired future, and do not have to repeat existing actions and established practices; they can develop new responses to problems. They use current knowledge to move beyond themselves and decide where they are now, where they want to be, and how to get there from where they are in the present (p. 984). The third dimension, the *practical–evaluative* element, views agency as the capacity of individuals to exercise contextual judgements. That is, prudent, intelligent, and practical decisions concerning which actions to perform in order to address problematic situations. Here, agency lies in agents' ability to read the present situation and make decisions in real time that may challenge a given state of affairs. This element sees participants increasing in their capacity to bring about change where the consequence of their actions cannot be structured or controlled. In effect, Emirbayer and Mische posit that human agency should be conceptualised as "a temporal embedded process of social engagement informed by the past (in its habitual aspect), but oriented towards the future (as a capacity to imagine alternative possibilities) and towards the present (as the capacity to contextualize past habits and future projects within the contingency of the moment)" (p. 963).

Emirbayer and Mische stress that these three dimensions of agency are analytical distinctions, and that all three can be identified in various degrees within any empirical instance of action. In Figure 2.1 below, I relate these three dimensions of agency with the moments of Bandura's analysis. As an individual proactively commits to bringing about a future action (intentionality), sets in place a course of action to bring about a future result (forethought), motivates themselves to see their plans through (self-reactiveness), and reflects on the adequacy of their actions (self-reflectiveness), this individual's thoughts and actions are seen to be able to transform or reproduce their structural environment, and can be informed by past habits, oriented towards an imagined future, or based on present judgments.

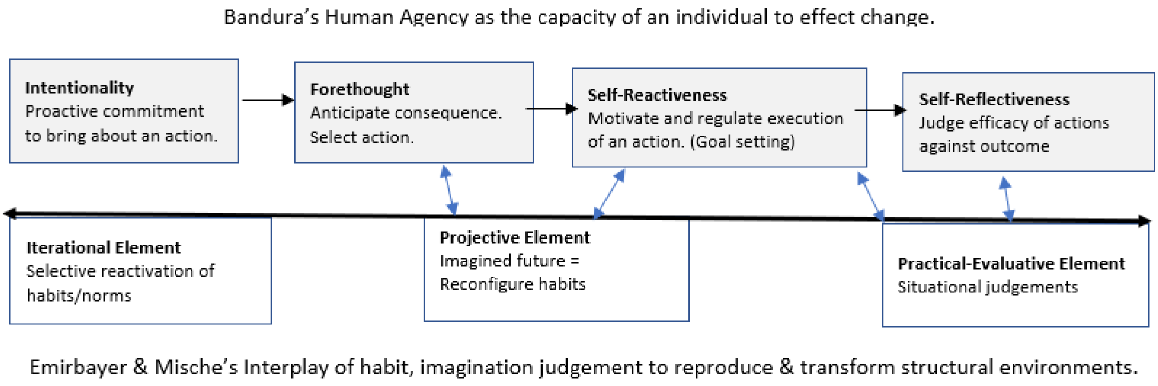


Figure 2.1 – Relating Emirbayer & Mische's (1998) Situational Agency to Bandura's Human Agency (2001)

These two perspectives on agency can be used to analyse the character of student engagement in a school classroom community. Bandura's view is relevant to the extent that it elaborates on the features that underpin students' actions as they strive to produce a desired outcome. His theory offers insights into how students can work interdependently with others to bring about outcomes that they cannot deliver independently. Emirbayer and Mische, on the other hand, contribute the insight that students can bring about desired outcomes by making ad-hoc decisions in the present that could transform the restricted structures in which they are acting. In the context of this study, these decisions might involve deviating from existing habits relating to how students should act in the classroom and finding new ways to develop mathematical knowledge, or indeed simply retaining good habits and traditions. The following section explores how students' decisions and actions can lead to knowledge.

2.1.3 Agency as Epistemic

Scardamalia (2002) argued that the notion of responsibility links human agency, as defined by Bandura (2001), to knowledge, which is the central focus of Scardamalia's conception of epistemic agency. Knowledge arises from choices for which the agent is responsible (Reed, 2001). To 'know', individuals or collectives need to be in control of their actions and have the ability to determine how to apply their will towards concrete forms of action. Individuals or collectives that take responsibility for their learning are aware of what they know or do not know and act on this awareness to advance their knowledge.

The idea that active engagement by participants is required for them to learn or construct knowledge has its roots in Vygotsky's social constructivism (Bereiter, 2002; Scardamalia, 2002; Valsiner & Veer, 2000). Constructivism is a philosophical and psychological perspective on learning that contends that individuals construct or form much of what they learn through their actions and interactions in the world (Packer & Goicoechea, 2000). The sociological applications of constructivism, which emphasise the influence of the social environment on learning, have driven contemporary discussions of agency, its meaning, and its expression in educational environments.

Marlene Scardamalia, a psychologist and educational researcher who is considered one of the pioneers of computer-supported collaborative learning, put forward the notion of epistemic agency (2002) in the context of knowledge-building pedagogy (Scardamalia & Bereiter, 2006). A self-described "deep" constructivist (Scardamalia, 2014), she distinguished between "shallow" constructivist methods such as guided discovery (Brown & Campione, 1994), in which teachers plan what the students are to discover, and "deep" constructivist methods, in which the highest-level capacities such as planning and the evaluation of learning – which, in our age, are typically accorded only to the teacher — are handed over to the students. Students not only construct their understanding but the whole space of invention, operating as a professional knowledge environment (Scardamalia, 2014, 2:20mins). Emerging from the context of this new learning environment, Scardamalia presents her notion of epistemic agency as the metacognitive ability concerning "goal-setting, motivation, evaluation, and long-range planning" (Scardamalia, 2002, p. 79). In her view, students with epistemic agency assume responsibilities typically left to teachers, and, *pace* Bandura, these students can have collective metacognitive abilities that are different from the mere combination of individual ones. Collectively, students who take responsibility for their learning, form ideas, relate them to others' ideas, and agree upon an ideal compromise. It is the collective contribution of students that results in and sustains the collective knowledge advancement.

Scardamalia did not provide a clear theoretical account of the concept of epistemic agency, nor describe how it can be identified in an educational setting. I consider the idea of epistemic agency to emerge from her work on collective cognitive responsibility (Scardamalia, 2002). Collective cognitive responsibility exists in groups such as medical teams that carry out knowledge-based work. These groups exhibit qualities such as flexibility, continued learning, collaboration, and rational thinking. Though each member has a specific duty and/or area of expertise, roles are not necessarily fixed. When problems arise, team members can take over from each other without relying on a higher level of authority. The group's success is distributed across all of the individuals, rather than attributed solely to the leader. In addition to the more tangible and practical aspects, individuals within these teams all take cognitive responsibility to acquire the knowledge that their activities require and ensure that everyone is adequately knowledgeable. In teams with collective cognitive responsibility, the individuals and the teams are more productive and innovative than those without such responsibility (Scardamalia, 2002). A classroom in which students develop epistemic agency exhibits the characteristics of collective cognitive responsibility. These classrooms act as a community, with collective contributions creating new knowledge and advancing collective knowledge.

2.1.4 Summary

This section has outlined three conceptualisations of agency that emanate from three related perspectives on knowledge, learning, and human development. The first perspective, from which Bandura's (2001) conception of human agency emanates, is Social Cognitivism. This perspective views learning as a reciprocal triad of personal factors, environmental variables, and behaviour. What is in students' minds (thoughts, beliefs) and the teacher's expectations (rules, procedures) influence students' actions and the outcome of these actions. The second perspective is Pragmatism, the perspective of Dewey (1900) and Mead (1932) that represents knowing as based on one's experiences in one's environment; this informs the relational pragmatist viewpoint of Emirbayer & Mische (1998) that represents the relations between ends and means as pre-eminently dynamic, and as unfolding and ongoing processes (Emirbayer, 1997). This view recognises that each student experiences the world uniquely and can react to this experience idiosyncratically as the situation changes for them. Finally, the third perspective of deep Constructivism (Bereiter, 2002; Scardamalia, 2002) argues in favour of students taking responsibility for what they know and do not know and creating knowledge from this process. These three perspectives of agency are compatible, and dovetail in the notion of epistemic agency, on which taking responsibility for what one knows or does not know transforms individual-situational agency into a new form of agency related to knowledge.

I hold the position that students have the capacity to change and adapt to an innovative pedagogy. While I recognise the agency of the individual students and that of students as a collective as they respond to their classroom learning environment and its pre-existing structures, I lean towards the notion of deep constructivism, appreciating that students can create knowledge as they take responsibility for their learning in a secondary school mathematics classroom. To supplement this perspective, I require a theory that reconciles the social character of learning with this interest in classroom practice.

2.2 Theories of Social Learning

Epistemic agency, as Scardamalia defines it, is, in the classroom context, a quality that sustains the creation of new knowledge by the collective contributions of students who take responsibility for their learning. Having established this, I can identify one goal of this study to be the designing of a pedagogy that supports students in the development of such agency. This innovative pedagogy, elaborated upon in chapter 5, restructures the classroom as an environment in which students can learn as a community. To this end, it draws upon Sfard's (1998) two metaphors of learning, the knowledge-creation metaphor (Paavola et al., 2006), and the social perspective of learning (Wenger, 1998); each of these connects the pedagogical environment with a notion of the community of practice. In the section that follows, I will review these theories to the extent that they underwrite the development of my own theoretical construct. This review will include an elaboration of the notion of community and power relation.

2.2.1 Metaphors for Collective Learning

Metaphors for learning respond to questions such as who the subject of learning is, the kind of knowledge they should learn, and how they learn it. They reveal certain essential features of learning by asking us to consider it in terms of other behavioural practices. In her article "On Two Metaphors for Learning and the Dangers of Choosing Just One" (1998), the mathematics educator Anna Sfard proposed two primary ways of thinking about how learning occurs: the acquisition and the participation metaphors. The acquisition metaphor depicts knowledge as the capacity of an individual mind, and learning as a process whereby the individual is guided in assimilating or constructing pre-given knowledge. Sfard's participation metaphor, on the other hand, focuses on "knowing" rather than "knowledge". Knowledge does not exist in individuals' minds or in the world, but is situated in the cultural practices of a community (Lave & Wenger, 1991; Rogoff et al., 1998; Wenger, 1998). Learning occurs as individuals participate in and are inculcated into the forms of life that constitute their community. Sfard’s presentation of the participation metaphor does not seek to inspire changes in pedagogical practice; rather, her focus is on mastering existing practices. However, thinking her participation metaphor together with the notion of a community of practice as discussed in section 2.2.2, it is clear that participants could, through active negotiation, develop a practice that is both historical and dynamic (Wenger, 1998, p. 53).

Indeed, Paavola, Lipponen, & Hakkarainen (2006) suggest an approach that relies upon but goes beyond the two metaphors mentioned above, highlighting the capacity for advancing collective knowledge. Their metaphor, that of ‘knowledge creation’ (p. 536), addresses the possibility of innovative learning activities for the creation of knowledge; taking it seriously requires a theory or model of learning that clearly emphasises innovation in relation to learning and knowledge. The knowledge-creation view of learning is connected with the theories of knowledge-building (Scardamalia & Bereiter, 2010) and knowledge creation (Nonaka, 1991) that I discuss in section 4.1 in order to examine what is vital in knowledge communities and innovations in learning, and, ultimately, in order to suggest new approaches to pedagogy.

This *knowledge-creation* approach to learning explicitly builds upon Sfard’s (1998) two metaphors for learning. The acquisition metaphor represents the “monological” view of human cognition and activity, according to which important events happen exclusively within the human mind. In contrast, the participation metaphor emphasises a “dialogical” view of human cognition, whereby important events such as learning occur as the individual interacts with culture, other people, and the surrounding environment. Finally, the knowledge-creation metaphor corresponds to a “trialogical” model (see Figure 2.2); emphasis is placed on the way individuals collaboratively develop shared knowledge objects and artefacts (Paavola et al., 2006, p. 539). In innovative knowledge communities based on the third model, learning occurs during collaborative practices that create shared objects of knowledge.

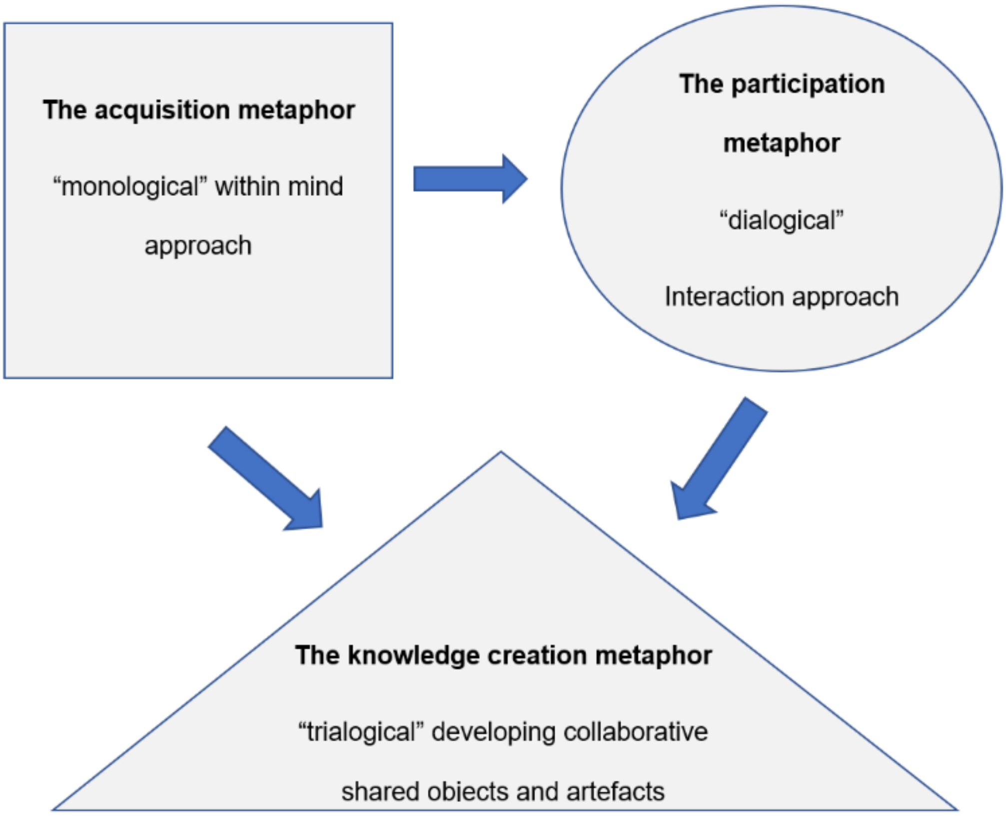


Figure 2.2 – The Three Metaphors of Learning (Paavola & Hakkarainen, 2011, p. 535 - 557)

This proposed innovative pedagogy stands in contrast with a conventional pedagogy (see section 2.3), which relies on the acquisition metaphor, considering the teacher to be in sole control of the transmission of knowledge, and rendering students as passive receivers of this knowledge, having no other role than to store the information received from the teacher. The participation metaphor suggests a pedagogy in which the students are not passive but are required to take an active role in their learning, and points to the idea of students learning as a community. Indeed, in most pedagogies that uphold this metaphor, such as the community of learners model (Brown & Campione, 1996; Rogoff et al., 1998), the classroom is organised as a community with the students working together, all serving as resources for each other and guided by the teacher's leadership.

The knowledge-creation metaphor allows for a further departure from this model, allowing me to describe the classroom and pedagogy that I propose in this study, as it examines learning in terms of the social structures it creates and the existing processes of collaboration that support innovation and knowledge advancement. This pedagogy views learning as a social process while still recognising the competencies and initiatives of the individuals that make up the community. It focuses on the process of innovation that occurs as people interact, rather than on the contents of individual minds, and brings the dynamics between individuals and environmental structures for creating new knowledge to the forefront. The individuals' initiative feeds the communal effort to innovate, while the social environment feeds the individual's initiative and cognitive development. Constructing shared objects of knowledge requires more than dialogue; it requires the interaction of individuals' contributions and collective contributions in a community learning environment. The proposed pedagogy will focus on students both individually and collectively taking responsibility for their own knowledge creation; the "knowledge-creation" metaphor underpins this pedagogy.

2.2.2 Communities of Practice

My proposal for a new pedagogy based on knowledge creation also requires a sufficiently dynamic conception of the community in which learning takes place. My thinking about community draws on ideas of communities of practice in the work of Etienne Wenger, wherein community relations are of mutual benefit to participants in achieving their shared goals and advancing their mathematical knowledge. In communities of practice, learning is not an individual experience, but rather a social phenomenon that occurs as individuals engage in activities that are essential to the community. Thus, knowledge is competent participation; knowing is the ability to participate in the community's endeavours, and learning involves the transition towards such competence, changing who a person is. Figure 2.3 below shows the components that characterise participation in Wenger's social theory of learning and knowing, and I will discuss them in turn.

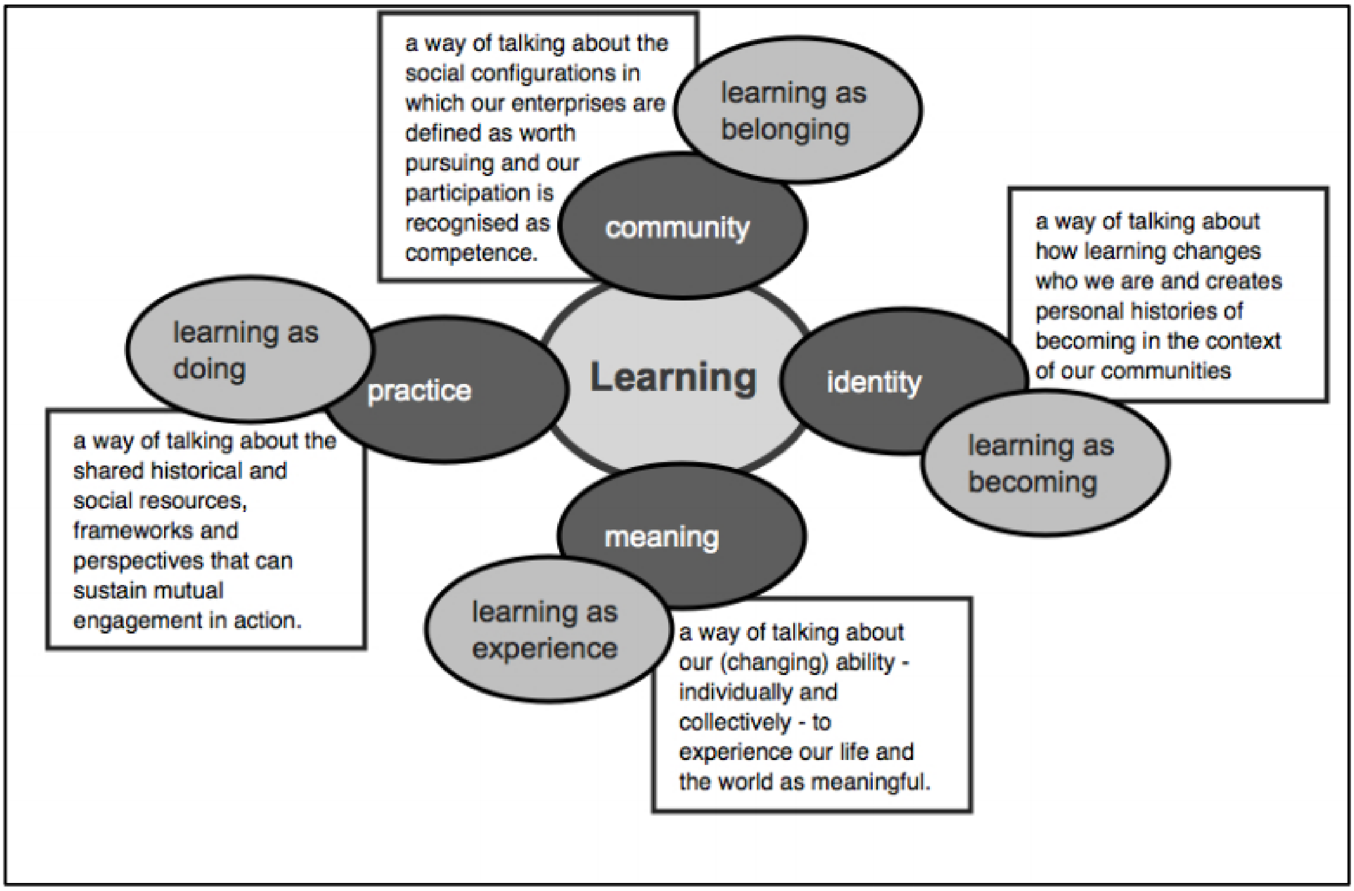


Figure 2.3 – Components of a social theory of learning (Wenger, 1998, p. 5)

2.2.2.1 Learning as Doing

A practice is a way of doing things developed over time by participants of a community of practice to fulfil their purpose of coming together. In a mathematics classroom viewed as a community of practice, the classroom participants, that is, the students and teacher, through their engagement (their doing) over time, develop ways of communicating and behaving that fulfil their aim of learning mathematics. These modalities of communication and behaviour could include students' knowledge of how to communicate with each other and with the teacher, or of how they access homework and receive feedback. These tacit and explicit classroom practices, negotiated over time, include actions and reifications (Wenger, 1998) that are unique to the participants of that classroom. The term "negotiation" intends "to convey a flavour of continuous interaction, of gradual achievement, and of give and take" (p. 53), emphasising that the participants' practice is a production of their individuality – of who they are as individuals and who they become as a community.

2.2.2.2 Learning as Experience

Our experience gives meaning to our participation in activity. Wenger used the concept of the "negotiation" of meaning to "characterize the way we as individuals experience the world we are in and how we experience our engagement in it as meaningful" (p. 53). For example, consider the case of students who attend mathematics classes. As they engage in their learning, their activities develop into patterns of action. It is the development of these patterns all over again, lesson after lesson, that constitutes the experience learning mathematics – of what mathematics means to them. The term negotiation is used in the sense of continuous interaction, of the continuous development of meaning through the interactions of participants with their practices.

Reification is a connected term that, in this study, functions to explain the role of material objects in the community of practice. The term refers to the capacity for abstract, distributed, complex ideas to achieve reality as material objects as they assume central functions within a practice. Thus, the curriculum is a reification, as is a lesson plan or a tick in a student's exercise book. In the classroom, reifications are products of students' experiences. It is the experience gained through their participation that gives meaning to what they do. A tick in the book, for instance, conveys to the students that they are correct, knowledgeable, and have given the right answer or solved the problem. A tick is authentic to the students because of the meaning it projects. The meaning of the tick is based on their experience of being in a classroom. Thus, when students say “give me a tick” or “my work was ticked” or “shall I tick it?”, the tick itself is only representative; it is the experience of its meaning, the idea of which it is a reification, that is really circulating. Summarily, reification is “the process of giving form to our experiences by producing objects that congeal this experience into ‘thingness’” (p. 58). The actions and reifications that form the practice of a mathematics classroom give this practice meaning – that is, make visible what learning mathematics is to the participants.

2.2.2.3 Learning as Belonging

The participants of the mathematics classroom negotiate what constitutes competence in the practice of learning mathematics. Competence reflects the actions and reifications that define belonging, that is, being a classroom community member (Wenger, 1998). The community determines competence; it is what the community recognises as competence that defines competence in their community. In some mathematics classrooms, competence is accorded to students who frequently answer questions or put their hands up, or else who complete the set work quickly. Wenger emphasised, however, that belonging to a classroom community requires more than competence alone; it also requires experience of participation. Experience of participation includes mutuality of engagement, establishing relationships with other participants, engaging with them, and responding to their actions and reifications. Accountability for the other participants includes doing what is required to learn in ways acceptable to the community and its participants. Competent members of a community show their belonging by participating in the practices of the community.

2.2.2.4 Learning as Becoming

As individuals participate in the pedagogy of a mathematics classroom, they build an identity that emerges from the negotiation of what it means to be a member of the classroom community and to engage in its practice of learning mathematics. A participant's identity in the classroom is who they become as a member of the classroom, how they influence the community practice, and how it influences their participation. As participants of a mathematics classroom engage in the practice of learning mathematics, other participants develop relations with them that reify them based on their competence; they are viewing, for example, as good at algebra, at explaining, or at showing their working. Identity involves how we experience our participation and how others project their reifications of our participation on us. Identity can be defined, then, as "a layering of events of participation and reification by which our experiences and its social interpretations inform each other" (p. 151).

A mathematics classroom operating as a community of practice can benefit from the mutual relations inherent in any community with a common purpose. The purpose of advancing their collective mathematics knowledge directs students' participation and practice. As they participate, they negotiate this practice and develop competence as mathematics learners. Their competent participation and participation experience make them belong to the classroom community; as such, participants develop identities and influence the identities of other members of the classroom community. While these ways of viewing learning in a community complement the aims of this study, Wenger's trajectories of participation (Wenger, 1998, p. 153) that legitimises unequal forms of membership in a community of practice is at odds with this study's view of community. In addition, Wenger has been criticised for his benevolent view of community and for not considering the detrimental impact power/knowledge relations can have on the members of a community (Creese, 2009; Paechter, 2003; Tusting, 2005).

2.2.2.5 The Notion of Community

The notion of community I propose for my mathematics classroom, to meet the aims of this study, draws from the work of the British philosopher John Macmurray. He views community as a mode of unity informed by relationships of the individuals intrinsic worth (McIntosh, 2015, p. 14). These communities are not created or sustained by force but emerge voluntarily and are sustained through friendship. He argued in his book Conditions of Freedom (Macmurray, 1950), that what differentiates these communities from society in general is that they are constitutive of equality and freedom (p.73-74). Akin to friendship, where there is equality of consideration and value, each member of the community has equal value, and their voice counts equally. This does not imply that the individuals are not different in terms of their natural disposition or their capabilities, rather in these communities, the relations between members overrides these differences. The individuals are free to be their authentic self's and express their uniqueness. In essence, equality and freedom are mutually inclusive, they are conditional of each other. Being equal means one can act in accordance with their nature and freedom of expression is made possible amongst equals. Macmurray's view of community suggests human relations in which the individual and the community are interdependent, "we enter into personal relations with others because it is through them that we can be and become ourselves" (Fielding, 2012, p. 685). The learning is learning to live as a community, both the teacher and the students voluntarily take responsibility for advancement of each other's mathematics knowledge and avoid exercising their freedom in a way that will limit the freedom or the voice of others (McIntosh, 2007, p. 75).

Though Macmurray was calling for education to focus on human fulfilment rather than personal gain, and did not give illustrations of the freedom he described, I can extend this mode of community to a mathematics classroom. The relationships between students and teachers and or between students and students do not depend on their individual functions, that is how they benefit each other, or individual achievement, rather, it is about reciprocal caring for how each other feel in the classroom as they learn together as equals.

I propose a democratic community where the relations of equality and freedom, that exist between students and with the teacher, include participation with a democratic stance (Vinterek, 2010); a classroom where the students trust and respect each other, and have the freedom to take control of how they learn and what they learn, they exhibit a "willingness to listen to others, to speak up and a willingness to give voice to their own thoughts" (p. 373). This proposed classroom community contrasts with the relationships of power that exist in society. Classrooms are microcosms of society, as such, if allowed, hegemony and relations of power, can impacts on the relations between teachers and students and between students and students.

2.2.2.6 Power Relations in Society and the Classroom as a Community

Relationships of power exist in all human interactions and structure human behaviour (Foucault, 1978, p. 96). As individuals are constantly interacting, power is constantly at play in these interactions. Foucault put forward a productive view of power as both positive and negative. He analysed it as something that is capillary, and circulates with individuals as vehicles of power, "not something that is acquired, seized, or shared, something that one holds on to or allows to slip away" (p.94). In effect we all exercise power and are subjected to power by others. In Foucault's view, power exists only when it is put into action. "In effect, what defines a relationship of power is that it is a mode of action which does not act directly and immediately on others. Instead, it acts upon their actions: an action upon an action, on existing actions or on those which may arise in the present or the future" (Foucault, 1982, p. 789). That is to say, the exercise of power directs the conduct of others, it opens possible actions or outcomes, that can be harmful or productive. It also implies a degree of freedom or the possibilities of resistance from others (Foucault, 1980, p. 780), otherwise, there would be no need to direct their conduct.

While all individuals or collectives are implicated in power relations, that does not mean that all have equal power. Foucault also posited that power circulates as knowledge and is visible in discourse and discursive practices, such as in the discursive practices of a mathematics class. With this conceptualisation of power in mind, in a classroom community, where teachers and students relate with each other relations of power are at play and could have a positive or negative impact on individuals and the community. This power that circulates will result from the innovative classroom pedagogy, the discourse of schooling that ascribes knowledge hence power to the teacher and from power ascribed to constructs such as race, gender, class, and socioeconomic factors that act to marginalise individuals in society. An awareness of the workings and source of power are important for this study, if I seek a democratic classroom community as described in the previous section.

2.2.2.6.1 Power Relations in Schools

Some sociologists have claimed that school are structured, designed and organised to mirror the divisions, ranks and hierarchies' existent in society (Giroux, 2011; Giroux & Penna, 1979). The interconnection between ideology, pedagogy and the curriculum acts as a tool to socialises students into society (Bernstein, 2009; Giroux & Penna, 1979). Bourdieu argues that cultural reproduction occurs in schools by normalising what constitutes as knowledge and truth (Bourdieu, 1990). He posits that schools subtly reproduce the power relations that exist in society through mediating the dominant culture that tacitly confirms what being educated means. Michael Apple

(2004, pp. 29–30) describes schools as agents of cultural and economic reproduction, maintaining the inequity of society. Hence factors such as race, gender, disability, sexual orientation, socioeconomic status, immigration that disenfranchise sections of our society from participating equitably and democratically (Fraser, 2012; Fraser & Sunkara, 2019; Wallace et al., 2022) can be mirrored in school and in classroom as students and teachers relate with one another.

Educational research (Boaler et al., 2000; Boaler & Greeno, 2000; Gore, 1995; Hargreaves et al., 2021; Smith, 2014; Solomon, 2009c) show that, beyond the inherent discursive practice of school, the power relations at play in society are evident in the mathematics classroom and act to exclude students from full participation. Class, culture and gender caused teachers to position students, in the mathematics classroom, as competent or not competent thereby, restricting student's access to mathematics knowledge and impacting student's self-belief in their ability to participate in mathematics (Solomon, 2007, 2009c). This positioning acts to limit access to good teaching for low-attaining students (Hargreaves et al., 2021), and limits girls take up of A level mathematics (Smith, 2014).

2.2.2.6.2 Power Circulating between Teacher and Student

The discursive practice of school generally places the teacher by virtue of their knowledge in a position of social dominance in the student teacher relationship, referred to by Bernstein as "control of the social base" , (Bernstein, 2000, p. 30). During student to student interactions, student can copy this teacher attribute and power as social dominance can circulate as mathematics knowledge limiting other students' contributions and mathematic meaning making (Langer-Osuna, 2017). Though the reason why the students mimic the teacher's behaviour could be related to broader institutional norms that focus on competition and comparison rather that community learning (Barron, 2000, p. 432). Understanding the power-relations at work in the classroom is essential if this research seeks to achieve its aims.

As previously stated, power circulates as knowledge, knowledge which Foucault posits is arbitrary. He argued that knowledge is a product of power relations asserting what truth is constructed and kept in place through strategies such as discourse that support and affirm it and exclude and counter other discourses (Foucault, 1978, pp. 100–101). Power operates in the processing of information that selects what is being labelled as fact, that is, in what the curriculum and teachers allow to be circulated in the classroom. This fact becomes the dominant discourse and other dominated discourses are excluded. Knowledge within schools and in society is carried out and kept in place through a wide range of strategies that affirm and support it such as practice, institutions and hegemony, where those who are dominated by others – such as students, take on board the values and ideologies of the dominant teachers in schools and accept them as their own: this leads to students accepting their position within the hierarchy as natural or for their own good. This internalisation of the dominant discourse by the dominated is the capillary form of power. In this sense, discourses, truth, power and knowledge are intricately linked. This interconnection may give an explanation why the power relations that exist between teachers and students are pervasive such that my attempts to change this dynamic in the classroom may be resisted by the students this study aims to empower.

I find Foucault's work on discourses useful in helping me think about how I know what I know; under what circumstances the information is produced, where it comes from and whose interest it serves. Thus, the discourses of teaching and pedagogy do not hold universal truth but are constructed and held in place by the practices of schooling, it is thus possible to think differently about practices and to trace how what we in schools accept as 'true' is kept in its privileged position.

Consequently, discourses can be seen as a means of resistance as well as a means of oppression. "Discourse transmits and produces power; it reinforces it, but also undermines and exposes it, renders it fragile and makes it possible to thwart it". (Foucault, 1978, pp. 100–101). Though transforming the pedagogy is an act of resistance it is equally an exercise of power because both the students and the teacher have the freedom to effect change. However, for change to be sustained, the students have to feel that it is purposeful, I have to make the logic of the innovative pedagogy clear to the students, and the aims of the study decipherable (pp. 94–95). To improve students' relationship with and learning of mathematics, it is possible for the students and I to interact on a basis of mutual authority and competence. In exercising power, we can direct each other's conduct towards respect and trust and through enacting an innovative pedagogy build a democratic community – empowering the students to take responsibility for what and of how they learn mathematics.

2.2.3 Summary

At the beginning of this section, I framed a goal of this study, which is to develop a pedagogy that would support students' achievement of epistemic agency. To develop responsibility for their learning requires a pedagogy in which learning is a social endeavour. Thinking about this pedagogy begins with a decision about the metaphor of learning used to describe who the subject of learning is, the kind of knowledge learners should learn, and how learners learn in the pedagogy. The knowledge-creation metaphor (cf. Paavola et al., 2006) provides a way of conceptualising learning in terms of innovative communities of knowledge that does not exclude learning as acquisition or learning as participation; instead, it emphasises how individuals collectively participate to acquire shared knowledge objects and artefacts. This metaphor of learning is of interest to this study as it prepares the context in which epistemic agency can develop and gives form to the goals of the innovative pedagogy that I am developing.

In the second part of this section, I outlined Wenger's social learning theory that discusses how learning can occur in a classroom that operates as a community of practice, I highlighted the notion of community that will support the aims of this study, considered the power relations at work in society and the relationships of participation that this study's innovative pedagogy aims to develop. However, while the theory outlined four ways of learning in a community – learning as doing, learning as experience, learning as belonging, and learning as becoming – it focuses on knowing rather than knowledge. This social learning theory that focuses of students participating in established practices, therefore, could be viewed as being at odds with a study that focuses on mathematics knowledge and innovative forms of learning within a classroom community. Thus, I hope to draw on the ideas of learning through participating in a community from Wenger (1998), while also moving beyond them by means of the ideas of collective learning from Paavola et al. (2006) in the design of my innovative pedagogy.

2.3 Pedagogy

This section focuses on aspects of pedagogy that will influence this study's innovative pedagogy design. The first sub-section will describe the conventional pedagogy alluded to in section 2.2.1 above. The following two subsections will introduce the constructs of authority and positioning. These two constructs show how the pedagogy can impact the students' experience of and relationship with mathematics in a classroom.

2.3.1 The Conventional Pedagogy

The notion of conventional pedagogy that I introduce here has its roots in my own experience (see section 1.1.3.2), as well as in Paulo Freire's critique of what he describes in his seminal book, *Pedagogy of the Oppressed*, as the "banking concept of education" (1970, p. 72). In this conception, education takes the form of depositing. The teacher, as the depositor, narrates knowledge to the student who acting as depositories, mechanically receives, memorises, and repeats the information. I consider this teacher-student relationship akin to the acquisition metaphor of learning introduced in section 2.2.1 above. In the banking model, according to which there is an asymmetrical relationship between the teacher and the students, the teacher controls the subject knowledge and its learning as outlined in table 2.1 below.

The Teacher	**The Students**
teaches	are taught
knows everything	know nothing
thinks	are thought about
talks	listen meekly
disciplines	are disciplined
chooses and enforces their choice	Comply
acts	have the illusion of acting through the action of the teacher
Chooses the program content	(who are not consulted) adapt to it
Confuses the authority of knowledge with his or her professional authority, which he or she sets in opposition to the freedom of the student	
is the subject of the learning process.	are mere objects

Table 2.1 – Attitudes and practices of the banking model of education. Quoted from *Pedagogy of the Oppressed* (Freire, 1970, p. 73)

Freire argues that this oppressive pedagogy prevents students' agency from being creative and transformative. He called for an equitable pedagogy based on inquiry in which "knowledge emerges only through invention and re-invention, through the restless, impatient continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other" (Freire, 1970, p. 53). Jacques Rancière, an influential French philosopher who continues to engage with social issues, also criticised the prevalent pedagogy of 1980s France that positioned students as of unequal intelligence to the teacher. He called for an emancipatory pedagogy in which the intelligence of students is recognised and not stultified by what he termed the "explanation logic" (Bingham et al., 2010, p. 3). He posited that schools presuppose students to be ignorant, and present knowledge as needing to be explained by teachers; instead of making students' intelligence equal to that of the teacher, this explanation perpetuates the myths that further explanation is needed, that students are unable to learn without the explanation of the teacher, and that, therefore, they are always of unequal intelligence. Both Rancière and Freire called for a pedagogy of equality, where the polarised view of teacher as knowledgeable and in control, and the students as ignorant and powerless, is replaced by a pedagogy in which students and teachers share authority in the classroom and learn alongside each other.

Although each of these critiques of education had as their contexts different parts of the world and moments in history, I see similarities between the banking model observed by Freire, the inequality of intelligence described by Rancière, and the pedagogy experienced by students in most parts of my school. This pedagogy is clearly based on an unequal relationship between the students and the teacher similar to that outlined in table 2.1 above; I will refer to this as the ‘conventional pedagogy’, and argue with Boylan (2010), Pratt & Kelly (2007), and Wright et al. (2020) that this is typical of learning mathematics across England.

Critical mathematics education research, such as that of Gutstein (2006) and Wright (2017), has drawn inspiration from the work of Freire (1970), and has developed mathematics pedagogies with social justice commitments that help students to understand the communities they live in and the ways inequality is contested and produced in the world. This study does not seek to assume a critical perspective on society, though there is an overlap with critical mathematics education in the fact of this study’s desire for equality in the authority relations between students and teachers.

An important difference in my aims here, compared with Freire’s and Rancière’s, is that, while these thinkers aimed at overhauling society to achieve equality and social justice, this study aims to achieve equality in the humbler context of the mathematics classroom, and aims above all at improving the student’s relationship with the subject in order to better facilitate their learning.

In the summary of the previous section, I mentioned that the innovative pedagogy based on a knowledge-creation metaphor of learning would aim to have students taking responsibility for their mathematics learning. Taking responsibility for learning requires a pedagogy in which learning is a collective community endeavour, and in which students participate in their learning actively; achieving this state of affairs is an aim of this study. In the conventional pedagogy, wherein the teacher has sole authority, these relations of authority can constrain students’ abilities to engage with mathematical ideas and reflect on their learning. In extreme cases, it interferes with their ability to obtain mathematical insights and solve problems in the first place (cf. Amit & Fried, 2005; Brubaker, 2012; Schultz & Oyler, 2006). Thus, in order to avoid these pitfalls, I turn to a consideration of the phenomenon of classroom authority itself in the preparation of my own pedagogy.

2.3.2 Authority in the Classroom

In an educational context, authority can be defined as a "social relationship where some people are granted the legitimacy to lead, and others agree to follow" (Pace & Hemmings, 2007). It is distinguishable from the form of power, which connotes subjugation of one individual to another's will by some form of coercion (see section 2.2.2.6). Instead, authority involves a relation of obedience and voluntary submission that is quasi-reciprocal rather than coerced. Authority "operates in situations in which a person or group, fulfilling some purpose, project, or need, requires guidance or direction from a source outside himself [sic) or itself" (Benne, 1970, p. 392). Those who lead and those who submit are both relevant to determining the claims to the legitimacy of the authority. Both can determine the extent to which the need for guidance is fulfilled and change the relationship accordingly. Authority requires legitimate claims to competence; otherwise, it becomes a power relationship that involves coercion, a pattern of *over* and *for,* rather than *with* (McNay in Brubaker, 2012).

Authority in education appeals to a value system or normative order that students uphold with their teacher, giving sense to their relationship. Authority cannot be disassociated from the idea of freedom as the students are free to acknowledge the legitimacy of the teacher's authority (Perry et al., 2008). If the students are coerced to accept the teacher's authority, the latter cannot claim their authority as legitimate. The students have the freedom to reject or resist the teacher's authority, but do not do so as they recognise its legitimacy (Goodman, 2010; Peters, 2015). This could be evidenced in a classroom in which students often moan about the relevance of a particular mathematics topic to their lives – "Miss, how will this help me in real life?" – but nevertheless capitulate to the curriculum requirements stressed by the teacher, knowing of the future benefits of a good mathematics grade. Having said this, in classrooms where the students exercise their freedom to reject or resist teachers' authority, they could expose themselves to negative consequences, and hence coercion, regardless of whether a given teacher has legitimate claims to competence (Hargreaves, 2017).

As stated in the previous section, in a conventional pedagogy, the teacher is the sole authority. Relevant to this study is the analysis of teacher authority as two interwoven but distinct dimensions of "content" authority and "process" authority (Oyler, 1996a). These dimensions of authority originated in Peters' (1966) view of the teacher as both "an" authority and "in" authority (p. 239-240).

The "content" dimension of authority refers to one who is validated as a knower and viewed as the legitimate possessor of knowledge (i.e., of content). A teacher is an authority carrying out their role as a teacher to teach their subject content. This content authority is referred to in this study as "epistemic" authority (Hargreaves et al., 2018). The use of the term "epistemic" as opposed to "content" is in keeping with

Solomon’s (2009a) use of the word epistemic that views mathematics knowledge as open to negotiation and knowers as creative negotiators of mathematics knowledge. Epistemic authority is attributed to the teacher by the definition of the “teacher” role. It presupposes that the teacher has studied to attain the subject knowledge, and is therefore employed by the school. However, the teacher has to demonstrate and establish this authority in the classroom for it to be legitimised by the students (Hargreaves et al., 2018; Wagner & Herbel-Eisenmann, 2014).

On the other hand, a teacher has “process” authority due to an aspect of the prevailing culture: how the knowledge is taught in the classroom in a given society. This process dimension of authority, synonymous with being ‘in’-authority (Peters, 1966), is best understood in terms of the notion of framing (Bernstein, 2000). Framing relates to how knowledge is communicated and the nature of the relations that go along with it. It relates to who is in control of selecting the knowledge to be communicated, the “how” of learning, its sequencing, its pacing, the instructional criteria, the control of the social base, the regulative criteria, and the dominant values of the society that make the communication of knowledge possible (Bernstein, 2000, p. 37). When the teacher is in control, such as in a conventional pedagogy, the framing is said to be “strong”. The teacher has authority over the processes of how the knowledge is communicated to the students. Theoretically, where the students are in control, the framing could be said to be weak; it is important to understand that this is not an evaluation of quality, but of the potency of individuals’ relations to the determination of practice.

As I view students' active participation in all aspects of their learning as necessary for the aims of this study – to improve student's relationships and learning of mathematics – this study requires a move away from the conventional pedagogy in which authority is in solely the teacher's possession in order to achieve its aim. Instead, it calls for a shared authority pedagogy, where the students participate in all aspects of their learning. As Oyler (1996a) notes, this is a more significant move than it would seem: "Sharing authority then is much more than offering activity choices; rather it requires that teachers and students develop and negotiate a common destination or agenda" (p. 23).

2.3.2.1 Shared Authority

The process and epistemic dimensions of authority are not the only ways to construe authority in an educational setting. Various authors identify a range of types of authority (Amit & Fried, 2005; Pace & Hemmings, 2007; Solomon, 2009b; Wagner & Herbel-Eisenmann, 2014). From my reading of Solomon (2009c), I would argue that the process/epistemic distinction points to what one has authority over, while the notion of shared authority addresses whether/how participants distribute authority amongst themselves. Shared authority, also referred to as "revised authority" (Amit & Fried, 2005, p. 151), is the authority characterised by co-participation that involves both the students and the teacher; in this case, the legitimacy of either the students or the teacher's authority comes from mutual interdependency where those involved, such as the teacher and the student, are continually learning and reaching beyond their present relationship to a relationship that “supports independence while acknowledging differences in knowledge, skill and status” (Benne, 1970, p. 401).

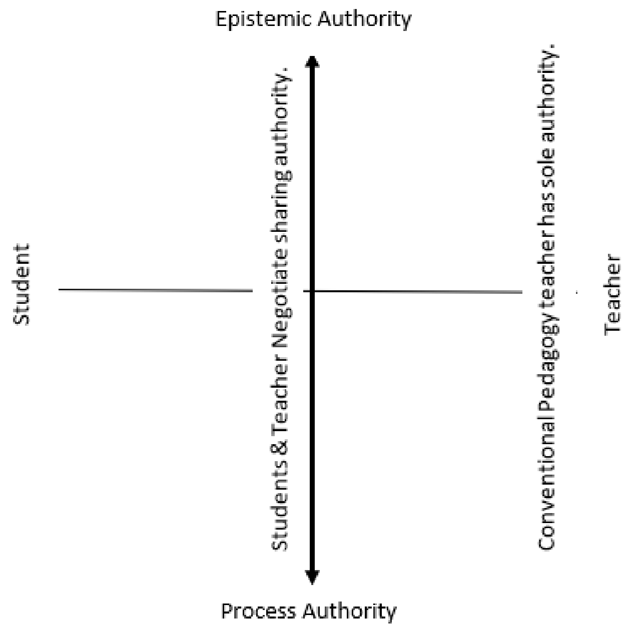


Figure 2.4 – Process/Epistemic dimensions of authority vs teacher/student’s authority distribution.

In classrooms with revised authority, students, and the teacher, through their participation, can negotiate how process and epistemic authority is shared (see figure 2.4). The revised authority shifts the focus of authority to negotiation and consent, and renders the relationships upon which authority supervenes as dynamic and fluid (Amit & Fried, 2005). The students do not blindly expect the teacher to be the expert, but see expertise in themselves and in each other (Brubaker, 2012).

Epistemic authority refers to who is viewed as legitimately knowledgeable and process authority refers to how the knowledge is taught in the class. However, teachers also have other relationships to knowledge that support their authority.

Shulman (1986, 2013) coined the term “pedagogical content knowledge” (PCK) to emphasise that a discussion of one’s knowledge of a subject is not sufficient to explain what is necessary for teaching. He suggests a trichotomy of categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. In this study, epistemic authority refers to subject matter content knowledge, and the notion of process authority subsumes pedagogical content knowledge and curricular knowledge. As the term suggests, subject matter content knowledge refers to the structure and amount of subject knowledge in the teacher’s mind. Pedagogic content knowledge refers to the generic principles of classroom organisation and management, the most useful representations of ideas that make them comprehensible to students’ preconceptions, and common misconceptions that students bring with them to topics. Curricular knowledge refers to the full range of topics required for the subject; it includes the sequence of topics, instruction material, and assessment requirements. From a mathematics perspective, pedagogical content knowledge and curricular knowledge can be conceptualised as mathematics knowledge for teaching (MKT) (J. Silverman & Thompson, 2008); in other words, as what is necessary for successfully teaching mathematics.

It is prudent to assume that teachers and students can share content authority and process authority in a pedagogy in which both students and teachers participate equally in a classroom community. However, as the teacher and students negotiate their practice (see section 2.2.1) to advance their mathematics knowledge, their mutual relations of interdependence would recognise that some aspects of process authority such as mathematics knowledge for teaching will best reside with the teacher. Due to their training and experience in the profession, the teacher is more likely to possess knowledge such as the exam board requirements that influence the questions students practice in class.

2.3.2.2 Positioning

In her book chapter "Doing Undergraduate Mathematics: Questions of Knowledge and Authority", Solomon (2009a) discussed how students are "positioned" in mathematics learning communities by their perceptions of authority. The positioning of students in the classroom can result from how the pedagogy distributes authority between the teacher and students in a mathematics classroom. Davies and Harré (1990) described positioning as the discursive process in which speech and action are used to arrange people in social structures by locating them in conversations as "observably and subjectively coherent participants in jointly produced storylines (discourses)" (p. 48). Storylines (discourses) "are the ongoing repertoires that are already shared culturally or they can be invented as participants interact" (Herbel-Eisenmann et al., 2015, p. 188). Interactions are communications, dialogue, or actions that occur among people, either face to face or through other media. Interaction occurs in a mathematics lesson between participants, whether between teacher and student or student and student. As participants interact, they assign positions for themselves and others participating in the interaction.

Positioning constrains what one may meaningfully say or do. With every position comes a connected discourse. In this way, positioning may "diminish the domain of what one does out of the possibilities of what one can do" (Harré & Slocum, 2003, p.

106). There are many positions available for the students and the teacher formed by their interactions in the discourse of schooling. A teacher standing in front of the class positions themselves as *in* authority (process authority) and consequently positions the students as subject to such authority. This positioning of the teacher constrains them to control the students' behaviour, while it expects the students to behave in a certain way, such as sitting quietly. Subsequently handing the whiteboard pen to the students, the teacher is able to position the student as an authority (epistemic authority) in a way determined by the particular context; having been so positioned by the teacher, the student is expected to answer correctly. In this sense, people are positioned through interaction with others, and this positioning tracks these interactions (Davies & Harré, 1990). Positions are responsive to context, and participants' relations to them are dynamic, as one can occupy more than one position and shift between positions.

To position someone is to establish what their duties and rights are, and to determine what they are obliged and allowed or not obliged and not allowed to do (Harré & Moghaddam, 2003; Harré & Slocum, 2003). A participant's rights constitute what others must do for them, and their duties constitute what they must do for others. Having been positioned, either interactionally by others or reflexively by themselves, a person "sees the world from the vantage point of that position" (Davies & Harré, 1990, p. 6). The position gives meaning to the participants' and others' speech, writing, and actions (Harré & Moghaddam, 2003). The meaning of a position is influenced by and influences the past, present, and future of the participants' interactions and participation; thus, in an educational setting such as a classroom in which the teachers are in authority, the conventional positioning of students can include or exclude them from participating in mathematics learning (Solomon, 2007, 2009c). Positions are defeasible (Harré & Slocum, 2003) and can be disputed over time or in the moment. This study aims to develop a pedagogy that challenges the teacher-student discourse that positions the teacher as knowledgeable and the students as not knowledgeable.

A useful distinction for my thinking is that between position and roles. In contrast to flexible and situation-specific positions, roles in interactions are static, though long-term positions approximate the status of a role (Harré, 2012). The static nature of a role can be understood when considering its close relationship with the function of a "job". A role, like a job, "represents a set of constraints and requirements that is rather pervasive in someone's life" (Harre & Slocum, 2003, p. 104). "Teacher" is a fixed role in a school, while the teacher themselves can, through their interactions, be positioned temporarily or lastingly as an authority or otherwise in different situations, dependent on the discourse. This study, by proposing a pedagogy that it takes to be innovative, follows the heels of other research that has tried to change mathematics classroom pedagogy in England by challenging existing authority relations. The pedagogy and its discourses determine the location of authority, as well as the roles and positions available to its subjects in the maths classroom.

2.3.3 Summary

This section examines the pedagogy critiqued by Paulo Freire and Jacques Rancière and its similarities to the conventional pedagogy experienced by students in many present day classrooms in England. The proposed innovative pedagogy will seek to facilitate co-participation and interdependence between students and teachers (Benne, 1970), as against the established forms. Students and teachers sharing authority in the classroom will learn from each other and negotiate how best to use their different skills and experiences in mathematics learning.

In the first three sections of this chapter, I have described aspects of agency relevant to the aims of this study, emphasising the usefulness of the deep constructivist notion of epistemic agency, according to which students take responsibility for their knowledge. I have also discussed the knowledge-creation metaphor, which represents learning as both an individual and collective endeavour; this metaphor prepares the way for the possibility of a dynamic pedagogy, where learning occurs as students interact, rather than where knowledge is merely transmitted into their passive minds by a teacher, as described in section 2.2. Wenger's social learning theory allowed me to examine how learning can occur through students' participation in a mathematics classroom. In this third section, I developed the notion of authority in the context of mathematics pedagogy. In the following two sections, I will begin to argue for the notions and concepts that I rely upon in working to achieve the aims of this study, first constructing the theoretical framework.

2.4 Theoretical Framework

The aim of this study is for the students in my mathematics classroom to actively participate in all aspects of their learning, and to thereby improve their relationship with and their learning of mathematics. To achieve this, existing constructs that have achieved similar aims to mine will be considered in order to help develop the theoretical argument that will underpin this study. The two focal ideas of knowledge building and shared epistemic agency will be introduced in this section as they build upon the previously discussed notions of agency – in particular epistemic agency – that made visible the possibility of students taking responsibility for their learning introduced in section 2.1. The theory of communities of practice reflects this study's interest in the classroom as a learning community and its possibilities for changing participants' relationship with mathematics; however, the community of practice alone cannot account for the acquisition of knowledge, such as mathematics knowledge, and has been supplemented with social learning theories.

2.4.1 Knowledge Building/Knowledge Creation

This section discusses in further depth Scardamalia and Bereiter's conceptualisation of knowledge building and Nonaka's contemporary account of knowledge creation. The concept of knowledge building is helpful for this study as it illuminates students' engagement with knowledge to the extent that it is useful to all classroom participants. It goes beyond the weak constructivist notion of learners' active construction of knowledge to include the two characteristics of intentionality and community knowledge (Scardamalia & Bereiter, 2010) addressed in sections 2.1 and 2.2 respectively. From a weak constructivist perspective, learning is personal and occurs unconsciously through engagement in activity. By contrast, the deep constructivist perspective of knowledge building considers students as intentionally producing purposeful and valuable knowledge; it furthermore concerns the creation of knowledge in the form of conceptual artefacts for the benefit and advancement of the community. Although individual learning could occur in the process, it is not the ultimate goal of the activity; the primary goal is to solve problems, develop new thoughts and ideas, and advance community knowledge.

Understanding knowledge building requires a prior understanding of conceptual artefacts (Bereiter, 2002, p. 64) and their role in collaborative knowledge building. Conceptual artefacts are abstract knowledge objects (e.g., ideas, theories, algorithms) that can be realised in some material form, typically through discussion or physical construction. Logical relations exist between conceptual artefacts; for example, one conceptual artefact could justify another, and be derived from yet another. Artefacts can be criticised, tested, and improved. Bereiter and Scardamalia claim that in order for conceptual artefacts to be treated as objects of new knowledge and credited as evidence of knowledge, they must: i) be of value to people other than the individual; ii) have value that endures beyond the moment in which it is conceived; iii) apply beyond the situation that gave rise to them; and iv) display evidence of a modicum of creativity in their production (Bereiter & Scardamalia, 2011, p. 3). For example, consider a situation in which an individual, through experience as a decorator, develops a good sense of symmetry. For Bereiter and Scardamalia, the individual has *acquired* knowledge, not built it. If the individual produces a short video that shows how images are reflected from one side to another, the individual would be said to have *created an artefact*. This artefact, though not conceptual, would enable others to access and acquire the tacit knowledge and skills that the individual has. For the artefact to be termed conceptual, the individual would have to produce a mental *theory* that explains how the symmetric image is produced. This theory is a conceptual artefact, and it can be treated as knowledge that is represented in the video, which therefore fulfils the criteria above. Developing the theory that supports the conceptual artefact is the process of knowledge building. When students build knowledge, they are actively engaged, as a community, to create conceptual artefacts. This collective approach to creation shares and advances the knowledge of the community.

Knowledge building therefore consists in the continuous collective production of improved forms of ideas (conceptual artefacts) that contribute to the advancement of knowledge in a community (Bereiter, 2002). It challenges learners to go beyond individual capabilities and to collaborate, with whom they share a common epistemic goal. Bereiter (2002) and Scardamalia & Bereiter (2014) derived knowledge building from an epistemological outlook that treats ideas as entities in their own right, independent of the mental states of individuals. In classrooms organised around knowledge-building pedagogy, individual students are recognised for their contributions to collective knowledge advancement rather than for what is “in their minds”. In these classrooms, students find respect and acceptance as contributors in knowledge creation (Scardamalia & Bereiter, 2006).

Thus, on the basis of their theory of knowledge building, Scardamalia and Bereiter proposed a pedagogy that encourages an individual to intentionally execute higher level cognitive processes on their own, without depending on their teacher, within a classroom community that further sustains knowledge advancement by providing opportunities for student-to-student feedback. The pedagogy is based on twelve principles (see Appendix 1) which deviate from currently prescribed procedures (Lai & Campbell, 2018; Scardamalia, 2002). Six of these principles align with the aims of this study and the innovative pedagogy I propose. The other are less relevant to secondary school mathematics pedagogy that follows the GCSE curriculum. I will here outline the principles that align with this study, and subsequently synthesise them with other active theories in order to produce my own characterisation of shared epistemic agency in a knowledge-building pedagogy. The relevant principles are:

- *Community knowledge, collective responsibility* that encapsulate the aim of knowledge-building pedagogy to produce knowledge that is useful to and usable by the participants of a classroom community (see section 2.2.2 on communities of practice).
- *Epistemic agency* (see section 2.1.3), which is essential for supporting the collective efforts of knowledge advancement beyond the individual performance of tasks.
- The *collective improvement of ideas* (see section 2.3.2.1)*.* There are no final truths; learners view every idea as having the potential to be improved. The improvement of ideas comes from the students as they seek to reconcile conflicting conceptions. There is the “continual application of a ‘make it better’ heuristic” (Scardamalia & Bereiter, 2014, p. 400).
- *Knowledge-building discourses* for the improvement of ideas (see section 2.2.1). Bereiter (2002) argues that classroom discourse should mimic professional science discourse. It should, in other words, be

cooperative and more concerned with creatively advancing the collective knowledge beyond what is currently known.

- The *democratising of knowledge* that is a result of such discourses (see section 2.3.2.1). In a classroom based on the knowledge-building paradigm, all participants are deemed legitimate contributors to collective knowledge.
- The *use of authoritative information*, such as multimedia resources, in these classrooms. In my classroom, this involves the careful use of such things as MathsWatch, textbooks, and other media in order to construct coherent knowledge from diverse representations.

The following section will discuss the concept of "knowledge creation", not to be confused with the "knowledge-creation" metaphor for learning described in section 2.2.1.

2.4.1.1 Knowledge Creation

Though distinct from knowledge building, Nonaka's (1991) concept of knowledge creation relates to the former in its focus on the ways in which a community can create new knowledge from within, through active engagement; this concept is useful for the secondary mathematics classroom in which students need to make mathematics knowledge and their problem-solving strategies explicit to each other. The distinction between knowledge building and knowledge creation is due to the different disciplinary commitments of the associated theorists: knowledge building was developed in the context of education, while knowledge creation is a dynamic that was initially identified in the context of the corporate organisation.

Nonaka's concept of knowledge creation is germane to the aims of this study to the extent that it recognises the value of knowledge as both explicit and tacit, placing an emphasis on the process by which personal knowledge is made available to others. Explicit knowledge is easy to articulate, while tacit knowledge is personal, hard to formalise, and challenging to communicate to others; it consists of mental models, beliefs, and perspectives (Nonaka, 1991). This concept can explain how, in the mathematics classroom, knowledge can be tacit or procedural, and students may find it difficult to articulate their reasoning and justify their solutions to problems; or else the knowledge can be explicit, in which case students will typically find it easy to communicate their thinking. Both types of knowledge are of value, and Nonaka's theory further reveals the process by which the two interact in a "spiral of knowledge" (p. 97) to generate innovations; that is, to create new knowledge. This presents the interaction between students as a process of knowledge creation

The knowledge spiral, which depicts the iterative transformation and sharing of knowledge from the level of the individual to that of the organisation, and even among organisations, is grounded in four complementary knowledge-creation stages that operate between individuals and groups in an organisation (Figure 2.5).

