Exploring student perspectives of Mathematical Project Based Learning

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I, Jessica Barnecutt confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Impact statement

The research had a substantial impact at several levels which I illustrate through the ‘Mathematics of Migration’ project. I co-created this project, based on my learning of what makes effective Mathematical Project Based Learning (MaPBL) from the literature. In particular, I increased the use of guidance and scaffolding and embedded assessment. This shows the impact my research had on my practice: I have transitioned to a more evidence informed approach and critically evaluate literature to see if the findings are likely to translate to my context. This enriches and informs my role as a school leader: it supports my decision making and makes my decisions more likely to have impact.

The ‘Mathematics of Migration’ project was developed collaboratively by several members of the department, who tried, tested, and refined the project. Colleagues’ informal comments suggest that being involved in a professional dialogue about teaching through MaPBL made them reflect on and develop their practice. One of the team wrote their own, award-winning, project. Further, the outcomes of my research were shared throughout the process with our ‘professional learning community’. These findings subsequently influenced project design, as well as the support the teachers provided and the challenges they attended to in the classroom.

The ‘Mathematics of Migration’ project has been recognised as being at the forefront of innovative maths education in the UK (Nesta changemaker award) and having the potential to address educational disadvantage (Let teachers SHINE). Through external funding I developed training to support other teachers’ understanding of how to use this project and pedagogical approach in their classrooms. The training was informed directly by the findings from my research. I knew what teachers in a similar context to my own were likely to find challenging about completing a project such as this and directly engaged with these issues in the training. Over 100 teachers completed the initial training. The training was then developed into an online course on Future Learn which was completed by over 500 teachers and educators. In this way, my research has had an impact on the way people teach both in the UK and around the world.

Additionally, the exploratory study fills a gap in existing research, and challenges some existing findings. It evidences wide-ranging student perspectives on mathematical, teacher-initiated PBL, with a focus on the relationship between students' attitudes to mathematics and MaPBL. It contributes to our understanding of the affective traits and skills students need to be successful with MaPBL, the challenges they faced and the pedagogical strategies that they perceived to be effective. Now as a deputy head in a large comprehensive school in Newham, I plan to use this research to help inform further curriculum design, including developing cross-curricular PBL. Finally, my BSRLM paper (Appendix 1) strengthened my identity as a bona fide member of the mathematics education research community in the UK. I believe work at the boundary of research and practice is vital to the impact of education research.

Abstract

The thesis reports on a real-world enactment of teacher-initiated Mathematical Project Based Learning (MaPBL) by teachers and students in one school in the UK. The thesis aims to illuminate our understanding of the relationship between these 12-15-year-old students' attitudes to mathematics and MaPBL, of the challenges they faced and the pedagogical strategies they perceived supportive, when leading their own learning during MaPBL. The study was conducted in an East London secondary school, which serves a community of high deprivation, whose dominant cultural background is British Bengali. It contributes to our understanding of some tensions, inherent in young people who live in an intersection of cultures, when learning mathematics in such ways.

The research adopted a constructivist grounded approach. Data were collected through lesson observations, student focus groups and surveys, and a teacher workshop and interview. The Covid-19 pandemic and national lockdowns impacted data collection: the study was more exploratory than originally envisaged. Two theoretical lenses, activity theory and complexity thinking, were employed to illuminate interpretation of the data.

The study offers a unique contribution in privileging student voice. It found that, in contrast to some existing literature, students' attitudes towards MaPBL, and the level of embraced autonomy, varied significantly with the nature of the projects, the actions of the teacher, and the beliefs of the students.

Much literature discusses the outcomes of MaPBL on students' affective traits and skills. This study offers a unique contribution to knowledge in suggesting that students require a variety of affective traits and skills **before they can embark on** MaPBL productively– but it is then very much worth doing. These include: self-efficacy, resilience, motivation, a relational vision of mathematics, self-regulated learning and working collaboratively. The thesis evidences pedagogical strategies that were perceived to support affective traits and skills. The study has implications for teachers and researchers wishing to work with a similar approach. Additionally, in line with the aims of a professional doctorate, there has been a symbiotic relationship between the research and my professional work.

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CHAPTER - INTRODUCTION

1.1 Why is this study important?

This study is focused on an aspect of mathematical project-based learning (MaPBL) that teachers in my prior research (Institutional Focus Study - IFS) had identified as one of their biggest challenges when teaching through MaPBL - that of facilitating student-led learning. The related literature is sparse. This study contributes to filling this gap by exploring students' perspectives towards leading their own learning in this context.

There is little evidence of 'real world' problems being given in the 11-16 classroom (Ghosh, 2014). This study provides a student account of an enactment of MaPBL by a group of teachers in a school, which involves students engaging with authentic problems.

Existing research into PBL frequently stops short at considering the student perspective (Grant, 2011). In contrast, this study puts student voice at the heart of research. It seeks to understand the challenges students perceive they face as well as the pedagogical strategies they perceive to be most effective. It explores the relationship between a student's attitude to mathematics and their attitude to MaPBL. To support an authentic interpretation of the students' experiences, a teacher workshop to probe a professional response was also completed. This study has implications for teachers at the school and other teachers who may wish to embrace a similar approach.

The study school is in East London and the dominant cultural background of the community the school served is British Bengali. Young people placed between cultures is a widespread phenomenon in the modern world. There is little known about these students' specific needs in melding their school mathematics learning with their background culture and wider learning needs. The reported work informs our understanding of that.

1.2 Teaching mathematics in the English context

The 2014 National Curriculum describes mathematics as a creative, interconnected discipline, that is essential to life and the foundation for understanding the world (Department for Education, 2013). The National Curriculum has three strands: students should be mathematically fluent, able to reason mathematically, and able to solve problems. This said, mathematics education in the UK in recent times has been dominated by an overarching pressure on schools to cover ensure high achievement at GCSE (Foster, 2013). This demand for grades has often left teachers feeling a lack of control, de-professionalised and uncreative (Perryman et al., 2011). Many teachers have “concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations but did not equip them for the next stage of education, work and life” (Ofsted, 2012, p. 9). Despite this paper being written almost 10 years ago, my experiences suggest that this is often still true today.

The 2014 National Curriculum’s renewed emphasis on reasoning mathematically and problem solving was associated with new specifications for the mathematics GCSE and A-Level examinations. However, previous such curriculum shifts did not bring a coherent change in pedagogy. Ofsted (2012, p. 13) showed that whilst teachers acknowledged students should develop problem solving for GCSE “they did not always recognise the implications for a shift in teaching methodology to ensure the best grounding for success with the (then) new specifications.” My professional interactions suggest the issue persists, and GCSE examiners continue to report GCSE mathematical problem solving and reasoning as under-developed (e.g. Pearson, 2017).

1.3 My beliefs, values, and experiences

My role as an insider researcher was central to this study; related affordances and tensions are discussed in 5.5. My beliefs, values, and experiences will have shaped the collection, analysis, and interpretation of the data:

My own experience of education favoured projects. My primary school experience in the late 1980s at the end of the ‘golden age’ of teaching (Le Grand, 1997), was rich in learning through topics or thematic projects, which I loved. In contrast, my secondary education appeared to be dictated by the National Curriculum and performing well in SATS tests. I found the learning somewhat tedious and compartmentalised.

When I became head of department, I was frustrated with how much time I devoted to supporting students to achieve exam success; as a predominantly social constructivist I was worried, that as Wake (2014) suggests, the current mathematics curriculum can leave students unprepared for the work place. Further, I didn’t feel that all students were developing productive mathematical dispositions (Kilpatrick et al., 2001) or the wider skills that would be required for them to be confident users of mathematics when they left school.

I had (and have) a strong drive to make students' education relevant and exciting. I wanted to foster curiosity and a love for learning, to empower students with critical thinking skills and an ability to lead their own learning that sets them up for life. To me, education is more than just learning about a discipline, it is about students having Human Agency, the ability "to influence intentionally one's functioning and life circumstances" (Bandura, 2006, p. 164). My response to these frustrations and challenges was to look for alternative pedagogies. I wanted to support the development of students' wider skills and engage them in authentic learning so they would be ready to apply their knowledge outside of the classroom. I wanted students not just to study mathematics, but also to study how to use their mathematics, as is a typical workplace practice (Wake, 2014).

Similar challenges are cited by complexity thinkers. Fuite (2005) argued that due to scarcity of time, lessons are often formed of direct centralised instruction, whilst Davis et al. (2006) posit that the curriculum is fragmented. Through a complexity lens, it can be argued that this militates against *self-organisation*. Davis et al. (2006) argue for educators to try to find a balance between structures that are so rigid that they don't allow for an innovative response and so loose that they don't support coherent activity. In Ch2 I discuss how I believe that MaPBL can create the conditions required for *emergence*.

1.4 Project Based Learning (PBL)

1.4.1 The PBL landscape

The concept of PBL was created over a hundred years ago with the "project method" designed by Kilpatrick (Holm, 2011). It has recently re-gained popularity as educators turn to pedagogies that engage and motivate students in learning and equip them with the skills that prepare them for life and work (Rogers & McGrath, 2021). It is comparatively popular in the US, where it is associated consistently with higher attainment (Innovation Unit, 2016; New Tech Network, 2020) and schools can readily access support. For example PBL Works has been running for over 20 years and provides a bank of PBL resources, offers training and is actively engaged in research on PBL (PBL

Works, 2019). However, still only about 1 percent of schools in America use this approach (Glenn, 2016).

PBL is viewed as a popular pedagogical approach in the UK (Menzies et al., 2016; Rogers & McGrath, 2021). Only a few schools in the UK use the approach for their whole curriculum. However, a larger number of schools use the approach for particular year groups, subjects or for projects delivered outside of the curriculum time (Menzies et al., 2016).

1.4.2 Defining characteristics of PBL

PBL is developed from constructivist theories: that knowledge is constructed or created by the individual from experiences and reflecting on these experiences (Hendry et al., 1999), that learning is a social process, and is context-specific (Cocco, 2006). PBL was developed from the work of psychologists such as Vygotsky, Piaget and Ausubel (Maaß & Artigue, 2013; Savin-Baden, 2014). There seems to be a lack of consensus on exactly what constitutes PBL (Condliffe, 2017). Indeed, institutions, and researchers often come up with their own sets of design principles to help them and educators decide when they are engaging in PBL (Condliffe, 2017).

Thomas' (2000) work is seminal. Thomas argues that despite the many approaches to project work in schools, a genuine PBL project has five key distinguishing features, discussed below: student-led, construction of knowledge, curriculum centrality, guiding question, authentic context. The study school incorporates these, together with embedded assessment, guidance and scaffolding, and working collaboratively. These characteristics are discussed in relation to how students learn mathematics in 3.1. In more detail:

Student-led: Students should be given the autonomy and responsibility to lead their own learning (a fuller definition is given below).

Knowledge is constructed through the projects: The teaching and learning pedagogies deployed should allow students to gain and construct knowledge through the project. Constructing new knowledge was frequently cited by participants in my previous research (IFS) as one of the hardest design features to enact. In the study school, teachers often used projects as a way of providing students with the opportunity to apply the knowledge they had previously learnt rather than allowing them to cover new content through a project. I argue that this application of learning is, in fact, a construction of new knowledge as they are re-constructing their existing knowledge to apply it into a different context.

Curriculum centrality: The project should be central to how the curriculum content is delivered. A project should not be viewed as an 'add on' to the curriculum but as an integral part. Students should be engaged in learning actively through PBL: examples provided, and experience given, through the project, to ensure the curriculum aims are met.

Driving question: The driving question should encourage the students to grapple with the underlying concepts and principles of a topic. In the study school, the teachers always set the driving question, to ensure full curriculum coverage. Students were allowed to lead their own learning within the sphere directed by the teacher.

Authenticity: The project should be authentic, allowing students to solve problems that are aligned with real world needs and to experience how they can use their mathematics. The projects used in the study school typically have a tangible outcome, which will often be presented to an audience to create a greater feeling of authenticity. Tenuta (cited in Cotič & Zuljan, 2009) argues that a student's "reality" includes the world of fantasy and imagination, and that the context of the project doesn't need to refer to the student's real everyday life. However, with a desire to support students use of mathematics in everyday life, the projects in the study school focused on contexts linked to students' lives outside of school.

Guidance and scaffolding: Kokotsaki's (2016) literature review recommended guidance and scaffolding to support successful enactment of PBL. The teachers in the study school utilise a range of different pedagogical strategies to do this.

Embedded assessment: Condliffe's (2017) literature review recommended that teachers should embed assessment throughout the project. The teachers in the study school aim to use this assessment to inform guidance and scaffolding.

Working collaboratively: The projects in the study school typically involve students working collaboratively in small groups of 2 – 3 students.

1.4.3 Related pedagogies

PBL has something in common with several other approaches to mathematics education, including problem-BL, inquiry-BL, mathematical modelling, and problem solving, so there is some transfer to be had between the related bodies of literature. I now outline the key differences.

Problem-BL, as distinct from project-BL, originated from the philosophies of Dewey. Dewey thought that education should begin with the curiosity of the student (Savery, 2006). The problem-BL approach was pioneered in the medical schools in the mid-1960s and since then has been used to teach many different disciplines and age groups (Hendry et al., 1999). The main difference between problem-BL and project-BL is, according to Savery (2006), the role of the teacher. In a project-based approach, the teacher will assume a role of a coach or instructor. They will guide and support the students and look for "teachable moments" where they can help further students' learning. In a problem-based approach a teacher will not provide information, placing a greater demand on the students to define the problem for themselves as well as to solve it. However, researchers such as Hmelo-Silver (2004), argue that for students who have not developed significant meta-cognitive skills, there is a place for direct instruction from the teacher in problem-based learning, which means the pedagogies are often similar.

Inquiry-BL is a student-centred pedagogy that introduces students to mathematical and scientific inquiry. Students pose questions, explore situations, develop their own methods and solutions and communicate and justify their explanations (Maaß & Artigue, 2013).

There is much overlap between project-BL, problem-BL and inquiry-BL. The words are sometimes used synonymously (Maaß & Artigue, 2013) or it is argued that one is a subset of another. When reading the literature, I have found that it is more helpful to read the definition and design features of a pedagogy and see how closely this aligns to the study schools working definition of MaPBL rather than to decide from the name an intervention is given whether the approaches will share much in common.

Mathematical modelling is often defined in different terms. Wake (2016) argues that fundamental to all the different descriptions is the idea that mathematical modelling involves connecting a problem in a non-mathematical context (e.g. business finance) to the world of mathematics. Students bring the context and mathematics together by both mathematizing (identifying the relevant mathematics that can lead to a solution or sense making of the problem) and interpreting the mathematical solution that has been found, back into the context of the problem. Often these processes are used cyclically.

Mathematical problem solving is viewed as confronting an activity where the problem solver does not immediately know how to get to the answer (Lester, 2013; Schoenfeld, 2016). Successful problem solving includes coordination of prior experience, knowledge, familiar representations, patterns of inference and intuition (Lester, 2013).

Broadly speaking, and consistent with Thomas (2000), I conceptualise MaPBL as often including elements of mathematical modelling or problem solving – but the more complex for often incorporating more than one of these, and because of student, not teacher, decisions. It requires that, through MaPBL, students grapple with the underlying principles and concepts of a topic; that students have the autonomy to choose how they will approach and solve the problem; and that they construct new knowledge. It is the student-led aspect of MaPBL that teachers in my prior research reported students finding particularly challenging, which I now discuss.

1.5 Overview of my prior research (IFS)

My Institutional Focus Study (IFS) explored the teacher perspective of the enactment of MaPBL in the study school and informed the focus of the thesis.

During the IFS the school was transitioning to teach with a hybrid of PBL and teacher-led pedagogies, with the year nine mathematics students. There is little research focused on such hybrids or the challenges teachers face in this transition. The study found that the biggest challenge to teachers was facilitating student-led learning. The IFS used a constructivist grounded approach to explore why teachers found this demanding. It highlighted the particular challenges they encountered and reports on the strategies they developed to aid student-led learning.

Data were gathered using reflective diaries, semi-structured interviews, a focus group, and an observation of a team meeting. This approach gave me the flexibility to explore the challenges that teachers raised in detail, the benefit of triangulating my data (Long & Johnson, 2000) and the ability to make comparisons between the different data which helped spark insights (Charmaz, 2014).

It added to existing research showing that facilitating student-led learning during PBL is a significant challenge for teachers. In the study mathematics department, this was reported as the single biggest challenge for teachers who were transitioning to teach through a hybrid of PBL and more teacher-led pedagogies.

The study also adds to the existing research on student-led learning. The participants reported finding it particularly difficult to facilitate student-led learning if they were inexperienced, became anxious about the work rate of the students, or were teaching lower attaining pupils. It would be useful to understand more about the teachers' approaches to and experiences of facilitating student-led PBL with lower attaining students as exploration of this area was limited in this study.

The IFS also reports on several strategies that the participants suggested would support the learning being student-led. Firstly, they planned carefully and often collaboratively to create an appropriately pitched project that they deemed easy to access, interesting and engaging. Secondly, they set up small groups of two or three carefully selected students who would provide mutual support and challenge within the project. Lastly, they carefully monitored the students' progress and regularly intervened to ensure that students were accessing the project and engaging mathematically at a correct level of challenge. The teachers intervened through small group and individual questioning, by using exemplars, tables and scaffolds, as well as conducting whole class discussions. The nature and timing of effective interventions is ripe for further study.

My thesis was developed to explore some of the areas highlighted in my IFS, particularly that of student-led learning, through a student lens. There is limited student voice on both MaPBL and student-led learning and there is a need to better understand the differences that these pedagogies might make to student skills, knowledge and affect.

1.6 Student-led learning

That a project is student-led is a defining feature of PBL.

> *Projects are student-driven to some significant degree. PBL projects are not, in the main, teacher-led, scripted, or packaged. PBL projects do not end up at a predetermined outcome or take predetermined paths. PBL projects incorporate a good deal more student autonomy, choice, unsupervised work time, and responsibility than traditional instruction and traditional projects* (Thomas, 2000, p. 4).

The terms ‘student-led’ or ‘student-driven’ are not usually well-defined or clearly differentiated between in the literature. In the study school, we felt that ‘student-driven’ described a slightly stronger position than we were aiming for: teachers should drive the overall direction to ensure curriculum coverage, but that students should lead their own learning within this. Therefore, the term ‘student-led learning’ was adopted.

The word ‘autonomy’ is often used synonymously with student-led or student-driven learning; Thomas (2000) uses it in his summary to stand for his definition of student-driven learning. Stefano (2013) views acting on choices as exercising autonomy. Whilst Deci and Ryan define autonomy as *“action that is chosen; action for which one is responsible*” (Deci & Ryan, 1987, p. 1025). I view student-led as constituting more than just students making choices and being responsible for these choices: students should have ownership of the way they carry out the work, the methods they use, and evaluate what they are doing.

Some of the more recent definitions of PBL, such as that given by PBL Works (2019), use the words “*student voice and choice*” in their criteria for ‘Gold Standard PBL’ rather than ‘student-led’. They define student voice and choice as allowing: “*students to make some choices about the products they create, how they work, and how they use their time, guided by the teacher and depending on their age and PBL experience*.” This definition seems to bypass some of the cognitive autonomy that I believe is of crucial importance. The type of autonomy that students are given has been found to create different outcomes. Stefanou et al. (2013) found that where autonomy is procedural, similar to the concept of ‘*student voice and choice*’, it created a high level of initial engagement in activities. However, where the autonomy is cognitive, similar to this study’s conceptualisation of student-led learning, the students demonstrated an ‘enduring psychological investment in deep-level thinking’ as well as the maximising of motivation and achievement.

Thomas’ definition of student-driven learning was viewed by the mathematics department in the study school as being underdeveloped and focused on what it was not, rather than what it should be. The department developed their own working definition of student-led learning in mathematics:

> *Student-led learning involves offering students the responsibility and autonomy to plan how they will respond to a problem using the methods of their choice, guiding the students to ensure that appropriate learning happens and supporting the students to solve the problem in the way they choose. When leading their own learning, students may make choices about, amongst other things: methods, sequence of activities, timeframe, allocation of tasks, or outcome of the project. Student-led learning may be demonstrated by: students showing an awareness of when they need help or more information; students seeking help from within their group, another group, the teacher, the internet, or other resources; students working with their group to plan their methods, discuss ideas, delegate tasks and challenge each other’s thinking. For student-led learning to be effective, students need to have self-motivation and engage fully in the project. They need the resilience to not be deterred if a solution is not immediately obvious. Lastly, they need self-regulated learning skills (Department meeting notes, September 2019).*

It is this departmental definition that I shall use for this study, and this definition that informed some of my reading for the initial literature review. The main difference in the conceptualisations is that Thomas defines student-led learning as incorporating ‘unsupervised work time’, whereas the department defines the teacher as having a guiding and supervisory role. This adaption to the definition is aligned with the more recent literature, which has found that PBL is only effective if students are adequately supported (Kokotsaki et al., 2016; Lazonder & Harmsen, 2016) and is in keeping with the modified definition of PBL that I adopt, which acknowledges that the teacher should guide and scaffold students’ work.

1.7 Local research setting

The research was conducted in a small inner-city secondary school where I was Head of Mathematics and then Assistant Head Teacher. Over a 4-year period, we adapted the mathematics curriculum for year 8 to year 10 students (13 to 15-year-olds) so that they experienced learning through some MaPBL as well as more traditional teacher-led pedagogies.

Below I attempt to provide a detailed contextualisation of the school as it allows other researchers/practitioners to see whether their context is similar and hence gives an indication of the extent the findings of this study may be generalizable to their context.

1.7.1 The school

The school was in an inner-city borough of London which in recent years has been a low-income, low employment, multicultural area. 62% of students are pupil premium, against a national average of 28%[1]. Education in the borough has recently improved significantly (East End Community Foundation, 2017) and GCSE results are above the national average[2] The same is true of the study school: in 2018, the progress results were the highest for a mixed comprehensive school in the borough. This success has allowed the school to develop a culture of ‘earned autonomy’ (Perryman et al., 2011): because the progress measures are consistently strong, departments are typically offered a high level of autonomy in the way they organise their curriculum and their use of pedagogy.

The high standard of education in the borough is, however, in stark contrast to the youth unemployment rate which is still very high at 25%, one of the highest of all local authorities in the country (East End Community Foundation, 2017).

1.7.2 The department

The department was fully staffed with mathematics specialists who, from conversations with them, appeared passionate about mathematics and teaching students; for over seven years, including the period of the study, the department performed in in the top 20% of schools nationally for student progress in mathematics. It also had a focus on developing a growth mindset in students.

Teachers in the department had established a ‘professional learning community’ (Hord, 1997). They regularly discussed teaching and learning, informally over lunch or formally in their weekly department meeting. In this way they provided a strong support for each other, and best practice was disseminated rapidly through the team.

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[1] Data from gov,uk 2020
[2] Data from gov.uk 2018

I became Head of Mathematics at the study school in 2013 and we introduced MaPBL from 2016. The projects were adapted and refined over time and more projects were constantly being added. The school won awards for their projects and a small grant to share the projects and pedagogy with other teachers. When I was promoted to Assistant Head Teacher in 2019 a new Head of Mathematics was appointed who was supportive of the department continuing to adopt a hybrid of MaPBL and more traditional teaching.

I was therefore well placed to explore the concept of student-led learning during MaPBL.

1.7.3 The students

Most of the students (over 90%) are British Bengali. These students are typically the children of 2nd and 3rd generation immigrants, so whilst 50% have English as an additional language the vast majority attended primary school in the UK and their English does not create an obvious barrier to learning.

The SES (socio economic status) indicators of Bangladeshi students in the UK are low (Strand, 2021). For example over 40% of Bangladeshi parents are long term unemployed or working in routine/semi-routine occupations, more than double that of white British parents (Strand, 2021), and 46% of Bangladeshis live in relative poverty compared to only 16% of white British families (Demos, 2015). My observations of the students in the school suggest they are typically hard working and driven. My experience of the British Bengali community's attitude to education is echoed in the wider literature: there is a strong belief in the importance of education (Smart & Rahman, 2009) and parents are supportive and actively encourage their children (Abbas, 2003). In a study of Bangladeshi girls in the borough, parents were found to have high, realistic and achievable ambitions of their daughters (Smart & Rahman, 2009). At the study school, this support includes many parents paying for private tuition. These attitudes appear to impact positively on students' education: once deprivation is accounted for, nationally, Bangladeshi students make significantly more progress than white British students (Strand, 2021).

Anecdotally, the students in the study school are often described by teachers as passive or wanting to be 'spoon-fed'. This accords with the literature that suggests that the dominant discourse of British Asian girls is that of being quiet and anonymous (Shain cited in Wong, 2012). Both in the mathematics department and across the school, teachers often talk about strategies needed to make the students more independent.

1.7.4 The intervention

The mathematics department at the study school amended their curriculum to include several departmentally-designed MaPBL units. The typical approach was for two members of the department to collaborate on the writing of a project before testing it on their classes, refining it and then it being used by the entire department.

The interviews that I carried out for an earlier study (M0E2) and discussions I had in the department suggested the teachers wrote the projects to achieve several goals. Firstly, they wanted to equip students with life skills: they wanted the students to be able to use their school mathematics beyond the classroom. Secondly, they wanted to develop students' capacity to problem solve, partially because of the increased problem solving in the new GCSE specification. Thirdly, they wanted students to have a rich and fun experience in their mathematics lessons. Lastly, they wanted students to experience large data sets to support with the statistics component if they proceeded to study A-Level maths and because of the increasing importance of data literacy.

The projects were created around the design principles discussed above. They were designed to be completed by small groups of students and largely student-led, to develop students' understanding of the mathematical content outlined in the National Curriculum and to allow student to apply their mathematics into an authentic context. Teachers would plan how to embed assessment throughout the projects and how to support and guide the students so that the planned mathematical outcomes were achieved.

In year 8 there was one MaPBL project which formed part of a larger school wide cross-curricular project. The year 9 curriculum had a series of weekly "skills builders" for the first term. These were short activities lasting from 30 to 100 minutes designed to develop the skills that the department viewed as being important for learning through MaPBL: being curious, collaborative, determined and thoughtful. This led into a further three MaPBL projects in year 9 and three MaPBL projects in year 10. Each project would typically last between one and two weeks. Some of the resources for these projects can be seen in appendices 7 – 9 and an overview is in Table 1-1.

1.8 The impact of Covid-19

Data collection was planned for December 2019 until September 2020. In March 2020 the Covid-19 pandemic gave rise to a national lockdown. Schools were closed to most students until September 2020. The study school chose not to deliver remote live teaching until the second lockdown. This limited my ability to complete lesson observations and student focus groups.

When schools re-opened in 2020 each school put in mitigations to try to reduce the spread of Covid-19. In the study school this included an expectation that teachers would not circulate, and students would sit in rows and face the front. This severely impacted the ability to complete group work. The country had a second national lockdown with school closures from January 2021 until March 2021. Students did not complete MaPBL from March 2020 until March 2021. After March 2021 the enactment of MaPBL was more limited than previously due to competing pressures on the department, including that of lost curriculum time. In Ch5 I outline the impact this had on the study.

1.9 The study

In my IFS, I found that participating teachers perceived one of their biggest challenges when teaching through MaPBL was that of facilitating student-led learning. Further, there appeared to be limited discussion in the PBL literature around student-led learning, with organisations such as PBLWorks focusing on specific elements of student-led learning: ‘student voice’ and ‘student choice’. A major review of the literature on MaPBL highlighted that many studies are not explicit about what student choice means in MaPBL, and that the issue of student choice is underdeveloped in the literature (Condliffe, 2017). The same seems to be true of the wider literature: I have found few studies that focus on student-led learning and particularly student perspectives on that.

Year	Project	Overview
8	Mathematics of Migration	Students choose a line of inquiry which is explored through a large data set. They present their findings using calculations, graphical representations, and infographics to communicate what they have learnt on a poster. Delivered as part of a wider cross curricular project on migration. Parents were invited in to view the outcomes.
9	Cake Bake	Students plan the cakes they will bake to sell in a charity bake sale. The goal is to make as much money for charity as possible within the constraints. Students scale recipes, optimise costings and timings, and understand the concepts of profit and loss. The project gives students the opportunity to solve a complex multi-step problem, work logically through a task, and break down a problem into simpler steps. Some students executed their plans and baked cakes for a charity bake sale.
9	Soma Cubes	Students are introduced to a Soma cube. They learn to draw the nets for different pieces before considering the optimum way to position the nets for the most efficient use of material. The project develops students' thinking in three dimensions and helps them build connections between 3D objects and 2D representations. It encourages learners to be logical and mathematically creative.
9	Design a Bag	Students embark on market research; they design a questionnaire and then collect their own data. They proceed to analyse this data using calculations and statistical techniques. The data analysis is used to design a bag.

10	Amazon Trader	Students use wholesale websites such as wish.com to find products that they can sell through Amazon FBA to make a profitable business, based on £250 start-up capital. They consider currency conversions, shipping costs, admin fees, VAT, income duty, the size of the product and the subsequent costs for storage and postage. The plan was to support some students to launch their own businesses, but due to the complexity of Amazon FBA this did not happen.
10	Income Support in a Model Society	Students collectively create a virtual micro-society of 24 families, each with complete profiles detailing family structure, living conditions, incomes, and expenditures. Students are then tasked with creating a policy to distribute income support benefits. They must create a clear, precise, mathematical model for distributing benefits and engage with issues of fairness, inequality, and efficiency to solve the problem.
10	Best Borough	Students use a large data set to decide which borough they think is the best in London. They produce a clear report which explains which borough is the best and why they think this. The report is evidenced with graphs and calculations.

Table 1-1 Overview of intervention

This study focuses on student perspectives of student-led learning during MaPBL, the challenges that students face and how teachers develop and use pedagogies and strategies to foster, encourage and support the development of student responsibility and choice in the classroom. To explore this area, I developed the following research questions for my thesis:

RQ1: What is the relationship between students' attitude to mathematics and their attitude to PBL, in particular the student-led aspect of that?
RQ2: What is challenging for students in leading their own learning during MaPBL?
RQ3: What pedagogical strategies are perceived to be most effective for supporting student-led MaPBL?

As will be discussed in more detail in Ch5, this exploratory study used a constructivist grounded approach to data collection, coding and analysis (Charmaz, 2014). Utilising a grounded approach allowed me to explore an area in which I found limited research, to follow emergent ideas and concepts, and to create and then explore my emergent inductive conjectures.

1.10 Thesis development and structure

Chapter 1 aimed to help the reader understand why PBL is a fundamental area of interest to me, how this may affect my analysis and interpretation of the data and introduces my research questions.

Chapter 2 provides an overview of the two theoretical lenses used to illuminate the interpretation: activity theory and complexity thinking.

Chapter 3 presents my view of learning mathematics – in broad socio constructivist terms. It explores some of the literature on self-regulated learning and around the affective traits that appear to be of particular importance for student-led learning. These conceptualisations provide a background that is drawn upon in Chapter 4.

Chapter 4 offers a review of literature for this research. This includes the literature I read to sensitise myself to the key concepts that surround student-led learning in PBL and literature that was read later as the core categories emerged from analysis of the data. Further literature is also drawn into the discussion in chapter 9. I end this chapter by identifying the gaps in knowledge and by presenting the research questions.

Chapter 5 provides an overview of the research methodology. It starts by discussing my use of a constructivist grounded approach and then highlights the impact that Covid-19 had on my study: how it became far more exploratory than originally anticipated. I end the chapter by discussing the ethical challenges.

Chapter 6 provides the methods and results for the first phase of the data collection. and includes a discussion of my approach to analysis and interpretation. I consider the rigor or trustworthiness of the study and the methodological challenges. I present the six emerging core codes: autonomy, self-regulated learning, mathematical resilience, working collaboratively, self-efficacy and goal orientation, together with the evidenced related challenges and the support strategies students suggested. I then offer an account of how I came to my emergent inductive conjecture: that students' attitudes towards MaPBL and experience of MaPBL are mediated by their self-efficacy, vision of mathematics and goal orientation. I illustrate my emergent conjectures with summaries of four vignettes that can be read in full in appendix 17.

Chapter 7 provides the methods for the second phase of the study. I present the quantitative data that were collected to explore my emergent inductive conjecture and offer a modified conjecture: that student have increased positive beliefs and emotions towards learning mathematics through MaPBL when they have a high self-efficacy and relations vision of mathematics.

Chapter 8 presents my interpretation of the data through complexity thinking and activity theory lenses.

Chapter 9 provides a discussion of the data, structured according to the research questions. The discussion considers how the suggestions the data provide complements, adds to, or contradicts the current literature. I end this chapter by summarising the tentative contributions to knowledge that the study offers.

Chapter 10 concludes this thesis by exploring implications, emerging from the findings, about how to best support student-led learning during MaPBL. I also provide recommendations for further research. I end this thesis by discussing personal learning experiences related to how this research journey contributed to my knowledge in becoming a teacher educator and researcher. I discuss how I have developed the capacity for autonomous research and how this will support my career in school leadership.

2 CHAPTER 2 – THEORETICAL LENSES

A theory describes the relationships between phenomena. It can help to explain or to predict and can also be used to guide interpretation. Using a theoretical lens was particularly important to me as an insider researcher: it had the potential to sensitise me to the consideration of previously unnoticed variables or make visible something that I had previously only sensed (Charmaz, 2014). I wanted a theoretical lens that could provide me with a new way of looking at reality (Prediger et al., 2010), that would alter my viewpoint (Charmaz, 2014), and that would 'make the familiar strange' (Brock-Utne, 1996).

It is impossible to separate theory and practice, as they are symbiotic. In completing my thesis, the theoretical lenses I read and utilised helped me to understand my practice more deeply and reflexively and as such they changed my practice. Similarly, my practice influenced how I used theory: theoretical lenses develop and transform as our studies progress (Prediger et al., 2010).