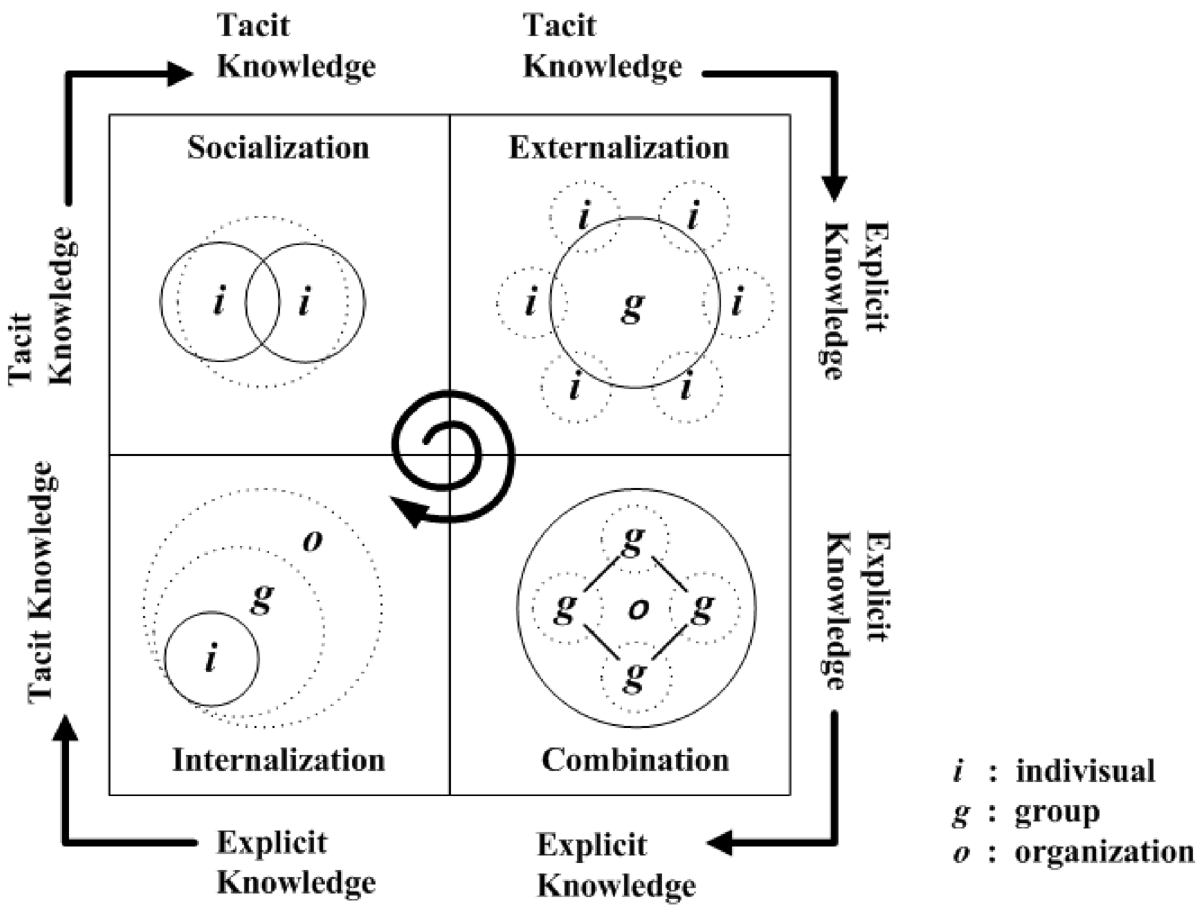


Figure 2.5 – The Knowledge Creation Spiral. Source: (Umemoto, 2002, p. 464)

The first stage involves the transmission of tacit knowledge from individual to group due to the sharing of experiences in the activity *socialisation*. It is essential to develop trust between individuals at this stage, as close interaction and collaboration are necessary for the effective sharing of the explicit knowledge over time. In the second stage, tacit knowledge is transformed into explicit knowledge through *externalisation*. In this stage, the tacit knowledge of a socialised group is made explicit through discourses, metaphors, diagrams, and concepts – that is, through artefacts. Thus, during externalisation, knowledge can be exchanged by means of what Nonaka refers to as a "metaphors, analogies and models" (p. 99), which is broadly analogous to Bereiter's conceptual artefact. In the third stage, the new explicit knowledge aggrandises itself through its *combination* with existing explicit knowledge, and is subsequently distributed throughout the organisation. In the fourth stage, explicit knowledge is transformed back into tacit knowledge, through *internalisation*, and begins to inform the practices of individuals. This implicit knowledge is then itself socialised, beginning the cycle anew.

Bereiter (2002) was critical of Nonaka's knowledge spiral on four counts, noting its exclusion of creativity, understanding, knowledge work, and collaborative knowledge building (Bereiter, 2002, pp. 175–177). He argued that as the model does not distinguish between "knowledge involved in productive work and knowledge that is a product of productive work" (Bereiter, 2002, pp. 177–178), it cannot promote learning that will contribute to a community's ability to create knowledge. He noted that the knowledge spiral could be a carrier of ritual and tradition, as it presupposes shared implicit understanding but does not necessitate understanding at the individual level. The individual does not become what he referred to as "a fully functioning member of a knowledge society" (Bereiter, 2002, p. 173). However, I argue that Nonaka's perspective on knowledge can contribute its thinking to structure the mathematics pedagogy that I seek to develop in this study. Supplementing her picture of the transformation of knowledge with capacities for discussion and shared problem solving evades Bereiter's critiques and contributes individuals' tacit knowledge to community knowledge. In this sense, student-to-student explication of mathematical knowledge fulfils the criteria of new knowledge, and I argue that it qualifies as knowledge building. Despite Bereiter's criticism, other authors such as Paavola et al., (2006), whom I discussed in section 2.2, and Damşa et al., (2010) whose work I describe in the next section, have also combined these two models.

In summary, knowledge building and knowledge creation orient the design of a pedagogy that focuses on the individual's engagement with knowledge for community benefit. Individuals can be seen to benefit from the pool of knowledge within the community from which they can draw. This picture of the synergy of the individual and the community is in agreement with Wenger's theory of community of practice (see section 2.2.2), wherein he argues that through participation, benefits such as accountability and mutual relations contribute to the advancement of a community's enterprise (Farnsworth et al., 2016). In the following section I describe the kind of agency I desire the pedagogy of this study to develop in the students. This agency is referred to as shared epistemic agency.

2.4.2 Shared Epistemic Agency

Shared epistemic agency, introduced by Damşa et al. (2010), is the central concept of this study. It is described by these authors as an emergent construct that builds on Scardamalia's (2002) notion of epistemic agency (see section 2.1.3), which they used to characterise undergraduate students' abilities to carry out complex, authentic collaborative projects. They conceptualised shared epistemic agency to include the notion of sharedness that presupposes intentionality (Bandura, 2001; see section 2.1.1), the collaboration between participants, the social-communicative processes that leads to new collective knowledge (see section 2.4.1), as well as the notion of an established community of practice – i.e. the mutual relations of participation that support coherence in a community (Wenger, 1998, and section 2.2.2). Shared epistemic agency describes the interdependency of partners (see section 2.3.2.1) and the collaborative actions that do not happen when individuals work on their own. It also draws on the knowledge-creation perspective of learning (see section 2.2.1) that situates learning as occurring during collaborative practices that create shared material knowledge objects.

Damşa et al.'s construct of shared epistemic agency, which lies within the knowledge-creation perspective (see section 2.2.1), depicts a specific form of epistemic agency (see section 2.1.3) that emerges during collaboration to create shared knowledge objects. In this sense, the shared knowledge object is both the outcome of the group's collaboration and the reason for the group's activity (Stahl, 2009, p. 64). Damşa et al., like Nonaka (1991), acknowledge the interaction between explicit and implicit knowledge as of value to knowledge creation, while arguing that shared epistemic agency goes beyond knowledge building. They argued that knowledge building emphasises collective collaboration for the improvement of singular ideas, whereas shared epistemic agency involves working on more than one idea to create knowledge through the advancement and development of complex knowledge objects (Damşa et al., 2010). These authors posit that learning occurs as students act to give conceptual artefacts a concrete form as material objects of shared knowledge, such as reports, essays, or software.

Shared epistemic agency can be understood as the "capacity that enables individuals, groups, or collectives to make appropriate judgments, to make plans, and to pursue these through purposeful action, in order to achieve the construction of knowledge" (Damşa, 2014, p. 446). In addition to sharedness, this definition emphasises epistemic productivity and negotiation within the community. The related notion of "temporality" refers to the emergent nature of the agency in question (p. 447); it suggests a certain kind of practice that is reflexive and iterative, considering past practices and experiences metacognitively to solve present problems and create plans that lead to future desired outcomes.

Shared epistemic agency is an empirical concept; in other words, it is a conceptualisation of observable phenomena and they expressed the intentions that materialise indicative of the agentic behaviour (Damsa et al., 2010, p. 155). The unit of analysis, is, therefore, the group-level actions that constitute the conditions for its emergence. These actions fall into two categories: the epistemic and the regulative. Epistemic actions are directed towards knowledge and the creation of knowledge objects. These include actions that serve to create awareness of the current knowledge situation within the group (e.g., brainstorming, discussing); that create shared understanding; that alleviate a lack of knowledge and gather information (e.g., researching, asking, discussing); and that generate collaborative actions (e.g., explanations, concepts) (Damşa & Andriessen, 2012).

Regulative actions are the processes that occur at the metacognitive level and that prepare the foundation for epistemic actions. They do not directly influence the creation of knowledge objects, although they make their creation possible. Regulative actions are based on the group's intentions (Bandura, 2001) to create the knowledge object, and consist in the procedures that occur as a result of this intention (Emirbayer & Mische, 1998); that is, they are the result of the meta-knowledge that the group has about the process and the progress of creating the knowledge object that informs the actions that the group takes. These actions, consisting of projective actions, the setting of a common goal, the creation of a plan of action, and proactive engagement, are required for successful outcomes. Regulative actions, such as monitoring the progress of the knowledge object and reflecting on it, and relational actions – the social aspect, i.e., validation and the acknowledgment of individual contributions – facilitate relations between individuals and the group, making possible the maintenance of their epistemic community. An overview of epistemic and regulative actions is offered in Appendix 5.

2.4.3 Summary

Knowledge building conceptualises a community learning environment in which students interact with shared intentions to improve on their ideas, creating new knowledge continuously. Shared epistemic agency is a conceptualisation of the capacity of individuals and collectives to perform collaborative actions, bringing together multiple ideas to create a knowledge object, which is the material realisation of their new knowledge. To achieve the aims of this study, I consider these concepts in the context of the capabilities of students. It is my intention to promote the emergence of shared epistemic agency amongst the students in my mathematics classroom, creating a learning environment in which they continuously develop new knowledge and control their own knowledge advancement.

Although Damşa et al. describe shared epistemic agency in terms of the epistemic and regulative actions that, over time, lead to the creation of a knowledge object, their empirical study reports only on undergraduate students engaged in one-off collaborative group work to produce an authentic knowledge object such as an instructional design project or a training and evaluation project (Damşa et al., 2010). Their research cannot be applied without modification to a secondary mathematics classroom, in which both participants and subject matter are considerably different from the original objects of the study. Thus, I proceed with my own study by apprehending and developing the notion of shared epistemic agency in this new context; I determine that the shared epistemic agency that I want to emerge is a quality of students that is an index of active participation in all aspects of their learning of mathematics and an improved relationship with mathematics, which leads to improved mathematics learning. Good GCSE grades will evidence this improved learning in the students' terminal secondary school examinations.

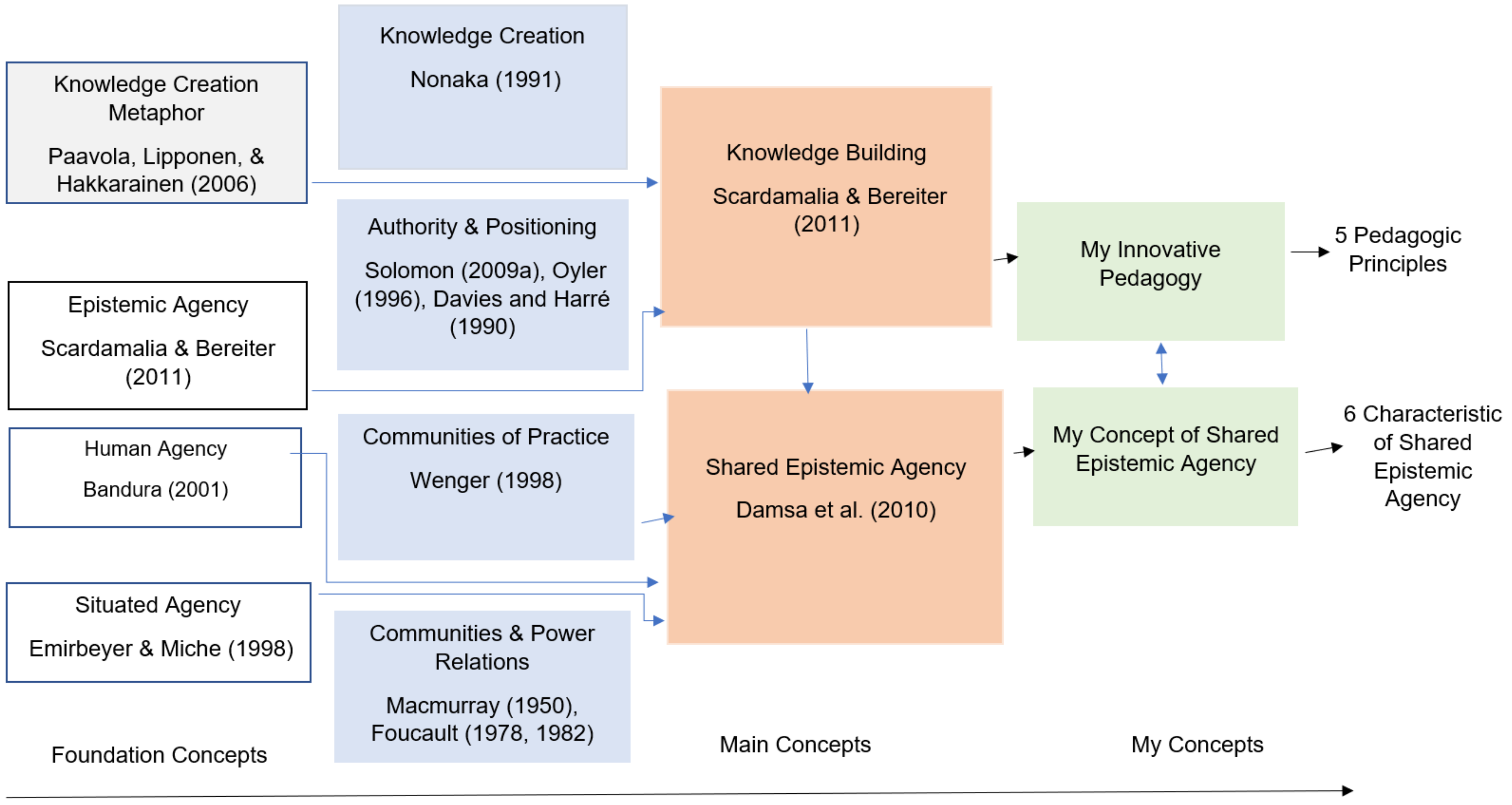


Figure 2.6 – Theoretical background: interconnection and relevance of concepts, notions, and perspectives.

On the strength of the theoretical background developed) in this chapter (see Figure 2.6), I can now characterise the specific kind of shared epistemic agency that I consider appropriate for the aims of this study. Its six characteristics are given as:

a) *Intention*. The agency will include intentionality: the proactive commitment to bring about a desired outcome (see section 2.1.1) that presupposes purposefulness and will include community knowledge (cf. Bandura, 2001; Damşa et al., 2010; Scardamalia & Bereiter, 2014).
b) *Extension*. The student deliberately focuses on going beyond existing knowledge. This notion originates in the theory of knowledge building (see section 2.4.1, first paragraph) that extends constructivism towards deep constructivism (see section 2.1.3), in line with which students control all aspects of learning (cf. Bereiter & Scardamalia, 2011).
c) *Explication*. This refers to purposeful dialogue that makes knowledge explicit so that it can be shared (see section 2.4.1.1). Drawing on Nonaka's knowledge spiral, shared epistemic agency will acknowledge sharing personal knowledge and the interaction between tacit and explicit knowledge that communicates mathematics knowledge through dialogue, advancing all students' knowledge in the classroom (cf. Nonaka, 1991).
d) *Expertise*. Students are considered to be expert learners who set themselves similar tasks to those typically imposed by mathematics teachers. This draws on Damşa et al.'s notion of regulative actions (see section 2.4.2) that depict the metaknowledge possessed by the group that allows them to manage and monitor the advancement of the knowledge object, requiring them to not to

rely solely on external sources such as the teacher (cf. Damşa & Andriessen, 2012).

e) *Mutual Relations*. In order to sustain epistemic agency, mutual relations between individuals must be established (see section 2.2.2). The application of my revised notion of shared epistemic agency will include a consideration of the mutual relations that support the coherence of the community in the project of fulfilling their common purpose of learning mathematics (cf. Wenger, 1998).

f) New Knowledge – This refers to learning through collectively developing ideas and explanations that are new to the students (see section 2.4.1) The final object of analysis will be the new knowledge students are able to create, in the form of a conceptual artefact that is the product of more than dialogue with the pedagogical authority, instead combining the collective and individual contributions of learners who are actively engaged in developing new ideas and explanations in the context of unfamiliar mathematical concepts (cf. Bereiter, 2002; Bereiter & Scardamalia, 2011).

The precise nature of these characteristics, in the specific context of the knowledge-creating classroom practices that are the object of my study, will be illuminated in the following sections. The actions and artefacts that are indicative of each of these six characteristics will also be identified by the end of this study. Henceforth, the term "shared epistemic agency" will encapsulate the six characteristics stated above. The wider construct originating in Damşa et al. (2010) will be referred to as "SEA" for differentiation. Therefore, a preliminary question that this study seeks to answer is: What are the indicators of shared epistemic agency in the mathematics classroom?

As previously stated, knowledge building requires a learning environment that could support the emergence of shared epistemic agency. The innovative pedagogy I propose draws on the concepts of knowledge building and knowledge creation to support the emergence of shared epistemic agency. The pedagogy will be based on the knowledge-creation metaphor of learning, according to which new knowledge is continuously and creatively produced from within the learning community. It will seek to reimagine the conventional teacher-student power relations by demonstrating the interdependence of authority (see section 2.3.2.1), and by redefining learning as a community endeavour. The pedagogy will draw on the six key principles of knowledge building, and will notably include reflection that leads to improvement (see section 2.5.1), as well as explicitly relying on the community relations that support the genuine advancement of knowledge.   Given my synthesis of the previous literature performed in this chapter, I clarify the principles of the innovative pedagogy I propose as stipulating that students are responsible for:

1. Building objects of mathematical knowledge (cf. Bereiter, 2002; Damşa et al., 2010; Emirbayer & Mische, 1998; Reed, 2001; Scardamalia, 2002).

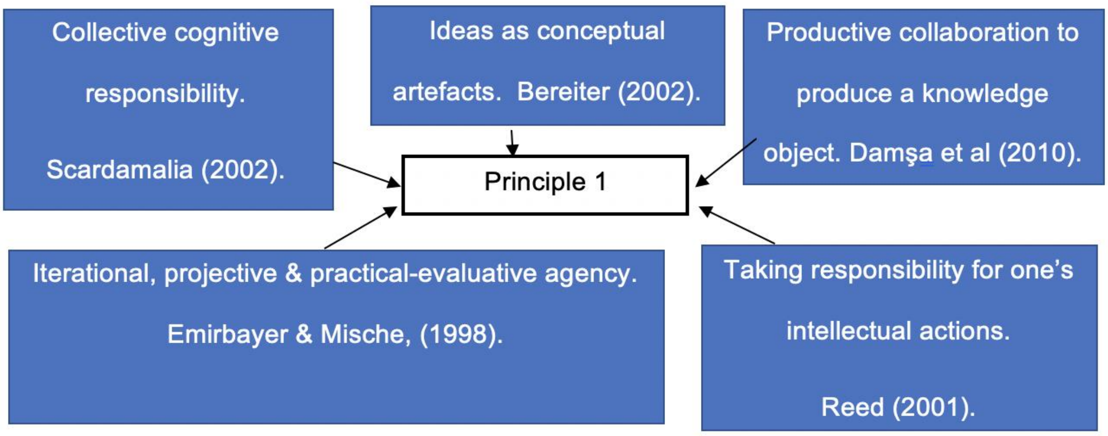


Figure 2.7 – Pedagogic principle 1

2. The process that makes this knowledge explicit so that it can be shared, internalised, and used by all the classroom participants (cf. Bandura, 2001; Damşa et al., 2010; Nonaka, 1991).

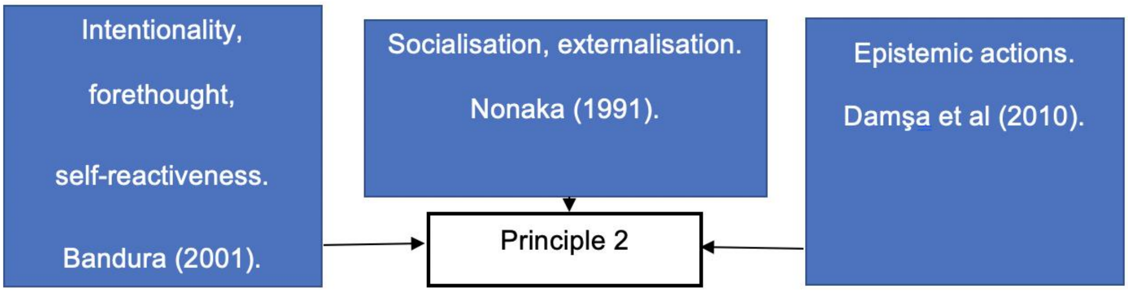


Figure 2.8 – Pedagogic principle 2

3. The discursive process that communicates this knowledge to the classroom community (cf. Emirbayer & Mische, 1998; Nonaka, 1991; Scardamalia & Bereiter, 2014).

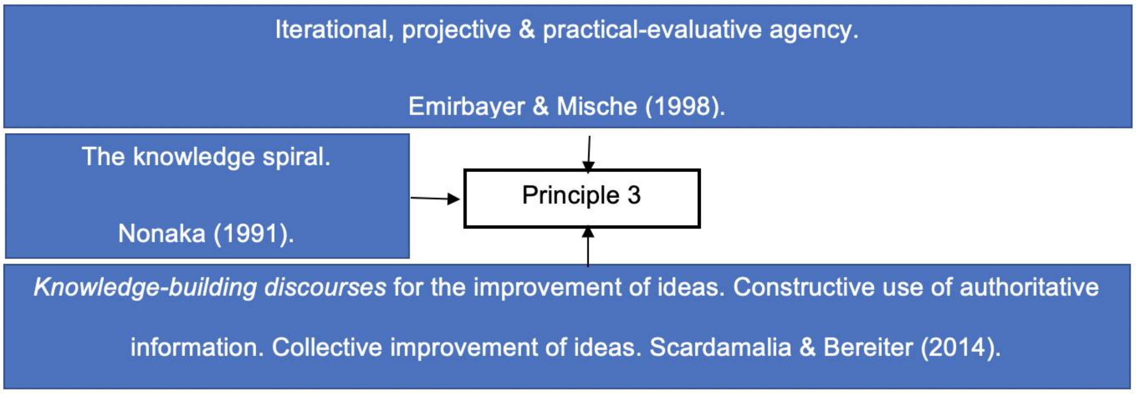


Figure 2.9 – Pedagogic principle 3

4. Maintaining the social relations and communicative processes that are conducive to the advancement of mathematical knowledge (cf. Bandura, 2001; Damşa, 2014; Damşa et al., 2010; Wenger, 1998).

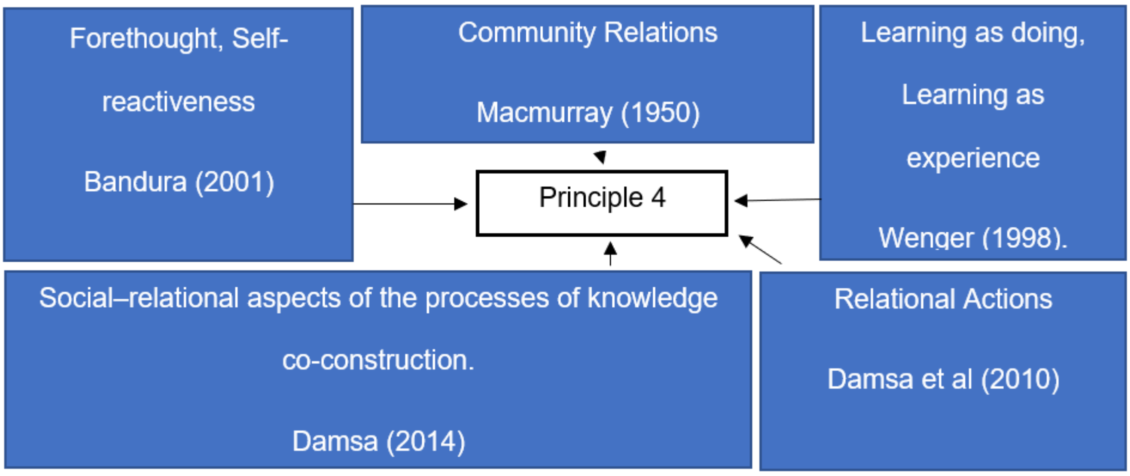


Figure 2.10 – Pedagogic principle 4

5. Reflecting on practice and making plans for the improvement of ideas and activities (cf. Bandura, 2001; Bereiter & Scardamalia, 1998; Emirbayer & Mische, 1998; Yang, Chen, et al., 2020).

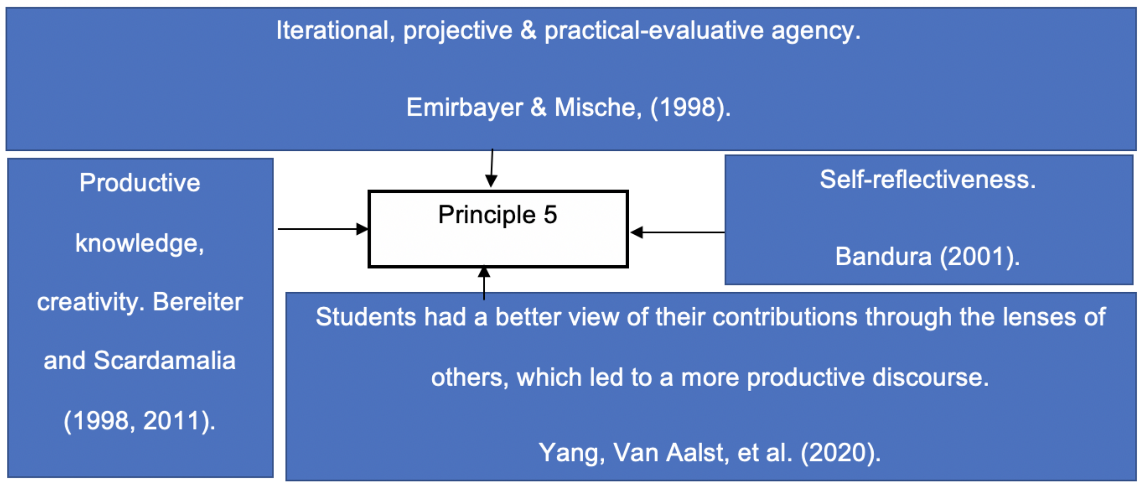


Figure 2.11 – Pedagogic principle 5

In the next section, I will investigate pedagogies that have turned control of learning over to the students, providing a touchstone for my own suggestion of a pedagogy that meets the aims of this study in the context of my mathematics classroom.

2.5 Researching Innovative Pedagogies

The emergence of shared epistemic agency requires more than groups of individuals learning collaboratively. Simply bringing students together to work on a joint task and pooling their knowledge together is not sufficient to create new knowledge (Barron, 2000; Scardamalia & Bereiter, 2010). As elaborated in the previous section, it requires an established community with customary practices negotiated over time (Damşa et al., 2010; Wenger, 1998). It necessitates an innovative pedagogy with a purpose, namely, which goes beyond collaborative learning to include the notion of productivity; that is, a knowledge-creating classroom. Having outlined the principles of my innovative pedagogy, in this section I investigate knowledge-building pedagogies and transformative mathematics pedagogies in England to inform the design of my own.

2.5.1 Knowledge-Building Pedagogies

This section describes three pedagogies (Moss & Beatty, 2011; Yang, Chen, et al., 2020; Yang, van Aalst, et al., 2020; Zhang et al., 2018) that are explicitly framed by the concept of knowledge building that I described in section 2.4.1. Though online technology, which is not a focus of my own study, heavily supports student interaction in these pedagogies, the findings are still relevant for their analyses of the ways in which the pedagogy was decisive in developing students' participation in the creation of new knowledge. In Moss and Beatty (2011), fourth-grade students collaborated on an online database that provided a communal space where students posted their ideas and read each other's, engaging in critical reflective activity. In this way, they all contributed to the community knowledge base. The database was entirely student-managed; the teacher's voice was not present, nor were answers or solutions provided from an external source. In other words, the students had collective cognitive responsibility for coming up with conjectures, and solutions, and negotiating the various approaches to mathematical problem solving.

This research illustrates how the knowledge-building principles of the democratisation of knowledge and epistemic agency (see section 2.4.1) can further a mathematics problem-solving and learning culture. Democratising knowledge requires that all participants within a community are legitimate contributors to the community knowledge and that their contributions are valued and acknowledged. Moss and Beatty's students working together to solve problems evidence their epistemic agency; they supported each other's suppositions and questioned when ideas or solutions were incorrect. In this way, the community was assured that the solutions provided were correct. In the absence of an external verifier, the students not only verified their solutions to the problems independently, but also routinely took responsibility for offering evidence and justification for their solutions, with the intention being to make sure that the whole community understood the proper solution to the problems. In this way, they took responsibility for the community's collective understanding. Moss and Beatty researched 8-to-9-year-olds across three schools; the intervention took the form of a one-off addition to the existing classroom pedagogy, which contrasts with my aim to change the overall learning experience of secondary students for a single subject over a whole year.

Moss and Beatty's research, however, does bear similarities to my study; its demographics were of a similar economic status, and a significant proportion of the students were categorised as low-achieving. Equally, the democratisation of knowledge and the quieting of the teacher's voice are outcomes of a knowledge-building pedagogy that resonate with this study's aims.

Yang, van Aalst, et al., (2020) conducted research with low-achieving ninth-graders who collaborated on an online platform. The findings from this research were similar to those of Moss and Beatty. They illustrate how "academically low-achieving" students could get involved in sustained collaborative and productive knowledge-building discourse and inquiry (p. 1253). In addition, in a manner that is particularly relevant to my study, the research illustrated that by engaging in reflections, students had a better view of their contributions through the lenses of others, which led to a more productive discourse. Reflecting on others' contributions to knowledge improvement did not lead to criticism, but became the community practice, the classroom norm. This research focused on developing a community in which the goal and focus of the classroom was knowledge-building collaboration that advanced collective knowledge; reflective assessment was not based on individual attainment, but on the progress made by the whole class.

Research by Zhang et al. (2018) sought to support student-driven inquiry within a socially organised pedagogy. The researchers worked with two upper-primary school classrooms on a knowledge-building initiative. The researchers sought to provide structure to the students' inquiries while still allowing the flexibility that enabled their agency and imagination to thrive. The researchers designed an inquiry-structuring, timeline-based web platform, ITM (Zhang et al., 2018, p. 401), that discovered emerging directions and interests in students' interactive discourses. ITM then formulated unfolding inquiry strands and made them visible to students to support ongoing participation and reflection. The reflective process, facilitated by the technological apparatus, shifted control of the inquiry from the teacher to the student's agency. While the research highlighted the value of reflection, and knowledge building pedagogy was the established science pedagogy for a twelve-week period, the teacher guided the students' inquiry to a larger extent than is proposed in this research. The research shed light on how to construct pedagogical structures with students to develop a classroom community that sustains the students' ownership of their collective thinking journey to support knowledge-building interaction, but I attempt to go further, in line with the renunciation of authority consistent with deep constructivism.

The three studies noted above show how a knowledge-building pedagogy can lead to the emergence of favourable characteristics in the classroom environment, such as the democratisation of knowledge, epistemic agency, the quietening of the teacher's voice, community learning, and improved participation in learning – however, in each case, a technology platform where ideas were shared was central to the pedagogy. In addition, the three studies took place outside of England. In the following section, I will therefore conduct a literature review to identify further research related to my study that has transformed pedagogies, without reliance on a technology platform, in English secondary schools.

2.5.2 Transformative Pedagogies in England

My literature review focuses on studies that have transformed mathematics pedagogies in secondary schools in England in the last ten years, as this frame bears close relevance to the context of this study (see section 1.1). I used the UCL library search facility and put in the terms: <*Any field (contains)* ***transformative pedagogies*** *AND Any field is (exact)* ***mathematics*** *AND Any field is (exact)* ***England***>**,** and I filtered for the Years: 2011-2021, Form: *Articles and Book Chapters*, and Topic: including Pedagogy. Two articles from the 145 results were of interest; the other 143 did not describe a mathematics pedagogy in England. However, on further reading, these two were not found to be germane to the specific aims of my study.

Ruthven et al. (2017) developed the “epiSTEMe” pedagogical model, which focused on improving student engagement with mathematics and science in the first year of secondary school education through exploratory dialogic conference. It was not relevant to this study, as the pedagogic measures it proposed retained the privileged position of the teacher as an authority, and it involved changing the nature of mathematics content as opposed to improving student agency. This research, if anything, further entrenches the roles of teachers as knowledgeable and students as requiring continuous guidance to be knowledgeable.

The “participatory pedagogy” of Lyndon et al. (2019) focused on pedagogic mediation and viewed the student as a social being with the capacity to construct their knowledge in collaboration with others. Despite the similar view of the student in my study, the research differed in context as it focuses on nursery school children, and was not mathematics-specific.

I altered the search term to: <*Any field is (exact)* **pedagogy** *AND Any field is (exact)* ***mathematics classroom*** *AND Any field is (exact)* **England**>***.*** I filtered for the Years: 2011-2021, Form: *Articles and Book Chapters*, and Topic: including Pedagogy, including Education & Educational Research. This produced 13 results; of interest was the work of Hofmann & Ruthven (2018); Watson & De Geest (2014); and Wright et al. (2020). The other 10 articles did not describe a mathematics classroom pedagogy in England.

Watson & De Geest (2014) carried out three-year ethnographic research with three secondary school mathematics departments in England, teaching students of a similar socioeconomic background to that of the students in my study. The departments sought to improve the achievement of their students. However, the transformation did not directly focus on improving student agency. Instead, it centred on changing classroom groupings to mixed-ability, expanding the mathematics tasks available to students, and developing teachers' confidence in their subject content knowledge. These changes are similar to those that have been discussed in my mathematics department and many others over the years; with this study, I propose something more radical: a change in our beliefs about students and the historico-cultural role assigned to them.

Hofmann & Ruthven (2018) were co-researchers on the epiSTEMe project, alongside Ruthven et al. (2017); indeed, the limitations of the project noted above apply to their study as well. I discuss Wright et al. (2020) below.

Altering the search term to <*Any field is (exact)* ***mathematics pedagogy*** *AND Any field is (exact)* ***student agency*** *AND Any field is (exact)* ***England***> produced one new result: Wright (2017)***.***

Further manipulation of the search terms revealed Foster (2013), who is critical of the reductionist approach to traditional mathematics pedagogy, and who and calls for a more holistic approach to mathematics tasks; however, his article was focused on critique, and did not put forward a pedagogy. My systematic search, therefore, resulted in the identification of two studies that share an interest in putting forward a pedagogy, based in an English secondary school, and focusing solely on mathematics. These are the works of Wright (2017) and Wright et al. (2020) from my literature review; the work of Solomon et al. (2021), which I discovered through a search of recent articles from researchers in my bibliography, was also useful.

Wright et al. (2020) adopted a critical model of participatory action research to transform mathematics classroom practice in a London secondary school. The mathematics pedagogy research project they undertook was a collaboration between Peter Wright, an academic researcher, and two secondary school mathematics teachers, who are also co-authors. The project's aims were twofold. The first aim was to investigate the effect of making a progressive mathematics pedagogy visible to students, leading to their appreciation of how to be successful mathematics learners. Progressive pedagogy in this research referred to a problem-solving teaching approach that was discursive, collaborative, and open-ended. The second aim focused on developing and refining the model. Wright's approach to pedagogical transformation focused on developing the teacher's practice.

Wright comes from the school of critical mathematics education, also influenced by Paolo Freire, whom I mentioned in section 2.3.1. Critical educators such as Gutstein (2006) introduced practices that reimagine the authority relations in the classroom and alter the mathematics teaching materials in a bid to help students to understand the society in which they live, and recognise how inequality is contested and produced in society. I do not advance a critical view of society; nor am I interested, in this study, in precipitating changes in the social at large. Though my study focuses on social justice in terms of wanting the students to be total participants in their learning, its ultimate aim is improving exam performance to offer students greater opportunities in life. Wright et al. indeed seek a reversal of historically inequitable academic outcomes by making the pedagogy more visible; in this way, their study and mine have a similar focus. However, though he argued for teachers and students to reflect on the implicit power relations in the classroom that prevent a relationship of trust, which would allow classroom rules to be negotiated and made clear to students, rather than the teacher relying on their authority to control students (Wright, 2017), Wright et al.'s transformation did not go far enough in my view. The researchers restricted student's agency to articulating the justification behind the teacher's intentions. The students did not participate in any decision-making, nor did they initiate or direct any change within the pedagogy; this leads me to question whether the intentions to involve students in negotiating classroom rules held the same social learning focus of developing a practice (see learning by doing in section 2.1), as my study intends to do. The locus of the participatory action-research practice was the relationship between the researcher and the teachers.

In Solomon et al. (2021), the research focused on introducing “Realistic Mathematics Education” (RME) to a group of low-attaining students who had not achieved the accepted pass grade in GSCE Mathematics. The development of the RME pedagogy is supported by “guided reintervention” that requires increased participation on the part of the students and particular practices by the teacher, both underpinned by a significant shift in responsibility and authority from the teacher to the students. The teacher orchestrated whole-class mathematical discussions for a specific goal (p. 175-6). The pedagogy positioned the students as knowledgeable and expected them to articulate and defend their solution strategies.

The research shares similarities with this study. It sought to increase students’ epistemic authority by shifting authority from the teacher to the students and positioning them as knowers responsible for articulating their thinking and solution strategies. However, the study was founded upon a curriculum-focused RME theoretical base, whereas my study is driven by pupil relationships with mathematics. I left the question of how the mathematics was to happen to the students, and our own resources built on workbooks and exam practice.

The literature review has shown that numerous researchers in mathematics education have sought and still seek changes to the conventional mathematics pedagogy. Both Wright and Solomon needed longitudinal studies to embed and research their pedagogy, and both were participatory in that they trialled new ideas in existing cultural settings, not labs. My study, however, stands alone in seeking an everyday pedagogy in which students take control of learning the mathematics curriculum in a secondary school mathematics classroom in England.

The need to change my classroom pedagogy started long before the commencement of this doctoral study. As described in the introduction, I had begun to consider how my actions in the classroom may constrain the students from engaging with mathematics logically. Prior to embarking on this research, I had started to allow the students to take greater control in the classroom and to teach topics to each other. I also allowed them to make decisions about the sequence of the teaching of topics. However, I knew that convincing other professionals to change the conventional pedagogy required a systematic study. I also needed to justify to myself the benefits of my pedagogy by rigorously collecting evidence.

I am aware that there must be other ways of designing a pedagogy that would lead to the emergence of shared epistemic agency in a mathematics classroom. This study's innovative pedagogy started to develop as my classroom practice for two years before the commencement of this study, when I had attempted to silence my authoritative voice as teacher in the classroom so that students could find their own ways of making sense of mathematics through their active participation. In this way, I believed they would respond more logically to problem-solving and ultimately do better in the GCSE terminal examinations.

2.5.3 Summary

My pedagogy will involve the students working collaboratively in line with the pedagogic principles I have established above (see section 2.4.3). The design of the pedagogy will be described in fuller detail in the following section. From my experience before this study, I found that the students act as both an epistemic support and motivator for each other's mathematics knowledge when the authority of the teacher is weakened. The kind of participation that I want my students to be engaged in will develop and change the teacher-student relationship over time. This directs this study towards an action-research methodology that seeks to answer the following questions:

1. What are the indicators of shared epistemic agency in the mathematics classroom?
2. What sustains the emergence of shared epistemic agency in the mathematics classroom?

3 METHODOLOGY

At the end of the previous chapter, I identified the need for a study that combined two interwoven strands: firstly, the design and enactment of an innovative pedagogy that promotes shared epistemic agency in a school context; and secondly, data collection and analytical methods that would enable me to answer my research questions about what indicates and sustains shared epistemic agency. My reading of methodology literature led me to combine these two strands under the auspices of action research, allowing me to engage in “a form of disciplined, rigorous enquiry, in which a personal attempt is made to understand, improve and reform practice” (Ebbutt in Cohen et al., 2018, p. 345). The first section of this chapter sets out my initial vision for what my pedagogy should achieve, informed by the literature introduced in chapter 2. The second section reviews how action research is justified as a research method both in general and for this specific project, and then introduces my plan for my own cycles of action research. The third section outlines the research design that combines the pedagogy stages that correspond to the teaching cycles and the research cycles that outline how data is collected. The fourth section discusses how enacting the pedagogy as part of the action-research methodology allowed me to continuously adapt the pedagogy, its enactment, and the design of the project to meet the aims of the study.

3.1 The Pedagogy

This research project investigates the emergence of shared epistemic agency amongst the students in a mathematics classroom organised around an innovative knowledge-building pedagogy. The innovative pedagogy is based around five principles that I have synthesised from the literature and summarised in chapter 2, as well as being informed by practices that I personally trialled in the classroom. As these principles stipulate a handing over of responsibility to the students, I will henceforth refer to students as “participants”, being faithful to the commitments of my innovative pedagogy (my role as a participant will be discussed later in chapter 6). This is to emphasis not only their responsibility but also their agency in advancing the collective mathematics knowledge of members of the classroom. The participants are responsible for:

1. Building objects of mathematical knowledge (cf. Bereiter, 2002; Damşa et al., 2010; Emirbayer & Mische, 1998; Reed, 2001; Scardamalia, 2002).

    My plan is to have pairs of participants take responsibility for teaching the other members of the class a mathematics topic (these pairs are therefore named “teacher participants”). They are responsible for planning and leading the discussion and learning of a mathematics topic. They make use of relevant information which is not supplied by myself, but discovered independently from other sources such as mathematics websites (MathsWatch, Corbettmaths, Maths Genie), the broader internet, or other individuals. The knowledge objects by which they will reify their mathematics knowledge is the PowerPoint lesson plan they are asked to produce for the lesson, and the answers to the mathematics questions the participants solve during the lesson.

2. The process that makes this knowledge explicit so that it can be shared, internalised and used by all the classroom participants (cf. Bandura, 2001; Damşa et al., 2010; Nonaka, 1991).

   My idea is that, as the teacher participants prepare their lesson plan to teach the rest of the class (the student participants), they consider and decide on how best to make the mathematics topic explicit so that the student participants will be able to make sense of it. This could involve deciding on how their exposition of the mathematics concept is structured and how the contents of the PowerPoint lesson plan support this exposition.

3. The discursive process that communicates the knowledge to the classroom community (cf. Emirbayer & Mische, 1998; Nonaka, 1991; Scardamalia & Bereiter, 2014).

   I intend for the participants of the classroom to engage in discussions to improve their knowledge of the mathematics topic being taught. Through this discussion, tacit knowledge is explicated, and participants ask questions and receive answers that help to clarity their knowledge. My idea is that as I am not the "mathematics authority", the participants must find their own ways to advance their collective knowledge, including sharing what they know and building on each other's knowledge.

4. Maintaining the social relations and communicative processes that are conducive to the advancement of mathematical knowledge (cf. Bandura, 2001; Damşa, 2014; Damşa et al., 2010; Wenger, 1998):

    I hope that as the participants take turns to collaborate with each other as teacher participants, and as they interact with other participants in the classroom, they will develop relationships in which they appreciate and value each other's contributions to the advancement of their mathematics knowledge. This appreciation and valuing of each other arises from their interdependence and from the empathy that comes from each participant, having experienced being both a teacher participant and a student participant at different times.

5. Reflecting on practice and making plans for the improvement of ideas and practices (cf. Bandura, 2001; Bereiter & Scardamalia, 1998, 2011; Emirbayer & Mische, 1998; Yang, Chen, et al., 2020).

    I built reflection time into the pedagogy. All participants, including myself, have time to reflect on our individual actions and those of other participants, considering how these actions impact the advancement of collective mathematics knowledge. The purpose of this process is for the participants to contemplate strategies for acting in future in order to improve the process of advancing their mathematics knowledge.

3.1.1 The Stages of the Innovative Pedagogy

The innovative pedagogy that I outline here is the initial design with which this project began. My proposals take place in cycles of four stages; the structure of these cycles is shown in Figure 3.1 below.

Stages of the Innovative Pedagogy		
Stage 1 **Select**	As a class, participants select: • Teaching pairs. • The mathematics topic to teach in line with the faculty curriculum map.	
Stage 2 **Plan**	This is followed by two or three planning lessons where TPs • Make sense of the mathematics. Then plan/design: • How to communicate knowledge. • Structure of the lesson and learning. • The knowledge objects (PowerPoint, answers to questions in the booklet).	
Stage 3 **Share**	TPs take turns to • Teach their mathematics topic to the class.	SPs are expected to • Come to each lesson with knowledge of the mathematics topic to question information and help others learn.
Stage 4 **Reflect**	After all TP pairs have taught their topic, the class spend one lesson reflecting on the teaching cycle (TC) and starting stage 1 of the next TC.	

Figure 3.1 – Stages of the innovative pedagogy

The four stages of the pedagogy are: Select, Plan, Share, and Reflect; these, in turn, have been developed with the guidance of the five pedagogic principles extracted from the outstanding literature. These four stages form a teaching cycle, and allow the participants to learn mathematics at each stage individually, in pairs, and as a community. In the first stage of each teaching cycle, the participants select their partners and the mathematics topics that they will later share with the classroom community. The pedagogy allows the students to examine their own and other participants' current mathematics knowledge and learning behaviours when selecting and deciding whom they will work with as teacher participants. The participants select other teaching partners in the first stage of subsequent teaching cycles; in this way, they work with different individuals from the class, and mutual community relations are further developed (pedagogic principle 5).

In the Plan stage, the pedagogy incorporates pedagogic principles 1 and 2. The teacher participants collaboratively produce a knowledge object in the form of a PowerPoint lesson plan that structures how they will make the mathematics topic explicit to the student participants during the lesson. They will also produce solutions to the mathematics questions that the student participants will solve during the lesson. This lesson plan and answers reifies, therefore externalises (Nonaka, 1991) the teacher participant's mathematics. Students have been observed to learn more effectively when they prepare to teach others (Bargh & Schul, 1980); such preparation awakens a need for explication and clarification that requires self-explanation (Chi et al., 1994). My expectation is that, just as SEA develops through the collaborative production of a knowledge object (Damşa et al., 2010; Damşa, 2014), the teacher participants' engagement in the Plan stage will lead to the development of SEA.   The key attribute of mutual relations emerges at this stage, as planning takes place across and between partners.

In the Share stage, the pedagogy incorporates learning through interaction to produce new knowledge. The teacher participants who selected and planed their mathematics topic share their knowledge, through actions and reifications, with the student participants (pedagogic principle 3). The student participants are expected to come to the lesson with knowledge to share of the mathematics topic. In the ensuing communicative process, as the participants interact to advance their mathematics knowledge and that of other participants (pedagogic principle 3), the characteristics of shared epistemic agency are expected to emerge. In the Reflect stage, the participants will meet with me individually or in pairs in an interview setting to discuss the lesson in which they acted as teacher participants. The process will require them to reflect on their lesson and the lessons of other participants to decide on improvements they could make to their next lesson. Reflection also takes place as a whole class activity, where the participants publicly share what they feel will lead to future improvement. While time is put aside for reflection so that practices can be continuously improved, I expect that, as the pedagogy design calls for the participants to repeatedly participate as teacher participants and student participants, the very nature of their learning will become reflective, causing them to implicitly and continuously improve what they do. This Reflect stage, nevertheless, explicitly strengthens the mutual community relations that enables the give and take of feedback.

Table 3.1 below incorporates the stages of the innovative pedagogy, the pedagogic principles that set it up, and the characteristics (see section 4.3) that I will use to analyse the emergence of shared epistemic agency.

Stage	**Learning through …**	**Pedagogic principles (the justification for the stages)**	**Expected characteristics of shared epistemic agency – (what I will look for)**
Select	Mutual community relations	Principle 5. Student responsibility for maintaining the social relations and communicative processes that are conducive to the advancement of mathematical knowledge (cf. Bereiter, 2002; Damşa et al., 2010; Emirbayer & Mische, 1998; Reed, 2001; Scardamalia, 2002)	• Mutual relations, including a participant aligning their thoughts and actions with those of others
Plan	Collaboration between pairs	Principle 3. Student responsibility for building knowledge objects (cf. Bereiter, 2002; Damşa et al., 2010; Reed, 2001; Scardamalia, 2002) Principle 2. Student responsibility for the process that makes knowledge explicit so that it can be shared,	• Intentions to develop one's knowledge and to share it with others • Extension – seeking to know from external source • Explication – making knowledge explicit to each other • Expertise – taking on the role of teacher • New knowledge – the knowledge to share with others reified as a PowerPoint lesson plan

		internalised, and used by all the classroom participants (cf. Bandura, 2001; Damşa et al., 2010; Nonaka, 1991) Principle 5. Student responsibility for maintaining the social relations and communicative processes that are conducive to the advancement of mathematical knowledge (cf. Bandura, 2001; Damşa, 2014; Damşa et al., 2010; Wenger, 1998)	• Mutual relations – working collaboratively
Share	Interaction with the community	Principle 3. Student responsibility for the discursive process that communicates the knowledge to the classroom community (cf. Emirbayer & Mische, 1998; Nonaka, 1991; Scardamalia & Bereiter, 2014) Principle 5. Student responsibility for maintaining the social relations and communicative processes that are conducive to the advancement of mathematical knowledge (cf.	• Intentions to resolve an unknowing • Extension – seeking to extend one's knowledge • Explication – making knowledge explicit to others • Expertise – process authority • New knowledge – resolution of the unknowing • Mutual community relations – developing relations that enable knowledge advancement

		Bandura, 2001; Damşa, 2014; Damşa et al., 2010; Wenger, 1998)	
Reflect	Reflection	Principle 4. Student responsibility for reflecting on practice and making plan1.1s for improvement of ideas and practices (cf. Bereiter & Scardamalia, 1998, 2011; Brown & Campione, 1996; Emirbayer & Mische, 1998; Yang, Chen, et al., 2020)	• Mutual community relations

Table 3.1 – The innovative Pedagogy – Learning, principles and characteristics of shared epistemic agency.

3.2 Rationale for Action-Research Approach

Action research fits the purpose of this study as I am seeking to systematically investigate and legitimise what I believe, from my experience as a teacher, would improve students' mathematics learning. As a rigorous practice-based methodology, it allows me, as the teacher-researcher, to study what happens in my classroom from within and continuously make modifications and evaluations as the research progresses. The findings of this project will be my subjective interpretations of the experiences and communications of my Year 10 mathematics classroom participants; the knowledge to be gained from this research is socially constituted, and emerges as a result of our actions and participation in the research.

3.2.1 History of Action Research

The tradition of action research can be traced back to Kurt Lewin's writings on social psychology (1946), which he based on his field work with communities during which he conceived action as emerging from a process of group interactions and exploration, rather than as the sole result of rational deduction; or, as in Dewey's theory of learning, as a product of our experiences of practice, rather than as a surrender to already-formed ideas (1973).

Lawrence Stenhouse's seminal work, *An Introduction to Curriculum Research and Development* (1975), whose purview was educational policy in the UK, makes an exceptional case for the usefulness of action research as a methodology for studying and improving the practice of teaching. In contrast, research informed by theories such as those of Lewin and Dewey contribute to the relevance of historical research methods as opposed to traditional scientific research (Stenhouse, 1981). The interests of scientific research lie in developing general and predictive laws and theories based on observed data (induction). These theories provide information about the context of our actions and allow us to apply them to predict the outcome of specific actions (Stenhouse, 1981, p. 105). Scientific researchers tend more towards a positivist perspective: they believe that knowledge ascertained from experience is certain and true (Somekh, 2006), and assume that there is an answer to everything, even if it is still “out there” waiting to be discovered. Once discovered, all possible answers will be commensurable, compatible, and agreeable to every one (Berlin, 1997).

Historical research, a category to which action research belongs, is concerned with the analysis of our experiences in terms of their context in time and space (Stenhouse, 1981). In contrast to positivist research, action researchers tend to take an interpretive epistemological position, assuming that knowledge is ambiguous and uncertain, and that there is no single answer to a given question; rather, multiple answers can be arrived at that could generate further questions (Berlin, 1997). They assume that knowledge can be created through dialogue with one another as well as through discovery. Answers can be provisional, tentative, and open to critique and modification. They can be incommensurable and unsolvable (Berlin, 1997; Mouffe et al., 2013).

This historical view of knowledge as pluralistic and historically-mediated is the basis on which teachers as practitioners are called to become researchers, as, since it holds the view that knowledge is not fixed, it allows that everyone has the capacity to create knowledge and develop theories. The kind of theory produced from within practices by practitioners who engage in action research is different from that produced by academics. It is personal and flexible, and of practical use in the day-to-day practice of teaching – e.g., in the classroom – where problems are interdependent on each other, and situations are flexible, consisting of changing and interacting factors. In contrast, theories developed by non-practice-embedded "experts" are abstract, and practices and concepts are spoken about from an outsider perspective (McNiff, 2013) – nevertheless, they continue to be techniques and models that need to be verified in the uncertain and complex environment of the classroom. Considering these two views of the production of theory, it is rational to expect that teachers should be encouraged to develop theories that improve their practice. However, this is not traditionally the case; critics of teachers who carry out action research in their setting have argued that research should be left to academics (cf. Hattie, 2016), and that tacit knowledge on the part of teachers can reduce their motivation to publish their findings or produce theories (Taber, 2013). Teachers are viewed as "doers" of educational theory, and their competence is considered to lie in the ability to improve the practice of teaching, while academics are viewed as thinkers who debate knowledge and explain how learning occurs. Stenhouse (2012) argued against this divide between academics and practitioners that legitimises the knowledge of academics and not that of teachers, advocating for action research as a basis for teaching(p.1). This is evident in his notion of the teacher as an "extended professional" (1975, p. 143), or as a reflective practitioner (Schon, 2008) who is not expected to take the conclusions of academics on faith, but who rather tests ideas against their real classrooms – the "laboratories" in which they command their own knowledge, and in which they are able to develop their own theories. This is what action research means: it is where the act of research cannot be separated from the research goals or from the justifications of the profession; where the knowledge gained is tested and modified by professional practice. The teacher, in turn, is expected to approach their practice from a research stance, viewing it as exploratory and provisional (Stenhouse, 2012, p. 133).

Other contributions to action research theory include Habermas' critical theory of communicative action (1991), on which the moral purpose and goal of human action is to understand each other. Communicative action adds to the pluralistic view of knowledge, as it seeks to create an ideal situation in which individuals have equal rights to speak and communicate their feelings, wishes, and views. This was the basis of the emancipatory action research of Carr and Kemmis (Carr, 1986). On their view, communicative action is the type of action people undertake when they "make a conscious and deliberate effort to reach (a) *intersubjective agreement* about the ideas and language they use amongst participants as a basis for (b) *mutual understanding* of one another's points of view in order to reach (c) *unforced consensus* about what to do in their particular situation" (Kemmis et al., 2014, p. 36). In line with Stenhouse's view of the teacher as an extended professional, here professional practice is understood as an endeavour undertaken by those who make independent and autonomous decisions, free form nonprofessional or external constraints, to commit to the wellbeing of their clients based on theoretical knowledge and research.

John Elliott's (2011) description of professional practice drew on Hans Gadamer's philosophy that viewed action as emergent from continuous self-reflection, and experience itself as consequently being “skepticism in action” (Gadamer in Somekh, 2006). Elliott conceptualised professional practice, including teaching practice, as a "practical science" (2011, p. 66), in which professionals, in order to be responsive to change and uncertainty in practical situations, exercise practical wisdom to give an appropriate response. These practical situations are typically complex, difficult to predict due to their fluidity, value-laden, and difficult to stereotype. These intelligent professionals exercise their "situational understanding" (p. 66) that is based on repertoires of experience; they do not simply apply or recall sets of abstract or theoretical propositions in these situations (Elliott, 2011, pp. 66–67). In Elliott’s view, self-evaluation and personal systematic reflection is part of the action research process. Professionals collect and interpret data, and base their actions on a situational understanding that integrates their moral commitments with practical aims.

3.2.2 What is Action Research?

Action research can be broadly defined as systematic inquiry made public (Stenhouse, 1981, p. 104), carried out by professionals to improve their practice. The “action” part of the term refers to "action disciplined by inquiry, a personal attempt at understanding while engaged in a process of improvement and reform" (Hopkins, 2014, p. 58). It includes communicative action (Habermas, 1991), practical wisdom, and situational understanding (Elliott, 2011). Stenhouse (2012) posits that the inquiry should be rooted in professional curiosity, acutely felt and systematic in that it is structured over time, continuously integrating both the experience and intellect of the practitioner in practice and the relevant thinking of others. It becomes research when it is published, inviting critical dialogue. The publication offers explanations and descriptions of what the professional has done, which in this context constitutes the “theory” (McNiff, 2013, p. 17). It is the publication of the theory that makes the research become a claim to knowledge.

Action research as a methodology is concerned with changing individuals and the culture of groups, institutions, and societies to which they belong (Kemmis, McTaggart cited in Cohen et al., 2018, p. 345). This view of action research aligns with the aims of this study: to concretely improve students' participation in their mathematics learning to improve their relationship with mathematics.

Action research is “a continuous process of problem posing, data gathering, analysis and action” (Wright, 2020, p. 329). It involves a spiral of self-contained cycles (Kemmis et al., 2014; Koshy, 2010; Lewin, 1946; McNiff, 2013). The authors of the British Educational Research Association (BERA) Close-to-Practice research project specified at least two action research cycles (Wyse, 2018 in Wright, 2020). In the first stage of a typical action research cycle, the researcher plans what they will do based on their existing knowledge. In the second part, the acting part, the researcher implements the plans they developed in the first part. The third part consists of observing the outcome of the actions, and in the fourth part, the researcher reflects on what they will do next based on their analysis of the data they have collected in this cycle, and on the new knowledge gained. This reflection part also forms the next planning stage. It is a responsive and systematic procedure meant to deal with concrete problems located in complex situations. The process is monitored constantly by a variety of mechanisms over varying periods. As the teacher-researcher, I can make adjustments, modifications, or even changes in direction where necessary, based on feedback, to benefit the ongoing process.

It is important to pay heed to the fact that researchers in the positivist tradition consider action research to be lacking validity, rigour, and transferability (Koshy, 2010; McNiff, 2013; Somekh, 2006; Taber, 2013). In the pursuit of rigour, action researchers should take care not to reduce the methodological principles of action research into a collection of static methods and procedures (Wright, 2020). In keeping with this, regarding validity and rigour, I will give detailed descriptions of the robustness of my data collection methods and my systematic analytical procedures at all stages of the action research process. This study will be published and open to criticism by the public.

In action research, the notion of “transferability” can be used in place of the term “generalisability” (Lincoln & Guba, 2003). The prevailing contention is that action research should not seek generalisable data, unlike most forms of social scientific research (Koshy, 2010; McNiff, 2013); rather, it is based on the belief that there are no definite answers to problems, nor theories that can be applied in all possible situations, but only personal theories that are open to modification by others in similar contexts. Being open to modification by others reflects the belief that learning continues, and is an invitation for others in mathematics education and education at large to contribute to their own experiences and knowledge. The outcome of this study should, therefore, not be judged on the basis of positivist criteria; rather, it should be judged in terms of its coherence: in particular, of my adherence to and successful propagation of my values regarding democratic participation in the mathematics classroom. Moreover, it should be remembered that the process of inquiry is as important as the outcome (Reason & Bradbury, 2008). In the context of my study, this process raises ethical issues, given the conflict between my role as a teacher to educate the participants and the research requirements. I will fully discuss the ethics of my research in section 3.3.4.

3.3 The Research Design

The research design aligns with the innovative pedagogy that dictates how mathematics learning will occur in the classroom. As previously mentioned, I structured the innovative pedagogy in four stages that make up a teaching cycle (see section 3.1). I collected data in two action research cycles (see section 3.2) that correspond to four and three teaching cycles, respectively.

3.3.1 The Teaching Cycles

The participants took part in seven teaching cycles during the research project. The five pedagogic principles underpinned the four stages of each teaching cycle (see table 3.1 and figure 3.1).

3.3.1.1 Teaching Cycle Stage 1

The Select stage is the first stage of each teaching cycle, wherein the paired participants chose the topics they want to teach from the curriculum map designed by the mathematics faculty. The mathematics faculty aims for all mathematics classes to keep pace with each other as far as possible. Hence, after each teaching cycle, the next set of topics of choice for participants continues a sequence prepared in the faculty curriculum map. The participants were assigned to pairs in the first teaching cycle. In subsequent teaching cycles, participants chose their own pairs, and I later placed restrictions on this selection process (see section 4.1.2 for an explanation of changes made to the pedagogic and/or research design during the course of this study). However, participants always had some choice in the selection, and, as expected, participants did work with whoever became their partner. Following the selection process, each pair selected a topic to teach. An adjudicator was selected at random to aid the topic selection process; should a dispute arise as to which pair was assigned to a given topic, the adjudicator decided on the final arrangement.

3.3.1.2 Teaching Cycle Stage 2

In stage 2, the Plan stage, the participants spent two or three lessons planning for the mathematics lesson they were to teach. On occasion, there was collaboration across pairs as required by the teaching sequence. For instance, in teaching cycle 2, similar 2D shapes were discussed by one pair and the next, who taught the extension of these shapes into similar areas and similar volumes. Providing an opportunity for the two pairs to collaborate allowed the second pair to build on the knowledge shared by the first pair; in this way, collaboration occurred both within and between pairs. This enacted pedagogic principle 4, that of mutual relations, as the participants had to develop ways of successfully sustaining the collaboration over time.