Below I give a brief overview of the two theoretical lenses that I use in this study, activity theory and complexity thinking. Activity theory is often used in educational research: it is concerned with social transformations. In my IFS I completed a brief interpretation with activity theory. I felt that it helped me to view the phenomena in a different way and had 'made the familiar strange', and so I was keen to use it again. However, I increasingly felt it was difficult to grasp the complexities of what was happening with only one theory. Further, activity theory focuses on the dialectic relationship between the individual and society: the interactions of the people who compose the activity system. Third generation activity theory does consider the interconnectedness of activity systems; however it does not consider overlapping or embedded systems (McMurtry, 2006) which I feel are significant influences on what happens in the classroom. Complexity thinking conceptualises systems as nested and therefore focused my attention onto the different systems that a classroom is embedded within. As I will discuss at the end of this section, I felt that these two theoretical lenses were complementary: they had similar units of analysis and underlying assumptions, however they provided me with varying foci (McMurtry, 2006) and complementary insights.

I felt that the impact of using these theoretical lenses on my interpretation was significant and reflected that in a published paper (Barnecutt, 2020; Appendix 1).

2.1 Activity theory

Activity theory is a holistic and contextual approach developed by Vygotsky in the early 20th century to emphasise the social origins of the action of individuals (Engeström & Miettinen, 1999). It is predominantly descriptive but can be explanatory and also developmental. It helps to reveal the social and material forces that are at play in a situation and supports attentiveness to the dialectic links between the individual and social structure. The focus is on understanding human interaction through tool and artefact. This focus was especially useful in this study for identifying and exploring the strategies that can be used to support students when learning through MaPBL.

Engeström (1999) developed a model of an activity system that built on the work of Vygotsky and others. It is often represented by a diagram consisting of three triangles (Figure 2-1). An activity system is viewed as consisting of the following elements:

- the *subject*(s) from whose position and perspective the analysis is conducted, and the *object* that motivates action and leads to an *outcome*;
- *mediating artefacts*, things that mediate action can include signs and tools, discursive practices and prior knowledge;
- *rules*, which can be visible or invisible such as beliefs and values;
- the *community*, the social group or environment in which the activity takes place;
- the *division of labour* which explores how the work is shared either horizontally, between people or people and tools in the community, or vertically, between people of power and status and others in the community (Engeström et al., 1999).

Activity theory is relevant when participants and their purposes and tools are undergoing constant and rapid change (Hashim & Jones, 2007), as is the case in this study with the introduction of a new pedagogy (MaPBL). These elements sensitised me to different phenomena that were occurring in the MaPBL classroom (Figure 8-1).

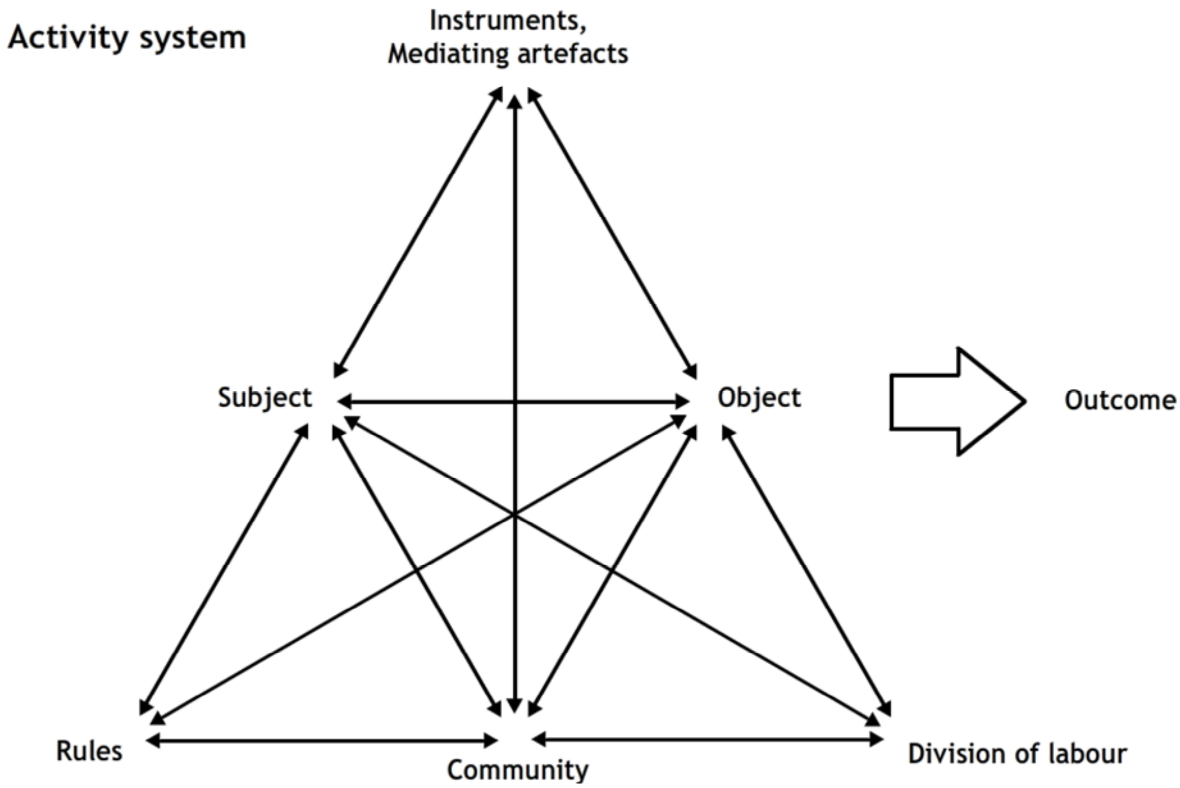

Figure 2-1: Activity system

2.2 Complexity thinking

Complexity thinking is less well-established and is a collection of ideas rather than a single theory. It is not explanatory but primarily descriptive. It is influenced by a broad range of literature, with its origins in the sciences, systems theory, cybernetics, and information science. It is being used increasingly to look at social areas including education. Similarly to activity theory, it places emphasis on the cultural and historical background of the system and is suited to exploring a system going through change. It is useful for analysing contexts, such as the one in the study school, where there are multiple interactions between different aspects of the context, and at least some of the 'players' are conceived as having agency that they may exercise in ways that are not always predictable (Davis et al., 2006). It views systems as being nested within other systems that constantly influence each other.

Complexity thinking puts an emphasis onto self-organising systems or *emergence.* A self-organising classroom will have the potential for expansive change. *Expansive change* is defined as change that creates a radically wider range of possibilities than was there before; instead of perpetuating the status quo, the space of the possible is expanded. According to complexity thinking, *expansive change* is possible only when, amongst other things, there is:

- *internal diversity*, enough difference and variation between agents that there will be a range of possible responses;
- *sufficient redundancy*, a common ground between agents for example a common language, shared responsibilities, or consistency of setting;
- *close neighbour interactions*, where the neighbours that must interact with one another are notions such as ideas, hunches and queries;
- *enabling constraints*, structural conditions that have enough coherence to ensure common purpose and enough randomness to ensure constant adjustment and adaption (Davis et al., 2006).

This concept of emergence made complexity thinking an ideal theoretical lens for me to use. My perception of the department-envisaged MaPBL is one that, in complexity terms, allows for emergence. Students are allowed to follow their own self-interest and obsessions which creates a diversity of interpretations and actions. Support and scaffolding from the teacher provide students with a clear purpose - complexity thinking does not view a common goal as necessary; a clear purpose is enough to create a complex system. The classroom can operate as a decentralized network (Figure 2-2) which Davis et al. (2006) argue is necessary for an intelligent system and creates the opportunity for emergence; where the space of the possible is expanded.

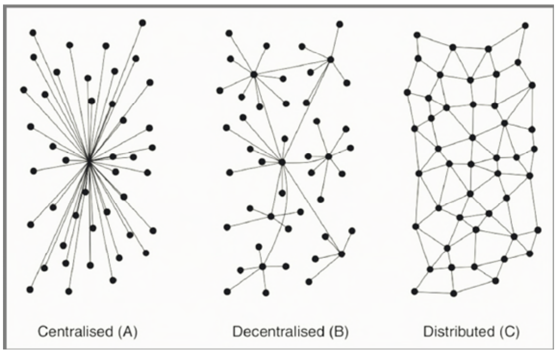

Figure 2-2: Centralised, decentralised, and distributed networks

2.3 The benefits and challenges of using multiple theoretical lenses

The adoption of two different theoretical lenses to interpretation of the data allowed access to a wider range of likely factors contributing to student attitudes, the challenges they experience, and their perceptions of strategies teachers provide for support. For example, complexity thinking drew my attention to the nested systems within which the classroom exists, such as the students' wider social context, whilst activity theory drew my attention to the vertical and horizontal divisions of labour. In this way, I viewed using multiple lenses as giving me complementary insights (Kidron et al., 2008). Using multiple theoretical lenses also helped me to develop and better understand my own emergent ideas. As I entered a dialogue and compared and contrasted my conjectures with other theories, I had a better understanding of my own conjectures.

Prediger et al. (2010) highlight how one of the challenges in using multiple theoretical lenses is that they may have different kinds of dialogues, the same words might have different meanings. Activity theory and complexity thinking have many similarities which I felt helped to negate these challenges and made the theories complementary. For example, both theories place a strong emphasis onto social systems and acknowledge the impact of the historical context and both believe that collective practices cannot be reduced to the sums of the individual parts – the sum of the whole is greater than the sum of the parts.

Kidron et al. (2008) highlights how challenges may occur if two theories have different underlying assumptions which may lead to contradictory interpretations. One of the key differences between activity theory and complexity thinking is the level at which they assume learning takes place. Activity theory seems to refute the idea of a collective knower (McMurtry, 2006), whilst complexity thinking puts an emphasis on the knowledge that exists within a complex system. I view these assumptions as being a product of considering processes that are working at distinct levels (Davis et al., 2006) and therefore they were not contradictory interpretations. Kidron et al. (2008) also highlight that challenges can occur when there are different units of analysis. In activity theory the unit of analysis is activity itself whereas in complexity thinking the unit of analysis is a nested system. Again, I did not feel any tensions arising in interpretation from this difference.

There was overlap between the theories, however I did not attempt to integrate them locally into a new framework, but rather combined them to give complementary insights (Prediger et al., 2010). I found that the theories fed back on each other; my interpretation grew symbiotically with them both in an ongoing dialogue. Insights that I gained from one interpretation aided and deepened the interpretation using the other lens.

3 CHAPTER 3 - LEARNING MATHEMATICS

To me, the goal of mathematics education is not just for students to learn mathematics, but to develop mathematical dispositions (De Corte, 2004) and to become users of mathematics. Schools need to prepare students to use their mathematics in “settings as yet undetermined” (Wake, 2014, p. 273).

Here, I present how I view learning mathematics as broadly social constructivist. I then explore some of the literature on self-regulated learning and around the affective traits that appear to be of particular importance for student-led learning. These conceptualisations provide a background that is drawn upon when I discuss the PBL literature (Ch4).

3.1 Learning mathematics

A social constructivist approach is commonly viewed as originating from the work of Vygotsky. In the constructivist paradigm, learning occurs when a student has an active role in the learning process, develops their own understanding of concepts and creates their own representations (Cobb et al., 1992), through interactive pedagogies (Erbil, 2020).

One of the defining characteristics of MaPBL adopted by the study school is working collaboratively. Ideas are constructed through interaction with the teacher or other students (Powell & Kalina, 2009). Learning arises when a student attempts to make sense of another person’s vision of the world (Sfard, 2006) and the student gradually acquires knowledge, and the characteristics and norms of the learning community (Liu & Chen, 2010).

A criticism of some pedagogical approaches based on social constructivism, is the lack of structure (Mayer, 2004). To mitigate for this, a defining characteristic of MaPBL in the study school is support and guidance from teachers. Vygotsky describes students as having a ‘zone of proximal development’ (ZPD), the area of activity students can advance through with the assistance of a ‘more knowledgeable other’ but not on their own (Vygotsky, 1980). I view a ‘more knowledgeable other’ inclusively, as teacher, peer, resources, activities, or classroom discussion (Siyepu, 2013). Within a mathematics class, the ZPDs of students are diverse (Suranata et al., 2018) and therefore students will require different scaffolding. How students’ learning is mediated by support and guidance is a key concern of this study.

Social constructivism views learning as situation specific, and context bound (Liu & Matthews, 2005). If students are to use their mathematics after school, they need opportunities to use their mathematics in authentic situations. To use their mathematics, students need to connect mathematics and context, something which is typically found to be challenging for students (Wake, 2015). Boaler (2000) found that when the students completed mathematical tasks within school that were more similar to those of the real-world, students were more willing and able to use the methods they had learnt at school to complete a real-world task, as well as being able to use this knowledge in other contexts such as their exams. Of particular importance in this discussion is how students view the authentic context.

A student self-regulating their own learning is a major component of constructivist learning (De Corte, 2004) which I now discuss:

3.2 Self-regulated learning

A recent EEF guidance report reiterated that metacognition and self-regulation are important for effective pupil learning (Quigley et al., 2018). Self-regulated learning has been shown to be a predictor of academic performance (Minnaert & Janssen, 1998) and to decrease student discipline issues (Corsi, 2010).

Self-regulated learning, metacognition and self-regulation are sometimes used interchangeably. Whilst there is much overlap, there are distinctions which can be traced back to their origins (Dinsmore et al., 2008). Self-regulation was initially concerned with behaviours and emotions, whilst metacognition is concerned with learner development. Metacognition views *'the mind of an individual as the initiator or trigger for subsequent evaluations or judgements'*, whereas self-regulation views it as the environment that *'stimulates awareness and their regulatory response'* (Dinsmore et al., 2008, p. 405). The concepts of motivation and cognition occur much more frequently in the self-regulation literature (Muijs & Bokhove, 2020).

To understand what happens when students are leading their own learning, I think we have to look to both internal and external triggers and therefore I view self-regulated learning in Dinsmore's (2008) terms – it encompasses both metacognition and self-regulation but is focused solely on learning.

The dimensions of self-regulated learning are under-theorised in the literature (Pellegrino & Hilton, 2012), which may pose a challenge for teachers trying to support students' development.

3.2.1 Metacognition

Metacognition includes both knowledge of cognition and regulation of cognition. Knowledge of cognition includes declarative, procedural, and conditional knowledge. There is a strong relationship between metacognition and outcomes, with a positive correlation between problem solving and meta-cognition (Mevarech & Kramarski, 2014).

When considering metacognition, it is important to remember that it is age specific and quite possibly domain specific, although that is contested (Mevarech & Kramarski, 2014). Consequently, when discussing metacognition later in the study I have drawn from the literature on metacognition in mathematics in secondary schools. It is possible to help students to develop their metacognitive skills (Dignath & Büttner, 2008). However, the metacognitive strategies that students use are task specific: students must adapt them for each task.

3.2.2 Self-regulation

'Self-regulation refers to self-generated thoughts, feelings, and behaviours that are oriented to attaining goals' (Zimmerman, 2000, p. 14). Students need to regulate cognition, motivation and affect, behaviour and context (Pintrich, 2000b). It is what a student uses to overcome challenges – internal challenges such as counterproductive impulses, or external challenges that may arise (Pellegrino & Hilton, 2012). Zimmerman (2002) describes three phases of self-regulation; forethought, planning and self-reflection.

Time management is a crucial cognitive regulatory aspect of self-regulation. It has been found to lead to higher academic achievement (Zimmerman & Risemberg, 1997) and have a sustainable impact (Liu et al., 2009). Whilst there is no agreement on an exact definition, time management is commonly viewed to include: determining needs, setting goals, prioritising and planning tasks, and monitoring.

Self-regulated learning is not viewed as being achieved naturally (Gidalevich & Kramarski, 2019). I conjecture that the sample students may have underdeveloped self-regulated learning skills as many students live in the intersection of results driven cultures, of the British Bengali community, the local context, and the national climate of performativity in education (Ch1). My knowledge gained as a senior leader in the school, suggests that students have been offered limited opportunities for student-led learning. It, therefore, seems unlikely that they have well developed self-regulated learning skills which may provide increased challenges for students when leading their own learning during PBL.

3.3 Affective traits that support student-led learning

Social constructivism highlights that there are close interactions between affective and cognitive factors (Op't Eynde et al., 2006): affective traits play a significant role in learning (McLeod, 1992). There is often overlap between affective factors and they are frequently mutually supportive. For example, students with high self-efficacy are likely to show stronger resilience (Borman & Overman, 2004) and motivation (Schunk, 1991). Here I briefly discuss the affective traits that are most relevant to the focus of my study: resilience, motivation, self-efficacy, and attitudes. They are discussed more fully in relation to the mathematical literature in appendix 2 and then in relation to PBL, in Ch4.

3.3.1 Resilience

Johnston-Wilder et al. (2010) suggest that students require a 'mathematical resilience': *'maintaining self-efficacy in the face of personal or social threat to mathematical well-being'* (Johnston-Wilder & Lee, 2010, p. 3). Mathematical resilience has been found to support problem solving (Attami et al., 2020) and group work (Hafiz & Dahlan, 2017), both of which are fundamental in MaPBL.

Borman et al. (2004) describe how resilience can derive from factors that come from a student's environment, specifically, positive teacher–student relationships, a safe and orderly learning environment, and a strong school community. They argue that students will show a stronger resilience if they have high mathematical self-efficacy, a positive outlook to school, high self-esteem, and a high engagement with academic activities.

3.3.2 Motivation

Here I discuss two theories of motivation: achievement goal theory and self-determination theory before considering how authentic contexts may motivate students.

3.3.2.1 Achievement goal theory

Achievement goal theory suggests that students can have a mastery goal, they want to develop competence, to understand new concepts and develop new skills, or a performance goal, they want to demonstrate competence, to do better than others or to surpass normative-based standards and to achieve with little effort (Ames, 1992). The students' wider school culture and home environments are likely to be performance driven (Ch1).

When studying mathematics, students with a mastery orientation have been found to have stronger self-regulated learning skills (Hidayat et al., 2018; Maretasani et al., 2016; Meece et al., 2006). This could be because students with a mastery orientation are more likely to use self-monitoring and organisational approaches (Hidayat et al., 2018), seek diagnostic feedback, and persist in the face of negative feedback (Schmidt & Ford, 2003). Studies have also found that resilience and mastery goal orientation are positively correlated (Splan et al., 2011).

3.3.2.2 Self-determination theory

Self-determination theory suggests that intrinsic motivation, learning and wellbeing are enhanced when students have three basic needs met: the need for autonomy, to follow their own values and interests; the need for relatedness, to relate to other people and to feel their activity fits with the values that exist within their culture; and the need for competence, to feel that they are able to be successful and meet their goals (Ryan & Deci, 2000). In 9.2.5, I explore the challenges of meeting the needs of autonomy, competence, and relatedness during MaPBL.

3.3.2.3 Authentic contexts

Research has found that problems set in authentic contexts can be motivating for students. However, Vos (2018) found that having an authentic question – a question that people in the 'real world' would pose – was more motivating for students than the context. Rellensmann et al. (2017) in a survey of 100 ninth graders, found that interest in “real world problems” in mathematics was lower than for problems with no connection to reality. This might be because they were ‘unauthentic word problems’ (Vos, 2018), that weren’t meaningful for students. In 9.2.5 I discuss how students viewed the authenticity of some of the projects.

3.3.3 Self-efficacy

Tilfarlioglu et al. (2011) claim that in order to support autonomy, you have to attend to beliefs. They found that there was a positive relationship between learner autonomy and self-efficacy. Self-efficacy as conceptualised by Bandura (1986) relates to a person’s beliefs about how well they will perform in relation to a specific activity. It is situational or project specific (Pampaka et al., 2011). Self-efficacy and achievement in mathematics have been found to have a strong reciprocal relationship (Sartawi et al., 2012; Williams & Williams, 2010).

Bandura (1986) stresses how it is important to differentiate self-efficacy from the more general concept of self-concept. Self-concept is how someone thinks about, perceives, or evaluates themselves. Beliefs regarding confidence are part of self-concept.

That self-efficacy and mathematical resilience are linked is well established in the literature. Students with low self-efficacy have been found to have less trust in their own abilities in face of adversity; rationalise problems as threats as opposed to challenges; experience a higher level of negative emotional arousal when completing demanding tasks; and show less perseverance when faced with challenging situations (Bandura et al., 1997).

The literature suggests that there is a significant link between self-efficacy and self-regulated learning (Coutinho & Neuman, 2008; Tian et al., 2018). Tian et al. argue that this was because self-efficacy creates a powerful motivation for self-regulated learning.

3.3.4 Attitudes

Attitudes are widely viewed as being fairly sustained emotional responses that are reflected in what you say and do (Hannula, 2012b). They are of moderate intensity and reasonable stability. Hannula (2012b) states that the most commonly used definition in mathematics education is based on that of Hart (1989): attitudes can be viewed as consisting of three components, beliefs, emotions and behaviour. I adopt this definition for this study and now focus briefly on each of beliefs and emotions.

3.3.4.1 Beliefs about the nature of mathematics

Beliefs are generally perceived amongst researchers to have an important influence on learning (De Corte & Op't Eynde, 2002). Skemp (1978) suggested that students have different beliefs about the nature of mathematics: relational or instrumental. Some students view mathematics as relational, as a connected body of knowledge that can be added to; they focus on understanding what to do and why, and especially, how that connects with and relates to what else they know. Other students view mathematics as instrumental, as something that can be learnt by rote, they just want to learn the rules without understanding why things work, so long as they can 'do it' for a particular purpose such as pass an exam. I view this as an over-simplification: students don't necessarily have one understanding or another (Di Martino & Zan, 2009) and the balance might vary cross different areas of mathematics.

3.3.4.2 Emotions

Alongside other affective processes, emotions are viewed by social constructivists to be an integral part of learning (Op't Eynde et al., 2006). Emotions are states of consciousness or feelings (Voica et al., 2020), that are highly contextual (DeBellis & Goldin, 2006) and that can be of limited stability.

Emotions are typically viewed as being positive or negative; not succeeding with a goal typically leads to negative emotions and succeeding with a goal typically leads to positive emotions (Hannula, 2014). However, Pekrun et al. (2007) posit that emotions not only have a positive and negative dimension, but they also have an activation dimension. For example, relief or relaxation are positive deactivating emotions, whilst boredom is a negative deactivating emotion.

3.4 Chapter summary

Above, I presented my theory of learning and conceptualisations of self-regulated learning and the affective traits of resilience, motivation, self-efficacy, and attitude. This provides a background for the literature review of student-led learning within PBL which I now discuss:

4 CHAPTER 4 - LITERATURE REVIEW

4.1 Introduction

I view this as a literature review for research (Maxwell, 2006). It can be considered a conceptual framework rather than a thorough review of the literature in the field; the literature was chosen based on its relevance to the study. For my initial literature review I conducted a literature search on student-led learning within PBL or other similar pedagogies in secondary schools (for a description of my approach to the literature search, see appendix 3). In searching for literature on student-led learning, I did not find any systematic review of the area, nor authoritative or seminal papers - they were largely small-scale. The literature on agency, did however, have something to offer.

In keeping with a constructivist grounded approach, as I began to develop my emergent codes and categories, I started to read further literature primarily from the field of mathematics and the wider PBL literature, but sometimes from different fields and disciplines based on my emergent themes. Some of this literature is in the literature review and some is drawn directly into the discussion (Ch9).

Mathematics is underrepresented in the literature on PBL (Condliffe, 2017). Condliffe suggests this could be because there is a lower level of implementation of PBL in the mathematics classroom. She posits that this could be because enactment of PBL in mathematics is more complex than for other subjects and because some teachers have difficulty integrating PBL into the mathematics curriculum. A phenomenon that my IFS and teaching experience, suggests is true.

Condliffe (2017) also highlights that whilst the most common type of PBL is teacher-initiated PBL, little is known about the most common approaches that are used or the challenges that teachers face within this. I found limited literature that was based on teacher-initiated PBL. My IFS contributed to filling these gaps and considered the challenges that teachers faced when transitioning to use PBL and the strategies that they used to support students.

In 4.2 I discuss student-led learning and the significant change in role for both the teacher and the students. I then explain how support and guidance is imperative for students during MaPBL, but that this support needs to be carefully tailored to students' age and prior experience and should be added or faded, or a combination of both as required. In 4.3 I discuss self-regulated learning, which I consider in Dinsmore's (2008) terms as composed of both self-regulation and metacognition, during MaPBL. In 4.4, 4.5 and 4.6 I explore the relationships of the affective traits of self-efficacy, resilience, and motivation, with PBL. In 4.7 I discuss attitudes, which I define in Hart's (1989) terms as a composite of their beliefs, emotions and behaviour, I consider how PBL can influence students' attitude to mathematics. I also present the beliefs, emotions, and behaviours that students are reported as experiencing towards PBL in the literature. In 4.8 I discuss how working collaboratively has been found to aid student-led learning and mathematical achievement and present ideas from the literature on how to engender positive collaboration. In 4.9 I present my research questions that were designed to contribute to the existing knowledge base. In 4.10 I discuss some of the limitations to the literature review. Specifically, that much of the literature that I draw on is in a different context and sometimes a different subject domain to the study. In 4.11 I provide a summary of the above discussions.

4.2 Student-led learning

The teacher and student roles in PBL are significantly different from that in a 'teacher-led' classroom. The teacher's role in PBL it to "*facilitate exploration, development, imagination, and communication of ideas and concepts"* (Özdemir et al., 2015): they need to engage throughout the process with students (Hoogenes, Mironova, Safir, Mcqueen, et al., 2015; Mergendoller et al., 2006) and provide guidance and support where required (Kokotsaki et al., 2016; Lazonder & Harmsen, 2016). This guidance can help to provide structure for students, enabling them to have autonomy over parts of the project, and can provide hints and supporting information (De Jong, 2006). Effective support and scaffolding can help reduce 'cognitive load' (Kokotsaki et al., 2016), ensure that students acquire the relevant knowledge (Mayer, 2004), help students to self-regulate their learning (Ge et al., 2016) and provide motivational support (Kokotsaki et al., 2016). As I highlight below, support for students needs to be adjusted according to the age of the students, how experienced they are in learning this way, and the curriculum content.

For PBL to be effective, a student must embrace the offered autonomy and act with agency. The literature on agency suggests that students are *“neither autonomous agents nor simply mechanical conveyers of animating environmental influences”* (Bandura, 1989, p. 1175). Action, cognitive affective factors, as well as environmental events are all interacting determinants of agency. Self-efficacy beliefs are viewed as pervasive: they mediate motivation, affect and action (Bandura, 1989). Bandura (2018) theorises three elements of agency: forethought, self-reactiveness and self-reflectedness, which require self-regulation. Self-efficacy again impacts a student’s self-regulation: a student’s self-efficacy will impact the type of goals a student sets for themselves (Bandura, 2018).

Promoting agency has been reported as a way to make mathematics more meaningful, engaging and relevant (Wright, 2017). However, there is an intrinsic context-dependency in such work (Nieminen et al., 2021) and agency needs to be developed over time. In a 3 year long design-experiment, Brown (2020) explored the development of agency in the secondary mathematics classroom. Brown found that through the process of argumentation – where the goal is to find collective solutions and build agreement and mutual understanding – students were able to act with more agency over time. He describes how students were initially submissive and deferred to the teacher to tell them what to do. He found that within weeks, students had started to become more confident, to focus on the task solution and recognise the contributions of their peers. Over time, he found that students became assured, they started to think for themselves and evaluate the quality of the groups thinking and learning. To engage and empower students in this way has been found to require a strong relationship between the student and the teacher and require a high level of trust (Wright, 2017).

Deciding the level of autonomy to give students can be complicated, and as Brown’s (2020) study suggests, may change over time. Barak (2004) who completed a small-scale study (N = 60) about Design Technology PBL at six secondary schools, suggested that students can be viewed as being at one of three stages, ‘show me’, ‘let’s think together’ or ‘trust me’, and that teachers should adjust their support accordingly. If students are at the ‘show me’ stage, they need direct supervision. If they are at the ‘let’s think together’ stage they need collaborative supervision and at the ‘trust me’ stage they need non-direct supervision. Teachers can be supported to foster the transition from being teacher explicit to student explicit and to make instructional decisions appropriate to their classes, through the use of a graphical representation that highlights the many ways this transition can occur (Cuevas et al., 2005).

Other researchers suggest that students should be given the autonomy to manage themselves (Mergendoller et al., 2006), they should seek the information they require from their teachers rather than be given it (Casey, 2011) and that the obligation is on students to seek clarification if they don’t understand (Brown, 2020). Kim et al.’s (2018), suggestion is to allow students to control the level of support that they receive during PBL. They suggest that scaffolding can be faded or added, or a mixture of both can be used. They suggest that giving students the opportunity to ask for help when they need it or reduce the support when they feel competent ultimately leads to students having a stronger control over their own learning. They acknowledge that students may misjudge their own ability, however they suggest that challenge can be moderated by teachers who can ask students to reflect on their choices.

Lazonder et al. (2016) completed a meta-analysis of 72 studies of inquiry-based learning in science or mathematics. They were interested in the impact of guidance during inquiry-based learning on learning activities, performance success and learning outcomes. They explored whether the effectiveness was dependant on the type of guidance or the learners’ ages. They found that guidance led to students acting more skilfully and performing more highly on tests after the inquiry. They also found that adequate guidance was just as effective as specific guidance for learning activities and outcomes. For example, simple prompts supported students as well as an extensive explanation. They found that highly specific guidance was only required when students needed to maximise their performance, for example to create a presentation for parents. They also found that this did not change with a student’s age.

When students are leading their own learning in MaPBL, it is especially important to ensure that the mathematical component of the project is realised. A study by Gresalfi et al. (2012) found that whilst teachers were using identical curriculum materials, the level of student mathematical response varied significantly. Students' responses were more likely to show a higher level of mathematical engagement when teachers clearly communicated the expected nature of response and made students accountable for being explicit about the mathematical components of their decisions. Similarly, a study of teaching mathematics for social justice found that where students were given more guidance, they used mathematics more effectively. One effective strategy was for students to compare mathematical and non-mathematical statements (Wright, 2017).

A similar situation has been observed in the technology PBL classroom where students focused on the processes not understanding the underlying concepts (Barak, 2012). This may be because, as found in a study on agency in the mathematics classroom, students need to learn the new social norms and how to interact positively together before it is possible to discuss or understand more rigorous concepts (Restani, 2021).

In summary, the literature suggests, support and guidance is imperative for student-led learning during PBL. The amount of support given needs to be carefully tailored to students' age and prior experience and should be added or faded, or a combination of both, as required. The department had discussed how to support student-led learning during PBL regularly in department meetings and have used many different strategies. It will be important to try to capture the different strategies that are used whilst completing the lesson observations and how students respond to them. The other research tools should attempt to ascertain students' perspectives on the different strategies.

4.3 Self-regulated learning

I conceptualise self-regulated learning as encompassing both metacognition and self-regulation (3.2). Self-regulated learning is required by PBL, as students solve ill-structured problems (Milbourne, 2016). Stefano et al. (2013), in their comparison of types of PBL, found that when the focus of the project was on applying or integrating knowledge, as is typical of most of the projects in the study school, rather than acquiring knowledge, the students experienced higher levels of regulation of cognition: they used deeper level learning strategies and stronger critical appraisal of effort and knowledge.

The demand that PBL places onto students' self-regulated learning can cause challenges as students' skills are sometimes limited (Mergendoller et al., 2006). However, PBL and approaches similar to PBL have been shown to develop students' self-regulated learning skills (e.g. with second chance adult learners Koutrouba & Karageorgou, 2013; in natural sciences Magdaş & Pop, 2015; in Biology Sungur & Tekkaya, 2006). But as self-regulated learning is likely to be discipline specific (3.2), these results may not transfer into MaPBL.

Lawanto et al. (2013), in their study of students completing a design project, argue what my experience suggests: teachers need to dedicate time to support students with self-regulated learning, especially planning. Some studies have utilised written scaffolds. Cuevas et al.'s (2005) study, of inquiry based science in an urban school with a large proportion of low SES students, provided students with an inquiry framework which had hints for the students to support them in planning what they needed to do. However, the literature suggests teachers do not always attend to supporting self-regulated learning. For example, Barak (2012) found that the teachers in their study of technology PBL often focus on improving students' subject knowledge rather than their self-regulated learning skills. They reported believing that students will naturally develop these skills.

English et al. (2013) in an American study of schoolwide PBL, theorise that each of the phases of self-regulation can be viewed as being matched by a phase of PBL. Forethought is linked to the project launch, performance to the guided inquiry or solution creation phase and reflection to the conclusion. They suggest 'fading' the support throughout a project: for the first phase, there should be significant teacher direction, but for the last phase, students should use their self-regulation skills and knowledge. My experience suggests that self-regulation will need to be developed over a longer period than one project and further reflection is of vital importance and should be supported. English et al. also suggest, and I agree, that greater understanding of the self-regulated learning processes that are being used within each phase of PBL is required.

Time management is an important self-regulation strategy (3.2.2), that students have been reported to struggle with during PBL. For example students have been observed to leave things until the last minute (Barak, 2012) and have reported that the least enjoyable aspect of PBL is the stress of time management (Magdaş & Pop, 2015). Eilam et al. (2003), studying students over a year-long inquiry in a Grade 9 Science class, found that it was particularly challenging for students to effectively manage their own time when they were also acquiring a new skill. Eilam et al. found that higher achieving students in the class were better at managing their time. They posit that this may be because: they have a stronger prior knowledge which enables them to understand the tasks that they must achieve and therefore to judge how long they should take to complete them; and they are likely to have more mental effort to devote to self-regulated learning, as they are more likely to have high self-efficacy and hence less likely to deal with other emotional disruptions.

Section 3.2 highlighted literature that found that a student's goal orientation mediates their self-regulated learning. The same has been found in MaPBL, Maretasani et al. (2016), in their study of 32, grade 10, Indonesian students, found that students with a mastery goal had a higher performance on every aspect of metacognition than students with a performance goal. They found that students with a mastery goal, had a stronger knowledge of cognition, better regulation of cognition, were able to meet all the indicators of problem solving and use all Polya's steps. Students with a performance goal were more likely to struggle to plan, carry out the plan and they rarely looked back to reflect on what they had done: they struggled with planning, monitoring, and evaluating. The study had several limitations including non-randomised allocation to the control group and a small sample size. Further, it seems that the assessment measures were researcher designed, if they were they may be less reliable and valid in comparison to published standardised measures.

In summary self-regulated learning is both required for, and can be developed through, PBL. The department, upon embarking upon teaching through MaPBL, identified some of the self-regulatory skills that students might find challenging. Many teachers regularly used written scaffolds or gave guidance to support students' self-regulated learning. For example, teachers planned the projects with students, or used starters that re-visited prior knowledge to support mathematical connections. That teachers already used a variety of strategies to support the development of students' self-regulated learning will allow me to explore students' perceptions of the strategies they experienced.

4.4 Self-efficacy

Self-efficacy is considered an important affective trait in the literature on agency (Bandura, 1989) and is a key concern of this study. Despite this, there appears to be a lack of literature that explores the impact of a student's self-efficacy on their ability to effectively enact PBL. However, some literature does suggest that PBL has an impact on self-efficacy, which I now discuss:

Some studies have reported that MaPBL has increased students' self-efficacy (Cerezo, 2004; Peranginangin et al., 2019). Peranginangin et al.'s (2019) study of Karo Culture PBL in Indonesia, does not provide the instrument used and therefore it is not possible to tell if it measures self-efficacy or a related construct. Cerezo's (2004) US study of 'at risk girls' enactment of mathematics or science PBL in a middle school, concluded that students felt more able to use the library, speak in class, have confidence in their organisation and ability to keep on task as well as a higher completion rate of homework. These phenomena suggest to me that students were showing an increase in self-confidence rather than in self-efficacy. Similar findings were made by Brosnan et al.'s (2010) US study on the impact of a MaPBL summer school, where students were found to have an increased self-confidence. They suggested this could be due to support of the 'community of practice' which was established, or the influence of students completing more demanding problems.

Students did not always seem to benefit from attempting demanding problems. Moyer et al. (2018) completed a seven year longitudinal study into inquiry mathematics, which has a similar definition to MaPBL. They found no difference in students' perceived competence, their belief that they have the attributes needed to succeed, between the control group (who had been taught more traditionally) and the experimental group. However, they noted that students' perceptions of mathematics had changed: all the students in the experimental group who reported lower perceived competence described this as being because they thought mathematics was difficult. A similar finding was made by Boone (2013): whilst the majority of students reported that MaPBL makes them feel more competent in mathematics, some students felt uncertain and unsuccessful when faced with these complex problems, and that led to them liking mathematics less.

Other studies have found no impact on self-efficacy. Sungur et al. (2006) used a quasi- experimental design, (N = 61) with a control group and experimental group both taught by the same teacher. The experimental group learnt Biology through Problem-BL, which has a similar definition to PBL. The experimental group did not report any change in their self-efficacy for learning and performance. The study also found that self-efficacy had a significant relationship with student's use of all the learning strategies surveyed such as critical thinking, metacognitive self-regulation, and peer learning. The only strategy that did not have this positive relationship with self-efficacy was asking for help.

It appears that MaPBL has the potential to improve students' self-confidence, but whether it improves their self-efficacy, their belief they will succeed on a specific problem, is harder to ascertain. Further, for some students, the increase in the complexity of the problems during MaPBL means that they perceive mathematics as being more difficult and their perceived competence does not increase.

4.5 Resilience

Ponton et al. (2000), in their consideration of the literature associated with the fields of self-directed learning and psychology, concluded that initiative, resourcefulness, and perseverance, factors that are required for resilience, all need to be fostered for autonomous learning to occur. The importance of resilience was also acknowledged by the teachers in the study school in their definition of student-led learning (1.6).

The literature that explicitly investigates the relationship between resilience and PBL is sparse (see appendix 3). Hafiz et al. (2017), in their comparison between MaPBL and guided discovery, found that MaPBL had significant effect on mathematical resilience. They found that mathematical resilience was improved by the motivation that students felt when completing PBL, this motivation meant that they were more likely to persevere when problem solving. Students who had learnt through MaPBL reported having: increased beliefs in mathematics being a useful tool for solving every-day problems; a curiosity for new knowledge; and a stronger understanding that succeeding in mathematics requires effort. However, a study by Hutauruk et al. (2019), found that mathematical resilience was not significantly higher for an experimental group, who learnt through MaPBL, than the control group, who experienced 'traditional' lessons. Both were small-scale studies that involved only a few classes and therefore neither set of results is generalizable. However, these papers highlight that is unclear as to whether MaPBL itself develops resilience.

4.6 Motivation

Self-determination theory has been found to provide important insights into motivation in PBL (Liu et al., 2009; Prigmore et al., 2016). Liu et al. in their quasi-experimental pre-test/post-test study of 767 secondary school students completing project work, a form of PBL, in Singapore found that students' motivation mediated their experience and learning during PBL. Students who were intrinsically motivated, who had high autonomy, relatedness, and perceived competence needs were likely to have a more positive experience and perceive that they experienced greater learning during PBL. This study did not use control groups and therefore these results may reflect students experience of school in general and not PBL.

The MaPBL literature often reports that students completing PBL show higher levels of motivation and engagement than in 'traditional' lessons (Canuteson, 2017; Cerezo, 2004; Langer-Osuna, 2015). A case study of MaPBL conducted by Langer-Osuna (2015) attributed the increased engagement in learning reported by a student to be due to the autonomy that he operated with, alongside the opportunity to draw on his own lived experiences. Langer-Osuna argues that through learning in a MaPBL environment, the student was able to re-construct his identity in the classroom and so better engage both with mathematics and within collaborative work. Belland's (2006) study of PBL, of three classes of students with special needs in a school in America, found the students reported that this increase in engagement was due to them working collaboratively.

The context of a project also appears to influence motivation. Daher et al. (2009), who completed a study of students' perceptions whilst learning mathematics through their mobile phones found that students are enthusiastic to learn about themselves and their environment, whilst Hung (2009), whose study is of PBL across subjects, found that students needed contexts that inspired curiosity and interest. Hung (2009) posits that relevance and proximity influence ownership.

Al-Balushi et al. (2014) in their quasi-experimental pre-test/ post-test study found that the projects that appeared to make the most impact had an enjoyable and unusual end product. They suggested that having an audience for the end product increased the work rate of the students. However, they also found that there was an observed decline in enthusiasm over time. They posited that the initial enthusiasm could have been due to the novelty effect of the projects.

Not every study found intrinsic motivation to be linked to the students. Lam et al. (2009), in a study in a secondary school in Hong Kong found that students' intrinsic motivation during PBL stemmed from the intrinsic motivation of the teacher. Teachers with a higher intrinsic motivation were viewed by students as providing more support during PBL lessons, which led to the students demonstrating higher intrinsic motivation. My observations of, and conversations with, the sample teachers suggest that they are intrinsically motivated which may have impacted students' motivation.

4.7 Attitudes

When I completed the initial literature search, I found only four papers that discussed how students report their attitudes towards and experiences of PBL. Similarly, Grant (2011) comments how little research has explored the student perspective of PBL whilst Saunders-Stewart (2008) highlighted the limited literature focused on what students think about the outcomes of PBL. Beckett (2005) argues that where research has considered the student opinion, it has often simplified the student perspective and has not tried to communicate the dilemmas that students face. The research that has focused on student opinion in PBL, has typically focused on PBL within science (Saunders-Stewart, 2008).

Considering student attitudes to the student-led aspect of PBL is pertinent to my context. My IFS (1.5) found that in the mathematics department where this study was situated, the biggest challenge for teachers who were transitioning to teach through a hybrid of PBL and more teacher-led pedagogies was that of facilitating student-led learning.

Below, I consider how MaPBL has the potential to influence students' attitude to mathematics. Then, using Hart's (1989) definition of attitude, I discuss the literature on students' beliefs, emotions, and behaviours towards PBL.

4.7.1 Attitudes to mathematics

Completing MaPBL has been found to have the potential to change students' attitude to mathematics. Some small-scale studies of MaPBL have reported on students' attitude to learning mathematics using pre and post-tests (e.g. Statistics: Koparan, 2014; Ratio, proportion and percentage: Özdemir et al., 2015; Geometry: Uyangör, 2012). One study used a control group; however, it was not randomised, and it is likely that the teachers in these studies were committed to this approach. These studies appear to be teacher-initiated MaPBL, therefore these findings are of particular relevance to this study. Other studies (Cotič & Zuljan, 2009; Moyer et al., 2018; Wade, 2013) found no reported change on students' attitude to mathematics when learning through PBL. However, as Cotič et al. (2009) highlight, these findings need to be considered in the context of the wider changes in learning students were experiencing: despite the fact that the students were working on harder problems, they maintained the level of positivity they had previously exhibited.

4.7.2 Beliefs

The wider literature has shown that students report believing that PBL: supports their achievement and application of knowledge (Saunders-Stewart, 2008); improves their self-regulated learning skills such as self-direction and time management (Koutrouba & Karageorgou, 2013; Meyer & Wurdinger, 2016); increases their self-esteem and confidence, concepts that share something in common with self-efficacy (Saunders-Stewart, 2008); develops willingness, persistence and initiative, socio-affective skills that share something in common with resilience (Koutrouba & Karageorgou, 2013); and increased their communication skills (Magdaş & Pop, 2015) and ability to work collaboratively (Meyer & Wurdinger, 2016).

Whilst this research might suggest that the study school students are likely to have positive beliefs about the outcomes of PBL, much of this research context is of PBL that is used across the curriculum. In the study school, only the mathematics department uses PBL, and students do not learn solely through

MaPBL. For this reason, these findings may not transfer into the context of the study school.

The MaPBL literature, whilst limited, has also found that students have positive beliefs about this pedagogical approach. Some small-scale studies (N<100) found that after learning through MaPBL, students reported: benefiting from using multiple solution methods and perspectives (Schettino, 2016; Wade, 2013), valuing the contextualisation provided by PBL as it activated their prior knowledge and their ability to make connections and be creative (Ubuz & Aydınyer, 2019) and believing that MaPBL developed their career related skills (Canuteson, 2017). However the students in Canuteson's (2017) study also reported feeling less confident about being prepared to pass exams. The study school students have high aspirations in terms of examination outcomes, and therefore I think it is possible, they may also have concerns around MaPBL supporting the learning they need for their exams.

Studies have also reported on aspects of PBL that students value: working collaboratively (Nugraha & Ridwan, 2019), being given freedom and autonomy (Buchanan, 2016; Grant, 2011), and satisfaction for their self-selected topic (Buchanan, 2016). Saunders-Stewart (2008), in a study of inquiry based learning (IBL) that has a similar definition to PBL used in the study school, found that students who do more IBL report higher levels of appreciation of personal responsibility.

Learning mathematics through instructional methods similar to PBL has been found to alter students' beliefs about the nature of mathematics. Clarke et al. (2004) completed a study (N = 173) of students being taught mathematics through IMP – an instructional method that shares many similarities with PBL - in three different schools in California. The teachers involved were all self-selected, however some rigor was provided through comparison with a control group, and a triangulation of methods. They reported that this instructional method engendered measurably different belief systems in students. It changed their perceptions of the discipline of mathematics - including what constituted mathematical activity, the origins of mathematical ideas and identification of themselves as mathematicians. They began to view mathematics not just as 'random rules', but as something that has arisen to fulfil the demands of society. They were also more able to see mathematics in wider life. Similarly, Moyer et al. (2018) found that of the students interviewed, a significantly greater percentage of students taught through inquiry-based learning, rather than a more traditional curriculum, had a relational as opposed to instrumental vision of mathematics. Moyer's findings particularly may not transfer, as only some mathematics lessons at the study school are taught using PBL.

In summary, studies have found that students believe that PBL helps improve their skills, however one highlighted that PBL makes some students feel less confident about passing their exams. Other studies have found that students reported valuing the freedom and autonomy offered in PBL. MaPBL also appears to have the possibility to alter students' beliefs about the nature of mathematics. However, care needs to be taken not to assume these findings will translate into the study context.

4.7.3 Emotions

The literature suggests that emotions towards MaPBL are mixed. Some studies have found that students appear to enjoy learning through MaPBL (e.g. Schettino, 2016), as they find it interesting and fun (Canuteson, 2017; Özdemir et al., 2015), are challenged to think differently (Cerezo, 2004), use their mathematics in an authentic context (Canuteson, 2017) and work collaboratively (Cerezo, 2004). However, Hsu (2019) claims due to the complexity and challenges of PBL, students will inevitably feel frustration, fear, uncertainty and anger. She describes how there can be an 'emotional dissonance' in PBL between the students and the teachers that arises from the change in roles. Whilst Hsu's research was based on a small-scale study in science, a similar finding was made by Canuteson (2017) in the mathematics classroom. Canuteson reported that students who were learning through MaPBL found that learning in groups made things easier, but that they were sceptical that they were being taught the full curriculum; they felt that they were missing things. Similarly, Muir et al. (2016) found that students reported they were concerned about curriculum coverage and felt that they were 'not learning how they should be learning'. Virtue et al. (2019) explain that students' uncertainty about curriculum coverage may be to do with the fact that they do not realise the depth of learning that they are gaining during PBL.

As MaPBL is a relatively new pedagogical approach for the study mathematics department and is not used elsewhere around the school, it seems highly plausible that the students in the study could feel an ‘emotional dissonance’ as teachers assume different roles. The study school has an established staff team, and a long history of strong exam results. When teaching in the study school I have always felt trusted by the students: they accept the position of authority of the teachers and, if a teacher can manage behaviour, the students appear to accept that the teachers will know what is best for them. However, students have a strong desire to achieve well and therefore it is possible that the findings of Canuteson (2017) and Muir et al. (2016) may transfer into this context.

Much of the literature reports that students find learning though MaPBL interesting and ‘less boring’ than more traditional lessons (e.g. Canuteson, 2017). However, other studies have reported that students found that learning through MaPBL was boring. In two of these studies, this was linked to students’ relationship to group work. For example, in a study conducted by Özdemir et al. (2015), the students who were bored reported not working well with their groups or not enjoying working by themselves. Similarly, Uyangör (2012) found that students who reported feeling bored also reported not working effectively in groups.

Habók et al. (2016) state that teachers are more careful with students’ emotions when teaching through PBL than when teaching through more traditional methods. They posit that this could be because when teachers are teaching from the front, it is hard to consider the students’ emotions, or teachers might consider that students’ emotions are more vulnerable in a PBL context. When discussing student emotions with teachers during the study, it will be important to remember that their own behaviour in relation to emotions might change during PBL.

In summary emotional responses to PBL in the literature are varied and include enjoyment, interest, boredom, frustration, and uncertainty. I view it as unsurprising that research has found students experience a variety of emotions during MaPBL, as emotions are viewed as being states of consciousness or feelings, that are highly contextual and of limited stability.

4.7.4 Behaviour

Typically, the studies I found suggest that students perceive MaPBL increases their engagement in learning in mathematics (Canuteson, 2017; Özdemir et al., 2015). Özdemir et al.'s study is of particular relevance as it is also teacher-initiated MaPBL. In a study of teaching mathematics for social justice, which aims to foster student agency, one of the teachers found that students with low prior attainment were most likely to show an increased engagement (Wright, 2017). However, it was not universally true that PBL increases engagement and engagement seems to be necessary, but not sufficient, for learning. Johnson et al. (2013), in their study of Chemistry PBL, found that in the PBL lessons students' behavioural engagement (including attendance, time on task and allowing others to stay on task) decreased slightly. Whilst Napitupulu et al. (2016) in a study of students in an upper secondary school in Indonesia, reported that there was some resistance in the classroom to learning through MaPBL. They attributed this reluctance to the fact that students were unaccustomed to this approach to learning. My experience at the study school suggests that engagement in MaPBL is dependent on the student and the project.

A study by Grant (2011) found that students reported choosing tasks in PBL that they viewed as 'easier', 'faster' and required less work. This is not typical of the study school students in their 'normal' mathematics lessons. I often find that when given choice students will choose something hard and subsequently get stuck, rather than picking the easier option.

4.8 Working collaboratively

Working collaboratively is a defining feature of PBL (Ch1) and aids student-led learning (Gillies & Nichols, 2015; Hamilton, 2018; Nugraha & Ridwan, 2019). I view collaboration within PBL in Remedios et al.'s (2008) terms: collaboration is all actions that are intended to increase the group's development of knowledge or that contribute to a deeper, more complex understanding. Working collaboratively is both a process of and an outcome of PBL (ibid). It has been found to have many benefits. A study of student-led discussions by DeJarnette et al. (2013) found that when students work together, they will provoke each other to extend prior knowledge, use multiple representations and connect the work they are doing to non-mathematical experiences.

The ideal composition of groups is not clear in the literature on PBL. Cheng et al. (2008) completed a study of 1921 students in eight schools in Hong Kong who had studied through PBL across subjects for a year. They found that the nature of the groupings, their gender composition, size, or heterogeneity, did not appear to relate to whether collective efficacy was higher than self-efficacy when group processes were of high quality. University students conducting inter-disciplinary projects have reported that they like working in pre-allocated groups rather than choosing their own (Harmer & Stokes, 2016). This could be to do with the issues of "friendships" that Le et al. (2018) highlight in a mixed age study of collaborative learning in Vietnam. They found that students of all ages may be less disciplined, more inclined to talk off topic and, importantly in complexity terms, be less critical when students work with friends.

In the departmental enactment of PBL students typically worked in groups of two or three students. There was limited discussion in departmental meetings around group composition. This is perhaps because it is already reasonably typical for students to work together and help each other in their mathematics lessons.

The literature suggests that students find it reasonably easy to distribute tasks during PBL. For example, students completing ICT PBL were found to naturally take on different tasks (Baser et al., 2017). Similarly, students completing 'structured' MaPBL in Brunei were found to comfortably delegate tasks (Botty & Shahrill, 2015). This may have been supported by the structure provided: the teacher emphasised that students should delegate tasks and a section about this was included on the 'PBL facts list'. However, this was not found universally. A study by Gibbes et al. (2014) of PBL at University found that some students worked independently rather than as part of the group.

Cheng et al. (2008) found that, when the four processes for effective group work outlined by Johnson et al. (1993) (positive interdependence, individual accountability, equal participation and social skills) were met, students generally had a higher collective efficacy than self-efficacy (as conceptualised by Bandura (1997)) based on self-reported measures. Similarly, Kokotsaki et al. (2016) in their review on the PBL literature gave six recommendations for effective PBL, one of which was the importance of having effective group work, with students having equal agency and participation. These processes can be challenging to facilitate (Cheng et al., 2008). Some studies of PBL have reported that students undertaking PBL do not participate equally (Gibbes & Carson, 2014; Nugraha & Ridwan, 2019).

Nugraha et al. (2019) suggests that having group roles may aid equal participation. Cohen (1994) in her review of the literature of group work found group roles to be helpful when students are working on ill-structured problems such as the ones they meet in PBL. For example, she found that if you had a facilitator this increased both communication and work rate of the group. Kim et al. (2018), in their theoretical discussion around scaffolding during PBL, suggest that roles based on ability level would help to provide scaffolding for students and enhance motivation.

In summary, working collaboratively has been found to aid student-led learning and collective efficacy. Positive interdependence, individual accountability, equal participation, and social skills all seem to support students to work collaboratively during PBL. However, the ideal composition of groups, and the form collaborative learning should take is not clear from the literature (Cheng et al., 2008; Condliffe, 2017).

4.9 Research questions

It appears that, whilst there is an extensive literature around PBL, the concept of student-led learning is underdeveloped in the literature and more research is required around how teachers support and scaffold students to lead their own learning during PBL. Research on student perspectives of PBL and MaPBL also has less prevalence in the literature. The literature has considered how PBL supports affective traits, but less literature has considered the affective traits required for students to be successful in PBL. These gaps in the literature informed the development of my research questions:

RQ1: What is the relationship between students' attitudes to mathematics and their attitude to PBL, in particular the student-led aspect of that?
RQ2: What is challenging for students in leading their own learning during MaPBL?
RQ3: What pedagogical strategies are perceived to be most effective for supporting student-led MaPBL?

4.10 Limitations of the literature

Most of the study school students are British Bengali and many come from backgrounds with a fairly high level of deprivation. This means that not all the literature will necessarily be transferable into the context of this study. Further, as some of the literature around PBL, especially in mathematics, is underdeveloped; I have had to draw from literature that comes from different disciplines, countries, and contexts, which has similar implications for possible challenges with transferability. Below, I highlight some of the different ways that the literature differs in context to that of the study.

Some studies focused on particular groups of students, for example at risk females, immigrant students and students with special educational needs. As the study school is a mixed inner London comprehensive, there are students that fall into all these groups and therefore, the findings are still of interest. However other studies are of students at public schools and hence the findings may be of less relevance. The same is true of studies with older or younger students.

As the definition for PBL was defined by the department in the study school, the exact definition is not shared with other studies. However, I took care to ensure that I felt that there was sufficient overlap within the definitions given in the included literature and that of the study school. I drew on the literature from other related pedagogies such as Problem-BL and inquiry-BL. In the study school the students only experience PBL in some mathematics lessons, so again this raises questions around transferability with the studies where PBL is embedded throughout the school and taught through multiple disciplines.

Askew et al. (2010) highlight the challenges with 'cherry-picking' elements of successful practice and not considering carefully how these elements have a relationship with aspects of the national culture, attitudes and policies. They also posit that high attainment may be less linked to specific mathematics teaching practices and more linked to cultural values. Therefore, careful consideration must be given as to whether the findings from other counties will transfer.

A further limitation to the literature on PBL is that many of the studies have a small sample size and few studies have random allocation of participants to a control group and an experimental group. This means that we cannot be sure that any of the reported outcomes are necessarily a product of PBL as other teaching and learning activities could have influenced the outcome of the results.

In summary, the findings in the literature need to be treated carefully and the limits to generalisability into the context of the study school need to be considered.

4.11 Chapter summary

I commenced the chapter by considering how I use the literature review in this study for research. I explored the literature on student-led learning and self-regulated learning. I then presented literature about how PBL develops some of the key affective traits that I view as supporting student-led learning: self-efficacy, resilience, and motivation. I drew mainly from the literature in PBL, but sometimes drew from the wider literature around student-led learning, such as that on agency in mathematics. I discussed student attitudes to PBL including how this effects their attitudes towards mathematics.

Above, I highlighted some of the gaps in the literature. I presented my research questions, which aim to frame a contribution to the gaps in the literature around MaPBL, specifically: the student perspective of PBL; student-led learning during PBL; and scaffolding and guidance that support students to take ownership of their own learning. In the next chapter, I provide an overview of my planned methodology to investigate the research questions.

5 CHAPTER 5 – METHODOLOGICAL OVERVIEW

5.1 Introduction

This study is framed by my IFS which found that teachers perceived facilitating student-led learning to be one of the most challenging areas of PBL pedagogy to enact, and reported a lack of existing literature (Barnecutt, 2019). This study aimed to explore students' perceptions of student-led learning. It attempted to provide a rich description of how students report their own perceptions of PBL and the challenges they experience in leading their own learning. It aimed to offer an explanation of why students face these challenges and a student perspective on strategies that teachers can employ to support students. Viewed through a complexity lens, predicting how students will behave is not possible in a complex system (Hetherington, 2013). However, when designing this study, I felt that providing rich description about the processes that were observed in the study school would be of value to other educators seeking to adopt a similar approach and would inform the development of MaPBL in the study school.

This chapter provides an overview of the approach I used, my intended fieldwork, and my amended fieldwork in light of the Covid-19 pandemic. I conclude by outlining my role as an insider researcher and the ethical considerations. In this and the subsequent chapters, the abbreviations seen in Table **5-1** are used.

LO	Lesson Observation
FG	Focus Group
Int	Interview
RQ	Research Question

Table 5-1: Table of abbreviations

5.2 Constructivist grounded approach

The study needed to be flexible in its design, due to the limited literature about students' perceptions of PBL and student-led learning in PBL. A grounded approach was ideal: it does not seek to test a hypothesis, but rather provides a grounded description of what is occurring (Charmaz, 2014). Further, a grounded approach helps to go beyond description and to explain what is happening (McCallin, 2003).

I chose to use the constructivist grounded theory approach proposed by Charmaz (2014), which is an interpretative theory. This reflects my understanding of the researcher's influence on data collection and analysis than that adopted in Grounded Theory (Strauss & Corbin, 1998) or Classical Grounded Theory (Glaser, 1978). I view the data as being co-constructed by me, with the participants in the study, through my observations and interactions with them. I acknowledge that my thoughts, ideas, preconceptions, and decisions affect the data I gather and how I interpret it, even if these differences are subtle (Ball, 1990). This is consistent with complexity thinking which views the researcher as being situated within, and constructing an understanding of, the classroom (Hetherington, 2013).

A constructivist grounded approach aligns well with both theoretical lenses that I use to aid interpretation (Ch2). Both lenses draw attention to social contexts and interactions. The unit of analysis used for this study was the individual student, who considered through an activity lens can be viewed as part of the social setting of the classroom (Engeström et al., 1999), and through a complexity can be viewed as nested within the complex system of their group, which is nested within the complex system of the classroom (Davis et al., 2006).

Using a grounded approach allowed data collected with different techniques to be analysed (Menon et al., 2008). The flexibility also proved beneficial when alterations were made because of the implications of the Covid-19 pandemic.

5.3 Overview of intended fieldwork

My original intention was to carry out lesson observations, followed by focus groups with the students I had observed, to collect deep and rich data, analysis of which would enable me to create emergent categories. This would be followed by a survey about the emergent categories to allow me access to wider responses, compare across classes, prior attainment levels and gender and provide greater contextualisation to the categories. This survey would include some qualitative responses. I then planned to obtain further selective data, through further lesson observations, focus groups and another survey to illuminate and define the properties of my categories and the relationships between the categories (theoretical sampling). I had planned to carry out a minimum of 4 lesson observations, 4 focus groups and 2 surveys alongside a teacher workshop, to probe a professional response. However, as is typical with a grounded approach, the design for data collection was flexible and I planned to collect data until I felt theoretical saturation was reached: new data were not providing new properties or insights about the categories.

The plan to use a mixed methods approach to data collection allowed me to be expansive and creative and did not limit my approach to data collection (Johnson & Onwuegbuzie, 2004). I planned to have an emphasis on the qualitative paradigm. Using the template provided by Johnson et al. (2004), the model I planned to use can be viewed in Figure 5-1 . Table 5-2 shows the fieldwork that constituted each part. However, as I discuss next, the impact of the Covid-19 pandemic altered my planned fieldwork.

QUAL → quan + qual → QUAL → quan

Note. "qual" stands for qualitative, "quan" stands for quantitative, "+" stands for concurrent, " ⟶" stands for sequential, capital letters denote high priority or weight, and lower case letters denote lower priority or weight.[1]

Figure 5-1: Intended mixed methods model

5.4 Overview of amended fieldwork

Fieldwork commenced as per the original plan. However, the Covid19 Pandemic and subsequent lock down meant that my later fieldwork had to be amended. See Table 5-2 for the alterations to the fieldwork. These changes also shifted the dominant paradigm. Instead of viewing the research as having an emphasis on the qualitative paradigm I viewed the qualitative and quantitative elements as having equal status.

An amended model can be seen in Figure 5-2. Table 5-2 shows the fieldwork that constituted each part. The impact of these amendments is discussed in 6.5.

Figure 5-2: Amended mixed methods model

QUAL → qual + quan → QUAN

Note. "qual" stands for qualitative, "quan" stands for quantitative, "+" stands for concurrent, " ⟶" stands for sequential, capital letters denote high priority or weight, and lower case letters denote lower priority or weight.[1]

Dominant Paradigm	Status	Planned fieldwork	Purpose and research questions to be addressed
QUAL	Intended Enacted	Semi-structured lesson observation **(L01).** Schedule focused on student-led learning.	To observe directly what is happening in the classroom. To triangulate what students report happens in the classroom with direct observations. RQ1, RQ2 and prompts to explore RQ3 in FG.
	Intended Enacted	Focus group **(FG1)** with students from LO1	Explore the emergent areas of interest from the initial coding of LO1. Explore RQ1, RQ2, RQ3.
	Intended Enacted	Semi-structured lesson observation **(L02).** Schedule focused on student-led learning.	As LO1.
	Intended Amended (1)	Focus group **(FG2)** with students from LO2	Explore the emergent areas of interest from the initial coding of LO1, LO2 and FG1. Explore RQ1, RQ2, RQ3
	(1)	I did not choose to complete FG2 as intended as at the time I did not view them as leading their own learning. Later, on reflection, I realised that this would have provided useful data. Lockdown meant this was not possible. **FG2** was completed with students who I had taught who I knew had led their own learning during MaPBL.	Explore the emergent codes from LO1 and FG1. Explore RQ1, RQ2, RQ3.
	Additional Enacted	Teacher interview with the teacher of L02	To investigate a teacher's perspective of RQ1, RQ2, RQ3. To gain an understanding of teacher embrace of student-led learning.
	(3)	A third lesson observation **(L03)** was carried out as LO2 was not viewed at the time as providing data appropriate to the	To observe directly what is happening in the classroom. To triangulate what students report happens in the classroom with direct

		focus. Lockdown was announced before a focus group could be carried out.	observations. RQ1, RQ2 and prompts to explore RQ3 in FG.
Qual + quan concurrent	Intended Amended (2)	**Survey** with a mix of Likert-scale questions and open-ended questions. Year 9 or year 10.	To investigate whether the views expressed in the FGs were felt more widely. To provide a greater contextualisation to the emergent codes. RQ1, RQ2 and RQ3.
	(2)	Survey sent to all year 8 to year 11 (47/360 participants). Low response rate attributed to lockdown.	As planned.
QUAL	Intended Amended (3)	Semi-structured lesson observation **(L03).** Schedule focused on emergent codes.	Illuminate and define the properties of the emergent codes. RQ1, RQ2 and prompts to explore RQ3 in FG.
	Intended Not Enacted	Focus group **(FG3)** with students from LO3.	Probe observation. Illuminate and define the properties of the emergent codes. RQ1, RQ2, RQ3.
	Intended Not Enacted	Semi-structured lesson observation (**L04**). Schedule focused on emergent codes.	To illuminate and define the properties of the emergent codes. RQ1, RQ2 and prompts to explore RQ3 in FG.
	Intended Not Enacted	Focus group **(FG4)** with students from LO4.	Probe observation. Illuminate and define the properties of the emergent codes. RQ1, RQ2, RQ3.
-	(5)	Teacher workshop completed remotely after survey1. Two teachers attended.	As planned.
quan	Intended Amended (4)	Survey of Likert-scale questions. Year 9 or 10.	To investigate whether the views expressed in the FGs were felt more widely. To provide a greater contextualisation to the emergent codes. RQ1, RQ2 and RQ3.

QUAN	(4)	Almost all students in year 8 to 11 invited (151/414 participants). Lock down meant students had completed limited MaPBL.	As planned.
-	Intended Amended (5)	Teacher workshop.	To probe professional response to the findings (RQ1, RQ2, RQ3).

Table 5-2: Summary of fieldwork linked to research question

5.5 Insider researcher

In this study I was an insider researcher who understands PBL. This gave me significant advantages over non-insiders. Being an insider researcher gave me insight into the phenomena being explored. Viewed through a complexity thinking lens, being an insider researcher meant that I was situated within and was able to construct an understanding of the 'real world' in the classroom. Hetherington (2013) highlights how this is a particularly helpful position when researching the "messiness" of the classroom. My interpretations were supported by my own professional experience of teaching through PBL and supporting student-led learning. However, I recognise that this is part of the tension with being an insider researcher: my experience might have led me to make wrong assumptions, or to stop noticing what was familiar.

Being a part of the professional learning community (1.7.2) in the mathematics department in the study school allowed me, in complexity terms, to form a perception of emergent shared meanings. I was involved in the discussions the department had about how they envisaged MaPBL to work in the classroom. Whilst I had a perception of an emergent shared meaning of the work of the department, I am aware that people's espoused beliefs are often different to their actions, that some people's opinions may not have been shared by the whole department and I may have made assumptions about the shared meanings.

An insider researcher can have a preconceived idea of what will be discovered (DeLyser, 2001). I focused on learning about what the participants were showing me (Charmaz, 2014) and kept a methodological diary which I used to reflect on and challenge my assumptions. I noted when I felt I was directing interviews too heavily or that I was forcing the data. I then went back through the data and reflected on whether this was happening.

In addition, I tried to find ways of 'making the familiar strange' (Brock-Utne, 1996) and to check that my understanding was shared by others. The focus groups allowed me to establish a shared understanding with students and sometimes challenged my preconceptions. For example, when the students said it would be better if the teacher explained what they had to do, I felt irritated. My natural position was to assume their teacher had explained. I had to take a step back and listen to what the students were saying. I reflected that the students might not have engaged with, understood, or remembered the teacher's explanation and therefore this was an area that needed more consideration.

5.6 Ethical considerations

The research followed the British Education Research Association ethical guidelines, UCL Data Protection Registration Number: Z6364106/2019/10/121, dated: 17/10/2019. I completed two further ethical amendments approved through UCL processes.

In appendix 24, I consider the following ethical challenges: informed voluntary consent and the right to withdraw, pseudonymisation, data protection and the processing of personal data, being an insider researcher, and the costs and benefits to participants.

5.7 Chapter conclusion

Above, I provided an overview of both my intended and amended methodologies. The Covid-19 pandemic had a serious effect on my ability to complete the planned methodology: persistent and prolonged engagement was limited, and with that, my attempt to provide thick description. This resulted in the study being much more exploratory than I had originally intended. Chapters 6 and 7 provide more detail about the methodology used in each phase, alongside a presentation of the results.

6 CHAPTER 6 – PHASE ONE - METHODOLOGY AND RESULTS

6.1 Introduction

Here I present the methodology and the results for the first phase of the study. These data were collected before and during the first national lock down. A summary of the data collection events can be seen in Table 6-1.

6.2 Fieldwork

Whilst this is a case study, grounded in a particular school at a specific time, the department had a strong commitment to trying to support student-led learning during PBL and as such, I tried to focus my observations on classrooms that were potentially a 'telling' sample (Mitchell, 1984). Rather than trying to portray what typically happens, this study seeks to provide the student perspective of a departmental enactment of partial PBL where professionals have made an espoused commitment to this approach and are supported by the wider department. For this reason, all three teachers observed had stated their commitment to the idea of teaching MaPBL in personal conversations and department meetings. They each had less than 5 years' teaching experience but were reasonably experienced in teaching MaPBL and had either run external training, or written their own, in one case award winning, projects.

Information about the students who participated is in appendix 11, and the lessons observed and the resources used in these lessons are in appendices 5, 7, 8 and 9. Below, I discuss the instruments used for data collection in phase one.