3.3.1.3 Teaching Cycle Stage 3

Stage 3, the Share stage, is where the teacher participants shared their mathematics knowledge with the student participants. Each pair of teacher participants communicated knowledge of the mathematics concept to the class explicitly while maintaining mutual relations. They negotiated with the rest of the class the number of lessons required to teach the topic; some topics lasted for one lesson, while others lasted for four. The emphasis is on sharing because, in line with the innovative pedagogy, I expected student participants to come to the lesson with some knowledge of the mathematics topic, and advance the knowledge of the classroom participants by sharing their knowledge through engagement in knowledge building (see section 4.1.2). Within this third stage of the teaching cycle, I shared my authority with that of the participants (see section 2.3.2.1), taking on the role assigned to me by the teacher participants. The assigned roles ranged from being a teaching assistant to being a student participant.

3.3.1.4 Teaching Cycle Stage 4

The fourth stage of the teaching cycle, the Reflect stage, occurred after all teaching pairs had taught their mathematics topic to the class. At the start of the corresponding lesson, the participants and I spent time reflecting on the completed teaching cycle. We collectively and informally discussed what we did well and what we could do better. The discussion was typically chaired by myself. I posed problems that arose from my reflections on the previous teaching cycle, and invited participants to offer suggestions on these problems. Participants on occasion brought forward problems and proposals of their own for improvement. Regardless of the source of the problem, together we arrived at an ideal course of action. In this way, our reflection informed the planning for the next stage of the teaching cycle.

I study this innovative pedagogy in the naturalistic setting of a mathematics classroom in a secondary school. The setting is essential, because the pedagogy needs to be enacted in a milieu of well-understood schools in order to isolate the influence of my experimental variables, answer the research questions, and meet the broader aims of the study.

3.3.2 The Research Cycle

The study took place over two action research cycles. Each cycle has five stages (see Figure 3.2) and comprises one or more teaching cycles. Stages 1-4 of the research cycle coincide with the corresponding stages of the teaching cycles, and are repeated as necessary before stage 5. The following sub-sections outline the research cycles and the respective data collection methods.

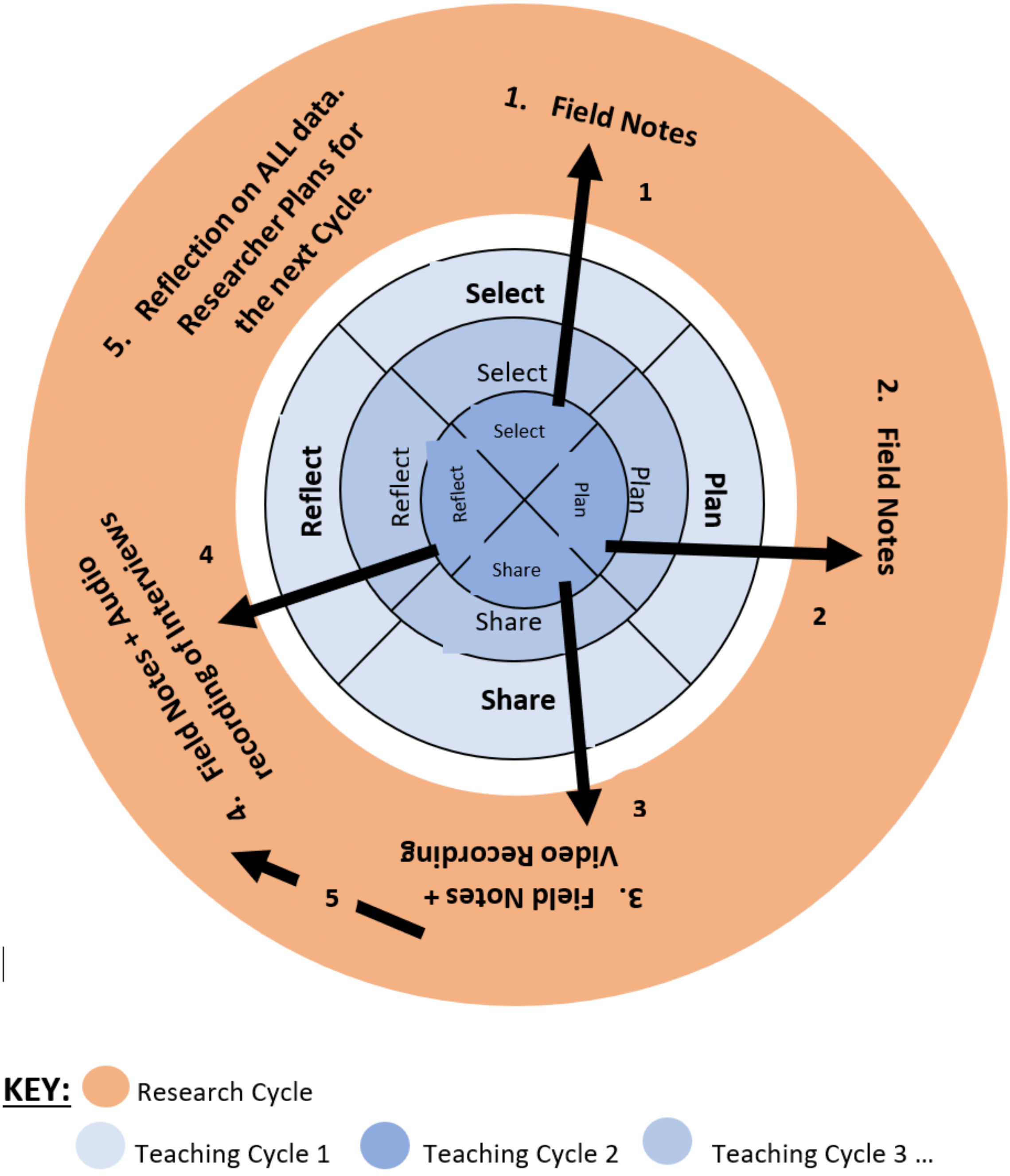


Figure 3.2 – Teaching cycles and research cycle interplay

3.3.2.1 Research Cycle Stage 1

This research stage coincides with the Select stage of the teaching cycle, wherein participants select their mathematics topic and their teaching pairs. The research focuses on recording how topics were shared, how pairs were formed and unformed, and the mutual relations exhibited by the participants. I kept field notes during this stage of each teaching cycle (Figure 3.2, Arrow 1) as part of my ongoing observation and reflection.

3.3.2.2 Research Cycle Stage 2

This research stage coincides with the Plan stage of the teaching cycle, wherein participants make sense of the mathematics knowledge and plan how to communicate it to other participants. The focus was on participants' Expertise, that is, the characteristic of shared epistemic agency that focuses on the process authority of the teacher participants as they plan the knowledge and reifications that will communicate their mathematics topic. Field notes recorded my observations, including descriptions of how the participants worked in pairs, what they did, how they extended their knowledge, and the reifications produced. I also noted what I did as the teacher to support the enactment of the innovative pedagogy (Figure 3.2, Arrow 2). My observations at this stage informed later interview questions (Figure 3.2, Arrow 5) and opened opportunities for me to document how participants experienced the pedagogy.

3.3.2.3 Research Cycle Stage 3

This research stage coincides with the Share stage of the teaching cycle, wherein teacher participants and student participants interact to advance their individual and collective mathematics knowledge. The focus was on illuminating the emergence of shared epistemic agency (in terms of characteristics A-F) in the classroom, and supporting the subsequent Reflect stage of the action research cycles. Observations were the primary means of data collection in this stage of the research cycle. I made video recordings of some of the lessons and kept field notes (Figure 3.2, Arrow 3) to provide valuable data on how the innovative pedagogy is enacted in real time to answer the research questions.

3.3.2.4 Research Cycle Stage 4

This research stage coincides with the Reflect stage of the teaching cycle, wherein participants reflect on the lesson they taught and their actions leading up to the lesson (Figure 3.2, Arrow 4). The focus was on participants' thoughts, perceptions, and ideas for future cycles (pedagogic principle 5); this is tightly linked to the reflection stage of the action research. Field notes record my observations of this stage for most teaching cycles. Semi-structured interviews took place for teaching cycle 3 and teaching cycle 5 that were part of research cycles 1 and 2 respectively. The interviews were “anchored-interviews” (Merriam & Tisdell, 2015, p. 139), as I based the questions on what I wanted to clarify, having reviewed the field notes and listened to video recordings of the 'share' stage (Figure 3.2, Arrow 5). Following the interviews, I transcribed the audio recordings.

3.3.2.5 Research Cycle Stage 5

This research stage does not coincide with the stages of the teaching cycle, but occurred at the end of each action research cycle. The focus was on my planning for the next research stage, which could be the next action research cycle or the analysis of the research data. This stage made use of all recorded data, and included my reflections on data collection methods and adapting either the pedagogy (see section 3.4.1.3) or the research design (see section 3.4.1.4). These adaptations did not always fall neatly at the end of an action research cycle; some adaptation occurred following reflection at the end of teaching cycles. However, they always informed the design of the following teaching cycle or research cycle. Reflection on all the data collected from all stages of the teaching cycle involved my watching the recordings, reading my field notes, noting what may need improvement, and taking the necessary action. It informed the plan that I subsequently fed back to the participants in the first stage of the subsequent action research cycle. Although the feedback originated from me, the researcher, participants negotiated its enactment.

3.3.3 Schedule of Action Research Cycles

The research project commenced on the first week of the academic year 2018-2019. In the first lesson, I explained the research to the participants and gave out consent forms to be signed by parents (see section 3.4.1.1). The first action research cycle started in week 1 of the academic year and ended in week 26. The second action research cycle started in week 27 of the academic year and ended in week 41 (see Table 3.2).

Weeks Beginning's of Academic year	Teaching Cycle (TC)	Stages of TC				Research Cycle		Action Research Cycle
		1	2	3	4	Data Collection	Stage 5	
1 – 03/09/2018	1	√	√					1
2 – 10/09/2018				√				
3 – 17/09/2018				√				
4 – 24/09/2018	2	√	√		√	Field notes (FN)		
5 – 01/10/2018				√		FN		
6 – 08/10/2018				√		FN		
7 – 15/10/2018				√	√	FN	√	
Half Term								
9 – 29/10/2018	3	√	√			FN		
10 – 05/11/2018				√		FN + video recording (VR)		
11 – 12/11/2018				√		FN + VR		
12 – 19/11/2018				√		FN + VR		
13 – 26/11/2018				√		FN + VR		
14 – 03/11/2018				√		FN + VR		
15 – 10/12/2018		Exams week					√	
16 – 17/12/2018	4	√	√		√	FN	√	
Christmas Break								
19 – 07/01/2019			√	√		FN + interview (I)	√	
20 – 14/01/2019				√		FN + I	√	
21 – 21/01/2019				√		FN	√	
22 – 28/01/2019				√		FN	√	
23 – 04/02/2019				√		FN	√	
24 – 11/02/2019				√		FN	√	

Half term							√	
26 – 25/02/2019				√		FN	√	
27 – 04/03/2019	5	√	√		√	FN		2
28 – 11/03/2019				√		FN + VR		
29 – 18/03/2019				√		FN + VR		
30 – 25/03/2019				√		FN + VR		
31 – 01/04/2019				√		FN + VR		
Holiday						FN + VR	√	
34 – 22/04/2019				√		FN + VR	√	
35 – 29/04/2019		√	√	√	√	FN + VR + I	√	
36 – 06/05/2019	6			√		FN + I	√	
37 – 13/05/2019	7			√		FN + VR		
38 – 20/05/2019				√		FN + VR		
Half Term								
40 – 03/06/2019				√		FN + VR	√	
41 – 10/06/2019				√	√	FN + VR	√	
46 – 15/07/2019								

Table 3.2 – Research design schedule of action research cycles

Answering my research questions required that I study the complex interactions of the participants in my classroom as they repeatedly enacted the pedagogy. The repetition of each stage of the teaching cycle allowed the participants (student and teacher participants), both individually and collectively, to renegotiate how to enact the pedagogy to meet the purpose of advancing their mathematics knowledge. This process of negotiation and renegotiation was carried out from lesson to lesson. As they enacted the pedagogy as both student participants and teacher participants, participants were able to experience the pedagogy from a unique variety of perspectives, and to involve this experience in negotiating the future of the practice.

As a teacher-researcher, I studied the emerging practice, and, from our reflections at the end of each teaching cycle (see section 3.3.1.4) and my reflection at the end of each research cycle (see section 3.3.2.5), the participants and I took the opportunity to adapt the pedagogy and enact the improvements, then reflect upon them once more.

Research design should suit the purposes of the research. Other research into SEA, such as Damşa et al. (2010), with whom the concept first originated, employed in-depth case studies to study the emergence of SEA. This design was suitable in its own research context, as it focused on studying a group of no more than four undergraduates' actions as they worked on an individual project over a 10-week period, throughout which the group met every other week. In contrast, answering my research questions involved studying the lesson-by-lesson interactions in the context of a secondary school classroom of eighteen participants and their teacher over 41 weeks, with four lessons per week. My action research design and methodology, therefore, are better suited to this research study into shared epistemic agency.

3.3.4 Ethics

Researching in my classroom, I was conscious of my role as a teacher whose purpose is to teach my students mathematics to the best of my ability. Improving the mathematics knowledge of my students continued to be my priority.

My two roles as teacher and researcher shared the same purpose, values, and processes, but my engagement with educational research has transformed my beliefs about the best way to realise these factors. I undertook this research degree in order to formalise and organise my investigations into how my students can engage more with their classroom mathematics. Above all, I wanted my students to realise that the mathematics classroom was not necessarily structured by a pedagogy in which I, the teacher, was the sole source and fount of mathematics knowledge that they are to passively receive. They are to be involved, and, ideally, to take control of and make decisions about their education.

Prior to starting my research degree, I changed the way mathematics learning took place in the classroom (see section 1.1.3.2); I felt that for the students to behave differently, the existing classroom pedagogy and my own role within it must change, so I sought ways for students to feel that the mathematics belonged to them. This change proceeded on the expectation that students learn for themselves and organise their learning sequence, using whatever learning tools they chose: the teacher, mathematics software, the internet, and fellow students. On occasion, the students led the whole classroom. As the teacher, I provided resources, explained misconceptions, and provided the mathematics curriculum map for the year, and was primarily the liaison between the students and the department. The outcomes for the students, in terms of the available measure of school assessments, was not significantly better or worse than for other classrooms in the year group; however, this particular evaluation tool was not suitable to provide evidence that I could share with fellow professionals. At this point, I decided to take on a research degree to study what goes on in my classroom in a more systematic and theoretical way. Furthermore, this would ultimately contribute to education and to knowledge.

While carrying out this study, I continuously sought feedback regarding the participants' learning using the faculty assessment process. I hoped that engaging in the research as a teacher-researcher enhanced my capacities as a mathematics teacher. As I sought to improve mathematics learning in secondary school classrooms in both roles, there was no conflict of interest.

I sought and obtained committee approval for this study in which I view the participants as competent individuals whose opinions and views are valid. I personally asked the participants and their parents to give consent to take part in interviews and lesson observations. There is, however, a distinction between the classroom pedagogical practice that determines the experiences of the participant and the teacher-researcher's reflection on and collection of data through interviews and lesson observations. I did not seek consent for students to participate in the planning and delivery of lessons or the design of assessments, as this is how I, the teacher, involve my classes in mathematics learning. The way teachers design the classroom pedagogy is at their professional discretion. The planning and delivery of lessons by students occurs in schools, and is not subject to parental consent. In sending students to school, parents are giving consent to the school (and therefore to the class teachers) to exercise good judgment in pedagogic design to the benefit of their children. I sought consent for interviewing and lesson observation as these are part of the research study and not part of day-to-day schooling.
Participants could opt out of video recording during lessons or the study analysis. Participants could also opt-out of their taught lesson being recorded or analysed as part of this research. However, participants who have opted out of the study may inadvertently be caught in video or audio recordings; data solely about them will not be analysed, nor will extracts containing their image or voice be shown to others. However, they may be present in the reflections and views of their partners, as well

as in the views of other participants in the class. Such reported data will be carefully anonymised.

The school has channels for students to request to move to another class, regardless of this research. One student opted to move to another classroom, and I received another student as a replacement. This new student did not complete the consent form, and was one of three students who did not take part in the study as they did not complete the consent form. Two of these students joined the class when the study was underway, while the third student opted out from the start of the study. As a result, I neither recorded their lessons nor interviewed them, and anonymised dialogue that pertained to them or made references to them. I refer to these students indiscriminately as Student A, or B.

A potential selection bias could arise from the students being allowed to opt out of the class. The students were allocated to my class based on the alphabetic order of their surname. The 90 students in the ability bracket were placed in alphabetic order and assigned to teachers (see section 3.4.1.1). I had no input into the allocation of students to my class.

I maintained confidentiality throughout the research. As part of my duty of care as a teacher, I prepared for the unlikely event of a participant disclosing information that makes me feel they are in danger. Should this have occurred, I would have followed the school's safeguarding policy.

Given the power differences between the participants and myself, ethical issues could have arisen during the interviews. The participants may have not wanted to upset me by making negative comments about the class, or may have said what they thought I wanted to hear. Asymmetrical power relations always exist between teachers and students, and this would be the case even if another teacher interviewed the students. In any event, the interviews did not occur until the end of the first action research cycle; by then, the participants had experienced sharing authority, and seemed to speak freely.

The structure of the pedagogy had the potential to facilitate and perhaps even intensify social hierarchies that may have existed among the students. The fact that the class had not existed prior to the start of the research mitigates against this. Working in pairs could mean that student may have to work with someone they did not want to; in addition, quiet students might have felt more vulnerable than ever when they had to present to the class; or confident speakers might have had even more opportunity, in a democratic classroom, to assert their predominance. Dividing the students into groups based on their personalities might have been unpleasant for some students, as certain pairings could have added to the anxiety of certain students. However, there was always an element of choice in the selection on pairings; in addition, the mutual relations that developed amongst the students meant that students were able to negotiate how they operated as teacher participants. They generally worked to their strengths. For instance, in some pairings one student focused on presenting while another focused on the PowerPoint and one-on-one interactions with the student participants. A further issue could be my reflexiveness as the researcher and the teacher. I have been explicit in subsequent chapters about how I analyse my data (see chapter 4) and how I reached my conclusions (see chapter 6) in order to reduce the impact of my values and beliefs in the research.

At the end of the study, I intend to provide the school with a verbal summary of the research findings. The summary will not refer to any individual students nor group of students. Furthermore, I ensured that all the data I collected was stored securely: I have stored video, audio recordings, and transcripts on an encrypted external hard drive with a backup copy on the cloud. I will store this data in the format in which I collected it for a further two years after my degree award; after two years, I will completely erase the recorded data, and will not archive it or use it for further research.

My position as a teacher and deputy headteacher could have had an impact on the participants' behaviour and my interpretation of outcomes. However, this study aims to turn control over to the students. As such part of the study involves how they deal with their behaviour in a classroom environment. Significantly, the school's behaviour policy was available to be used by both the students and by myself. The interpretation of the research data is based on my subjective experience as a mathematics teacher for over two decades, as well as the knowledge gained from my critical engagement with the literature and with contemporary research.

3.4 Enacting the Research Design

The research involved studying a group of eighteen 13-to-14-year-old participants and one teacher – myself – in the mathematics classroom of one secondary school in London, UK. The participants in the mathematics classroom were in Year 10, and at the beginning of a two-year GCSE Mathematics curriculum. Data collection commenced at the beginning of the 2018/2019 academic year and lasted for the whole year. Having previously agreed with the school that Year 10 was the appropriate age group and curriculum for the research, the head of the mathematics faculty allocated participants to the classroom (see section 3.4.1.1). This allocation meant the start of a new relationship between the participants and me, as most of the participants were not in the same mathematics classes as each other during the previous academic year, nor had any of them been taught by me in previous years. I introduced the mathematics pedagogy (see section 3.4.1) to the participants at the start of the academic year, and I sought consent to participate in the research from parents and the school at the start of the academic year (see Appendix 2). I collected data from two action research cycles spread across the academic year (see Table 3.2). The spread of each action research cycle was intended to time for shared epistemic agency to emerge to a significant extent. Practically speaking, it allowed me to analyse the data collected in each cycle in order to inform the next cycle.

3.4.1 Action Research Cycle 1

Action research cycle 1 comprised four teaching cycles, as shown in Figure 3.1. The mathematics topics taught by the participants and the duration of lessons are represented in Table 3.4 below. The topics broadly followed the sequence of the curriculum map laid out by the mathematics faculty. In line with the schedule, I did not collect any data during the first teaching cycle. At this early stage, the participants and I were coming to terms with the practicalities and realities of the research, such as the participants' anxiety about teaching lessons, or the delay in submitting consent forms. I started writing field notes in the second teaching cycle, but encountered unforeseen difficulties in sourcing video recording equipment and becoming acquainted with its proper operation and implementation. This difficulty delayed the event of the first video recording until the third teaching cycle. Recorded interviews and data analysis were carried out during the fourth teaching cycle, bringing the first research cycle to an end.

3.4.1.1 Selecting Participants

The 18 participants in my mathematics class that took part in the research study were assigned to my class at the end of the previous academic year by the head of the mathematics faculty (HOF). There were thirteen Year 10 mathematics teaching classes. The HOF ranked students from highest to lowest based on their end of Year 9 mathematics assessment scores in order to assign them to a mathematics class. The highest-achieving 25 students were placed in one class, and the students with the lowest scores were placed in two classes. Of the 180 remaining students, the top 90 were arranged in alphabetical order by surname in 5 teaching groups, belonging to a group known as the “upper higher band”. The process was repeated for the lower-achieving 90 students, referred to as the “lower higher band”. My class was in the lower higher band. Teachers were assigned to classes by the HOF and the Assistant Headteacher who had timetabling responsibility. I had not previously taught any of the students, so our relationship as participants started on the first lesson of the academic year, in September 2018.

18 students were given a consent form (Appendix 3), to be signed by themselves and their parents, and I verbally explained the research project to them. 15 participants returned completed consent forms and took part in the research. Of the 3 students who did not return completed consent forms, 1 opted out and I did not interview this student, nor were any recordings made of their lessons. 1 participant moved to another class and the replacement student was expected to bring in the forms but did not; I did not interview this replacement student, nor were any of their lessons recorded. The third student joined the class later on in the Autumn term, and did not complete the consent form. However, these 3 students participated in enacting the innovative pedagogy. All 3 participants were aware that they might be unintentionally included in the data analysis as part of the class but would not be identified, and effort was made, as far as possible, not to focus the camera on them. Any reference to them was as Student A, or B. All participants who consented to participate in the research were referred to by their chosen pseudonym, as shown in table 3.3 below.

#	**Pseudonym**	#	**Pseudonym**	#	**Pseudonym**
1	Adam	7	Jayzee	13	Tom
2	Beyoncé	8	Jevonte	14	Ty
3	Crimson	9	No Miss	15	Jon
4	Daniel	10	Pearl	16	A/B
5	Deepz	11	Roan	17	A/B
6	James	12	Teesh	18	A/B

Table 3.3 – Participants' selected pseudonyms

3.4.1.2 Selecting Teacher Participants

My initial thought was that the make-up and selection of the participant teaching partnerships would not impact the outcome of this research, as the focus was on sharing mathematics knowledge and learning as a community. Thus, in the first teaching cycle, the participants selected one or two teaching partners without any restriction. In the first teaching cycle, I noted uneven participation within the three-partner teaching participants. This uneven participation started in the planning stage and carried through to the teaching stage. The classroom layout (see Figure 3.3) placed physical restrictions on participants' movement, making working in a group of three especially difficult. In addition, the uneven distribution of friendships within the group tended to exclude a participant, as exemplified in an extract from field notes (see Appendix 4). I posed this problem to the participants during our collective and informal discussion at the reflection stage (see section 3.3.1.4). The outcome was that pairs became the optimum size for teaching partnerships. After the first teaching cycle, to ensure participation in all aspects of the pedagogy, in subsequent cycles I gave participants the option to change teaching partners and explicitly limited teaching partners to pairs. Pairing up was not always straightforward; in teaching cycle 2, after participants had chosen their partners, I made the last 2 participants partners as no other participant chose them as a partner, nor did they make a choice.

TEACHING CYCLE ONE			
Teacher Participants (TP)	**Topic Taught**	**Start Date**	**# Days**
All	Allocation/Selection/Planning	07/09/18	3
Crimson + Student A	Inverse/Direct proportions	13/09/18	1
Jayzee + Beyoncé	Proportions Recipes and ratios Questions	14/09/18	1
Teesh + Student A	Exchange rates	17/09/18	1
No Miss + Student A	Best Buys	18/09/18	1
Deepz+ Ty + James	Sharing ratios	20/09/18	1
Adam + Roan + Pearl	Ratios and fractional problems	21/09/18	1
Jevonte +Daniel + Tom	Percentage change	24/09/18	1
TEACHING CYCLE TWO			
All	Allocation/Selection/ Planning	25/09/18	3
Crimson + Pearl	Compound interest and depreciation	01/10/18	1
Beyoncé + Jayzee	Reverse percentages	02/10/18	1
No Miss + Student A	Similar shapes	04/10/18	2
Student A + Teesh	Inverse and direct proportions	08/10/18	2
Deepz + James	Speed, distance, and time	11/10/18	1
Jevonte + Tom	Area of similar shapes	12/10/18	1
Roan + Adam	Volume of similar	15/10/18	1
TEACHING CYCLE THREE			

All	Planning	16/10/18	2
Student A + Student B + Ty	Linear equations	01/11/18	2
Deepz + Jevonte	Solving quadratic equations	05/11/18	3
Teesh + Pearl	The quadratic formula	08/11/18	1
Daniel + Jayzee	Completing the square	12/11/18	2
James + Adam	Inequalities	15/11/18	3
A + No miss	Forming equations	19/11/18	2
Tom + Beyoncé	Linear and quadratic simultaneous equations	22/11/18	4
Crimson + Roan	Solving simultaneous equations graphically	29/11/18	2
All	Sketching and drawing quadratic equations	03/11/18	2
Myself	Regions	06/12/18	2
TEACHING CYCLE FOUR			
All	Planning	19/12/18	2
Tom + James	Rearranging formulae	09/01/19	2
Roan + Crimson	Algebraic fractions (+/-)	11/01/19	4
Deepz	Simplifying algebraic fractions	18/01/19	4
Beyoncé + Jayzee	More algebraic fractions (x/÷)	25/01/19	4
No Miss + Pearl	Surds	04/02/19	4
Student A + Student C	Solving algebraic fractional equations	11/02/19	3

Teesh + Daniel	Iteration	15/02/19	1
Teesh + Daniel	Iteration	26/02/19	2
Jevonte + Adam	Algebraic proof	01/03/19	2

Table 3.4 – Teaching Schedule for teaching cycles 1-4

In the third teaching cycle, the problem I posed to the participants concerned extending our experiences of participation. The outcome was that I selected the teacher-participant pairs to support learning by experience (see section 2.2.2) by getting participants to work outside of their usual friendship groups. This experience improved mutual relations, an essential characteristic of shared epistemic agency (see section 2.4.3).

In the fourth teaching cycle, I once again allowed participants to choose their partners. I wanted them to be as comfortable as possible with their partner, to improve the collaboration, and to reduce limitations. For instance, it was easier for friends or participants in the same tutor group to meet up outside the lesson to finish their planning.

As stated in section 3.3.3 above, the research design allowed for a continuous cycle of reflection and improvement to the innovative pedagogy to answer the research questions and improve the student's relationship with and their learning of secondary school mathematics. Hence, I experimented with the pairings in order to best encourage teacher participants to work together (pedagogic principle 4), extend their knowledge (pedagogic principle 1), and share this knowledge effectively to advance the community knowledge (pedagogic principle 2).

3.4.1.3 The Quality of Mathematics Knowledge

Following teaching cycle 1, I observed that teaching participants were unsure of the limits of the topic or the content they were to teach; they did not have the mathematics knowledge for teaching (see section 2.3.2.1) that teachers develop through their experience in the profession. This experience produces knowledge of such factors as the topic sequencing and types of examinations questions. Field notes extract 3.1 gives an example of the teacher participants' lack of knowledge of typical GCSE questions.

Date: 21/09/2018 (TC1). Topic: Ratio & Fractional Problems

Teachers: Adam + Roan + Pearl

The participants researched the topic during the planning stage (Vignette 1) using mainly 'MathsWatch'; this exposed them to an understanding of ratios as fractions such that they included in their lesson questions such this:

1. In a box of chocolates, the ratio of pink chocolates to white chocolates is in the ratio of 2:5.

   What is the fraction of pink chocolates in the box?

The participants prepared a work sheet with similar questions involving the concept of ratios as a fraction of a whole. In essence, they did not extend mathematics beyond what was taught in the previous lesson (see Table 3.4).

The reason was that they did not know where to research. They typed in the topic and did not go beyond what was available on the MathsWatch package. This left the participants in the class unable to solve the now routine ratio problem where they are expected to combine ratios – questions such as:

Given that A: B = 1: 6 and B: C = 2: 5

a) Find the ratio of A: B: C

Give your answer in its simplest form.

Field notes extract 3.1 – Teacher participants knowledge of GCSE questions

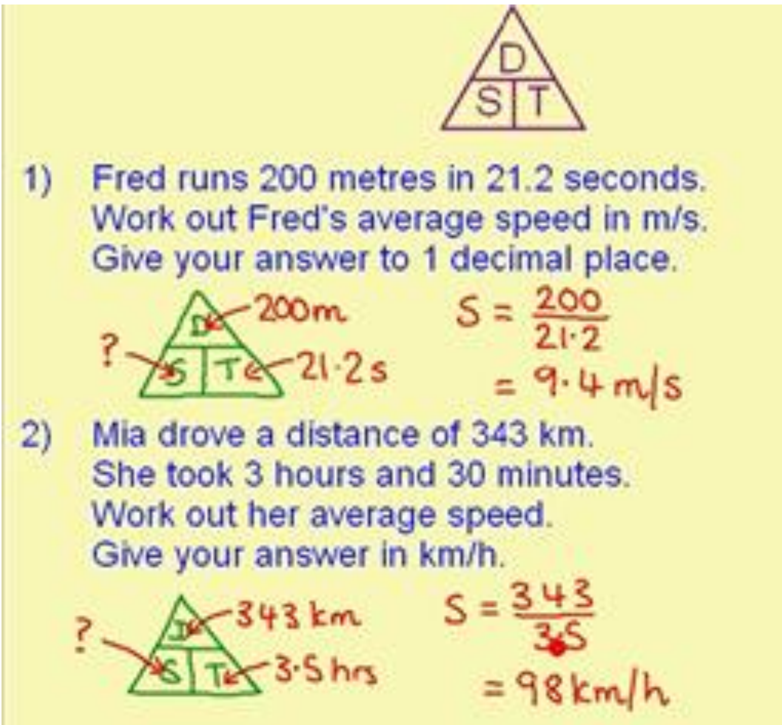
To make this knowledge available to the teacher participants, from TC2 onwards I provided the topics' mathematics questions, as well as further content, extensions, and problem-solving exercises. I experimented with various ways of doing this while still sharing authority with the teacher participants. Producing a booklet of questions at the start of each teaching cycle was most efficient. It allowed participants to use their limited planning time to focus on gaining knowledge rather than sourcing questions – a lesser priority, as teachers have historically most often used mathematics textbooks with answers at the back to prepare lessons. Most teacher participants came to their lessons with solutions to the questions in the booklet: a reification of their expertise as participants.

Reflecting on teaching cycle 2, I observed that some teacher participants copied and pasted worked examples and used these as part of their explanations. This copying method adversely impacted the quality of mathematics knowledge shared, as the teacher participants tended to focus on procedural knowledge at the cost of conceptual knowledge, limiting their ability to problem solve, as shown in field notes extract 3.2.

Date: 11/10/2018 (TC2). Topic: Speed, Distance, and Time.

Teacher Participants: Deepz + James

This is a screenshot of what Deepz and James, the teacher participants, explained on the board.



The process was explained, but the reasoning behind the process was not. The student participants were able to answer similar questions; however, the teacher participants' lack of knowledge was exposed when the students encountered questions such as question 5 below:

Question 5

A car takes 15 minutes to travel 24 miles. Find the speed in mph.

In calculating the solution to question 5, the teacher participants used the time in seconds without appreciating that it needed to be converted into hours.

Field notes extract 3.2 – Screenshot of worked examples.

To mitigate this, and in keeping with pedagogic principle 2, we discussed the problems of copying and pasting worked out examples during the reflection stage; I encouraged teacher participants to work out and explain questions in real time.

In the third teaching cycle, I found time before each teacher participant pair's lesson to assess their mathematics knowledge to assure the quality of the mathematics. This pre-meeting ultimately proved unnecessary. The teacher participants had prepared sufficiently and knew their content, at least as far as I could decipher in the short meeting. Moreover, in the classroom, participants asked questions that I could not have anticipated; in essence, meeting before the lesson was of no benefit. I discontinued this practice.

It was my professional responsibility to maintain the proper pacing and quality of mathematics study in my classroom. The futures of the participants, myself, and the school depended on the GCSE Mathematics examinations' results in May 2020 (21 months from the start of the project). My class was not in isolation; their performance would impact my appraisal as a teacher. In a faculty of thirteen Year 10 mathematics classes, my class's performance was to be judged against that of other classes, as was the performance of the mathematics faculty judged against that of other faculties. Introducing a new pedagogy was a risk; thus, the selection of the teaching pairs, the questions, and the explications had impact beyond the interests of research, and required considerable thought, commitment, experimentation, and adaptation during the initial teaching cycles.

3.4.2 Data Collection Methods

Answering the research questions required studying the participants as they enacted the pedagogy over and over again, lesson by lesson. Hence, methods of data collection employed needed to suit that purpose, and consider participants' reflections on their enactment. To this end, the two methods of data collection employed were lesson observations and participant interviews. The observation included video recording and written field notes; these methods were sufficient to collect data that provided evidence for the emergence of shared epistemic agency and answer the research questions. I collected data over 10 months, from September 2018 to June 2019.

3.4.2.1 Observation

Observation is a suitable strategy for this research. It is a data collection method that is more than just looking at a social situation; it involves the researcher systematically noting people, events, settings, behaviours, routines, and how the observed phenomena exist in their natural social setting (Cohen, 2018,p. 542). It allows the researcher to collect valid and authentic data that can reveal mundane routines and activities, provide rich contextual information, and offer opportunities to document verbal, non-verbal or physical phenomena that occur as the classroom participants enact the innovative pedagogy.

As my research was over an academic year, observations allowed me to collect rich, first-hand, and *in situ* data about the complex interactions of participants as they engaged in the classroom practice of the innovative pedagogy. This data, in the form of video recordings and field notes, identified the characteristics of shared epistemic agency as they emerged over time.

Video recording as an observation method of data collection involve using a camera to record the observed phenomena, that is, the epistemic interactions in my classroom, in real time. Video recording had the advantage of giving me a more "external" view of what occurred in the lesson than if I had to base the data on my recollections alone. It also has the advantage of allowing me, as a teacher-researcher, to carry out my responsibilities as a teacher in the classroom while still collecting data from our interactions. Video recording collects both visual and audio information, including body language, gestures, and facial expressions that are important for interpreting participants' communications as they interact and give meaning to their actions and reifications (Silverman, 2016) that illuminate the characteristics of shared epistemic agency. The limitations of this data collection method are that the camera cannot be everywhere at once or record all interactions in the classroom. I note in the following section that it was preferable to manipulate the positioning of the camera to collect useful data rather than record all interactions from one perspective during the Share stage of teaching cycle 3.

3.4.2.1.1 Classroom Layout and Decisions about Video Focus

A description of the classroom layout will give context to some of the decisions made regarding video recording. The camera I used was fixed, instead of a roving camera, as the focus was on the details of participants' social interaction (cf. Heath, 2010).

However, the design of the learning environment – the classroom and the seating positions of individual participants – affected the interactions that were to be captured in the camera's view, and therefore influenced decisions regarding the camera's position. Figure.3.3 below shows the classroom layout where I recorded the Share stage of teaching cycle 3.

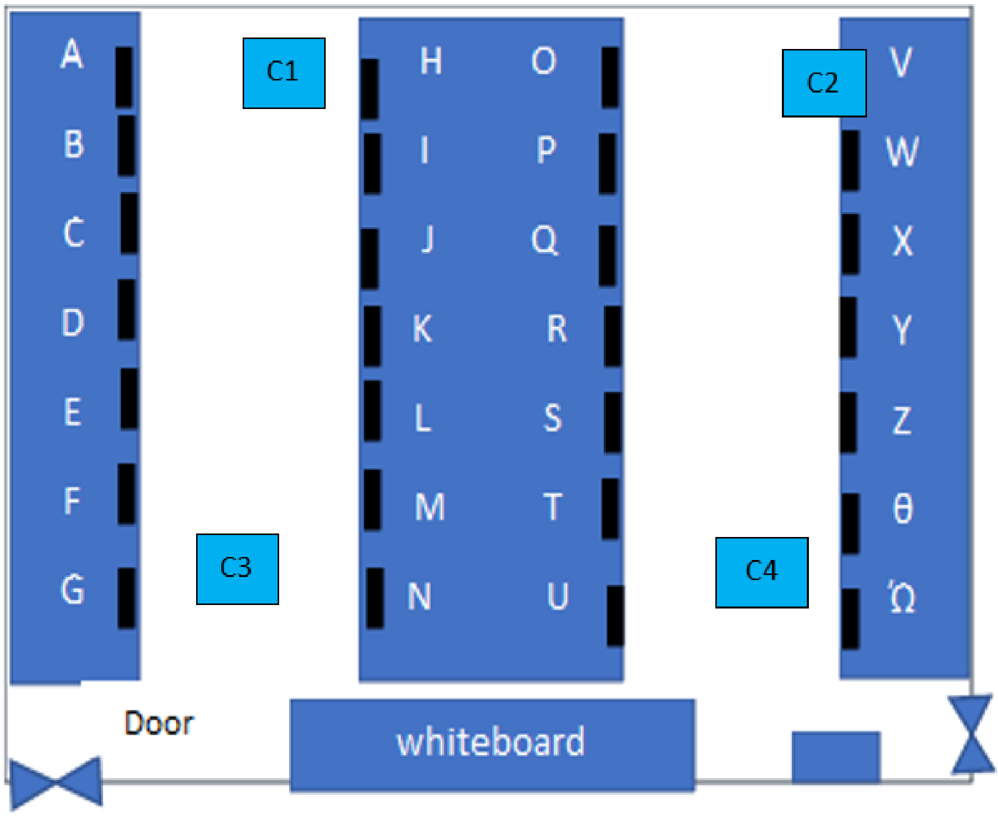
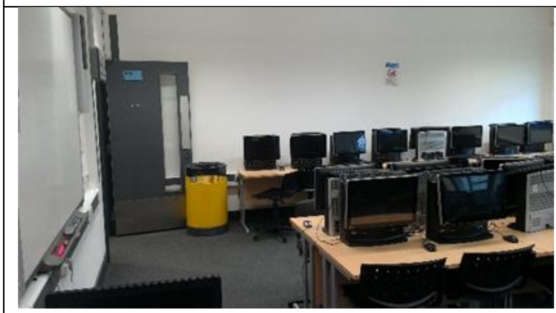


Figure 3.3 – ICT Classroom layout with seating positions (A-Ώ) and possible camera positions (C1-C4)

The classroom was a computer suite, with 28 computers in 4 columns of 7 computers each. As shown in the diagram below, this room layout created two corridors in the classroom: one directly opposite the entrance, and the other after the third column of computers. The 18 participants exercised choice over where they sat during the lesson. Over time, most participants became identified with a particular seating position, as is typical in a conventional classroom. The majority of the participants sat in positions O to Ώ, along the second corridor, though some participants, typically those who were less inclined to speak up, preferred to sit elsewhere.

Given that I desired to record what happens on the board, and but also to avoid obstructing participants' view, the layout and the seating positions of participants limited the positions available for placing the single recording camera to either of the two corridors. Photo 3.1 below shows the view from each of the four available camera positions.

	
Camera position C1	Camera Position C2
	
Camera Position C3	Camera Position C4

Photograph 3.1 – Camera positions for the ICT Suite recordings.

In teaching cycle 3, the first recording cycle, I experimented with different camera positions to find the optimum position. Over the 10 recorded lessons, the camera was positioned at C1 twice, C2 three times, and C3 four times. The camera was placed in position C4 once in cycle 3 when its connector to the tripod was missing, and I needed to keep it safe from interference. Another consideration was to avoid recording the 4 participants and teaching assistants who had not signed the consent form.

In line with the research design, following recording, I prepared for an interview with the participants to reflect on the previous cycle and plan for the next cycle. The preparation necessitated my watching over the video recordings. Watching the recordings from the first recording cycle allowed me to judge the positions against a revised criterion. Recordings from camera positions, C1, C3, and C4 had limited visibility of the more interactive participants. C2 was the optimum position. The camera had in its scope the majority of the class, and especially the most interactive participants. I realised that valuable data came from participant interactions, thus it was important to position the camera so that I could observe where this interaction was more likely to occur and be heard and seen. The trade-off was that some participants were excluded from the data in the seated position, but came into view as teacher participants. This is appropriate and inevitable; in a naturalistic setting, it is essential to maximise to the greatest extent possible the quality of the data (Heath, 2010), accepting trade-offs if they offset losses with greater gains.

The presence of the camera in the classroom and the awareness of the participants that I was recording them could have caused them to act differently, thus contaminating the data. However, from my experience and from the testimony of other researchers, I note that participants cannot sustain a change in behaviour in a social setting unless it is extreme, and will ultimately return to their natural behaviours (Creswell, 2018). The camera became all but invisible in the classroom after the participants became accustomed to its presence.

3.4.2.1.2 Field Notes

Field notes are research diaries that are used to chronicle the researcher’s thoughts and reflections and record what happens during the research project. They can be highly descriptive or less so, depending on the research design. In the best case, field notes should be written as the phenomena being observed are unfolding, or at least soon after the event. Being a participant-researcher, I could not write notes during lessons. In line with best practice, I wrote up my notes as soon after each lesson as was possible. The field notes were especially helpful for recording the participants' interactions that were not recorded on video. The notes were not a detailed account of the lesson, but rather a record of occurrences that I judged to be significant at the time. A limitation was my lack of awareness of which phenomena would be most relevant during the final analysis or the write-up of the research. Moreover, field notes are highly subjective; human perception can be unreliable, and must be selective given the ubiquity of data (Patton, 2015). The field notes I kept were nevertheless descriptive and helpful as extracts to support some of my findings and discussions, and to illuminate aspects of the thesis.

3.4.2.2 Interviews

Interviews are a way of collecting data through conversation, by asking questions and listening to the answers. It is a conversation with a purpose and structure determined by the interviewer. Interviews are used to determine what is "in and on someone else's mind, what we cannot directly observe" (Patton, 2015, p. 426). They are thus useful for discovering people's experiences, hopes, and feelings – information about the world they live in or about the past that cannot be replicated or clearly discerned in other ways.

My research design included interviewing the participants at the end of each action research cycle. As part of the reflection stage, the interviews aimed to gain insight into participants’ experiences of and perspectives on the innovative pedagogy – that is, what they have learnt, what they would avoid, and what they would do in the following action research cycle. As the study aimed to improve their relationship with and learning of mathematics, it was essential to gain information from the participants enacting the pedagogy and who had experienced and were experiencing other pedagogies in their schooling. In addition, answering the research questions necessitates observing participants’ perspectives on their increased participation and responsibility for their own and others’ learning.

My interviews were semi-structured, as the questions I asked required the participants to explore their action and thoughts; though I thought out the questions beforehand, as participants gave their individual perspectives on events, I had to ask follow-up questions to clarify information. After the first action research cycle, I interviewed participants and transcribed the interviews from the audio recordings in preparation for the analysis.

The first set of interviews took place as planned, after teaching cycle 3. In line with the research design, the idea was to complete the interviews before the end of the action research cycle, so that information from the interview could form part of the planning for the following action research cycle. The interviews took place in the morning, during the school day to ensure attendance. I chose participants from the class list based on the proximity of their form room to the interview location. I started by interviewing the participants whose form room was nearest to my location, and I withdrew them from their morning registrations and the non-curricular Physical, Social, and Health Education (PSHE) lessons. In this way, I minimised the impact on their curriculum time. Registration and PSHE are consecutive lessons on Thursday mornings, so I had 75 minutes to conduct the interviews; these took place over two consecutive Thursdays (see Table 3.5).

Action Research Cycle	**Date**	**Participant interviewed**
1	10/01/2019	Jayzee, Nomiss, James
	17/01/2019	Jevonte + Deepz, Crimson
2	02/05/2019	Adam, Daniel, Jevonte
	09/05/2019	Pearl, Teesh, Tom

Table 3.5 – Interview timetable for action research cycle 1

Five participants had individual interviews, while Jevonte and Deepz were interviewed together in the second week. I decided to interview a pair of teacher participants together, reasoning that such participants may trigger each other to remember more or that the ensuing discussion may provide more insightful answers. The participants agreed with each other's accounts of events or responses to questions. Jevonte only spoke if I asked him to speak first; barring that, Deepz dominated the interview and Jevonte agreed with him. As this paired interview did not generate fruitful insights, and seemed to be affected by social and personal factors, I decided on individual interviews only in the next research cycle. To prepare for the interviews, I watched the lessons and selected parts of the recordings that I intended to bring to the attention of the participants being interviewed for clarification and discussion. This elaboration did not materialise. The participants generally did not like watching themselves, so after two of them requested not to watch themselves in the first week, I stopped showing clips of video recordings during interviews.

The interview questions (Appendix 5) focused on eliciting the teacher participants' experience of and perspectives on their enactment of the characteristics of shared epistemic agency, with a particular focus on the characteristics of Extension and Expertise. I based my questions on Damşa et al.'s (2010) discussion of actions that indicate SEA; hence, my focus was on the teacher participants and their preparation for the lessons, in keeping with pedagogic principles 1, 2, and 5. Following reflection on the research, I changed this approach to interviewing; I discuss the changes in the following section.

3.4.3 Reflecting on Action Research Cycle 1

At the end of the first action research cycle, that is, during stage 5, I reflected on the innovative pedagogy and on the data collection methods. This reflection involved watching the video recordings and reading the field notes and interview transcripts. The purpose of the reflection was to evaluate the innovative pedagogy and its enactment, and, from this evaluation, to make necessary adaptations to the next teaching cycles in order to support the emergence of shared epistemic agency. This is where my research aims most influenced the pedagogy, as I wanted to enhance the emergence of shared epistemic agency in order to improve the students' relationship with and learning of mathematics. I also evaluated the data collection methods and adapted them to the particular environment of our secondary school to improve the quality in the data in the next action research cycle, gearing its collection towards answering the research questions.

3.4.3.1 Reflecting On the Pedagogy

Following the first action research cycle, having watched the video recordings, read the field notes, and listened to the audio recordings of the interviews, in order to help with answering the research questions, I decided on two aspects of the pedagogy that required a greater focus at stages 2 and 3 of the teaching cycle, and posed this to the participants during the discussion at stage 1 of the fifth teaching cycle. The first focus, concerning stage 3, the Share stage, was to improve the quality of epistemic interactions. This required improving the knowledge that the student participants brought with them to the lesson (pedagogic principle 3), which would better facilitate the emergence of shared epistemic agency, as they could engage in more productive dialogues with the teaching participants. The second focus, concerning stage 2, was for teacher participants to include strategies to assess student participants' learning in their planning. This focus referred to pedagogic principles 1, 2, and 3. These two foci, I hoped, would improve the mathematics knowledge shared by both teacher and student participants during their epistemic interactions, which make visible the characteristics of shared epistemic agency. Making the characteristics of shared epistemic agency visible contributed to answering the research questions.

3.4.3.2 Reflecting on the Data Collection

Initially, the research design included collecting audio recordings of participants' actions and reifications during the select and plan stage of the research cycle (see Table 3.6).

Stage	TC	Planned Research Cycle Activity	Actual Research Cycle activity
1	Select	Field notes + Audio recording	Field notes
2	Plan	Field notes + Audio recording	Field notes
3	Share	Field notes + Video recording	Field notes + Video recording
4	Reflect	Field Notes + Interviews – Audio recorded and transcribed	Field Notes + Interviews – Audio recorded and transcribed
		Reflection on all data +planning	Reflection on all data +planning

Table 3.6 – Planned and actual data collection

The rationale behind this decision was the desire to collect various forms of data across the teaching cycle similar to other research into SEA (cf. Damşa et al., 2010; Damşa, 2014); in addition, collecting data from different sources should increase opportunities to trace the emergence of shared epistemic agency. However, my study differs from those of Damşa et al. and others, as it is a study of participants' epistemic interactions across more than 150 one-hour lessons over an academic year, as opposed to a group of 4 students over five lessons. Over time, I came to realise that the amount of data I was collecting was unmanageable.

At the end of the first research cycle, I listened to the audio recordings of stages 1 and 2 (see Table 3.2) and reflected upon them. While I found the recordings interesting, as they gave me insight into what the participants deem necessary about learning mathematics, two main issues arose that caused me to discontinue the recordings, namely, with selecting participants and with the data's reflexiveness and usefulness.

Before the first research cycle, I had trialled recording audio in classroom discussions. Listening to the recording and transcribing the audio recording made me realise the importance of where the recording device is placed; I trialled different positions. Carrying it on my person made it difficult to hear conversations between participants. When I stood near participants to hear what they were saying, my presence disrupted and changed the content of their conversation. Placing it at the front of the class rendered the conversations at the back of the class inaudible. The solution I came to was to choose a pair of participants as the focus of the recording and place the recording device near them.

At the start of the first teaching cycle, I chose participants randomly, as I could not develop a fairer selection criterion, and placed the recording device near them. In the first planning lesson, I placed the recording at position U, next to Roan, and on the second planning day, I placed it at position Y between Beyoncé and Jayzee (see Figure 3.3).

Listening to the recording of the transcribed below in Transcript Extract 3.3, I realised the significance of the participants' use of the computer on audio recordings.

Date: 25/09/2018 (TC2). Topic: Reverse Percentages

Teacher Participants: Jayzee + Beyoncé

Planning Session 1

Jayzee: "It's basically take-away…"

Beyoncé: "Ah, you take away that by that, ah like how miss showed us on the board."

Jayzee: "Some people think you find 10% of that and you add it on, I'm guessing you take it away …"

Beyoncé: "Yeah"

Jayzee: "… because they're asking for the original price so …"

Beyoncé: "Yeah…"

Transcript Extract 3.3 – Transcript of audio recording during the planning session.

During the planning stage, participants learned from a video and discussed what was on the screen. In the second line of Extract 3.3, Beyoncé said “Ah, you take away that from that … “ ; the conversation referred to what I could not see. The computer was a central focus of the communication, and I had no access to that part of the conversation. Resorting to using memory and experience to fill in the gaps in the conversation affected the validity of the data, as I would have to have made assumptions.

In addition, discovered early on that recording audio stage one of the teaching cycle would not be feasible. It was a whole class activity, and the recording device could not pick up all the participants’ contributions. The device recorded conversations of those within range while not recording those out of its range. Ultimately, I decided to limit data collection in the first two stages to field notes.

In hindsight, I could have overcome these issues, but at that time, I was overwhelmed by the amount of data I was collecting, the different technologies in use, and my roles as teacher, school leader, and researcher. I became concerned that the research would become unmanageable, so by the end of the first research cycle, I decided that the data for the research would come from the video recording of the lessons and the strict verbatim transcription of the audio-recorded interviews.

3.4.4 Action Research Cycle 2

The second action research cycle began on 5 March 2019 and lasted eight teaching weeks. It consisted of three teaching cycles and ended on 13/06/2019. The faculty curriculum map dictated the topics to be covered, the delivery sequence, and the placement of assessments. Some of the topics, such as surds, which required up to four lessons, in conjunction with the home learning quiz that took up half of the Friday lessons, meant that I had to extend the research period from seven teaching cycles over two terms, as originally planned, to seven teaching cycles across the entire academic year (Table 3.2).

The end-of-year assessment added a sense of urgency to the research project. Teaching cycle 5 ended on 03/05/2019, leaving four teaching weeks until the first exam. Circle theorems and revision of all the mathematics topics taught from the start of Year 9 had to be covered within this period, and necessitated my changing the structure of the teaching cycles. I cut out in-class planning time and required teacher participants to plan outside of lesson time, and also restricted the Share stage to a single lesson per topic.

During the first stage of teaching cycle 5, I shared with the participants the new foci from the reflection on action research cycle 1. The foci aimed to improve the advancement of mathematics knowledge of the classroom participants through epistemic interaction. Epistemic interactions were strongly supported by participants coming to each lesson with prepared mathematics knowledge. In line with these foci, the plan agreed upon by the participants during the first stage of teaching cycle 5 was for teacher participants to inform the student participants of a question they had to attempt before the lesson.

3.4.4.1 Selecting Teacher Participants

By the fifth teaching cycle, the classroom practice had become established, as evidenced by the effort that both student participants and teacher participants put into the Share stage. To bolster the communicative abilities of teacher participants and their authority within the classroom, I negotiated with the participants a final change to the pair selection process that was based on each participant's personality.

I discerned from the video recordings that participants fell into two broad categories: the quiet participants and the confident speakers. This distinction had more to do with their Expertise as teacher participants than with their mathematics knowledge. The quiet participants were soft-spoken, introverted, and communicated best with those closest to them. They were very good at working with participants individually, but appeared overwhelmed in the classroom, with the 17 other participants vying for attention. Ethically, as a teacher, I wanted the best opportunity for all participants to learn, and felt that this would be realised if I could prevent the pairing of two quiet participants.

Following a discussion during the selection stage, an agreement was reached that each pairing should have a single confident speaker. To this end, I split the class into 2 groups: quiet participants and confident speakers. I suggested that each teaching pair should constitute one participant from each category. The participants organised themselves into seven pairs; five of the seven possible pairings agreed with the suggestion while two groups did not, as one group comprised two quiet participants and the other two confident speakers (see Table 3.7)

TEACHING CYCLE 5			
Participants	**Topic Taught**	**Start date**	**# Days**
All	Planning	05/03	2
Deepz, Ty	Bounds	11/03	2
Crimson + Beyoncé	3D Pythagoras's theorem	14/03	3
No Miss + Student A	Sine rule	19/03	2
Teesh + Student A	Cosine rule	22/03	2
James + Crimson	Home learning	26/03	1
Tom + Daniel	SOHCAHTOA	28/03	2
Roan + Jevonte	3D trigonometry	01, 04	2
All	Exact values	02/04/	1
Jayzee + Pearl	Area of any triangle	23/04	2
Adam + James	Functions	29/04	3
TEACHING CYCLE 6			
Student A + Teesh	Circle theorem 1	06/05	1
Tom + Daniel	Circle theorem 2	06/05	1
Roan + Jevonte	Circle theorem 3	07/05	1
Pearl + Adam	Circle theorem 4	07/05	1
Crimson + James	Circle theorems 5 & 6	09/05	1
No Miss + Student A	Circle theorem 7	09/05	1
Deepz + Ty	Circle theorem 9	10/05	1
TEACHING CYCLE 7			
James	Tree diagrams	13/05	1

Roan + Daniel	Similar area & volume	14/05	1
Deepz + Ty	Area & perimeter of sectors	16/05	1
Adam + Jevonte	Regions	17/05	1
Jayzee + Beyoncé	Proportions	03/06	1
Daniel + K	Recurring decimals	04/06	1
Tom + Jevonte	Quadratic sequences	06/06	1
Wilmer + Deepz	Completing the square	07/06	1
D + No Miss	Angles in polygons & parallel lines	10/06	1
C + Teesh	Rearranging equations	11/06	1
Crimson + Pearl	Algebraic fractions	13/06	1

Table 3.7 – Teaching schedule for teaching cycles 5-7

3.4.5 Data Collection Methods

In the second research cycle, the data collection methods used in research cycle 1 continued – observations and participant interviews. However, changes in the school's requirements and offerings impacted the data collection.

3.4.5.1 Observations

Written field notes and video recordings continued into the second research cycle. In the fifth teaching cycle and the seventh teaching cycle, I recorded the Share stage. This was an adaptation to the original research design in which I had intended to have three action research cycles; however, as the academic year was coming to an end, there was no time for reflection after the second research cycle, so I amalgamated teaching cycle 7 into the second action research cycle.

3.4.5.1.1 Video Recordings

As discussed in section 3.4.2.1.1, a limitation of video recording is the position of the camera. Learning from research cycle 1, position C2 (see Figure 3.3) was the favoured recording position of the camera, as it allowed me to see the teacher participants and epistemic interactions amongst participants, in which shared epistemic agency is visible.

In teaching cycle 7, the class moved to a different classroom, and the position of the camera impacted data collection. In the new classroom layout shown in Figure 3.4 below, the room was wider than it was long. The participants sat in groups around the peripheral areas of the classroom, and this made it difficult to hear what was being said; this impaired my ability to follow participant conversations, especially as they moved around the classroom freely.

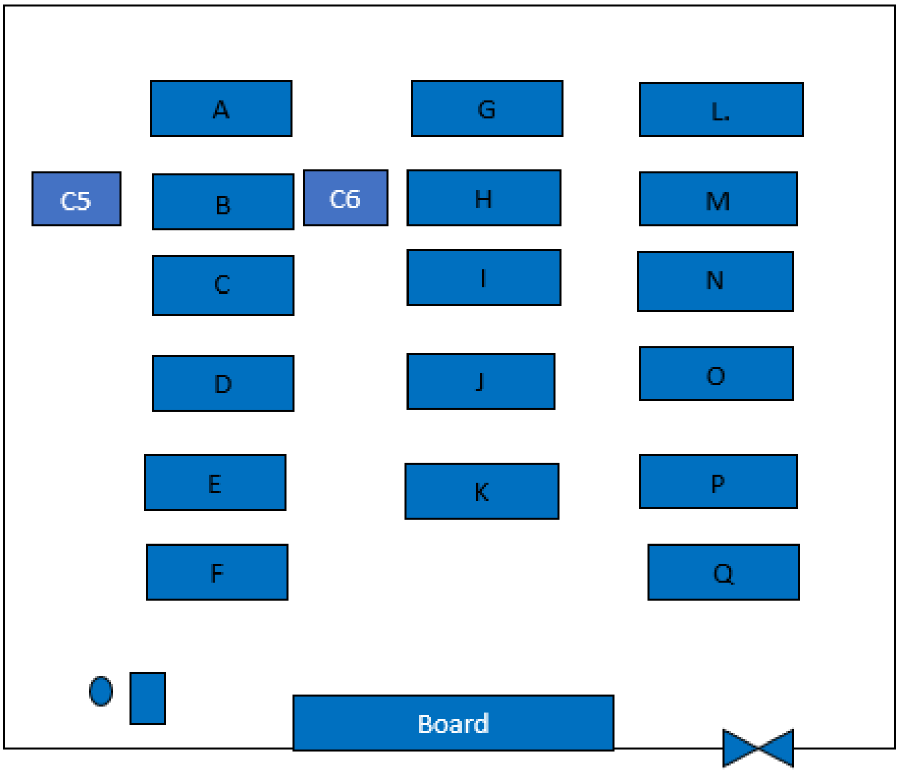


Figure 3.4 – Layout of teaching cycle 7 classroom

	
Camera View from Position C5	Camera View from Position C6

Photograph 3.2 – Camera positions for the teaching cycle 7 classroom

My responsibilities as a senior leader meant that my focus during that period was on Year 11 attendance to the GCSE examinations; I believe this caused me to forget the lessons I had learnt about positions of the camera, and even when I did attempt to rectify this by moving the location of the camera, the participants were too spread out to be able to both hear and see complete interactions. When located at positions C5 and C6 (see Photograph 3.2), I had a good view of the more interactive participants seated at positions N, O, and P; however, I could not hear their interactions due to the room's width, as the camera was too far away. When the camera was at position C5, I could only see and hear the interactions at seating positions C and D. In both positions, I could hear most of the conversations at the board.

The impact on the research was that it limited the quality of recordings. The viable recordings used in this paper were those where I could both see and hear participants’ interactions. Of the four Episodes (see section 4.1) selected for this teaching cycle, three were at the board, and one occurred around seating position D when the camera was at position C5.

Teaching cycle 7 was the last recording for the academic year, as end-of-year examinations followed soon after, and there were no more recording opportunities. Had there been more recording opportunities, I may have taken decisions to improve the quality of the recording by taking control of the seating arrangement and clustering the participants together. This change could create a viable position for the camera such that I would both see and hear more participant interactions. Nevertheless, when I realised that the participants were not in the full view of the camera, I did not change the seating arrangements, as I was concerned that it might compromise the research had hitherto been achieved; additionally, I would be teaching the same class through the next academic year, and would have liked for the shared epistemic agency to continue to emerge. Having spent the whole year enacting a pedagogy that required participants to take responsibility for their learning, it included giving the participants choice and freedom of movement within the classroom; this had become an essential feature of the practice.

3.4.5.2 Student Interviews

Following the second action research cycle, I conducted individual interviews using the same time slots as in the first interview, eliminating the potential for impact on other curriculum subjects. The main change in the two interviews was my interview questions. I had conducted the first round of interviews at an early stage of the research, when I was less certain of its direction and which questions would be helpful. By default, I focused on what I would like to know; as a teacher and as a researcher, I asked practical questions about how they prepared and why they did what they did (see Appendix 6). The interview was informal and semi-structured, as I followed up on participants' responses, and asked further questions about particular things that happened in their lessons.

The interviews positively impacted the research from a pedagogic and ethical perspective, giving me insight into how the participants thought about the roles of teachers and students. When interviewed, the students all said that they learnt more and worked harder as teacher participants; when asked the question "What is the purpose of learning?", all students' responses identified this purpose as the achievement of good grades so that they could have a promising future. These responses constantly reminded me of my ethical responsibility as a teacher and how the research supported this responsibility.

Regardless of these positives, by the end of the first research cycle, I became increasingly concerned about the value and rigour of these interviews with respect to the research, which had an impact on their usefulness for analysis. Unlike the lessons and their recordings, which offered many opportunities to put improvements into practice, I had to get the interviews right the first time I carried them out. I found myself in a catch-22 situation. I was carrying out action research with a methodology of incremental improvements towards an uncertain outcome, but as a result, I was carrying out interviews to shape this research without knowing their final contribution to it.

While sharing the transcripts with my supervisors, they pointed out that I interviewed the participants as a teacher rather than as a researcher, and this was true. On reflection, the participants were apprehensive about the interviews, and, subconsciously, I wanted to reassure them that they were doing a great job. I became the teacher. Interview Transcript Extract 3.4 is a section of the interview with Adam, a quiet but hardworking student.   When he taught his lesson, he showed his strengths as a hardworking and caring individual. Line 1 and line 7 of the extract show me as the teacher trying to build his confidence, and answering the questions for him rather than, as a researcher, asking in a bid to extend my own knowledge.

1	Me	Yeah, and then you did. This last one was. What was this last one? Functions.
2	Adam	Yeah, functions.
3	Me	Good, so we're talking about functions. So, talk to me about that lesson. The planning, how did you plan it and everything?
4	Adam	Erm I, so I went home and researched on MathsWatch and tried to understand the clips and did some questions as well. And then I just like, put some questions in the PowerPoint, and then I just tried and told myself how to teach the class, but I didn't understand some of the questions, and then… But James… James helped me and then, yeah, I understood the questions and I could help everyone.
5	Me	Do you think the lesson went well?
6	Adam	Yeah.
7	Me	Good, so what was your plan? I see how you prepared, and I guess you were preparing so that you would be able to help people understand.
8	Adam	Yeah.

Interview Transcript Extract 3.4 – From transcript of interview with Adam

Having received this feedback, and having decided on Episodes as the unit of analysis, I made the decision to focus only on analysing the video recordings of lessons, though it had taken a long time to decide on a legitimate method for analysing them, and de-emphasising the interviews. For one, there would need to be different a unit of analysis for the interviews, and it is not evident how the two data sources would inform each other. I also questioned whether the interviews would add substantially to the research. Considering these two points, the interview transcripts did not form part of my analytical framework.

In hindsight, I should not have scheduled the interviews at the points in the research at which I did initially. With the wisdom of hindsight, having gone through the research, the interviews would have been best placed at the end of the project, if they were to effectively contribute to the research findings. Having said this, the interviews did serve a purpose for both the participants and myself. Most notably, they were an opportunity for me to find out the lengths to which the participants went to prepare for stage 3 of the teaching cycles. It also gave me the opportunity to acknowledge them individually and let them know that they were doing very well. It was, above all, my capacity as a teacher that was more concerned for how the participants felt that led to my decision not to analyse the interview data.
Following action research cycle 2, I began analysing all the elected data. The analytical methods I employed are outlined in the next chapter.

4 ANALYTICAL METHODS

This chapter reports the analytical method developed for application to the data collected from observations (see section 3.4.5.1) – that is, the video recordings and field notes. I viewed the data from the video recordings, and I identified Episodes of students' epistemic interactions within this data. These Episodes of shared epistemic agency became the units of analysis. I did not analyse the data collected in the field notes in the same way as I did the recordings; instead, extracts from the field notes were used in the writing of the research to exemplify or explain information. Where I use extracts in this way, I identify them as emanating from the field notes. I did not analyse interview data beyond the part of this research concerned with enacting the pedagogy. I have discussed my intentions and subsequent decisions about interviews in the final section of the previous chapter.

4.1 The Unit of Analysis

An Episode of shared epistemic agency is a snapshot of participants' interactions in which the six characteristics of shared epistemic agency interplay to produce new knowledge. An Episode begins with an intention to resolve a state of unknowing and ends with the production of new knowledge formed during the knowledge-building process. In essence, an Episode consists of three distinct parts: the Intention, the knowledge building (comprising four patterns of action), and the New Knowledge (see Figure 4.1).

INTENTION → KNOWLEDGE BUILDING → NEW KNOWLEDGE

Figure 4.1 – The three parts of an Episode

Episodes do not have a specified time frame; they begin with the emergence of agency in the form of an Intention to advance knowledge, and result in an outcome, new knowledge, as a consequence of this Intention. Through participants' interactions and exercise of their agency, knowledge-building practices transform an Intention into New Knowledge.

The idea of an Episode as the unit of analysis came from Clarke's (2001) method for analysing classroom interaction, which focused on the “object of interest” (p. 36). In my study, the object of interest is shared epistemic agency. The notion of an Episode allowed me to select relevant moments from the hours of video data, and to organise it in the terms of a theory (cf. Dowling & Brown, 2010). As a snapshot of shared epistemic agency operating within the classroom practice, an Episode is valid as the primary unit used to analyse the data gathered in this project.

The value of an Episode of shared epistemic agency as a unit of analysis is that it focuses on the analysis of shared epistemic agency as an encapsulation of the six characteristics (see section 2.4.3) identified in the literature in use to advance the learning of the classroom community. It also indicated how the aims of the study are being met, in that it shows the participants in control of their knowledge advancement, indicating a relationship of active participation. I decided on this approach, as opposed to the analysis of isolated characteristics of shared epistemic agency exhibited by individuals or groups of participants. Analysing a given characteristic in isolation would have enhanced knowledge of that characteristic, but shared epistemic agency is most productive as an interplay of six characteristics. This decision enabled me to focus on the productive nature of shared epistemic agency; this is the purpose of this research. The analysis could then attend to the structure and development of Episodes in a bid to answer the research questions:

1. What are the indicators of shared epistemic agency in the mathematics classroom?
2. What sustains the emergence of shared epistemic agency in the mathematics classroom?

In essence, an Episode allows the analysis to focus on the interplay of the six characteristics employed by the participants, as part of the classroom practice, for their knowledge advancement. In this way, the outcome of the analysis, that is, the findings and discussion, will point to the purposeful and productive enactment of shared epistemic agency, answering to the research questions and meeting the aims of the study.

The Intention to advance knowledge is a response to a state of unknowing. This unknowing, tacit or explicit, is identified by the individual or group of individuals who expresses the intention to gain knowledge, or by an individual or group or individuals making a judgment about others' lack of knowledge. The Intention responds to the state of unknowing in a bid to resolve it. Knowledge building is the process that leads to the resolution of the unknowing, with New Knowledge being the resolution of the unknowing into a form of knowing. By this definition and the definitions offered above, an Episode is productive of new knowledge.

While an Episode corresponds to a single intention, there is no limit to the number of times knowledge building can produce new knowledge within an Episode. This New Knowledge can recursively lead to further knowledge building that produces further New Knowledge without an explicit change of Intention, as exemplified in Figure 4.2 below.

INTENTION → KNOWLEDGE BUILDING ↔ NEW KNOWLEDGE

Actions/Reifications — *Actions/Reifications* — *Conceptual artefacts*

Figure 4.2. – Intention = 1, knowledge building & new knowledge ≥ 1

Each part of an Episode is made visible through actions (dialogical and physical interactions) and reifications, which are the expression of an intention, itself the proactive commitment to resolve a state of unknowing. Knowledge-building interactions are interpreted by means of the observation of actions or reifications. The third part, new knowledge, the product of an Episode, is made visible as a conceptual artefact (see Figure 4.2).

In the remainder of this section, I will explain how I have connected the characteristics of shared epistemic agency to the notion of an Episode of shared epistemic agency, and how I recognise and define this in my data. To aid in this explanation, I refer to this annotated transcript of Episode 19 throughout my discussion of intentions and knowledge building. I will also refer to another annotated transcript to explain the third part of an Episode, new knowledge. I explain the coding of these annotated transcripts below.

Extract 4.1 – Unit of Analysis – Part 1 and 2 – Episode 19

Context: Daniel and Tom are walking around the class helping students and checking their work, and Daniel is using a booklet with solutions compiled by Tom. Daniel walks to Crimson and checks his work, and Daniel compares Crimson's solutions to Tom's solutions.

Part	Line	Participant	Actions/Reifications	Codes
INTENTION	1	Daniel (to Tom):	"Are you sure it's 11.3?"	I – Xpt.
	2	Said as Daniel walks over to Tom, he puts his hand on Tom's shoulder, and they both look at the solution in the booklet; discussion ensues. After studying their solution, they both walk back to Crimson.		MR solidarity
KNOWLEDGE BUILDING	3	Daniel (To Crimson):	"Did you put the 15 over 3?"	Ext
	4	Crimson:	"7 over 15."	Exp
	5	Daniel:	"Where did you get 7?"	Ext
	6	Crimson shows Daniel the work in his booklet, Daniel studies it.		
	7	Crimson:	"What you do … (Inaudible discussion. Crimson explains to Daniel and Daniel appearing to question and challenge.)	Exp
	8	Discussion ensues with Crimson outlining his solution.		Exp

		9	Daniel (points to a line in the booklet):	"How do you know that's 2?"	Ext
		10	Crimson (pointing along the solution):	"This one is 6, minus this one …" (Inaudible explanation.)	Exp
		11	Roan, who is sitting nearby, joins in listening to the dialogical interaction. He comments.		
		12	Daniel (to Roan):	"I wasn't talking to you. Sit back down." Roan sits down.	MR
		13	Daniel keeps on studying the solution in the booklet. Tom walks up to him.		
		14	Daniel (places hand on Tom's shoulder):	"Technical difficulties…" (pointing at Crimsons work). "He's right."	Xpt, MR, NK
		15	Daniel (pointing through Crimsons working out):	"That's 4 … (inaudible explanation)"	Exp
		16	Tom:	"Why did he do it like that?"	Ext
		17	Inaudible discussion with Daniel explaining to Tom.		Exp
		18	Daniel (giving booklet to Tom):	"You correct yourself."	Xpt.
		19	Tom:	"No, I'm not going to" (Daniel holding onto the booklet)	
		20	Daniel points out something in Crimson's booklet; discussion ensues between the three, with Crimson explaining. Tom is		

			questioning and challenging Crimson's explanation; they point to the solution as the interaction goes on.		
		21	Crimson (Pointing at the question, with raised voice):	"It's not this line, it's this line!"	Xpt Exp
		22	Daniel (points to a spot on the page):	"The one here, bro."	MR
		23	Crimson (pointing at the booklet):	"The one here is 15, this one is 3."	Exp
		24	Crimson:	"You do six times four."	Exp
		25	Daniel:	"Because four is this line."	Exp
NK		26	Discussion ends with Tom taking his booklet from Daniel to correct the solution.		NK

4.1.1 Intentions

Episodes of shared epistemic agency begin with an Intention (coded as I), a proactive commitment to bring about a future outcome (Bandura, 2001, p. 6). An Intention originates in an individual's thoughts and manifests as an action, and it is the action that the Intention produces that makes it visible. These actions can be either dialogical interactions or physical interactions. Dialogical interactions (i.e. dialogues) are verbal communications in the classroom that express the Intention, while physical interactions refer to what the participants do with their bodies to express the intention.

In Extract 4.1, I indicate where each of the characteristics of shared epistemic agency were made visible. In this way, I “coded” (cf. Saldaña, 2013, p.5) line 1 and line 2 as expressing Daniel’s Intention to resolve his state of unknowing. The question “Are you sure it’s 11.3?” initiates a dialogical interaction; he is verbally communicating his intention to Tom at the same time as walking over to him, a physical interaction. This verbal communication is a successful interaction as it generates a response from Tom. An intention can also be made visible by physical action; a teacher writing the working out on the board could express an intention to resolve a presumed unknowing by explaining knowledge to others. While actions make visible a current intention, reifications can make visible a previous Intention.

As noted in chapter 2, the term “reification” in this research draws on Bereiter’s concept of conceptual artefacts (2002) and Wenger’s concept of reification (1998). Artefacts denote human creations created to serve a particular purpose, while reification refers to “the process of giving form to our experience by producing objects that congeal this experience into ‘thingness’” (p. 58). Reifications are our projections of meaning onto the material world, which we then perceive as existing in the world and having a reality of their own. The booklet of solutions compiled by Tom that is referred to in Extract 4.1 is a reification. It represents preparedness and mathematics knowledge, and Expertise as a teacher participant. The booklet congeals within it a previous intention and previous actions to advance the mathematics knowledge of the class. While Tom’s previous actions were not visible, as he produced the booklet in the past and outside the lesson, its presence in the lesson serves as a reification that illuminates these previous actions that resulted from a previous intention (not the one that initiated this Episode). Reifications can also be non-concrete objects, such as the mnemonic device "SOHCAHTOA" is also a reification; it is an acronym, a scholarly creation in which are congealed the trigonometric ratios, such that its use is indistinguishable from the use of that of which it is a reification. In addition, a reification can be a symbol, such as that formed by raising a hand in the classroom. While this could be considered an action, in the classroom context, the action gives a material form to an abstract call for attention.

4.1.2 Knowledge Building

Knowledge building is the second part of an Episode. It refers to the interaction between participants to respond to an Intention to resolve a state of unknowing and produce new knowledge, which is the final outcome of the interaction.

I consider interaction as knowledge building if it proceeds from an intention to advance mathematics knowledge, and if the participants exhibit all four of the knowledge-building characteristics of shared epistemic agency. These characteristics are Extension, Explication, Expertise, and Mutual Relations.

4.1.2.1 Extension

Extension (coded as Ext) focuses on the actions and reifications of the individual participants as they strive to extend their existing mathematics knowledge**.** It elaborates on what the participants do to go beyond their existing knowledge. These actions and reifications implicate awareness of what is unknown and the seeking of ways to improve, interrogate, and challenge their existing knowledge. In Extract 4.1, line 3, Daniel seeks to extend his knowledge by the action of asking Crimson, "Did you put the 15 over 3?". It is this action, in the form of dialogical interaction, that makes the characteristic of Extension visible in this Episode. Though not exemplified in this extract, an Extension can also be made visible by a reification. Showing one's working out for the teacher participant to highlight your error is an Extension. In this instance, it is made visible through the working out, which reifies the existing knowledge one wants to extend. The working congeals within it one's existing knowledge and unknowing.

4.1.2.2 Explication

Explication (coded as Exp) focuses on the actions or reifications that make knowledge in the form of concepts, processes, ideas, or formulae explicit to another participant. Explication could be a phrase, sentence, exposition, or even a diagram that clarifies the knowledge to make it useable to another. In Extract 4.1, line 4, Crimson's dialogical contribution of "7 over 15" in response to Daniel's Extension is an Explication, and points to Crimson's knowledgeability, or his epistemic authority (Oyler, 1996b, p. 149). It is the beginning of Crimson's effort to make the mathematics knowledge reified in his booklet and inhering in his mind explicit to Daniel. It is his action, in the form of a dialogical interaction, that makes the Explication visible. For instance, lines 7, 8, 10, and 21 indicate Crimson's continued efforts to explicate his solution. Explication can also be made visible by a reification, such as in line 15, where Daniel uses Crimson's working out as the Explication of the solution. The working out captures the solution to the problem and the knowledge that is presently unknown.

4.1.2.3 Expertise

Expertise (coded as Xpt) focuses on the participants expressing process authority (Oyler, 1996b, p.149) in the classroom community. Expertise places the participant in control of the learning culture of the classroom, the selection, pace, sequence, criteria of the mathematics knowledge, and the social base (Bernstein, 2000) that makes advancing community knowledge in the classroom possible. In Extract 4.1, lines 14, 18, and 21 show the participants assuming authority within the learning culture; this is how the learning should occur. Line 21, which I labelled as both Explication and Expertise in this context, takes into account the tone of Crimson's voice. The Explication "it's not this line, it's this line" points to his epistemic authority as he explains which line should be considered for the calculation. But it is his raised tone of voice, expressing the belief that the teacher participants should know this already, that points to his Expertise.

In the context of his role as teacher, Daniel demonstrated expertise in lines 14 and 18, as he is behaving in a manner consistent with his responsibility for the learning in the classroom. I saw his actions in both lines as part of his bid to ensure that the knowledge reified in the booklet by fellow teacher participant Tom was accurate. His actions in line 18 show his authority as he says to Tom, “you correct yourself”. These acts of authority make Expertise visible. Though not exemplified in this extract, Expertise can also be made visible by a reification. Other participants will interpret a participant placing a finger to their lips as an instruction to desist from a particular unwanted behaviour, be quiet and focus on advancing mathematics knowledge. The finger to the lips reifies the instruction “be quiet” and the authority of the participant who produces the reification.

4.1.2.4 Mutual Relations

The concept of Mutual Relations (coded as MR) focuses on the relationships between participants in the community that enables them to interact to advance their mathematics knowledge. It refers to how they relate to and create an environment that they find conducive to knowledge-building interactions. In Extract 4.1, line 2, Daniel walks over to Tom and puts his hand on Tom’s shoulder, both look at the solution in the booklet, and a discussion ensues. In the narrow context of the Episode, Tom putting his hand on Tom’s shoulder is evidence of a mutual relation in the form of a physical action. Tom and Daniel are the teacher participants. Tom prepared the answers to the questions in the booklet that the classroom participants were using. Daniel, having discovered that Tom’s solutions may not be accurate, walks up to him to share the news that is not positive. Placing a hand on Tom’s shoulder communicates solidarity between educational partners, enabling Tom to be open to hearing the need for correction.

Similarly, in line 22, the use of the word “bro” reified friendship and concern for another’s feelings. Though not exemplified in the extract, like Explication and Expertise, Mutual Relations can also be made visible by reifications, such as through the issuing of an achievement point to a participant who is performing well. Issuing an achievement point reifies a positive relationship to one’s teacher, peers, and learning, and, in the school’s context, it acknowledges the student’s potential for becoming a prefect.

For the next section on New Knowledge, I will introduce the annotated transcript of Episode 9 to explain this third part of an Episode. In section 4.2, I use this and the other annotated transcript introduced earlier in this chapter to explain the final stage of the data analysis.

<u>Extract 4.2 – Unit of Analysis – New Knowledge – Episode 9</u>

Context: Teacher participant James is at the board introducing the concept of ‘less than’ and ‘greater than.’ He is using his PowerPoint lesson. Student participants are focused on the board, listening to his exposition.

	Participant	Actions/Reifications		Codes
1	Student A (calling out from the back of the class):	“Ja1mes …”	*INTENTION*	MR – Trust
2	James: (turns to Student A)	“Yo!”		MR
3	Student A:	“… I’ll show you something easier?”		I, Exp
4	As she speaks, student A starts to come towards the board.			I, MR
5	James stops writing and turns towards where the student participant is sitting; as she comes forward, she takes the pen he is offering and writes on the whiteboard.			Xpt, Exp
6	4 7		Knowledge Building	Exp -
7	Deepz:	What is that?		Ext
8	Student A: (pointing to the board)	“Look, 4 and 7” (pointing to the four then to the 7), “4 is less than 7.”		Exp
9	(She gives the pen back to James and walks back to her seat.)			MR
10	Deepz:	“Oooh, that’s smart.”	**New Knowledge**	MR, NK
11	Other participants:	“Ahhhh.”		
12	James (nodding in acknowledgment):	“That’s smart, that’s smart.”		
13	James (pointing to the board):	“That’s a good way to remember it”		Xpt, MR
14	Deepz:	“You see that, 4 and 7.”		
15	Student B:	“Greater than and less than, so four is less than 7?”		Exp

16	(James at the board sits down to allow B to explain, and for the class to talk about it.)				Xpt
17	(Class chatter.)				
18	James:	"Everyone understands that?"			Xpt
19	(Acknowledging noises and gestures.)				NK
20	Student A (says happily, with a big smile on her face):	"You see that little trick there!"			End
	James gets up and continues his explanation.				

4.1.3 New Knowledge

New Knowledge (coded as NK) is the resolution of the Episode's Intention, the product of knowledge building. As previously mentioned, an Intention is a bid to resolve an unknowing; the resolution of the unknowing is New Knowledge. This New Knowledge takes the visible form of a conceptual artefact (Bereiter, 2002, p. 64).

Conceptual artefacts (see section 2.4.1) are abstract knowledge objects such as discussable ideas, theories, algorithms, and concepts that are represented in some material form; this material form could include an expression. These artefacts can be used as knowledge and credited as New Knowledge only if they fulfil the criteria of being of value to people other than the individual, having a value that endures beyond the moment, having application beyond the situation that gave rise to it, and displaying some measure of creativity in their production (Bereiter & Scardamalia, 2011, p. 3). The knowledge that they express can be criticised, improved upon, or used to further develop other knowledge.

In Extract 4.2, the digits “4” and “7” are fundamental reifications, as symbolic human fabrications that reify different quantities; this intersubjectivity exists beyond the mathematics lesson. The symbols for “greater than”, “less than”, and “equals to” are also reifications, of the bigness, smallness, or sameness of one quantity compared to another. What was met with approval by the class because it mnemonically superimposed the symbols of greater than and less than upon an example of their functioning, was New Knowledge, which the students found hard to express with more complex symbols, and which in this form was more readily accepted as fact.

The New Knowledge was not what Student A wrote on the board; the New Knowledge was the knowledge that was reified by the special use of the digits “4” and “7”. The numbers 4 and 7 are the artefact, and the knowledge is the corresponding concept. The conceptual artefact in this Episode is knowledge of what the symbols “<” and “>” mean individually, and of when to use them in a mathematical context. It is credited as New Knowledge as it fulfils the criteria mentioned above. It is of value to all participants; the knowledge exists beyond this lesson on inequalities, as the participants will use it in other contexts, such as when solving quadratic equations. The use of the digits “4” and “7” is a creative way to remember the symbols. The New Knowledge is what the participants now know, the knowledge they have gained, which is an improvement on existing knowledge.

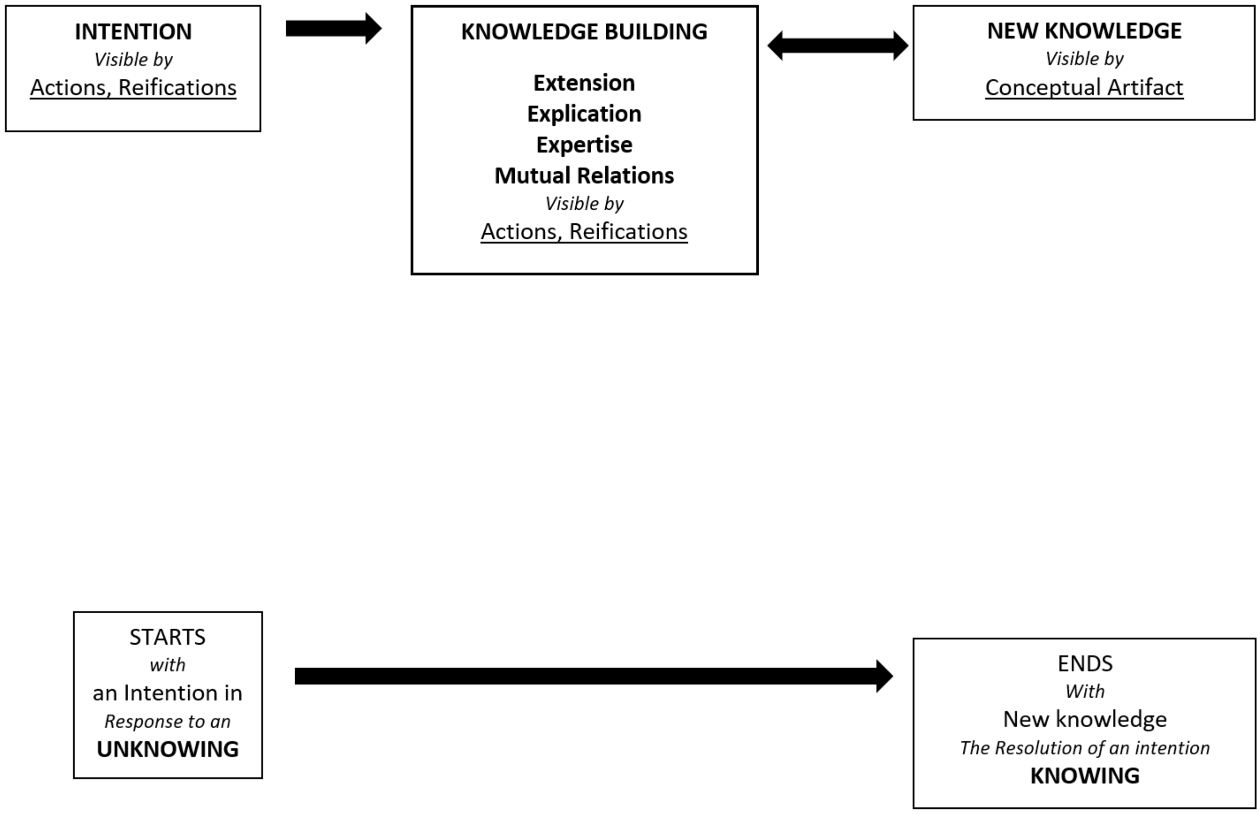


Figure 4.3 – The process of an Episode, the unit of analysis

Having demonstrated the structure of an Episode, as shown in Figure 4.3, I conclude by reiterating that the six characteristics of shared epistemic agency are made visible through actions and the production of artefacts, as the participants interact to advance their mathematics knowledge and that of the classroom community. As is expressed in both extracts, the three parts of an Episode can overlap as they occur. In Extract 4.1, line 14, Daniel had resolved his unknowing but continued interacting with Crimson and Tom until the latter also had New Knowledge. In extract 4.1, the New Knowledge was not as explicit as in extract 4.2. However, New Knowledge was created because Tom corrected the question in the booklet, marking the end of the Episode. The following section will explain how I selected Episodes in the research.

4.2 Episode Selection

The process of identifying Episodes commenced after all the data was collected. In practice, reflection occurred at the end of each action research cycle, and data analysis did occur, though this did not result in the selection of Episodes. It took time and effort for me to develop a reliable method to analyse all the data in such a way as to answer the research questions. This was the contribution of the second action research cycle. Further reading, discussions, and feedback from my supervisors helped me develop the notion of an Episodes of shared epistemic agency that I employ as the unit of analysis.

4.2.1 The Selection Processes

Having decided on what constitutes an Episode and how to identify it, I set about re-watching all 39 hours of recordings in chronological order. While watching, I was looking for instances of an Intention expressed by the classroom participants other than myself. When I observed an Intention, I asked the following questions of the Intention in the lesson context:

1. Is it epistemic, i.e., directed towards mathematics knowledge?
2. Is the Intention resolved?
3. Does the resolution result in new mathematics knowledge?

4. Is there evidence that more than one participant is involved in stating, demonstrating, or validating the new mathematics knowledge?
5. Are all the four characteristics of knowledge building – Extension, Explication, Expertise, and Mutual Relations – demonstrated by the participants?

If the answer to all five questions was yes, I had identified an Episode. Upon such an identification, I reviewed the recordings and filled in an Episode summary sheet. The summary sheet contains the details of the Episode. An exemplar of a completed summary sheet from Extract 4.2 is shown below in Figure 4.4. In the next section, I will explain how I completed this summary sheet.

<table>
<tr><td>9.</td><td>Intentions
(Explication)</td><td colspan="2">Knowledge Building</td><td>New Knowledge</td></tr>
<tr><td colspan="2" rowspan="2">Who: Student A
What: to show an easier method
Action: calls out, walks to board</td><td>Extension</td><td>Who: Deepz, other participants (inaudible discussion)
Action: questions</td><td rowspan="2">End time: 00:56</td></tr>
<tr><td>Explication</td><td>Who: Student A, Student B, other participants (inaudible discussion)
Reification: “4” and “7” on board</td></tr>
</table>

Start time: 00:17		**Action:** explanation of reification	**Conceptual Artefact:** New way to remember what greater than and less than symbols represent
	Expertise	**Who:** Teacher participant James **Action:** allows student participants to share authority, gives up pen, allows discussion time	
	Mutual Relations	**Who:** Student A, James, Deepz **Action:** Trust, informal language, acknowledgment from other participants, respect for others	

Figure 4.4 – The summary sheet of Episode 9, Extract 4.2

4.2.1.1 Completing the Summary Sheet

I used the summary sheets to document information about each Episode to enable further analysis without requiring a re-watching. I designed the summary sheet in three columns for the three parts of an Episode. In the first column, I record information about the Intention. In Figure 4.4., the Extract 4.2, the Intention in line 3 orients the student towards Explication; the statement "…I'll show you something easier?" is born out of Student A's desire to resolve an unknowing through

Explication. Student A is the '"who", and the "what" is a concise description of the who's Intention. The Intention is made visible by her dialogical interactions of calling out, and her physical action of walking towards the board (lines 1-4). I documented these, along with the start and end times of the Episode, in the "action" section of the summary sheet. In the knowledge building column of the summary sheet, I filled in a concise description of the "who", and their actions or reifications that make visible each of the four characteristics of shared epistemic agency that make up knowledge building. In the above example:

- Extension was visible in lines 7, Deepz being the "who", with other students being the "who" in line 17. These instances of Extension were made visible through the dialogical modality of questioning.
- Explication was visible in lines 5 and 8, where student A was the "who", and in line 15, where Student B was the "who". These instances of Explication were made visible through the reification – the digits "4" and "7" on the board, followed by dialogical interaction.
- Expertise was visible in lines 5, 13, 16, and 18, wherein the teacher participant James was the "who". His actions that made Expertise visible, through dialogical and physical interaction, were allowing Student A to share his authority by handing over the board pen, sitting down to let her explain to the classroom participants, and controlling the pace of learning.
- Mutual relations were visible in lines 1, 2, 9, 10, 13, and 16 in which Student A, James, the teacher participant, other participants, and Deepz were the "who". Actions that made the Mutual Relations between them visible include in line 1,

when Student A calls out from the back of the class, reifying her trust that James would not stop her from doing what she wanted to do; this is further highlighted by her simultaneously getting up from her seat and walking towards the front of the class (line 5). The informal response of “Yo!” from James (line 2) reifies equality between participants. Deepz, James, and other participants show approval for Student A’s epistemic contribution (lines 9-11).

In the third column, I recorded information about the New Knowledge ascertained. I recorded the end time of the Episode when the unknowing was resolved, along with a brief description of the conceptual artefact. I completed a summary sheet for all thirty-six Episodes. Table 4.1 below gives concise information about the Episodes I selected.

TC	**Episode**	**Start Time**	**End Time**	**Recording Reference**	**TP**
3	1	1:47	2:30	1-JEDE TC3 061118.MP4	Jevonte + Deepz
	2	3:57	5:08		
	3	07:40	13:33		
	4	33:20	1:58	1-2JEDE TC3 061118.MP4	
	5	00:23	3:22	2-TEPE TC3 091118.MP4	Teesh + Pearl
	6	13:29	18:30		
	7	20:48	21:38		
	8	27:20	28:16		
	9	00:17	00:56	4-JAAD TC3 151118.MP4	James + Adam
	10	01:01	2:27		
	11	26:07	27:44		
	12	31:33	32:08		
	13	33:28	00:16	4-2JAAD TC3 151118.MP4	
	14	18:56	25:27	5-BYJA TC3 221118.MP4	Jayzee + Beyoncé
	15	08:44	10:06	5-2BYJA TC3 221118.MP4	

5	16	09:16	11:16	8-Daniel Tom TC5. MP4	Daniel +Tom
	17	31:39	32:13		
	18	1:59	4:16	8-3Daniel Tom TC5b. MP4	
	19	2.44	4:22		
	20	2:33	3:13		
	21	4:44	5:52		
	22	4:50	7:10		
	23	0:40	2:53	9-Adam James TC5.MP4	Adam +James
	24	6:25	7:52		
	25	8:36	11:40		
	26	20:46	23:06		
	27	25:04	28:29		
	28	11:49	16:50	9- 3Adam James TC5.MP4	
	29	6:25	11:05	10-Deepz Ty TC5.MP4	Deepz +Ty
	30	17:34	20:52		
	31	4:04	5:13	11-Pearl Jayzee TC5.MP4	Pearl + Jayzee
	32	13:09	29:30		
7	33	5:40	12:32	13-Adam TC7.MP4	Adam
	34	3:22	5:19	14-Deepz Ty TC7.MP4	Ty + Deepz
	35	11:28	29:22	15-Tom Jevonte TC7.MP4	Jevonte +Tom
	36	31:57	11:15	16-Adam Deepz TC7.MP4	Adam + Deepz

Table 4.1 – The thirty-six Episodes identified across the research

The first column (TC) identifies the teaching cycle of each Episode. The second column is the Episode number, the third column identifies the start time of the Episode, and the fourth column indicates the end time of the Episode. The fifth column, the lesson recording reference, identified the exact recoding file for validity. I titled the recordings of each teaching cycle with a distinctive method for easy identification. In teaching cycle 3, for instance, “1-JEDE TC3 06118” begins by indicating the recording number in chronological order. The capital letters indicate the first two letters of names of the teacher participants, the teaching cycle, and the date of the recording. In teaching cycle 5, “8-Daniel Tom TC5” identifies the recording number in chronological order of recording, the pseudonyms of the teacher participants, and the teaching cycle.

4.2.1.2 Barriers to Episode Selection

The thirty-six Episodes identified in Table 4.1 are not exhaustive of all Episodes of shared epistemic agency that occurred across the 102 lessons; they account for the Episodes I was able to identify in the video recordings. Episodes of shared epistemic agency could have occurred during the non-recorded lessons, and Episodes of shared epistemic agency could have occurred during the recorded lessons but out of shot of the camera. The position of the camera constrained what was observable (see section 3.4.2.1.1). These constraints, in turn, limited the field of selection, which means that more Episodes occurred during the recording, but for this research, I only identified Episodes from the data collected by the camera.

My participation in any stages of an identified Episode could cause me to deselect the Episode if there is evidence that my authority hampered participant agency. For instance, in Extract 4.3 below, I was too quick to interject from line 23, so the New Knowledge produced was not solely down to the teacher participants' agency or the student participants. I habitually assumed authority.

Extract 4.3. Teaching Cycle 3. Date: 08/11/2018. Time: 06:00 – 07:28 Topic: Quadratic Formulas. Lesson 1. Teachers: Teesh & Pearl		
1	Crimson: "You know where it says minus …"	
2	Daniel: "Where …"	
3	Teesh (to Daniel): "Wait …"	
4	Crimson: "… where it says minus 'b', erm, and it says minus ten would you say minus and a minus is a positive or would you say …"	
5	Teesh: "you tell me …"	
6	Teesh (turning away from the board): "erm so copy …"	
7	Me: "Erm sorry, sorry, I can't let that pass by, he did ask a valid question …"	
8	Teesh: "Erm miss, I don't know."	
9	Me: "Then you say you don't know."	
10	(Classroom chatter.)	
11	Jevonte: "Go on MathsWatch."	

12	Pearl: "Guys we don't know so we have to come up with an answer together."	
13	Me: "Thank you."	
14	Jevonte: "Wow."	
15	Other voices: "Wow."	
16	(Class chatter.)	
17	Teesh: "Crimson repeat your answer."	
18	Crimson: "it says minus b and b is minus 10 so does a minus and a minus become a positive?"	
19	Pearl: "minus and a minus …"	
20	Teesh: "well to be honest a minus and a minus (inaudible) … but if you're writing it in the calculator …"	
21	(Continuous classroom chatter as they discuss.)	
22	Crimson: (inaudible)	
23	Me: "I can't let this pass by …"	
24	Teesh: "But miss wait, if you have a minus and a minus it's got to be a positive but …"	
24	Me: "Wait, because its crucial point, when you say minus and a minus, what's the 'and'?"	
26	Teesh: "Times …"	
27	Me: "Say that then … so a minus times a minus is a what?"	
28	Crimson: "Plus."	

29	Me: "It's a plus, there you go, so there it should be plus ten."	
30	Teesh: "So I was saying, you get two separate answers …"	

Extract 4.3 – Example of a deselected Episode

My assumption of epistemic authority also caused me to fail to identify Episodes in the recordings of Daniel and Jayzee's lessons. As Jayzee lost confidence and doubted her knowledge, this affected the interaction between participants, and I was called on more to take on epistemic authority, which further reduced the teacher participants' authority. In essence, my assumption of authority on that occasion meant that I could not identify an Episode where all the interactions were based on the participants' knowledge. Moreover, I did not select Episodes from the lessons of the three participants who did not hand in the consent forms (see section 3.4.1.1), as I did not record their lessons. Above all, the primary barrier that caused me to not select Episodes was my assumption of authority and failure to blend it with the authority of other participants.

This research does not require every Episode of shared epistemic agency to be addressed and form part of the analysis. The research questions are about discovering what indicates and sustains shared epistemic agency in a mathematics classroom, and there is ample room for this discovery in the thirty-six Episodes that I consider in this study.

4.2.2 Transcribing an Episode

After selecting the thirty-six Episodes and completing the individual summary sheets, I then transcribed each Episode. Transcribing the Episode entailed listening to and watching the recordings repeatedly, pausing, rewinding, and re-watching to note what was said, as well as any gestures and inflections of the tone that may bear on communication.

4.2.2.1 Explaining the Extract Heading

Each extract starts with a heading such as the heading of the first transcript shown below:

Unit of Analysis – Parts 1 and 2 – Episode 19

The first part of the heading identifies what is being shown by the transcript. In this chapter, I used the first transcript to explain the units of analysis (parts 1 and 2). The second part of the heading identifies the Episode to which the transcript refers; in this example, it is Episode 19. The lesson recording reference that identifies the exact recording and the pseudonyms of the teacher participants can be found in Table 4.1 above.

Below the heading is the context of each Episode or extract. This describes what is happening in the classroom to help situate the Episode or extract within the classroom environment in which it took place.

4.2.2.2 Coding the Transcript

The first step in coding each line of the transcript of an Episode required re-watching the recording and reading any field notes from the lesson to place each action and reification in context. Considering the transcript of Episode 19 in section 4.1, the dialogical interaction in line 1 shows the Intention that underwrites the Episode. I understood this Intention to advance knowledge in relation to what had transpired before the interaction. It was in response to an unknowing that the teacher participant Daniel came to identify regarding the solution that fellow teacher participant Tom had recorded in the booklet.

The second step required identifying the end of the Episode and coding it. This identification requires carefully observing at what point in the Episode the unknowing that was the object of the Intention was resolved. The resolution of the unknowing that constitutes New Knowledge does not occur at the same temporal point for every participant in the Episode. For instance, in the transcript for Episode 19, line 14, Daniel ended the dialogue by stating that Crimson was right, and the unknowing was resolved for him at this point. However, it took until line 26 for the unknowing to be resolved for Tom. The same is the case for the transcript of Episode 9 (see Extract 4.2). While Deepz's dialogical interaction in line 10 indicated that the unknowing had been resolved, it was not until line 19 that other participants indicated, by noises and gestures, that the unknowing had been resolved at a shared level. The green rectangle labelled New Knowledge indicates this in both transcripts.

The third step involved reading through each line of the Episode in its context, and deciding which of the four knowledge-building characteristics of shared epistemic agency the action or reification makes visible at which points. On reaching such decisions, I coded each line.

A line of an Episode can be coded as one or more characteristics of shared epistemic agency – for instance, line 14 in the transcript of Episode 19. There were three different actions coded as three different characteristics of shared epistemic agency that emerged at this point in the Episode. The line began with Daniel placing a hand on Tom's shoulder. In the context of the Episode, this physical interaction of placing the hand expresses solidarity on the part of Daniel with Tom, just before the former, indicated that Crimson's solution to the problem is correct, meaning that Tom's solution is incorrect. This is coded as an expression of Mutual Relations. Following Daniel's placing of a hand on Tom's shoulder, he said, "Technical difficulties" … "He's [Crimson's] right", indicating a decision regarding the quality of the mathematics knowledge that is coded as Expertise. This dialogical interaction is also coded as New Knowledge as it indicates that Daniel has resolved his unknowing.

Transcribing and coding each Episode helped me become more familiar with my data and how each characteristic of shared epistemic agency is made visible. This understanding helped with the second level of analysis that I present in the next chapter, which considers my findings.

5 FINDINGS

In this chapter, I take a closer look at the units of analysis, that is, the thirty-six Episodes of shared epistemic agency I identified from the data (see Table 4.1). Transcribing and coding each Episode allowed me to perform a more detailed consideration of how the characteristics of an Episode of shared epistemic agency manifest in the classroom. Elaborating on the Episode and on how participants interact as they direct their agency towards the characteristics of shared epistemic agency will offer insight into what is indicative of this agency and how it is sustained in the classroom. It will thereby contribute to answering the research questions.

As stated in chapter 4, an Episode of shared epistemic agency comprises three parts: Intentions, knowledge building, and New Knowledge (see Figure 5.1). An Episode starts with an Intention to resolve a state of unknowing, a lack of knowledge, and ends when the production of New Knowledge that remedies this lack (see section 4.1).

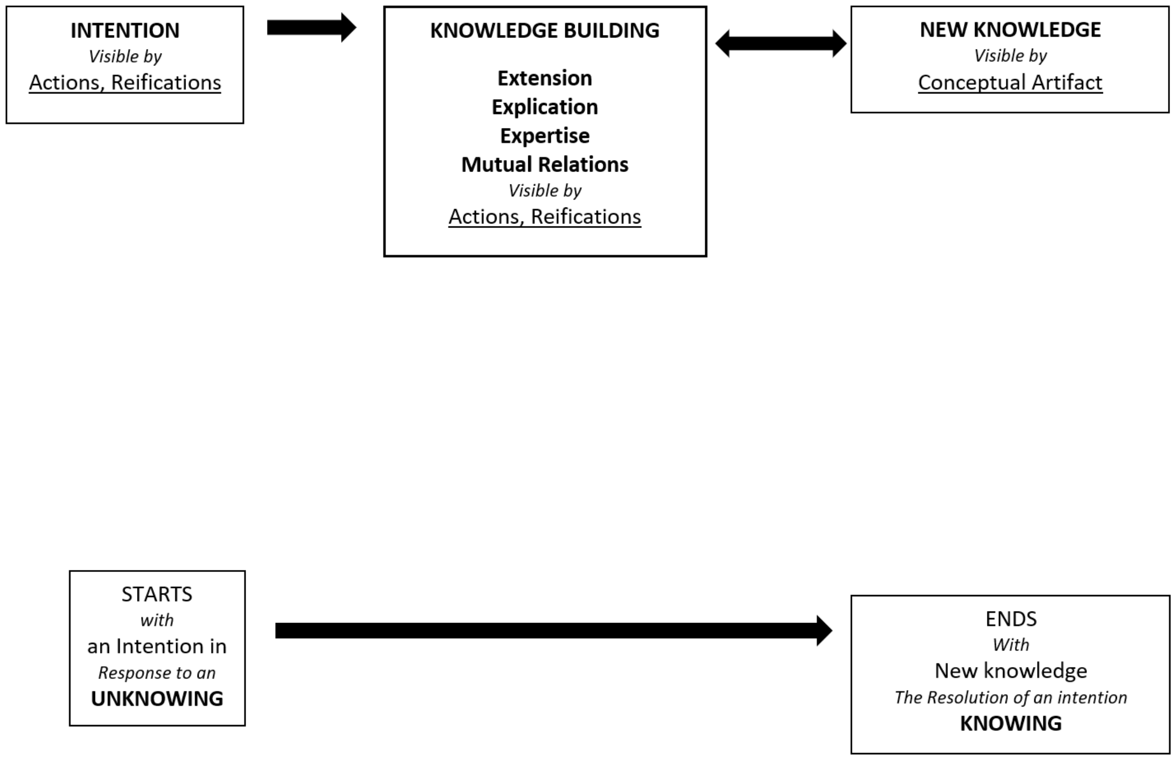


Figure 5.1 – The unit of analysis

Throughout this chapter, I will use extracts from Episodes to elaborate my findings. When explaining each discovery, I will select the Episode that I feel best demonstrates it, and I will also select Episodes based on how interesting they are and how well they show the unique personalities of the participants. While I sought to include all participants across my choice of Episodes, my priority was to explain each finding clearly. This means of selection resulted in my using extracts from sixteen out of the thirty-six Episodes. I repeated some Episodes, such as Episodes 1, 10, and 23 twice, while I used three different extracts from Episodes 2 and 6 across the chapter. The repetition of Episodes demonstrates that Episodes were complex interactions between participants that pointed to illuminate multiple aspects of participation.

The first part of this chapter will elaborate on the three parts of an Episode: Intentions, knowledge building, and New Knowledge. In the second part of this chapter, I will elaborate on participants' interactions in the classroom, their positionings during interaction, and how they expressed their authority. In this way, this chapter will highlight the findings related to each of the six characteristics that encapsulate shared epistemic agency, as well as further findings that emerge from the participants' interaction. The themes that arise from these findings will provide answers to the research questions.

5.1 Elaborating on the Unit of Analysis

An elaboration of an Episode of shared epistemic agency was made possible by the transcription and coding of each Episode. In this section I will describe a more nuanced conception of each part of an Episode. This description will identify the modes of Extension, Explication, and Expertise, elaborate on Mutual Relations, and discuss how an unknowing is resolved as New Knowledge.

5.1.1 Intentions

The Intention (see section 4.1.1) part of an Episode is the start of the Episode; it is the proactive commitment to resolve a lack of knowledge, an unknowing. An Intention orients its bearer towards any of the characteristics of Extension, Explication, or Expertise as it expresses this intentionality. Suppose the Intention relates to a participant striving to know, to extend their existing knowledge. In this case, it will orient the participant towards Extension; if it expresses their aim to make knowledge explicit to other participants, it will orient them towards Explication; and if it expresses their consolidation of process authority, it will orient them towards Expertise. Unlike in the case of the other knowledge-building characteristics, I found no empirical evidence to show that an Intention can orient towards mutual relations. I attribute this lack of evidence to the fact that Episodes in this research are all epistemic, focusing on mathematics knowledge, while the presence of Mutual Relations is a characteristic that only indirectly supports knowledge building in this context.

An Intention can be made visible by participants' actions, that is, dialogical interactions and/or physical interaction and/or reifications (see section 2.2.2.2). It can be triggered either by a teacher participant's (TP's) or student participant's (SP's) lack of knowledge, or when a teacher participant or student participant make a judgment about an individual or group's lack of knowledge. We will call the former "Identified unknowing", and the latter "Assumed unknowing". All these elaborations of an Intention – the three ways it is made visible, in the three different orientations towards action – as well as the two types of unknowing, and the participant who initiated the Intention, will be exemplified with the extracts from four Episodes below.

Extract 5.1 - Intentions (Ext, Dialogic Interaction, Identified) - Episode 1

Context: The question $2x^2 + x - 21 = 0$ was placed on the board by teacher participants (TP) Deepz and Jevonte for the student participants to solve. This was the start of the second lesson on factorising quadratic equations. Pearl, a student participant (SP), initiated the dialogue with the question in line 1.

Part	Line	Participant	Action/Reification	Code
Intention	1	Pearl (SP)	"How can we use the same method with the x?"	I (Ext)
Knowledge Building	2	Deepz (TP)	"It's the same thing that we did yesterday."	Xpt
	3	Student A (SP)	"But what do you times together to get x?"	Ext

In this extract from Episode 1, the Intention orients Pearl towards the shared epistemic agency characteristic of Extension. The Extension orientation identifiable at line 1 that commenced the Intention was an expression of Pearl's desire to extend her existing knowledge. The Intention, once initiated, was externalised: Pearl asked, "How can we use the same method with the x?". Pearl's apprehension of her lack of knowledge triggered the Intention; thus, this Episode was triggered by an Identified unknowing.

Extract 5.2 – Intentions (Exp, Dialogical/Physical Interaction, Assumed) – Episode 9

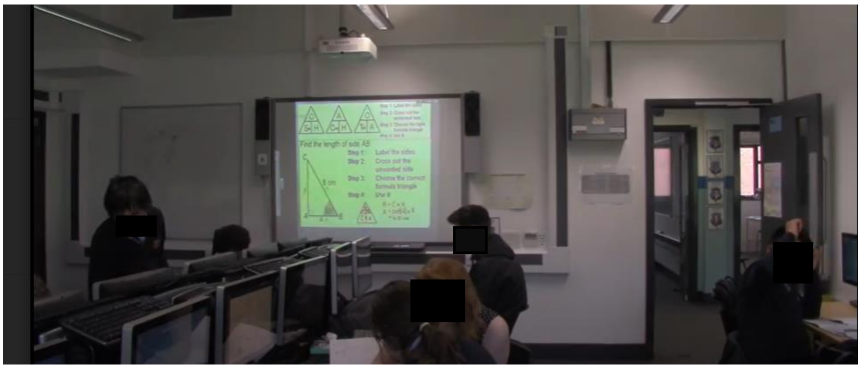
Context: Teacher participant James is at the board introducing the concept of "less than" and "greater than" using a PowerPoint lesson prepared earlier. Student participants are focused on the board, listening to his exposition.

Part	Line	Participant	Actions/Reifications	Code
Intention	1	Student A (SP)	(Student A calls out from the back of the class)" James …"	MR Trust
	2	James (TP) (turning towards Student A):	"Yo"!	MR Solidarity
	3	Student A (SP):	"… I'll show you something easier?"	I (Exp)
	4	As she speaks, student A comes towards the board.		I

In this extract from Episode 9, the Intention orients Student A towards the shared epistemic agency characteristic of Explication. This orientation is because line 3, in which the Intention commenced, was an expression of Student A's desire to make the concept of "greater than" and "less than" explicit to the classroom participants. The Intention was thus initiated by Student A, a student participant, and made visible by the dialogic interaction in lines 1-3 and the physical interaction of walking up to the board in line 4. It was initiated by Student A, who assumed that a lack of knowledge existed amongst the classroom participants. Hence, this Episode was triggered by an Assumed unknowing.

Photo 5.1 – Intentions (Ext, Dialogical Interaction/Reification, Identified) – Episode 17

Context: The students are working on the questions in their booklet. Adam, a student participant, is working independently at position G. At 31:28, he raises his hand (see Photograph 5.1 below).



Photograph 5.1 – Intention (Ext, Dialogical interaction/reification, identified)

In this Photo from Episode 17, the Intention orients Adam towards the shared epistemic agency characteristic of Extension. Adam's raised hand initiated the Intention, as it was an expression of his desire for Tom, the teacher participant, to extend his existing knowledge. The Intention was made visible by Adam raising his hand, which was also a reification of the call for attention. Adam, having identified his lack of knowledge, triggered the Intention; hence this Episode was triggered by an Identified unknowing.

Extract 5.3 – Intentions (Exp, Dialogical/Physical Interaction, Assumed) – Episode 19

Context: Teacher participants Daniel and Tom are walking around the class helping students and checking their work. Daniel is using a booklet with solutions compiled by Tom. Daniel walks to Crimson and checks his work. Daniel compares Crimson's solutions to the solutions Tom has prepared.

<table>
<tr><th>Part</th><th>Line</th><th>Participant</th><th>Action/Reification</th><th>Code</th></tr>
<tr><td rowspan="2">Intention</td><td>1</td><td>Daniel (TP)
(to Tom):</td><td>"Are you sure its 11.3?"</td><td>I
(Xpt)</td></tr>
<tr><td>2</td><td colspan="2">Daniel walks over to Tom and puts his hand on his shoulder; they both look at the solution in the booklet, and discussion ensues. After studying their solution, they both walk back to Crimson.</td><td>MR
Solidarity</td></tr>
<tr><td>Knowledge Building</td><td>3</td><td>Daniel
(to Crimson):</td><td>"Did you put the 15 over 3?"</td><td>Ext</td></tr>
</table>

In this extract from Episode 19, the Intention orients towards the shared epistemic agency characteristic of Expertise. This orientation is because line 1, which commenced the Intention, is an expression of Daniel as a teacher participant exercising his process authority, controlling the learning process by seeking to ensure that the answers in the booklet are correct. The Intention was thus initiated by Daniel and made visible by the dialogical interaction, “Are you sure it's 11.3?”, in line 1, and the physical action of walking over to Tom and placing his hand on his shoulder. It was initiated by Daniel, who assumed that a lack of knowledge was operative in Tom’s calculations. Hence, this Episode was triggered by an Assumed unknowing.

These four extracts are summarised in the table below.

Episode	**Orientation**	**Initiated by**	**Visibility**	**Episode Trigger**
1	Extension	Student participant	Dialogical interaction	Identified unknowing
9	Explication	Student participant	Dialogical interaction Physical Interaction	Assumed unknowing
17	Extension	Student participant	Physical interaction Reification	Identified unknowing
19	Expertise	Teacher participant	Dialogical interaction Physical Interaction	Assumed unknowing

Table 5.1 – Elaboration of Intentions

The elaboration of all Episodes (see Appendix 7) demonstrates that about half of the Episodic Intentions oriented participants towards Extension, followed Explication, with the lowest number of Intentions oriented towards Expertise. All Intentions that were triggered by an Identified unknowing were initiated by a student participant identifying their own unknowing, with the exception of Episode 30 (see Extract 5.18), where the student participants identified another participant's unknowing, and Episode 14 (see Extract 5.7), where a teacher participant Identified a student participant's unknowing. Episodes triggered by Assumed unknowings were initiated by both student and teacher participants.

5.1.2 Knowledge Building

The knowledge building part of an Episode is where the participants exercise their agency through their interactions to resolve the Intention. Their agency manifested as the interaction of four characteristics of shared epistemic agency: Extension, Explication, Expertise, and Mutual Relations (see section 4.1.2).

Analyses of the empirical data led me to develop a more nuanced conception of the characteristics of Extension, Explication, and Expertise from the diverse ways these characteristics of shared epistemic agency were made visible by the participants in the enactment of the innovative pedagogy. I refer to them as "modes" of each characteristic, and I classify them in the following sections. What is significant about these modes is not that they do not occur in other classrooms, but that they occur in my classroom during knowledge building as part of an Episode to resolve an unknowing.

Having completed summary sheets (see section 4.2.1.1) for each of the thirty-six Episodes, I realised that I had described how each characteristic was made visible differently on each sheet; I had written down the action or reification of the participants as observed on the video recording, and noted the diversity of expressions. I compiled all the different descriptions of the actions or reifications of each characteristic and grouped them into discrete modes to aid further analysis. Table 5.2 gives an example of four of the original descriptions of actions or reifications that I identified as indicating Extension; I have grouped these into a single mode, “Articulates unknowing”.

Action and reification noted on the summary sheet	**Episode**	**Classified mode of Extension**
“That’s not what I got”	6	Articulates unknowing
Teesh asks what others got	13	
Jayzee explains knowledge limit	24	
Crimson acknowledges unknowing	26	

Table 5.2 – Classifying a mode of Extension

Having identified these modes, I designed an appropriate summary sheet (see Appendix 8), and re-watched the recordings of each Episode to identify how many times each of the modes occurred in each Episode as an indication of the mode's relevance to the research.

5.1.2.1 Modes of Extension

Extension is the characteristic of shared epistemic agency by which participants direct their agency towards striving to know in a bid to extend their existing knowledge (see section 4.1.2.1). From the analysis of the recordings, I identified five distinct modes of Extension by which participants sought to extend their existing mathematics knowledge. The five modes are as follows: Questions, Seeks affirmation, Requests, and Articulates unknowing. I identified over 170 instances of these five modes of Extension across the 36 Episodes, with Questions being the most common, followed Seeks affirmation, Requests, Challenges, and Articulates unknowing being the least (see Appendix 9).

5.1.2.1.1 Questions

Epistemic questions (referred to as Questions) are direct questions asked by a participant of another participant (who is thereby deemed to have epistemic authority, whether a teacher or student participant), or else openly presented to an audience, in a bid to extend the former's existing knowledge. These questions are usually prefaced by "why", "what", "where", "how", or "when"; they are epistemic, as they relate to mathematics knowledge. Importantly, not all sentences that have the form of grammatical questions qualify as Questions in the Episodes.

Extract 5.4 – Extension (Questions to Student Participant) – Episode 18

Context: Student participants James and Jayzee were seated next to each other. It was the beginning of the second lesson on Trigonometry. They were to finish off the booklet of questions that was started in the previous lesson. The following Knowledge building interaction commenced with James declaring his lack of knowledge.

Part	Line	Participant	Action/Reification	Code
Intention	1	James (SP):	"I do not know what to do" (Identified unknowing).	I (Ext)
Knowledge Building	2	James (SP):	"What's ACB? … Oh, do you need to calculate angle ACB?"	Ext Question
	3	Jayzee (SP) (holding up her booklet and pointing to the diagram):	"The angle is always the middle letter …"	Exp

<u>Extract 5.5 – Extension (Questions to Teacher Participant) – Episode 29</u>

Context: Deepz, the teacher participant, explained bounds to the classroom participants with a worked example. He then asked the student participants to copy it from the board. He asked them to ask him if "they didn't understand". At 8:56, Daniel walks up to the board and points to the working out. Teacher participants Deepz and Ty were at the teacher's table. While Daniel was asking the question, James joined him at the board. Other participants could hear the discussion.

Part	Line	Participant	Action/Reification	Code
Knowledge Building	1	Daniel (SP) (pointing at a place on the board):	"Hey Ty, where did you get this from?"	Ext Questions
	2	Deepz (TP):	"Well like, listen, you always divide by two because … look (he comes up to the board) when you are finding the upper bound or lower bound its always plus or minus five"	Exp MR Trust
	3	Daniel and James (SPs) ask further questions at the board (inaudible).		
	4	Daniel (SP):	"Ain't the answer seventy-six?"	Ext

Line 2 from Extract 5.4 and line 1 from Extract 5.5 are Questions, as they are grammatical questions that concern mathematics knowledge, and serve the purpose of extending a participant's existing knowledge. In Extract 5.5, Daniel, a student participant seeking to extend his knowledge, directed the Question to Ty, the teacher participant, whom he viewed as knowledgeable, but Deepz, the other teacher participant, assumed epistemic authority and responded to the question. In extract 5.4 line 1, the student participant James initiated the Episode with an expression of his Identified unknowing. He openly shared his unknowing with the participants who sat around him. In line 2, he directed a Question to Jayzee, the student participant who sat next to him, and Jayzee assumed epistemic authority as a knowledgeable participant. Questions are not the same as other modes of Extension that include questions such as those that request help or that are posed to seek affirmation of knowledge.

5.1.2.1.2 Seeks Affirmation

Seeks affirmation occurs when a participant seeks validation of their knowledge to ensure that they are proceeding in the correct manner. The question posed requires a ‘yes’ or ‘no’ response from a knowledgeable participant. I identified this as a mode as Extension because the participant seeking affirmation is striving to know, which in this case is evident from their desire to be confident in the knowledge they have gained.

Extract 5.6 – Modes of Extension (Seeks Affirmation) – Episode 10

Context: James, the teacher participant, is introducing the concept of representing inequalities on a number line. Student participants are joining the exposition. This Episode was initiated by James' dialogical interaction in line 1 and the reification of the number line. The Episode's Intention was triggered by an Assumed unknowing. James assumed that all or some of the student participants did not have knowledge of representing inequalities on a number line. As such, the Intention was oriented towards Explication.

Part	**Line**	**Participant**	**Action/Reification**	**Code**
Intention	1	James (TP):	"If you want to plot this here, so we know that it's less than, so we put a circle …" (Assumed unknowing)	I (Exp)
Knowledge Building	2	Student B (SP):	"… and you colour it in, right?"	Ext Seeks Affirmation
	3	James (TP):	"… yeah, you colour in the circles because its less than …"	Exp
	4	Crimson (SP):	"… and then you draw an arrow down …"	Exp
	5	James (TP):	"Then you draw an arrow down."	Exp

	6	Student A (SP):	"Wait, wait I got a question!"	MR Trust
	7	James (TP):	"Yes?"	MR
	8	Student A (SP):	"So, if its more than you draw an arrow that (pointing towards her right) across the way?"	Ext Seeks Affirmation
	9	James (TP):	"Yes."	Exp

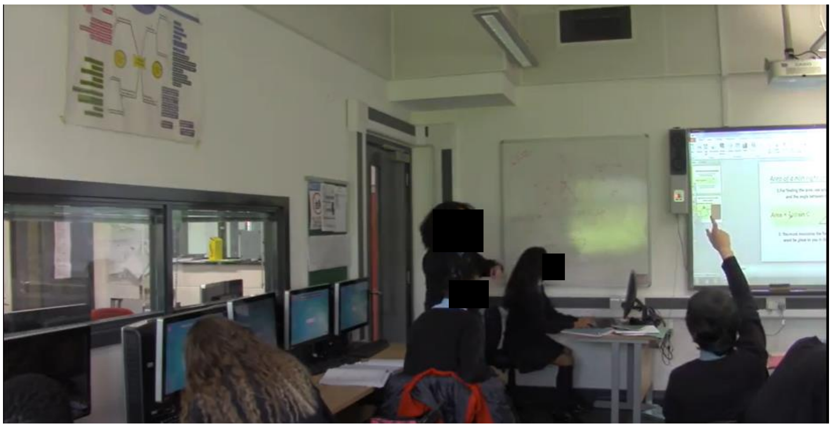
In the dialogical interaction between the participants in Extract 5.6, Student B (in line 2) and Student A (in line 8) asks James a question by which they articulate their existing knowledge, and seek from James further affirmation of their knowledge. The questions are such that James can provide a "Yes" or "No" response (see lines 3 and 9).