6.2.1 Semi-structured LOs

LOs allowed me to observe directly what occurred in the classrooms and enriched and supplemented other data collection methods (Simpson & Tuson, 2008). LOs align with the focus on interactions that is found in both activity theory (Engeström et al., 1999) and complexity thinking (Hetherington, 2013). The LOs were semi-structured: I made written field notes throughout prompted by a schedule (appendix 61). The schedule was informed by my prior work in my IFS, the departmental definition of student-led learning and the initial literature review. The schedule was only semi-structured to ensure, in complexity terms, that an open space was maintained to respond to diversity.

Data	Date Collected	Class	Participants
L01	10/12/2019	Mr Drew Year 10 Set 2/6	3 year 10 students
FG1	18/12/2019	Students from LO1	2 year 10 students
LO2	5/2/2020	Mr Jafri Year 9 Set 3/5	5 year 9 students
FG2	4/3/2020	Taught by Ms T (year 9 and myself (year 10 and 11) Set 1/6	8 year 11 students
L03	12/3/2020	Mr Robinson Year 10 Set 6/6	8 year 10 students
Teacher interview	27/05/20	n/a	Mr Jafri
Survey1	8/5/2020 – 24/7/2020	All year 8, 9 and 10	15/120 year 8 students 26/120 year 9 students 6/120 year 10 students
Teacher Workshop	28/07/20	n/a	Mr Drew and Mr Robinson

Table 6-1: Summary of phase one data collection events

Before each LO, I asked the teacher to set the scene, so I understood where the lesson sat in the planned learning and the teacher's intentions for the lesson. As someone new to conducting LOs for research purposes, I initially found that I spent too much time considering and trying to analyse what I was seeing. As I progressed through the data collection, I learnt to simply write down what I noticed. Writing what I noticed, alongside using a fairly structured framework, helped to generate reasonably rich data (Daas & McBride, 2014).

Completing the LOs allowed me to triangulate what teachers reported was happening in the classroom, with what was actually happening in the classroom. For example, in LO2, I found that the teacher had structured the project so thoroughly, that students were completing a series of application of mathematics tasks in an authentic context and not completing PBL (discussed further in Ch8).

6.2.2 Interviews

Interviews are aligned with a grounded approach as they give the researcher access to a participant's subjective world view and allow the researcher a level of control over the data they generate (Charmaz, 2001). Viewed through an activity lens, they provide an opportunity to understand why the participants act as they do, based on the assumptions of the participant consciousness and agency. Through a complexity lens they offer the possibility of viewing unpredictable and emergent phenomena. Complexity thinking places an importance on discourse and from an interview setting, what the words tell you about a culture.

6.2.2.1 Student focus groups

As a senior member of staff in school, there was a power imbalance between me as the researcher and the students as the participants. I chose FGs to help dilute this power imbalance (Barbour, 2005) and to give students greater confidence to contribute.

The participants of FG1 were students I observed in LO1. The interview schedule for FG1 (appendix 10) was developed to explore the emergent areas of interest from the initial coding of LO1. To help students remember what they had experienced in the lesson I highlighted potential statements that might later trigger a reflection (Kawulich, 2005). The participants of FG2 were students that I taught, the interview schedule was based on the coding from LO1 and FG1 and designed to explore emergent codes as well as provide rich data around the research questions.

An advantage of FGs is that the views of the participants can be compared and contrasted through careful questioning. During both FGs, I tried to listen carefully, paraphrasing what I thought students had said and asking them for clarification if there were any areas of ambiguity. I tried to ask participants if they agreed with what another participant was saying and to add to or challenge another participant's statement.

6.2.2.2 Teacher interview

I initially disregarded LO2 as the teacher appeared to have offered limited autonomy. However, when I considered LO2 through an activity lens, I realised I was viewing a *contradiction* (McNicholl & Blake, 2013): the *mediating artefact*, the project, was being used in a markedly different way to how it was intended. The project was used to give students an opportunity to apply their mathematics rather than as a PBL project, with opportunities for student-led learning. Activity theory can provide transformational understanding when the researcher tries to resolve contradictions. As I wanted to explore this contradiction, I interviewed the teacher and discussed in more detail his embrace of student-led learning.

6.2.3 Teacher workshop

The teacher workshop took place after most of the data had been collected. During the workshop, I co-constructed a positive professional response in relation to the core codes that I had developed. It allowed me to triangulate the data and question whether I had developed an authentic interpretation. As discussed under rigor (appendix 23), this helped to increase the reliability and validity of the study as it established a means of peer validation.

6.2.4 Survey 1

The LOs and the FGs allowed me to collect a limited amount of reasonably rich data, analysis of which enabled me to create emergent codes and conjectures. To investigate whether the views expressed in the FGs were felt more widely and to provide a greater contextualisation to the codes, I designed a survey. The questions included a mix of Likert-scale questions, so I could quantify and compare responses, and open-ended questions. The open questions predominantly focused on how students experienced leading their own learning as the FGs had generated limited data on this, although the responses to these were still limited.

I used a number of surveys from other researchers to help with my initial survey design: two mathematics related belief questionnaires (Hannula, 2012; Op't Eynde & De Corte, 2003), students' perceptions of life skill development questionnaire, designed to be used in PBL schools (Meyer & Wurdinger, 2016), and a measurement of affective mathematics engagement designed to compare STEM and Non-STEM PBL (Lee et al., 2019). Where relevant, I tried to ensure that some questions for each construct were phrased positively and some negatively. As this was the first time I had written a survey I shared an initial draft of the questions with my supervisor and some of my EdD peers, who all offered suggestions on how to improve the questions. The development of the survey can be seen in appendices 20 and 21.

The survey was sent to all students in years 8, 9 and 10. The low response rate and limits to reliability are discussed below.

6.3 Approach to analysis

Constructivist grounded theory employs well-described and established approaches on how to collect data and complete coding and analysis (Jones & Alony, 2011). I tried to avoid method slurring (Cutcliffe, 2000) and so followed the methods outlined by Charmaz (2014):

1. I commenced coding as soon as possible after each set of data were collected. Data collection and analysis were then conducted simultaneously in an iterative process.
2. The initial line-by-line coding was completed with NVIVO. Using line-by-line coding helped me to question what was being said or observed and to analyse participants' views critically rather than just agree, as can be a tendency of insider researchers (Merriam et al., 2001). I used a heuristic device suggested by Charmaz (2014) of coding with gerunds. This focused my coding on processes and actions rather than structures and themes (see appendix 14).
3. I used comparative methods, comparing across data to try to find similarities and differences, patterns, and contrasts. This helped me to establish analytical distinctions and see the familiar in a new light.
4. I then moved into focused coding by drawing on the data to develop conceptual categories or core codes. The core codes I devised subsumed numerous initial codes and were designed to try to categorise my data completely and to give the most coherent interpretation. The names of these core codes changed slightly over time (see appendix 15) as I made further interpretation of my data and developed a stronger theoretical sensitivity from reading wider literature. The core codes developed to become student-led learning, self-regulated learning, resilience, working collaboratively, self-efficacy and motivation.
5. Throughout the process, I aimed to create inductive emergent categories and later conjectures that were grounded in the data and systematic data analysis rather than attempting to fit the codes to analytic pre-conceptions or existing theories.

6. After establishing the core codes, I progressed to theoretical sampling. There is a tension between theoretical sampling and obtaining valid data (Charmaz, 2014): if you begin theoretical sampling too early, collecting and analysing data linked to your core codes only, then you may miss other factors that are important. I therefore attempted to gather data that had broad and deep coverage of categories from a variety of sources before I developed the core codes and commenced theoretical sampling. Due to the Covid-19 pandemic, this was more limited than I would have hoped for. In theoretical sampling, I went back through the data that had been collected and recoded the data based on my core codes, seeking pertinent data that would help me to develop my emergent conjectures. As I collected more data, I thought carefully about how it would help me to elaborate and refine my core codes, and specifically how I could help to demonstrate and develop the links between them.
7. Throughout the theoretical sampling, I searched for variation in my core codes – to try to understand how, when, and why my codes varied.
8. I defined and checked my codes, explained the relationships between them and the variation that exists.
9. I arrived at a point where I felt that collecting new data would not reveal new theoretical insight or further properties of my codes. I used a series of questions offered by Charmaz (2014) to support with checking my codes were saturated.

6.4 Approach to interpretation

I used two complementary lenses to aid my interpretation: activity theory (2.1) and complexity thinking (2.2). Both lenses are suitable for use with complex social networks: the classroom, or groups of students, can be viewed as identifiable systems. They view the individual and environment as a dynamic unity: they draw attention to the individual systems of cognition, whilst considering the systems which such cognition emerges out of and is nested within.

6.5 Methodological challenges and limitations

6.5.1 The impact of Covid-19

As explained earlier, my methodology was amended in light of Covid-19 disruptions. These changes meant that the original aims of persistent observation, prolonged engagement and thick description were limited: the study became far more exploratory than I had originally conceived.

6.5.2 Rigor or trustworthiness

Rigor is a key concern in constructivist grounded theory (Jones & Alony, 2011). It is necessary to consider the rigor, or worth, of a study as this stops time and effort being wasted (Long & Johnson, 2000). It was my intention to ensure that the study demonstrates trustworthiness. The Covid-19 pandemic and subsequent school closures meant that this was not possible to do this to the extent I had envisaged. A full discussion of the rigor or trustworthiness of the study can be viewed in appendix 23.

6.5.3 Student voice

As I see it, there is a moral imperative to consider and act upon the opinions of students, moderated by professional knowledge, experience and discernment. The United Nations Convention on the Rights of the Child (United Nations, 1989) states that students have a right to be heard and also a right to be involved in the decisions that affect them. Furthermore, student voice can be empowering for students: it helps them to feel respected and engaged in what is happening to them (Cook-Sather, 2006). Mansfield (2018) argues that listening to and acting upon student voice is 'indispensable to ethical leadership responsibilities', and highlights how students can provide valuable insights into many areas of school life such as student performance, retention and how decisions are made.

Pupil voice is not without its challenges: power relations are unequal and problematic (Robinson & Taylor, 2007). Apart from the ethical considerations, discussed below, I as the researcher had to undergo a cultural shift, to disrupt the typical balance of power and be ready to listen attentively to what students were telling me (Cook-Sather, 2006). When I listened to students' opinions, I tried to be conscious of ideas that they were suggesting that I might not want to face (Cook-Sather, 2006). In my research journal I noted and came back to these ideas. I tried to ensure that I didn't dismiss suggestions that I viewed as being imagined by the students (Harfitt, 2014), or suggestions that I viewed as arising from students not having a full enough understanding of their own education. I also tried hard to ensure that I didn't see student voice as singular: it is complex and contextually based.

6.5.4 Accessing beliefs and emotions

A common characteristic of belief-related research is the challenge in reliably accessing beliefs (Leder et al., 2006). With teachers, there is often a difference between their espoused values, and their practice (Breunig, 2017). I view it as likely that the same may be true for students. Some researchers hold the conviction that beliefs have observable behavioural consequences (Rokeach, 1968). I was not able to imply students' beliefs from their behaviours in lessons, however I was able to use their observed behaviours as a springboard for discussions on their beliefs.

Similarly researching emotions is viewed as complex, not least because many emotional reactions are not conscious. De Bellis et al. (2006) posit that the lack of research on emotions in mathematical problem-solving is because it can be difficult to design reliable empirical studies of affect and because mathematics is often understood as a purely rational subject. Di Martino et al. (2011) highlight how challenging it can be to research emotions, and stressed the limitations of instruments such as interviews or questionnaires in attempting to capture feelings that may be unconscious and fleeting. In the LOs that I completed, the students' emotions were not often apparent. Research on problem solving in mathematics has found that students can be reluctant to express emotions or frustrations in this context (Voica et al., 2020). The range of emotions may therefore have been greater than was reported.

6.5.5 Survey1

Survey1 had a low response rate (47/360). The insufficient data mean that I am unable to say with any confidence how widespread the findings were. However, it exposed in some depth the range of student attitudes around PBL, consistent with what I observed in classrooms. Many of those attitudes are likely to be found in other classrooms, but the degree to which these findings transfer to other contexts, and their rate of occurrence there, is unknown.

Analysis of the survey produced some unexpected correlations. This could suggest that students' views of PBL are not strong or stable. Alternatively, it could suggest that the students did not fill the survey in carefully (perhaps because they were working remotely) or wanted to respond in the way that they felt they were expected to – the students appeared to have a strong positive response bias - and therefore limited reliability should be placed in this tool.

6.6 Results

The data suggested students' attitudes to PBL were varied, diverse and sometimes contingent: some students reacted differently to different projects, and different groupings. Both students and teachers appeared to find it hard to enable student-led learning and it was rare to observe students taking control of all aspects of their learning. However, there were many instances when students were able to lead a part of their own learning, for example making choices about who would do what, the timeframe for completion of a task, or the methods they used. Despite the challenges, students typically reported valuing the opportunity to have some autonomy and offered many suggestions around how teachers can support them to do this.

The six core codes that I developed through the coding process were: student-led learning, self-regulated learning, self-efficacy, mathematical resilience, motivation and working collaboratively. Each of these codes has sub-codes relating to aspects of PBL that students found challenging and aspects that students felt were supportive in helping them lead their own learning during PBL. Here I present a short synopsis of each code, with some illustrative quotations. I present further data organised to demonstrate my emergent inductive theory around attitudes: that a student's self-efficacy, vision of mathematics, and goal orientation appear to mediate that student's attitude towards PBL. This data is presented as 4 vignettes, each of a student or students who seemed to demonstrate different beliefs. This emergent inductive theory is investigated in phase two. There is a fuller description of each of the codes in appendix 16, which includes some overlap with the data presented here, but in particular, gives details of survey data relating to each code. In chapter 9, I discuss my findings organised by research question, rather than by the key themes or the emergent theory that I developed from the coding process.

6.6.1 Student-led learning

Student-led learning is a defining feature of MaPBL (Thomas, 2000) and part of the departmental envisaged plan for MaPBL as discussed in team meetings.

Mr Robinson and Mr Drew reported that MaPBL required students to make many more choices and decisions than they would in their usual lessons:

> *With… [a] data project there are… so many different ways, so much freedom…, there's no real right way of doing things: advantages and disadvantages, rather than a particular correct answer* (Mr Robinson, TW).

He explained, however, that this level of autonomy varied between projects, reporting that in some projects there is still a common route through:

> *With… Amazon Trader apart from picking their own products I guess there is a set way you should really do it* (Mr Robinson, TW).

It appeared that the level of offered autonomy and student embrace of autonomy varied significantly, due to the nature of the projects, the actions of the teacher and the attitudes of the students. Which I now discuss. Further, sometimes when students were given autonomy, their decisions were not made mathematically.

Students were not always offered autonomy by their teacher. In LO2, Mr Jafri did not appear to offer significant autonomy to the students. However, several students who were observed in the lesson assumed autonomy despite not being offered it. Sometimes this occurred when students challenged the teacher and asked that they could follow their own interests, and at other points this happened with students simply ignoring the teacher's requests. For example, Mr Jafri briefly brought the class back together and stated that they shouldn't show their methods on their final piece of work. When some of the students remonstrated, Mr Jafri repositioned what he had said as a 'suggestion', allowing students to present their work in the way they chose. Later in LO2, students ignored the fact that they were supposed to be working on the same task and instead worked on different parts of the project; this was viewed by me, as the observer, as being to better complete the task.

Sometimes students did not embrace the offered autonomy. In LO1, students had the freedom to make many of the choices themselves. They had an overall brief (choose the products they wanted to sell and work out if they could sell them for profit, factoring exchange rate, referral cost, delivery cost and Amazon fees) but could choose how to enact this. However, the observed students did not fully embrace the offered autonomy. Both groups observed chose to use an optional writing frame, and appeared to use similar methods, collectively going through one product at a time, working through the calculations the writing frame required. The use of these scaffolds meant that students were viewed as exercising much more limited autonomy, due to the amount of structure the scaffolds provided.

The Survey1 data seem to contradict this observed lack of embrace of autonomy, with most students responding that they always or often chose how to solve each of the tasks (used their own methods). However, there was variation between the projects (Cake Bake – 22/30, Maths of Migration – 29/36, Amazon Trader – 6/6). This may be because students were unaware of how much a writing frame restricted their choice of methods.

Survey1 also suggests that students varied both in their adoption of the choices made available, and in their perceptions of how challenging they found different process choices, for example, 29/33 students reported always or often choosing the order in which to complete the different tasks in Maths of Migration whilst 19/27 did for Cake Bake.

A product of student autonomy is that students did not always appear mathematically focused. In LO1, the choices that the students were observed making seemed primarily to be about selecting the products that they wanted to sell. Group V did not appear to be using the project as a mathematical optimisation task but were instead choosing their products based on their perception of what would sell. When asked about whether they had chosen their products based on what would be best mathematically, Anjum responded:

> *I don't think we really thought like that. We did think about jewellery. I thought that that price could easily go up because jewellery…, you could sell it for more because it might look better...*

The following core codes offer suggestions as to how the student embrace of autonomy can be fostered during MaPBL.

6.6.2 Self-regulated learning

Self-regulated learning, as conceptualised in 3.2, comprises of both metacognitive and self-regulatory skills. Self-regulated learning became a core code as FG1 students reported that they found the most challenging aspects of student-led learning during MaPBL to be that of conceptualising the problem and devising a plan, both metacognitive skills. Furthermore, from the LOs it was clear that students struggled to manage their own time, a self-regulatory skill. Some FG students (4/10) reported appreciating metacognitive support from their teacher or the resources that their teacher provided, and half (5/10) suggested that teachers should provide further support. In this section, I discuss the reported challenges that students experienced with self-regulated learning as well as the nature of the support that students and teachers reported to be effective.

Students reported requiring support to conceptualise the problem and to see connections between the project and their prior mathematical knowledge: "*I feel that the first lesson, we didn't really do anything as we didn't understand*" (Anjum, FG1). Anjum described how she felt that having a first lesson that was focused on ensuring that the students understood, that was interactive, and where students could ask lots of questions would be helpful in aiding understanding. She described it as a lesson where: *"…you can just be carrying on asking the teacher questions, quite an interactive lesson*". This was strongly supported by the results of Survey1, with all but one student (strongly) agreeing with the statement: I am able to complete projects more easily if the teacher spends more time at the beginning explaining it (46/47).

As I discuss in the first vignette, in FG1, students described how challenging they found it to see the connections between the mathematics that they had previously learnt, and could do, and knowing how they could use this mathematics within the projects. Both students in FG1 described finding it useful to complete starter activities that allowed them to practice some of the skills required for the project and see the connection to what they had learnt before. Anjum explained: "*then you could get into it after*". This aligns with the results of Survey1 where most students reported finding it extremely, very, or moderately helpful to complete starter activities that help them practice the mathematics they might use in the project (40/45).

Students reported finding it challenging to devise a plan, especially if it was a large and complex project. Many students in FG2 (5/8) stated that the teacher should provide support with devising a plan. Students reported that they found it easier when they were older as they had stronger problem-solving skills. One teacher suggested that mathematical creativity may help students to devise a plan more easily. Other strategies that the teachers suggested can help are giving students a writing frame and showing them what they are aiming for by providing exemplars. Students wanted to maintain the teachers support even when devising their own plan: more students reported finding it extremely, very, or moderately helpful for a teacher to check their plan (41/45), than reported finding it helpful to write a plan (36/45). In the LOs, when students couldn't get support from the teacher, they were observed turning to their planning sheet to support their thinking in what to do next.

In LO1 and LO2 students in each group were observed to regularly check their work with their peers. They also reported other strategies for checking their progress, including checking how many of the tasks had been completed and checking how they were doing in relation to the goal of the project. This was supported by Survey1 with most students (strongly) agreeing with the statement: when working on a project I normally check each thing I do with others in my group (35/40). One teacher reported that it was possible to get students to self-check their work by reading aloud what they had written and asking them to consider if it was correct. Checking, reviewing, and re-writing were reported by Mr Jafri to help improve the outcome of the projects.

In LO1, the students had been given the task and the overall timeframe, but it appeared that they were able to choose how they allocated that time, with some reminders from the teacher - for example, reminding students when they should move onto the report write up. In conversation with the teacher after the lesson, he informed me that not all the groups had finished the project within the given timeframe. In LO2, the students were given a time within which each stage of the project must be completed. After this time was up, the next phase of the project was shown. In this lesson, the observed students typically appeared to do what they had been asked to, however in one instance a group ignored the teacher's instructions and took ownership of their own timeframe. It was presumed by me, as the observer, that they did this to complete the task more effectively.

Survey1 supports the finding that students find it challenging to manage their own time. A much lower proportion of students stated that they always or often made their own decisions around how much time to allocate each task (Cake Bake – 16/29, Maths of Migration – 20/36, Amazon Trader – 6/6) than allocating tasks, choosing the methods to use, or choosing which order to complete tasks. The vignettes present some of the student comments on how they struggled to make choices around time and how this can be supported.

6.6.3 Self-efficacy

Self-efficacy relates to a person's beliefs about how well they will perform in relation to a specific activity (3.3.3). It is situational or project specific (Pampaka et al., 2011). I deemed self-efficacy to be a core code as students reported being more able to take control and lead their own learning when they were secure in what they needed to do:

> *We kind of took more control because Sir came at first and explained it to us* (Anjum, FG1).

In this way, self-efficacy links to autonomy: students who seemed to have more self-efficacy took more control; and teachers reported that students require more self-efficacy when there isn't a definitive 'right answer'. The data suggest that self-efficacy also links to working collaboratively: working collaboratively has the potential to increase available self-efficacy and resilience, students with higher self-efficacy showed stronger resilience. There were also some tentative links in the data between metacognition and self-efficacy: the student with perceived low self-efficacy reported many challenges with metacognitive strategies.

Similarly to resilience, the data suggest that students require self-efficacy in order to lead their own learning to make decisions that they can't be sure are correct and to take risks. Student self-efficacy during projects seemed to be supported by giving students an outline plan, asking them to present their work, having midway check-ups and working collaboratively, as I now discuss.

Teachers reported thinking it was important that students felt confident enough during projects to take risks. Mr Drew, with agreement from Mr Robinson reported:

> *Some students might find it a bit overwhelming because it's something they have no idea about so (they need) some of that confidence to put (them)self in the danger zone… and just see what happens.*

Similarly, when discussing the autonomy he offered students in LO2, Mr Jafri reported that he structured the lessons as he felt that some students in his class had "*a real fear of getting things wrong*".

Some students in the focus group (2/10) stated that they disliked being confused and not knowing what to do.

In the survey, 31/40 students (strongly) agreed that they sometimes found projects overwhelming because they didn't know what to do at the start. This is juxtaposed with 30/37 students (strongly) agreeing that they feel confident that they are doing the right thing when they are completing a project. These results may indicate that students struggle at the beginning of a project but feel more confident when they have commenced working. Further, some of the strategies that I discuss below may have helped support students' confidence as they progressed through the projects.

As analysed under self-regulated learning, many students felt having an outline or checklist would support them and give clear guidance about what was expected from them and therefore increase their belief that they could complete the project successfully. Ammara explained how she would have liked an outline with "*boxes to tick and it gives you an exact plan to follow, so you know where you're going, and you won't get confused as you would just like starting from scratch.*" There was strong support for this idea in FG2. However, as I discuss in vignette 4, the students wanted it to be "*vague*" and allow for student "*creativity*". I interpret this as the students wanting to feel reassurance that they were doing what was expected of them, without losing the freedom and the autonomy that the projects provided. The survey is aligned with the concept of students appreciating having a checklist, with 42/46 students reporting they would find this extremely, very or moderately helpful.

Students also seemed to adopt a higher self-efficacy when working with someone whom they thought could complete the project:

> *If you give us another project to do…, I wouldn't know because I didn't get enough from the project that we did, and **she did**. So, I'll be like, "Okay, I'll do this"* (Tanzia, FG1).

6.6.4 Mathematical resilience

Teachers reported that mathematical resilience (defined in 3.3.1) is required if students are to succeed in completing projects, as there are many difficulties and challenges that students must overcome, so it is a core code. Mr Jafri described resilience as "*the number one thing*" (Mr Jafri, Int). As discussed below the right context appeared to be one of the factors that support students to have more mathematical resilience. Students reported further factors that affected their mathematical resilience, which I now discuss.

Most of the groups, when observed working independently from the teacher on a project, showed strong resilience (7/9 groups observed). This determination to work appeared to be so strong from one student that even when her teammate tried to take her off task, she remained focused. However, two others did not show this level of engagement. When discussing one of these students, Mr Jafri reported that he felt that student was out of his ‘comfort zone’, and therefore he gave up during the lesson:

> *I’ve moved where he sits three times…, not because he's badly behaved, it’s just trying to find someone… he's going to work well with, that… fires him up... you've seen his book, he's really messy… but … he's often answered a lot of questions…when it’s straightforward… he's fine…, so I guess MaPBL is way out of his comfort zone.*

One student reported several situations where they were likely to have low mathematical resilience and give up: if they didn’t feel they would be successful, didn’t understand the mathematics or felt overwhelmed by the amount of work. I discuss these in vignette 1, as they were all experienced by the student who had low-self efficacy.

Mr Robinson reported that in the projects, some choices had more consequences than others and students lacked mathematical resilience when they felt that they had made a wrong choice:

> *I don't think I've ever done a project where kids are… confidently making decisions all of the way through. In the migration (project), when they choose their own line of inquiry…, that maybe goes smoothly because there is less of a repercussion if they make a (mistake)…. Whereas later on… they have to start thinking about which graph (they) should… choose…, so, I think those sorts of decisions when it is clear that they've chosen the wrong thing, maybe that's the point where they sort of retreat a bit. You have to really sort of push them to talk about it with you and that’s the only way they make that decision.*

The survey also showed that students’ work ethic was varied and dependent on the project. For example, in Cake Bake when asked how much they worked hard, 9/29 stated always, 12/29 often and 8/29 occasionally. This is more consistent than Maths of Migration where 8/37 reported always, 23/37 often, 3/37 occasionally and 3/37 rarely.

Students described several strategies that they used to overcome barriers, including getting help from the teacher. Ammara reported how when someone helps and supports you, this not only helps you overcome that obstacle, but also encourages you to try harder:

> *If you got confused…, having a figure to tell you, "Oh you can move on by doing this. You can overcome this by doing this" it will encourage you further and you want to achieve your goal.*

This is supported by the survey which suggests that nearly all students reported that getting support from the teacher was extremely, moderately, or very helpful (44/47).

In the LOs, students were observed overcoming barriers by working collaboratively (4/5 groups). Group W were observed apparently hitting difficulties: for example, they were sometimes unsure of what different words and concepts meant or how to calculate what they needed. When this happened, they were observed to work together to figure out the problem. They did this by posing questions to each other which they then tried to answer. Students were also observed overcoming barriers by using reference sheets that had been given by the teacher (2/5 groups). For example, in LO1 when Group W struggled to work out an answer collaboratively, Ismaeel's partner tried to get the teacher. When the teacher didn't come over, they continued trying to solve the problem themselves by looking at the reference sheets they have been given to help them. This is supported by the survey which suggests that nearly all students (41/45) reported that getting support from others in their group was extremely, moderately, or very helpful.

6.6.5 Motivation

Motivation became a core code as students had distinct opinions about how they appraised the learning from the projects. 3.3.2 describes how students can be viewed as having mastery or performance goals, how this will mediate how they appraise a task: whether they perceive the task as being personally relevant and attractive, and thus will affect their motivation and subsequent engagement.

Fahiza and Anika (FG2), both demonstrated a clear exam orientation stating that they considered mathematics 'useful' if it is the mathematics that "*will come up on your exams… that will directly help you get the grades"* (Anika, FG2*).* In vignette 2 I present Anika's account of MaPBL, which was largely negative and suggests she did not have a strong motivation for action during MaPBL.

Students' orientations were not always distinct. For example, as discussed in vignette 4, some of the students who clearly reported valuing the learning in the projects also indicated that they felt it was important to ensure that they learnt the mathematics required to pass their examinations. Survey1 suggests that the main reason students try in mathematics is because it is an important exam subject, with 38/43 (strongly) agreeing. However, 24/36 (strongly) agreed with the statement that their main interest was in learning maths; doing well in the exam was a spin off. This suggests that maybe students care about both but succeeding in exams dominates their thinking.

Having a mastery goal orientation also appeared to influence what motivated a student*.* The students in the FG who appeared to have a mastery goal orientation reported placing a high value on learning within and about many authentic contexts.

Over half of the students in the FGs, and many of the students in Survey1, reported valuing using and applying their mathematics to solve authentic problems:

> *[What did you like about the project?] We had a real problem to work with (rather than working out how many chocolate bars Harry gets)* (9M2, Survey1).

Most FG students (8/10) made comments about how they valued the contextual learning in the projects, beyond mathematics:

> *I think the immigration project was very interesting because it showed us what other people around the world are going through (Ammara, FG2).*

Many students in the FGs (7/10) and the survey (41/45) reported working harder when the context appealed. For example, Mehdi, said Amazon Trader inspired him "*to do much more*" (Mehdi, FG2).

Myesha felt that studying lots of different contexts increased motivation:

> *If you have more off topic kind of projects it will give you a drive.*

Fahmida explained how she felt that a project would be more interesting if it the context was familiar to her:

> *I think things that are relevant to the students. Like the [Design a] Bag project, we didn't really know much about it, so it wasn't as interesting… to… some people.*

Mr Robinson reported that he felt having an authentic context that was *familiar* could work as a ‘hook’ into the project as it allowed students to start work on the project more easily:

> *The initial hook of them having to choose what products they wanted… They could all join in that conversation… maybe that was why the rest of the group work went well.*

The nature of the context was observed to create challenges for the students. For example, in LO1, the students appeared to find it challenging to understand things such as VAT or the ‘fulfilment fee’. Authentic contexts sometimes created tangential conversations; this was observed happening in over 50% of the groups (4/7). For example, in LO1, Group W started to talk about advertising campaigns, whilst in LO3 both groups discussed their own grades.

Another aspect of performance goal orientation that seemed to mediate a student’s *object motive* was that of competition. Some students in the FGs (3/10) reported that when the projects were structured so that they were working in a group, a slight competitiveness with other groups made them more engaged in projects:

> *… having groups made it kind of a competition. Personally, being really competitive, it drove us a lot more* (Myesha, FG2*).*

The teachers also reported that they felt that having a tangible output motivated students to work harder:

> *The motivation is a lot higher because they are going to have to present this…* (Mr Robinson, TW).

6.6.6 Working collaboratively

Working collaboratively was a key theme in FG1 and FG2 with almost all students (9/10) commenting about working with others. In 4.8 I defined ‘collaboration’ as all actions that are intended to increase the group’s development of knowledge or that contribute to a deeper, more complex understanding (Remedios et al., 2008).

In LO1 and LO2, students were observed showing effective collaboration including: questioning each other (Groups W and V); challenging each other’s thinking (Group W); seeking reassurance from each other (Group Y); and working together to collaboratively solve a problem (Groups V, W, Y, Z).

One of the students in FG2 highlighted how having an effective group was an important part of successfully working on a project. In an effective group, you have "*people that are motivating, they encourage you to do the work*" *(Myesha, FG2).* This was supported by the survey1 with 38/40 (strongly) agreeing with the statement that they think that group work helps the learning of mathematics. The reasons students gave for viewing collaboration positively were that it allowed them to share ideas and help each other. Students reported that collaboration was more effective if every student in their group worked hard, the students motivated each other, and communication was good.

The distribution of tasks appeared limited. In Group V and Group W (LO1), students were observed working together on the same task, although they contributed differently. For example, in Group W one student completed all the calculations whilst the other student questioned him. I viewed this as an unequal distribution of work which was also seen in groups xxx. In LO2 (Groups X and Y) the students were sometimes observed distributing the tasks: students would work on different tasks simultaneously.

Anjum described how having a stronger understanding of the requirements of the project would allow them to distribute the work more easily:

> *If we had a whole first lesson of everything that we'll have to do…, so we already had an understanding,… then we could divide it by our group members.*

This links to the sub theme of students wanting greater support with timeframes. Myesha described how in her group they realised that to complete the project on time, they would have to distribute the work:

> *We were kind of forced to complete it even if it was really hard. Everyone had to split the work and it had to be rushed. So, it wasn't properly done.*

The results from survey1 around distributing work could be viewed as being unclear. When asked how often they chose who in the group would complete each task, 6/6 students reported often or always for Amazon Trader, 22/29 for Cake Bake and 31/37 for Maths of Migration. However, 27/38 reported (strongly) agreeing that 'when working on a project in a group, we normally all complete the same task together'. The teachers in the FG speculated that this discrepancy could come from students thinking that *"working together is… how they should attack this problem*" (Mr Drew, TW). Equally, it could be that students simply chose to all work on the same task together, which as highlighted above, they reported sometimes working effectively.

Some students may have chosen to work together as it was more suitable for them in that situation. For example, one student described how she felt that when they were doing something new and challenging it was better to work together: if they worked on separate things then they might not know how to complete the task and therefore wouldn't be able to get on with the work.

The students had not been given formal roles in any of the lessons observed. However, often students appeared to take on different roles within the group. For example, in LO1, Ismaeel completed all the calculations and wrote everything down. However, his partner constantly asked him questions, which he responded to, and in this way, his partner helped to drive the thinking. Some students in the FGs (4/10) reported taking on different roles within their teams quite naturally. Myesha explained: "*We basically just saw what everyone was strong with, what they thought they could do.*"

One student thought that they might have been able to be more organised if they had been given pre-determined roles (Vignette 3). However, this didn't appear to have much traction with other members of the focus group; when asked if anyone else thought it would be helpful to have them, the only student to answer was Fahmida who reported that as the students were working in small groups of only 2 or 3, they weren't necessary. However, this student also reported that in her team they all worked on the same task together and did not typically distribute the work.

6.7 Emergent inductive conjecture

The data from the first phase of the study suggest that students experience of, and attitudes towards, MaPBL were mediated by at least four dimensions: having a mastery or performance goal orientation, a student's self-efficacy and a student's vision of mathematics. I used this to create the model that can be seen in Figure 6-1. I view this as a conjecture as there is not enough data to say confidently that this is what happens. However, from the data in the first phase of the study, it looks as though it might be what is occurring.

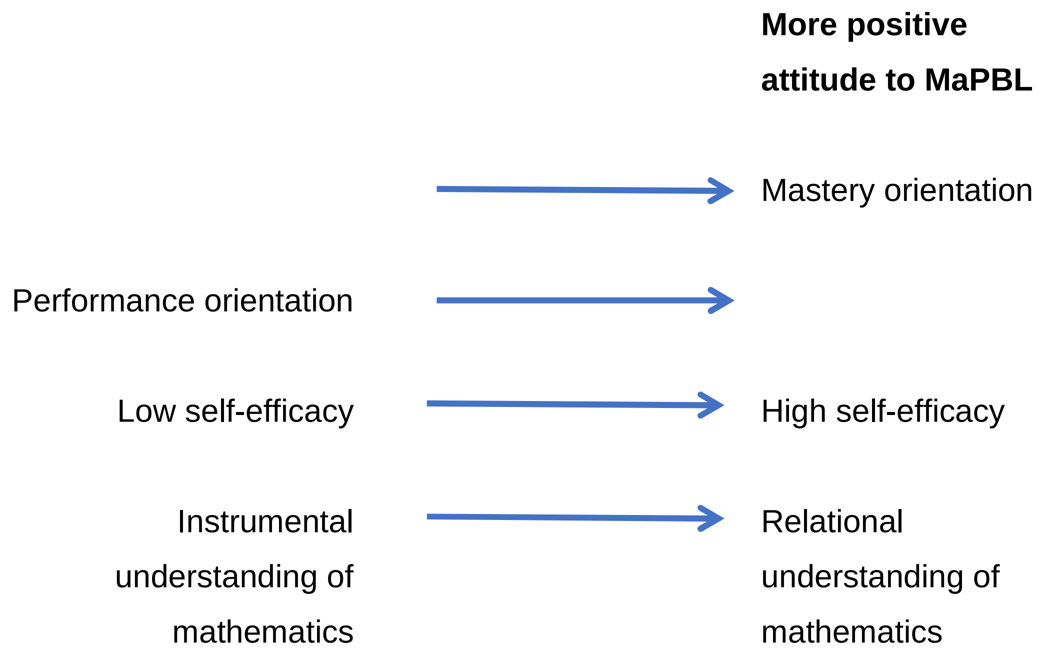

Figure 6-1: Emergent inductive conjecture - factors mediating attitude to MaPBL

I illustrate this conjecture with four vignettes, each of a pseudonymised student, or students, with different perceived beliefs or goal orientation. I give a summary of each vignette below (they can be viewed full in appendix 177). In each, I present an interpretation of the student's attitudes towards learning through MaPBL, the challenges they reported facing and the strategies for support they reported perceiving as helpful. Where appropriate, I give an indication of whether these attitudes and opinions were shared by other students.

Perceived low self-efficacy: Tanzia's account of MaPBL was largely positive. However, she experienced passing, in the moment, emotions such as frustration which led to her demonstrating low mathematical resilience and exhibited some off task behaviour.

Tanzia reported finding challenges with: drawing connections between prior mathematical knowledge and the mathematics in the project; understanding the order that she needed to carry out mathematical processes and understanding how to interpret the answers she was getting. She also struggled to approach the project mathematically. She used all support offered and appeared to most value personalised support from the teacher.

An instrumental vision of mathematics and performance orientation: Anika's account of MaPBL was largely negative. She reported not enjoying learning through MaPBL as she did not feel it helped her to gain a knowledge or understanding of the mathematics on the exam. She did, however, acknowledge that the contexts of some of the projects meant that they were interesting and that some students in the class may enjoy the projects in a way that she did not. Another student implied that perhaps students like Anika did not work to their fullest during MaPBL.

Anika reported finding it challenging to know what to do during MaPBL and described feeling confusion, which she viewed as a negative emotion. She suggested that to help with this, teachers could provide an outline of what the students needed to do. She also reported that it would be challenging if you did not have a group who communicated well. However, she felt that a good group would provide support in completing the work.

An instrumental vision of mathematics and a mastery goal: Shaffat's account of MaPBL was largely negative. He reported not enjoying learning through projects as he felt he learnt better in a more traditional classroom. He reported finding it challenging to be organised when learning through MaPBL and ensuring that the work was finished in a timely manner. The strategies for support that he suggested were often connected to supporting his organisation during MaPBL, such as a checklist with suggested timings and having group roles. He also reported that he felt that a midway check-up and strong problem-solving skills, especially the ability to understand which mathematics to use, would aid students during MaPBL. Despite not enjoying MaPBL, he seemed to respect the autonomy that this style of learning provided.

Students with perceived high self-efficacy and mastery goal orientation: Nadman and Fahmida's account of learning through MaPBL was positive. They reported valuing the wider learning and found the context of some of the projects interesting. They reported that they felt that the projects helped them mathematically and that they worked harder during MaPBL. Fahmida found it challenging to manage her own time properly, to understand the project well enough to distribute the work between her team members, and to focus on the mathematics. She reported that support such as a checklist may help students; however, she wanted it to be vague to allow the students independence. Both students appreciated the support that they got from their peers and being able to work collaboratively.

The vignettes helped to illustrate how students' beliefs and goal orientations may have been mediating their attitude towards and experience of MaPBL.

6.8 Chapter conclusion

Above, I discussed how I collected my data in phase one of the study, my approach to analysis and interpretation and the methodological challenges and limitations. This included the impact that Covid-19 had on my data collection and consequently on the trustworthiness of the study: the study became far more exploratory than I had originally intended.

I presented the results of the first phase of the study under the core codes student-led learning, self-regulated learning, self-efficacy, motivation, resilience and working collaboratively, including the evidenced related challenges and the support strategies students suggested. Some of these strategies bring with them tensions for the teacher since greater support often brings with it reduced opportunity to develop aspects of mathematical working that teachers are trying to promote.

I explained how I viewed the data collected in the first phase of the study as suggesting that successful student-led learning is highly dependent on students' beliefs and goal orientation. I then offered my inductive emergent conjecture: that students' attitudes towards MaPBL are mediated by their self-efficacy, vision of mathematics and goal orientation and presented summaries of four vignettes to illustrate this. I wanted to see if these phenomena were being experienced more widely and so, in phase two of the study I completed a survey to investigate my emergent conjecture.

7 CHAPTER 7 - PHASE TWO – METHODOLOGY AND RESULTS

7.1 Introduction

The results from phase one of the study suggested that self-efficacy, goal orientation and vision of mathematics mediate students' attitudes towards MaPBL. Phase two was designed to explore whether these phenomena that I had interpreted in the first phase of the study, that informed my inductive emergent conjecture, were experienced more widely in the school population. To do this I designed a survey (appendix 21) with question batches on mathematical self-efficacy, goal orientation, vision of mathematics and attitude to MaPBL, which I used to create subscales, alongside a question on whether students enjoyed the more independent learning in the projects.

Data	Date Collected	Class	Participants
Survey2	21/5/2021 – 07/07/2021	All year 8, 9, 10. Approximately half of year 11	28/133 year 8 students 14/112 year 9 students 75/111 year 10 students 34/58 year 11 students

Table 7-1: Summary of phase two data collection events

7.2 Method

The first section of Survey2 contained three small batches questions on: self-efficacy, goal orientation (mastery and performance) and vision of mathematics (instrumental or relational). The questions were adapted from pre-existing instruments (see appendix 20 for the instruments I considered). The last section contained questions about students' attitudes towards PBL. These questions were grounded in the responses that students had given in the FGs. All responses were given using a Likert scale of strongly disagree (1) to strongly agree (5), with negatively phrased questions reverse coded.

I tried to use my learning from Survey1 to inform the design of Survey2: I limited the number of questions; ensured that there were a higher number of 'reverse questions'; used and adapted existing instruments wherever possible; and ran pilot tests and cognitive interviews (appendix 19) to ensure that students understood the questions.

7.2.1 Mathematical self-efficacy

The questions on mathematical self-efficacy were taken directly and unamended from a Canadian study completed by Blotnicky et al. (2018), who had a similar conceptualisation of mathematical self-efficacy. Blotnicky et al. found the questions to be unidimensional and reliable through a confirmatory factor analysis [KMO = .698, $p < .01$, $\alpha = .72$].

7.2.2 Vision of mathematics

Yackel ((1984) cited in Cifarelli et al., 2010) created an instrument that assessed whether a student's vision of mathematics favoured a more instrumental or relational understanding. This instrument is based on the work of Skemp (1978), whose work I drew on in this study. Researchers who used this instrument found strong internal reliability, Quillen (2004) removed four questions, ($\alpha = 0.89$), whilst Cifarelli et al. (2010) used all questions ($\alpha = .73$).

Yackel designed the instrument for use with university students. I felt that there were too many questions, and some might be challenging for younger students to understand. I learnt from my original survey the importance of limiting the number of questions, not least to help students remain focused throughout. I therefore removed some of the questions I judged to be more complex.

7.2.3 Goal orientation

The questions on mastery and performance goal orientation were a subset of questions from a survey by Elliot et al. (2001) which were reduced to better reflect my definition of this construct. The concept of avoidance goals was not highlighted through the FGs and therefore, due to the limited scope of my thesis, I removed these questions. Whilst this is a relatively old instrument, I found the questions had a comparable level of detail and granularity to that my qualitative data had provided. More modern instruments designed to measure this construct typically split a mastery goal orientation into sub strands. Whilst this may reflect a more up to date thinking of achievement goal orientation, for my purposes I chose questions that were reflective of the comments that the students had made in the FGs.

I amended some questions, to simplify the language and to reflect the emphasis on exams that students in the FGs had reported. When this survey was written, performativity would not have been such a widespread phenomenon and this alteration of the questions reflects a shift in the culture of education: exams are important gatekeeper qualifications and I felt that there would be greater construct validity if more questions were exam-goal oriented. For details on the amendments see appendix 20.

7.2.4 Attitudes towards MaPBL

The questions to explore student opinion on PBL were based on the qualitative responses' students gave in the FGs: the questions were grounded in the data, with a view to establishing how widespread the evidenced attitudes were. I designed two questions for each element of attitude: belief, emotion, and behaviour, one phrased positively and one negatively.

7.3 Participants

Most of the students in years 8 to 11 completed the survey as part of routine School Self Evaluation. 151/414 either had parental consent (year 8 and 9) or reported they had spoken to their parents about the research (year 10 and 11). They also gave their own consent. This low rate of parental consent was disappointing but is reflective of a common challenge in the school. I made every effort within the bounds of what myself and my supervisor considered ethical, to boost the rate of permission, including regular reminders to students and text messages to parents.

134 out of the 151 (89%) also gave consent for their responses to be linked to their background data. The contextual data about these 134 students (Table 7-2) shows that they were largely representative of the wider school population, however the participants were slightly skewed towards female, non-school free school meal students without an SEN.

	Gender		Prior attainment			SEN		FSM
	Male	Female	Low	Mid	High	Support	EHCP	
Sample	41%	59%	18%	50%	32%	13%	2%	50%
School	48%	52%	15%	50%	35%	17%	4%	60%

Table 7-2: Contextual data about Survey2 participants

7.4 Methodological challenges and limitations

7.4.1 The impact of Covid-19

When schools re-opened after the Covid-19 pandemic lockdowns the study school put in mitigations that included an expectation that teachers would not circulate, and students would sit in rows and face the front. The ability to complete group work was severely limited. This, alongside the lost curriculum time, meant that from March 2020 until the survey, the enactment of MaPBL was limited in the study school. The survey included text to remind students about the projects, however most students would have completed much MaPBL for over a year prior to completing the survey.

7.4.2 Survey design

All responses were given using a Likert scale of strongly disagree to strongly agree. A major limitation of this method is that the scales are not intrinsically equal-interval, but there was not an equal-interval scale available.

I designed the attitude to projects question batch. It is likely that this 'researcher designed measure' was less reliable and valid in comparison to published standardised measures. However, I could not find a published measure for this construct. I tried to mitigate this by completing a pilot survey and cognitive interviews. However, there is a limitation to cognitive interviews: they are not experienced in the same way as the survey.

7.4.3 Method factors

Some of the question batches had two factors when this wasn't expected. It is possible that these factors could be distinct due to a "method factor": the variance can be attributed to students' 'mis-response', rather than the question batch measuring two separate constructs. Some participants responded on the same side of neutral on a Likert scale for both reversed and non-reverse items suggesting that the mis-response could be due to acquiescence (Baumgartner & Steenkamp, 2001).

7.5 Results

The responses to the questions were scored from 1 (strongly disagree) to 5 (strongly agree). Questions worded negatively were reverse scored (indicated by 'RS' in the rotated factor matrices). The suitability of each batch of questions for principal component analysis was evaluated. As is discussed below, one factor for each batch of questions was chosen for further analysis. Two question batches, vision of mathematics and goal orientation, had an item removed as they did not load onto a factor. The KMO for each batch (after these items were removed) were between .65 and .75 falling within the 'mediocre' to 'middling' range according to Kaiser and Rice (1974). Bartlett's test of sphericity confirmed that the correlation matrices for each question batch contained non-zero items (see Table **7-3**). Together, these constitute evidence for the presence of adequate structure within each batch of questions for performing a principal component analysis (Field, 2013). Hence participants' responses were entered into a principal component analysis using principal axis factoring and a varimax rotation.

I then assessed the internal reliability of each of the factors by calculating Cronbach's alphas, which measure internal consistency by indicating how closely related sets of items are as a group. In my case, high alphas would imply that the items are dimensions of the same construct. The alphas for each factor are shown in Table 7-3, they all comfortably exceeded Cohen et al.'s (1996) minimum recommendation of .65.

	KMO	**Bartlett's test of sphericity**	**Cronbach's alpha**	**Variance**
Mathematical self-efficacy	KMO = .75	[$\chi^2(10) = 112$, $p < .001$]	$\alpha = .7$	One factor accounting for 46% of the variance
Vision of mathematics	KMO = .65	[$\chi^2(21) = 135$, $p < .001$]	$\alpha = .7$	Two factors accounting for 53% of the variance.
Goal orientation	KMO = .67	[$\chi^2(15) = 149$, $p < .001$]	$\alpha = .68$	Two factors accounting for 68% of the variance
Attitude to projects	KMO = .72	[$\chi^2(15) = 236$, $p < .001$]	$\alpha = .77$	Two factors accounting for 65% of the variance

Table 7-3: Summary of initial analysis of question batches

Dancey et al. (2007) define weak correlation as between $r = .1$ and $r = .3$, I assume this definition, however, I caveat that this is a rule of thumb and not contextualised within the discipline. The classroom is a complex place and that there are many factors that influence students' engagement with, and attitudes to, PBL. The literature in this area typically only reports weak or moderate correlations between factors (e.g. Hann, 2019).

Subscales were created by averaging the scores of the items that loaded onto each factor for each participant. The mean average scores for each subscale, were checked for outliers and normality. This is because with a small sample size, outliers can affect the power of significance tests and the assumption of normality is made when computing confidence intervals, completing significant tests, or estimating model parameters (Field, 2013).

For confidence intervals and significant tests to be used reliably, the level of variance for a particular variable needs to be consistent across the different groups, so each subscale was checked for normality and linearity. Similarly, as you need to assume that the responses were independent, homoscedasticity or homogeneity of variance and independence were checked (appendix 22).

Each scale was checked to see if there was a significant variation for any of the background contextual variables: free school meals (FSM), prior attainment band, gender, Key Stage, and SEN. I used a significance level of $p = 0.05$, and below this I rejected the null hypothesis (Field, 2013).

Some of the subscales appeared to be related to each other. Self-efficacy had a weak correlation with relational vision of mathematics (significant at the 0.01 level (2-tailed) $r = .31$) and a weak correlation with mastery orientation (significant at the 0.01 level (2-tailed) $r = .23$). The relational vision of mathematics had a correlation with mastery orientation (significant at the 0.01 level (2-tailed) $r = .56$). No factor showed a correlation with the positive attitude to MaPBL factor. However, as I discuss below, their appeared to be correlations with some of the items that constitute the positive attitude to MaPBL factor Table 7-4.

I now discuss each subscale in turn.

7.5.1 Mathematical self-efficacy

The 5% trimmed mean of the participants in this study was 3.26 for the mathematical self-efficacy subscale, suggesting they had a moderate mathematical self-efficacy. This conclusion is based on the interpretations of Blotnicky et al. (2018) whose instrument I replicated for this study.

Correlations

	Mathematical self-efficacy subscale	Relational vision of mathematics subscale	Mastery orientation subscale
Mathematical self-efficacy subscale	1	.314	.231
Relational vision of mathematics subscale	.314	1	.556
Mastery orientation subscale	.231	.556	1
Positive attitude to projects subscale	.004	.104	.022
Projects are an effective way to learn mathematics.	-.004	.149	.138
I enjoy learning through projects.	.187	.135	.086
I often feel confused when learning through projects (RS).	.052	-.068	-.104
I work harder in projects that I do in 'normal' maths lessons.	-.188	-.020	-.182
Projects don't help me learn the mathematics I need for my exams (RS).	.004	.019	-.009
I give up more often during projects than 'normal' lessons (RS).	.017	.038	-.032
I enjoy learning more independently in projects.	-.084	.200	.074

Table 7-4: Correlations between subscales and items from the attitude to projects question batch

The 54 male participants (*M* = 3.4, *SD* = .66) compared to the 79 female participants (*M* = 3.14, *SD* = 0.59) reported significantly higher self-efficacy, [$t(131) = 2.15$, $p = .03$] and the 34 students in key stage 3 (*M* = 3.6, *SD* = .54) compared to than the 100 KS4 students (*M* = 3.1, *SD* = 0.64) reported significantly higher self-efficacy, [$t(132) = -3.63$, $p < .001$].

Mathematical self-efficacy had a weak correlation with students agreeing with the statement "I enjoy learning through projects" ($r = .19$). However mathematical self -efficacy showed a low negative correlation with students agreeing that they work harder in projects than they do in 'normal' maths lessons ($r = -.19$).

7.5.2 Vision of mathematics

Two factors were extracted, Table 7-5 shows the factor loading after rotation. Factor 1 represents an instrumental understanding and factor 2 a relational understanding. Whilst the items clustered in accordance with the research-based definitions, prior research (Quillen, 2004) suggests that there would be one factor. It is possible that these two factors could be distinct due to a "method factor" discussed above. An alternative possibility is that, as discussed in Ch3, the conceptualisation that a student's vision of mathematics is on a continuum from instrumental to relational could be an over-simplification. In fact, students could view mathematics as both instrumental and relational.

The 20 participants who had low prior attainment (*M* = 3.85, *SD* = .63) compared to the 92 participants who had middle or high prior attainment (*M* = 3.44, *SD* = .68) demonstrated significantly higher relational vision of mathematics scores, [$t(110) = 2.46$, $p = .02$]. This result surprised me as often I presume that students with a lower prior attainment are more likely to have a more instrumental vision of mathematics.

A relational vision of mathematics had a weak positive correlation with students reporting projects are an effective way to learn mathematics (significant at the 0.05 level (2-tailed), $r = .18$) and with students reporting enjoy learning more independently in projects (significant at the 0.05 level (2-tailed), $r = .19$).

Rotated Factor Matrix[a]	Factor 1	Factor 2
Doing mathematics consists mainly of using rules (RS).	.664	
Learning mathematics mainly involves memorising procedures and formulas (RS).	.567	
Getting the right answer is the most important part of mathematics (RS).	.528	
Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works (RS).	.452	
I usually try to understand the reasoning behind all of the rules I use in mathematics.		.701
When I learn something new in mathematics I often continue exploring and developing it on my own.		.687
Mathematics involves relating many different ideas.		.458

Table 7-5: Rotated factor matrix for vision of mathematics

7.5.3 Goal orientation

Two factors were extracted from the question batch on students' goal orientation. Table 7-6 shows the factor loading after rotation. The items that clustered on the factors I anticipated from the literature: factor 1 represents a mastery goal for mathematics and factor 2 represents a performance goal for mathematics. This could have been due to a "method factor" or because it is possible for a student to have both a mastery and performance orientation (3.3.2); they are not on a continuum. The items for the performance orientation factor did not capture all of a performance orientation and also showed a negative average covariance among items which violates reliability assumptions. This factor was not used in further analysis.

The survey suggests students in the school had a reasonably strong mastery orientation towards mathematics (M = 4.13). There was no significant effect on the mean average scored for the mastery orientation subscale for any of the contextual variables.

There was a weak negative correlation with students working harder in projects than they do in 'normal' maths lessons (significant at the 0.05 level (2-tailed), r = -.19).

Rotated Factor Matrix[a]	Factor	
	1	2
My goal in mathematics is to get a better grade than most of the other students (RS).		.770
I want to learn as much as possible from my mathematics lessons.	.434	-.434
It is important to me that I understand all of my mathematics lessons.	.798	
It is important to me to take time to understand new ideas in mathematics.	.654	
It is important for me to do well compared to others in my mathematics class (RS).		.562

Table 7-6: Rotated factor matrix for mastery orientation subscale

7.5.4 Attitude to projects

Two factors were extracted from the question batch on attitudes to projects, the rotated factor matrix can be seen in Table 7-7**Error! Reference source not found.**. Factor one contains the items I designed to show a positive attitude to projects, whilst factor two shows the items I designed to show a negative attitude to projects. I would have expected these to be on a continuum. However, as explored above, one could be a method factor. Equally, whilst I had designed the questions to be around the concept of a positive and negative attitude to MaPBL, on reflection the questions that indicate a negative attitude to mathematics could be viewed as students simply finding learning through MaPBL challenging, but not indicate a negative attitude. For this reason, I did not use the negative attitude to mathematics subscale.

None of the background contextual variables appeared to have a significant effect on students' attitudes to projects. The positive attitude to projects factor did not correlate with any of the other factors.

Students tended towards agreeing that they enjoyed learning mathematics through projects (*M* =3.71, *SD* = 1.06) and that projects are an effective way to learn mathematics (*M* = 3.64, *SD* = 1.02). They were slightly more likely to agree that they work harder in projects than they do on 'normal' maths lessons (*M* = 3.25, *SD* = 1.08). Whilst students were slightly more likely to disagree with the statements: I often feel confused when learning through projects (*M* = 2.9, *SD* = 1.00) and I give up more often during projects than 'normal' lessons (*M* = 2.75, *SD* = 1.00). Students on average appeared to be neutral about whether projects help them learn the mathematics they need for exams (*M* = 2.99, *SD* = 1.06).

Rotated Factor Matrix[a]	Factor	
	1	2
Projects are an effective way to learn mathematics.	.879	
I enjoy learning through projects.	.722	
I work harder in projects that I do in 'normal' maths lessons.	.511	
Projects don't help me learn the mathematics I need for my exams (RS).		.775
I give up more often during projects than 'normal' lessons (RS).		.634
I often feel confused when learning through projects (RS).		.485

Table 7-7: Rotated factor matrix for attitude to projects subscale

7.5.5 Student-led learning

Students did not appear to have strong feelings about the question "I enjoy learning more independently in projects" (*M* = 3.09, *SD* = 1.10). The 79 female participants (*M* = 3.3, *SD* = 1.0) compared to the 54 male participants (*M* = 2.69, *SD* = 1.2) reported a significantly more positive opinion about enjoying learning more independently in projects, [*t*(131) = 3.3, *p* = .001]. The 66 FSM participants (*M* = 3.3, *SD* = 1.0) compared to the 68 Non FSM participants (*M* = 2.85, *SD* = 1.1) reported a significantly more positive opinion about enjoying learning more independently in projects, [*t*(132) = -2.13, *p* = .035].

As reported above, enjoying learning more independently showed a low significant correlation with the relational vision of mathematics subscale (*r* =.20). It showed a low significant correlation with one of the mathematical self-efficacy statements "I feel powerless doing mathematics problems" (*r* = 0.19). It also showed a low, but significant correlation with two of the statements: "Projects don't help me learn the mathematics I need for my exams" and "I give up more often during projects than 'normal' lessons".

7.6 Modified conjecture

As discussed above, Survey2 did not always suggest that there was a relationship between the subscales that might have been expected from the phase one data. However, it did support some of the elements of my emergent inductive conjecture. Firstly, there was a relationship between mathematical self-efficacy and a student's enjoyment of MaPBL. Secondly, having a relational vision of mathematics, appeared to mediate a student's beliefs and emotions towards MaPBL: students with a more relational vision of mathematics seemed to be more likely to report that projects are an effective way to learn mathematics and to report enjoying learning more independently in MaPBL.

Based on the results of Survey2, I modified my emergent inductive conjecture. It can be seen in Figure 7-1 and illustrates how the study suggests mathematical self-efficacy and a relational understanding of mathematics mediate a student's beliefs and emotions towards MaPBL.

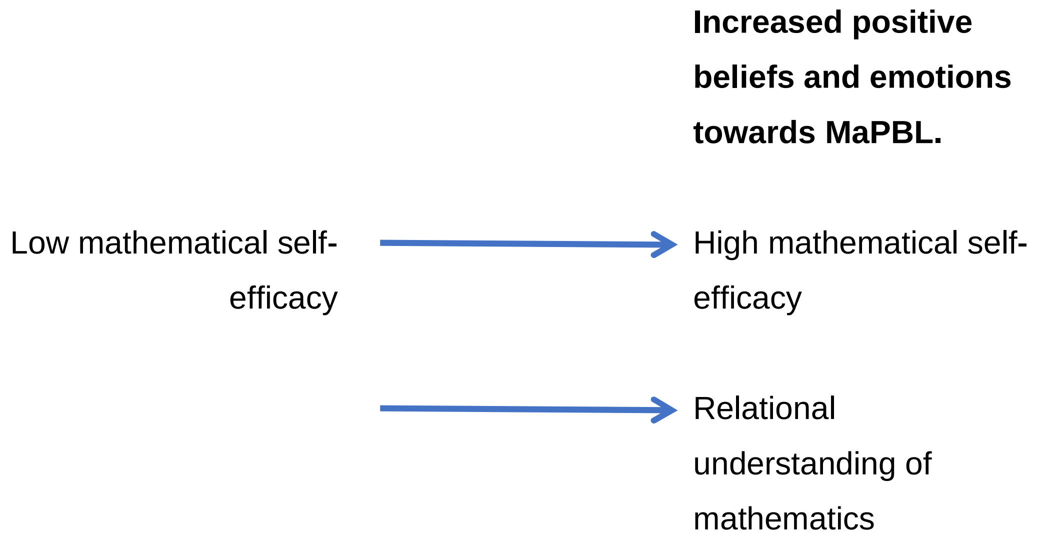

Figure 7-1: Modified conjecture – factors mediating attitude to PBL

7.7 Chapter conclusion

This chapter provided an overview of the research process and the results from the second phase of the study. I explained how I created a survey with question batches on mathematical self-efficacy, vision of mathematics, goal orientation, and attitude to projects. I provided a summary of the contextual data of the participants and discussed the methodological challenges and limitations.

Survey2 allowed me to investigate whether the phenomena that the students were reporting in the first phase of the study were experienced more widely in the student body. Due to the larger sample size, it also allowed me to explore whether contextual variables, such as gender and FSM appeared to mediate students' perspectives. I presented these results and suggested the data provide support for a modified conjecture: that students with a high mathematical self-efficacy and relational understanding of mathematics were more likely to show increased positive beliefs and emotions towards learning mathematics through MaPBL.

The methodology used in phases 1 and 2 opened up possibilities to make tentative contributions to knowledge. These will be discussed later and include considering how students' mathematical self-efficacy, vision of mathematics and goal orientation mediate their attitude towards MaPBL, and the challenges students experience and the strategies that support them when leading their own learning during MaPBL.

One of the biggest strengths of this methodology is that it puts student voice at the heart of the research. Presenting the student perspective offers a contribution to knowledge and originality. Student voice may inform future iterations of PBL, which has the potential to be transformative for teachers and the school community (Robinson & Taylor, 2007). It may provide a "*powerful and constructive trigger for enacting teachers' pedagogical change and developing our understanding of students' learning processes*" (Harfitt, 2014, p. 212).

8 CHAPTER 8 – INTERPRETATION

8.1 Introduction

Viewed through an activity theory lens, classroom communities are rarely considered flexible enough to allow for activities that break with the traditional *rules* of the classroom being a teacher-controlled space (Jonassen, 2002). However, the study shows that it is possible for this to occur in secondary mathematics classrooms in the UK. Furthermore, students reported enjoying being afforded the opportunity to learn both more independently and through MaPBL. Here I provide an interpretation of the data that I collected in both phases of the study through activity and complexity lens (Ch2).

8.2 Students' attitudes to MaPBL

The sample students generally reported a positive attitude to MaPBL in Survey2. From a complexity thinking perspective it is important to consider the *nested systems* that the students operate within. In the next sections, I discuss how there may be factors, possibly specific to the context of the school, that stem from the ambiguously bounded systems that the students were within that may have helped to foster this positive attitude to this way of working. Mathematical self-efficacy, working collaboratively and the authentic contexts all seemed to act as *mediating artefacts* on student's engagement with and enjoyment of MaPBL. Which I now discuss.

Some students in the focus group stated that they disliked being confused and not knowing what to do, although this was not a widely reported phenomena (Survey2). Confusion is sometimes viewed in the literature as an important part of the problem-solving process. However, these students appeared to experience this confusion negatively, which I interpret as being at least partly because they had low self-efficacy towards the task and did not believe they could be successful. Viewed through a complexity lens, this situation can be viewed as having the potential to create a *negative feedback loop.* If a student with perceived low-self efficacy thinks they are struggling with a task, they have a desire to give up. If they give up (as was observed in LO1), or even just put less effort in, then they are less likely to be able to complete the task, leading to lower self-efficacy. This could explain why mathematical self-efficacy showed a significant positive correlation with enjoying learning through MaPBL.

Whilst students with a higher mathematical self-efficacy, reported increased enjoyment at learning through projects, they also reported that they worked harder in their 'normal' mathematics lessons. It would be helpful to understand what students think hard work means. In my experience, students' *rules* of learning often equate hard work to the written output that they create and not to the level of thinking that they have completed. It is also of note, that the subscale is for mathematical self-efficacy whereas the core code was framed as a MaPBL self-efficacy. Students who have a strong mathematical self-efficacy may not necessarily have a strong MaPBL self-efficacy.

The data suggest many students valued and enjoyed the collaborative aspects of MaPBL. Viewed through a complexity lens, learning is a series of networked interactions that are dynamic, evolving, and unpredictable. Viewing learning as networked suggests that students will learn from their peers as well as the teacher.

The authentic context itself can also be viewed as a *mediating artefact* that influenced a student's object motive. Authentic contexts that are interesting, relevant, and competitive appeared to increase student engagement. However, what students perceive to be interesting and relevant differed between students and authentic contexts sometimes appeared to create complexity for students and lead to tangential conversations. The students that I interpret as having a strong performance goal orientation did not view the projects as useful as they could not see the connection between the mathematics on the exam and the mathematics in the projects, although they still found some of the contexts interesting. Viewed through a complexity lens, student interest can be viewed as having the potential to create a *positive feedback loop*: students find a topic interesting, so work harder, and in finding out more about the topic their interest grows. This *positive feedback loop* can amplify the potential impact of an interesting topic.

A familiar context may provide *redundancy* for the students: a familiar context may provide a common language around the problem, aiding interaction between the students. What students felt was relevant or familiar to them or others, though, differed. Again, this creates a conundrum for teachers: whose interests should be privileged in designing projects, and what should be the balance between authenticity and mathematical goals, for example.

A student's tacit *rules* of mathematics, whether they had a relational vision, appeared to mediate their beliefs and emotions towards MaPBL: students with a more relational vision of mathematics seemed to be more likely to report that projects are an effective way to learn mathematics and to report enjoying learning more independently in MaPBL. Viewed through an activity lens, this may be because the student and teacher were more likely to have a common ground between their perspectives on the *object* of MaPBL if students had a relational understanding of mathematics. Engeström (2005) suggests that a common assumption is that an *object*, the student, and teacher's goal for action, will have the same centrality and appeal for both the teacher and the student and gaining this partially shared *object* is a crucial challenge. I posit that if a student views mathematics as relational, the student is more able to see how MaPBL will support their learning and hence have a shared *object* with the teacher.

I now consider how the study suggests that teachers varied in the level of autonomy that they offered to students and students varied in their adoption of the choices made available. Alongside the support students need to make mathematical choices.

8.3 Not being offered autonomy

In my initial assessment, I felt that the department had optimum conditions to foster student-led learning. The *rules* of the department were supportive of teaching through MaPBL: the department's espoused beliefs were that of the value of MaPBL and students exercising autonomy and independence; the projects were mapped into the scheme of work, so there was an expectation that they would be used; and due to high exam results, teachers had the 'earned autonomy' (Perryman et al., 2011) to teach with innovative and progressive pedagogies. There was a *division of labour* between the members of the department, who sourced, wrote, and adapted projects that the entire department then used. The teachers also received support through the department as a *community*; they encouraged and supported each other to take risks and shared their experiences.

Considering the wider *community* of the school and the Borough reveals some reasons why it may have been challenging for teachers to allow student-led learning. The school's most recent inspection report (Ofsted, 2017) said that learners were often passive in lessons and lacked independence and whilst the Borough has shown a huge improvement in academic results and has high progress 8 scores, youth unemployment remains one of the highest in the country (East End Community Foundation, 2017). Seemingly, students appear to have academic skills, but lack the wider skills required for student-led learning. One of the teachers reported that students became anxious when expected to lead their own learning, which made the teacher anxious about giving students autonomy. In this sense, the students' context influenced teacher embrace of MaPBL.

The teachers seemed to find facilitating student-led learning one of the most complex parts of MaPBL to enact. I hypothesise that for a teacher to facilitate student-led learning, the students and the teacher require *expansive change*. Consideration of the conditions for *expansive change* helps to provide some suggestions for why in some classrooms, teachers demonstrated a more limited embrace of student-led learning during MaPBL than was envisaged by the department.

The expertise about MaPBL came from within the department. The only external input came from reading research and as noted previously, there is little literature about the student-led learning element of MaPBL. This led to limited *close neighbour interactions* between teachers and the concept of student-led learning. Further, many members of the department had trained within the department, whilst others had been at the school for a significant period; this may have created a lack of *internal diversity*. This lack of *close neighbour interactions* and *internal diversity* may have meant that there were similar notions of what made good MaPBL and a more limited pool of ideas about how teachers can support student-led learning.