5.1.2.1.3 Requests

Requests to enable Extension (referred to as Requests) are actions or reifications directed by one participant towards another participant with epistemic or process authority, requesting an action or reification that they feel would enable the Extension of their existing knowledge; or, an action or reification directed by one participant towards another participant with authority, requesting permission to carry out an action or reification that they feel would enable the extension of their knowledge. The participant with authority could be either a teacher participant or a student participant. I have exemplified Requests in photograph 5.2 below and in Extract 5.7. In the photograph, the student participant Requests the teacher participant, who has epistemic authority, to carry out an action; while in Extract 5.7, the student participant requests the teacher participant with process authority for permission to carry out an action themselves. Requests are not direct questions about mathematics knowledge, and this differentiates them as belonging to a different mode of extension from that of Questions.

Photograph 5. 2 – Modes of Extension (Requests) – Episode 31

Context: Teacher Participants Pearl and Jayzee stand in front of the class, explicating the concept of finding the area of any triangle. They were explicating how to label a triangle's sides and angles in response to Crimson seeking affirmation. Roan raised his hand at 4:50.



Photograph 5.2 – Modes of Extension (Requests)

Raising his hand, in this context, was a reification of a request for the Extension of knowledge. Roan wanted either of the teacher participants, whom he viewed as epistemic authorities, to come over to him and help with an aspect of the topic. The field of Requests, as a mode of Extension, does not include all grammatical requests made by participants, but is specific to requests to enable Extension.

<u>Extract 5.7 – Modes of Extension (Requests) – Episode 3</u>

<u>Context:</u> The question $2x^2 + x - 21 = 0$ was placed on the board by teacher participants (TP) Deepz and Jevonte for the student participants to solve. The previous lesson was on factorising quadratic expressions. Teesh, a student participant, initiated the Episode. The Episode's Intention was triggered by her Identified unknowing. She knew how to factorise quadratics, but did not know how to solve them. As such, the Intention was oriented towards Extension, as she sought to extend her knowledge to the solving of quadratic equations. This was the basis of her dialogical interactions.

Part	Line	Participant	Action/Reification	Code
Intention	1	Teesh (SP):	"Let me do it on the board" (Identified unknowing).	I (Ext) Requests
Knowledge Building	2	Deepz (TP):	"No, but that's not it though."	Xpt
	3	Teesh (SP):	"Ok, but then when I do it, I show you what I can do …" (inaudible).	Ext Requests
	4	Deepz (TP):	(Inaudible response.)	
	5	Teesh (SP):	"I will do what I can do then …"	Ext Requests
	6	Deepz (TP):	"No but …" (inaudible).	
	7	Teesh (SP):	"That what I said, I will do what I can do then you do the rest."	Ext Requests
	8	Deepz (TP):	"Ok, come up and do it" (Deepz starts to prepare the board for Teesh to write on).	Xpt Controls Manages

In line 1 of the extract, Teesh, the student participant, Requests of Deepz, the teacher participant with process authority, to be allowed to come to the board and extend her knowledge by showing what she knows. The Requests to extend her knowledge continued in lines 3, 5 and 7. The action that Teesh felt would extend her knowledge was her coming to the board and publicly starting to factorise the quadratic equation. She was confident that knowledgeable participants would explain how to proceed with solving the quadratic equation, and she trusts that she will extend her knowledge in the process.

5.1.2.1.4 Challenges

Epistemic challenges (referred to as Challenges) occur when a participant challenges the veracity of knowledge presented to them by another participant with epistemic authority, in a bid to extend their existing knowledge. In this mode of Extension, though the participants seek to extend their existing knowledge, their current knowledge is sufficient to challenge the knowledge presented to them, though it requires Extension to move beyond its current point.

Challenges are coded as a mode of Extension where their essence in context is to extend knowledge. Challenges A and B below show the difference between a challenge coded as Extension and a challenge coded as Expertise (see section 4.1.2.3).

Extract 5.8 – Modes of Extension (Challenge A) – Episode 2

Context: The participants were engaged in a discussion regarding factorising the equation $2x^2 + x - 21 = 0$. Crimson, a student participant, seeks from Jevonte, the teacher participant, confirmation of the procedure for factorising quadratic equations. I selected this extract for the ensuing discussion where Pearl, another student participant, put forward the suggested solution of the numbers 6 and 7, but was still unsure about the negative numbers. This extract shows her challenging the logic of what Crimson was saying.

Part	**Line**	**Participant**	**Action/Reification**	**Code**
Knowledge Building 4:30	1	Crimson (SP):	"So it has to add to make minus forty-two, so it will be minus six."	Exp
	2	Pearl (SP):	"Not add."	Ext Challenges
	3	Crimson (SP):	"it's minus six, minus six plus one equals minus six."	Exp
	4	Pearl (SP):	"What!"	Ext Challenges
	5	Crimson (SP):	"Add minus six plus seven."	Exp
	6	Pearl (SP):	"Minus six plus seven doesn't give you forty-two, though!"	Ext Challenges

Extract 5.9 – Modes of Extension (Challenge B) – Episode 14

Context: Tom the teacher participant, was about to show the class how to solve simultaneous equations involving a plus and a minus (6x – 3y = 3 and x + 3y = 11). Student A, a student participant, asked if she could do the question on the board. Tom ascertained her knowledge first, then let her work the question out on the board. As part of her procedure to eliminate y, she added the two equations. Tom walked up to her and whispered line 1 of the dialogue below at 21:32.

Part	**Line**	**Participant**	**Action/Reification**	**Code**
Intention	1	Tom (TP):	"You made a mistake; you're supposed to take away." (Identified unknowing)	I (Xpt)
Knowledge Building	2	Student A (SP):	"No, you're not; this (pointing to working out) will give you a minus."	Exp A challenge
	3	Tom (SP):	"No, it won't."	Xpt

In the context of extract 5.8 (challenge A), Pearl was seeking to extend her existing knowledge, and Crimson had taken up epistemic authority. Pearl Challenges Crimson's knowledge in line 2, 4, and 6. This dialogical interaction that culminates in line 6 shows how Pearl Challenges the veracity of what Crimson told her in seeking to extend her knowledge.

Challenge B (see Extract 5.9), in contrast, does not highlight a mode of Extension. The dialogical interactions performed by Tom, the teacher participant, initiated the Episode; his Intention was triggered by what he identified as Student A's unknowing. He was not seeking to extend his existing knowledge when he challenged Student A's solution on the board. He was taking up his process authority as a teacher participant to check the quality of the knowledge that Student A sought to Explicate to the classroom participants. Student A responded with a further challenge to Tom in line 2. This challenge was an Explication, as student A attempted to make the knowledge more explicit to Tom and was not seeking to extend her own existing knowledge.

5.1.2.1.5 Articulates Unknowing

Articulates unknowing occurs when a participant identifies their lack of knowledge in a bid to have a knowledgeable participant extend their existing knowledge. I highlight this mode of Extension in the extract from Episode 6 below.

<u>Extract 5.10 – Modes of Extension (Articulates Unknowing) – Episode 6</u> <u>Context:</u> Teesh, the teacher participant, was at the board showing other participants how to use the quadratic formula to solve the questions she had posed to the class. Crimson, a student participant, was calling out the answer, and Teesh was writing the answers on the board. At 14:39, Student A, a student participant, made a dialogical interaction.

Part	Line	Participant	Action /Reification	Code
Knowledge Building	1	Student A (SP) (to the class):	"I didn't get that, but I got the same calculation in my calculator."	Ext Articulates Unknowing
	2	Jayzee (SP):	"What did you get?"	Xpt
	3	Student A (SP) (to Jayzee):	"I got ..."	Ext Articulates Unknowing
	4	Student B (SP):	"Are you sure, student A, because it happened last time …"	Xpt
	5	Teesh (TP):	"Everyone got this, yeah?"	Xpt
	(Classroom chatter.)			
	6	Student A (SP) (to Teesh):	"I didn't get it."	Ext Articulates Unknowing
	7	Crimson (SP):	(Turning to Student A with surprise) "Oh, you didn't? What did you get?"	Xpt
	8	Student A (SP):	"I put this in my calculator" (she passes her calculator to Crimson, who studies it).	Ext Articulates Unknowing

	9	Teesh (TP) (to the class):	"So, who got the one with the minus then?"	Xpt
	10	Crimson (SP) (to Student A):	"You did two minuses, Student A."	Exp MR
	11	Crimson (SP) (to Student A):	"It is not minus; it's ordinary five."	Exp

Student A sought to extend her knowledge in Extract 5.10 lines 2, 3, 6 and 8 by publicly articulating her lack of knowledge, expecting this to trigger another participant to act to extend her existing knowledge.

Not the articulation itself, but the demonstration of confidence that the statement will trigger another participant to act, marks it as a mode of Extension. As Teesh was writing the solution to the quadratic equation on the board, line 1 was spoken loudly in the presence of the whole class, directed to no one in particular and everyone in general. Student A made the statement with the subjective belief that it would trigger a knowledgeable participant in the class to act to extend her knowledge.

5.1.2.1.6 Summary

The five modes of Extension by which participants sought to extend their mathematics knowledge are summarised in Table 5.3 below.

Mode of Extension	**Description**
Questions	• Epistemic • Directed towards a knowledgeable other • Usually prefaced by “why”, “what”, “where”, “how”, or “when”
Seeks affirmation	• Participant seeks to confirm existing knowledge • The question posed requires a “yes” or “no” response
Requests	• Participant requests an action or reification from another participant, or • Participant requests to be allowed to carry out an action or reification • Directed towards a participant with authority
Challenges	• Participant challenges the veracity of another’s’ epistemic authority • Not to be confused with its function as a mode of Expertise (Challenge B)
Articulates unknowing	• Participant articulates their unknowing • Directed towards knowledgeable participant(s)

Table 5.3 – Modes of Extension Summary

5.1.2.2 Modes of Explication

Explication is the characteristic of shared epistemic agency by which participants direct their agency towards making mathematics knowledge explicit to another participant or group of participants (see section 4.1.2.2). The knowledgeable participant assumes epistemic authority if they decide to Explicate mathematics knowledge to another participant, and they have authority bestowed upon them if another participant asks them to Explicate mathematics knowledge. I elaborate on this distinction in section 5.2.1. From the analysis of the recordings, I identified four distinct modes of Explication employed by the participants in this research: Clarifies, Affirms, Tells, and Explicates unknowing. I identified over 200 instances of Explication across the 36 Episodes, Clarifies being the most occurring mode, followed by Affirms, Tells, and Explicates unknowing being the least occurring (see Appendix 9).

5.1.2.2.1 Clarifies

Clarifying knowledge for another (referred to as Clarifies) occurs when a participant acts to help another participant make meaning of a mathematics concept during knowledge-building interactions. The extract from Episode 23 below is an example of this mode of Explication.

Extract 5.11 – Modes of Explication (Clarifies Knowledge) – Episode 23

Context: This was the second lesson on composite functions. The previous lesson ended with participants working out solutions to the questions in their booklets on the board. At the start of this lesson, as a continuation of the previous day's lesson, teacher participant James calls on Crimson, a student participant, to come to the board and show the class of participants how to solve a question on composite functions.

Part	**Line**	**Participant**	**Action/Reification**	**Code**
Knowledge Building	1	Crimson (SP) (writing on the board):	"So, when you're given this question, you always look to the one here (pointing) to the left."	Exp Clarifies
	3	Student A (SP):	"Yeah …"	
	4	Crimson (SP):	"So, if its gf you will look at the g because it's the one on the left, ok … always remember that, so whenever you get a question like this you want to find out what's here (pointing) in this case it's f, so we	Exp Clarifies

			know straight away that we will be using g ….”	
	5	Student A (SP):	“Yeah …”	
	6	Crimson (SP):	“… and putting it here …”	Exp Clarifies

In this extract, lines 1, 4, and 6 reveal Crimson's actions, including the reification of the mathematical working out on the board, and his dialogical interaction is aimed at helping the other participants to make meaning of the concept of composite functions. This action and reification by Crimson is to support the advancement of the mathematics knowledge of participants in the learning community.

5.1.2.2.2 Affirms

Affirms occurs when a participant acts to affirm the mathematics knowledge of another participant, to support the advancement of the other participants' existing mathematics knowledge. In the data from the thirty-six Episodes, Affirms always occurred in response to another participant seeking to extend their mathematics knowledge by seeking affirmation. Extract 5.6 from Episode 10, presented in section 5.2.1.5 above, is an example of the emergence of this mode. In lines 2 and 8, Student B and Student A sought to extend their existing mathematics knowledge by seeking affirmation from James, the teacher participant. In lines 3 and 9, James affirms their knowledge in a dialogical interaction. This interaction serves to affirm

Student A and Student B's existing knowledge; in this way, James advanced the quality of their mathematics knowledge by remedying their uncertainty.

5.1.2.2.3 Tells

Tells as a mode of Explication occurs when a participant offers up mathematics information as a statement to other participants to support existing mathematical knowledge, without explaining the mathematical principles that underpin the information, as exemplified in the extract below. This extract is again from Episode 6, suggesting the complexity of epistemic interactions in each Episode.

Extract 5.12 – Modes of Explication (Tells) – Episode 6

Context: Teesh, the teacher participant, showed participants how to use the quadratic formula to solve the question she had posed to them. Crimson, a student participant, was calling out the answers, and she was writing it on the board. The rest of the classroom participants were engaged in comparing their work to what was written on the board. This Episode was initiated by Teesh, the teacher participant, and the Intention was triggered by her Assumed unknowing. She Assumed that all or some of the participants did not have the knowledge required to solve the question she had posed to them. The Intention oriented towards Expertise, as Teesh sought, through the dialogical interaction, to control how Crimson explicated his knowledge.

Time	Line	Participant	Dialogue	Code
Intention	1	Teesh (TP):	"So, Crimson, what did you…?" (Assumed unknowing)	I (Xpt)
Knowledge Building	2	Crimson (SP):	"Do you want me to say the whole thing?"	
	3	Teesh (TP):	"Huh?"	
	4	Crimson (SP):	"Do you want me to give you the equation?"	
	5	Teesh (TP):	"Yes, tell me how you wrote it."	Xpt
	6	Crimson (SP):	"Minus five…"	Exp -Tells
	7	Teesh (TP):	"Minus five …" (writing on the board).	Xpt
	8	Crimson (SP):	"Plus five squared."	Exp - Tells
	9	Teesh (TP):	"Plus, five squared" (writing on the board).	Xpt
	10	Crimson (SP):	"Plus, and then the square root" (gestures square root in the air).	Exp - Tells

In this extract, Crimson, in lines 6, 8, and 10, offers up mathematics information as a statement to the teacher participants to support the advancement of the mathematics knowledge of the classroom participants, without explaining how or why he has arrived at this information. However, in the context of the Episode, Teesh, the teacher participant, repeats what Crimson says and shows the working out on the board, publicly checking and confirming the mathematics knowledge.

5.1.2.2.4 Explicates Unknowing

Explicates unknowing occurs when a participant makes explicit the unknowing of another participant to support the advancement of the other participants' existing mathematics knowledge. I exemplify this mode in the extract from Episode 1, which I repeat below.

Extract 5.13 – Modes of Explication (Explicates) – Episode 1 Context: The question $2x^2 + x - 21 = 0$ was placed on the board by teacher participants Deepz and Jevonte for the student participants to solve. This action was at the start of the second lesson on factorizing quadratic equations. Pearl a student participant initiated the Episode with the dialogue interaction in line 1.

Part	Line	Participant	Action/Reification	Code
Intention	1	Pearl: (SP):	"How can we use the same method with the x?" (Identified unknowing)	I (Ext) Questions
Knowledge Building	2	Deepz (TP):	"It's the same thing that we did yesterday."	Xpt
	3	Student A (SP):	"But what do you times together to get x?"	Ext Questions
	4	Deepz (TP):	"You do twenty-one times minus two equals minus forty-two."	Exp Clarifies
	5	Pearl (SP):	" No, no … you see how we split it; what do we split the x?"	Ext Questions
	6	(Inaudible chatter between Teesh, Pearl, and Student A)		MR
	7	Teesh (SP):	"Oh, I see what you mean … Deepz, you know what she's trying to say? You see how there's usually a number in the middle; she's saying, how do you split it if there's only an x?"	Exp Explicates Unknowing
New Knowledge	8	Jevonte (TP):	"There's a one in front of it."	Exp Clarifies
		Deepz (TP):	"So, it's one basically; x is one."	

In this extract, Pearl, in lines 1 and 3, Questions to extend her existing knowledge. In line 4, Deepz, the teacher participant, Clarifies in response to Pearl. However, this Explication received by Pearl did not extend her existing knowledge, as neither Deepz nor any of the other participants engaged in the epistemic interaction offered the desired Explication that would enable Pearl to solve the quadratic equation. Pearl wanted to know the coefficient of the x in the equation. She did not realise that the coefficient of the x was one. It took Teesh in line 7 to make Pearl's unknowing explicit, and it was after Teesh had made this unknowing explicit that Jevonte, the teacher participant, could resolve the unknowing in line 8.

5.1.2.2.5 Summary

The four modes of Explication by which participants make their mathematics knowledge explicit to another participant are summarised in Table 5.4 below.

Mode of Explication	**Description**
Clarifies	Makes mathematics knowledge meaningful for another
Affirms	Affirms the knowledge of the participant who Seeks affirmation General response is "Yes" or "No"
Tells	States mathematics information Does not explain the mathematics underpinning the information
Explicates unknowing	A participant makes another's unknowing explicit. Aids the advancement of another's knowledge

Table 5.4 – Modes of Explication Summary

5.1.2.3 Modes of Expertise

Expertise is the characteristic of shared epistemic agency by which participants direct their agency towards expressing process authority (Oyler, 1996 p. 6) in the classroom community. The participant takes control of the learning culture of the classroom (see section 2.3.2), including of how the learning is to take place and of the learning behaviours of the participants. The three distinct modes of Expertise employed by the participants are Controls learning behaviour, Checks current knowledge and Manages learning resources. These will be referred to as Controls, Checks and Manages for brevity. I identified over 130 instances of these three modes of Expertise across the 36 Episodes, with Controls occurring the most, followed by Checks, and Manages being the least occurring (see Appendix 9).

5.1.2.3.1 Controls

Controls learning behaviour (referred to as Controls) occurs when a participant assumes authority over how knowledge is advanced in the classroom community, including how other participants behave, to ensure that the mathematics knowledge of all participants is advanced. This authority includes such functions as controlling the pacing and sequence of the lesson in line with the pre-prepared lesson plan. For example, in Episode 8, Teesh said, “I’m going to start moving on because you people are taking long” (27:41). Teesh, the teacher participant, wanted the class to finish solving the question on the board independently so that they could go over it together as a class. The statement shows her in control of the pace.

In other Episodes, such as in Episode 1 Extract 5.1 above, in line 2, Deepz responded to Pearl's Extension with the statement "it's the same thing that we did yesterday." By this statement he was attempting to control Pearl's learning process. The statement reified what the school considered good practice; that is, Deepz encouraged her to go back over the previous day's work and make an effort to remember what she had learnt previously. A similar example is Episode 6 Extract 5.10 above: in line 5, Teesh, the teacher participant, controlled how Crimson presented his mathematics knowledge to the class. In this way, she controlled the learning behaviour.

5.1.2.3.2 Checks

Checks current knowledge (referred to as Checks) occurs when a participant inspects the current mathematics knowledge of another participant in order to ensure that the process of knowledge advancement is taking place. This is exemplified in the extract from Episode 25 below.

Extract 5.14 – Modes of Expertise (Checks) – Episode 25

Context: This is the second lesson on algebraic functions. Jayzee And Beyoncé are seated next to each other, working on composite functions. This extract from the Episode shows how Beyoncé takes on responsibility for Jayzee's knowing, by checking Jayzee's current knowledge at each stage of her Explication.

Time	**Line**	**Participant**	**Action/Reification**	**Code**
Knowledge Building	1	Beyoncé (SP):	"I don't know if this is right, but this is what I did" (she puts her booklet in between them and points to her working out).	Exp Clarifies
	2	Beyoncé (SP):	"You see how x is first" … (she pauses) ... "yeah?" (she looks up at Jayzee).	Exp Clarifies. Xpt Checks
	3	Jayzee (SP):	(Nods her head showing agreement.)	
	4	Beyoncé (SP):	(Beyoncé continues with her explanation) "and this is second …" (she looks up at Jayzee again).	Exp Clarifies
	5	Jayzee (SP):	(Nods in agreement.)	
	6	Beyoncé (SP):	"…yeah?"	Xpt Checks

	7	Beyoncé (SP):	"in this one, x is first" (she pauses and looks at Jayzee, who does not nod in agreement).	Exp Clarifies Xpt Checks
	8	Beyoncé (SP):	"You see how x is first?" (she pauses and looks at Jayzee, Jayzee doesn't nod in agreement).	Exp Clarifies Xpt Checks
	9	Beyoncé (SP):	"What don't you get?"	Xpt Checks
		(Explanation continues.)		
	11	Beyoncé (SP):	"What don't you get, like where?"	Xpt Checks
	12	(Explanation continues.)		
	13	James (TP) walks up to them and stands behind, listening to Beyoncé's (SP) explanation.		Xpt Checks
	14	Beyoncé (SP) (to James):	"Am I right?"	Ext Seeks Affirmation
	15	James (TP) nods in agreement and continues to listen.		Exp Affirms
	16	James (TP) walks away to another student.		Xpt

In this extract, Beyoncé, in lines 2 and 6, checks Jayzee's knowledge using the dialogical interaction of "yeah?", and awaits a response from Jayzee. The word “yeah”, with an interrogative tone, was used by Beyoncé to check that the knowledge advancement due to her Explication was taking place. In lines 4, 7, and 8, Beyoncé checks Jayzee’s knowledge by the physical action of looking at her and waiting for a nod. The physical action of looking at Jayzee is a reification of the implicit phrase “yeah, does the explanation make sense to you?”. In lines 9 and 11, Beyoncé explicitly asks what Jayzee does not understand.

5.1.2.3.3 Manages

Managing learning resources (referred to as Manages) occurs when a participant manages the resources that help advance mathematics knowledge in the classroom community. The resources include the concrete learning resources such as the interactive whiteboard, PowerPoint lesson plans, equipment such as worksheets and booklets, and human resources such as myself, the classroom teacher. This mode of Expertise demonstrates how the teacher participants direct their agency towards utilising resources to advance community knowledge. This mode of Expertise is not commonly observable within an Episode, as it involves processes external to the lesson that set up the learning. The extract below that exemplifies the mode is taken from the start of a lesson and not an Episode.

Extract 5.15 – Modes of Expertise (Manages) – Recording 7

Context: This takes place at the beginning of a lesson. Crimson, the teacher participant, arrives before the other participants and sits at the teacher's table. Beyoncé, the second teacher participant, along with another participant, arrived next, stood by the teacher's table, and proceeded to engage in conversation while others walked directly to their usual seating positions and sat down.

Time	Line	Participant	Action/Reification	Code
1.22	1	Me:	"Is there a reason why there is a delay? Can I do something?"	
	2	Crimson (TP):	"No, no, no, it's fine, its fine"	Xpt– Controls
	3	(I walk up to the teacher's desk, and conversation ensues between Crimson and me.)		
	4	Crimson (TP):	"All right, guys, can you get your books out, your green book and your normal book."	Xpt – Controls
	5	Deepz (SP):	"Don't we need a booklet or something?"	
	6	Crimson (TP) (to Daniel, who had been	"You sit down …". (Inaudible. Daniel goes to sit down.)	Xpt – Controls

		standing at the teacher's desk):		
	7	Crimson (TP):	"Ms Mezue is going to hand out the booklets."	Xpt – Manages
	8	Me:	"No, everyone has their booklets."	
	9	Crimson (TP):	"Everyone has their booklet."	
	10	Deepz (SP):	"I handed mine out to someone yesterday."	
	11	Crimson (TP):	"Well, sorry, everyone has their booklet if you don't have yours well …"	Manages
	12	(Crimson manipulates the wall plug and wires in a bid to get the white board to function.)		Xpt – Manages
	13	Deepz (SP):	"I wasn't given a booklet	
	14	Crimson (TP):	"What do you mean you weren't given a booklet?"	
	15	Deepz (SP):	"cos I wasn't here last Friday, remember last Friday …	
	16	Crimson (TP):	"Well, you have to ask Ms. Mezue."	

	17	Crimson (TP):	"Get your green books and your normal books out?"	Xpt – Controls
	18	Deepz (SP) (to me):	"Miss, I wasn't here on Friday when they gave out the booklet; I wasn't here last Friday."	
3:26	19	Crimson (TP):	"Copy down the title."	Xpt – Controls
	20	Pearl (SP):	"Just the title?"	
	21	Crimson (TP):	"If you want to copy down the others …"	Xpt – Controls
	22	Student A (SP):	"Crimson, Can I have paper, please?"	
	23	Crimson (TP):	"Paper" (he gives her paper).	Xpt – Manages
6:00	24	(I return to the class with a booklet.)		
	25	Crimson (TP) (to me):	"Miss, can you sit down please and do the work."	Xpt – Manages
	26	(I give the booklet to Deepz and sit down. While Crimson goes around to check on the questions, the participants are working on from the board.)		Xpt – Checks

In this extract, Crimson, in lines 11, 12, 23, and 24 manages the concrete resources (booklets, paper, the whiteboard) necessary for the learning process to take place in the mathematics classroom. In lines 7 and 25, he manages the human resources (me). In line 7 he directs me to give out the booklets and in line 25 he directs me to sit down and do the work as a student participant. Other examples of participants managing concrete resources to enable the process of learning include in Episodes 3 and 4, in which the teacher participant prepares the whiteboard so that a student participant can show the class how to solve a mathematics question. In Episode 11, in which the teacher participants James and Adam each manage the PowerPoint when the other is explicating a mathematics concept to the classroom community, is another example. Lastly, a further example is evident in Episode 14, in which student participant Pearl assumed the role of teacher participant and supported Tom with his PowerPoint presentation, as his partner Beyoncé was absent on the day.

5.1.2.3.4 Summary

The three modes of Expertise, by which participants directed their agency towards expressing process authority in the classroom community, are summarised in Table 5.5 below.

Mode of Expertise	**Description**
Controls	• Controls how participants learn such as; ○ The pace of learning ○ Participants learning process
Checks	• Checks participant's current knowledge
Manages	• Manages concrete resources • Manages human resources

Table 5.5 – Modes of Expertise Summary

5.1.2.4 Mutual Relations

Mutual Relations is the characteristic of shared epistemic agency that highlights the ways participants channel their agency towards relating with other participants in the classroom community. Actions and reifications coded as Mutual Relations can be contextual or non-contextual, as well as being conducive or non-conducive to the advancement of mathematics knowledge

5.1.2.4.1 Mutual Relations as Contextual or Non-contextual

The identification and interpretation of Mutual Relations requires an internal perspective on the context and the participant expressing it. As an ethnographic participant observer, I bring to this part of the study my awareness of the flexible interpersonal relationships in my classroom. Various actions and reifications can correspond to the same Mutual Relation, while the same action or reification can correspond to multiple distinct relations. For example, in Episode 9 (see Extract 5.2), where I considered the actions and reifications of James to be solidarity in line 2, the physical interaction of James turning towards Student A, and the dialogical interaction of the word “Yo!” shows James’ solidarity with Student A. Using the informal and affectionate address “Yo!” meant that even though James was the teacher participant, and Student A was the student participant, they were both participants in learning mathematics. This act of solidarity encouraged Student A to come forward and make a contribution knowledge to the community’s knowledge.

A participant could demonstrate solidarity through other actions or reifications, such as in Episode 19 (see Extract 4.1), in which Daniel’s physical actions, described in lines 2 and 14, of placing his hand on Tom’s shoulder also showed solidarity within the context of that Episode and the relations Daniel had with Tom, regardless of the fact that Daniel was inferring, in line 2, the possibility that Tom might be incorrect in his mathematics solution. The placing of his hand on his shoulder acted to soften the dialogical interaction in line 1, allowing both teacher participants to resolve the unknowing.

These actions and reifications are Individual and contextual because it was not just the hand on the shoulder that identified the Mutual Relation; the context of the action was part of the identification, as placing the hand on a participant's shoulder could also be viewed as an act of aggression in another context and between different participants.

Trust was also demonstrated in various ways across episodes. For instance, in Episode 9 (see Extract 5.2), in line 1, while James, the teacher participant, is introducing the lesson, Student A calls out from the back of the class. The dialogical interaction of the call showed the relation of trust between Student A and James. Student A trusted that she could call out and be listened to. The strength of this trust is recognised in relation to the conventional classroom that Student A and James experience in most other subjects. In these classroom environments, students seek permission to speak out or to make a contribution. In contrast, Student A calls out from the back of the class and starts to walk towards the front of the class even before James responds. This trust enabled her to proceed to initiate an Episode; in this way, it was conducive to the advancement of knowledge. Trust was also shown by the actions of James and Daniel in Episode 29 (see Extract 5.5). In line 3, both student participants come up to the board to extend their mathematics knowledge. Daniel had directed his agency toward Extension as he Questions Deepz in line 1. When Deepz, the teacher participant, went to the board to explicate the mathematical concept of bounds, Daniel and James followed him. This physical interaction showed their trust in the learning community, that they could act in whatever ways they needed to in order to extend their mathematics knowledge, including going up to the board without the permission of the teacher participant. Their trust was conducive to the advancement of their mathematics knowledge and that of the other participants who were listening.

Not all Mutual Relations were contextual to the participants and the Episode. For example, in Episode 2 (see Extract 5.17), after Crimson Tells in line 5, Teesh responds with the dialogic interaction “Smart. It is!” This constitutes a positive reinforcement, directed at a member of the learning community, that would hold the same meaning in any Episode with any of the participants. Similarly, in Episode 30 (see Extract 5.18), in line 14, Deepz’ dialogical interaction “Everyone makes mistakes, that’s why we’re here” is a motivational message that holds the same meaning regardless of the context.

Nevertheless, as a result of the contextual nature of most actions and reifications coded as Mutual Relations, I did not see fit to categorise the actions and reifications that made this characteristic visible into modes.

5.1.2.4.2 Mutual Relations as Conducive or Non-Conducive for the Advancement of Knowledge

Mutual Relations are said to be conducive when the corresponding actions and reifications contribute to the advancement of mathematics knowledge, and non-conducive when they do not contribute to the advancement of mathematics knowledge.

The previous section addressed how Mutual Relations can be conducive to the advancement of mathematics knowledge. An illustration of how Mutual Relations can be non-conducive can be found in Episode 19 (see Extract 4.1). As Daniel, Tom, and Crimson interacted in lines 2-10, Roan, who was seated next to Crimson, had been listening. In line 11, Roan stood up and made a comment. Daniel responded in line 12 with the dialogical interaction, “I wasn’t talking to you, sit back down”. This utterance is rude in any context, and reveals the presence of undesirable Mutual Relations that are non-conducive to the advancement of mathematics knowledge, as it clearly prevented Roan from offering a contribution. Table 5.6 below outlines the Mutual Relations that I identified across Episodes, and the total number of observed actions and reifications corresponding to Mutual Relations that were conducive or non-conducive to the advancement of mathematics knowledge.

Mutual Relations conducive to knowledge Advancement	Mutual Relations non-conducive to knowledge advancement
Equity, solidarity, persistence, respect, empathy, trust, helpfulness, confidence acknowledgement	Rudeness, anger, frustration, rejection, distraction, disrespect
Total across Episodes: 90	Total across Episodes: 32

Table 5.6 – Classification of Mutual Relations

5.1.3 New Knowledge

I coded, as New Knowledge, the part of an Episode wherein mathematics knowledge emerges from knowledge building as a resolution of the unknowing that triggered the Intention. Given that it resolves an unknowing, this knowledge is new to the participants involved in the Episode, and is considered legitimate if it is able to resolve the unknowing for all participants involved in the Episode. I consider two issues in this section that arise from this fact, and which highlight the rigour of this study and its participants' agency. These two issues are: how the end of an Episode is indicated and how the New Knowledge is built. I note that it is not the emergence of the New Knowledge that brings the episode to an end, but rather, it is the participants acknowledging that the New Knowledge has resolved an unknowing, thereby advancing their mathematics knowledge, that brings the Episode to a close. I tracked back through each episode to find the source of the New Knowledge and observed that it results from the participants appealing to the mathematical principles themselves or to a knowledgeable participant.

5.1.3.1 Acknowledging the resolution of an episode

Acknowledging the resolution of an Episode takes the form of action and/or a reification. In Extract 5.13 (see section 5.1.2.2.4), the Episode ended with lines 8 and 9, when Jevonte Clarifies and resolves Pearl's unknowing with the dialogic contribution, "There is a one in front of it", and when Deepz confirmed this in line 9 with the statement, "So it's one basically". This brought the Episode to an end, as Pearl and Student A then had the knowledge required to solve the quadratic equation. They did not direct their agency towards further Extension, but rather proceeded to solve the equation. In Episodes in which more participants were involved, all participants acknowledged that the unknowing has been resolved to bring the Episode to an end. For example, in Extract 4.2 (see section 4.1.2.4), the Episode entered its final stages when Deepz, in line 10, said "Oooh, that's smart", expressing his appreciation of Student A's New Knowledge. This was followed by other participant's dialogic interactions, "Ah", in line 11, expressing their acknowledgement of the New Knowledge. However, the Episode does not end until line 18, when the teacher participant Checks by asking, "Everyone understands that?", and, on receiving acknowledgement in line 19, resumes his explanation. He had paused this to allow Student A to present her New Knowledge, and to confirm that the rest of the participants shared this knowledge too.

Thus, acknowledging the impact of New Knowledge, the end of an Episode can be seen to occur when participants no longer direct their agency towards further Extension. This is also exemplified in Episode 6, in which the Intention was triggered by an Assumed unknowing (see Extract 5.12). Teesh, the teacher participant, was consequently presenting the step-by-step process for solving quadratic equations using the quadratic formula to all participants. From the extract of the end of this same Episode (see Extract 5.16 below), the Episode came towards its conclusion in line 36, when student participant Crimson Tells a value of x. In line 35, Teesh then Checks with the dialogic Interaction, “Who else got this?”. The Episode ends in line 42, when no further participants direct their agency towards Extension, indicating that all unknowing has been resolved. A new question was then placed on the board for all participant to try.

A reification, such as clapping, can indicate participants acknowledging the New Knowledge, thus the resolution of the Intention staged in Episode 11 (see section 5.2.3.2). The Episode neared its conclusion when participants recognised the connections between solving linear equations and solving linear inequalities and several participants said, “ah” which reified the resolution of their unknowing. This was followed by some participants clapping, some saying “ok”, and Student A, who taught an earlier lesson on solving linear equations, saying, “That’s what I taught you guys”. The Episode ends when no participant directs their agency towards Extension. The teacher participants then moved onto a slide with new inequality problems for the participants to solve.

5.1.3.2 Building New Knowledge as Dimensions of Appeal.

New Knowledge is developed through participants’ interactions during the knowledge-building phase of an Episode. I tracked back through the whole of each Episode to find the source of the New Knowledge; I ascertained that it is concretely produced during knowledge-building by participants' appeals to conceptual knowledge, a knower, or procedural knowledge. These appeals could be viewed as dimensions, as some Episodes involve a combination of two or more appeals.

5.1.3.2.1 Appeal to Conceptual Knowledge

The appeal to conceptual knowledge resolves an unknowing by the implicit or explicit understanding of the principles governing a domain of mathematics (cf. Rittle-Johnson & Alibali, 1999). Episode 1 (see Extract 5.13 above) exemplifies this resolution of an Episode.

In line 1, Pearl, a student participant, identifies her unknowing. The question that the teacher participants Jevonte and Deepz had placed on the board was:

$$2x^2 + x - 21 = 0$$

In the knowledge-building part of the Episode (lines 2-8), Teesh explicated Pearl's unknowing, making it accessible to the other participants. Subsequently, in line 9 Jevonte resolved the unknowing by resorting to the conceptual understanding of a principle of algebra, according to which, when a variable has a coefficient of one, the digit "1" is not written. Explicating this knowledge was how Jevonte resolved Pearl's unknowing in line 9, and allowing her to factorise the quadratic equation. This resolution brought the Episode to an end.

An appeal to conceptual knowledge also resolved Episode 9 (see Extract 4.2), where the implicit understanding of the domain of inequalities and the explicit understanding (see Figure 5.2 below) of mnemonic device resolved the Assumed unknowing that triggered the Intention.

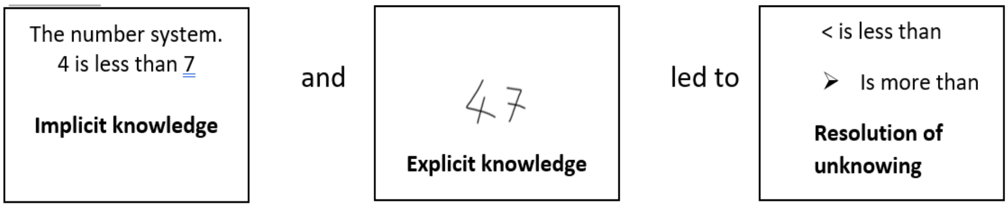


Figure 5.2 – Appeal to conceptual knowledge in Episode 9

5.1.3.2.2 Appeal to a Knower

The appeal to a knower resolves an unknowing by means of the mathematics knowledge possessed by a particular participant. This has the coincidental effect of validating the participant as knowledgeable (cf. Wagner & Herbel-Eisenmann, 2014). The second paragraph of section 5.1.3.1 (acknowledging the resolution of an Episode), in which the resolution of Episode 6 is described, exemplifies how New Knowledge is built through an appeal to a knower. The Episode's Intention, shown in Extract 5.12 above, was triggered by Teesh's Assumed unknowing. She assumed that some participants did not have the mathematics knowledge required to solve quadratic equations by using the quadratic formula. Teesh, the teacher participant, asked Crimson, a student participant, to provide the step-by-step process he used to solve the quadratic equations using the quadratic formulas to all participants. Extract 5.16 shows Teesh writing Crimson's solution on the whiteboard, and demonstrates her appeal to Crimson as a knower.

Extract 5.16 – Appeal to a Knower – Episode 6

Context: Teesh showed how to use the quadratic formula to solve the question she had posed to the class. Crimson, a student participant, calls out the answer, and Teesh, the teacher participant writes it on the board. Student participants were engaged in comparing their work to what Teesh was writing on the board.

Part	Line	Participant	Action/ Reification	Code
Knowledge Building	10	Crimson (SP):	"Plus, and then the square root" (gestures square root in the air).	Exp-Tells
	11	Teesh (TP):	"Yeah."	Xpt-Controls
	12	Crimson (SP):	"Five squared."	Exp-Tells
	13	Teesh (TP):	"Yeah."	Xpt-Controls
	14	Crimson (SP):	"Minus four."	Expertise-Tells
	(The epistemic interaction continues, with Teesh acknowledging each step.)			
	15	Crimson (SP):	"Over two times six."	Exp-Tells
	16	Student A (SP):	"Twelve, basically."	Exp-Clarifies

13:59	17	Teesh (TP):	“And what did you get?”	Xpt-Checks
	18	Crimson (SP):	“Err, zero-point-two-nine-five-three-three-three.”	Exp-Tells
	19	Student A (SP):	“He’s chatting rubbish!”	MR
	20	Crimson (SP):	“This is” (inaudible).	
	21	Jayzee (SP):	“To two decimal places.”	
14:07	22	Crimson (SP):	“Miss said write the whole thing then do two decimal places.”	Exp-Clarifies
14:39	See section 2.1.4			
15:35	23	Crimson (SP) (to Student A):	“You did two minuses, Student A.”	Exp-Clarifies
15:36	24	Crimson (SP):	“It’s not minus five, its ordinary five” (checking her calculation).	
15:53	25	Crimson (SP);	“Yes, you got zero-point-three.”	Exp-Tells
	26	Crimson (SP) (publicly to Teesh):	“Yes, Student A got it.’	

		27	Teesh (TP):	"Can someone tell me what they got for the minus one?"	Xpt-Controls
		28	Crimson (SP):	"The minus one is one point …"	Exp-Tells
		29	Daniel (SP):	"Wait, slow down, slow down."	Ext-Requests
		30	Teesh (TP):	"Tell me, how did you put it."	Xpt-Controls
		31	Crimson (SP):	"In the same way."	
		32	Teesh (TP):	"Tell me, then."	Xpt-Controls
		33	Crimson (SP):	(sighs) "Minus five, minus five squared …"	Exp-Tells
		34	Teesh (TP):	"Yeah"	Xpt-Controls
			(The epistemic interaction continues.)		
16:56		35	Teesh (TP):	"So, what did you get?"	Xpt-Controls
17:29		36	Crimson (SP):	"Minus one-point-one-two-six-six-nine-seven-nine."	Exp-Tells
		37	Teesh (TP):	"Who else got this?"	Xpt-Checks
		38	Student A (SP):	"Wait, Teesh, wait, Teesh, let me clarify what Crimson wrote" (as she works out on her calculator).	Ext-Requests
		39	Pearl (SP):	"Jayzee, why, what was your problem?"	Xpt-Checks
		40		(Jayzee discusses with Crimson.)	

		41	Pearl (SP):	(Walking towards Jayzee from where she was seated at the teacher's desk) "Let me tell you what the problem is" (she holds out her calculator and explains the problem to Jayzee).	Xpt-Checks
18:30 New Knowledge		42	Pearl (SP):	(Pointing to the new slide on the board) "Try these ones."	Xpt-Controls
		43	Teesh (TP):	"Try these ones."	

In Extract 5.16, Crimson's epistemic authority resolved Teesh's Assumed unknowing that triggered the Episode. Lines 10 to 36 show the knowledge-building epistemic interactions between participants, in which Crimson has epistemic authority. The step-by-step explanation of his process, which was written on the whiteboard for all participants to see by Teesh, the teacher participant with process authority, resolved the Assumed unknowing in which Teesh was suspended, facilitating the learning of the other classroom participants in their solving of quadratic equations using the quadratic formula.

5.1.3.2.3 Appeal to Procedural Knowledge

The appeal to procedural knowledge resolves an unknowing by executing action sequences for solving mathematics problems (cf. Rittle-Johnson, 2017; Rittle-Johnson & Alibali, 1999). This resolution of an Intention is exemplified in Episode 2; see Extract 5.17 below.

Extract 5.17 – Appeal to Procedural Knowledge – Episode 2

Context: The lesson started with the teacher participants requiring the student participants to factorise the equation $2x^2 + x - 21 = 0$. This Episode was initiated by Crimson, a student participant. The episode's Intention was triggered by his Identified unknowing of the method for factorising quadratic equations. His dialogical interaction in line 1 oriented the Intention towards Extension. Seeking confirmation from Jevonte, the teacher participant, of the procedure for factorising quadratic equations is evidence of Crimson's uncertainty.

Part	**Line**	**Participant**	**Action/Reification**	**Code**
Intention	1	Crimson (SP):	"Jevonte, Jevonte."	I
	2	Jevonte:	"Yeah?"	
	3	Crimson (SP):	"So, it has to add to make one and times to make minus forty-two?"	I(Ext) Seeks affirmation
	4	Jevonte:	"Yeah."	Exp-Affirms
4:09	5	Crimson (SP):	"Seven and minus six … "	Exp-Tells
	6	Teesh (SP):	"Smart. It is!"	MR
	7	(Incoherent chat, with many voices agreeing and giving their solutions.)		

4:30	8	Crimson (SP):	"It has to add to make minus forty-two, so it will be minus six."	Exp-Clarifies
	9	Pearl (SP):	"Not add."	Ext-Challenges
	10	Crimson (SP):	"It's minus six, minus six plus one equals minus six."	Exp-Clarifies
	11	Pearl (SP):	"What!"	Ext-Challenges
	12	Crimson (SP):	"Add minus six plus seven."	Exp-Clarifies
	13	Pearl (SP):	"Minus six plus seven doesn't give you forty-two, though!"	Ext-Challenges
	14	Crimson (SP):	"Minus six times seven gives you minus forty-two."	Ext-Clarifies
	15		(More chatter.)	
5:04	16	Teesh (SP):	"Crimson, tell me what you said."	Ext-Requests
New Knowledge	17	Pearl (SP):	"What?"	
	18	Crimson (SP):	"Minus six times positive seven makes minus forty-two."	Exp-Clarifies
	19	Pearl (SP):	"Is that not what I said?"	
	20	Deepz (TP):	"Crimson, Crimson, it's correct."	

This extract depicts the resolution of an unknowing by an appeal to procedural knowledge. Crimson sought to extend his existing knowledge in line 3 by asking the epistemic question, "So, it has to add to make one and times to make minus forty-two?" The question he asked concerned the "how" of factorisation: the order of the product and the sum in relation to the coefficients of a, b, and c in the quadratic equation $ax^2 + bx + c = 0$.

In the ensuing knowledge-building interaction, the students had to use their knowledge of multiplication, addition, manipulation of negative numbers, and factors to find two numbers that multiplied to produce -42 and add to produce +1. The discovery of the two numbers satisfied the procedure, thus resolving the Intention and ending the Episode; it was correctly applying a procedure to arrive at the solution that constituted the New Knowledge.

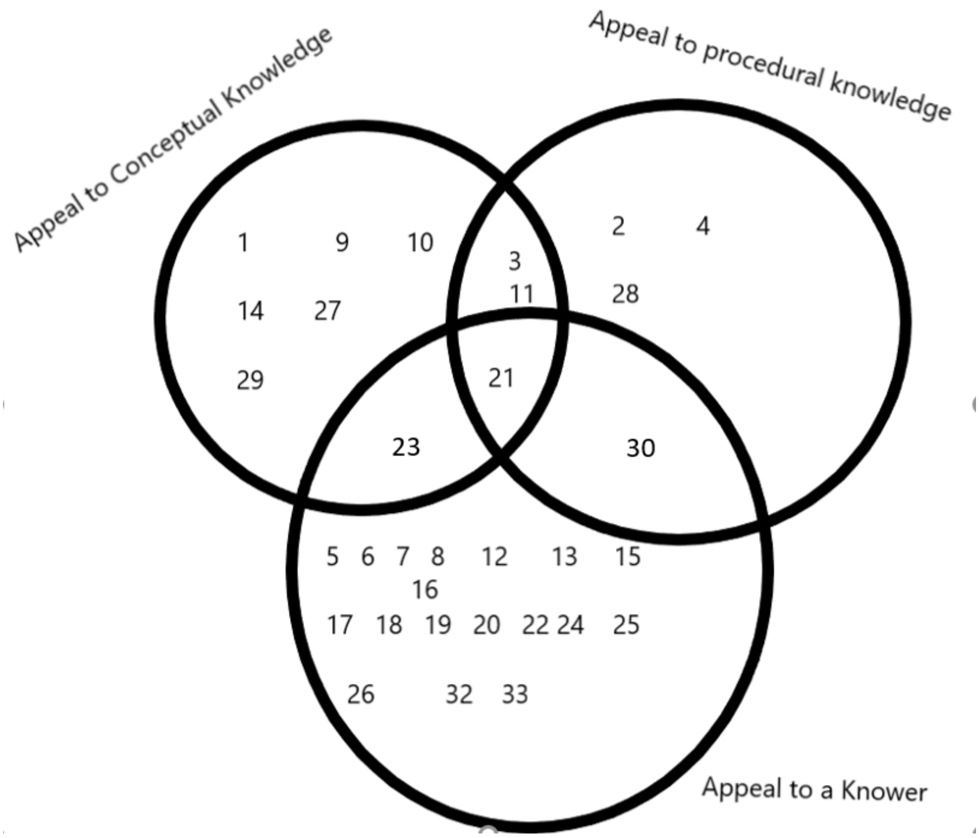


Figure 5.3. – New Knowledge – The dimensions of appeal in all Episodes

I have illustrated each of the three types of appeal; however, Figure 5.3 that shows that in some Episodes there are a combination of two or more types of appeal. In the next section I will illustrate this combination of appeals with a single Episode.

5.1.3.2.4 Appeal to a Knower and Procedural Knowledge

As previously stated, New Knowledge can result from a combination of appeals during the knowledge-building part of the Episode, as is illustrated in Extract 5.18 from Episode 30 below.

Extract 5.18 – Appeal to a Knower and Procedural Knowledge – Episode 30

Context: This is the second lesson on Bounds. Jevonte, a student participant, had volunteered to work out question 3a on the board. As he was writing on the board, Crimson initiated the Episode. His Intention was triggered by an unknowing he had identified on the board. His dialogical interaction and physical interaction of pointing oriented the intention towards Expertise.

Part		Line	Participant	Action/Reification	Code
Intention		1	Crimson (SP):	(Pointing to board) "He's doing it wrong" (Identified unknowing).	I (Xpt) Checks
		2	Deepz: (TP)	"Who, who?"	
		3	Student A (SP):	"Apparently, you're wrong Jevonte!"	Xpt-Checks
	Knowledge Building	4	Jevonte (SP):	(turns from the board) "Who?"	

		5	Student A (SP):	"Apparently, you're wrong, duh."	Xpt
		6	Jevonte (SP):	Jevonte: "How …?"	Ext-Questions
		7	Deepz (TP):	"Listen, let him have …"	Xpt-Controls
		8	Jevonte (SP):	"I literally just wrote out the box yeah, what do you expect me to do?"	
		9	Student A (SP):	"Yeah, apparently, its wrong."	Xpt
		10	Jevonte (SP):	"How is it wrong?"	Ext-Questions
New Knowledge		11	Crimson (SP):	"It's not zero-point-five, it's two-point-five. It is to the nearest five metres; you have to do five divided by two which is two-point-five. Fifty-three is right."	Exp-Clarifies
		12	Pearl (SP):	"Read the question, read the question."	Xpt-Controls
		13	Deepz (TP):	(Goes to the board and speaks to Jevonte then turns to Crimson) "You're right, you're right."	Xpt-Checks
		(Interaction continues.)			
		14	Deepz (TP):	"Everyone makes mistakes, that's why we're here."	MR-Motivation

		15	Daniel (SP):	“Why is it one-hundred-and-two-point-five?	Ext-Questions
		16	Jevonte (SP):	“Ask him,” (points to Crimson) “that’s what he told me.”	
		17	Daniel (SP):	“Why are you listening to him?”	
		18	Deepz (TP):	“It’s right.”	Xpt-Checks
		19	Crimson (SP):	“It’s right, it’s to the nearest five metres, so you divide by two.”	Exp-Clarifies
		20	Deepz (TP) (to Jevonte at the board):	“Right, to the nearest five metres you do five divided by two, so the lower bound is one-hundred-and-two-point-five.”	Xpt-Controls
		21	Pear (SP):	“Wait ... Shouldn’t it be one-hundred-and-three-point-five?”	Ext-Questions
		22	Deepz (TP):	“No, you are taking away two-point-five.”	Exp-Clarifies
		(Discussion continues.)			
		23	Deepz (TP):	“Who doesn’t understand It?”	Xpt-Checks
		24		(Inaudible response.)	
		25	Deepz (TP);	“What bit don’t you understand? Do you understand why we did five divided by two? Do you understand that?”	Xpt-Checks
		26		(Inaudible response.)	

	New Knowledge	27	Deepz (TP) (to the class):	"Say if we are trying to the nearest 8 metres what will we do?"	Xpt-Checks
		28	Pearl (SP):	"Twelve, you add and subtract four."	Exp-Tells
			Deepz (TP):	"Yes, Who has done 3b?"	

In Extract 5.18, the New Knowledge was realised by Crimson in line 11. Crimson, as an epistemic authority, Clarifies the relevance of the phrase 'the nearest five metres'. This New Knowledge was the result of an appeal to Crimson as a knower, and had the potential to resolve an unknowing. However, the Episode did not end, as not all participants as yet acknowledged the resolution of Crimson's Identified unknowing; knowledge building was still underway for the rest of the class. Further acknowledgement was as a result of an appeal to procedural knowledge, that is, to the adding and subtracting after dividing by 2 as illustrated in line 19. In line 20, Deepz, the teacher participant, asked Jevonte to write it on the board for all participants to copy down. It was this appeal to procedural knowledge that allowed all participants to acknowledge the resolution of the unknowing bring about the end of the Episode. The New Knowledge was, therefore, ultimately the result both of Crimson Explicating his mathematics knowledge as a knower and Deepz appealing to the procedural knowledge of adding and subtracting following a division by 2, both of which together allowed the other participants to acknowledge the resolution of the unknowing.

5.2 Elaborating on Participants’ Interactions

In this section, I elaborate on the findings that emerged from the interactions between participants in the mathematics classroom as they directed their agency towards the learning of mathematics. As stated in the introduction, this research addresses the emergence of shared epistemic agency amongst the participants of my classroom as they enacted my innovative pedagogy. The framework established through my literature review suggested that shared epistemic agency was encapsulated by the six characteristics summarised in section 2.4.3. However, I was particularly interested in the interactions of these characteristics as participants enacted the innovative pedagogy, and how the interactions highlighted what was unique about the participation in my classroom as an indicator of how the research was meeting the aims of the study. To do this, I focused on three features of the pedagogy indicated in the literature (see section 2.3.2), considering the positioning of participants during interaction, how process authority manifested in the classroom, and how participants’ epistemic authority impacted on the way mathematics knowledge was advanced in the classroom.

5.2.1 Positioning

As participants interacted in the classroom for the purpose of learning mathematics, they assigned positions to themselves and to other participants in the process. Positioning can be descried as the discursive process by which speech and action are used to arrange people in social structures through locating them in conversations as participants in jointly-produced ongoing repertoires that are elements of a shared culture, or which can be invented as participants interact (see section 2.3.2.2). Positioning someone establishes what others must do for them or what they must do for others. Positions differ from roles; while positions are context-specific and flexible, as participants can occupy more than one position and shift between positions, roles are static in their interactions.

The roles of student participant and teacher participant have been hitherto used to distinguish between the functions of participants in each Episode as they enact the innovative pedagogy. However, as shown in the extracts used to elaborate upon the various modes, the knowledge-building characteristics of shared epistemic agency are not specific to these roles. Extension is not restricted to student participants; neither is Explication nor Expertise restricted to teacher participants, as one would expect when considering the conventional relationship between students and teachers. As exemplified in Extract 5.19 below and as is typical in other Episodes, enacting the innovative pedagogy made available to participants the positions of the learner, knower, and facilitator.

The position of learner is associated with Extension; the position of knower is associated with Explication; and the position of facilitator is associated with Expertise. I borrow the term “facilitator” from Kolb et al. (2014), and use it in the sense of actively supporting learning – that is, creating conditions that enable others to learn and removing obstacles that prevent others from learning (p. 7). The difference between a position and the associated characteristic of shared epistemic agency is that while a participant directs their agency toward one of these characteristics, the position is more than the actions and reifications of participants, and is also about how a participant is viewed by others during interaction, what others must do for them, and/or what they must do for others whilst in these positions. While a participant can direct their agency towards explicating their mathematics knowledge, and their actions or reifications will be coded as Explications, the positioning can occur prior to as well as during these a participant's actions and reifications, as they can be positioned by another participant before they begin speaking.

The findings demonstrate that a participant can be positioned interactionally, in a given moment, by others as a knower or as a facilitator. They can be positioned reflexively, in a given moment, by themselves as a learner, knower, or facilitator, and can be positioned institutionally by the pedagogy as a facilitator when ascribed the role of teacher participant. The interactions in Episode 2 best demonstrate these positions, as illustrated in Extract 5.19 below.

Extract 5.19 – Positioning – Episode 2

Context: The teacher participants, Jevonte and Deepz, positioned institutionally by the pedagogy as facilitators, had asked the student participants to factorise the equation $2x^2 + x - 21 = 0$.

Part	Line	Participant	Action/Reification	Code	Positioning
Intention	1	Crimson (SP):	"Jevonte, Jevonte" (Identified unknowing).	I (Ext)	Crimson positions himself as a learner. Crimson positions Jevonte as a facilitator. Jevonte has been positioned as a facilitator by the pedagogy.
	2	Jevonte (TP):	"Yeah"		Jevonte accepts the position of a facilitator.
	3	Crimson (SP):	"So, it has to add to make one and times to make minus forty-two?"	I (Ext) Seeks Affirmation	Crimson positions himself as a learner. Crimson positions Jevonte as a knower.
	4	Jevonte (TP):	"Yeah"	Exp Affirms	Jevonte accepts the position of a knower.
4:09	5	Crimson (SP):	"seven and minus six …. "		Crimson positions himself as a knower.
	6	Teesh (SP):	"Smart. it is!"	MR Non contextual conducive	Teesh positions Crimson as a knower.

	7	(Incoherent chat, many voices agreeing and giving their solutions.)			
4:30	8	Crimson (SP):	"It has to add to make minus forty-two, so it will be minus six."	Exp-Clarifies	Crimson positions himself as a knower.
	9	Pearl (SP):	"Not add?"	Ext-Challenges	Pearl positions herself as a learner.
	10	Crimson (SP):	"It's minus six, minus six plus one equals minus six."	Exp-Clarifies	Crimson positions himself as knower.
	11	Pearl (SP):	"What!"	Ext-Challenges	Pearl positions herself as a learner.
	12	Crimson (SP):	"Add minus six plus seven."	Exp-Clarifies	Crimson positions himself as a knower.
	13	Pearl (SP):	"Minus six plus seven doesn't give you forty-two, though!"	Ext-Challenges	Pearl positions herself as a learner.
	14	Crimson (SP):	"Minus six times seven gives you minus forty-two."	Exp-Clarifies	Crimson positions himself as a knower.
	15		(More chatter.)		

5:04	16	Teesh (SP):	"Crimson, tell me what you said."	Ext-Requests	Teesh positions herself as a learner, and Crimson as a knower.
New Knowledge	17	Pearl (SP):	"What?"		
	18	Crimson (SP):	"Minus six times positive seven makes minus forty-two."	Exp-Clarifies	Crimson positions himself as a knower.
	19	Pearl (SP):	"Is that not what I said?"		
	20	Deepz (TP):	"Crimson, Crimson, it's correct."	Xpt-Checks	Deepz positions himself as a facilitator. Deepz confirms Crimson in his position as a knower.

Extract 5.19 shows that positions are flexible as a participant can be positioned or position themselves as either a learner, knower, or facilitator from moment to moment. Crimson initiated the Episode as a learner, and by line 5 has positioned himself as a knower. In line 1, Crimson positions Jevonte as a facilitator, and Jevonte accepts this positioning in line 2. By seeking affirmation from Jevonte in line 3, Crimson positions him as a knower with epistemic authority. Having been positioned as a knower by Crimson, Jevonte accepts the position in line 4 and Affirms the latter's knowledge. This demonstrates how one participant can position another. In line 5, Crimson also starts to position himself as a knower, and in line 6, Teesh acknowledges this positioning. By line 16, Teesh, in seeking to extend her knowledge, positions Crimson as a knower; Deepz further confirms this positioning in line 20.

This interplay of positioning extends the possibility for describing participants as student participant learners (SL), student participant knowers (SK), student participant facilitators (SF), teacher participant learners (TL), teacher participant knowers (TK), or teacher participant facilitators (TF) in the context of a given situation within an Episode. Although I am referring to a participant who is learning, knowing, or facilitating, I am not reducing knowledge building to the individual experiences associated with these positions; rather, I am emphasising the publicly-recognised capacities and criteria for being a learner, knower, and facilitator. In this way, one's positioning is constituted by the community, or sections therein.

5.2.1.1 Positioning as a Learner

Enacting the innovative pedagogy, the participants positioned themselves as learners by seeking knowledge from an external source such as another participant, as shown in Extract 5.19 above. While, in theory, a participant can, for instance, be positioned as a learner by another participant, in the discourse of conventional pedagogy, a student is institutionally positioned by the teacher as a learner in the classroom by default. This positioning did not emerge in the practice developed in this study. Instead, while a participant could themselves assume the position of learner, a participant could not position another participant as a learner. A participant positioned as a knower in an epistemic interaction does not automatically confer the position of learner upon the other participants. The learner has to position themselves.

This reflexive positioning as a learner could show that in an epistemic interaction, being a learner or making the decision to learn in order to extend one's knowledge is a decision an individual makes for themselves in the moment. This difference in positioning, when compared with the situation in a conventional pedagogy, will be discussed further in chapter 6.

5.2.1.2 Positioning as a Knower

My analysis shows that positioning as a knower is based on who claims to be knowledgeable, or who represents themselves as having epistemic authority in a given moment. The data shows that if participant A is seeking knowledge from participant B, it can be taken to show that participant A sees participant B as knowledgeable, and therefore as a knower. This positioning contrasts with a further scenario in which participant A decides to share their knowledge with participant B; in this scenario, participant A sees themselves as knowledgeable, and positions themselves in their interactions with participant B as a knower.

5.2.1.3 Positioning as a Facilitator

The analysis of the video recordings showed that the participants of my classroom directed their agency towards expressing process authority (see section 5.2.3), and, in this way, facilitated the advancement of mathematics knowledge in the classroom community. The modes of Expertise – Controls, Checks, Manages (see section 5.1.2.3) – suggest the ways participants control the learning behaviours of participants in the classroom (see section 5.1.2.3.1), check their current knowledge (see section 5.1.2.3.2), and manage the learning resources (see section 5.1.2.3.3). These actions and reifications exemplify the participant as a facilitator of mathematics learning; by directing their agency towards expressing process authority, they facilitate the "how" of the learning in the classroom community.

As can be seen in Appendix 8, which presents the tally of the attestations of knowledge-building characterisations from my review of the video recordings, teacher participants were positioned or positioned themselves as facilitators twice as many times as student participants. This difference is evidently the result of the pedagogical measures that require the participants to take on the role of teacher participants, urging them to assume process authority while in this role. In the role of teacher participants, they are expected by the pedagogy (see section 3.1) to plan the structure of the lesson structure, and to take responsibility for how knowledge is made explicit and communicated to other participants so that their mathematics knowledge is advanced. Having been positioned by the pedagogy as a facilitator, teacher participants were also interactionally positioned as facilitators by other participants within the lesson, while, on occasion, student participants also reflexively positioned themselves as facilitators. There was no evidence of a student participant being positioned as a facilitator by another participant. A logical explanation could be that participants associated the teacher participants with the conventional role of the teacher, and expected them, while they occupied the role, to take responsibility for the learning culture in the classroom, but did not expect this of other student participants. Thus, there were three observed ways in which positioning as a facilitator can occur: institutionally (teacher participants, by the pedagogy), interactionally (teacher participants, by other participants), and reflexively (student participants, by themselves).

5.2.1.4 The Learner and the Knower as Productive Agents

An Episode of shared epistemic agency is productive, as the outcome is New Knowledge (section 4.1.3). The knowledge-building phase is that in which the Intention to resolve an unknowing through epistemic interaction leads to the production of this New Knowledge. The findings show that the positionings of learner and knower iteratively and reciprocally constitute each other during this phase, as exemplified in Extract 5.20 below.

Extract 5.20 – Learner/Knower as Productive Agent – Episode 23

Context: This is the second lesson on composite functions. The previous lesson ended with the working out of solutions to the booklet questions on the board. At the start of this lesson, as a continuation of the previous day's lesson, teacher participant James called on Crimson, a student participant, to come to the board and show the class how to solve a question on composite functions.

Part	Line	Participant	Action/Reification	Code	Positioning
Knowledge Building	1 1:52	Crimson (SP):	"…the answer you get from 'g,' you put into 'f,' you understand …"	Exp-Clarifies	Crimson positions himself as a knower.
	2		(Murmurs of acknowledgment from participants.)		Participants positions Crimson as a knower.
	3	Pearl (SP):	"Couldn't you start with ern …"	Ext-Seeks affirmation	Pearl positions herself as a learner. Pearl opens up the position of knower.
	4	Student A (SP):	"… 'gh'?"	Exp-Explicates unknowing	Student A positions herself as a knower.
	5	Daniel (SP):	"No."	Exp-Affirms	Daniel positions himself as a knower.
	6	Crimson (SP):	"Yes, you could if 'f' wasn't there …"	Exp-Affirms & Clarifies	Crimson positioning self as knower.
	7	Pearl (SP):	"Yes, in front of 'h'…"	Exp-Articulates knowledge	Pearl positions herself as a knower.

	8	Student A (SP):	“Couldn’t you start with ‘fh’?”	Ext-Seeks Affirmation	Student A positions himself as a learner. and positions Crimson as a knower.
	9	Crimson (SP):	“You could do five squared minus equals twenty-five.”	Exp-Affirms & Clarifies	Crimson positions himself as a knower.
	10	Pearl (SP):	“5 squared? Why you started with 5 squared ...?”	Ext-Questions	Pearl positions herself as a learner.
	11	Student A (SP):	“‘Cause you put it into the …”	Exp-Clarifies	Student A positions herself as a knower.
	12	Pearl (SP):	“Yeah, yeah …”	Exp-Clarifies	Pearl positions herself as knower.

Pearl positioning herself as a learner in line 3 prepared the opportunity for other participant to position themselves (and be positioned by her) as knowers in lines 4-7. Furthermore, it led to Student A positioning herself as a learner in line 8. Each line in the interaction acted to produce the next as part of the knowledge-building interaction. Crimson (SK), in line 1, shares his knowledge of composite functions with the other participants. This sharing causes Pearl, in line 3, as a learner, to seek affirmation. In this moment, Student A, as a knower, acts to explicate Pearl's unknowing, and by this dialogical interaction, helps other participants to make sense of Pearl's Extension. This exchange leads to Daniel positioning himself as a knower, but he does not follow through with his Explication, and this opens up the opportunity for Crimson to position himself as a knower. Positioning herself as a knower, having been the learner who started off the interaction, Pearl finishes off Crimson's Explication in line 6. In this way, Pearl was able assume an active role in her own knowledge advancement and that of her peers.

The productive quality of the interaction is referred to as "productive agency" (Schwartz & Okita, 2004), in that, when Pearl sought to extend her knowledge in line 3, she did not know what impact it would have, but Crimson, Student A, Daniel, and herself actively (that is, as agents) built on each other's knowledge and modified it, ultimately producing New Knowledge. This productive interaction is the "sharedness" of shared epistemic agency that advances the knowledge of the classroom community.

The research questions are concerned with the characteristics of shared epistemic agency as it emerges, and positioning during epistemic interaction is a significant example of these. The characteristics are more complex during the interaction than is suggested by their definitions alone; Extension does not simply signify a lack of knowledge, and could be a form of authority; moreover, the corresponding positions have a productive impact on each other. I will draw on these qualities of the positions in the discussion chapter.

5.2.2 Process Authority in Interaction

The process dimension of authority refers to who is in control of the culture of learning in the classroom – that is, of how the learning takes place (see section 2.3.2). Building on my analysis of process authority through participants' interactions, I highlight three issues: a blending of authority amongst participants as they enacted the innovative pedagogy; a freedom to pursue dialogical and physical interactions in the classroom; and the emergence of the learner as having implicit control over the other participants' behaviour.

5.2.2.1 Blending of Process Authority

As facilitators, participants did not take on all the responsibilities associated with the conventional teacher; in facilitating the advancement of knowledge, they blended their authority with mine. Enacting the innovative pedagogy made clear that advancing the knowledge of other participants in a secondary mathematics classroom community required more than the subject content knowledge that constitutes epistemic authority. The teacher participants did not have the mathematics knowledge for teaching (see section 2.3.2.1) that accompanies the possession epistemic authority. As stated in chapter 2, process authority in this study subsumes pedagogic content knowledge and curricular knowledge that is conceptualised as mathematics knowledge for teaching. Mathematics knowledge for teaching includes knowledge of the scope of the mathematics topic to be taught in a given lesson, the prerequisite understanding required to engage with the topic, and the relationship between the topic and the examination requirements.

In teaching cycle 2, teacher participants Deepz and James were required to teach the topic “speed, distance, and time”. Their primary source in their preparatory research was the MathsWatch virtual learning environment (VLE) to which the school subscribes. Hence, they focused their lesson on the time-distance graphs that they encountered on the platform; they did not extend the topic to questions on speed, distance, and time calculations, which were the more typical foci of examination questions. They did not have the additional knowledge of exam requirements, nor of the scope of the topic.

On the other hand, teacher participants Adam and James, who taught inequalities in teaching cycle 3, extended their discussion of the mathematics topic to solve linear inequalities and quadratic inequalities using sketches of quadratic graphs. They did not have the mathematics knowledge for teaching that causes a teacher to structure an instructional sequence in terms that are intelligible to the learners by laying the foundations for learning other ideas. This knowledge would have positioned the drawing of quadratic graphs as a prerequisite for solving quadratic inequalities, and assimilated this technique into a network of ideas that are important to students' reasoning.

I remedied this situation by bringing my mathematics knowledge for teaching to enable the participants to advance the community's mathematics knowledge. In this way, there was a blending of process authority between the participants and myself. I contributed my mathematics knowledge for teaching to support the participants' enactment of the innovative pedagogy without usurping their authority. From teaching cycle 3 onwards, I produced a booklet of mathematics questions for each teacher participant, which became our reference material. This booklet, a reification of the appropriate mathematics knowledge corresponding to each mathematics topic, equipped the teacher and student participants with a representation of the boundaries of the relevant mathematical knowledge. The possession of the booklet placed the teacher participants in the same position as conventional mathematics teachers who use a textbook. The teacher participants could focus on advancing community knowledge rather than on preparing resources, which became my primary role. Producing the booklet evidences how the participants and I negotiated the blending of process authority over time; I produced the resource while they themselves managed this and other resources.

The emergent blending of process authority led to a change in my role on a lesson-by-lesson basis, dependent on the procedures and requirements of the teacher participant. Institutionally positioned as facilitators by the pedagogy, the teacher participants directed whether I was to take on the role of teaching assistant or student participant in the lesson, asserting their authority by renegotiating my authority, while, at the same time, calling upon my authority as the teacher when they chose to do so. For instance, in Extract 5.15 (see section 5.1.2.3.3), in line 1, in my role as the teacher, I asked Crimson about the cause of the delay in starting the lesson; he responded by stating that there was no reason for the delay (positioning himself as a facilitator). In line 7, he announced to the class that I would hand out the booklets (positioning me as a facilitator), though I reminded him that the students already had booklets (see line 8). As Deepz did not have a booklet, I went into the office and got him a spare booklet (positioning myself as a facilitator). Upon my return to the classroom, Crimson asked me to sit down and do the work, saying, "Miss could you sit down please?" (positioning himself as a facilitator by repositioning me as a student). I handed the booklet to Deepz, reminded Ty to focus on his work (asserting my facilitator position), and sat down to act as a student (accepting Crimson's positioning of me as a facilitator).

5.2.2.2 Control of Social Behaviour (Freedom of Dialogical and Physical Interaction)

Participants took individual control of their dialogical and physical interactions in the classroom. Positioned institutionally as facilitators, teacher participants often controlled the epistemic behaviours of other participants. However, individual participants also took control of their own social behaviours as they sought to advance their mathematics knowledge. As epistemic interactions occurred from moment to moment in the classroom, the spontaneous, liberal performance of physical interactions around the classroom and dialogical interactions with other participants became a central aspect of the classroom practice. Attending to these interactions reveals that participants physically moved around the classroom to interact with other participants; without restriction, they entered into or initiated dialogical interactions with other participants as they saw fit, in order to advance their individual knowledge and that of other participants. Extract 5.21 shows how participants engaged in epistemic interactions without restriction.

Extract 5.21 – Control of Social Behaviour – Episode 10

Context: The teacher participant James explicates knowledge to the classroom community concerning the representation of inequalities on a number line. Student participants were focused on him and his Explication. The Episode was initiated by James, the teacher participant, whose Intention was triggered by an Assumed unknowing. He assumed that a lack of knowledge of inequalities existed amongst the classroom participants. As such, his Intention was oriented towards Explication. Adam, the other teacher participant, was at the teacher's computer, managing the learning resource – the PowerPoint lesson plan.

Part	Line	Participant	Action/Reification	Code	Positioning	Movement/ Communication
Intention	1	James (TP):	"If you want to plot this here, so we know that its less than, so we put a circle …" (Assumed unknowing).	I (Exp)-Clarifies	James positions himself as a facilitator	
Knowledge Building	2	Student B (SP):	"… and you colour it in, right?"	Ext-Seeks affirmation	Student B positions herself as a learner	Student B calls out from seating position
	3	James (TP):	"… yeah, you colour in the circles because its less than …"	Exp-Affirms then Clarifies	James positions himself as a knower	
	4	Crimson (SP):	"… And then you draw an arrow down…"	Exp-Clarifies	Crimson positions himself as a knower	Crimson calls out from seating position

	5	James (TP):	“Then you draw an arrow down.”	Exp-Clarifies	James positions himself as a knower	
	6	Student A (SP):	“Wait, wait, I got a question!”		Student B positions herself as a learner and opens up the position of knower	Student A calls out from seating position
	7	James (TP):	“Yes?”		James positions himself as a facilitator	
	8	Student A (SP):	“So, if its more than you draw an arrow that,” (pointing towards her right) “across the way?”	Ext-Seeks affirmation	Student A positions herself as a learner	Student A calls out from seating position
	9	James (TP):	“Yes.”	Exp-Affirms	James accepts the position of knower,	

					and positions himself as such	
1:19	10	Crimson (SP):	“But if it’s not equals to, don’t colour it. You see, when it says equals to you, colour in the dot. If it does not say equals to, you don’t colour in the dot.”	Exp-Clarifies	Crimson accepts the position of knower by positioning himself as such	Crimson talks across to student A seated two positions away
		Student B (SP):	“I get that.”			
1:19	11	Student B (SP):	“Can I do the question on the board, please?”	Ext-Requests	Student B positions herself as a learner, opening up the position of facilitator	Student B stands up and comes to the board; James holds out the pen to her

	12	James (TP):	"Yes, sure, do you know how to do it?"	Xpt-Controls & Checks	James positions himself as a facilitator	
1:30	13	Daniel (SP):	"Do you have to draw a number line?"	Ext-Seeks Affirmation	Daniel positions himself as a learner	Daniel calls out from the seat
1:39	14		(Jevonte stands up, walks over to communicate with a participant, and walks back.)			

The last column of this extract exemplifies the learning activities that became typical of the classroom community. Participants communicated with each other when they saw fit to do so. In line 2, Student B, by positioning herself as a learner, contributes to the Explication by interjecting with the question "…and you colour it in, right?" Though ostensibly a question with which she is seeking affirmation of her knowledge (Extension), this phrase also contributed to community knowledge as it was asked and responded to publicly. Crimson, in line 4, continues James's Explication with "and then you draw an arrow down." James repeats his exact phrase in line 5, while correctly drawing the line to the left. He thus appears to understand that by "down", Crimson actually meant "to the left". So, by drawing the line to the left, he legitimised Crimson's contribution. Student A contributed to the Explication with her presentation of a question in lines 6 and 8. Her question further legitimised the term "down" as meaning "to the left". In line 10, Crimson further contributes to knowledge advancement by Explicating knowledge to Student B. In the recording, James becomes inaudible towards the end of his sentence in line 3, and did not finish expressing his thought. Crimson may have felt responsible, as his interjection in line 4 interrupted James; this may be why he decided to repeat himself more clearly in a public statement in line 10, setting the process of collective knowledge advancement that he had threatened to disrupt back on track.

Physical movement also occurred at will in the classroom. In line 11, Student B stands up and walks to the board, wanting to extend her knowledge by publicly working through a solution. James' acceptance of her behaviour is evidenced by his giving the pen to her. In line 14, Jevonte walks across the class, communicates with a student, and then returns to his seat. The reasoning behind this interaction is unclear, but he moved of his own volition and did not distract participants from their learning.

5.2.2.3 The Position of the Learner as Authority

My analysis of the data points to the possibility of considering the learner position as a source of process authority in the classroom. This authority manifested as the ability to cause other participants to behave in specific ways. Analysing participants' actions when positioned as a learner, and their impact on other participants during epistemic interactions, is pertinent to the research questions. Process authority is in the possession of the participant who seeks to extend their existing knowledge by causing other participants to act in ways that allow the participant in question to extend their existing knowledge. This is clear in Extract 5.10, wherein the whole class goes over a question on the board. The teacher participant Teesh asked the student participants what they did at each point of the working out, and she wrote down their responses on the board; some students were checking their work against these answers. In line 1, Student A reflexively positions herself as a learner, publicly declaring to the class, "I didn't get that." This caused the lesson to come to a halt, with all attention devoted to helping Student A investigate where she went wrong. Line 4, where Student B say, "Are you sure, Student A, because it happened last time," indicates that this act of publicly articulating her unknowing and causing the class to pause had happened on a previous occasion. Student A displayed the same authority in line 38, visible in Extract 5.16 of the same Episode, where she said, "wait Teesh, let me clarify what Crimson wrote"; again, by making a public statement, she halted the pace of the lesson, and having checked her work, Student A indicated that the lesson could continue. Similarly, in line 29, Daniel positions himself as a learner, saying, “wait, slow down, slow down,” causing the teacher participant to pause and not write anything on the board for a few moments so that Daniel could copy what was already written.

Positioning oneself as a learner can position another participant as a knower, thereby requiring the other participant to Explicate their mathematics knowledge. For instance, in Extract 5.19 (see section 5.2.1), in line 3, Crimson, in positioning himself as a learner seeking affirmation, implicitly positions Jevonte as a knower. Crimson self-positioning can be said to have caused Jevonte to act in a certain way – that is, to accept the position of a knower and to Explicate knowledge.

This evidence of the learner position as a source of authority points to the relational agency of the participants (cf. Edwards, 2005). Relational agency is the ability to align one's thoughts with those of other participants, to recognise what they need to achieve their goals, to interpret other participants’ problems, and to respond to this interpretation. This ability marks the classroom environment as a safe space wherein participants are free to share their lack of knowledge, with the trust that the community will do what it takes to help them know.

5.2.3 Epistemic Authority in Interaction

The epistemic dimension of authority refers to who is validated as a knower, i.e., who is viewed as legitimately knowledgeable (see section 2.3.2). In my analysis of epistemic authority in participants’ interaction, I highlight three issues: knowledge as a prerequisite for extension; a disregard for ability labels; and the individual and communal responsibility for knowledge advancement.

5.2.3.1 Knowledge as a Prerequisite for Extension

The data points to knowledge as a prerequisite for knowledge-building interaction; this prerequisite is inclusive of Extension, the characteristic of shared epistemic agency that focuses on extending a participant's existing knowledge. For a participant to direct their agency towards Extension requires the possession of certain background knowledge.

Knowledge is required for all modes of Extension, as exemplified in Extract 5.8 (see section 5.1.2.1.4), line 3: “So, it has to add to make one and times to make minus forty-two?” This example of Seeking affirmation as a mode of Extension is made possible by the presence of some procedural factorisation knowledge, however incomplete. From my personal assessment of the participants at this point in time, I discerned that Crimson had some knowledge of factorising an expression ax^2+bx+c. He knew that the coefficient “a” needed to be multiplied by the constant “c”. He also knew that the solution lay in the multiplication and addition of the correct figures. His unknowing, which needed resolution, was whether the product or addition gave the coefficient of “b” or the constant “c”. Teesh’s assessment of Crimsons’ solution as “smart” was based on her personal confirmation that his solution was correct, by checking that expanding the factorisation resulted in the original expression. This acknowledgment also required knowledge of factorisation.

Extract 5.1 (see section 5.1) provides another example of knowledge being a necessity of Extension in knowledge-building interactions. In line 3, Pearl's Extension by means of the epistemic Question, "But what do you times together to get x?" could not have been made if she did not have knowledge of factorising quadratic equations with a coefficient besides the integer 1. It is this knowledge that allows the participant who seeks to extend their existing knowledge to challenge an Explication that does not advance their existing knowledge, and to recognise when their knowledge has been advanced. Extract 5.7 from Episode 3 (see section 5.1.2.1.3) illustrates how Extension in the mode of a Request also requires certain prerequisite knowledge. In this extract, Teesh requests to Extend her knowledge of solving quadratic equations. In lines 3, 5, and 7, she requests permission from the teacher participant to solve the quadratic equation by doing what she knows and building on it in front of the class; this requires at least a provisional grasp of the associated principles. Summarily, Extension, in all four of its modes, does not indicate a total lack of knowledge, but in fact reveals a comprehension of certain prerequisite principles.

5.2.3.2 Disregarding Presumed Ability Labels

On entry, the school assigned students to ability bands based on their performance in the standardised assessments taken by all students in the UK at the end of their primary school education. If this information is unavailable, the school will assign a band from performance in the school's entry assessments. These bands indicate students' predicted range of attainment at the end of their secondary schooling.

Students could be assigned to any of the 1-2 (foundation), 2-4 (lower), 4-6 (middle), 6-9 (higher), or 7-9 (higher plus) attainment bands. My mathematics class comprised a selection of students from the lower, middle, and higher attainment bands.

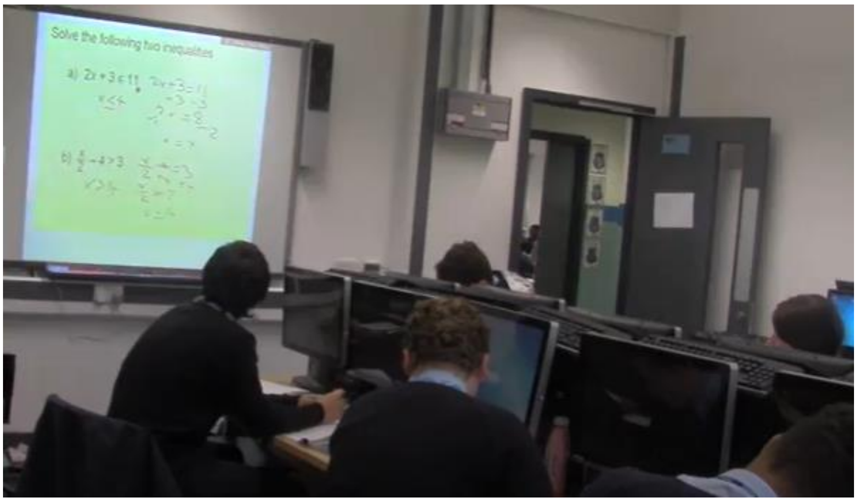
Regardless of the band to which they were allocated, all participants enacted the pedagogy as both teacher sand student participants. During the Select stages of the pedagogy, wherein participants selected their mathematics topic to teach (see section 3.1), all topics were available for selection. I did not consider the presumed level of difficulty of the topics or the participants' ability band, nor did participants appear to do so. This lack of consideration shows that the pedagogy and its enactment did not recognise the ability levels of the participants; nor did the participants consider the associated labels in proceeding with their learning.

At the end of each teaching cycle, when I reflected on the pedagogy (see section 3.4.2.3) I found no link between the quality of the enactments and the presumed ability of the participants; hence, I implemented no measures in subsequent selection stages to constrain which participants selected which mathematics topic. It remained open and democratic.

That participants participated in all stages of the pedagogy indiscriminately and collaboratively is evidence of their rejection the ability labels. They all prepared for the lessons, creating knowledge objects in the form of the PowerPoint presentations that reified their mathematics knowledge; they all shared their mathematics knowledge and reflected on each other's performance. Participants' acknowledgment of each other's performance is further evidence of this rejection (see Photograph 5.3).

<u>Photograph 5. 3 – Disregarding Ability Labels – Episode 11</u>

<u>Context:</u> Teacher participants Adam and James take turns to lead a discussion of different aspects of inequalities. Adam demonstrates the method of solving inequalities by going over the solutions for equations that have been taught previously. At the end of Adam's Explication, as shown in Photograph 5.3, the student participants spontaneously start clapping.



Photograph 5. 3 – Disregarding presumed ability labels

When questioned, the participants responded that the clapping showed appreciation for how Adam connected their previous knowledge of solving linear equations with the solving of inequalities. The clapping represented the ease with which they could now advance their mathematics knowledge, and their appreciation of Adam for making this happen. Unbeknownst to them, Adam was in the lower ability band;

their enthusiastic appreciation of the mathematics knowledge that he shared shows the disregard, on his part and the part of others, for the mathematics ability labels imposed upon them by the school.

5.2.3.3 Individual and Community Knowledge and Responsibility

Enacting the innovative pedagogy dictated by the pedagogic principles (see section 2.5.1), the participants took responsibility for advancing their mathematics knowledge. The pedagogy prescribed that the teacher participants take responsibility for advancing the knowledge of the student participants during the Plan and Share stages of each teaching cycle (see section 3.1). However, what was not prescribed by the pedagogy, but which nevertheless emerged and evidenced in the data, was the fact that the community took responsibility for individual participants' knowing and unknowing.

I use Extract 5.22 as an example. This extract is an expanded representation of the classroom chatter indicated between lines 5 and 6 of Extract 5.10 (see section 5.1.2.1.5). This classroom chatter is numbered as 5.1-5.8, and shaded in Extract 5.22 below.

<u>Extract 5.22 – Individual and Community Knowledge and Responsibility – Episode 6</u> <u>Context:</u> Teesh shows how to use the quadratic formula to solve the question she earlier posed to the class. Crimson calls out the answers, and Teesh writes them on the board. At 14:39, Student A performs a dialogical interaction.

Part	Line	Participant	Action /Reification	Code
Knowledge Building	1	Student A (SP) (to the class):	“I didn’t get that, but I got the same calculation in my calculator.”	Ext
	2	Jayzee (SP):	“What did you get?”	Xpt
	3	Student A (SP) (to Jayzee):	“I got ...”	
	4	Student B (SP):	“Are you sure, student A, because it happened last time …”	Xpt
	5	Teesh (TP):	“Did everyone get this? … someone got this, yeah?” (waits for responses).	Xpt
	5.1	James (SP):	“Now I got it.”	
	5.2	Teesh (TP):	“Eh?”	
	5.3	James (SP):	“I got it now.”	
	5.4	Crimson (SP):	“Did you get it?”	
	5.5	Daniel (SP):	“How did you get ...”	
	5.6	Daniel (SP):	“…Oh yes! Squared!”	
	5.7	Crimson (SP):	“Oh yes, Student A …“	
	5.8	Teesh (TP):	“Everyone got this?”	

	6	Student A (SP) (To Teesh):	"I didn't get it."	Ext
	7	Crimson (SP) (turning to Student A with surprise):	"Oh, you didn't? What did you get?"	Xpt
	8	Student A (SP):	"I put this in my calculator," (she passes her calculator to Crimson, who studies it).	Ext
	9	Teesh (TP) (to the class):	"So, who got the one with the minus then?"	Xpt
	10	Crimson (SP) (to Student A):	"You did two minuses, Student A."	Exp
	11	Crimson (SP) (to Student A):	"It is not minus five; it's ordinary five."	Exp

Teesh demonstrates her responsibility for the community knowledge by her repeated inquiry in lines 5, 5.8, and 9; she wanted to gauge and make clear what each participant knew and what they did not know. Extract 5.12, the continuation of this Episode, demonstrates her encouraging Crimson to articulate his knowledge in a structured step-by-step way for the advancement of participants' knowledge.

This extract shows how knowing or not knowing was important to the community. In lines 5.1-5.8, the public demonstration of participants' knowing and unknowing corresponded to a practice that had become standard in the classroom; this practice is evidence that the sharing of knowledge was considered to be valuable by all participants, whether teacher or student.

In line 1, Student A states that she did not get the answer written on the board. What follows indicates how the community took responsibility for ensuring that Student A's unknowing was alleviated. In the first instance, Teesh, the teacher participant, pauses her explanation of the calculation on the board. Notably, no participant complains either about Student A's interruption or about Teesh pausing her explanation, even though it appears, as line 4 suggests, that Student A had stopped the lesson before for an unjustified reason. In addition to this display of patience, the Explication performed by Teesh and discussed in lines 2, 4, 5.4, 5.7, and 7 represents the willingness to help Student A to know on the part of the other participants.

This extract demonstrates that a participant's knowing and unknowing was considered to be the community's property, and that the community's unknowing and knowing was the individual's responsibility.

5.3 Summary

This chapter detailed the findings of the analysis of the thirty-six Episodes of shared epistemic agency. The chapter is divided into two sections, with the first elaborating on the unit of analysis, and the second elaborating on participants' interaction as they enacted the innovative pedagogy. The first section discussed the six characteristics that encapsulate the shared epistemic agency that this study seeks to awaken amongst the participants. Analysis of the first characteristic, Intention, showed that Intentions orient toward the knowledge-building processes of Extension, Explication, or Expertise, and are triggered by a participant's desire to resolve an Assumed or Identified unknowing. The unknowing could be the participant's own, or that of another participant or group of participants.

The findings from the second part of this section, which discussed knowledge-building practices, showed how each of the characteristics was more nuanced in their enactment than is suggested by the literature. These nuanced depictions of the characteristics – which reveal the modes of Extension, Explication, and Expertise – showed how the participants operationalised the characteristics as they enacted the innovative pedagogy. The different qualities and values of Mutual Relations were also discussed. Analysis of the third part of an Episode led to the determination of the end of an Episode, and of the different ways of resolving unknowing that resulted in the achievement of New Knowledge.

The second section of this chapter addressed on participant interaction, and highlighted how positioning, process authority, and epistemic authority and their interactions were evidenced in the classroom. It gave an indication of the active participation and participants relationship with their mathematics as was the aim of the study.In particular, the participants' process authority was seen to emerge as the result of a blending of authority, which itself arose from the mutual interdependency of participants' experiences and skills. Command over dialogical and physical action was dependent on agency of participants in their project of advancing their individual and collective mathematics knowledge, and Extension emerged as an unexpected means of controlling and managing the behaviour of others.

As participants interacted to enact the innovative pedagogy, their relationships to epistemic authority revealed that certain background knowledge was required in order for a participant to direct their agency towards Extension. Moreover, it was determined that participants enacted the pedagogy regardless of the ability labels assigned to them by the school; as a group, participants took responsibility for their individual and collective knowledge advancement. In the following chapter, I will explicitly apply these findings to the research questions.

6 DISCUSSION

At the beginning of this thesis, drawing on the existing literature, I presented the concept of shared epistemic agency as emerging from the interplay of six distinct characteristics (see section 2.4.3). In the analysis section 4.2, I showed how I could identify thirty-six Episodes of participant interaction, each of which exhibited all six of these characteristics. This forms the data and findings that allow me to claim that my innovative pedagogy, based on the knowledge creation principles and iteratively refined through action research, produces shared epistemic agency as it is represented in the literature. What also emerged from the analysis is a different way of looking at shared epistemic agency. I have moved from seeing shared epistemic agency as a discrete set of distinctive behaviours to a more holistic view of its inseparable connection with student participation and community practice. These two themes, of the student and their community, guide this chapter and contribute to the answering of the research questions.

I have organised this chapter into three sections. In section 6.1, I respond to the first research question by considering the themes of “the student as a participant” and “the concept of a learning community”. Section 6.2 responds to the second research question, taking a deeper look at the second theme of the new learning community. Section 6.3 presents my holistic reflections on the action research, and the chapter concludes by reviewing in a broader context the key features of the innovative pedagogy that changed the student and the community, and that could be adapted by other researchers and educational practitioners.

6.1 Answering Research Question 1

What are the indicators of shared epistemic agency in the mathematics classroom?

I present a new conception of the student and their learning community in terms of the indicators of shared epistemic agency in my mathematics classroom. I contend that this emergent conception of a student as a Participant is connected with the positions of the learner, knower, and facilitator in the new Learning Community. Participants in the study developed a practice of learning through interaction, created knowledge from within the community through their agency and their experience, and democratised their participation.

6.1.1 Theme 1: The Concept of "Student as a Participant"

At the start of chapter 3, I introduced the students in this study as "participants" in order to emphasise their active participation in creating and enacting the innovative pedagogy, as well as their roles within the action research methodology of reflection and improvement. In addition, the term "participants" suggests liberation from the conventional view of a student in the classroom (see section 2.3.1). In this section, on the strength of my own experiences of participant enactment, and of the taxonomy of the types of participation which I provide in chapter 5, I present a new conception of the Participant.

The Participant that emerges, as an index of shared epistemic agency, is multi-faceted. This Participant is a learner in their capacity to Extend their knowledge, a knower in their capacity to Explicate their knowledge, and a facilitator in their capacity of Expertise (see Figure 6.1).

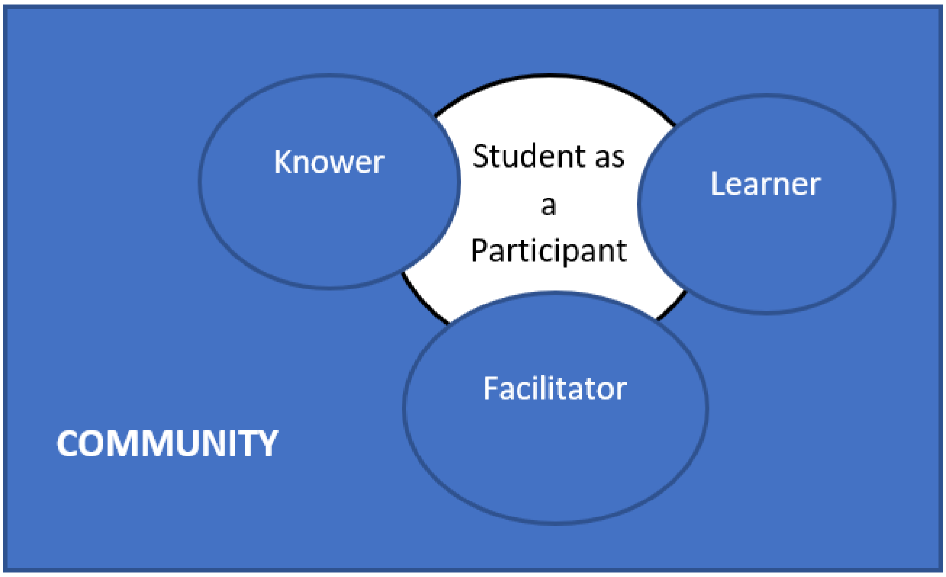


Figure 6.1 – The multi-faceted student as a Participant

My findings (see section 5.2.1) make it evident that the terms “teacher participant” and “student participant”, used to refer to participant roles in the enactment of the pedagogy, were not by themselves sufficient to describe the epistemic interactions that took place amongst participants. Initially, I designed the teacher and student participant roles under the assumption that the teacher participants for each lesson alone would act as knowers, with the remained of the participants being learners. The only disruption to this assumption was the convenient expectation – set by myself as the class teacher – that all participants would arrive at the lesson with some knowledge of the specific mathematics topic (see section 3.3.1). My later analysis reveals that, during the Episodes of shared epistemic agency, both teacher and student participants sought to extend their mathematics knowledge (see section 5.1.2.1), and so both groups acted as learners. Both teacher and student participants made knowledge explicit to others (see section 5.1.2.2) as knowers, and both controlled the learning culture of the classroom (see section 5.1.2.4) as facilitators.

In this way, all participants made advances to the mathematics knowledge that they brought with them to their lessons. Thus, in enacting the pedagogy, temporary positions of “learner”, “knower” (having epistemic authority), and “facilitator” (having process authority) became available to be taken up reflexively or interactionally by participants. A participant could be either a student participant knower (SK), student participant learner (SL), student participant facilitator (SF), teacher participant-earner (TL), teacher participant knower (TK), or a teacher participant facilitator (TF), at any given moment.

6.1.1.1 The Participant as a Learner

The Participant, in their capacity as a learner, is an individual in a community who seeks to extend their existing knowledge; that is, a Participant who directs their agency towards Extension. A Participant can assume the position of a learner at any moment in the pedagogical process. In this sense, being a learner is not a permanent state that a Participant occupies, but a flexible identification. In the aspect of a learner, the Participant takes control of their knowing and unknowing, is productive of epistemic interactions, and is not knowledge-less.

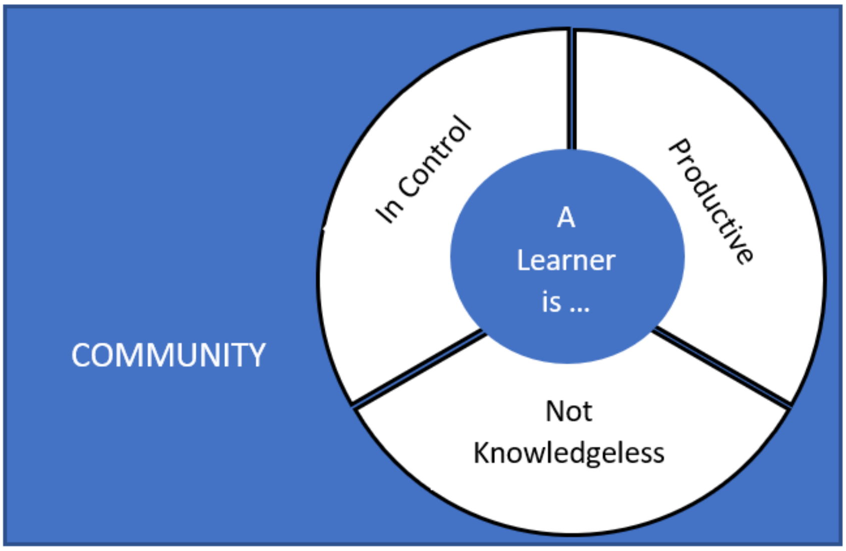


Figure 6.2 – The features of the Learner position

6.1.1.1.1 A Learner Can Take Control of their Knowing and Unknowing

I define a learner by the characteristic of Extension, towards which they direct their agency. In stating that the Participant "directs their agency" towards Extension, I am highlighting the fact that the Participant can, on their own terms, decide upon and seek to extend their mathematics knowledge as a learner. The Participant can wilfully assume the position of a learner, as evidenced by the findings that the learner is a reflexive positioning (see section 5.2.1.1). A Participant can position themselves as a learner by seeking to know, but there was no evidence to suggest that another Participant can position a Participant as a learner. I argue that this reflexive positioning, which suggests that being a learner is an identification that individual takes upon themselves, places the learner in control of their knowing and, conversely, their unknowing; this quality is most deeply connected with the concept of agency in general, and challenges the conventional views on which the learner is neither empowered nor in active control of their epistemic status.

The learner is not only in control of their knowing and their unknowing, but can also determine how they seek to extend their knowledge. The findings show five different modes by which the learner seeks to extend their existing knowledge. The frequency of modes (see Appendix 8) reveals that while Questioning is the predominant mode of Extension, the learner can also demurely make a Request or tenaciously Challenge the knowledge presented to them by another Participant as they seek to know. Figure 6.3 represents these diverse modes of Extension (see section 5.1.2.1), pointing up the learner's capacity to be adaptive in their quest for knowledge.

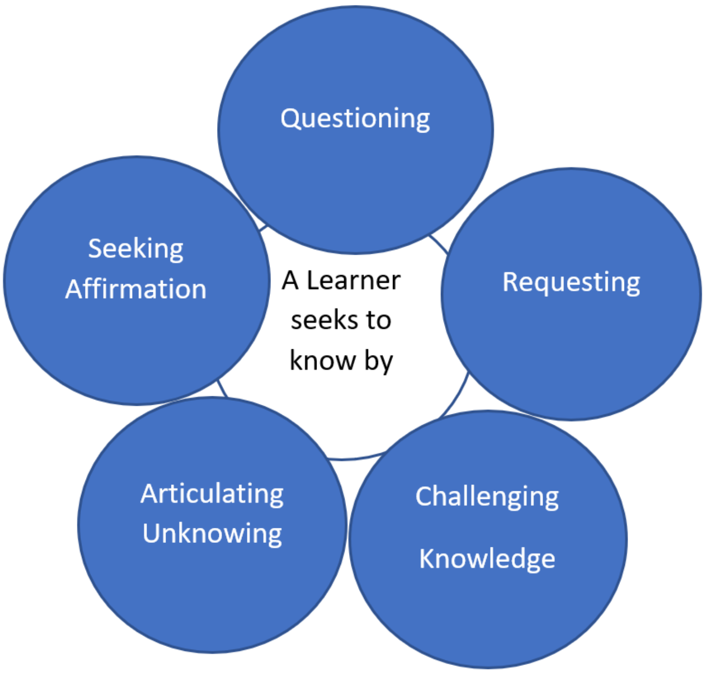


Figure 6.3 – The modes of Extension that point to a learner in control

By articulating their unknowing, the Participant as learner can also control the actions and reifications of other Participants in the community. The findings point to the learner position implicating a form of process authority that impacts the actions and reifications of other participants (see section 5.2.2.3). It is important to note that while I am unable to avoid speaking about the learner as an individual, the Participant's position is always a function of their interactions within a community; the learner seeks to extend their existing knowledge in a community, not in isolation; The learner seeks to extend their knowledge *from* the community, other participants in the community are the motivation for the position, and that is what makes it productive as whether directly or indirectly, this seeking also helps fellow learners within this community to achieve knowledge of their own

6.1.1.1.2 A Learner as Productive of Epistemic Interactions

The learner position that emerges from this study is productive in that, in seeking to resolve their unknowing, they set in motion a series of actions and reifications on the part of other Participants that leads to the creation of New Knowledge. This is because the learner and knower positions are iterative and reciprocal, and provide evidence of productive agency (see section 5.2.1.4). It is important to reiterate that the knowledge they attain is new because it is new to the students, and not new to the world (cf. Bereiter & Scardamalia, 2011, p. 4).

In reflexively positioning themselves as a learner, Participants typically position another Participant interactionally as a knower, as shown in Extract 5.20, wherein Crimson, in positioning himself as a learner by directing an epistemic question to

Jevonte, positions Jevonte as a knower. The Participant, thus positioned, had the opportunity to accept their conferred status, and make their knowledge explicit to the other Participant, beginning an epistemic interaction based on an iterative relationship between the learner and knower positions (see section 5.2.1.4). The original Participant continues seeking to know, and other Participants seek to explicate their knowledge until they resolve the unknowing. This capacity of the learner to sustain epistemic interactions as they tenaciously seek to know makes them productive.

6.1.1.1.3 A Learner is not Knowledge-Less

The findings show that knowledge is a prerequisite for Extension (see section 5.2.3.3). A Participant positions themselves as a learner by seeking to know. For a Participant to seek to know, however, they must first be aware of what they do not know. It is conceivable that the learner could seek to know everything about the mathematics topic, confirming that they are knowledge-less concerning that particular topic, though this is improbable in a subject with interlinked content – in any event, one requires such abilities as basic arithmetic in order to even approach any secondary mathematics question. The modes of Extension, which elaborate upon the nuances of its expression (see section 5.1.2.1), confirm that in the position of a learner, Participants can articulate what they do not know about the mathematics topics, and can also challenge the knowledge and Explications of other Participants. This shows that the learner is not knowledge-less, but knowledgeable.

As Participants position themselves as learners during their epistemic interactions, the corresponding unknowing could reside in the learner's mind, and accompany them to the lesson, or else it could arise from participation in the innovative pedagogy. The question of where the unknowing originates suggests parallels with the acquisition and participation metaphors of learning (Sfard, 1998) discussed in section 2.2.1, wherein the acquisition metaphor is seen to represent knowledge as the capacity of one's mind, and the participation metaphor is seen to represent knowledge as being situated in the cultural practices of a community (Lave & Wenger, 1991). Moreover, the fact that an Intention is initiated by participants during interaction, and triggered by an Assumed or Identified unknowing (see 5.1.1), shows that participation in the pedagogy creates the awareness of the state of unknowing; in this way, the pedagogy is productive to the extent that it creates an awareness of what one does not know, which itself constitutes knowledge.

I have outlined so far how the Participant can position themselves as a learner within the community. In this capacity, they take control of their knowing and unknowing, and can affect the actions and reifications of other Participants; they are productive of epistemic interactions, and are inherently knowledgeable. These characteristics are indicative of how the Participant behaves as a learner when realising shared epistemic agency. The conception of the learner that I present in this study is in conflict with that which is assumed in the dominant discourse on education. Gert Biesta (2010) argues that the term “learner”, when used to describe young people in schools, is understood in terms of a lack; the learner is considered to be an individual who is missing something that they need to learn. In this sense, the learner is not yet complete; they are not yet knowledgeable, not yet skilful, not yet competent; they need to learn in order to know. Biesta was not, in making these observations, arguing for a change to the term; he was questioning the assumptions we (teachers, parents, policymakers, and children) make about learners. This study testifies to the fact that the young people in our classrooms are not lacking in knowledge, and are fully capable of taking control of what they know and do not know. I argue, in the next chapter, that educators should assume the capability of learners as a general principle.

6.1.1.2 The Participant as a Knower

The concept of the Participant as a knower, presented in the study as an indicator of shared epistemic agency, refers to an individual who can explicate their mathematics knowledge to one or more Participants in the classroom; this Participant directs their agency towards Explication. Similar to the position of a learner, the knower position is temporary, flexible, and community-based. It differs from the reflexive position of a learner as it is an interactive positioning (see section 5.2.6.1.1); this means that a Participant can position themselves as a knower in their actions or reifications in order to explicate knowledge to another Participant; or, another Participant can position them as a knower in a bid to have them to explicate their knowledge to one or more Participants. The positioning of a knower is based on an interaction with another Participants in the community; it does not occur in isolation, but instead, its assumption constitutes the continuation of an interaction with one or more fellow Participants. A Participant does not position themselves as a knower to explicate knowledge to themselves; this position always maintains an orientation towards others. In this position, a knower has epistemic authority, is responsive to unknowing, and is ultimately productive.

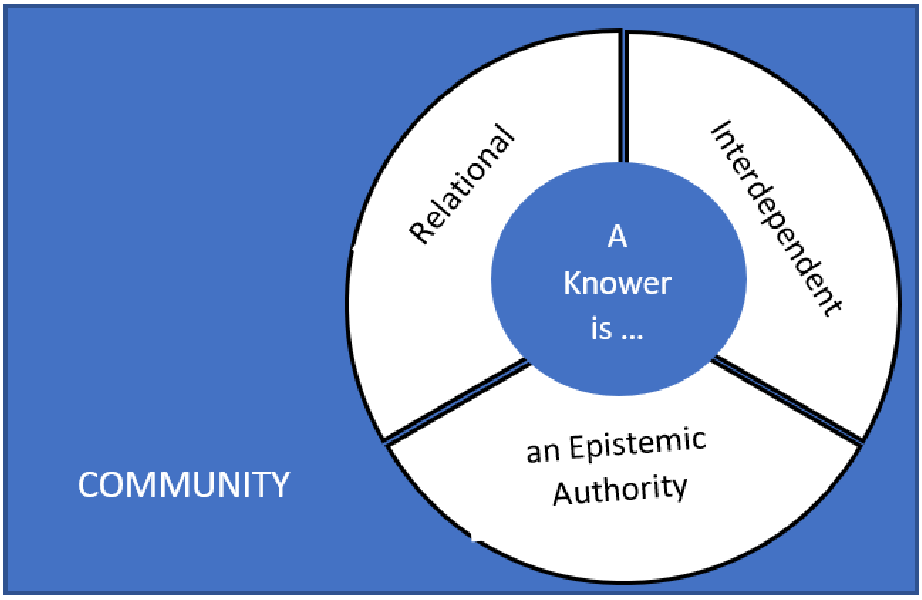


Figure 6.4 – The features of the knower position

6.1.1.2.1 A Knower has Epistemic Authority

Participants attribute epistemic authority (see section 2.3.2) to an individual when they validate them as a knower, and thereby recognise them as legitimately knowledgeable. In the recorded data, Participants are seen to recognise another Participant's epistemic authority based on their previous competent participation in epistemic interactions in the classroom and the quality of their current knowledge. For instance, in Episode 2, Extract 5.17 (see section 5.1.3.2.3), wherein Crimson positions himself as a learner at the start of the Episode, having given the correct factorisation of the quadratic equation, he is later positioned as a knower by Teesh in her bid to extend her own knowledge. In contrast, in Episode 23, Extract 5.20 (see section 5.2.1.4), in line 5, Daniel positions himself as a knower, following Pearl opening up of this position in line 3. Daniel was not validated as legitimately knowledgeable in that moment; his contribution was ignored. However, Crimson's affirmation in line 6 and his subsequent clarification was validated as knowledgeable, and the explication continued the interaction with him once more assuming the position of a knower. Participants determine the competence of previous participation in line with the standards of effectiveness set by the classroom community.

The findings attest to the competence of the knower, as they reveal that the appeal to a knower was the most frequent way in which New Knowledge was produced within the community (see Figure 5.3).

6.1.1.2.2 A Knower is Interdependent on a Learner and Relational

In addition to being positioned, Participants can position themselves as knowers to resolve other Participants' unknowing. When a Participant positions themselves as a learner, they open up the position of knower for another Participant to occupy, as demonstrated in Episode 23 (see Extract 5.20), wherein Pearl, positioning herself as a learner by seeking affirmation of her mathematics knowledge, opened up the position of knower to be occupied by a participant willing to provide this affirmation. Hence, the position of knower is assumed in response to a learner-Participant, and is inherently interdependent upon it.

Having been positioned by another Participant as a knower, it is up to the knower to accept the position by explicating their mathematics knowledge. This suggests the relational aspect of the knower position. A Participant who accepts the position reveals their capacity to recognise others' needs and respond to them; thus in this pedagogy, acting as a knower also enacts Mutual Relations within the community.

I identify in this study three factors that may motivate the knower to take up the position made available by another learner, or to position themselves as a knower. These factors are the responsibility Participants have for the advancement of other Participants' knowledge (see section 6.1.2.1.1 below), the feeling of being valued by the community (see section 6.2.1 below), and their accountability to the classroom practice (see section 6.2.2 below). These factors, which I discuss in detail later in this chapter, could explain why the knower appears to respond to an unknowing and to seek to resolve it. This response was evidenced as Participants became knowers; for example, in Extract 5.13, wherein Teesh positioned herself as a knower to explicate Pearl's knowledge. This, once more, is connected with the reciprocal and iterative nature of knower and learner positions in a knowledge-building interaction (see section 5.2.1.4), which I attribute to the tenacity of the learner who seeks to know, and which causes the knower to respond with Explication. The capacity of a knower to align their thoughts with those of other Participants, to interpret what is needed to resolve their unknowing, and to make decisions regarding best how to respond were identified in the analysis of the modes of Explication (see section 5.1.2.2). Participants made decisions on clarifying knowledge, explicating the unknowing of another Participant, telling another Participant, and affirming the knowledge of another Participant; the capacity for making such decisions can be considered as relational agency (cf. Edwards, 2005).

Jacques Rancière (1991) has claimed that the assumptions that inform teachers' behaviours, such as that of the logic of Explication (see section 2.3.1) that consider the students as in need of the teacher's explanation in order to learn, has resulted in “student” being equated with "stultification" (p. 7), as the student's intelligence is seen as subordinate. In his view

> To explain is to arrange the elements of knowledge to be transmitted in accordance with the supposed limited capacities of those under instruction. … Explanations are needed so that the one who is ignorant might understand the explanation that enables his or her understanding. … its primary function is to infinitize the very distance it proposes to reduce. To explain something to one who is ignorant is, first and foremost, to explain that which would not be understood if it were not explained. It is to demonstrate an incapacity ("On Ignorant Schoolmasters" in Bingham et al., 2010, p. 3).

This comment contests the relational bases of Explication that I put forth in this section. Rancière views Explication as part of the pedagogy myth of schooling, serving the purpose of demonstrating incapacity, and perpetuating the unknowing of the learner. Rather than advancing knowledge and bridging the inequality between the one who knows and the one who does not know, it performs the function of enacting, even inaugurating, and confirming an inequality. It consolidates the learner’s status as requiring Explication from another in order to know. Rancière views the learner as the product of the knower rather than a condition of the learner. Through my experience as a teacher, I can identify with Rancière’s critical view of the learning environment. The conventional pedagogic relations between the teacher (knower) and the students (learner) do not support possibility that students could learn without a teacher, instead installing the teacher as a necessary presence for providing students with instruction.

The knower I present in this study, however, is a product of the learner, who is themselves reflexively positioned to extend their knowledge. While I cannot attest to what lies in the Participants' hearts, it should be clear enough in my findings that the knower position in this study is flexible and temporary. The learner is a past knower, and can be a future learner or knower; as such, participants can identify with both positions. Can the knower seek to demonstrate their incapacity? I suggest, on the strength of my findings, that both knower and learner participate in epistemic interactions for knowledge advancement. However, when the knower positions themselves as a knower based on the Assumed unknowing of another participant, are they, through their Explication, replicating the behaviour of the traditional schoolmaster and inadvertently demonstrating the incapacity of the learner? I do not think so.

The evidence of this study conflicts with Rancière’s view, and allows an alternative view of Explication within the context of this innovative pedagogy, in which the function of the Explication is not to demonstrate the learner’s incapacity; rather, it is the learner to whom the knower’s purpose is subordinate. The relationship between the explicating knower and the learner most notably produces positive outcomes for the latter, not for the former. Rather than demonstrating incapacity, it demonstrates the productive nature of the two interdependent positions. Moreover, Rancière's challenges the role of the teacher are connected with a call for something resembling autodidacticism; I hold that while self-learning is of value to educational practice, I present in this study the possibility of learning as co-participation, and the benefits of such a style of learning, as discussed in detail in the next chapter.

6.1.1.3 The Participant as a Facilitator

The third Participant position indicative of shared epistemic agency is that of the Participant with the capacity to control the learning culture in the classroom (see section 5.2.1.3). This Participant has process authority, with which they direct their agency towards Expertise. The Participant can be "teacher-like", taking on the responsibilities that are typically associated with the role of the teacher (see section 5.1.2.3).

Figure 6.5 – The features of the facilitator position

6.1.1.3.1 Process Authority Facilitates Learning

In developing the pedagogy, I expected the Participants to share their mathematics knowledge with other Participants in the mathematics classroom, leading to the interactive positioning of Participants as learners and knowers with epistemic authority. There was initially no explicit requirement built into the design of the pedagogy for Participants to control the learning culture of the classroom. However, I recognised this necessity in the literature, observing Scardamalia's (2002, pp. 3–5) argument for the necessity of giving students control for the strategic activities involved in learning if they are to take responsibility for advancing their collective knowledge. This led to my inclusion of Expertise as one of the characteristics of shared epistemic agency. Consequently, the pedagogy implicitly demanded that certain Participants take control the learning environment as they occupy the role of teacher participant. The modes of Expertise identified by the analysis (see section 5.1.2.3) support this inclusion, confirming that both teacher and student participants directed their agency toward process authority by managing resources, checking students' knowledge, and/or controlling the learning behaviour. In this way, I hold that, in directing their agency towards Expertise and taking on the process authority in the classroom, the Participants were interactionally positioned by the pedagogy or by themselves within the pedagogy as learning facilitators.

6.1.1.3.2 Process Authority as Negotiated

This study determines process authority to be the product of a negotiation between Participants. This negotiation involves the recognition of the skills, experience, personalities, roles, and knowledge of all the Participants in the classroom, as well as an awareness of how these can contribute to advancing collective mathematics knowledge. The ability of participants to negotiate their participation is evidenced by the blending of authority (see section 5.2.2.1) between the teacher participants and myself, their recognition of my "mathematics knowledge for teaching", and the blending of authority amongst the Participants in their roles as teacher participants, wherein they made decisions in response to the learning behaviour of the other Participants; in each case, they assumed control over knowledge advancement. Facilitators took control of the learning behaviour, as illustrated in the analysis (see section 5.1.2.3.1), but did not control social behaviour, allowing dialogical and physical interaction to take place at participants' will (see section 5.2.2.2), promoting knowledge building as learners and knowers. These decisions regarding the distribution of control crystallised over time and became essential aspects of the practice. The innovative pedagogy, for which my initial framework was therefore only the seed, can thus be described as having established and consolidated itself through negotiation amongst Participants.

6.1.1.3.3 Summary

The "student as a Participant" can temporarily and flexibly position themselves reflexively as a learner, knower, or facilitator. They can be positioned interactionally as knowers or facilitators, and institutionally by the pedagogy as facilitators in the classroom community. These positions are indicative of the qualities of shared epistemic agency. As a learner, the Participant can control their knowing and unknowing and the ways they extend their knowledge; the learner is knowledgeable of what they do not know, and productive of knowledge building interactions that lead to the creation of New Knowledge. As a knower, Participants have epistemic authority based on their competence in epistemic interactions, and are relationally responsive to an Assumed unknowing. When positioned as a facilitator of learning in the classroom, Participants blended their control of the learning process with the authority of other Participants in the classroom, and negotiated control of the learning process based on their relative skill and experience in a given context.

I consider the Participant described by this study as competent in the learning environment. This competence is not the emancipatory result of their having surmounted societal oppression, the oppression that gives rise to the banking model of education (Freire, 1970) that I consider to bear troubling similarities to the institutional methods I have experienced in my school, and which other researchers agree is the predominant pedagogy in mathematics classrooms in England (see section 2.3.1). Their competence is not a result of their achieving intellectual equality with myself, the teacher, that Rancière (1991) argues is necessary for the emancipation of learners. However, this is not to say that the Participant is not empowered; indeed, this study points to qualities of agency that are explicitly considered to be empowering (see section 6.3).   My study, I hope, reveals that the Participant is competent to the extent that they possess the agency required to assume control, responsibility, and authority for their knowledge through their mutual and self-positioning. This competence, which renders Participants as equals within a democratic classroom community, is not hard-won; it does not require a sweeping social movement or radical change to the structure of the role of the teacher to surface, but only needs to be made visible by such practices as I have implemented in my pedagogy.

In presenting the Participant as competent, I have partly achieved the aims of this study; I argue for considering the students in my class as active Participants in the classroom who take responsibility for their learning, and who both contribute to and are constituted by a learning community. The wider aim was to present to the mathematics education community a different approach to classroom pedagogy in the context of the school curriculum. In this study, I hope it is clear that the Participant is trustworthy, responsible for and expected to possess knowledge; and, if nurtured, could transform education.

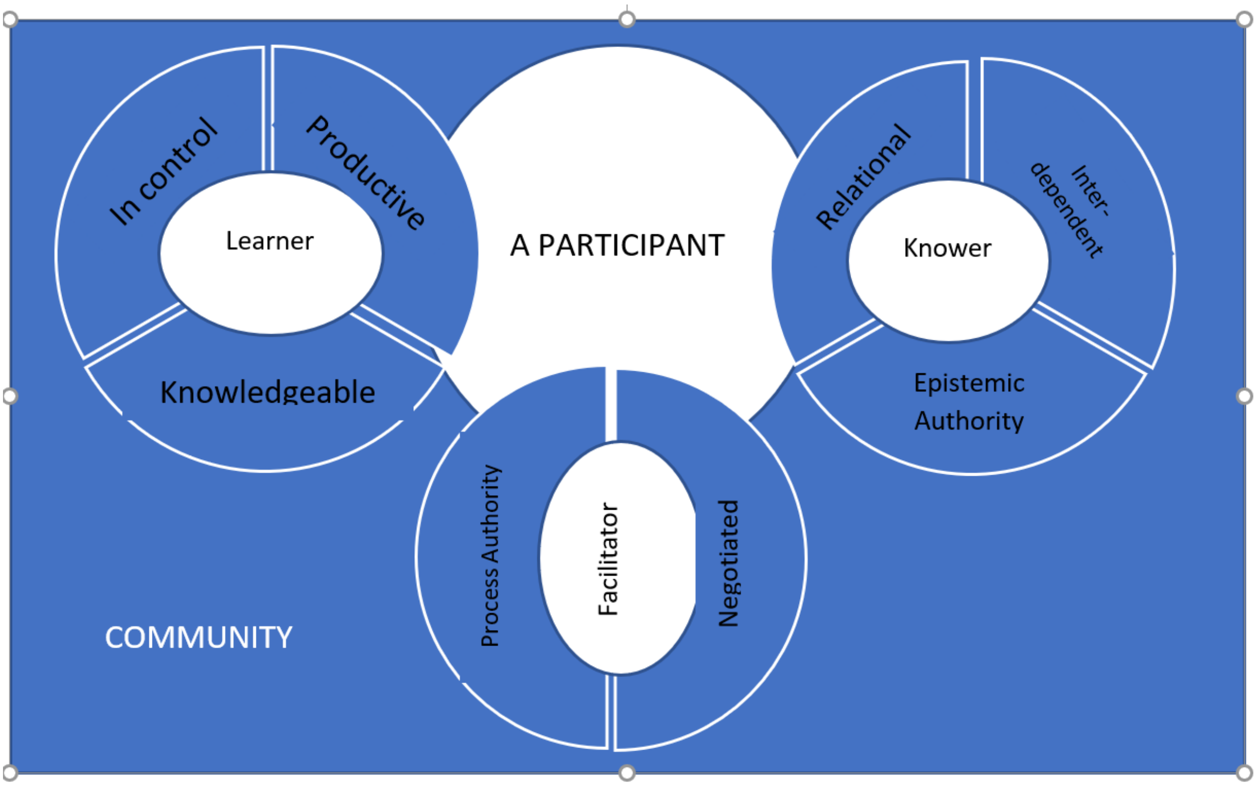


Figure 6.6 – The features of the Participant as a learner, a knower, or a facilitator

The conception of the student as a Participant raises the question of what I am as a member of the classroom, and I will discuss the specific nature of my participation in section 6.3.1.

6.1.2 Theme 2: The Concept of a "Learning Community"

The Participants in my mathematics class acted as a community that supported the advancement of each other's mathematics knowledge. They were more than a group of 18 individual students in a classroom learning mathematics; they became a community bound together by the Mutual Relations that developed around them as they endeavoured to learn the subject. It was these relations that enabled them to sustain their learning and achieve what they did.

Over time, the group emerged as a Learning Community focused on enacting the pedagogy, but the Participants did more than this alone: they brought their unique selves into the enactment, and forged their own unique way of learning mathematics. I could not have planned the Learning Community that emerged; it was its own organism, a community of Participants who were interactive in their knowledge building, democratic in their valuing of each other's participation in the innovative pedagogy, and productive of New Knowledge within the community (see Figure 6.7). This section will discuss what was noteworthy about the community that my mathematics class became and how these qualities of note indicate the shared epistemic agency of the Participants.

Figure 6.7 – The features of the Learning Community

Figure 6.7 shows the Participant within the Learning Community. This section will show how the Learning Community is interactive, productive, and democratic, and how this is an indication of shared epistemic agency. In the subsequent section, addressing research question 2, I will describe how the community's definitions of competence and accountability sustain the emergence of such agency.

6.1.2.1.1 The Interactive Classroom Practice

In section 6.1.1, I introduced the student as a multi-faceted Participant. The Participants learnt mathematics as they interacted with each other in the mathematics classroom, and these interactions defined their practice of learning mathematics in a community. Though I discussed the facets of the Participant independently, their continued existence was made possible only by their interaction.

There was a constant buzz of activity in the classroom; the most appropriate description would relate an *ecology* of epistemic interaction. All Participants mutually engaged in epistemic endeavours. They continuously and spontaneously interacted with each other from moment to moment in the classroom; they stood up, moved around the class, called out to each other, and worked independently; they formed groups and pairs, dissolved them, and regrouped or formed the same or different pairs; in these ways, they learnt mathematics (see section 5.2.2.2). The collective mathematics knowledge was advanced primarily because the free pursuit of dialogical and physical interaction supported the spontaneity of the actions and reifications that are characteristic of the learner, knower, facilitator positionings.