Complex systems have *short range relationships* - complexity thinking suggests that *"the teacher must find ways to foster the local exchange of information"* (Davis et al., 2006, p. 104). To support student-led learning, teachers would want to facilitate local exchange of information between students around how to lead their own learning. However, when I considered the discourse on student-led learning – it was limited. Students and teachers had little vocabulary to discuss how students led their own learning. This lack of discourse around student-led learning may have constrained what could be said, thought and done. In the study school, the MaPBL projects were typically introduced to students in year 9. Whilst the students completed a series of activities at the beginning of the year to help build their skills, for the students who were observed, MaPBL was still a relatively new setting. In this way, it is possible there wasn't *sufficient redundancy*: the students and teachers didn't have a common language or consistency of setting. This lack of redundancy, or sameness, would have hindered interactions between agents which would have meant that they were less able to compensate for another's failings.

Complexity thinking suggests that the most effective way to work is with a *decentralised network* (Figure 2-2), this allows individuals to follow their own self-interests, inspiring more diverse interpretations and actions, creating *intelligent groups*. However, *a centrally distributed network*, where the teacher retains control of the knowledge may seem like a more time efficient way of completing the curriculum.

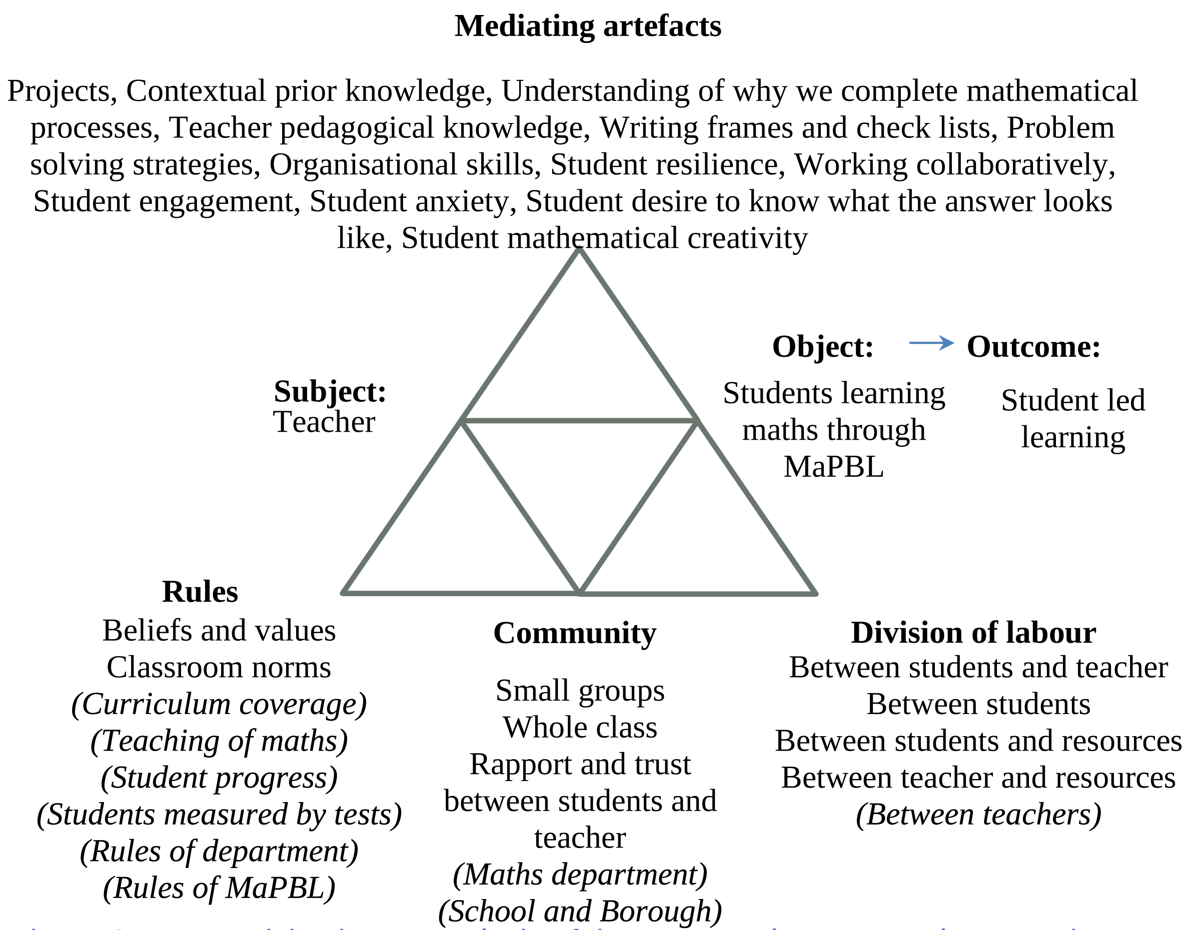

Figure 8-1: An activity theory analysis of the MaPBL classroom. Elements that agents bring to the classroom with them are in brackets.

In summary, despite my initial assessment that the department had optimum conditions to facilitate student-led learning, the teachers still faced many challenges in trying to teach in this way. However, even when students weren't offered autonomy, there were still times when students took autonomy for themselves, which I discuss next.

8.4 Assuming autonomy that wasn't offered

I interpret students assuming autonomy that had not been offered as being a product of their self-efficacy in the task and in the structure and support that was given to them by the teacher. From a complexity thinking perspective, the structured nature of the lesson, where students assumed autonomy, can be viewed as a *'complexity reduction'* in Biesta's (2010) terms. The teacher reduced the number of available options that students had for action by providing guidance on how to proceed and giving temporal boundaries. Biesta argues that *complexity reduction* makes the connection between action and consequence more predictable and secure. It could have been this feeling of security that allowed students to ignore the teacher's directions and choose a route that they felt would help them gain stronger outcomes. However, many lessons in the school provide structure and a *complexity reduction*, and yet it is not typical practise in the school for students to attempt to assume autonomy over their work. Through a complexity lens I view this as the nature of the project creating enough *randomness* to enable students to feel empowered to have a flexible and varied response. I hypothesise that the nature of the project and the fact that students had already been offered, and had assumed, some autonomy meant that they had adjusted and adapted from how they normally worked into a position of wanting to assume more autonomy. Complex systems operate *far-from equilibrium* and I hypothesise that this initial offering of autonomy, alongside the nature of the project, worked as a *positive feedback loop* and empowered the students to assume more autonomy.

It appears that the structured approach that the teacher provided in this lesson, the nature of the projects and the offering of a small amount of autonomy, may have empowered students to assume further autonomy and make their own decisions.

8.5 Supporting the embrace of autonomy

Viewed through a complexity lens, an autonomous student can be viewed as an *emergent system*. As discussed in Ch2 there are several necessary conditions for *emergence* that offer an interpretation of the challenges students appeared to face and the pedagogical strategies that the study suggests are supportive of students embracing offered autonomy.

As discussed below, many students experienced challenges with their self-regulated learning skills. For emergence, there needs to be sufficient redundancy. In this instance I suggest that students needed a *redundancy* in both their self-regulatory and metacognitive skills; these skills needed to be developed enough to provide a stability for the students, to enable them to cope with stress and uncertainty.

Emergence also requires *diversity*. I suggest that to work autonomously students needed to have greater *diversity* of ideas. When working autonomously it is not possible to predict the variation of thought that will be needed for intelligent action. Therefore, students need creativity and a wide range of possible responses to draw upon. I posit that if students had a greater *diversity* of responses, they would be more likely to embrace the offered autonomy and less likely to use the support offered such as the optional writing frames.

One further challenge that occurred as a product of student-led learning is that students were not always mathematically focused. Within an authentic task, sometimes other considerations are as important for students as the mathematics. However, as a defining feature of MaPBL is that projects should be central to the curriculum (1.4), it is important that students are exposed to the mathematical elements of the project, to ensure curriculum coverage. Starter activities can be viewed as increasing the number of *close neighbour interactions* between the students and the mathematical concepts they will need in the project.

Studying through MaPBL gave students a different way to engage with the classroom community. Complexity thinking suggests that for an effective complex system everyone should act as independently as possible, and that disagreement and contest are more effective than compromise or consensus –

which often excite nobody (Davis & Simmt, 2003). Davis et al. argue that an emergent collective can generate insights that are greater than the sum of the insights of the individuals. They argue that this arises from bottom-up individualised actions that are co-specified: if you have a collection of diverse individuals exploring a range of possibilities, then together they will be able to debate the merits of different actions and select the most well suited. The students may have experienced less challenges with completing projects on time and worked, in complexity terms, as a more intelligent group, if they were able to distribute work more effectively. The data suggest that students needed a greater coherence to ensure a focus or common purpose. The students didn't always have a strong understanding of what the project required: they found it challenging to conceptualise the problem and create a plan, which made it difficult to allocate the tasks. In this sense, the challenge can be viewed as stemming from students' lack of self-regulated learning skills, rather than their collaborative skills. I also suggest that they lacked the randomness needed to create a more flexible and varied response in the way they worked collaboratively. This could be because students at the study school were reasonably new to this way of working.

Interpreting how to support student embrace of autonomy through an activity lens also has something to offer. That students reported challenges with self-regulated learning skills during PBL can be viewed as students struggling with the *vertical shift of labour* from teacher to student that occurs during PBL. For example, in most of their lessons in school, the *rules* around time management are that teachers dictate how long students spend on any one task. This differs from the projects as envisaged by the department, where students manage their time within the timeframe given. At department meetings, there was little discussion about how to support students with this change in the *division of labour:* I hypothesise that teachers had not considered what a significant change this was for students.

Students, who are used to operating in largely teacher-controlled spaces, will take time to adjust to the *division of labour* moving from the teacher to the student. I view written scaffolds and plans, that students highlighted as being supportive, as an important intermediatory in the transference *of labour*. They provide a *horizontal division of labour* between the students and the scaffold.

Written scaffolds and peer support also facilitate a *decentralised network* (Ch2) which means that the students are not solely reliant on the teacher for support.

Further, I interpret the data as suggesting that some students lacked the self-efficacy for this shift in *the division of labour*. Plans and writing frames, in complexity terms, act as *enabling constraints*. I hypothesise that the students who wanted this support did not feel confident in having responsibility for leading their own learning. These scaffolds offer structure to students which helps to reassure them that they are approaching the project correctly. They provide students a common purpose that works to focus the group and as an enabler of emergence. They also create greater redundancy, as they provided a common frame of reference for students to work from.

Enjoying learning more independently in projects showed a weak significant correlation with one of the mathematical self-efficacy statements “I feel powerless doing mathematics problems.” A possible interpretation of this is that students who are more likely to feel powerless are empowered by the independent aspect of the problems. It also showed a low, but significant correlation with: “Projects don't help me learn the mathematics I need for my exams” and “I give up more often during projects than 'normal' lessons”. Interpreted through an activity lens, this could be viewed as the students’ *rules* of education encompassing the need to develop autonomy; perhaps with an underdeveloped skill set so they are aware that they struggle to take autonomy when it is offered. Considering this through a complexity lens, I am reminded that the system of the classroom is nested within other systems. It is probable that students are exposed to the importance of developing autonomy. Initiative, self-direction, decision making and responsibility are all considered important 21st century skills (Claro & Ananiadou, 2009). However, as indicated in the first phase of the study, students wanted support to take this responsibility. Perhaps students value autonomy, but don’t always enjoy having it.

In summary, for effective student-led learning, students to accept a transference in *the division of labour* from teacher to student and thus need strong self-regulated learning skills and high self-efficacy. Teachers can support this transference in the *division of labour* by providing options for a *horizontal division of labour* such as between student and resource.

8.6 Chapter conclusion

Above, I discussed students' attitudes to MaPBL viewed through activity and complexity lenses, both of which sensitised me to different aspects of the data. The consideration of the conditions for *expansive change* and an activity analysis proved useful frameworks to explore why students were not always offered autonomy. The concepts of *complexity reduction* and the *randomness* created by MaPBL gave an interpretation of why students assumed autonomy that wasn't offered. Whilst the concept of the transference of the *division of labour* during MaPBL highlighted how a *horizontal division of labour* can be created through resources, the classroom community, and the development of self-regulated learning skills.

In the next chapter I discuss the findings in the context of the literature.

9 CHAPTER 9 - DISCUSSION

This discussion is structured according to the research questions: in 9.1 I interpret students' attitudes to learning MaPBL; in 9.2 I consider what is challenging for students when leading their own learning during MaPBL; and in 9.3 I discuss the teacher strategies that are perceived to support student-led learning during MaPBL. In this study, because of the grounded nature of the analysis, it is inevitable that some areas of interest were not identified prior to data collection and were therefore not incorporated into the initial literature review. I contextualise these emergent areas of interest within the wider literature as I develop the discussion below.

9.1 RQ1 - What is the relationship between students' attitude to mathematics and their attitude to PBL, in particular the student-led aspect of that?

9.1.1 Beliefs about MaPBL

Both phases of the study found many students believed that projects are an effective way to learn mathematics; students with a more relational vision of mathematics were more likely to report this. This contrasts with the reported beliefs of students who appeared to have an instrumental vision of mathematics who stated MaPBL did not increase their skills, and the students with a performance orientation, who reported that learning through MaPBL was not optimum for their learning.

Students' beliefs about the value of learning through MaPBL appears to be an under researched area (Ch4) and none of the literature I found suggested that students' opinions about the value of learning through MaPBL was dependent on their view of mathematics. I now discuss why I think the context of the study school may have supported this phenomenon.

Complexity thinking attends to the dynamic nature of nested systems and how they act on and influence each other. I suggest that some of the nested systems the students operate within (Figure 9-1) may have contributed to a tension with students learning through MaPBL. The current national education agenda in the UK is one of accountability and performativity (Perryman & Calvert, 2020). Schools are held accountable for their GCSE results and this pressure is, in my experience, often transferred to the students. The community that the school serves, has a high level of deprivation (Gov.uk, 2021). However, whilst the students may come from disadvantaged backgrounds, there is no poverty of aspiration (P. Canavan, personal communication, June 6, 2021). Most of the sample students are of Bengali heritage, the children of second and third generation migrants who put a strong emphasis on education. As Ahmed (cited in Smart & Rahman, 2009) suggests, for many in the wider community, education and exam results will be viewed as a pathway to social advancement. There is evidence that South Asian families who are working class, have similar educational aspirations to the wider middle classes (Abbas, 2003).

It seems probable that learning within these nested systems will mean most students desire academic success. It is therefore of critical relevance whether the way they learn is in keeping with what they view as the *rules* of mathematics. I propose that for students with a relational vision of mathematics, MaPBL is more likely to fit with their *rules* of mathematics and therefore they place a stronger value on learning in this way: they view MaPBL as supporting their mathematics and hence their ability to achieve academic success.

Conversely for students with a more instrumental understanding MaPBL did not fit with their rules of mathematics. Having an instrumental understanding of mathematics is often linked to rule-following and passing a test (Pampaka & Williams, 2016). These students did not seem to recognise that in completing MaPBL they may have developed a stronger understanding of mathematics and made connections across topics in mathematics and between mathematics and their everyday knowledge, which are important elements of effective mathematics learning (Pampaka & Williams, 2016).

The students who expressed a strong exam orientation reported that they thought they didn’t increase their skills when learning through MaPBL. Some of these students reported mathematics as only being ‘useful’ if it was the mathematics that was on the exam. Even students who reported finding MaPBL helpful for their learning reported thinking that in year 11 students should prioritise projects that covered the material that was on the exam paper.

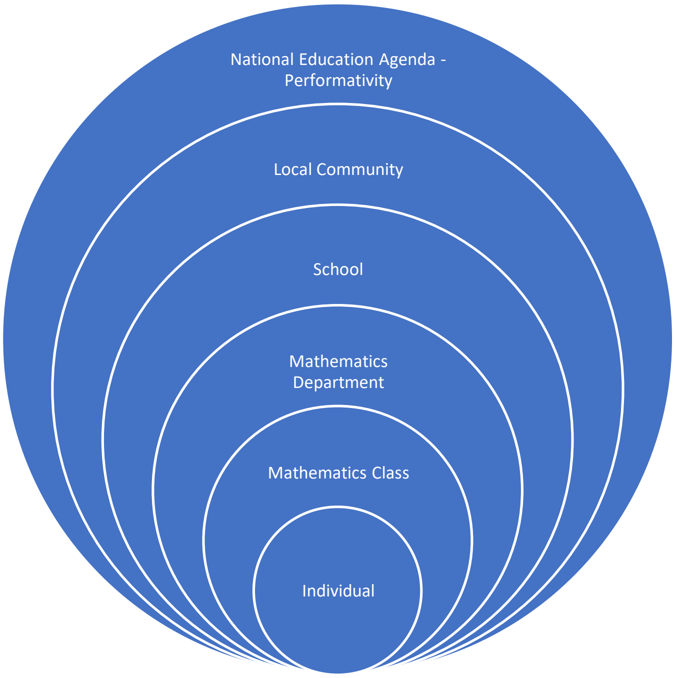

Figure 9-1: Nested systems

The idea that students who place a strong emphasis on exams do not always value learning mathematics in ways that don't emphasise learning for the exam is echoed in the wider literature. A UK study about the use of coursework in the mathematics classroom by Hernandez-Martinez et al. (2011) found that some students who did well in exams did not like coursework. The authors suggest that even though these students did not like coursework, it was still good for developing their mathematical understanding. Similarly in a Portuguese study of students completing problem solving tasks in groups, Ponte et al. (1994) found that younger students enjoyed engaging in interdisciplinary work in projects, whereas those in grade 10 reported feeling negative towards this learning, they 'expressed concern for their academic progress'.

Ciani et al. (2010) posit that it is challenging to maintain a mastery orientation in an education climate orientated towards students passing high stakes examinations. Their study of 15 mathematics classes in the US, found that giving students autonomy could work as a significant buffer against a classroom with a perceived performance-orientated goal. This may explain how the study school manages to foster a mastery orientation in many of the students despite the dominant ethos of performativity.

The suggestion that the nested systems within which these study students learn, and their consequent drive for academic success, means that MaPBL is not universally valued in the study school, could explain why this study has somewhat different findings from those in the literature. The studies that reported students valuing aspect of MaPBL were conducted in independent schools (Schettino, 2016; Wade, 2013) where students may have a view of success that is wider than the passing of exams.

One of the reasons given by students with claimed high self-efficacy and a relational understanding of mathematics for valuing learning through MaPBL, was valuing having autonomy over their learning. This valuing of autonomy, taking satisfaction in being able to follow your own interests and values when working, is, according to self-determination theory (Ryan & Deci, 2000), one of the key factors for intrinsic motivation, as discussed under ‘motivation’ below.

In the study school, even though some students tried to protect the autonomy they were being offered, there were a smaller number of students who wanted their lessons to be more teacher- structured. One of the students explained that they felt that the projects wasted their time. Similarly to the students who did not value learning in this context, this student had a performance orientation and felt that mathematics should be learnt in a structured linear fashion. This finding is similar to that of Ponte et al. (1994), where students problem solving in groups were negative about the lack of textbooks, and perhaps, the structure to learning that textbooks can provide.

In summary, this exploratory study suggests that whilst many students believed that MaPBL was an effective way to learn mathematics this was not universally true: this belief was more likely to be reported by students with a relational vision of mathematics. Furthermore, phase one of the study also suggests that it depends on students’ self-efficacy and goal orientation. I suggest these beliefs may be highly contextual to this situation due to the students’ desire for academic success.

9.1.2 Emotions towards MaPBL

Students reported a variety of emotions during PBL including enjoyment, fun, lack of enjoyment, frustration, anxiety, feeling overwhelmed, satisfaction and pride. Experiencing a range of emotions during problem solving in mathematics has been documented elsewhere, for example, in the case study of “Frank”, a student with high motivation who wanted to succeed. In a single problem solving task, Frank exhibited happiness, confidence, worry, relief, frustration, anger and nervousness (Voica et al., 2020).

Phase two of the study found that enjoying learning through MaPBL correlated to self-efficacy and enjoying learning more independently during MaPBL correlated to self-efficacy and a relational understanding of mathematics. This appears to be an unreported phenomenon in the PBL literature. Despite not often being considered in activity theory, emotions play a dialectical role with motives in activity (Leont'ev, 1978; Roth, 2007). I suggest that the emotions that the sample students experienced can be at least partially explained by considering their *object motives* and whether they believed they would be successful: students with a higher self-efficacy are more likely to think they can realise their motives, and students with a relational understanding of mathematics are more likely to think that MaPBL supports their motives. I now detail these different emotional responses contextualised within the wider literature.

It is notable that the FG students with an instrumental understanding of mathematics reported not enjoying the projects. Not enjoying PBL appears to be an under reported phenomenon in the literature. Similarly to the explanation I proposed for students not valuing PBL, I hypothesise that this phenomenon could be specific to the *nested systems* that the students are operating within. The sample students live in the intersection of cultures - the performativity of schools and the high aspirations of their families. Their view of success can be exam focused, and thus, if they don't have a relational vision of mathematics, view MaPBL negatively as it doesn't appear to contribute to their overall goals for education.

All the FG students reported that, if they felt their group was effective, they enjoyed working in groups. This accords with the literature (Ch4). Finding something fun could signify that the student was having social interactions that diverted them from the main task (Goldin et al., 2011); for example, several tangential conversations were observed. However, students typically reported effective collaboration and expressed feelings of frustration and irritation when this did not occur. In this study, I interpret enjoyment as being an 'activating emotion' in Pekrun et al.'s (2010) terms: enjoyment has the potential to focus attention, promote interest and intrinsic motivation and help facilitate self-regulation.

Some students, with a perceived high self-efficacy and relational vision of mathematics, reported feeling satisfaction and pride in the tangible outcome that was their original work. When students are asked to present their work in a format that is beyond the classroom, students become ‘emotional stakeholders’ (Rule, 2006) and have the potential to outperform their own expectations (Baser et al., 2017).

There were instances when the students with perceived low self-efficacy reported feeling frustrated and a desire to give up. Frustration is not necessarily negative; it could and should indicate that the problem is interesting and non-routine and can act as a catalyst for a student to know that they need to change their heuristic strategy (DeBellis & Goldin, 2006). It seems plausible that other students also felt frustration but did not report it as they experienced frustration as a positive catalyst to change their strategy.

The FG1 student with perceived low self-efficacy appeared to feel increased anxiety when leading their own learning and often reported wanting the teacher to check things for them. In a small-scale study by Pekrun et al. (2010) anxiety was found to be the most frequently reported emotion during problem solving in mathematics. Pekrun et al. viewed anxiety as complex as it can induce a strong motivation to avoid failure, or it can lead to task-irrelevant thinking, as happened with this student. In the study, one of the teachers reported perceiving that some students find it ‘painful’ when they are unsure what is required. Whilst this seems like a strong reaction, it is in keeping with some of the findings with the literature. DeBellis et al. describe how students can have a “deep, vulnerable, emotional engagement” (2006, p. 132) with mathematics that they term ‘mathematical intimacy’.

Boredom is a commonly reported emotion in mathematics lessons (Pekrun & Stephens, 2010). Students in this study did not report feeling bored at any point during MaPBL. This may be because, as one of the students reported, the projects are interactive. I interpret the lack of reported boredom as indicating that the structure seems to be engaging even while students identify some personally experienced limitations to the approach.

In summary, this exploratory study suggested that, in accordance with expectations and prior research, students experience a variety of emotions during MaPBL. It adds to the literature on PBL by suggesting that students' emotions are mediated by their self-efficacy and their vision of mathematics.

9.1.3 Behaviours during MaPBL

Behaviours are believed to have a reciprocal relationship with beliefs. I view this relationship as it is hypothesised by Goldin et al. (2011): beliefs can influence how a student construes a situation, which will influence the motivation a student feels, which will influence their engagement and action, and will then act on beliefs. The data in this study suggest behaviour is at least partially mediated by three beliefs: a student's self-efficacy, a student's vision of mathematics, and, perhaps to a lesser extent, a student's goal orientation. It is beyond the scope of this study to consider how behaviour, conversely, impacts upon these beliefs. Similarly to beliefs and emotions towards PBL, the students in this study showed a range of behaviours during PBL: some students were highly motivated to work harder in this context, whilst other students became demotivated and gave up. Viewed through a complexity lens, students are part of a complex interconnected system and as such, behaviour is likely to be influenced by a multitude of variables that are interwoven, interdependent, and interacting (McMurtry, 2006).

The MaPBL literature often reports that students completing PBL show higher levels of engagement and motivation than in 'traditional' lessons (Canuteson, 2017; Cerezo, 2004). However, in this study this was not found to be universally true: data from the first phase of the study suggest that students with a lower self-efficacy, an instrumental understanding of mathematics or a performance orientation did not report this increased motivation. Whilst data from the second phase of the study suggest that students with a higher self-efficacy and mastery goal orientation were more likely to work harder in 'normal' lessons (although I posit this is to do with how students interpreted 'hard work', see 8.2). It may be that the researchers who are studying PBL are educators who believe in the value of PBL and therefore, may be less likely to notice or report some of the more negative behaviours that were also identified in this study.

Students' motivation and subsequent engagement in MaPBL is discussed more fully below.

9.1.4 Conclusion

In conclusion, this study suggests that the students' attitudes, their beliefs, emotions, and behaviours, towards PBL were varied. The exploratory study adds to the current literature in suggesting that the sample students' attitudes are affected by their self-efficacy beliefs, their vision of mathematics and to a lesser extent, their goal orientation. It also offers evidence that challenges other literature, drawing attention to lack of enjoyment, and other negative responses to PBL, evidenced by some study students, despite their teachers' commitment to the approach. Such mixed responses can be understood within the nested eco-system in which the study students operate.

9.2 RQ2: What is challenging for students in leading their own learning during MaPBL?

Students reported, and were observed to have, several challenges with leading their own learning during PBL. Here I contextualise these challenges in the broader literature and highlight how some of the challenges seemed to be experienced by students almost universally, whilst other challenges appeared to be mediated by a student's self-efficacy, their vision of mathematics and their goal orientation. I discuss the challenges under the headings of the core categories: student-led learning, self-regulated learning, self-efficacy, motivation, mathematical resilience, and working collaboratively.

9.2.1 Student-led learning

One of the biggest challenges that students faced was not being offered autonomy by their teacher; teachers don't always afford students the possibility of becoming *self-organising* systems. This was contrary to envisaged department practice during PBL, as stated in department meetings, however, is a common phenomenon in the literature. A teacher's espoused values, often differ from their practice (Breunig, 2017), further, teachers typically adopt the pedagogies they were taught with (Lunenberg & Korthagen, 2003). Transitioning to more student-led pedagogy has been found to be particularly challenging (Hamilton, 2018); teachers can fail to see opportunities when their students can take the lead (Lunenberg & Korthagen, 2003), intervene in ways that limit students' agency (Restani, 2021) and rarely give students a high level of autonomy and ownership in lessons (e.g. in Science, Bencze al., 2006; in Geography, Bermingham, 2016).

Another challenge for students was that of embracing offered autonomy. The concept of student-led learning during PBL is underdeveloped in the literature (Ch4). I have not found studies that comment on students' lack of embrace of offered autonomy, or the tendency observed in this study of students to default to use all offered support, without considering whether it is required.

One of the challenges for teachers during student-led learning is that of ensuring students focus on the mathematics (Gresalfi et al., 2012; Wright, 2017):

9.2.1.1 Mathematical decision making

As analysed earlier, students often based decisions on their experiences or preferences, rather than on mathematics and optimising solutions. Whilst students may gain valuable skills and the decision making may be viewed as "authentic", this doesn't support students to get the required mathematical exposure. It is professionally important to me to ensure that the students remain mathematically focused. There is a tension between the need to help students succeed in their exams and the need to help them develop mathematical understanding, or even enjoy learning mathematics (Pampaka & Williams, 2016). I believe, as teachers, we have a responsibility to support all these outcomes and balancing these is a key professional issue. Under RQ3, I consider in more detail how we can support students during MaPBL to ensure they cover the curriculum content, as well as develop wider skills and understanding.

Observations suggested that some students were not as mathematically focused as I expected. It is possible that I underestimated the level of mathematical complexity- to use and apply mathematics from one area to another can be challenging for students, especially if their knowledge of the processes lacks depth or conceptual understanding (Hiebert, 1999). These students may not have the mathematical resilience required to solve the problem with a higher level of mathematical thinking. Moala et al. (2019) argue that students with low resilience dislike, and sometimes avoid, problems that challenge them mathematically. An alternative interpretation is that students may not have understood that a mathematical interpretation is wanted. Cooper et al. (1998) showed that learners from low socio-economic backgrounds were less likely to appreciate the mathematisation and pseudo-reality of contexts that had been internalised by middle class students. They were more likely to answer problems with their ‘everyday’ knowledge. As discussed under RQ3, teachers may need to make the mathematical interpretations more explicit.

9.2.2 Self-efficacy

The literature has reported that PBL increases students’ self-confidence and possibly their self-efficacy (Ch4). However, that students require strong self-efficacy to be successful in PBL appears to be an underreported phenomenon. The study adds support to the literature in finding that self-efficacy mediates a student’s self-regulated learning skills and resilience (discussed below) as well as their attitude to MaPBL. Attending to self-efficacy was suggested by the study to be a core concern for teachers.

9.2.3 Self-regulated learning

Self-regulated learning skills are a necessary *mediating artefact* that students require when learning through MaPBL. In Ch8 I discussed how I viewed many of the reported challenges with self-regulated learning as arising from the change in vertical shift of *division of labour* from teacher to student. This accords with the wider mathematical literature (Shilo & Kramarski, 2019) and PBL literature from other disciplines (4.3) which suggests that the ability to activate, control and regulate prior knowledge does not arise spontaneously, but needs to be developed.

The study suggests that time management was one of the most complex self-regulatory strategies for students to enact during PBL, perhaps because it is one of the students most under-developed skills. It supports the growing recognition in the literature of the importance of time management (Claessens et al., 2007), by highlighting its importance in a MaPBL context.

In this study, students’ self-efficacy and goal orientation appeared to mediate their self-regulated learning skills. This study supports the wider mathematics literature (3.3.3), that has found that students with perceived low self-efficacy find self-regulated learning more complex to enact by suggesting that the same appeared to be true in a MaPBL environment. Similarly, it supports the wider literature (3.3.2) that has found that students with a mastery goal orientation have stronger self-regulated learning skills and suggests the same is true in a MaPBL environment.

9.2.4 Mathematical resilience

Having strong resilience was found to be a challenge to some students' successful enactment of MaPBL. Here, I explore how the study suggests for the sample students, mathematical resilience was mediated by their self-efficacy, but not by their goal orientation or vision of mathematics.

The students with perceived high self-efficacy typically showed high levels of mathematical resilience when working on the projects. However, the student with perceived low self-efficacy reported finding it challenging to continue learning when she faced difficulties. This supports the current literature that has found self-efficacy mediates resilience in the classroom (3.3.3), by suggesting the same to be true in the MaPBL classroom. The literature suggests students with low self-efficacy are likely to avoid difficult situations (Schwarzer & Warner, 2013), as was observed with this student.

Students with a perceived high self-efficacy reported strong mathematical resilience, regardless of their goal orientation or vision of mathematics. This seemed somewhat surprising to me initially, as resilience and mastery goal orientation have often been found to be positively correlated (Ames, 1992; Splan et al., 2011). However, the wider education literature offers several explanations. Some studies have found that performance goals as well as mastery goals are predictors of academic resilience (Jowkar et al., 2014). Another possibility is that the students who had strong exam orientations also had a mastery orientation. These goals are not opposing: students can value learning and understanding whilst also desiring to do better than others (Pintrich, 2000a). Pintrich found that students with the highest self-efficacy were those with a high performance and high mastery orientation. Due to the grounded nature of the study, the importance of the construct of goal orientation was only understood after the focus groups and therefore whilst there is a clear performance orientation for some students, it is not clear whether these students also had a mastery goal orientation.

9.2.5 Motivation

As discussed in 3.3.2, intrinsic motivation is thought to increase when the three basic needs of autonomy, relatedness and competence are met (Ryan & Deci, 2000). The study suggests that there can be challenges with meeting these needs. Further, my data support the view these three needs are experienced differently depending on students' self-efficacy, their vision of mathematics and their goal orientation.

9.2.5.1 Relatedness

As noted under emotions, the sample students typically reported feeling enjoyment that they were able to work with others. Working collaboratively helped satisfy a need for relatedness. However, relatedness also requires your actions should be consistent with your cultural values. As discussed above the sample students are influenced by a mix of cultures, which seem to give students a strong desire to achieve academically. Hence, if students do not view MaPBL as supporting them with passing their exams, it may not be consistent with their cultural values.

9.2.5.2 Competence

As noted under RQ1, the student with a perceived low self-efficacy reported different beliefs, emotions, and behaviours from those with perceived high self-efficacy, their motivation changed throughout the project. Sometimes, when they felt they could be successful, they were motivated and engaged. At other times, when they felt that they wouldn't be successful, they became de-motivated and wanted to give up.

9.2.5.3 Autonomy

According to self-determination theory, to sustain intrinsic motivation students need to have autonomy, to follow their own interests and values. Katz (2007) posits that is it more important that students have tasks that they regard as relevant, than it is to be able to choose the task. This study suggests the same was true for the participants: students reported working harder when they found the context of a project interesting. However, the current study also found that what is relevant or interesting to some students it not relevant to others. Viewed in activity theory terms, relevance is culturally mediated. There is little consensus in the wider literature on what it means to make education relevant (Albrecht & Karabenick, 2018). Albrecht et al. (2018) suggest that relevance must be considered from the perspective of the student.

9.2.6 Working collaboratively

The data suggest the key challenges that limited students' ability to work collaboratively as an *intelligent* group were their self-regulated learning skills, distribution of the work and equal participation.

The challenge of unequal participation, found in this study is a frequently cited challenge in the PBL and wider literature (Cohen, 1994; Le et al., 2018). For example, in a study of primary students working collaboratively in Vietnam, more than 2/3 of the participants commented that there was a disparity in students' contributions and that some students 'free ride' (Le et al., 2018). Whilst these students were younger than the students in this study, a similar phenomenon appeared to occur, just to a lesser extent.

Having unequal participation was not reported in FG2. This could be because engagement in group work is often dictated by students of high 'status' (Cohen, 1994) and power and participation is usually held by more dominant students (Skinner et al., 2016). The students who attended FG2 were self-selecting and came from the top set. I suggest these students did not experience these frustrations as they worked in groups of high 'status', dominant students.