There was often a transparency or public element to their participation; for instance, a Participant positioning themselves as a learner provided an opportunity for other Participants to be share in the Intention towards resolving an unknowing, hence advancing their knowledge as a collective (see section 5.1.2.1.5). Public positioning as a knower or learner opened up the possibility of epistemic interactions, and other Participants join these with their own actions or reifications, or by simply listening in. Any given Participant's knowledge or lack thereof was a communal property, and, conversely, the community's unknowing and knowing were the responsibility of the individual Participants (see section 5.2.3.3).

The language used for meaning making functioned in its own way within the classroom context, and was not drawn from the standard mathematics vocabulary. My observations of epistemic interactions purposefully did not specify the quality of the mathematics communication in terms of received ideas about the curriculum, encouraging creativity and innovation. In Episode 1 (see section 5.1.2.2.4), wherein Pearl's use of the word "split" to describe the process of factorising quadratics was further elaborated by Teesh as "splitting the number usually in the middle" – this is not typical mathematical language; a discussion of performing arithmetical operations on the coefficient of b would have supplied the "proper" description. The Participants, however, clearly did not suffer from lacking the appropriate mathematics vocabulary; instead, their focus was on meaning and logic, and Participants in the classroom were able to support each other in their own informal and idiosyncratic ways – they understood each other's meanings. This understanding was facilitated by the Mutual Relations amongst the Participants that developed throughout their participation. They saw themselves in each other, and so could take risks with their learning, ask for support, and offer support to each other confidently. Once more, this mutual support should not be seen as the result of dependency or incapacity (see section 6.1.1.2.2); instead, it demonstrated the strength of the relations between collaborating agents in the Learning Community. Through their participation, they learnt how to be with each other, their personalities, their likes, their moods, and in this way, they were able to get the best out of each other; this is perhaps best demonstrated in Episode 25 (see section 5.1.2.3.2). The Participants showed solidarity with and trust in each other, praised each other (see section 5.1.2.4.1), and were sensitive to each other's perspectives. On occasion, they were also rude to each other as shown in Episode 19 (see section 4.1), but they repaired relations, and through further interactions, learnt to work with each other more patiently and effectively towards their common goal of learning mathematics.

My Participants inhabited an autonomous Learning Community; all were participating in an epistemic activity, and each interacted with the others and their knowing or unknowing as they moved between positions from moment to moment. I present this interaction as indicative of shared epistemic agency, which, given the particulars of my framework and analysis, is distinct from, if initially based upon Damşa et al.'s notion of SEA, which emphasises collaboration where I emphasise interaction.

Collaboration is "the action of working with someone to produce something" (*Dictionary Online - Google Search*, n.d.). In the context of education, there appears to be a lack of consensus on how collaborative learning should be defined; however, as a learning approach, there is agreement that it minimally involves groups of learners working together to achieve a common goal (Barron, 2000; Erickson, 1996; Stahl, 2016). In comparison, interaction involves "communication or direct involvement with someone or something; reciprocal action or influence" (Dictionary Online - Google Search, n.d.).

The difference lies in the context and the pedagogy; in Damşa's studies (Damsa et al., 2010; Damsa, 2014), the project involved undergraduate students in groups of 5-7 working together on a specific design project for a real-life client. The course leaders had defined the purpose of the collaboration from the start, and all group members were aware of the requirements of the design project. The groups had a clear outcome of producing a co-constructed knowledge object that guided their collaboration (Damsa & Ludvigsen, 2016; Oshima et al., 2018). The collaboration was evidenced to be productive of knowledge. Similarly, in collaborative or group work that occurs in mathematics lessons described in previous research (Bearison et al., 2002; Schwarz et al., 2021; Stahl, 2016), the problem to be solved is precisely defined and determined before the collaborative or group work, and each group member is aware of it. Typically, the collaborators or group members occupy fixed groupings for the duration of the collaboration. This description suggests that for an activity to be termed collaborative, it should be goal-directed, deliberate, and have a desired outcome and agreed purpose. In my study, the Plan stage of the teaching cycle, wherein the teacher participants produced the PowerPoint lesson plan can be said to be collaborative in this sense. Producing the lesson plan was the clearly defined goal of the fixed groupings that the Participants had agreed upon at stage 1 of the teaching cycle. This goal sustained the collaboration.

The remaining phases of my innovative pedagogy, however, were different. The epistemic focus was on the moment-by-moment epistemic interactions that occur in the classroom, and not on the narrow results of a collaborative effort. Consider each Episode of shared epistemic agency (see section 4.1): the Participants involved had no predetermined goal that determined the interaction, and neither was there a predetermined group for the duration of the Episode. Instead, an Episode started with a spontaneous Intention to resolve an unknowing, leading to the further spontaneous interactions of other Participants. There was a fluidity in the Participants' engagement in their interactions that sets it apart from that which arises within the rigid determinations of a merely collaborative learning environment. Participants could continuously change their positionings during the interaction and opt in and out of it at will. The knowing and unknowing of each Participant was the basis of the spontaneous interaction of the following Participants; there was dynamic epistemic reciprocity between the positionings that was productive of New

Knowledge. As is clear, this knowledge was the product of interactions (see section 5.1.3), and not of collaboration alone.

The distinction between Damşa's SEA construct and the shared epistemic agency indicative of the Participant in the learning environment is essential to note. A "pedagogy of interaction" reveals what is useful in a secondary school classroom for the emergence of shared epistemic agency – the benefits of which I hope to have shown – while a pedagogy of collaboration indicates what is useful for groups collaborating to achieve a specific project.

6.1.2.1.2 A Productive Community

The Participants learnt mathematics in line with the National Curriculum by enacting a deep constructivist pedagogy (see section 2.1.3) that expected them to take responsibility for higher-level capacities of classroom practice (Scardamalia, 2002; Scardamalia, 2014), such as planning the learning (see section 3.3.1.2), managing the learning (see section 5.1.2.3.3), checking their own and each other's learning (see section 5.1.2.3.3), and explicating knowledge (see section 5.1.2.2) – functions which are usually left to the teacher. By assuming these responsibilities, they advanced the community's mathematics knowledge by relying on their agency. Particularly indicative of shared epistemic agency was their capacity to enact the innovative pedagogy (see section 3.3.1) interchangeably as both teacher participants and student participants (see Photograph 5.2). In the Plan stage of the pedagogy, the Participants, in pairs, prepared to teach a mathematics topic to the class. To meet this requirement, they demonstrated their ability to use available resources, such as knowledgeable people beyond the classroom, the MathsWatch platform, and other media, to come to know their mathematic topics and share this knowledge with other Participants. I did not collect data during the planning stage, but it was possible to identify planning practices, drawing on my observations and the actions and reifications produced on each the teaching day. Participants exhibited a capacity to alleviate their unknowing by finding the mathematics knowledge relevant to their topic, make sense of this knowledge, use it to answer the questions in the booklet (see Extract 4.1), think about how to explicate it to the other classroom participants (see Photograph 5.3), and work in collaboration with their teaching partner to plan their PowerPoint lessons (see Photo 5.1 that shows Tom and Daniel's lesson plan on the board) and position themselves as knowers and facilitators in their lesson. Damşa et al.'s research presented the actions of groups in collaboration to produce a knowledge object (see Appendix 5); it is reasonable, in this study, to attribute these actions to the teacher Participants.

In the Share stage of the pedagogy, teacher participants came to the lesson with the questions booklet and a PowerPoint lesson. As was the practice, community knowledge was advanced through Participant epistemic interaction as they positioned and repositioned themselves. It is important to note that before the lesson, part of the classroom practice was that the teacher participants gave student participants a question from the new topic. In this way, the revised pedagogical design expected the student participants to arrive at the lesson with at least some knowledge. Both teacher and student participants bringing prior knowledge to the lessons contributed to the productive nature of knowledge-building interactions, as this knowledge was shared to build new knowledge (see section 5.1.3), challenged (see section 5.1.2.1.4), and modified through the interaction of the learner and knower as, who exercised their productive agency (see section 5.2.1.4).

6.1.2.1.3 Democratic Participation

Another indicator of shared epistemic agency was the democratic participation of all Participants in the pedagogy, regardless of their presumed ability (see section 5.2.3.2) The Learning Community did not consider ability labels, and all Participants' contributions were valued and acknowledged. Nevertheless, the Participants started the academic year with their mathematics ability labels, which were based on their performance in primary school terminal examinations, and which outlined their predicted grades at the GCSE examinations. The Participants encountered these labels and their discourse as students in other subject areas across the school, such as when studying English, Mathematics, and Languages. However, as they engaged in the practice of learning mathematics in the classroom, lesson after lesson, and enacted the pedagogy as both teacher and student participants, experiencing the different positions available to a Participant (see section 6.1.1), their participation expressed and constituted a rejection of these labels (see section 5.2.3.2). Participants did not limit their own or each other's participation based on these labels. Labels of presumed ability limit and constrain the mathematics made available to students (C. Morgan, 2013; Smith & Morgan, 2016); however, the experience of participation in the Learning Community clearly did not involve such constraints, and these labels were implicitly rejected. Thus, the innovative pedagogy and its discourse of democratised participation did not recognise or consider these labels as reifications of students' abilities.

6.1.2.2 Summary

My mathematics classroom emerged as a community of Participants with a practice of learning mathematics through interaction. This interaction was democratic, because all Participants experienced taking part in all aspects of the innovative pedagogy, and the contributions of each were valued and acknowledged. This Learning Community was productive of new mathematical knowledge based on the Participants' exercise of agency in exploring external sources, which allowed them to arrive at the classroom with the seeds of knowledge for shared development. Through epistemic interaction with other Participants, they shared, challenged, and modified this knowledge, and in this way, made sense of the mathematics. The Learning Community created knowledge from within itself to advance their knowledge as a whole.

This Learning Community that was interactive, democratic, and dependent on Participants’ capacity to take responsibility for advancing their mathematics knowledge was a product of how the community interpreted the enactment of the pedagogy. The Pedagogy did not prescribe specific practices for the Learning Community; it emerged by itself through Participants’ enactment of the pedagogy, and is an index of their participation.

As I mentioned throughout section 6.1.1, the learner, knower, and facilitator positions existed only in relation to the Learning Community that resulted from enacting the innovative pedagogy. The learner positioned themselves for a knower to emerge from the interactive, democratic, and productive Learning Community. The facilitator was so positioned by the pedagogy, and Participants positioned themselves during interaction in the Learning Community. The students as Participants in a Learning Community were interdependent non each other, and together instantiated a shared epistemic agency in a mathematics classroom.

This learning through interaction shows that learning mathematics was, and could be more than the process of one individual offering explanations to another, as it appears in the conventional pedagogy; in my study, it involved the productive democratic epistemic interactions that take place between the Participants as they position themselves as learners, knowers, and facilitators that promote lead to individual and collective knowledge advancement.

Returning to the research question, I argue for considering the student acting as a Participant and the emergence of a Learning Community, defined below respectively, as two strong indicators of shared epistemic agency.

This Participant is identifiable in their capacity as flexibly and interchangeably:

- A learner who controls their knowing and unknowing, is productive of epistemic interactions and is not knowledge-less. This capable learner has the potential to be transformative.
- A knower with epistemic authority who is relational in their response to an unknowing, and interdependent on a learner.

- A facilitator with process authority who can negotiate the blending of their authority with that of other Participants to support the advancement of collective knowledge.

The new Learning Community is identifiable as a classroom community that is:

- Interactive – the Participants learn mathematics through epistemic interactions wherein their participation positions them as learners, knowers, and facilitators
- Productive – the Participants create mathematics knowledge through the enactment of the pedagogy and their epistemic interactions therein
- Democratic – the Participants are presupposed to be able to participate, and their participation justified this presumption

6.2 Answering Research Question Two

What sustains shared epistemic agency in the mathematics classroom?

This question originated from my readings of the literature and my encounters with other researchers (Moss & Beatty, 2011; Scardamalia & Bereiter, 2010; Zhang et al., 2018) who endeavour to sustain student engagement in long-term, high-level knowledge-building activities and discourses. Each of these researchers acknowledged that the students' initial curiosity was fleeting for many individuals, and overall not sustainable in the long term. In one strand of the earlier research, a virtual “knowledge forum” platform was used to develop a knowledge-building culture that sustained a sense of belonging (Moss & Beatty, 2011; Scardamalia & Bereiter, 2010). More recently, this platform, enriched by reflective assessments supported by analytic tools (Yang, van Aalst, et al., 2020), and the related virtual “Idea thread Mapper” (Zhang et al., 2018), have sustained students' engagement in knowledge-building interactions by co-organising their inquiries and helping them to monitor the emerging directions of their learning.

In this research, I did not make use of a virtual platform to help develop a sense of community; I wanted to find out what could be capable of sustaining students' engagement in the classroom environment – what kept them coming back lesson after lesson, giving of themselves for the advancement of the community knowledge. I developed my innovative pedagogy in order to locate and intensify what sustained the students’ shared epistemic agency within their community. Based on my findings, I assert that it was a Learning Community developed through Participants’ interactions, that sustained shared epistemic agency. It was what the community defined as competence and their resulting accountability to the practice that sustained the emergence of shared epistemic agency anew in each lesson; I discuss these two factors below.

6.2.1 Competence

Competence is experienced and manifested by Participants during their engagement in the practices of a community. Counterintuitively, it should not be understood as the property of an individual, and though the community defines it, it cannot be awarded by the community to anyone in particular. As Wenger (1998) puts it, "It is not merely the ability to perform certain actions, the possession of certain pieces of information or the mastery of certain skills in the abstract" (Wenger, 1998, p. 136).

Competent membership of a community includes the ability to engage with other members and respond to their behaviours. A competent member understands the community's purpose to such a degree that they take responsibility for engaging with its purpose and its continued negotiation. They are familiar with and part of the community's actions and reifications through their participation in its practices (Wenger, 1998). In a conventional pedagogy, competence in mathematics is attributed to whomever gets answers questions correctly or performs a problem the fastest (cf. Darragh, 2013; Lambert, 2017). In my class, Participants defined competence as participation in epistemic interactions with other Participants, and not in terms of personal ownership of knowledge, that is, of knowledge stored in a learner's mind. I arrived at this interpretation of competence because my analysis of Episodes revealed that epistemic interaction was what Participants continuously did (see section 5.1), what they expected each other to do (see section 6.1.2), and what appeared to define being in the mathematics classroom for them (see section 5.2). It was also how they learnt mathematics, and learning mathematics was the purpose of their coming together as a class to begin with. Wenger's regimes of competence (Farnsworth et al., 2016; Wenger, 1998) equate a valued community member with one who participates meaningfully in what matters to the community; the community values a member of a community because of their competent participation in the community. In essence, this was represented in my study by becoming a valued Participant, and taking part in epistemic interactions as a learner, knower, and facilitator, that sustained the community and the emergence of shared epistemic agency.

6.2.1.1 Competence as a Knower

The interactional positioning of a knower (see section 6.1.1.2), that is, a Participant ascribing the knower's position to another Participant, resulted from the Participant seeing competence in the other Participants, the competence to contribute to the Extension of their existing knowledge. The findings on positioning (see section 5.2.1) show how Participants who positioned themselves as learners sought the Extension of their existing knowledge from another Participant. The learner sought this Extension from the other Participant because they judged the knower capable of extending their knowledge; they judged them to be competent. A knower is positioned due to another Participant perceiving their competence.

My analysis show that the learner did not typically expect the knower to resolve the learner’s unknowing by themselves; instead, the learner expected the knower to participate in the Extension of their knowledge. Competence lay in being part of the process, and taking on the position of a knower in that moment.

Positioning oneself as a knower is also viewed as competence, as it leads to the advancement of community knowledge. This positioning contributes to resolving an Intention triggered by an Assumed unknowing (see section 5.1.1), the first part of an Episode of shared epistemic agency. An Episode of shared epistemic agency leads to New Knowledge, and this matters to the community. The proportion of Episodes resolved by an appeal to a knower (see section 5.1.3.2.2 & Figure 5.3) further proves that a knower's competence should be understood in terms of their value to the community.

6.2.1.2 Competence as a Learner

As explained in section 6.1.1.1, positioning oneself as a learner was also seen as competence in my classroom community, as it was a requirement of the epistemic interactions that led to the creation of New Knowledge (see section 5.2.1). Participating in the community as a learner was meaningful to the classroom community as it helped achieve what mattered to them; it helped them to individually and collectively learn mathematics. Uncertainty and conflict have been observed to be trigger conditions that stimulate the initiation of learning (Clarke, 2001). This is consistent with my findings, which show, firstly, that Episodes of shared epistemic agency that led to the creation of New Knowledge were triggered by Participants' Intention to resolve an unknowing (see section 5.1), and secondly that the learner position was productive to epistemic interactions (section 6.1.1.1.2). Participants positioned themselves as learners publicly, again and again. In this way, they positioned others and themselves as knowers, again and again. This acknowledgment of each other's competence, leading them to position each other as knowers, alongside their own competence as learners, contributed to their sense of belonging to the classroom learning community (Wenger, 1998, pp. 178–179).

The idea that an expression of unknowing as representative of competence and authority (see section 5.2.2.3) – moreover, even that it is desirable in a classroom – is at odds with the dominant discourse of education. This discourse has come to consider expressions of students' uncertainty as ignorance, as demonstrated by the actions of students in other mathematics classrooms; it is connected with the mathematics classroom culture that begins with the teacher's exposition of the topic and the enforces the expectation that students listen to learn from this exposition to solve similar questions. Indeed, this expectation renders students who cannot solve problems independently as lacking. Conversely, in my experience, in-school appraisers working with teacher capabilities and performance management (DfE, 2012) judge a teacher's expositions; the implicit message is of a correlation between the quality of a teacher's exposition and the speed of students' understanding. Expressing uncertainty when following a teacher's exposition is not viewed as competence, but rather as evidence of a lack on the part of the teacher or student.

In the Learning Community that emerged in my classroom, extending one's knowledge was beneficial to the community; thus, seeking to do so, however humbly, was viewed as competent behaviour.

6.2.1.3 Competence as a Facilitator

Participants positioned by the pedagogy or by themselves as facilitators within the pedagogy were viewed as competent by the community. The pedagogy positioned the Participants as competent by means of the expectation that they take on the role of teacher participant and deliver mathematics lessons to their classmates. The expectation involved a bestowal of knowledge, capability, and authority, usually reserved for the “teacher” role, upon the Participant. When positioned as a facilitator by the pedagogy, the teacher participant was expected to check participants’ mathematics knowledge and facilitate the learning in the classroom by assuming process authority.

6.2.1.4 Competence as Productive Interaction

Competent community members engaged in what mattered to the community (Wenger, 1998), as was evidenced by the epistemic interactions between learners and knowers in the classroom. The Participants publicly asked Questions, Identified unknowing, made epistemic Requests, and Sought affirmation. In response, other Participants publicly directed their agency towards Explication, Clarifying, Affirming, and Articulating knowledge. The notion of productive agency (Schwartz & Okita, 2004) describes how individuals' agency can alter their environment in such a way as to cause adaptations to existing practices. The knowledge of an individual – whether complete or incomplete – when shared with the community, invites other individuals to use it; they modify it, causing a change in the original individual and producing collective learning. This explains the productive nature of Participants' interactions, that is, how their epistemic interaction leads to further epistemic interaction (see section 5.2.1.4), and can also explain why an individual Participant did not need absolute knowledge to establish competence. Participation in epistemic interaction was in itself productive. This conception of a process of productive interaction bears some resonances with (Stahl, 2016) notion of group cognition. Though his research relates to group work and this study relates to community interaction, his notion of group cognition nevertheless demonstrates the mechanism by which epistemic interactions are assigned value in the Learning Community. It suggests that through interaction, Participants collectively accomplished what they could not have accomplished on their own.

Competent participation is what matters to the community. Epistemic interaction mattered to the community, as was evident when the class moved out of the computer suite to a new classroom. This movement was during the GCSE exam period, during which the ICT suite became a study room (see section 3.4.5.1.1). Upon occupying the new classroom, the Participants changed the seating arrangement: they organised the tables in groups of four. Each table of two Participants faced another table of two Participants, allowing pairs to become a group of four and two groups of four to become eight. Participants working on their own was no longer the established practice.

Sociomathematical norms are the established practices of a mathematics classroom (Yackel et al., 2000); as Yackel notes, individuals' beliefs about mathematics learning, their own role, those of others, and the classroom sociomathematical norms are mutually constitutive (Yackel et al., 2000); they develop with each other. Thus, it can be inferred that classroom interactions defined not only what it meant to be a competent member of my classroom, but also what it meant to learn mathematics (i.e., the sociomathematical principles by which the classes were routinely conducted).

One of the emerging themes of the innovative pedagogy was its sustenance of multiple ways of enacting competence in the classroom: by being a temporary knower with epistemic authority, by being a learner who creates a need to know, or by being a facilitator who exercises their process authority in interactions. I have argued that moving between these positions of competency and facilitating others to do so sustained the emergence of shared epistemic agency.

6.2.2 Accountability

Our perception of ourselves (our identity) has a powerful impact on interacting, engaging, behaving, and learning within a community (Wenger, 1998), and is central to students' beliefs about their role in the classroom and their potential. Different pedagogies are not just vehicles for more or less knowledge, but shape the identities which students develop in the mathematics classroom through the practices in which they engage (Boaler, 2002b; Boaler & Greeno, 2000). My innovative pedagogy viewed students as Participants; as they participated in learning mathematics as competent members of the Learning Community, they individually and collectively negotiated their identity as such (Bishop, 2012; Wenger, 1998). Reified by other Participants as competent members of the learning community, they developed identities of belonging that made them accountable to the classroom practice (Farnsworth et al., 2016). I believe that this participation, belonging, and accountability process, as illustrated in Figure 6.8, sustained the emergence of shared epistemic agency.

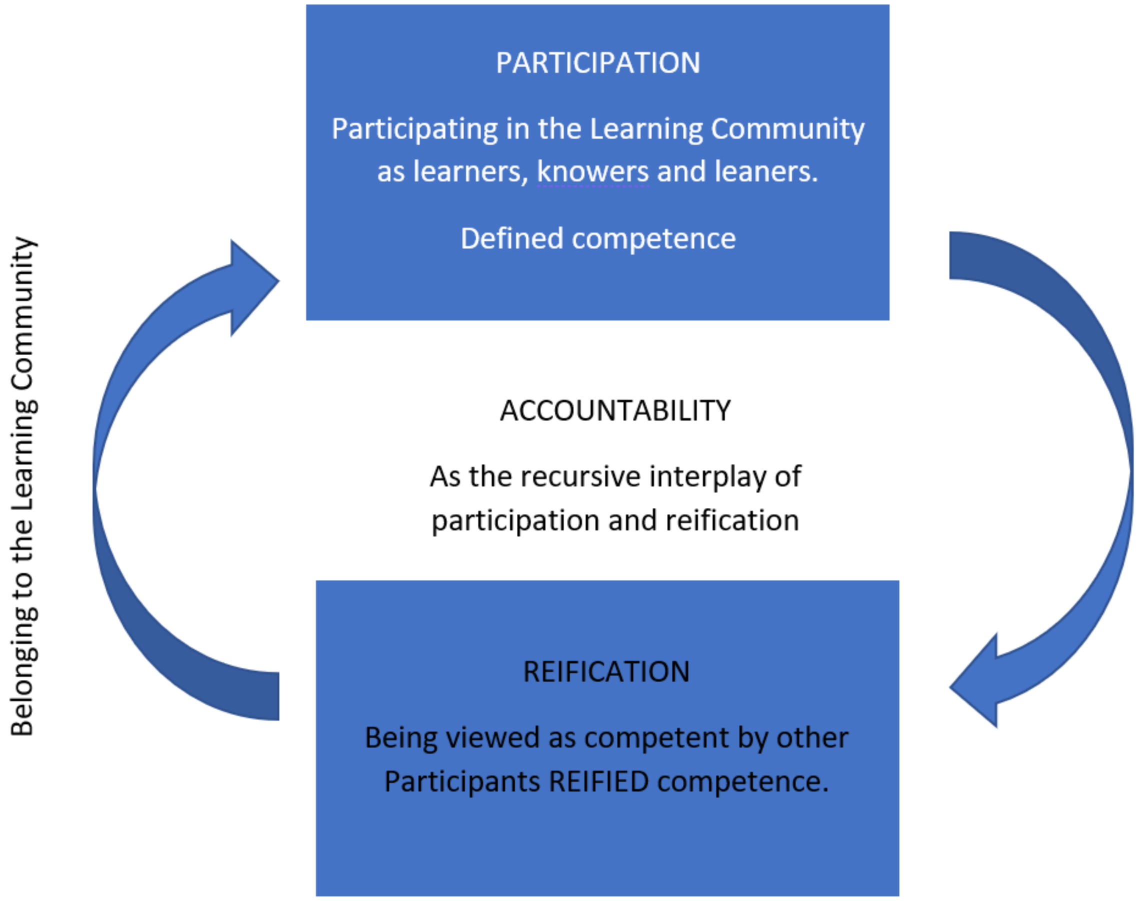


Figure 6.8 – Interplay of participation and reification

For instance, consider a Participant who takes up the position of knower; they identify themselves as competent in that moment. For the identity to be meaningful in the norms of this pedagogy, they have to participate as a knower by directing their agency towards Explication. In the same way, the innovative pedagogy positions Participants as facilitators once a teaching cycle identifies their potential competence, but it is their actions and reifications of competence that makes meaningful their identity as a valued member of the classroom, whose participation matters (Farnsworth et al., 2016; Wenger, 1998). The more opportunities for Participants to take up the positions of competence and feel valued, and the more

Participants reify their participation as competent, the more the identity of belonging emerges amongst them. Consider Recording 7 (see section 5.1.3.2.3), wherein Crimson was positioned as a facilitator by the pedagogy. He participates as a competent member of the classroom community as a facilitator and a knower. However, it is the way other Participants reify his participation by positioning him as a knower, or accepting his control of the learning behaviour, or making an affirmative statement such as “Smart, it is!” (see section 5.1.3.2.3) that establishes his identity as a competent member of the group.

Participating in the innovative pedagogy led to the emergence of Participant identities of belonging through an interplay of reification and participation particularly, reification on the part of other Participants, who identify competence and participation in the individual. This relationship is iterative and reciprocal: the more the individuals that participate in epistemic interactions and facilitate learning, the more other Participants see them as competent. Further, the more Participants see them as competent, the more they feel valued and that they belong to the community, and becoming accountable, as illustrated in Figure 6.8. The role of identity in sustaining the community is a continuous negotiation of participation and reification. Participants were continuously renegotiating their identities of belonging; consequently, they established their accountability before the community through their participation anew in each session.

Accountability to the practice of the Learning Community can explain the relational and interdependent quality of the knower positioning (see section 6.1.1.2.2) – why the Participants position themselves as a knowers again and again in response to other Participants positioning themselves as a learner. It can also explain why Participants' intentions are triggered by the identified or assumed unknowing of an individual or a group of individuals (see section 5.1). Participants do not have to respond to the unknowing of another Participant, but in the Learning Community they continuously did, as demonstrated by the thirty-six Episodes identified in the data (see section 4.2.1.1). An Episode did not end until all participants acknowledged that the unknowing had been resolved (see section 5.1.3.1). Finally, that Participants accepted that the unknowing of all Participants needed to be resolved can be explained by their accountability to the Learning Community. Summarily, accountability to the community can explain Participants' continued participation in enacting the pedagogy, and why they continued to position themselves as learners, knowers, and facilitators in the Learning Community.

6.2.3 Sustaining the Community

Competence and accountability are connected through Participants' participation in the innovative pedagogy, and, in my classroom, sustained the Learning Community that emerged (see Figure 6.9 below).

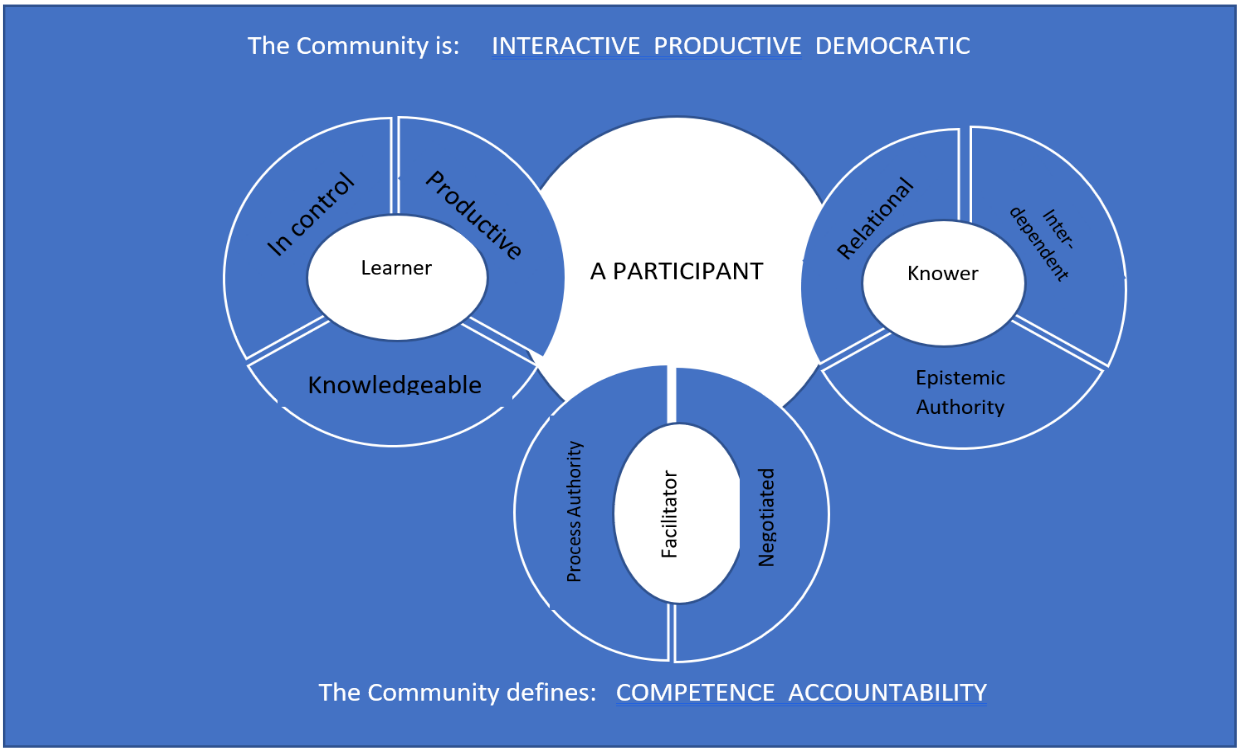


Figure 6.9 – The Participant in the Learning Community

To complete my response to research question 2, I provide evidence that the Learning Community was sustained throughout the study by citing Participants' achievement in the GCSE Mathematics terminal examinations. This data, available a year after the end of my study, can be taken as evidence that Participants sustained their accountability to the practice of the community through the following academic year. Wenger's (1998) trajectory of participation can explain this sustenance. Our identity, he explains, is temporally continuous, possessing a coherence through time that connects the past, the present, and the future (Wenger, 1998). When negotiating our current identities, we incorporate who we were in the past and who we can be in the future. It can be surmised that Participants internalised the positive experiences of the innovative pedagogy, and that this formed a significant aspect of their past; looking towards the future, the Participants also considered the trajectories of the competence of past students as they took on new identities of competence in their subsequent mathematics classrooms. There is a discourse prevalent in England according to which good grades in mathematics are a gateway to future prosperity (C. Morgan, 2013). In my secondary school, this discourse connects hard work in the mathematics classroom and good grades in the Mathematics GCSE to future economic freedom. The school reinforces this discourse by celebrating the career paths of past students who achieved good grades in mathematics. Considering these former students opens up future trajectories for the Participants in my mathematics classroom; it shows them who they can become if they work hard in mathematics. Thus, they connect future trajectories with their present identity of competent participation in a classroom.

The following academic year, the Participants of my mathematics class became my Year 11 class. I did not follow the original format of the innovative pedagogy, as the mathematics faculty curriculum map focused on past exam papers. No longer upholding a systematic research framing, I personally observed that the Participants still valued each other, took responsibility for their learning, and were accountable to each other, but I did not realise how different they were from other students until the last mathematics lesson, as revealed in Extract 6.2 below.

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Year 11 lesson recollection, Thursday 12/03/2020, written as field notes and expanded upon for clarification

On this Thursday, I combined my students with the students from another maths class, as the number of staff and students in attendance was reduced due to fears associated with COVID-19. I went with my 6 students to the teaching room of 11H (presumed to be of higher mathematics ability than my class) to form a class of 13 students.

As Year 11 students, the group was preparing for their terminal GCSE examinations. I placed a question on the board for them to solve. After about 4 minutes, I noticed that the students from 11H were sitting quietly and working, while Roan, James, and Tom from my class had gotten up and were in discussion.

I presumed that the students from 11H had solved the question, so I invited any of them to come up to the board and share the answer; none of them wanted to get up. I called one of them to come up to the board and start solving it, but he said that he had no idea how to. The other students said the same. When I asked them why they had done nothing to help themselves, they responded that "Sir" (meaning their teacher) wanted them to depend on their own knowledge. One of the students then asked me to help them.

James then said, "Miss will not help you, you have to try." James came to the board and did what he thought, with Roan and Tom interjecting at points, and eventually the question was solved.

Field notes extract 6.2 – Sustaining accountability

Extract 6.2 shows that being the teacher for a combined class made visible how the Participants' experience of the pedagogy had changed them to the extent that their shared epistemic agency had become normalised and taken for granted in our classroom setting. It points to the students as learners, knowers, and facilitators a year later. After reading the question, my class Participants immediately got up and discussed the question with other Participants. They acted as learners by first identifying their knowing and directing their agency towards seeking to extend their knowledge, and subsequently got up to speak to others and engage in epistemic interactions to this end. They acted as facilitators by directing other students in 11H to be agentic and try to solve the problem, and they acted as knowers by contributing their knowledge when James came to the board. On the contrary, the students from 11H, on realising they could not solve the question, sat quietly and waited.

Except in exceptional circumstances, all 15-to-16-year-olds in the UK take an external examination, the General Certificate of Secondary Education (GCSE), at the end of their secondary schooling. This examination has implications for both schools and individual students. For the students, it is a gateway to further education, if they achieve at least a grade 4 in Mathematics and English, which is considered a pass. The accountability measures used by the DfE see them rate schools on their students' performance at the GCSE examinations. However, the Participants of this study graduated in June 2020, during the COVID-19 pandemic. On 18 March 2020, the Secretary of State cancelled the Summer 2020 exam series to help fight the spread of COVID-19, and schools awarded students who were due to sit the exams a Centre Assessed Grade (CAG), "centre" denoting the school at which a given student is studying. The national expectation was that faculty and school leaders would assure that the awarded grades would be based on a holistic professional judgment, balancing different sources of evidence. Each school faculty ranked the students (Jadhav, 2020) by their performance and then awarded a grade. Following the directive, in my school, the subject head used evidence from two mock examinations already submitted on the school system to arrive at the rankings and thereby the CAGs. Individual class teachers were not involved in this process. The centre heads further standardised all subject results.

Table 6.1 below shows the results and ranks of the Participants. Column 2 is the minimum expected grade for each student, based on their end of primary school data. Column 3 shows their ranking before starting this study, based on their end of Year 9 internal examinations (EOY9) that took place in June 2018. Column 4 shows their rankings in June 2020, and column 5 is their Centre Assessed Grade for GCSE Mathematics.

1	2	3	4	5
Participant	**Min Expected Grade**	**EOY9 Rank 2018 (239)**	**CAG Maths Rank 2020 (230)**	**CAG Maths**
1	*4*	*58*	*12*	*9*
2	*6*	*73*	*23*	*8*
3	*4*	*184*	*67*	*7*
4	*4*	*85*	*70*	*7*
5	*4*	*N*	*73*	*6*
6	*4*	*74*	*76*	*6*
7	*4*	*188*	*100*	*6*
8	*4*	*113*	*104*	*6*
9	*4*	*165*	*107*	*6*
10	*4*	*71*	*112*	*6*
11	*4*	*181*	*129*	*5*
12	*4*	*138*	*137*	*5*
13	*4*	*143*	*146*	*4*
14	*2*	*133*	*168*	*4*
15	*2*	*180*	*172*	*4*
16	*4*	*Not yet on roll*	*175*	*4*
17	*2*	*165*	*186*	*3*
18	*2*	*N*	*208*	*2*

Table 6.1 – Participants GCSE outcomes

I view this table as evidence that the Participants benefited from the innovative pedagogy. Most of the students improved their ranking over the two years; the mean change in ranking was calculated to be -22.8 places. Furthermore, the rankings compare the Participants to the other students in the cohort who experienced a conventional pedagogy. It is clear from the data that the innovative pedagogy did not adversely affect the Participants. Rather, when comparing the students' minimum expected grades to their CAG's, 83% of the class, except for three students (who met expectations), exceeded presumed expectations, compared with 73% of the whole cohort.

6.2.4 Summary

Research question 2 originated in my encounters with other researchers' endeavours to sustain student engagement in long-term, high-level, knowledge-building activities and discourses. While an online learning culture sustained students' engagement in these researches, I wanted to develop an innovative pedagogy without reliance on technological means, and subsequently to find out what kept the Participants coming back lesson after lesson and with their classroom community in order to advanced their mathematics knowledge. I contend in this section that:

- The Learning Community defined competence as participation in what mattered to them.
- Advancing mathematics knowledge mattered to the Participants in the Learning Community.

- Participation in epistemic interaction as learners, knowers and facilitators was productive of mathematics knowledge.

Thus, participation as a learner, knower, and facilitator constituted competence in the Learning Community, and also shaped what it meant to learn mathematics. It is how the Learning Community defines competence that sustains the emergence of shared epistemic agency.

I further argued that:

- As Participants engaged with the Learning Community, they continuously negotiated their identity of belonging to it.
- They developed identities of belonging when their competent performance was reified by other Participants.
- Identities of belonging made Participants accountable to the practice of the Learning Community.
- As the Participants participated again and again, and as their participation was reified as competent again and again, their identity of belonging to the practice of the Learning Community was renegotiated again and again.

Hence, Participants' identities of belonging, which were cautiously negotiated by the interplay of participation and reification, established their accountability to the practice of the Learning Community, and therefore to its aims of advancing collective knowledge.

Returning to research question 2, I argued that the Learning Community, by its definition of competence and accountability to the practice of how Participants learnt mathematics, sustained the continued emergence of shared epistemic agency, even as students continued their studies in a new environment in Year 11.

6.3 Reflecting on the Action Research

Having answered research question 2, I realised the need to give an account of the development of shared epistemic agency that focused particularly on my role as a participant and the innovative pedagogy. This section records my reflections on the research to this end.

My reflections on the development of shared epistemic agency have, as their bases of evidence, a different kind of data from the systematic Episodes that informed the rest of the study. While such data was neither rigorously collected nor systematised, it reflects upon my experience as a researcher and teacher in a manner that is faithful to the process of action research. It draws upon any evidence that is available to me through means other than formal methods of data collection; thus, it primarily consists in my personal recollections and recognitions from the period of the holistic action research process (McNiff, 2017, pp. 25–26).

As a supplement to the modalities of participation in epistemic interaction that I have discussed in the previous two sections of this chapter, I contend in this section that two further factors developed the Participant and the Learning Community: those of my role as a participant and of the innovative pedagogy itself.

6.3.1 My Role as a Participant

As I enacted the innovative pedagogy that expected Participants to take responsibility for higher-level capabilities of classroom practice, such as planning what is learnt (as a knower) and how it is learnt (as a facilitator), organisation of the classroom learning (as a facilitator), and evaluation of their learning (as a learner and as a knower), I myself also had to participate differently. Handing such responsibilities over to students went against the dominant discourse of teaching, as well as against what I learnt and experienced in the position I held as a mathematics teacher for over two decades. Teaching can be taken to mean "everything that teachers must do to support the learning of their student" (Loewenberg Ball et al., 2008, p. 395). Though I was the teacher of the mathematics class, with the corresponding ethical responsibility for the students' mathematics learning, I had to extend my ethical responsibility as a classroom teacher of mathematics beyond this narrow definition, to that of an Educator of mathematics. This extension of my role took my responsibilities beyond those of simply "supporting" students' current mathematics knowledge, towards empowering them to control their mathematics knowledge and unknowing even beyond secondary education. The word "educate" comes from the root word "educe", which means to "bring out or develop (something latent or potential)" (Dictionary Online - Google Search, n.d.). As an "Educator", a term I borrow from (Kolb et al., 2014, p. 207), my role was to draw out from my students their latent potential. This term emphasises the aspects of teaching that enable others to continuously learn (Pelletier, 2012), and suggests the implicit belief that learning is most effective when students participate in knowledge creation through the use of their own intelligence, experience, experiments, persistence, and attentiveness (Biesta, 2017; Engels-Schwarzpaul, 2015, pp. 1254–1255). As an Educator, I want my students to acknowledge what they do not know, and to have the capacity and will to use the resources at their disposal to extend their knowledge. As an Educator who sought to empower the Participant, and as a researcher into students' mathematics learning, I also became a Participant in the Learning Community, and I myself enacted the innovative pedagogy as a learner, knower, and facilitator.

6.3.1.1 My Position as a Learner

In order to realise the notions of the Participant and the Learning Community, I positioned myself as a learner in the innovative pedagogy; however, I was not learning secondary school mathematics. I was learning to be an Educator. This required me to learn to trust the Participants, their mathematical knowledge as knowers, their participation as facilitators, and their desire to be successful in mathematics as learners. This trust enabled the Participants to enact the innovative pedagogy, and, on my end, to answer the research questions and fulfil the aims of the study.

Learning to trust was essential in my journey towards becoming an Educator. By trusting, I overcame my fear that the Participants would not learn mathematics well without me being in authority; I was concerned that the Participants could not be as able a teacher as I was. Trust emerged over time as I experienced Participants' enactment of the innovative pedagogy. In the earliest teaching cycles, I met with the teacher participants before they taught their lessons to prevent my fear from becoming a reality. Meeting with the Participants made me realise how prepared and organised they could be, and halfway through the second teaching cycle, I stopped meeting with them. I began to trust in their capabilities as learners to enact the Plan stage of the innovative pedagogy without supervision.

I also learnt to trust Participants as knowers, enabling me to deal with uncertain epistemic situations during the Share stage of the innovative pedagogy. These situations included those in which I had to decide how to respond when a Participant could not answer a mathematics question, or when they gave an incomplete or unexpected answer or reasoning. This negotiation of uncertainty is one of the reasons teachers find it frustrating to share authority with students; they find it overwhelming to examine and to act on a possibly confusing or unanticipated mathematical contribution without preparation, preferring to control the dialogue through direct instruction (Sullivan et al., 2020). When a Participant could not give an answer to a mathematics question, I learnt from my experience the negative consequences of undermining the Participants by publicly doubting their capacity as knowers and seizing epistemic authority as a knower (see Appendix 9). I learnt to be patient and to give Participants in the Learning Community the opportunity as knowers to respond to the uncertainty by themselves. For instance, in Extract 4.3 (see section 4.2.1.2) from teaching cycle 3, when the teacher participant Teesh could not answer the question asked by Crimson and Pearl, the other teacher participant made the statement, “Guys we don’t know, so we have to come up with an answer together”; prior to that, student participant Jevonte said “go on MathsWatch” to find out the answer. Even as the Participants were coming around to a response, I was not patient, I interjected, and, in teacherly fashion, steered them towards the answer. In contrast, my patience in subsequent cycles allowed Participants to come up with suitable answers upon which that they all agreed without my intervention, as shown in Extract 5.13 (see section 5.1.2.3.3), wherein I did not seize epistemic authority when Pearl expressed her unknowing, allowing Teesh to emerge as a knower.

A strategy I learnt to employ in response to Participants' public unknowing was to privately pose a question to challenge an individual participant's mathematics knowledge. An extract from Episode 30 (see section 5.1.3.2.4) exemplifies this strategy, encouraging, without disrupting or preventing, the emergence of a knower. Deepz was the teacher participant for the mathematics topic of bounds; Jevonte was at the board finding the upper and lower bound to the nearest 5 metres. Using the method presented by Deepz, Jevonte was adding and subtracting 0.5. The Participants who were all looking at the board did not challenge the inconsistency on the board. As an Educator, in order not to undermine Deepz and other Participants' claims to the position of knower, I questioned Crimson individually as to the difference between rounding to the nearest metre and rounding to the nearest 5 metres. As a learner, Crimson, having conducted his own research on the internet, challenged Jevonte. The epistemic interaction that ensued resolved the unknowing and produced New Knowledge. In this way, I was able to help the Participants, without compromising their status as knowers.

Trusting the Participants as knowers enabled me to learn, as it challenged what I believed about mathematics and its education. For instance, Crimson and Beyoncé taught the basic principles of Pythagoras' theorem during teaching cycle 5 in order to introduce their application in 3D space. They started the lesson with the explanation that Pythagoras could be solved either algebraically or non-algebraically. They explained that the non-algebraic method was "square, square, add, square root" or "square, square, subtract, square root". I had up to this point only experienced solving the Pythagoras theorem in terms of the algebraic equation $a^2 + b^2 = c^2$. This experience with Crimson and Beyoncé as teacher participants challenged me to truly commit to my belief that mathematics is dynamic and subjective, and that it should be always be relevant to the context in which it is used. I readily accepted the Participants' knowledge as legitimately mathematical; their non-algebraic method made sense, and was effective for solving the relevant problems.

Teachers have to confront the subjective nature of their beliefs about mathematics and what constitutes its proper practice. While I questioned the mathematics behind the non-algebraic method that Crimson adopted from an external source, and questioned my students using the non-algebraic method, the experience challenged me to reconsider what it means to "do" mathematics, and what is valid and invalid. While I have not fully resolved whether the non-algebraic method that I allowed Crimson to use in the class is totally mathematically sound, the Participants' challenge to my beliefs positioned me as a learner seeking to extend my knowledge of what constitutes mathematics.

6.3.1.2 My Position as a Knower

As a knower enacting the innovative pedagogy, my participation was to validate the Participant as a knower. To this end, I had to view myself as a co-knower in the

Learning Community. That meant that my mathematics knowledge was not the dominant knowledge in the classroom; it was a voice like any other voice, and I was not the sole epistemic authority.

In the knower position, my mathematics knowledge was challenged by other participants as they sought to extend their own. For example, when Deepz, as a teacher participant, was introducing the concept of bounds using the strategy of halving the place value, Participants such as Daniel and James, who walked up to the board, asked many questions as they sought to extend their knowledge (see Extract 5.5). Deepz, the knower, directed his agency towards Explication as they engaged in an epistemic interaction. Positioning myself as a knower, I went to James and explained another way of finding the upper and lower bounds (it was the way I had taught it for years). James listened to me, but expressed a preference for Deepz's Explication; he went back to Deepz and continued an epistemic interaction until he resolved his unknowing.

In the Learning Community, in service to the emergence of the Participant, I was a knower in the background so that Participants could be knowers publicly. As an experienced mathematics teacher, I was able to contribute my mathematics knowledge for teaching (see section 2.3.2.1), including my knowledge of the mathematics curriculum, the sequencing of mathematics topics, and the examination requirements, to prepare the booklets that offered Participants guidance on the boundaries of the mathematical knowledge required for completing each topic (see section 3.4.1.3). In this way, as knowers, the Participants and I were interdependent.

6.3.1.3 My Position as a Facilitator

In my capacity as a facilitator, in order to develop and enact the innovative pedagogy that sought to change my students' perspective of what it means to be a learner, I had to change how I behaved in the classroom to enable the Participant to emerge.

To become a Participant, the students had to overcome their initial resistance to what they viewed as taking on the teacher's role, and commit to participation in the innovative pedagogy. It was my responsibility to facilitate this change, and I used my authority as the classroom teacher to initiate the students' undertaking of the first teaching cycle. Following their enactment of the first teaching cycle and reflection on their participation, the emerging Participants did not resist engaging in the second or subsequent teaching cycles; they appeared to recognise their competence and become willingly accountable to the Learning Community.

I view the Educator as one who has the will to

> Forbid the supposed ignorant one the satisfaction of what is known, the satisfaction of admitting that one is incapable of knowing more. Such a teacher forces the student to prove his or her capacity, to continue the intellectual journey the way it had begun. This logic operating under the presupposition of equality and requiring its verification (Rancière in Bingham et al., 2010, p. 5).

Encouraging the students to participate in the first teaching cycle, indeed appeared to allow them to recognise their capabilities in enacting the innovative pedagogy, and their reflections seemed to verify the success of this participation.

I negotiated how I behaved in the Learning Community, my actions, and my reifications with the Participants. Sharing authority requires more than the teacher giving up some of their authority; it is also about the students' willingness to pick up the slack. Giving up my authority and taking up certain aspects was negotiated in the Learning Community on a lesson-by-lesson basis. The authority negotiations were situational; every lesson was a new day, subject to such considerations as the personalities and performances of the teacher participants, the mathematics topic, the questions, the weather, and, perhaps, what happened the day before and in previous lessons. The sharing of authority from my perspective was more of a gradual and nonspecific blending. It required a recognition that in this pedagogy, authority was not mine to share with the Participants; instead, based on our different strengths and our interdependence, we negotiated on each day and in each moment what each Participant would or would not do. This required me to decide how I should behave (cf. Blasco et al., 2021; Kolb et al., 2014) in each moment. This perhaps gives sense to Wagner and Herbel-Eisenmann’s provocative, if gnomic, statement: "for a teacher to share *authority* is not like sharing a cookie, where if half is given away, only half is left" (2014, p. 872). Instead, when authority is shared (and not divided up), it circulates amongst the Participants.

6.3.2 The Pedagogy of Trust

In chapter 1, I attempted to convey my motivations for undertaking this study. I had experienced the way in which students taking responsibility for what they know and do not know changes their relationship with mathematics (see section 1.1.3.2), and I wanted that to become the standard experience of the students in my mathematics classrooms. I started this study to discover the best environment for allowing students to participate in all aspects of learning mathematics, in the belief that this participation would improve their relationship with the subject and empower them to achieve the grades that would avail them of better opportunities in life. I designed the pedagogy to fulfil this purpose: to advocate for the participation and empowerment of my students. What makes my pedagogy different from conventional forms is that rather than changing the concepts of study, the curriculum, or the classroom environment, it focused on changing its perception of the subject of the pedagogy to achieve the desired outcome. The subject of the pedagogy is the student; the pedagogy described this student within the existing structures of the school. In saying that the pedagogy *described* the student, I mean to suggest that I designed the pedagogy to be enacted by students who already owned the qualities it aimed to produce, rather than designing a pedagogy that would produce the desired qualities in students. In this way, the latent potential already inhering in the student had to emerge to meet the expectations of the pedagogy. For this reason, I refer to my pedagogy as a “pedagogy of trust”. I trusted the subject of the pedagogy to embody its purpose; I trusted in the student's ability to enact the pedagogy, and, as a result, achieved a new arrangement which transformed the student into a Participant, the teacher into an Educator, and the classroom into a Learning Community.

6.3.2.1 The Student, a Participant

The student was not solely an individual seeking to learn mathematics; the student in the innovative pedagogy was a Participant: a competent, knowledgeable individual operating in a Learning Community. This Participant and their peers could collectively and interdependently advance their mathematics knowledge.

As the focus of the pedagogy, the Participant is responsible for what they know and do not know, and also takes responsibility for the knowing and unknowing of their fellow Participants. This agency situated the Participants as autonomous agents in the Learning Community, directing how they learnt with the participant Educator who was in dialogue with them. Mathematical knowledge was available and accessible to Participants, and the pedagogy presupposed that they could access and make sense of this knowledge. Conversely, conventional pedagogies view the teacher as essential to students' acquisition of knowledge, and do not presuppose students' capacity to be knowledgeable before interacting with the teacher. In such the pedagogies, epistemic authority lies with the teacher, and is intrinsic to their role; on the other hand, in my study, knowledge tracks and locates authority. Given that the Participant is in possession of knowledge in the innovative pedagogy, it opens up the possibility of authority lying with the student.

6.3.2.2 The Teacher, an Educator

The innovative pedagogy presumes the existence of an Educator; it does not describe the teacher. This lack of description could be a limitation in contrast to conventional pedagogy, which focuses on how teachers behave. However, it points to the teacher's function as situationally positioned in relation to the student – as dynamic and adaptive to new contexts. The assumption is that the Educator, as responsible for the students' education, should make a situational decision regarding how they should act in order to best enable the students to fulfil the purpose of the pedagogy. This description of the Educator contrasts with that of the teacher, whose purpose is to use the knowledge they have gained from institutional teacher education, awareness of policy, and practice in order to instruct students on what they should learn and how they should learn it.

The contrast between teacher and Educator is broadly analogous to the contrast between scientific research and historical research discussed in section 3.2.1. Scientific research aims at objectivity, believing that knowledge is certain and true, and that general rules can be applied universally to produce uniform outcomes. In contrast, historical research is subjective, embracing a pluralistic view of knowledge as contextual, uncertain, and open to modification.

The conventional pedagogy expects the teacher to act in conventional ways. The purpose of education is universal for all students. The teacher is not called upon to use their situational understanding in response to the actions and reifications of the students; instead, the teacher seeks for the students to adapt to the established practice.

The innovative pedagogy, on the other hand, accepts that the Participants will act in unique and unforeseeable ways in the Learning Community to advance their mathematics knowledge. It expects that the Educator, as an intelligent professional, can use their "situational understanding" (Elliott, 2011, pp. 66–67), established by a repertoire of experience, to decide how to respond to the actions and reifications of the Participants as they produce them – actions and reifications that may be difficult to stereotype and taxonomize. The Educator cannot rely on conventional rules and practices, but recognises that the Participants' attitudes towards producing actions and reifications are flexible. Hence, the Educator needs to select, from several possible actions, how best to respond to Participants’ behaviour to sustain a positive learning environment.

Similarly, Carolin Kreber (2013) refers to the Educator as one who acts “phonetically” (pp. 149–150). To act phonetically is to recognise that human beings are unpredictable, and that the engagement required for them to achieve genuine knowledge does not consist solely in the reception of scientific knowledge; rather, the Educator must accurately assess a situation and make an appropriate decision while abandoning the security of regulations and rules.

Mutual uncertainty appears as a characteristic of the pedagogy. As a teacher-researcher, I started the enactment of the pedagogy unsure of how it would be enacted by the participants and the impact it would have on their mathematics learning. I had to through the action research process constantly renegotiate its terms and structures in response to developments in the shape of daily interactions learning. At the same time, the Participants engage in the epistemic interactions uncertain of their mathematics knowledge but resiliently building on each other’s knowledge. Dealing with uncertainty contrasts with the structured and established practice of the conventional pedagogy. However, I argue that what is required is for teachers to accept that a pedagogy based on uncertainty is of interest.

6.3.2.3 The Classroom, a Learning Community

The pedagogy did not describe the Learning Community; it emerged by itself through the actions and reifications of the Participants as they enacted the innovative pedagogy, and the through the actions and reifications of the Educator as I responded to the Participants. I infer that the particular learning community that emerged in this study resulted from the Participants and their unique enactment of the innovative pedagogy. A significant contrast with conventional pedagogies that becomes clear here is that that this Learning Community was enabled to emerge. In conventional pedagogies, the sense of community is not essential to the mathematics classroom; nor is any particular knowledge-producing environment encouraged to realise itself – rather, the objective is to have students assimilate and be assimilated into a prescribed practice.

In conventional mathematics classrooms, learning is primarily an individual endeavour, as demonstrated by the students in 11H (see Field Notes Extract 6.2). While students may help each other on occasion as they work in pairs or groups, the purpose of the activities that take place in the mathematics classroom activity is to have each student learn for themselves. In these learning environments, students are recognised for actions that contribute to their individual knowledge. In the innovative pedagogy, the Learning Community emerged autonomously as the Participants sought to advance their individual mathematics knowledge and that of other Participants; in this pedagogy, personal knowledge was the community's property, as Participants shared their knowledge from moment to moment. In the Learning Community, learning occurs as a process in which the boundaries between individual and community learning become blurred and permeable.

6.4 Summary

In this chapter, I present answers to the research questions posed at the start of this study. These answers were arrived at by considering the two themes that culminated out of the findings outlined in chapter 5. These themes were of the student as Participant and the Learning Community. Reflecting on the action research process, I presented the notions of the teacher as an Educator and the innovative pedagogy as the necessary means for initiating the development of the Participant and their Learning Community.

The new conceptualisation of the student as a Participant and the classroom as a Learning Community are presented as indicative of shared epistemic agency. The Participant can be positioned reflexively as a learner, interactionally as a knower or facilitator, and institutionally as a facilitator in a Learning Community that is interactive, democratic, and productive.

The Learning Community defines competence and Participants' accountability to its practice of learning mathematics through the enactment of the innovative pedagogy. The manner in which the Learning Community defines competence triggers the negotiation of the terms of Participants' belonging within the community and their accountability to its practice; this process takes place through participation and its reification on the part of Participants, sustaining the emergence of shared epistemic agency.

I present the teacher as an Educator whose purpose is to draw out from Participants their latent potential to take responsibility for the advancement of their mathematics knowledge, as a necessary condition for the development of their status as a

Participant in a Learning Community that is indicative of their shared epistemic agency. In this sense, I present the Educator as necessary for the development of shared epistemic agency, and ultimately, as serving the purpose of the pedagogy.

The innovative pedagogy, with its aims of installing participation and empowerment, recognises that each Participant engage with the learning process differently; that is, they will bring their own personalities, experiences, and background to the enactment. This uniqueness could give the impression that the findings of this study could apply only in the context of the participants of this study; however, I contend that this study's contributions to the field of mathematics education points to its generalisability. I present these contributions in the next chapter.

7 CONCLUSION

This chapter reviews the central argument of the thesis, which highlights students' competence as Participants in their mathematics learning, and points to the potential that the study maintains for transforming mathematics education. In the first chapter, I described the origins of my motivation for conducting this study: how my experiences as a mathematics teacher led me to critically consider the established practices and discourses surrounding students and their learning, and especially the manner in which they directed my actions in the classroom. I described how my professional interventions, which sought to enhance students' participation, highlighted the need for a more rigorous study. In the second chapter, I detailed my embarkation on a search for a theory upon which to base this study, and established a theoretical grounding for the possibility that students could create their own mathematics knowledge. I established that shared epistemic agency is the quality that I wanted to emerge in my students, and identified the concepts of knowledge building and knowledge creation that would accompany this emergence, in order to give form to the kind of pedagogy that will enable this agency in a secondary school classroom. I synthesised six characteristics from the literature that substantiate my conceptualisation of shared epistemic agency. Researching existing transformative pedagogies suggested initial design principles, and revealed a gap in previous experimental designs which could be filled by an innovative pedagogy that would drive the student agency I sought. On the strength of this chapter, I realised the questions the study needed to answer were:

1. What are the indicators of shared epistemic agency in the mathematics

classroom?

2. What sustains the emergence of shared epistemic agency in the mathematics classroom?

These two questions oriented the research towards describing the shared epistemic agency that was operational in my secondary school mathematics classroom in England.

The following three chapters focused on answering these questions. The knowledge I gained from chapter 2 informed the pedagogic principles on which I based my design of the innovative pedagogy outlined in chapter 3. I put forward my justification for using a qualitative action research methodology to systematically identify the characteristics of the shared epistemic agency as they emerged amongst the Participants. This chapter on methodology presented my design for the pedagogy in terms of four-stage teaching cycles that allowed continuous adaptation to the pedagogy and two research cycles that allowed data collection and reflection to improve the next cycle. The chapter also discussed the measures I took to assure the quality of mathematics knowledge acquired by students, and the ability of the research to answer its questions: by my responses to reflection after each teaching cycle and after the first research cycle.

In Chapter 4, I introduced a method of analysis that, in combination with the research and pedagogy designs, forms one of my original contributions. I needed to find a systematic way of selecting from a huge quantity of video data. I developed the notion of an Episode of shared epistemic agency (the unit of analysis) based on the six characteristics that I had synthesised from the literature. Chapter 5 developed a more nuanced characterisation of shared epistemic agency, and presented findings by analysing students' interactions. The findings of the study are summarised below.

Knowledge building

- Students can advance their individual and collective mathematics knowledge by epistemic interaction by which new knowledge is built in response to an identified or assumed unknowing.
- Students can take responsibility for their individual and collective knowledge advancement.
- During epistemic interaction, students seek to know from other students by asking epistemic questions, seeking affirmation, making requests, challenging knowledge, and articulating their unknowing. They explicate their mathematics knowledge by clarifying, affirming others' knowledge, telling, and explicating others' unknowing. They facilitate the learning of others by controlling the learning behaviour in the classroom, checking each other's learning, and managing the learning resources. The mutual relations that developed amongst participants were conducive for the advancement of mathematics knowledge.
- The students built new mathematics knowledge by appealing to procedural knowledge, a knowledgeable participant, conceptual knowledge; or by a combination of two or all three of these appeals.

Positioning

- As students interact to advance their mathematics knowledge, they can be positioned flexibly as a learner, knower, or facilitator from moment to moment.

- The students can reflexively position themselves as a learner, knower, or facilitator; they can be interactionally positioned by others as a knower or facilitator; and can be institutionally positioned by the pedagogy as a facilitator.