The literature suggests that students typically delegate tasks easily during PBL (Ch4). This study did not find this: students frequently worked on the same task. As suggested by the data, this could have been because students felt this was the most effective way to work.

9.2.7 Conclusion

This exploratory study offers a contribution to the gap in the literature of student perspectives on PBL (Grant, 2011), by signposting the challenges to successful enactment of MaPBL that students might experience. Principally: being offered and embracing autonomy, mathematical resilience, self-regulated learning skills, the projects satisfying a student's need for relatedness, competence, and autonomy, and distributing work and equal participation when working collaboratively. As discussed above, some of these challenges are mediated by a student's self-efficacy, vision of mathematics and goal orientation. I propose that if teachers attend to these challenges, students will be more able to assume autonomy when it is offered to them. I now discuss the strategies that this study suggests will be supportive of the students:

9.3 RQ3: What pedagogical strategies are perceived by students to be most effective for supporting student-led MaPBL?

Guidance and scaffolding is essential for effective PBL (Ch4). Below, I discuss the strategies that students and teachers suggested supported students during MaPBL and contextualise that in the wider PBL and mathematics literature. I consider the strategies for support under the headings of: student-led learning, self-regulated learning, self-efficacy, mathematical resilience, and working collaboratively.

9.3.1 Student-led learning

As discussed under RQ2, not being offered autonomy undermined student-led learning. It is beyond the limits of this study to consider how to support teachers to offer students autonomy. Instead, I focus on the pedagogical strategies that were perceived to be helpful to students. To foster student led learning, teachers must attend to the challenges that were highlighted under RQ2. As found in my data, different students and different projects will require different guidance and support.

Written scaffolds can be viewed as a tool for outsourcing learning: students can do things with the tools that they could not do otherwise. The tools can be viewed as creating a transference in the *division of labour* from the teacher onto the tool, with the aim that when the tool is later removed, the *division of labour* will shift to the student. Students who are used to operating in largely teacher-controlled spaces will take time to adjust to the division of labour moving from the teacher to the student.

I view the ultimate purpose of scaffolds and guidance as being to support the students to progress to a point where they need it minimally and only on their request. Over time, students would become aware of what they know and what they need to know and will use a variety of sources, such as the teacher, their peers, or a scaffold to access the help they need. The concept of ‘fading’, the gradual removal of support, and ‘adding’, students choosing to get extra support (Ch4), is vital to this. Probably because the sample students had had only limited experience of MaPBL, there is little discussion of ‘adding’ or ‘fading’. Similarly, when offered support, students typically accepted it (Ch8).

9.3.1.1 Mathematical decision making

Students sometimes struggled to make decisions based on mathematical considerations. An activity lens draws attention to the many *mediating artefacts* that are already used to support PBL. As suggested by one of the teachers, these can easily be adapted to support students with sustaining a mathematical focus. For example, if teachers give students a project outline or checklist, these should include clear expectations of the mathematical components that should be included in the project. This increases the number of mathematical *neighbour interactions,* the number of times students meet or have their attention drawn to mathematical concepts. That teachers need to give clear expectations of the mathematical components of a project is supported by the literature (Gresalfi et al., 2012). However, I think it is important that students' first, 'everyday', interpretation of a problem is also valued. Teachers should recognise this, as well as steering students towards a mathematical model, so that links between them are made. Teachers may find it easier to support these mathematical interactions if, as Wake et al. (2016), in their study of mathematical problem solving, suggest, teachers identify the possible problems and progression routes through the mathematics in advance.

As noted under RQ2, students, particularly those with a strong exam orientation often did not see the mathematical value in what they were doing, as the projects did not fit with what they viewed as the *rules* of mathematics. Teachers can increase interest by increasing the value of a topic, for example by explaining its interest or utility (Rellensmann & Schukajlow, 2017). For the sample students, I suggest that highlighting how the projects support the GCSE exam specification may have helped students understand more fully how what they are learning through projects is applicable to the exams they value.

9.3.2 Self-regulated learning

The literature suggests that students self-regulated learning skills can be developed through PBL (Ch4). Below I discuss some pedagogical strategies which the data suggest help support students' self-regulated learning skills. I suggest that this support can be 'faded' in time.

The study suggests students required support to understand and conceptualise the problem. One suggested pedagogical approach was to maintain some didactical elements in the MaPBL lessons; allocating time at the beginning of a project to support students to understand what is required, or modelling parts of the projects to students. Whilst this could be viewed as teachers not giving students autonomy, a recent literature review of PBL recommended that teachers should balance didactic learning and independent inquiry for successful PBL (Kokotsaki et al., 2016). Similarly, in a UK study of college mathematics, Wake et al. (2008) observed a teacher who they perceived to have unusually student-centred pedagogy. They found that whilst much of her observed practice was student centred, this would change when she wanted to expose a misconception or ensure students constructed meaning around ‘more advanced mathematics’. In these instances, the classroom became much more teacher centred until this new knowledge became operationalised by the students, at which point the teacher would withdraw the support.

Structured starter tasks that revisited the prior knowledge potentially needed during the project were also highlighted to be effective, especially for students with lower self-efficacy as they helped students see connections with previous work more readily. As discussed in 3.1, learning is context specific: even crossing from using mathematics in a more traditional mathematics lesson, to being able to apply that mathematics into a new problem can be complex for students (Boaler, 2000).

The most requested support was a checklist, which some students suggested could have time indications on it. Students reported that this would help reduce confusion, especially at the beginning of a project. Reiser (2004) views the main purpose of cognitive scaffolds as being to help students to structure the task. A checklist informs the learner of the necessary elements of a task and helps them to plan what they will do. One teacher suggested how the support the support that is given from a checklist could be ‘faded’ (Ch4). Initially teachers could give students a checklist, then they could create it collaboratively as a class, then students create a checklist that the teacher checked and then finally that they are able to create their own.

9.3.3 Self-efficacy

Some of the strategies suggested under self-regulated learning also appeared to engender self-efficacy, for example giving students a checklist. This could be because, as found by Valencia-Vallejo et al. (2019), when students are given metacognitive scaffolds which require them to evaluate their own knowledge and adjust their goals, this aids students in viewing themselves as someone who can achieve their own goals. Further, both the data and the wider literature suggest students will show increased mathematical resilience if they have a strong maths self-efficacy (Borman & Overman, 2004).

Another suggested strategy was using exemplars. Exemplars have the potential to create a *positive feedback* loop by raising students' aspirations, making them work harder, feel more success, and thus further raise their aspirations. The literature suggests that exemplar work created by other students in the school, may have the potential to impact students' self-efficacy. Self-efficacy is influenced by a student's peers (Schweinle & Mims, 2009) and therefore if students see other similar students succeed on a task then they will be more motivated to complete the task themselves (Schunk & Pajares, 2002).

9.3.4 Mathematical resilience

In the study, students were observed using different strategies to help them overcome the barriers that they faced in the project. For example, students received help from other members of their group, the teacher, by looking at the reference sheets and looking at their plans. The teachers stressed that the nature of the help that students requested was important – students shouldn't just say that they need generic help but should articulate what they didn't understand to get the specific help they required. Similar findings were made in the wider mathematics literature in a study by Moala et al. (2019): students need to be supported to set clear goals about what it is they need to know and how to structure their requests for help.

The PBL literature also suggests that being motivated by a project will support mathematical resilience (Hafiz & Dahlan, 2017). Therefore, to support resilience it is important to attend to the strategies suggested under motivation. This study also suggested that getting support from a teacher will increase motivation, a strategy favoured by the student with low-self efficacy. Both this student and other students commented on how when a teacher helps them it motivates them to try harder and they show increased mathematical resilience.

Some students in the study reported feeling overwhelmed. Moala et al. (2019) in their grounded study of 101 aged 9 students in New Zealand found that for students to show resilience, they should first expect mathematics to be challenging and they need to accept that sometimes they will be in a state of not knowing and feel overwhelmed by this. To support students' mathematical resilience, it is important to attend to students' beliefs that might undermine that.

9.3.5 Motivation

As discussed under RQ2 it can be challenging to ensure that the authentic context is motivating for the students. Most students in the study commented that they enjoyed, and found interesting, the "real world" aspects of the projects; learning about authentic contexts through mathematics and being able to understand how mathematics is used by people in their working life. This seemed to be particularly strong when students felt that the context was something that could be helpful to them in later life or was linked to their career goals.

It is important students can understand the relevance of the project. In the 'real world' people often make designs informed by market research, as was true for the 'design a bag' project, but some of the students in the focus group could not see the point in this project- they did not yet grasp how market research could be used. An authentic context that students don't feel will be useful in their life works to demotivate (Vos, 2018). In this way, this study adds to the existing literature in PBL (Daher & Baya'a, 2009; Hung, 2009; Maina, 2004) that has found that contexts that are interesting and relevant to students can increase student motivation and hence engagement in PBL.

The study also found, similarly to other PBL literature (Belland et al., 2006), that students were motivated by working with others. The study also suggests that introducing an element of competition, either an explicit competition or through having a team element that implied competition, supports students' motivation.

In the study the students' goal orientation and self-efficacy also appeared to affect their motivation and therefore strategies that support self-efficacy or support students to understand how the project fits with their goal orientation will also support motivation.

9.3.6 Working Collaboratively

This study suggests that working collaboratively has the potential to support students' motivation, self-efficacy, and resilience. Here, I explore the suggested pedagogical strategies, of group composition and group roles, and contextualise these in the wider literature. The study also highlights that pedagogies supportive of self-regulated learning may also improve students' ability to collaborate effectively (discussed above).

Students and teachers reported that carefully constructed groups can support a highly motivating environment. This has been found by studies of PBL (e.g. Willis et al., 2002) and of group work in mathematics (Terwel, 2011). As highlighted in 4.8 the ideal composition of groups in PBL is not determined. One student argued that students should choose their own groups, so they feel 'comfortable' with the people they are working with. This is not supported by the literature, which suggests that when students work with friends they may be less disciplined, more inclined to talk off topic and, importantly in complexity terms, be less critical (Le et al., 2018). This student reported high engagement throughout the project. However, complexity thinking suggests for effective decisions, disagreement is better than compromise (Davis et al., 2006). It is possible, that had he worked with people he knew less well, he may have been exposed to more challenge and therefore made better decisions.

One student suggested that it might be helpful to allocate group roles, to support students' organisation through the project. I hypothesise that predefined roles may help students to distribute the tasks more easily. As discussed previously (Ch4), Nugraha et al. (2019) suggest that having allocated group leaders could help to create more equal workloads, whilst Cohen (1994) reported that a facilitator can increase both the communication and work rate of a group. Complexity thinking suggests that it is futile to impose group roles on students as *self-organisation* is a bottom-up phenomenon. Some of the students suggested that this *self-organisation,* at least around how they worked collaboratively, occurred and they reported naturally assuming different roles. However, I think that for students who are newer to this approach, group roles can help to provide *redundancy* by creating greater clarity around their shared responsibilities.

9.3.7 Conclusion

This exploratory study offers a contribution to the gap in the literature around effective support and guidance during PBL (Strobel & van Barneveld, 2009) and specifically the student perspectives of this (Grant, 2011). It signposts the support and guidance students reported to be effective, principally: having an interactive introductory session, exemplars, modelling, a checklist, support with planning and time management, starters to activate prior knowledge, clear mathematical expectations, carefully constructed groups, an acknowledgment that MaPBL supports the learning required for exams, and an authentic context that is familiar, interesting and provides a tangible output.

9.4 Chapter conclusion

Much of the literature addresses the impact of PBL on students' attitudes and the outcomes they achieve. This study adds to, and sometimes challenges, the literature in considering how students' affective traits and skills mediate their experience of MaPBL. It suggests that students' attitudes towards PBL, what they find challenging and the strategies they think their teachers can use to support them, are mediated by their self-efficacy, vision of mathematics and to a lesser extent their goal orientation. The findings of this study add to the current literature by finding that many of the phenomena experienced when teaching through other pedagogies in the mathematics classroom are also found in the MaPBL classroom. But it also surfaces some more negative responses to PBL, not usually found in the literature, and apparently rooted in students' nested eco-systems.

Very little research has explored the student perspective of PBL (Grant, 2011; Saunders-Stewart, 2008). Beckett (2005) argues that where research has considered the student opinion, it has often simplified the student perspective and has not tried to communicate the dilemmas that students face. This exploratory study offers some suggestions as to student perspective on MaPBL and as such contributes to this gap in the literature.

This interpretation is subjective and is mediated by my experiences, beliefs, and values. It is, though, supported by own professional experience, although being an insider researcher comes with its own tensions (5.5 and Appendix 24).

10 CHAPTER 10 – CONCLUSION

Here, I summarise the findings and highlight the possible contributions this study has made, consider further research that may be of value and the strengths and limitations of the study. I acknowledge the fundamental subjectivity of the research, representing one account of student views on MaPBL in the study school.

10.1 Summary of findings

In this study I analysed student accounts of learning through MaPBL with a focus on student-led learning, adding to our understanding of student perspectives on MaPBL. The study underlined the breadth and layers of challenge that students experience during MaPBL. I exposed how a performativity agenda can undermine the embrace of a MaPBL approach and suggested that some of the wider claims made about PBL may be more context specific than expected.

This study offers a contribution to knowledge in evidencing a wide range of student responses to MaPBL, many at an individual level. It finds that the MaPBL classroom is highly complex, even compared with a normal classroom. Importantly, it offers a unique contribution to knowledge in suggesting that students require a variety of affective traits and skills **before they can embark on** MaPBL in a productive way – but it is then very much worth doing. These affective traits and skills include self-efficacy, resilience, motivation, a relational vision of mathematics, self-regulated learning and working collaboratively. The thesis also evidences pedagogical strategies that were perceived to support these.

Each of the theoretical lenses I used, activity theory and complexity thinking, offered sensitisers to the different phenomena reported. They suggested some explanations as to why students experienced MaPBL differently and why they faced challenges. Activity theory was particularly helpful for drawing attention to the significance in the shift in *the division of labour* in PBL, whilst the concepts of *dynamic nested systems* and *emergence* in complexity thinking were particularly illuminating.

RQ1: What are students' attitudes to MaPBL, and in particular, to student-led aspects of that?

This exploratory study challenges much existing literature in suggesting that the students' attitudes, their beliefs, emotions, and behaviours, towards MaPBL were varied and dependent on the student's beliefs, the task, and the context they were working in.

This account suggests that whilst many students believed that MaPBL was an effective way to learn mathematics, this was not universally true: this belief was more likely to occur with students who have a relational vision of mathematics. Furthermore, phase one of the study suggests that it also depends on a student's self-efficacy and goal orientation. This adds to our understanding of the relationship between beliefs and MaPBL and suggests that teachers should attend to students' wider beliefs. I suggest students' beliefs about MaPBL may be highly contextual to this situation due to the students' desire for academic success. The study suggests *an* explanation for some student beliefs about MaPBL:

- Valuing autonomy was more likely to be reported by students with perceived high self-efficacy and a relational understanding of mathematics.
- Students with an instrumental understanding of mathematics appeared not to enjoy having autonomy for their own learning: they reported missing structure. This supports previous studies which found some students did not value learning through grouped problem solving and desired the structure that a textbook provides.
- Students with a perceived high self-efficacy and strong exam orientation reported not to value working in this way. They could not understand how it would be useful to them in passing their exams; this supports the literature about students' beliefs about coursework.

This study contributes to our understanding of student emotions during MaPBL: students experience a variety of emotions, that have the potential to change both within and between projects, as has been found in the mathematical problem-solving literature. This account adds to our understanding by suggesting that students' emotions during MaPBL could be mediated by their self-efficacy beliefs and vision of mathematics. The study found:

- A relational understanding of mathematics and mathematical self-efficacy both have a significant weak positive correlation with students enjoying MaPBL.
- Female and FSM students expressed more positive opinions about enjoying learning more independently in MaPBL.
- All the FG students reported feeling enjoyment towards working collaboratively, as is often found in the literature about group work.
- Some students with a relational understanding and perceived high self-efficacy also reported feeling pride and satisfaction in their work, as is found more generally in the literature.
- The student with perceived low self-efficacy felt anxiety, frustration, and a desire to give up.

This study informs our understanding of students' behaviour during MaPBL. It suggests it is influenced by their motivation and is dependent on a multitude of variables:

- Students appeared to demonstrate increased motivation when: the context allowed then to follow their own interests; they were able to work with others; the work was in keeping with their values; and they felt that they could succeed.
- Contrary to much of the literature, the study suggests that students did not always experience stronger motivation and engagement when completing PBL.
- Students with a lower self-efficacy, an instrumental understanding of mathematics or a performance orientation did not appear to report increased engagement when learning through PBL.
- This study supports the current literature on self-determination theory, in that the results of other studies on motivation are also found when studying MaPBL.

RQ2: What is challenging for students in leading their own learning during MaPBL?

This exploratory study raises important issues concerning the student experience of MaPBL and offers a contribution to the gap in the literature by providing a student perspective (Grant, 2011). It signposts challenges that students might experience during MaPBL. Phase one of the study suggests that the challenges that the students experienced were mediated by their self-efficacy, their vision of mathematics and their goal orientation. The aspects that students reported perceiving most challenging were:

- Being offered and embracing autonomy. The study supports the existing literature in suggesting that enacting PBL is hard for both teachers and students as it requires significant changes to practice.
- Making mathematical decisions. Further, as found in the literature, the students did not always realise the mathematical relevance of the work they were doing.
- Having sufficient self-regulated learning skills. This study supports the wider mathematics literature that has found that students with performance goals and low self-efficacy find self-regulated learning more complex to enact. This study suggests that the same appeared to be true in a MaPBL environment.
- Feeling motivated by the projects, especially for students with a strong exam-orientation. This study supports the current literature on self-determination theory, in that the results of other studies on motivation are also found when studying MaPBL.
- Having strong mathematical resilience. The study suggests that self-efficacy is a mediating factor in students' mathematical resilience. This supports the current literature that has found self-efficacy and resilience to be linked, by suggesting the same phenomena may occur in the MaPBL classroom.
- Ensuring equal participation. This accords with expectations and previous research about working collaboratively.
- Distributing the work amongst the team members. This is a potentially unique suggestion that may warrant further exploration as it highlights

what appears to be a previously unreported potential challenge of working collaboratively.

RQ3: What teacher strategies are perceived by students to support them with leading during MaPBL?

In accordance with expectations and previous research, the sample students appeared to need and welcome support for student-led learning during MaPBL. They reported valuing the offered autonomy, so whilst they wanted support and structure, they didn't want to lose this.

The study adds to a gap in the literature about how to support PBL through scaffolding and guidance (Strobel & van Barneveld, 2009) by suggesting that teachers should attend to the challenges highlighted under RQ2, as well as students' goal orientation and vision of mathematics.

It informs our understanding by providing a student perspective of the strategies that can be used to support student-led learning. Whilst this is one account, it is likely other students in a similar context would also find these pedagogical strategies helpful:

- Strategies to support self-regulatory learning, including having plenty of time to understand a task, being able to ask questions, having teachers model parts of the project, being given exemplars, having structured starters that re-capped prior knowledge, and having a checklist with suggested timings. Many of these strategies also supported students' self-efficacy.
- Strategies to aid their mathematical resilience, including working collaboratively, being aware of the specific help that they need and having a variety of ways to draw on support, being motivated by the project and through support from the teacher, and having a high self-efficacy or scaffolds that support their self-efficacy.
- Having different strategies to turn to for support.
- Having projects with familiar, authentic contexts and tangible outputs that help to provide intrinsic motivation and supports students' mathematical resilience.
- Strategies to support effective collaboration, including having carefully constructed groups and possibly allocating group roles.

- Strategies to support intrinsic motivation as suggested by self-determination theory including: contexts that allow them to follow their own interests; being able to work with others; having work was in keeping with their values; and they felt that they could succeed.

The study also:

- Supports the existing MaPBL literature in highlighting the need to ensure that students are being explicit about how the decisions they make are mathematical.
- Adds to the literature by suggesting that students with a strong exam orientation need to be aware of how the projects are related to the exam specification and should be supported to see the benefits of learning in this way.
- Supports the wider mathematical literature in finding that students with perceived high self-efficacy appear to show more mathematical resilience when completing MaPBL, as has been found to be true in the more traditional mathematics classroom.

10.2 Possible contributions

This exploratory study researches an under-represented area of the literature: the student perspective of mathematical, teacher-initiated PBL (Beckett, 2005; Condliffe, 2017; English & Kitsantas, 2013; Grant, 2011; Saunders-Stewart, 2008; Strobel & van Barneveld, 2009), It reports on student accounts of the challenges they face and practical strategies that support student-led learning in MaPBL, triangulated in a teacher workshop. I now consider the different facets of the possible contribution this study makes.

10.2.1 Theoretical

This study provides an account of MaPBL in a school where many students live in an intersection of cultures. It appears to confirm what other evidence tells, that a performativity culture can undermine the pursuit of widely valued medium-term goals for education (e.g. Ofsted, 2012) and, in the context of MaPBL, received performativity ethos and cultural background may serve to limit the embrace of the approach: students' responses to MaPBL may reflect the nested eco-system in which they work. This means that some of the wider claims made about PBL in the literature may be more context specific than acknowledged. The study also surfaces more subtle issues about how ’performativity’ may take different forms, with a different profile, within different self-efficacy contexts.

The literature has found that PBL impacts on students’ beliefs and MaPBL has the potential to impact their vision of mathematics. This exploratory study adds to our understanding in suggesting that students’ beliefs and vision of mathematics also mediate the way they experience MaPBL, the challenges they have and the pedagogical strategies they find supportive. Much literature discusses the outcomes of MaPBL on students affect. This study offers a unique contribution to knowledge in suggesting that students need a variety of affective traits and skills before they can embark on MaPBL in a productive way.

It challenges some of the wider claims in the literature that PBL is engaging for all students and points to a need to understand and engage with students’ views about the purposes of learning mathematics.

10.2.2 Methodological

The methodology used in this study is not uncommon in our field but suggests a minor contribution in finding complexity thinking a helpful lens for considering the student perspective.

10.2.3 Professional

A fuller discussion of the professional contribution can be found in the impact and reflective statements. In summary, this study has had a profound effect on my professional work and the work of the department. The department has created award winning projects and run training sessions on teaching through MaPBL for other schools. This work has been a product of the ongoing symbiotic relationship that exists between the literature I have read and shared, the research that I am doing and the conversations that I have had, both formally and informally, within the department. The department practices have developed because I was working on this research and the research has disturbed what might otherwise have happened and the attitudes and perceptions that might otherwise have been present.

10.2.4 Implications: schools

Undertaking MaPBL has been shown to be a complex endeavour and a significant change of practice for these comparatively well-placed teachers.

Teachers need to offer students autonomy as many students value and can embrace autonomy when supported. To enable such embrace, teachers need to attend to the challenges summarised above, giving particular attention to student beliefs about themselves, about the nature of mathematics, and their goal orientation. Teachers also need to consider how they gradually transition the *division of labour* from teacher to student via *tools*.

Whilst this is an account and these challenges and strategies for pedagogical support may be specific to the students in this school, it is likely that related challenges would be faced in another school embarking on this approach.

10.2.5 Implications: policy makers

Policy makers would likely say they value this way of learning. Each challenge that students reported is linked to a characteristic required for a student to become an effective independent user of mathematics. The study suggests that if we value that outcome, then we need to re-structure systems to support students in achieving this.

10.3 Further research

This is an exploratory study that represents one account of student perspectives of MaPBL in the study school. It provides tentative findings in answer to the RQs. As an under researched area, these RQs would benefit from further study with larger samples, over an extended period and in different contexts.

Previous research has reported on the outcomes of MaPBL, but not what students require to engage effectively in student-led MaPBL. The study raises important questions about the beliefs, affective traits, and skills that students require to be successful. This area is ripe for further study.

The study also revealed some specific areas that may benefit from further research. For example:

1. Understanding how to address the impact of the performativity culture which undermines the embrace of MaPBL and the pursuit of widely valued medium-term goals for education.
2. Greater theorisation of the skills that are required for student-led learning during PBL, often termed 21st century skills. This study would have benefited from being able to draw on wider research in this area.

3. What makes a project authentic to students?
4. Whether distributing work is a common challenge in working collaboratively as this study suggests.

10.4 Strengths and limitations

The Covid-19 pandemic caused serious limitations to the extent of the study, making it far more exploratory than initially envisaged. Most notably, the school closures curtailed the collection of data in the first phase and the necessary changes to pedagogy on return to school meant that the phase two data were collected a significant time after students had completed MaPBL. Nevertheless, the study provides an important function in that it prioritises student voice and allows the student perspectives of this pedagogy to be given a platform.

As an insider researcher, and a proponent of this pedagogy, there is a danger of me finding what I expected to see or looking for what I wanted to see. I repeatedly tried to find data and research that contradicted what I thought was happening. I also asked teachers to comment on student perspectives to get a wider professional response. Furthermore, as acknowledged under professional contributions, completing my doctorate fundamentally influenced what happened in the department.

As an exploratory study, that provides an account of the data, I make no case for the generalisability of these findings. However, I argue that they still have use and value: I would expect that other teachers introducing MaPBL and possibly PBL more widely in a similar context may find that students experience similar challenges and benefit from similar strategies to the ones highlighted by the students in this study. The study therefore provides a suggestion as to the considerations that teachers should make when embarking on teaching through this approach.

10.5 Concluding remarks

I embarked on my EdD journey as I had become dissatisfied with primarily teaching students to pass exams and wanted to focus on the use of alternative pedagogies. Completing my thesis has exposed me to a wealth of literature on PBL and mathematics learning and this has fundamentally shifted my practice.

My research and the literature have confirmed what I already thought, that teaching with MaPBL is a complex endeavour. However, they have also imbued a greater awareness of what I need to consider when teaching through this approach, and practical strategies to apply, so that I am a better PBL practitioner.

The EdD has also impacted my efficacy as a school leader (discussed in my impact statement). Completing a small-scale mixed methods research study has given me an excellent grounding in research and helps me to do much more comprehensive and trustworthy evaluations of in-school initiatives. I now read literature more critically and consider whether findings are likely to transfer into my context. Focusing on the qualitative paradigm has taught me the explanatory power of this research and has shifted how I would design future evaluations.

In short, the EdD has had a huge influence on my development, and I now consider myself an educator-researcher with the capacity for autonomous research. As a school leader this will be invaluable to my practice and the impact that I have within education.

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Post Script – Reflective Statement

My reason for embarking on a doctorate was simple: I wanted a personal accountability structure that would compete with the drive for results that I felt dominated my efforts in education, to ensure that I focused on what I felt mattered. What I didn't expect was a shift in my world view. My doctorate has been an incredible journey that has taken me on a path from a somewhat simplistic belief that truth resides in measurable facts, that there is a linear cause and effect, to understanding the world is made up of complex, nested and interconnected systems that exist in a state far from equilibrium. Writing this statement has provided me with an invaluable opportunity to reflect on my journey to becoming a researcher, and consider the developments in my thinking. The influences on me have been many and include constructivist grounded theory, activity theory and complexity thinking.

In this reflective statement I detail how my doctorate has supported my understanding of the current climate in education, my understanding of research, my ability to complete research and the impact on my personal trajectory.

Understanding the current climate of education in the UK

My first assignment, Foundations of Professionalism (FoP), fostered a deep level of thought about the context of education in the UK. It helped me to understand the pressures on teachers and schools and how these developed from the historical and political context. It highlighted to me how in a culture of performativity and accountability, teachers need a high level of professional autonomy to teach through PBL. I explored how the transition to project-based learning in the UK mathematics classroom requires a 'new professionalism' that is both active and ethical.

Whilst immersed in the literature for FoP I began to realise how teachers' beliefs impact the decisions they make about how to teach. I learnt that teachers' beliefs were likely to be influenced by the context they worked in, how they themselves were taught, and the wider community. Through my FoP assignment, it became clear that PBL requires a significant change in practice for teachers and needed to be supported with a high level of ongoing CPD. My first small piece of research, Methods of Enquiry 2 (MOE2) developed my understanding of beliefs and helped me to realise that people's espoused beliefs are not always enacted. This greater understanding of beliefs helped me acknowledge the influences on my own beliefs as an educator, and to understand how it could be complex for teachers to transition to teach with a new pedagogy.

More recently, my understanding of learning has deepened. This has helped me to understand (and begin to critique) the current dominant narrative in UK schools that privileges a knowledge curriculum. It appears that this is built on a cognitive constructivist view of learning. It has increased my concerns that learning is being viewed as only occurring within this paradigm.

Understanding the nature of research

In Methods of Enquiry 1 (MOE1), I planned a research study which I later used as the first draft of my plan for my Institutional Focused Study (IFS). I read widely around the methodological approaches that I could adopt. During this phase of my doctorate, I leant heavily towards Strauss and Corbin's (1995) approach to grounded theory. It was later, during MOE2, that I started to realise that my beliefs were more in keeping with those proposed by Charmaz (2006, 2014) in constructivist grounded theory: that I view my interpretation of data as subjective, and influenced by my own experiences, especially as an insider researcher. I now conceptualise my interpretations as offering an account of the data, grounded in time and context and having relevance for other researchers and educators in similar situations.

In writing my MOE2 assignment I began to develop a stronger conceptualisation of MaPBL, coming to define the characteristics in very similar terms to the ones that I used in my thesis. I used the literature to briefly highlight the benefits and challenges of this pedagogical approach. Reading back through this now illuminates how much my understanding of research has developed. There was no critique on the methodology or trustworthiness of the studies: I had little appreciation of the importance of considering reliability, validity and the contextual nature of research. I have now become much more discerning and have a stronger appreciation of the necessity of considering whether findings from other studies will transfer to my context.

My understanding of ethical considerations started in MOE1. This was especially important for me as an insider researcher, and particularly through my thesis as I was working with young people. I found the ethical tensions in research fascinating and continued to hold them in the forefront of my mind throughout my doctorate. The impact of Covid-19 and the subsequent changes to my methodology added further ethical complexities which I had to navigate.

My understanding of using theoretical frameworks has developed throughout my doctorate. In my IFS I completed a brief interpretation with activity theory, which allowed me to understand how using a theory can develop one's sensitivities to different phenomena in the data. It drew my attention to some of the less visible interactions between the participants (subjects), the rules and the community in the study school and specifically helped me to understand the conditions that supported an enactment of MaPBL.

Reading about complexity thinking for my thesis had a significant impact on my world view and resonated with my experiences in the classroom, especially the idea that we operate within nested systems which are far from equilibrium. These systems have the potential for feedback loops which can amplify change and tipping points which can radically shift a system. I was particularly drawn to the concept of emergence, and the potential for expansive change, where the space of the possible is expanded. I felt that the impact of using these theoretical lenses on my interpretation was significant and reflected that in a published paper.

Becoming an educator-researcher

I have benefitted hugely in this process from both the structure of the course and the wisdom and guidance of my supervisors. Working, for the most part full time, whilst completing my thesis has been a complex endeavour. I typically oscillated between term time educator and holiday researcher. The structure of writing three assignments and then a 'mini-thesis' supported my development and suited my somewhat sporadic way of conducting research. It allowed me to develop my understanding, research skills and self-efficacy, in bite sized chunks, before embarking on thesis.

I took a short sabbatical from work after collecting most of my data. Having four months focused on my research was hugely helpful as it offered me the time to really immerse into the different constructs I was discussing and to understand how my data supported and challenged the existing literature.

Throughout the process I was avid in my use of a research journal. I found it an effective mechanism for spotting when I put my own bias too heavily onto the data. My research journal highlights how I re-coded entire chunks of text after reflecting that I was seeing what I wanted or expected to see. Further, my research journal has allowed me to track the development in my thinking and to see how it has changed. This gives me a positive feeling that I grounded my coding within the data.

Completing lesson observations based on observable student actions rather than teacher effectiveness, was an important step in my own development: I began to focus my thinking on the student experience. In my career I have completed hundreds of lesson observations that have focused on teachers and what they are doing, but I have never prioritised the student experience. In my research journal, after the first lesson observations, I recorded how fascinating it was to observe a lesson from the perspective of a student. I reflected on how this impacted my perspective on teaching and learning as a professional and that this is something I should have done previously in my career.

My writing has slowly developed. Reading extensively aided this process considerably. Not only has this development in writing aided my thesis, but it has also been hugely beneficial to my professional work. As I finish my doctorate, I hope to engage in a significant way in online discussions around curriculum - not least, to provide a counter narrative to the dominant voices in education that currently privilege knowledge over development in skills and students' dispositions.

With the encouragement of my supervisor, I have presented at several conferences. This has been hugely developmental for my ability to articulate my work and to receive feedback from the wider research community. I have subsequently written two papers for BSRLM. This supported my learning in facilitating my understanding of writing for a purely academic audience and having a greater precision in my work.

My supervisors supported my growth along this journey in a multitude of ways, not least by modelling what an educator/researcher should be. My primary supervisor's passion for supporting students (including me), was inspiring. She had completed an EdD herself and understood from the outset how a linear path from taught courses to IFS to thesis would greatly ease the transition: a carefully chosen route through FoP, MOE1, MOE2, IFS thesis has meant that each step built on the work of the last, something that I am hugely grateful for.

Next steps:

Findings from the first phase of the data collection were fed back to some of the members of the department through a teacher workshop and, at a higher level, to the entire department in a department meeting. I changed schools before I had analysed the data from the second phase of the study. I still need to share these results with the department so that they can inform their enactment of MaPBL. I would also like to disseminate the findings of this research more widely, both within the mathematics education community and possibly the wider PBL community.

Education seems to go in 'fads', with schools often viewing the latest approach as being a panacea. Prediger's (2010) work on networking theories has helped me to deconstruct this notion and to understand the possibility of combining theories. It is of vital importance to me that there are voices that challenge the current dominant narrative in education in the UK (and what I view as its over-reliance on cognitive constructivism). I hope that the understanding of learning and education that I have gained from my doctorate will give me the grounding to do this effectively.

As I complete my doctorate, I want to initiate plans for a free school whose pedagogy and curriculum approach will be driven, at least partly, by the knowledge and understanding that I have gained throughout my studies. The school will be evidence led, data informed and based on the needs of the students and the community. It will, I hope, have a form of PBL at its heart – one that recognises and works with the attitudes that students arrive with, acknowledging that these will affect a student's experience.