Process authority

- Process authority emerges during epistemic interaction, and constitutes a blending of authority. This blending of authority arises from the mutual interdependency of the experiences and skills of the students and the teacher. Students controlled their dialogical and physical actions in the learning environment during epistemic interaction
- The student is an authority, and can influence the behaviour of other participants when in the learner position. The student is also knowledgeable in this position.

Epistemic authority

- Participation in a learning community is democratic, and proceeds without regard for the ability labels ascribed to students by the school. As students interacted to enact the innovative pedagogy, their epistemic authority highlighted that mathematics knowledge was required for a participant to direct their agency towards Extension. Students enacted the pedagogy irrespective of the ability labels assigned to them by the school, and they took responsibility for their individual and collective knowledge advancement.

Finally, the innovative pedagogy that presumed the competence of its participants supported the emergence of shared epistemic agency.

Responding to the research question, in the sixth chapter, I reconceptualised shared epistemic agency from a set of discrete types of behaviour towards a more holistic view of student participation and community practice.

In the remainder of this chapter, I outline the potential contributions of this study, both to research and to educational practice in general. It is organised into four sections. In the first section I outline my contributions to the field of mathematics education, and how my contribution challenges current policy and practice. The second section discusses my contributions to research methodology, and in the third section I present the contribution this study makes to the theory of education. In the fourth section I discuss some limitations of the research and suggest avenues for further study.

7.1 A Contribution to the Field of Mathematics Education

In this section, I present my contribution to the field of mathematics education in two parts. In the first part, I present an alternative perspective of the Participant and Educator, and observe how these conceptualisations of the roles of student and teacher challenge current educational policy and practice. In the second part of this section, I present my innovative pedagogy that is purposeful for the emergence of the Participant in the Learning Community, and discuss how this Learning Community presents a challenge to educational policy.

7.1.1 A Contribution – The Participant and The Educator

I present an alternative perspective on the student and the teacher in the mathematics classroom before the field of mathematics education. This study started with my desire to improve students' participation in their learning by breaking down the crystallised power-relations within the classroom that frame and limit students' participation in the secondary school classroom. Having now completed the study, I am now able to demonstrate that students can indeed transcend the confines of the conventional teacher-student roles and take charge of their learning. In doing so, they exhibit the power to change the existing notions of “student”, “teacher”, and “mathematics classroom”. What emerged from this study is a transformative conceptualisation of the student as a Participant (see section 6.1.1), the classroom as a Learning Community (see section 6.1.2), and the teacher as an Educator (see section 6.3.1).

I identify the essential feature of the Participant in the Learning Community as their competence (see section 6.2.1). This Participant can be a learner, knower, or facilitator at any given moment in the interactions of the Learning Community as they seek to advance their knowledge and that of other Participants. Positioned as a learner, the Participant commands what they know and what they are yet to know. This command is the result of their reflexive positioning as a learner. A learner cannot be positioned as such by another Participant or by the discourse of schooling, but only by their own actions and reifications. One of the most significant insights from this study has been my claim that the Participant is an authority in this position

– not just because they are knowledgeable in their awareness of what they do not know, nor because they can control how they seek to extend their knowledge; rather, their authority is sustained by their ability to set in motion the actions and reifications on the part of their fellow Participants that lead to knowledge creation. The learner is productive because as they position themselves as learners, they position another Participant or cause another Participant to position themselves as a knower.

Unlike the learner position, which is always reflexive, a Participant may either position themselves as a knower or be so positioned by another Participant. Taking up the position of a knower in response to the positioning of a learner, the Participant demonstrates the interdependence of the two positionings; they also further show they can be relational in the learning community, recognising and responding to the epistemic needs of other Participants. The knower has epistemic authority in the Learning Community as they explicate their mathematics knowledge. However, I have shown that this Explication does not stupefy the learner, because, unlike in classrooms implementing the typical teacher-student relationship, in the Learning Community the learner and knower treat each other as being of equal intelligence; they see themselves in each other, having themselves taken up the positions of learner and knower that circulate amongst Participants. This positioning as knower and learner then forms the basis of a process of epistemic interaction (knowledge building) that leads to the creation of New Knowledge.

A Participant may position themselves as a facilitator, or may be so positioned by another Participant. Unlike other positionings, the Participants were also positioned institutionally by the design pedagogy as facilitators of learning. In this position, they exercised process authority in controlling how the learning was organised in the

Learning Community. In this position, the Participants continuously negotiated process authority with other Participants, including myself. My own role as a participant and my authority were open to negotiation within the Learning Community, leading to a blending of authority that recognised our interdependence; it was negotiated lesson by lesson, moment by moment, and was not rigidly determined either by the design or by fiat.

Through negotiation, a practice emerges in which all Participants take control of the dialogical and physical interactions necessary for the advancement of mathematics knowledge; in this way, authority circulates amongst participants. This practice, in which Participants control their own epistemic interactions, stands in contrast to the conventional practice, in which students sit in silence working independently, or in which they only briefly undertake group work orchestrated by teachers. The control that circulates amongst Participants lies in their production and management of spontaneous movements or dialogues that fulfil an immediate epistemic need.

In addition to my reconceptualization of the Participant, I contribute from my reflections (see section 6.3) the possibility of conceiving of the teacher as an Educator. The Educator's role is to draw out the Participants' latent potential. The Educator recognises that Participants behave in unpredictable ways; thus, the rules and regulations that underpin conventional educational policy and practice are recognised as being ultimately provisional, and unable to account for differences in individuals and environments; the Educator, who does not rely on such conventions, is rather required to possess situational understanding, and consistently making contextual judgments in order to empower Participants to take responsibility for their own advancement. The development of the Participant, I discovered, is conditional on presence of the Educator, who constantly verifies their capabilities. In my position as an Educator, I participated as a learner, knower, and facilitator (see section 6.3.1), but assumed these positions in a manner different to that in which other Participants did so. As a facilitator, I made situational judgments of my actions and reifications at every moment to ensure that, while fulfilling my ethical role as a teacher – as a knower – I also validated the epistemic authority of other Participants by refusing to exert my authority over theirs; most importantly, however, I positioned myself as a learner, and learnt to trust in Participants' competence as learners, knowers, and facilitators

I describe the Participant and the Educator as interdependent equal partners on an educational journey; each was knowing, each was learning, and each was facilitating the advancement of mathematics knowledge. The teacher brought their experiences, while the students brought their capacity to renew, revise, and transform mathematics learning. Contrary to the notion of mathematics learning as the presentation by the teacher of a fixed set of rules to be memorised and practiced by the student, I present a picture on which mathematics knowledge emerges within a Learning Community, in a manner that is unique to the subjectivities of the participants (see section 6.3.1.1), and which belongs to both the teacher and students as they blend their epistemic authority. Through my experience I believe it is possible to bring about a widespread reconceptualization of all students as Participants.

7.1.1.1 A Challenge to Educational Policy

The notion of the Participant challenges the dominant discourse of the subjects of education in the UK. I have characterised the Participant as competent and an authority as a learner, knower, and facilitator in the Learning Community. However, the dominant discourse of the learner used to describe the subjects of education is construed in terms of a deficit (see section 6.1.1.1.3). Educational policy contributes to the notion of the student as being equated with “stultification’ (see section 6.1.1.2.2), as it presents the pupil as of lesser intelligence compared with the teacher, and as incapable of taking responsibility for directing their learning.

The National Curriculum for England Mathematics program of study, which uses the term “pupils” to describe the subjects of education, aims for pupils in England to become fluent in the fundamentals of mathematics, to reason mathematically by following a line of inquiry, to develop an argument, justification, or proof, and to solve problems (Department for Education, 2014). What I consider a missed opportunity is that the curriculum does not describe the desired behaviours of the pupils who are the subject of the document. A description of the pupil could influence the discourse in schools, or at least start a much-needed conversation about how those in education can nurture the pupil the policy desires for the UK.

In addition, policy undermines the importance of the role of the Educator as vital for the empowerment of the Participants, as demonstrated in this study. Rather, it is explicit in its demands for instituting the supreme authority of the teacher. My search of recent government documents identified one that focused on pupils' behaviours in schools; however, rather than assuming a sympathetic view of the pupil and their potential empowerment in their education, the policy was explicit in prescribing behaviour policies for schools, and explained the powers that members of staff have to discipline and to manage their behaviour both inside and outside of school (*Behaviour and Discipline in Schools*, 2016).

In contrast, the Scottish Government considers learners' desired behaviour in its curriculum for excellence (Curriculum for Excellence - A Statement for Practitioners from HM Chief Inspector of Education (August 2016), n.d.). The document explicitly uses the term “learners” instead of “pupil” to "signify a major change in relations between children and young people, their teachers and the curriculum" (Reeves, et al., 2013). In the document, a successful learner is described as a person "with enthusiasm and motivation for learning, determination to reach high standards of achievement, openness to new thinking and ideas, and able to: use technology for learning, think creatively and independently, learn independently and as part of a group, make reasoned evaluations, link and apply different kinds of learning in new situations" (Curriculum for Excellence - A Statement for Practitioners from HM Chief Inspector of Education (August 2016), n.d.). While the motivation for using the word “learner” in the document is to represent the student as actively involved in their learning, the document does not indicate how those in the profession of education can or ought to nurture this learner. Observing its detailed descriptions of the responsibilities of the teacher, I contend that educational policy in England and Scotland inadvertently contributes to a deficient view of the student, and, in this way, that it hinders the development of the competent student whose emergence the government nevertheless appears to desire in mathematics education.

This study started from a supposition of the competence of students to participate in all aspects of their learning. This supposition – that the students already possessed the agency that the study sought to produce – informed the structure of the innovative pedagogy. Enacting this pedagogy successfully led to the emergence of the student as a multi-faceted Participant who is both competent and an authority in the Learning Community. Thus, my contribution to policy is the recommendation and imploration, supported by my research, that it presupposes the subject of education as competent. This presupposition could change the dominant discourse of the learner, pupil, student towards one that recognises their empowerment and agency. I hope to have shown that that this empowered learner can emerge, and that this emergence does in fact and improve mathematics learning.

7.1.1.2 A Challenge to Educational Practice

In presenting this Participant as Competent rather than incapable, I cast into question practices by which teachers take responsibility for the learning process, such as through the exposition of subject knowledge, classroom differentiation, and determining the role of questioning in the classroom.

In schools in England, the language of government policy positions the students as incapable of directing their learning by recommending questioning as a teaching strategy to develop pupils' "higher-order thinking skills" (Great Britain Department for Education and Skills, 2004, p. 3). Research shows that children as young as 2 years old exhibit these skills, and children of this age are attested as even asking a series of questions on a particular topic; they are able to build on the answers they receive to pursue other lines of inquiry, refine their ideas, and build up their stock of information about the world (Harris, 2020; Wellman, 2020). They actively seek explanation and when dissatisfied with an answer, will repeat their original question, disagree with the response, or provide their explanation. In a familiar setting, they ask more questions. Children also learn from both the explanations they give to others and the explanations they receive (Wellman, 2020). However, from around 10 years old, children are no longer avid questioners (Kuhn et al., 2020). This decline could be because schools today seek conformity and instruction from children, rather than eliciting the autonomy that encourages them to ask questions. In conventional pedagogies, teachers use questions to access students' knowledge. This use of questioning conforms with the discourse in which students are considered to be incapable and needing the teacher in order to learn, and in which the teacher is positioned as knowledgeable, gauging the extent to which the student can feed back what has been imparted to them; the students are positioned as performers, merely displaying their knowledge (Oyler, 1996b).

The GOV UK Education inspection framework: Overview of research (Ofsted, 2019, p. 15) that oversees school inspection included, as part of the research that underpins their inspection framework, a section on effective questioning. While the section acknowledges the various sources of questions in class, including those delivered by student to teacher and student to student, there is both a lack of information on techniques for elicit more questioning from students, and an emphasis on teacher-directed questioning; these two factors direct schools away from their focus on students' spontaneity and towards a policy of conformity to teachers' instruction. The Rosenshine principles of instruction (Rosenshine, 2012), emanating from Ofsted and used for teachers' continued professional development, states that "the most successful teachers spent more than half of the class time lecturing, demonstrating, and asking questions" (Rosenshine, 2012, p. 33). In addition, the structure of the conventional mathematics classroom that views as competent whomever gets the answer right or works fastest, can further position the student who questions as ignorant. In my experience as a teacher, this mitigation of students' spontaneous questioning has resulted in students shying away from asking questions in class; they do not what to appear “dumb” in front of their peers.

In this study, Participants asked questions spontaneously as they continuously sought to extend their existing knowledge; questioning presented itself as an inherent reality of the classroom, in which competent students sought information. The conception of the student as incapable has infiltrated the discourse of education and impacted recommended strategies; while these strategies seem to act in order to improve the education of “incapable” students, they can, as in the case of questioning, arrest children’s natural propensity to learn. The mathematics Program of Study (Department for Education, 2014) that aims to have pupils reason mathematically and apply their mathematics knowledge, needs students who are aware of their unknowing and seek to know. Students who are creative in extending their knowledge in adaptable ways interact with others to create knowledge and create a learning community wherein everyone's knowledge is advanced. It is my contention that this student is in every classroom, in front of every teacher, ready to be empowered.

7.1.2 A Contribution – The Innovative Pedagogy

I contribute a pedagogy purposeful for the emergence of the Participant who is competent and an authority in a Learning Community. The pedagogy is a full-time everyday pedagogy that 14-to-15-year-olds can enact in a secondary school mathematics classroom in England. It is based on the knowledge creation metaphor that depicts learning as occurring when individuals collectively create New Knowledge in the form of conceptual artefacts. I refer to the pedagogy as innovative; this is because it sought to change the established teacher-student power relations, transform the mathematics learning environment by demonstrating the interdependence of authority, redefine learning as a community endeavour, and challenge the existing discourse that defines the practice of mathematics learning. The pedagogy demonstrated the mutual interdependence of the authority of teacher and student in its expectation that the students participate fully in the advancement of their mathematics knowledge. This expectation informed the pedagogy design, in which the student took on responsibilities for their learning that are usually the preserve of the teacher, such as selecting their mathematics topic, planning the learning, sharing this knowledge with their peers in the mathematics classroom, and reflecting on their actions and reification. Being allowed to take on these responsibilities, the Participant emerged as competent in directing their learning. The Educator emerged as necessary for developing the Participant and validating this competence. The Educator contributed their experience, referred to as mathematics knowledge for teaching, and the Participants brought to the Learning Community the knowledge of their capacity to learn and the ways they can direct their learning and that of other Participants. The pedagogy demonstrated that authority in the classroom does not need to be imposed by the teacher; it demonstrates Benne's notion of anthropological authority (Benne, 1970) (see section 2.3.2.1) that focuses on negotiation and consent, and considers the relationship of authority in the learning environment to be flexible and fluid. This mutual interdependence of authority empowers the Participants, as it points to learning as co-participation; both the Participant and the Educator see Expertise in each other (as facilitators), continuously learn (as learners), and continuously seek to support each other (as knowers) to reach beyond their existing knowledge.

I have demonstrated through this study that students in a mathematics class in an English secondary school can interdependently control their classroom learning while raising their achievement in conventional assessments. This capacity corresponds to a monumental transformation, as it shows that mathematics learning need not and should not be based on the one-way conveyance of knowledge from the teacher to the students; instead, it demonstrates that learning occurs during both student-teacher and student-student epistemic interactions.

The pedagogy that points to such co-participation transforms mathematics learning from an individual endeavour into a community endeavour. It substitutes the image of the individual student striving to acquire (master/memorise through practice) mathematics knowledge for their benefit with the image of a community in which each individual student's knowledge is available to every other member of the classroom, and each student is accountable to the task of advancing their knowledge and that of their peers. This Learning Community sustains participation by its redefinition of competence. This redefinition of competence as valued participation in the community's practice of learning mathematics causes Participants to emerge that belong to the community and who are accountable to the advancement of the community knowledge.

The pedagogy that defines learning as a community endeavour wherein participation in epistemic interaction constitutes competence is essential in mathematics education, especially in light of the common continuation or desire to continue with Mathematics study beyond secondary education. In a Learning community where students' identity of belonging fosters accountability (see section 6.2.2) to the practice of learning mathematics, this sense of belonging can reduce exclusion from mathematics. Research has shown that identities contribute to exclusion in secondary school mathematics education; this is particularly the case of the low number of girls that continue with the study of mathematics beyond secondary education, regardless of their high performance at GCSE mathematics (Smith, 2014; Solomon, 2007). I argue that inclusion in mathematics is more decisive than ability when deciding upon participation in its learning, and call for a redefinition of what constitutes “success” and “failure” in mathematics classroom; I also argue for the need to shift focus away from the individual student and their personality as the cause of their “failures” in mathematics, and towards how educators can address the endemic “failure to belong” (Boaler et al., 2000) to the community of mathematics.

7.1.2.1 The Learning Community as a Challenge to the Educational Policy of Mastery

The notion of mathematics mastery was brought to the fore in compulsory education in England following the publication of the Programme for International Student Assessment (PISA) rankings in 2012. A small group of high-performing East Asian countries consistently dominated the top of positions in these international “league tables”. This publication led to a move for England to emulate the teaching methods and approach to mathematics mastery of practiced in these Asian countries. Accordingly, after Shanghai ranked 1st out of 65 countries in the PISA 2012 mathematics rankings, the Teaching for Mastery (TfM) programme adopted by the National Centre for Excellence in Teaching Mathematics in the UK was influenced by Shanghai's mathematics teaching approach (Boylan, 2019). As a secondary school mathematics teacher, I have noticed that the implementation of this programme is no longer a government priority, and this could be due either to its impact on learning progress being of negligible effect (Demack et al., 2017) or the difficulty in its enactment by teachers; the discourse is ambiguous (C. R. Morgan, 2017).

The notion of "mastery", either in the discourse of the students’ "learning for mastery" or of the teachers’ "teaching for mastery" (Boylan, 2017), originates with the idea that a student can simply learn all of a subject's content and store it in their mind. This idea is conceptually obsolete in this age in which mathematics knowledge is advancing and diversifying at a pace with which learners could not hope to keep up. In practice, the mathematics mastery programme aims to teach individual students the curriculum contents up to a certain standard, with periodic assessments to measure competence. Competence becomes a measure of what is in their minds; individual students focus on their knowledge and acquire as much as possible. I question whether it is realistic or desirable to expect individual students to learn and know everything in a subject's curriculum, as is the current expectation. Learning occurs through interactions; (Bereiter, 2002; Bereiter & Scardamalia, 2011; Moss & Beatty, 2011); other people's knowledge helps clarify and improve what one knows, as with mathematicians in the professional community (Bereiter, 2002). A deep constructivist approach posits that schools must acculturate students into the real world of professionals, wherein knowledge creation occurs as one takes someone else's knowledge further through active epistemic interaction, not individual mastery. In this sense, mastery discourse contradicts the evidence of students' creative and innovative abilities to problem solve and reason mathematically. If students do not learn to problem solve and reason authentically in classroom activities, where they can seek to solve their mathematics problems and support in the solution of others, it would be unreasonable for educational policy to expect students to develop these skills. This study proposes that as students actively participate in what matters, such as the Participants of my mathematics class participated in learning mathematics as competent mathematicians, they built on each other's knowledge, supported each other's learning, and collectively made progress. I contribute the notion of a pedagogy that creates a Learning Community as essential to develop the student necessary for success in mathematics education and beyond school.

7.1.2.2 The Pedagogy as a Challenge to Educational Practice of Ability Differentiation

The view of Participants as having the capacity to advance their collective knowledge through enacting the pedagogy stands in opposition to the discourse of ability prevalent in UK secondary schools. The basis of this prevalent discourse is the ideology according to which students have inherent, fixed, context-independent cognitive abilities (Oakes et al., 1997) over which the teacher has no control. In this discourse, assessments and ability settings place students on an ability spectrum in mathematics classrooms (see 1.1.3.1). Teachers consequently view students as "able" at one end of the spectrum, as of middle ability, and at the other, as of "low ability". Most UK schools, as reported by (OECD, 2013), teach 95% of 15-year-old students in subject-specific ability groups.

However, labels or the ideology of ability do not in themselves reduce attainment in students. It is teachers' belief in the labels and the ideology that reduces attainment (Hallam & Ireson, 2003; Marks, 2016), altering teachers' behaviours towards the students, for example, through their interactions with and expectations of the students. The discourse of ability also limits the mathematics made accessible to the students (cf. Morgan, 2013; Smith & Morgan, 2016).

While some research shows that positioning students by means of these ability labels or other differences can lead to low student confidence at both ends of the spectrum, especially in students positioned as of "lower ability" (cf. Boaler et al., 2000; Snell & Lefstein, 2018), other Educators believe that differential instruction holds positive benefits (Konstantinou-Katzi et al., 2013) to students. The rationale is that "students learn best when their teachers effectively address the variance in students' readiness levels, interests, and learning profile preferences" (Tomlinson, 2005, p. 263). While addressing the variance in readiness levels, students' interests, and learning profile preference sounds laudable, if it can at all be achieved in a whole class setting, it calls for the teacher to subjectively decide what mathematics is made available to each particular student, thereby limiting the mathematics to which some students are exposed. In my experience, these decisions are based on students' social behaviours or performance in previous assessments, both of which are not accurate indicators of an individual's ability to learn something. Objects of knowledge that have not yet been encountered offer a new opportunity for individuals to engage with them, and individuals should always have such opportunities, rather than being limited to what a teacher allows them to access.

In designing the pedagogy, what I took from my experience were the beliefs that the student has the competence to make decisions about their learning and that the proper purpose of schools is to verify this competence. I did not consider ability; I had learnt from a prior experience of going wrong in my expectations of a student (see section 1.1.3.1) that any notion that places students in a knowledge hierarchy, be it a criterion of differentiation or of ability, can be unfair, and that "It is ignorance of this 'knowledge of inequality' that is supposed to prepare the way to reduce inequality" (Bingham et al., 2010, p. 4). The outcomes of this study confirm that, without any reference whatsoever to presumed ability labels, students can democratically and competently take responsibility for their mathematics knowledge advancement.

I do not naïvely deny that some students have barriers that prevent their learning from easily progressing. My point is rather that as an Educator, I should not start from a presumption of the abilities of all students based on an ability spectrum which defines how I behave towards them or what knowledge I make accessible to them. Instead, I should presume that *most* students can learn and make sense of knowledge. The design of the pedagogy and the relations within the learning community can make a difference to how students relate to mathematics.

7.1.2.3 The Pedagogy as Empowerment

The Pedagogy of Trust that presupposes competence (see section 6.3.2), was purposeful for the emergence of the Participant and the emergence of the Learning Community. I contend that this emergence can be construed as a process of empowerment. The empowerment lies in the development of a democratic community and the relationships within it (see section 2.2.2.5), and in the power relations (see section 2.2.2.6) the Participants exercised in enacting the innovative pedagogy – hence, resisting and transforming the prevailing discourse of a conventional mathematics classroom.

Participation in the Learning Community was democratic and productive of mathematics knowledge (see section 6.1.2). The four stages of the innovative pedagogy specified what the students were expected to do (see section 3.1.1). However, to fulfil the principles that underpinned the innovative pedagogy (see section 2.4.3), that required the students to take responsibility, the pedagogy did not specify *how* it was to be enacted neither did it specify the *student behaviours* that were required for its enactment. This lack of specification was empowering as it gave the students' the freedom to bring their authentic selves to the enactment and to express their uniqueness. That the students could bring their authentic selves to the learning of mathematics was a validation of their intrinsic worth and competence in the act of learning mathematics (Macmurray, 1950). This empowered student emerged as a Participant that participated more democratically in the Learning Community. Participants' freedom of expression and freedom of behaviour (see section 5.2.2.2), the relationships of trust (see section 5.1.2.4), equality of participation (see section 5.2.3.2) and responsibility for each other's knowledge advancement (see section 5.2.3.3) were evidence of the democratic community. The Learning Community was not forced into existence, but was a consequence of Partcipant's freedom and it points to a pedagogy that empowered students to became authors of their own world (Ellsworth, 1990, p. 309).

Participation in the Learning Community was interactive and productive of mathematics knowledge. As the Participants interacted with each other, power relations were at play that structured their actions and reifications in the Learning Community (Foucault, 1978). In taking responsibility for the circulation of mathematics knowledge in the Learning Community, the Participants were no longer only subject to the thoughts and actions of the teacher. As vehicles of power, they could control their own actions and reifications and could determine how to apply their will towards the process of the community's mathematics knowledge advancement. That the actions and reifications of each Participant acts upon the actions and reifications of others in the Learning Community; as they positioned and were positioned by each other (see section 6.1.1), negotiated authority (see section

6.1.1.3.2), and defined competence and accountability (see section 6.2), is evidence of their productive relationships of power (Foucault, 1982). Ultimately, the Participant in the Learning Community transformed the view of the students from a dependent, constrained and passive receptor of mathematics knowledge to a Participant, who can take responsibility for what they know and don’t know and acting on this awareness, take control for their process of learning mathematics as a community.

7.2 A Contribution to Theory

The construct of shared epistemic agency originated from a study undertaken in the context of undergraduate collaboration (Damşa et al., 2010). In the outcome of this research, agency was defined as the "capacity that enables individuals, groups or collectives to make appropriate judgments, to make plans and to pursue these through purposeful action, in order to achieve the construction of knowledge" (Damşa, 2014, p. 446); the study presented an overview of epistemic and regulatory actions that indicate this construct. My research provides an opportunity to observe the interaction of secondary school students to develop its own derivative conceptualisation in this context.

In light of my wider reading, I synthesised six characteristics of shared epistemic agency that shaped my analytic framework. Through my empirical actions, I have refined, operationalised, and made the construct relevant to a secondary school classroom. I offer these contributions to theory.

7.2.1 Shared Epistemic Agency is a Manifestation of Who the Participants Become

I have transitioned, in the course of this research, from seeing shared epistemic agency as a set of discrete types of behaviour to a more holistic view of student participation and community practice that involved 14-to-15-year-old students and their teacher as they enacted an innovative pedagogy for learning mathematics over one year. I have shown in chapter 6 that one of the themes that emerged from the study's findings was a new conceptualisation of the student as a Participant. This Participant emerged as distinct from the conventional notion of student with which my class started at the beginning of the academic year – suggesting that the emergence of shared epistemic agency changed the mathematics students into Participants, and myself as the teacher into an Educator.

This becoming can be explained in terms of Wenger's (1998) discussion of identity. He suggests that our identity is a product of our lived experiences of participation in specific communities; he describes it as a "layering of events of participation and reification by which our experience and its social interpretation inform each other" (p. 151). As the Participants participated in enacting the pedagogy as learners, knowers, and facilitators, and as their relations with other Participants reified their competence, they began to see competence in themselves, making them accountable to the practice. This accountability drives further competent participation that other Participants reify; this reciprocal, iterative process of participation, layered over time, develops students' identities into those of mathematics Participants. This identity is flexible; it is constantly being negotiated through competent (or non-competent) participation.

An alternative way of conceptualising this change is offered by the positioning theory of Davies & Harré (1990). This theory recognises how discursive practices are directed, how individuals are positioned, the context of these practices, and how these positions affect the individual. Who we become manifests in social interaction through how we are positioned or how we position ourselves. Once a Participant takes up an available position, they see the world from the vantage point of that position and in terms of the discourses and behaviours directed towards them due to the position. Applied to the findings of this study, the flexible positionings are a result of Participants' interaction. This interaction constitutes and reconstitutes the Learning Community and the Participants that reify the various positions of learner, knower, and facilitator. Therefore, who a Participant becomes shifts in line with the positions they take up in the practice. As Participants are positioned as learners, knowers, and facilitators repeatedly, they begin to clearly see themselves as learners, knowers, and facilitators in these moments. Thus, mutually-inquiring agents in an epistemic community becomes who they are.

Both Wenger's description of identity and Davies & Harré's positioning theory are consistent with each other, and germane to the purpose of describing the becoming of the Participant. While the emphasis is not on the equality of positions, Wenger focuses on how participation in the practice of a community can lead to a change in individuals' identity and who they see themselves to be; at the same time, the explanation presented by positioning theory points to the discourse practices of the community (this includes how it defines and reifies competence) that open up positions for individuals.

This contribution extends the original construct of shared epistemic agency to include the continuous and spontaneous interactions that take place in a secondary school classroom enacting a knowledge creation pedagogy, in a contrast, Damşa et al.'s construct, which was based on the collaboration that occurs during specific group activities. I put forward terms that indicate this distinction between the two constructs of shared epistemic agency: collaboration and interaction. I propose that Damşa's construct, which is observable in the context of group collaborations to produce a knowledge object, be referred to as epistemic agency through collaboration. On the other hand, this research has identified a different kind of shared epistemic agency that applies within a secondary school context wherein students are engaged in spontaneous interaction to create New Knowledge. I propose that the construct identified by this research should be referred to as "shared epistemic agency through interaction".

7.2.2 Shared Epistemic Agency is the Practice of a Type of Learning Community

When applied to a secondary classroom, shared epistemic agency through interaction suggests a specific type of Learning Community. This Learning Community is not fully described by the construct of communities of practice I presented in chapter 2, but goes beyond them: it suggests a Learning Community that is both interactive and democratic, as described in chapter 6. While Wenger's community of practice could be extended to render it democratically interactive, my stipulations are not specific requirements of a community of practice.

Interactivity is more than the Participants' spontaneous and continuous actions and reifications, which are the bases of epistemic interactions and knowledge creation. The interactivity also needs to include the idea of freedom – the freedom of participation. This freedom of participation recognises the capacity of Participants to make decisions about how they should act in the classroom for knowledge creation, and, equally, the freedom of the teacher to make situational decisions regarding how to participate in the learning community. This idea of freedom is not freedom from societal oppression (cf. Freire, 1970), nor the emancipation of the individual from social inequalities (Rancière, 1991). It acknowledges that unique individuals with unique experiences, skills, and personalities, and diverse ways of knowing exist within each Learning Community. Hence, within each Learning Community, a different practice should emerge of its own accord. The idea of freedom that I posit as necessary for the Learning Community is the freedom from the unilateral authority of the schoolmaster, and requires a blending, and freedom of the teacher from the authority of the conventional and normative discourse of pedagogy, allowing situational judgments.

The Learning Community suggestive of shared epistemic agency through interaction also requires that participation be democratic. In this study, democratic participation resulted from a recognition of the interdependent capabilities of Participants rather than of mathematics ability measured in terms of a hierarchy of performance.

Research into how to sustain student agency has focused on developing a classroom culture that can sustain student interaction. Some research has explored more open-ended learning designs to support student inquiry (Zhang et al., 2018), while others have focused on the use of technology such as “knowledge forum” (Scardamalia & Bereiter, 2014), an online platform where students can collective discuss ideas, AsCRA, a reflective assessment tool (Yang, Chen, et al., 2020), and ITM, a time-line based inquiry structuring platform (Zhang et al., 2018) to develop a community culture in the classroom that sustains and develops students’ agency over a period of time. I contribute to this research, demonstrating that shared epistemic agency through interaction can be sustained by a Learning Community in which freedom of participation and democratic participation are maintained, and which does not require the development of special technologies.

7.3 A Contribution as a Teacher-Researcher

Having completed this study, I advocate action research as a methodology suitable for teacher professionals to undertake in order to transform their classroom practice and the educational profession at large. As noted in chapter 1, I am a mathematics teacher who recognised how the conventional mathematics pedagogy I employed excluded students from bringing their agency and capacity for independent thinking to mathematics learning. This limited their capacity to solve mathematics problems logically. I sought to improve my students’ participation in their learning and hence their relationship with mathematics.

In the same vein, as a secondary school teacher, I felt excluded from contributing my personal knowledge (McNiff, 2017), gained from years of developing strategies to improve student's learning, to the profession. Personal knowledge is often tacit, subjective, and intuitive, coming from contact with the world. It can be difficult to articulate and rationalise; sometimes, we just know what we know. However, this study is testament to the fact that, just as the Participants in this classroom as an interactive community demonstrated the capacity to explicate others' tacit knowledge (see section 6.1.2.1.1), a culture of action research amidst a community of professionals could enable teachers turn their personal knowledge into knowledge that can be shared.

I am of the opinion that the teacher in the classroom has the agency and competence to transform the conventional pedagogy, and I believe that this is already happening in classrooms such as mine. I present action research as a methodology that can enable teachers to make knowledge claims and present the adaptations they make to their practice as theoretical interventions. Sharing what we know can transform our practice and how our young people learn from within the educational profession, without relying on policy and external research.

Being a teacher-researcher necessitated the selection of a methodology that would allow me to study the transformation of my existing practice while still fulfilling my ethical responsibility as a mathematics teacher. My contribution is a particular action research methodology that is dynamic, as it allows for change and improvement; participatory, as it values the contributions of the Participants; and empowering and authentic (the inverse of a top-down approach to change), as it legitimises the combined contributions of my practice and my research towards a knowledge claim.

7.3.1 A Particular Action Research Methodology

Action research has been described as a meta-methodology (Attwater, 2014), allowing flexibility in its cyclical oscillation between the action and reflection. This flexibility allowed me to design a methodology wherein I could use my personal knowledge or trial and error to implement changes to the classroom practice within the action research process.

My research design, which repeats itself within each action research cycle, consists of one or more teaching cycles (TC) (see Figure 7.1). It can be thought of as cycles within cycles.

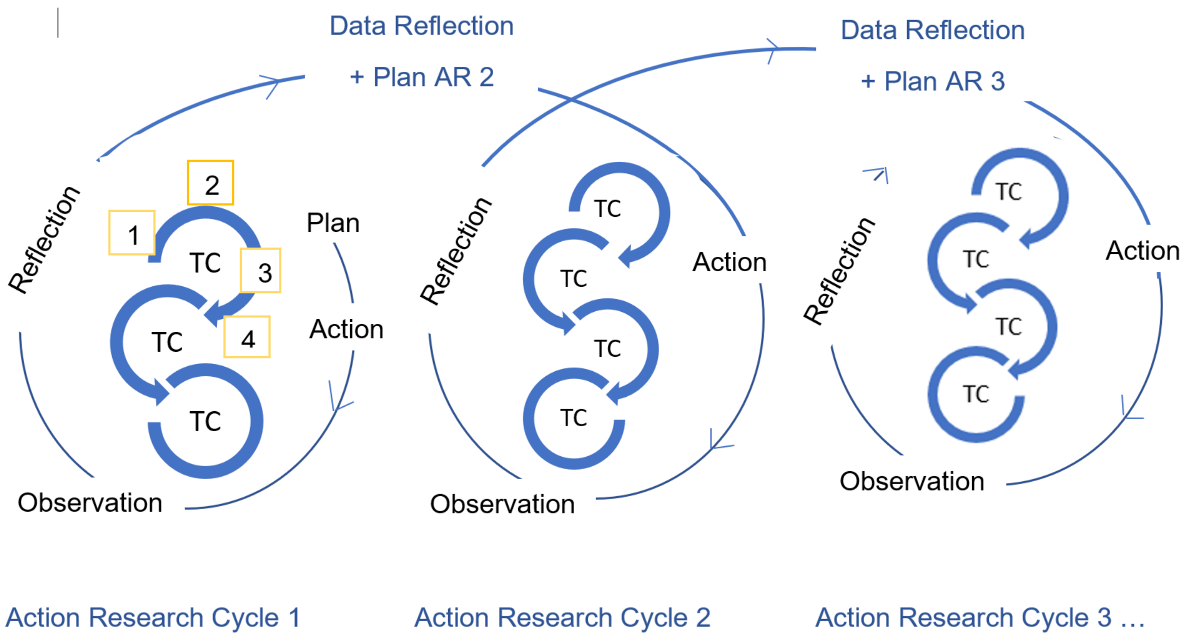

Figure 7.1 – My action research cycles inclusive of teaching cycles

Within each action research cycle, I nested miniature teaching cycles that were structured by the typical action research spiral of iteration and improvement. Each teaching cycle was designed based around the change I wanted to make in my classroom. The teaching cycle allowed me to (1) Plan, (2) Act, (3) Observe, and (4) Reflect on what I observed (see section 3.2.2). The duration of each teaching cycle is flexible in this type of study. In the case of this study, the duration of each teaching cycle was based around the requirement that each student have a turn as a teacher participant. However, in principle this structure can be adapted to fit the aims of innumerable types of research. For example, the teaching cycles could be weekly or could coincide with a mathematics topic or teaching module.

The final teaching cycle in each action research cycle was the data collection cycle. The first four stages of this teaching cycle aligned with the stages of the research cycle (see section 3.3). In the fifth stage, all data collected was reflected on, and any changes to the next research cycle or to the pedagogy were planned. Following the reflection stage of each teaching cycle, the participants and I acted to implement changes to the pedagogy for the next teaching cycle. The changes were based on the tacit knowledge I had of teaching, the participants' responses and explicit input, our context, and the aims of the study. Having a number of teaching cycles within each action research cycle allowed me employ such knowledge and make changes to the subsequent teaching cycles. In this way, I was a researcher adapting to the demands of the setting of my study, but also a teacher who could still act to fulfil my responsibility and ensure that the pedagogy served my students.

Working in a school environment is open to uncertainties from many directions. These includes structural uncertainties such as the length of teaching time needed for a topic, school closure days, assessments, illnesses, interruptions that can alter planned schedules, and uncertainty caused by undesired outcomes of plans. In the complex classroom environment, in which there are 18 co-participants whose agency impacts situations, it is not easy to predict and judge the outcomes of events. However, the methodology, arranging the research in spirals of self-contained teaching cycles, allowed for adaptation to parts of the research and pedagogical designs. I could make changes to best meet my ethical responsibilities as a professional while also answering the research questions. In a complex environment such as a classroom, it is often difficult to foresee the impact of actions until one engages with them; in addition, "you cannot understand a system until you try to change it" (Schein, 1996). Following Lewin (1946), Schein argued that it was a fundamental error to think that the notion of a diagnosis can be separated from that of an intervention. It is by engaging with the situation that one can determine what works and what does not. Having teaching cycles within each action research cycle, I could evaluate what worked in our classroom context and what did not, again and again, within each teaching cycle, learning and preparing for future stages as the innovative pedagogy was enacted.

The flexible nature of the methodology was beneficial to myself, the participants, and to the research outcomes, because I was able to be part of the research. I was not an outsider studying the setting or participants, such as in a phenomenological approach wherein the researcher seeks to thoroughly capture and study another's "lived experience" (Patton, 2015, p. 115). I was researching my own practice and bringing my experiences as an intelligent professional to the research. I could initiate changes during the research process and reflect on their impact; the possibility of effecting change through reflection was always reserved. Elliott (Elliott, 2011) referred to this type of reflection as “reflexive”, and considers it to be an essential part of the practical science of any professional (see section 3.2.1). This methodology allowed me to be reflexive, as the reflection was highly personal. As I reckoned with my own place in the research, the assumptions, experiences, and beliefs that underpinned my practice, along with my practical attitudes towards my profession, became open to reconstruction, and revealed possibilities for further intelligent action as a professional. The flexible nature of the methodology enabled me to improve my day-to-day practice as a professional and, ultimately, to make my systematic inquiry public (Stenhouse, 1981) to the field of mathematics education in the form of this project.

7.3.2 An Authentic and Empowering Methodology

The methodology can be captured by the designation “authentic methodology”, as I designed it ad-hoc to meet the research purpose; I chose to insert the uniqueness of this study and my creativity within the existing framework of my professional relations and within the research norms of academia (Kreber, 2011; Taylor, 2018). Bound by a professional obligation to do what is right for students, and the courage to seek to change what needed to be changed, this methodology legitimised my actions. I put my courage at the disposal of my students, availing us of the possibility of immediate benefit to the current practice rather than waiting for the research to be completed to effect change.

As opposed to positivist methodologies, wherein theoretical protocols dictate methods for designing the research and collecting and analysing data, in the action research methodology that legitimised my experience, my increasing knowledge had a recursive effect on my experience, which positively impacted the research. My readings of the literature enabled me to understand the history and original motivations of my practice, and therefore to consider alternative possibilities and approaches. Over the single year it took to complete the data collection, this knowledge impacted the thought process that underpinned my actions in the classroom and the research. I became both a better professional and a better researcher; as my knowledge and experience improved, I was able to design the data analysis to fit the research design that had changed in the course of the research. The unit of analysis that I developed, the Episode (see section 4.1), set my research apart from that of Damşa et al., in which actions were the unit of analysis.

This difference was due to context; Damşa et al. studied collaboration between undergraduates, while I studied my secondary school students' interactions (see section 6.1.2.1.1). In Damşa's case-study approach (Yin, 2014), she sought to observe and analyse the behaviours of groups of undergraduates as they collaborated to produce a knowledge object; in this research, on the other hand, the context was more flexible (see section 7.2.1), and the students and I were continuously interacting in different groupings towards the production of New Knowledge. In addition, as we engaged in enacting an innovative pedagogy that was changing who we were, our classroom actions also changed. Interactions in the classroom went beyond our individual actions, and it was the products of our intersubjective experiences that impacted the research and required analysis. As I see it, these contrasts highlight the fact that my original approach has particularly much to offer to other researchers concerned with epistemic agency.

Finally, I note that my research methodology, which legitimised my authenticity as a continuously evolving professional with a developing understanding and discerning application of theory, is both professionally and personally empowering. Indeed, my knowledge continues to grow in the wake of my discoveries; I have personally moved from seeing shared epistemic agency through interaction in terms of a set of discrete types of behaviour towards a more holistic view of student participation and community practice – for example, I am determined, in future practice and theoretical interventions, to go beyond thinking of the Episode as the unit of analysis, preferring an analytical method better suited to this change in commitments.

7.3.2.1 A Participatory Methodology

The methodology that legitimised my experiences also gave a voice to the 18 Participants of my classroom, and, by its advocacy, to students at large. As they enacted the innovative pedagogy, my students made it meaningful and purposeful for them. To make it meaningful and purposeful for mathematics learning, they adapted their performance, contributing to the adaptation of other Participants' performance; cycle after cycle, their participation spoke through the chapters of this study.

My students' voices, hence, their participation, came through in the dialogical and physical communications and reifications they made every day throughout the academic year. Their participation is woven into the field notes, the video recordings, and the transcripts of Episodes. In this way, they too were included in the transformation of how they (and hopefully others) learn mathematics in their classroom.

Researchers such as Schon, 2008; Stenhouse, 1981 have canonically called upon teaching professionals to conduct research to improve and change their practices (see section 3.2.1). Engaging in research is even more crucial in mathematics education if students are expected to be fluent in the fundamentals of mathematics, reason mathematically by following a line of inquiry, develop an argument, justification, or proof, and solve problems. Mathematics teachers need to adopt an alternative to the conventional pedagogy that is typical across England (see section 2.3.1) to promote the emergence and sustenance of problem-solving and agency in the mathematics classroom (Boaler & Greeno, 2000; Wright, 2021). However, it would be especially difficult to bring about this change in mathematics pedagogy if teachers themselves are not engaged in research to effect change. Change to the classroom's day-to-day practices comes from a teacher's belief that such change is needed, and their ownership in implementing this change (Beck & Young, 2005). Moreover, a top-down approach from policy or academia may make it difficult for changes to be enacted and or sustained in the long term (Ball, 2003, 2012). A lack of ownership of change could explain why conventional school mathematics has remained largely the same in the UK, despite the seemingly numerous reforms to the national curriculum (C. Morgan, 2010). Extending the argument that teachers as professionals need to be involved in changes to their profession if the change is to be impactful, students as subjects of the change also need to be involved, and even direct the change, as it is ultimately, they who will benefit.

As a teacher-researcher, I present a methodology that positions the teacher as an intelligent professional, that allows them to bring their authentic selves to the research, and that recognises the necessity of hearing the voices of the students if the outcome of the research (both in academic and professional contexts) is to be transformative for the mathematics student. My contribution is a particular action research that separates the cycles of the pedagogy from the research cycles so that changes to the pedagogy can be implemented as an action immediately while still carrying out the research.

7.4 Limitations and Suggestions for Future Research

This study has explored how shared epistemic agency through interaction can emerge in a secondary school Learning Community driven by a purposeful pedagogy built on the assumption of the interdependent competence of the Participants. This definition builds on and advances Damşa et al.'s (2010) descriptive concept of SEA through collaboration.

The data collection and analyses were focused primarily on the interactions that occurred as Participants enacted the Share stage of the pedagogy in the mathematics classroom. The other stages of the pedagogy were not analysed in the same manner. This was a consequence of the limited data collection methods that were available for making this stage visible, and the data is as such not reflective of all the learning that occurred as the Participants enacted other stages of the pedagogy. Additionally, the recordings were only of three teaching cycles. I opted to record every other teaching cycle (excluding the first) – the third, fifth, and seventh. I also made assumptions about the collaborative stage of the pedagogy, that is, the Plan stage, believing that since the teaching Participants produced a conceptual artefact, and that its later sharing represented shared epistemic agency through interaction, their collaboration could also be said to represent shared epistemic agency through collaboration. This is conjecture on my part, and it is possible that, if more comprehensive data on collaboration during the Plan stage had been collected and analysed, different findings than those of shared epistemic agency through collaboration may have been produced.

I did not analyse the learning that occurred through reflection at the end of each teaching cycle; neither did I analyse how this learning influenced the Participants; nor did I look at changes in the Participants of the learning community. I acknowledge that there is much more to be learnt from this study, and further opportunities for improving education for young people and making them feel that they are indeed good at maths.

In chapter 3 I highlighted how the camera positionings (see section 3.4.4.1.1) and my student interview techniques (see section 3.4.4.2) limited the data collection. The position of the camera during teaching cycle 7 limited the number of recordings made available for analysis. Further recordings from this teaching cycle in a different classroom setting may have provided evidence of how different environments impact shared epistemic agency through interaction. I excluded the student interviews from systematic review, and limited my analysis of data to what was observable on the camera recordings of the Share stage (see section 3.4.5.1.1). Student interviews at the end of the action research cycles, when Participants would be able to reflect on their entire enactment of the pedagogy, may have further supported the findings of this study, or evidenced other findings.

An area of further research that I put forward is the link between institutional authority and student's authority. In my experience, discipline in schools is imposed on students by teachers and by the institution of schooling's view of how students should behave to learn. In this research I found that in giving student authority, there was a reduced need for the conventional view of discipline in the classroom. In taking process authority, students negotiated classroom behaviour. Anecdotally I believe that giving the students process authority, reduced undesirable classroom behaviour. Or it could be that there is a divergence in what students view as undesirable behaviour in the classroom compared with teacher'++s view. Therefore, there is scope for further research into a revised view of discipline in schools.

An area for further thought is how I can be a true participant if I am not learning what the students are learning, can I really establish equity if they are seeking to gain their mathematics qualifications and I am in service to this?

Nevertheless, and in spite of the limitations, further research or thought, by the undertaking of this study, I have shown as a counterexample what is possible in a secondary school mathematics classroom in which authority circulates amongst the participants. The student emerges as a competent individual who forms a community with other students who, through their agency, advance their collective mathematics knowledge. It presents the mathematics classroom as a democratic community in which both the teacher and the students can learn, know, and facilitate each other’s education, with each bringing their unique skills and experiences in a blending of authority.

This has implications for how the students in the study viewed the field of mathematics education and education in general. The conventional view of the teacher as a necessary authority is fundamental to the discourse of schooling, and reflects the beliefs that most potently informs government policy, professional development, and teacher training. Against this, the views of the student and their competence advanced in this study relate a call for reforming of the pedagogy and the institutional ways in which teachers interact with students in classrooms and schools. It also raises the question of what further potential may be possessed by our students, waiting to be drawn out.

To answer this question requires other teacher-researchers to carry out similar research in their classrooms; I believe I have shown that it is more than possible to transform the pedagogy within the structures of the mathematics curriculum in a secondary school. While the uniqueness and narrow focus of this study may lead to its non-replicability, I believe that within the body of evidence that I present, sympathetic teachers and researchers can decipher the principles of my pedagogy and adapt them to their contexts, bringing about a comparable pedagogy of trust designed to be enacted by students who already own the qualities it aims to produce – a pedagogy that, in an appropriately egalitarian manner, believes in the competence of students, allowing this competence to emerge in its own way. Thus, I call on teachers to become researchers, as I believe that the process of applying the principles of this study to further research areas will provide yet more evidence of the immeasurable competence of the young people we teach.

My hope is that “not only scholars of teaching but also those whose learning experiences they intend to support, would seek to renew our common world” (Kreber, 2011, p. 866). To this end I hope that this study contributes a kind of answer to the question that educators persistently ask of themselves: “What works, what is and what is possible?” (Hutchings, 2000).

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Appendix 1 – The 12 Principles of the knowledge-building communities model. (Lai & Campbell, 2018 adapted from Scardamalia (2002))

Principle	**Explanation**
Community knowledge, collective responsibility	The aim of a knowledge-building community is to produce new ideas and knowledge which are useful to, and useable by, the community. Knowledge building is a community process.
Epistemic agency	Students take charge of their knowledge-building journey. They are responsible for setting plans, planning goals, and for evaluating their progress in knowledge creation.
Knowledge-building discourse	The explicit goal of the community is to advance knowledge which is achieved through discursive practices.
Real ideas, authentic problems	Ideas developed by the community are considered objects that can be manipulated. Students in a KBC will address problems they 'really care about' (Scardamalia, 2002, p. 75).
Improvable ideas	Ideas can be improved. There is always room for further discovery.
Idea diversity	Ideas need to be connected to a broader knowledge base – 'to understand an idea to

	understand the ideas that surround it (Scardamalia, 2002, p. 75)
Rise above	Diverse ideas are synthesised to 'higher planes of understanding' (Scardamalia, 2002, p. 76)
Democratising knowledge	All students are empowered in knowledge creation as legitimate contributors
Symmetric knowledge advancement	The advancement of knowledge is facilitated by exchanging knowledge between communities.
Pervasive knowledge building	Knowledge building is a way of life, not just done in the classroom. It also means developing confidence and dispositions of contributing ideas and knowledge for today and tomorrow.
Constructive uses of authoritative sources	Knowledge builders use critically knowledge produced in the past.
Concurrent, embedded, and transformative assessment	Internal and external assessments are part of the knowledge-building process.

Appendix 2 – Overview of actions indicative of Damşa's SEA (Damşa et al., 2010, p. 175)

Dimension	Category (of Action)	Action
Epistemic (Knowledge related)	Creating Awareness	Identifying a lack of knowledge
		Identifying problems
	Alleviating lack of knowledge	Examining given resources
		Collecting additional information
	Creating shared understanding	Sharing information (from source)
		Giving meaning to new concepts
		Discussing misunderstanding
		Sharing ideas and knowledge
		(Re)framing the problem
	Generative collaborative action	Generating new ideas
		Engaging in collaborative uptake of ideas
		Negotiating new ideas
		(Re)framing the object
		Engaging in shared construction of (intermediate) objects
		Revisiting ideas and object drafts
		Using feedback constructively

Regulative (process related)	Projective	Setting common goals
		Creating a joint plan of action
		Engaging in proactive conduct
	Regulative	Monitoring object advancement
		Coordinating object-related activitiesref
		Reflecting on actions, ideas, and problems
		Adjusting the groups current strategy
	Relational	Transcending (social) conflict
		Redirecting critical feedback
		Creating space for other’s contribution

Appendix 3 – Parent and student consent form

Permission for Participation in Educational Research

Researcher's name: Ijeaku Mezue

Topic: Knowledge building in a mathematics community.

Dear parents and carers

In addition to being the Senior Deputy Headteacher at Gladesmore Community School, I am carrying out a research degree (PhD) at the University College London (UCL) under the supervision of Professor Candia Morgan and Dr. Cathy Smith. This requires that I carry out a year – long research at Gladesmore Community School into the extent of students' participation in the mathematics classroom.

I have obtained the permission of the Headteacher Mr. T. Hartney to collect data through interviewing students in my Year 10 mathematics class and video recording some lessons for my analysis.

Q: **Who can participate in your research?**

A: The participant of my research will be the students in my Year 10 mathematics class.

Q: **is participation compulsory?**

A: I would like all students to take part as I feel the experience will be enjoyable and enriching for students, however participation is voluntary. Students and parents must give their consent prior to participation. Students may withdraw from participation at any point of the research before it is published. Their decision to participate or to withdraw will have no impact on their schooling.

Q: **What will participation involve?**

A: Participation will involve students taking part in up to three 15- minute interviews across the academic year. During the interview session students will be asked about their experiences in mathematic lessons. Interviews will be voice recorded for purposes of data collection facilitation. All interviews will take place at the school, at a convenient time outside lessons (during lunch, assembly time, before or after school).

Video recording of some lessons will take place as part of the research. The recordings will be viewed by the students being interviewed, by the teacher and by the doctoral supervisors. All analysis from the video and audio recordings will anonymise the participants. That mean no student can be identified from the thesis.

I hope to show some short video excerpts to an audience of teachers and researchers, to illustrate my findings. I will only use an excerpt including your child if you give permission below. I will obtain the Headteacher's permission for the specific video excerpts that I show.

Q: How will data be stored, and confidentiality be maintained?

A: All research data will be confidential. I will protect the data obtained during the research by storing it in a password-protected USB-stick locked away when not in use. Data will be subsequently destroyed according to ethical research requirements. When I report the data and excerpts from student interviews they will be anonymised, so that information given cannot be linked to any student. Where consent is given, short video excerpts of classroom situations could be used when I talk about my research to researchers and teacher.

The completed research thesis will be submitted to UCL for the purpose of obtaining a PhD.

Q: **What happens if I want to ask for more information?**

A: If you need any more information or would like to ask any questions please contact me through the school.

Parental Consent

☐ I give consent for my daughter/son to participate in this research study which will involve her/him taking part in interviews and being recorded within a lesson.

I have read the foregoing information, or it has been read to me. I have had the opportunity to ask questions about it and any questions that I have asked, have been answered to my satisfaction. I consent voluntarily for my child to participate as a participant in this study and for relevant data to be stored as described above

☐ **Please tick this box If you give permission for me to show short video excerpts showing your child in a classroom situation when I talk about my research to researchers and teachers.**

Print Name of Parent or Carer__________________

Signature of Parent of Carer___________________

Date ___________________________

Day/month/year

Student Consent

I have read the information in this form, or it has been read to me. I have had the opportunity to ask questions about it and any questions that I have asked, have been answered to my satisfaction. I consent voluntarily to participating in this study **and for relevant data to be stored as described above**.

Print Name of Student ___________________

Signature of Student____________________

Date ____________________________

Day/month/year

Appendix 4 –Limitations of having three teacher participants – An extract from field notes

Start of Teaching Cycle One (TC1). Date: 07/09/2019

In the first planning lesson there were two groups of three; Adam, Roan and Pearl who sat in positions U, T, Ώ respectively and Jevonte, Daniel and Tom who sat in positions V, W and X. The classroom being the faculty computer suit, the I-desks were fixed in position this constrained the participants in a group from all facing each other.

As this was the first week of a new academic year, only some were familiar each other. In the former teaching group, Roan and Pearl knew each other from the previous year, having been in the same maths class, this made Adam the outsider. Thus, throughout the planning stage, Roan and Pearl sat next to each other and discussed, while Adam who sat opposite backed them so found himself the outsider in the discussions.

Adam preferred to sit on his own and research the topic than engage in a three–way conversation with Pearl and Roan. Though it appeared, during the planning stage, that the group was working together, they had not planned their explanation to the class as a group of three. Pearl and Roan ended up explaining the topic, and Adam did not take part as a teacher.

Jevonte suffered a similar fate, while planning he discussed with Daniel who was sitting next to him but had very little conversations with Tom, the separation from the group was exasperated by Tom taking authority of the planning. This meant that Jevonte was given instructions and told decisions but was not part of the discussion behind the decisions, this meant that he did not have in depth knowledge of the topic.

Two day before the group was to teach their lesson, Daniel was excluded. That meant that the lesson was to be taught by Jevonte and Tom. These two had the least contact during the planning stage, on the day, Tom had epistemic authority, while Jevonte operated the power point lesson plan.

For Jevonte and Adam, being in a group of three, limited their epistemic authority as teacher participants, indirectly limiting their contribution to advancing their mathematics knowledge and that of other participants.

Appendix 5 – Interview one questions

1. What did you do to prepare for the lesson?
2. Looking back, do you think that you had enough and were prepared for the lesson?
3. Why did you structure the lesson the way you did?
4. Do you think the students learnt? How do you know?
5. What would you do differently next time?
6. What was the advantage of working with your partner?
7. What do you think we can do to improve as a class?

Appendix 6 – Interview two questions

1. Talk to me about your last lesson, how you planned it
2. Do you feel it when well? Why?
3. What have you learnt from this or any of the other lessons?
4. Are there any things you will do differently next time?
5. Does being a teacher or being a student affect how you behave in the classroom?
6. Have you improved as a mathematician?
7. What do you feel is the purpose of learning?
8. Who do you feel can be a teacher?

Appendix 7 – Elaboration of Intentions across all Episodes

Episode	**Orientation**	**Initiated by**	**Visibility**	**Episode Trigger**
1	Extension	Student participant	Dialogic interaction	Identified Unknowing
2	Extension	Student participant	Dialogic interaction	Identified Unknowing
3	Extension	Student participant	Dialogic interaction Physical Interaction	Identified Unknowing
4	Expertise	Teacher participant	Dialogic interaction	Assumed Unknowing
5	Expertise	Teacher participant	Dialogic interaction	Assumed Unknowing
6	Expertise	Teacher participant	Dialogic interaction	Assumed Unknowing
7	Extension	Student participant	Physical interaction	Identified Unknowing
8	Extension	Student participant	Dialogic interaction	Identified Unknowing
9	Explication	Student participant	Dialogic interaction Physical Interaction	Assumed Unknowing
10	Explication	Teacher participant	Dialogic interaction Reification	Assumed Unknowing

11	Explication	Teacher participant	Reification	Assumed Unknowing
12	Extension	Student participant	Dialogic interaction	Identified Unknowing
13	Extension	Student participant	Physical interaction	Identified Unknowing
14	Expertise	Teacher participant	Dialogic interaction	Identified Unknowing
15	Extension	Student participant	Dialogic interaction	Identified Unknowing
16	Extension	Student participant	Dialogic interaction	Identified Unknowing
17	Extension	Student participant	Physical interaction Reification	Identified Unknowing
18	Extension	Student participant	Dialogic interaction	Identified Unknowing
19	Expertise	Teacher participant	Dialogic interaction Physical interaction	Assumed Unknowing
20	Extension	Student participant	Physical interaction	Identified Unknowing
21	Expertise	Teacher participant	Physical interaction	Assumed Unknowing

22	Extension	Student participant	Dialogic interaction	Identified Unknowing
23	Expertise	Teacher participant	Dialogic interaction	Assumed Unknowing
24	Extension	Student participant	Dialogic interaction	Identified Unknowing
25	Explication	Student participant	Dialogic interaction	Assumed Unknowing
26	Explication	Student participant	Dialogic interaction	Assumed Unknowing
27	Explication	Teacher participant	Dialogic interaction	Assumed Unknowing
28	Explication	Student participant	Reification	Assumed Unknowing
29	Explication	Teacher participant	Dialogic interaction	Assumed Unknowing
30	Expertise	Student participant	Dialogic interaction	Identified Unknowing
31	Explication	Teacher participant	Dialogic interaction	Assumed Unknowing
32	Extension	Student participant	Reification	Identified Unknowing

33	Extension	Student participant	Dialogic interaction	Identified Unknowing
34	Extension	Student participant	Dialogic interaction	Identified Unknowing
35	Explication	Teacher participant	Reification	Assumed Unknowing
36	Explication	Teacher participant	Reification	Assumed Unknowing

Appendix 8 – A summary sheet inclusive of modes

Episode Number		Start		Recording	End	
9		00:17		4	00:56	
INTENTION	EXTENSION	S	T	EXPLICATION	S	T
S	Questions			Clarifies	4	
Orient: Exp	Seeks Affirmation			Affirms		
	Requests			Tells		
	Challenges			Explicates Unknowing		
	Articulates Unknowing	1				
NEW KNOWLEDGE	EXPERTISE	S	T	MUTUAL RELATIONS	S	T
	Controls		1	Conducive (Solidarity, trust	4	2
	Checks	1	1	Non conducive		
	Manages					

Appendix 9 – Frequency of modes table

EXTENSION	**Participants**			**EXPLICATION**	**Participants**		
Modes	Student	Teacher	Total	**Modes**	Student	Teacher	Total
Questions	67	3	70	Clarifies	61	70	131
Seeks Affirmation	45	2	47	Affirms	14	21	35
Requests	23	0	23	Tells	9	10	19
Challenges	16	4	20	Explicates Unknowing	13	2	15
Articulates Unknowing	18	0	18				
Totals			178				200
EXPERTISE	**Participants**			**MUTUAL RELATIONS**	**Participants**		
Modes	Student	Teacher	Total		Student	Teacher	Total
Controls	15	51	66	Conducive	40	50	90
Checks	26	35	61	Non conducive	22	10	32
Manages	1	10	11				
Totals			138				122

Appendix 10 – Jayzee and Daniel – Extract from field notes

Completing the square (TC3). **Date: 12/11/2018**

Daniel and Jayzee where to teach completing the squared. I knew that the focus was to use this as a method of solving quadratic equations, and I presumed that this knowledge was shared. They started their introduction by saying that Jayzee would start with the basics. Jayzee started to introduce the concepts of writing an expression in the form of $(x+a)^2+b$. I interjected that the focus was on solving equations (3-DJ TC3.MP4 2:06). This threw their plan into turmoil and undermined Jayzee. This error of mine was brought home to me when I watched the recording, and reflected that she did not come to school the next lesson and I wonder how far reaching my impact was. I am mortified by this experience but I am determined to be better