APPENDICES

1 Appendix 1 – BSRLM PAPER

PBL (PBL) in the mathematics classroom: teacher embrace of facilitating student-led learning

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My thesis explores perceptions of students leading their own learning during PBL (PBL) in the secondary mathematics classroom in the UK. In this paper I describe the contributions of two theoretical lenses, activity theory and complexity thinking, to my interpretation of *teacher* embrace of facilitating student-led learning. I consider the role of a theoretical lens for interpretation, and outline how I view the two lenses as being complementary aids to help me gain understanding of an empirical phenomenon whose complexity may have been more difficult to grasp with only one lens. I describe each theory and detail their specific influence on my interpretation.

Key Words: PBL; PBL; student-led learning; activity theory; complexity thinking; theoretical lenses

1.1 The study

The research is being conducted in a small inner-city secondary school where I am an Assistant Head Teacher. Over the last few years, we have adapted the mathematics curriculum for 13 to 15 year old students so that they experience a hybrid of PBL (PBL) and more traditional teacher-led pedagogies. That a project is student-led is a defining component of PBL (Condliffe, 2017). The PBL projects that we use are designed to be taught as a complete unit, with students given the autonomy to navigate their own ways through the problem. Our designed approach is that students lead their own learning by: having choice in how they solve a problem and the methods they use; sequencing the tasks; allocating the tasks within the group; allocating the time within the overall time given and some choice in the way they present the outcome of their project. This doesn't mean there is no teacher input - quite the contrary: teachers should support and guide students throughout the process. This will include whole class discussions as well as intervening with individuals and groups.

In my earlier research, I found that participating teachers perceived one of their biggest challenges when teaching through PBL to be that of facilitating student-led learning. There is limited discussion in the PBL literature around student-led learning, with some of the large PBL organisations focusing on specific elements of student-led learning: 'student voice' and 'student choice'. A major review of the literature on PBL highlighted that many studies are not explicit about what student choice means in PBL, and that the issue of student choice is underdeveloped in the literature (Condliffe, 2017). The same seems to be true of the wider literature: I have found few studies that focus on student-led learning. To explore this area I developed the following research questions for my thesis:

> RQ1: What are students' attitudes to leading their own learning in this context?
> RQ2: What are students' perceptions of what is challenging in leading their own learning?
> RQ3: What strategies employed by teachers do students feel support them with leading their own learning?

This paper focuses on teacher embrace of facilitating student-led learning; how teachers develop and use pedagogies and strategies to foster, encourage and support the development of student responsibility and choice in the classroom. The study used a grounded approach with parallel data collection and initial coding. The data that I interpret in this paper is drawn from three lesson observations of students undertaking PBL in the mathematics classroom, two focus groups of students, and an interview with one of the teachers I observed.

1.2 What is a theory?

A theory is a series of statements about the relationships between phenomena. It can help to explain or to predict. Using a theory can help to guide interpretation: it may sensitise to consideration of previously unnoticed variables, or may make visible something that had previously only been sensed (Charmaz, 2014). It may provide a new way of looking at reality (Prediger et al., 2010) and it can alter your viewpoint (Charmaz, 2014). It is impossible to separate theory and practice, as they are symbiotic. Theory helps us to understand our practices more deeply and reflexively and as such will change our practice. Similarly our practice will influence how we use theory: theoretical frameworks develop and transform as our studies progress (Prediger et al., 2010).

1.3 What are the benefits and challenges of using multiple theoretical lenses?

I found many benefits of using multiple theoretical lenses. The phenomena that I am studying are complex, and it might have been difficult to grasp the complexities of what is happening with only one theory. Using different theoretical lenses offered me different ways of approaching the phenomena and encouraged me to take into account different things such as the students' social context or the beliefs and values of the teachers. In this way, using multiple lenses gave me complementary insights (Kidron, 2008). Using multiple theoretical lenses also helped me to develop and better understand my own emergent theories. As I entered into a dialogue and compared and contrasted my theories with other theories, I had a better understanding of my own theory.

The literature identifies challenges in using multiple theoretical lenses. The theories may have different kinds of dialogues, the same words might have different meanings, or what in one theory may be called 'epistemic,' might not seem to correspond to what is called 'epistemic' in another (Prediger et al., 2010). Similarly, the two theories might have different underlying assumptions which may lead to contradictory interpretations (Kidron, 2008). The theories may also use different units of analysis, which may prove challenging.

These challenges were not significant in my interpretation. There was overlap between the theories, however I did not attempt to integrate them locally into a new framework, but rather combined them to give complementary insights (Prediger et al., 2010). I found that the theories fed back on each other; my interpretation grew symbiotically with them both in an ongoing dialogue. Insights that I gained from one interpretation would aid and deepen the interpretation using the other lens.

1.4 Activity theory

Activity theory was developed by Vygotsky in the early 20th century to emphasise the social origins of the action of individuals (Engeström et al., 1999). Engeström developed a model of an activity system that built on the work of Vygotsky and others. It is often represented by a diagram made of three triangles. Figure 1 demonstrates this model being used to complete an activity theory analysis with the unit of analysis, the PBL classroom.

Activity theory is a top down deterministic theory: it views events as being determined by previously existing causes. It helps to reveal the social and material forces that are at play in a situation and supports attentiveness to the dialectic links between the individual and social structure. It is predominantly descriptive but can be explanatory and also developmental. An activity system is viewed as consisting of the following elements:

- the *subject(s)*from whose position and perspective the analysis is conducted, and the *object* motivating action and leading to an *outcome*;
- *mediating artefacts*, things that mediate action can include signs and tools, discursive practices and prior knowledge;
- *rules*, which can be visible or invisible such as beliefs and values;
- the *community*, the social group or environment in which the activity takes place;
- the *division of labour* which explores how the work is shared either horizontally, between people or people and tools in the community, or vertically, between people of power and status and others in the community (Engeström et al., 1999).

1.5 Interpretation through an activity theory lens

Activity theory helped me to notice the social context that teachers brought to the classroom that supported them to foster student-led learning. The *rules* of the department were supportive of teaching through PBL: the department believed in the value of PBL and students exercising autonomy and independence; the projects were mapped into the scheme of work, so there was an expectation that they would be used; and due to high exam results, teachers had the ‘earned autonomy’ to teach with innovative and progressive pedagogies. There was a *division of labour* between the members of the department, who sourced, wrote and adapted projects that they all used. The teachers also received support through the department as a *community*; they encouraged and supported each other to take risks and shared their experiences.

Considering the wider *community* of the school and the borough reveals some reasons why it may have been challenging for teachers to allow students to lead their own learning. The school’s most recent inspection report said that learners were often passive in lessons and lacked independence and whilst the borough has shown a huge improvement in academic results and has very high progress

8 scores, youth unemployment is one of the highest in the country. Seemingly, students appear to have academic skills, but lack the wider skills required for leading their own learning. One of the teachers reported that students became anxious when expected to lead their own learning, which made the teacher anxious about giving students autonomy. In this sense, the students' context influences teacher embrace of PBL.

When student-led learning appeared to work well in the classroom, the *division of labour* often shifted, at least partially, from teacher to resource. The teacher used tools such as writing frames or checklists to help give students structure to what they were doing. Whilst this labour did not shift onto the student, but rather the resource, transitioning the labour in this way may be a necessary step towards students leading their own learning. This idea is explored further in the complexity thinking interpretation.

When I completed my initial activity theory analysis, I was interested in what supported student-led learning and so I disregarded the two lessons I had observed where student-led learning had not taken place. However, when I considered the lessons through an activity theory lens, I realised that what I was viewing was a contradiction (McNicholl, 2013): the *mediating artefact*, in this case the project, was being used in a markedly different way to the one in which it was intended. The projects were used to give students an opportunity to apply their mathematics rather than as a PBL project, with student-led learning. Activity theory can provide transformational understanding when the researcher tries to resolve contradictions. As I wanted to explore this contradiction, I interviewed the teacher and discussed in more detail his embrace of student-led learning.

Activity theory thus aided my interpretation and also prompted me to collect further data. However, it didn't always enable me to access the subtle and wider changes that were occurring in the classrooms, which is why I looked to complexity thinking.

1.6 Complexity thinking

Complexity thinking is less well-established and is a collection of ideas rather than a single theory. It is not explanatory but primarily descriptive. It is influenced by a broad range of literature, with its origins in the sciences, systems theory, cybernetics and information science. It is being used increasingly to look at social areas including business, economics and education. Complexity thinking is based on a pragmatist philosophy: ‘truths’ that are viable, reasonable, relevant and contingent (or changing). It views the researcher as part of what they are trying to understand. It is useful for analysing contexts where there are multiple interactions between different aspects of the context, and at least some of the ‘players’ are conceived as having agency that they may exercise in ways that are not always predictable (Davis et al., 2006).

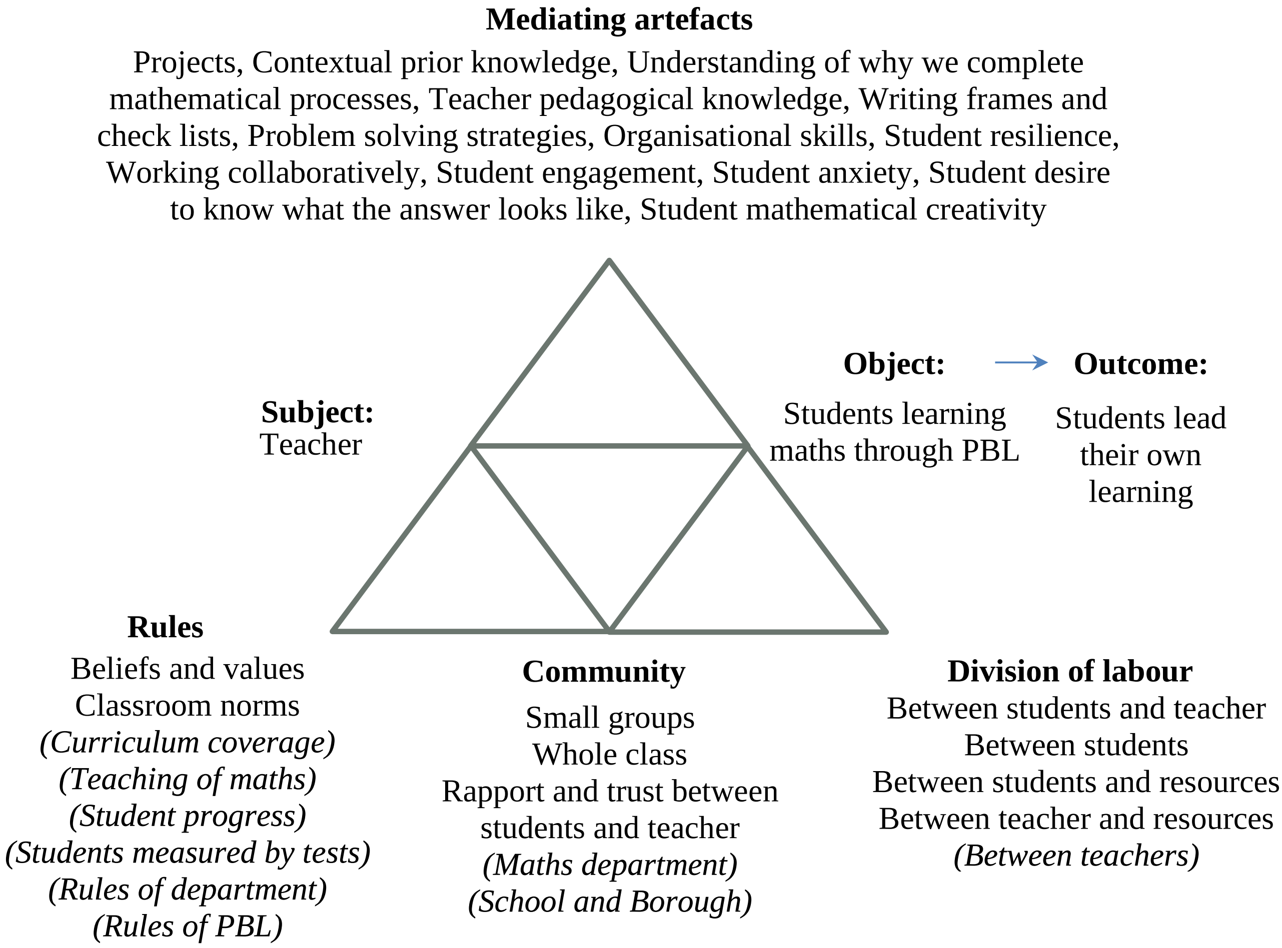

Figure 1 – An activity theory analysis of the PBL classroom. Elements that agents bring to the classroom with them are in brackets.

In complexity thinking, expansive change is defined as change that creates a radically wider range of possibilities than was there before; instead of perpetuating the status quo, the space of the possible is expanded. According to complexity thinking, expansive change is possible only when, amongst other things, there is:

- *internal diversity*, enough difference and variation between agents that there will be a range of possible responses;
- *sufficient redundancy*, a common ground between agents for example a common language, shared responsibilities or constancy of setting;
- *close neighbour interactions*, where the neighbours that must interact with one another are notions such as ideas, hunches and queries;
- *enabling constraints*, structural conditions that have enough coherence to ensure common purpose and enough randomness to ensure constant adjustment and adaption (Davis et al., 2006).

1.7 Interpretation through a complexity thinking lens

As analysed earlier, the teachers seemed to find facilitating student-led learning one of the most complex parts of PBL to enact. For a teacher to facilitate student-led learning, the students and the teacher require expansive change. Consideration of the conditions for expansive change helps to provide some suggestions for why in some classrooms, teachers demonstrated a more limited embrace of students leading their own learning during PBL.

The expertise about PBL came from within the department. The only external input came from reading research and, as noted previously, there is little literature about the student-led learning element of PBL. This led to limited *close neighbour interactions* between teachers and the concept of student-led learning. Further, many members of the department had trained within the department, whilst others had been at the school for a significant period of time; this may have created a lack of *internal diversity*. This lack of *close neighbour interactions* and *internal diversity* may have meant that there were similar notions of what made good PBL and a more limited pool of ideas about how teachers can support student-led learning.

Complex systems have *short range relationships* - complexity thinking suggests that, “the teacher must find ways to foster the local exchange of information” (Davis et al., 2006, p104). To support student-led learning, teachers would want to facilitate local exchange of information between students around how to lead their own learning. However, when I considered the discourse on student-led learning – it was very limited. Students and teachers had little vocabulary to discuss how students led their own learning. This lack of discourse around student-led learning may have constrained what could be said, thought and done. In the study school, the PBL projects were typically introduced to students in year 9 (their third year in the school). Whilst the students completed a series of activities at the beginning of the year to help build their skills, for the students who were observed, PBL was still a relatively new setting. In this way, it is possible there wasn't *sufficient redundancy*: the students and teachers didn't have a common language or consistency of setting. This lack of redundancy, or sameness, would have hindered interactions between agents would have meant that they were less able to compensate for another's failings.

The use of resources, such as a writing frame or checklist, was analysed as supporting students to lead their own learning. Through an activity lens they can be viewed as shifting the *division of labour*. From a complexity thinking perspective, they can be viewed as *enabling constraints*; they provided structure and gave a common purpose. They may also have created greater *redundancy*, as they provided a common frame of reference for students to work from.

Rather than viewing phenomena as having a cause and effect, complexity thinking suggests that the accumulation of small events will trigger a cascade of incidents. Complexity thinking helped me to consider the divergence from department-planned practice as being a series of incidents triggered by many previous events.

1.8 In summary

The adoption of two different theoretical approaches to interpretation of data allowed access to a wider range of likely factors contributing to limited enactment of student-led learning as originally conceptualised in department meetings. The value of identifying these possible explanations is they are now available for discussion in a teacher workshop where they can be further explored.

Acknowledgement: With thanks to Dr Jennie Golding and Professor Jeremy Hodgen.

2 Appendix 2 - Affective Traits

2.1 Resilience

Resilience is an affective ability that enables students to overcome difficulties and challenges (Hutauruk et al., 2019). It is supported by self-regulation. Johnston-Wilder et al. (2010) argue that a particular kind of resilience is needed in mathematics because of the type of teaching, the nature of mathematics and the pervasive beliefs about a person's 'fixed ability' in mathematics. They suggest that students require 'mathematical resilience': "maintaining self-efficacy in the face of personal or social threat to mathematical well-being" (Johnston-Wilder & Lee, 2019, p. 3). The need for mathematical resilience is highlighted by PISA 2012: 59% of 15 years olds across the countries in the study, reported experiencing worry about the level of difficulty in maths classes and 30% experienced feelings of helplessness when solving mathematical problems (Organisation for Economic Co-operation and Development (OECD), 2013).

Borman et al. (2004) describe how resilience can derive from factors that come from a student's environment, specifically, positive teacher – student relationships, a safe and orderly learning environment and a strong school community. They go on to argue that students will show a stronger resilience if they have high mathematics self-efficacy, a positive outlook to school, high self-esteem, and a high engagement with academic activities.

Mathematical resilience has been found to: address mathematical anxiety (Mackrell & Johnston-Wilder, 2020), lead to better problem solving as students are better able to coach themselves through difficulties (Attami et al., 2020), lead to better interactions between students hence a stronger ability to collaborate effectively in groups (Hafiz & Dahlan, 2017). Having a low resilience has been found to make students lazy and stops them from taking risks (Attami et al., 2020).

Suri et al. (2020) posit that there are four correlating factors that support the building of mathematical resilience. Firstly, students believe in the value of learning mathematics and that everyone can achieve. Secondly, students have a growth mind-set – they are prepared to struggle through the mathematics, believe in the value of working hard and acknowledge that mistakes can serve as learning opportunities. Thirdly, students have a belief in the possibility of growth – that it is possible to develop their abilities and find other solutions to a problem. Lastly that students realise that struggle and failure are all part of learning and that it is important to use support where required.

In summary, resilience has been found to support problem solving and group work, both of which are fundamental in MaPBL.

2.2 Motivation

Here I discuss two theories of motivation: achievement goal theory and self-determination theory. Achievement goal theory suggests that students can have a mastery goal, or a performance goal (Ames, 1992). Students with a mastery goal are viewed as wanting to develop competence, to understand new concepts and develop new skills. Students with a performance goal, are viewed as wanting to demonstrate competence, to do better than others or to surpass normative-based standards and to achieve with little effort. A student's goal orientation will affect how they learn; students with a mastery goal are typically more willing to engage in the process of learning, whereas those with a performance goal, are focused on achievement. Having a mastery goal is a necessary mediator of self-regulated learning as with a mastery goal a student will increase the length of time spent on tasks, show increased persistence when work is challenging and be more ready to take risk (Ames, 1992).

Ames (1992) highlights how a student's goal orientation can be influenced by the classroom learning environment, specifically the task, where the authority lies, and in how evaluation is conducted and recognition provided. She suggests that tendency towards a mastery goal will be enhanced if, amongst other things: teachers focus on the meaningful aspects of tasks and help students to develop short term goals to which they can self-reference; if the authority does not rest with the teacher, but students actively participate in the decision making and are supported with self-management and monitoring; and teachers focus on recognising effort, provide opportunities for improvement and try to ensure that evaluation is private not public.

Self-determination theory is a useful framework for considering students' actions and the support that teachers can provide to increase engagement (Prigmore et al., 2016b). Intrinsic motivation is key to self-regulated learning and further, awareness of motivation and emotion helps students to self-regulate (Järvenoja et al., 2019). Intrinsic motivation, according to Ryan and Deci (2000), occurs when students have their three basic needs met: the need for autonomy, to follow their own values and interests; the need for relatedness, to relate to other people and to feel that everything fits with the values that exist within their culture; and the need for competence, feel that they are able to be successful and meet their goals. When these needs are met, Deci and Ryan claim that this enhances motivation, learning and well-being.

In summary, according to achievement goal theory a student can have a performance goal or a mastery goal which impacts on their motivations. It will be interesting to consider students' goal orientations and how this impacts their motivation during MaPBL. The mathematics department has a focus on helping to develop mastery goal orientation in students, for example working on growth mind-set. However, the wider school culture and home environments are likely to be performance driven. Self-determination theory suggests that students will achieve their highest intrinsic motivation when they have autonomy, relatedness, and competence. MaPBL, as envisaged by the department, has the potential to meet all these needs.

2.3 Attitudes

Attitudes are widely viewed as being fairly sustained emotional responses that are reflected in what you say and do (Hannula, 2012). They are of moderate intensity and reasonable stability.

Hannula (2012) states that the most commonly used definition in mathematics education is based on that of Hart (1989): attitude is made of three components: beliefs, emotions and behaviour. I adopt this definition for this study and now focus briefly on each of beliefs and emotions.

2.3.1 Beliefs

Beliefs are generally perceived amongst researchers to have an important influence on learning (De Corte & Op't Eynde, 2002). They are considered to influence a student's decision making processes and hence, it is commonly assumed, impact on metacognitive skills and influence behaviours (Di Martino & Zan, 2011). Beliefs are fundamentally social; they are determined by the broad social historical context in which students are situated (Op't Eynde et al., 2006).

As discussed in Ch6 a common characteristic of belief-related research is the challenge in reliably accessing beliefs.

I now discuss students' beliefs about mathematics and students' belief in their ability to achieve success in a task, their self-efficacy, as these beliefs relate directly to my research focus.

2.3.1.1 Beliefs about the nature of mathematics

Skemp (1978) suggested that students have different beliefs about the nature of mathematics: relational or instrumental. Some students view mathematics as relational, as a connected body of knowledge that can be added to; they focus on understanding what to do and why, and especially, how that connects with and relates to what else they know. Other students view mathematics as instrumental, as something that can be learnt by rote, without understanding why things work, so long as they can 'do it' for a particular purpose such as pass an exam. Students with an instrumental understanding of mathematics are viewed as just wanting to learn the rules. I agree with Di Martino et al. (2010) who view this as an over-simplification: students don't necessarily have one understanding or another and the balance might vary across different areas of mathematics.

Since the mid-1980s mathematics educators have often used a different, but similar framework: Hiebert's 'conceptual' and 'procedural' knowledge (Lerman, 2014) and usually argued that students need both. Hiebert (1986) defines procedural knowledge as being made up of the formal language and symbolic representation and rules or algorithms. The procedural knowledge system is structured with an arranged hierarchy: procedures are made up of sub procedures. Hiebert defined conceptual knowledge as a 'connected web of knowledge' that is rich in relationships. He deemed students to have conceptual knowledge only when the student could recognise the relationship between this knowledge and other knowledge. However, Hiebert didn't view procedural and conceptual knowledge as being completely distinct. He felt that there was a critical relationship between them. Haapasalo et al. (2000) highlighted that most knowledge has both conceptual and procedural features and further that the distinction between them is both content and context dependent: what may be procedural for one student may be conceptual for another. I view procedural and conceptual knowledge as both being necessary in the mathematics classroom and in my experience of observing students in the classroom, they often appear to develop symbiotically.

In this study, I use Skemp's (1978) definition of 'instrumental' and 'relational' understanding of mathematics. Whilst Hiebert's 'conceptual' and 'procedural' knowledge is now more commonplace, I am interested in what students think acting mathematically is about, rather than what sort of knowledge they privilege, and therefore Skemp's work, linked to students' vision of mathematics, seems more appropriate. A student's vision of mathematics will also include beliefs about the role of both the teacher and the student and the social context. However, these do not form the focus of my study.

2.3.1.2 Self-efficacy

Self-efficacy as conceptualised by Bandura (1986) relates to a person's beliefs about how well they will perform in relation to a specific activity. It is situational or project specific (Pampaka et al., 2011). There is no consensus on how we measure self-efficacy in educational settings (Usher & Pajares, 2009) and a further complexity when assessing degree of self-efficacy is that students in developed nations seem to have a higher self-perception when judging their own mathematical ability and work rate (Askew et al., 2010). Askew et al. assumed that this comes from increased positive messaging from their teachers, who believe positive comments motivate students.

Self-efficacy and achievement in mathematics have been found to have a strong reciprocal relationship. For example, in a study by Sartawi et al. (2012), over 20% of students' variance in grades could be explained by the predictors of self-efficacy and motivation towards mathematics, in a bi-directional manner. Similarly Williams et al. (2010), in a study of 15 year old students across 33 countries, found that the relationship between self-efficacy and achievement in mathematics was usually direct and reciprocal. However, there was a cross cultural variation: for the UK, achievement impacted on self-efficacy and not the other way round.

The literature suggests that there is a significant link between self-efficacy and self-regulated learning. Coutinho et al. (2008) in their study of 629 undergraduate mathematicians found that self-efficacy was the strongest predicator of metacognition. Whilst these students were older than the students in the current study, similar findings have been made with younger students. For example, , Tian et al. (2018), in their study of 569 grade 10 students in China, found that metacognitive knowledge was mediated by self-efficacy. They argue that this was because self-efficacy creates a powerful motivation for self-regulated learning.

2.3.2 Emotions

Alongside other affective processes, emotions are viewed by social constructivists to be an integral part of learning (Op’t Eynde et al., 2006). Emotions are states of consciousness or feelings (Voica et al., 2020), that are highly contextual (DeBellis & Goldin, 2006) and that can be of limited stability.

Research in problem solving in mathematics is typically focused on cognition; much less research has focused on affect or the relationship between affect and cognition, despite the fact that emotions are viewed as affecting cognitive processes and are seen to play a key role in adaptation and decision making (DeBellis & Goldin, 2006; Di Martino & Zan, 2011). De Bellis et al. (2006) posit that this lack of research on emotions in problem solving is because it can be difficult to design reliable empirical studies of affect and because mathematics is often understood as a purely rational subject. Di Martino et al. (2011) highlight how challenging it can be to research emotions, and stressed the limitations of instruments such as interviews or questionnaires in attempting to capture feelings that may be unconscious and fleeting. Considering how to access the emotions that students experience during MaPBL will be an important consideration for the methodology of this study.

Emotions are typically viewed as being positive or negative; not succeeding with a goal typically leads to negative emotions and succeeding with a goal typically leads to positive emotions (Hannula, 2014). However, Pekrun et al. (2007) posit that emotions not only have a positive and negative dimension, but they also have an activation dimension. For example, relief or relaxation are positive deactivating emotions, whilst boredom is a negative deactivating emotion. Feeling relief, relaxation or boredom may mean that a student does not engage as fully with the work as they would if they felt activating emotions, such as fun, enjoyment - or frustration. The positive activating emotions are more likely to enhance performance as, Pekrun et al. argue, they may lead to increased interest in the study, increased effort or increased self-regulation. Similarly, negative activating emotions such as frustration can also be supportive of learning. For example DeBellis et al. (2006) describe what they view as an idealised pathway for heuristics in terms of emotions. They suggest that a student should start with curiosity and puzzlement which inspires exploratory work in trying to find heuristics, which will motivate them to better understand the problem. This can then lead to bewilderment as the student reaches a cognitive impasse and a lack of insight. Following this, the student may feel frustration which will inspire strategic re-thinking. When successful, this re-thinking will result in pleasure, elation, and satisfaction. In this way frustration is not viewed as negative, but as a sign to modify the strategy which aids the problem-solving process. As discussed above, the students in the study school are often viewed as passive by the teaching staff. It is possible, therefore, that the students in this study may be more likely to experience more deactivating emotions than activating emotions.

Positive emotions do not imply that students will attain more highly. Askew et al.'s (2010) study found that the relationship is complex, and indeed, can vary between different contexts or cultures: when making between-country comparisons, they found mathematical enjoyment and mathematical attainment to be negatively correlated, though some in-country studies did not show such a relationship.

Students' ability to control their emotions is discussed in the literature as meta-affect. Few studies have addressed how students regulate emotions in class (Hannula, 2014). DeBellis et al. (2006) posit that it is better for a teacher to foster students' meta-affect, than it is to provide activities that directly alleviate frustration, fear or anxiety. They suggest that if teachers can create a safe space where it is acceptable for students to make mistakes, then these negative emotions can have positive outcomes. Hannula (2012) found that whilst all individuals experience similar emotions during problem solving in mathematics, experts are better able to control their emotions.

It is assumed that emotions influence learning and that learning influences emotions: there is a reciprocal causation between emotion and learning and this create feedback loops (Pekrun et al., 2007). Emotions are assumed to influence motivation, cognitive resources, use of strategies, and self-regulatory strategies

(Pekrun et al., 2007). The effects of emotions on achievement are assumed to be moderated by these factors. Achievement, in turn acts back on emotion.

In this study it will be important to consider emotions carefully and to remember that negative emotions can have positive outcomes.

3 Appendix 3 – Initial literature search

The initial literature search was conducted using BEI (British Education Index), ERIC, SCOPUS and the Web of Science core collection. Search terms were chosen both from the literature and also by using the thesaurus on BEI and ERIC. In keeping with Boote and Beile's (2005) idea of a systematic and thorough literature review I now justify why research has been included or excluded from the review. "Student projects" (ERIC), "project method" (BEI) or "PBL" (Scopus and Web of Science) were searched alongside the following search terms: "student voice", "student choice", "autonomy", "student-led", "student-driven", "self-management" "self-culture", "self-regulated", "self-determination" (or another appropriate term found in the database thesaurus). Only studies of students of secondary school age were included. This gave 186 abstracts to read. On reading the abstracts, papers were also excluded for the following reasons: not about PBL, or with a significantly different definition of PBL; PBL delivered entirely online; projects where there was no academic curriculum focus for example about PE, outdoor learning, community service, crime or the environment. 73 abstracts were viewed as being relevant. 9 could not be found and 15 were duplicates. The remaining literature, 49 papers (appendix 4.1), was then read and the bibliographies checked for further literature of interest. The Interdisciplinary Journal of Problem-Based Learning was also used to identify further appropriate literature (appendix 4.2).

A second search was done after coding had commenced. Searching with the terms "PBL", or thesaurus defined equivalent, and "resilience" in Scopus, web of science, BEI and ERIC, yielded only 47 results. All but two of these related to higher education and were discarded. Of the remaining papers, one was in Spanish, and one could not be found. Two other papers about PBL and resilience were found when exploring the literature on mathematics and resilience, the findings of which I discuss in my thesis (4.5).

ERIC							
PBL and…	**Initially**	**Reduced to**	**Reasons for eliminations**	**Selected after abstract**	**Not found**	**Already found**	**Saved**
Independent study	229	10	AND secondary education NOT ICT NOT homework projects (also eliminated: independent research projects where students work on them individually)	6	4	0	2
Personal autonomy	84	18	AND secondary education or high schools (also eliminated: PE and community service)	5	0	1	4
Self management OR self care OR self regulation OR self monitoring	63	25	AND middle school/secondary school/ high school (also eliminated: engineering, different project definitions, ICT, peer teaching, anxiety)	8	0	1	7
Student responsibility	41	10	AND middle school/secondary school/ high school (also eliminated: crime, University, environment)	4	1	2	1
Student attitudes/ perceptions/ thoughts/ beliefs	1076	13	AND middle school/secondary school/ high AND Mathematics (also eliminated:Not PBL, conference proceedings)	4	0	0	4
TOTAL	**1493**	**76**		**27**	**5**	**4**	**18**
Project method in teaching and…	**Initially**	**Reduced to**	**Reasons for eliminations**	**Selected after abstract**	**Not found**	**Already found**	**Saved**
Independent study	3	3	Self assessment centres or teacher focus	1			1
Learner autonomy	1	1	Field work	0			

Self management OR self care OR self regulation OR self monitoring	0						
Responsibility in children	0						
Student attitudes/ perceptions/ thoughts/ beliefs	87	22	AND High School (also eliminated: attitude to subject not PBL, too young)	3		1	2
Self directed learning/ self directed	1	1		1			1
TOTAL	**92**	**27**		**5**	**0**	**1**	**4**
BEI							
Student Projects and…	**Initially**	**Reduced to**	**Reasons for eliminations**	**Selected after abstract**	**Not found**	**Already found**	**Saved**
Independent study	24	9	Lifelong learning 16 – 19 year olds	5	1	1	3
Student-led learning	0	0					
Student choice	1	0	NOT Higher education				
Student voice	29	16	NOT Governance (many ignored as not to do with PBL)	5		1	4
Student attitudes	2	0	NOT university NOT A level				
Self directed learning/ self directed	5	1	NOT university OR computing				
Self management OR self care OR self regulation OR self monitoring	3	0					
Personal autonomy	0						

Self regulated	3	3		3			3
Student responsibility	6	0	NOT university				
TOTAL	**73**	**29**		**13**	**1**	**2**	**10**
Scopus							
PBL and…	**Initially**	**Reduced to**	**Reasons for eliminations**	**Selected after abstract**	**Not found**	**Already found**	**Saved**
Student voice	5	0	Wrong age group				
Student choice	6	0	Wrong age group/ detsinations/ technology				
Student decision making	0	0					
Autonomy	73	16	NOT higher education, university, engineering, computer science, ESL or EFL	4	1	0	3
Student-driven	?	5		5	1	2	2
Student-led/ student-led	19	4	Wrong age group	2	0	1	1
Self management/ Self-management	12	5	NOT engineering NOT higher education	2	0	1	1
Self-culture/ self culture	0						
Self regulated	32	1	AND secondary school	0			0
		4	AND high school	2			2
Self determined							
Voice and choice	0						
TOTALS	**147**	**35**		**15**	**2**	**4**	**9**
Web of Science							
PBL and…	**Initially**	**Reduced to**	**Reasons for eliminations**	**Selected after abstract**	**Not found**	**Already found**	**Saved**
Student voice	3	0	NOT Higher education				
Student choice	2	0	NOT Higher education NOT ICT				

Autonomy	74	0	AND Secondary NOT ICT				
		5	AND High School NOT ICT	4	1	1	2
Student-driven	2	0	NOT Higher education				
Student-led/ student-led	11	3	NOT Higher education	2		1	1
Self management/ Self-management	8	0	NOT Higher education				
Self-culture/ self culture	0		NOT Higher education				
Self regulated	23	0	AND secondary NOT ICT				
		1	AND high school	1	0	0	1
Self determined	9	6	NOT Higher education	5	0	1	4
Voice and choice	2	1	NOT Higher education	1	0	1	0
TOTALS	**134**	**19**		**13**	**1**	**4**	**8**

4 Appendix 4 – Literature included in the literature review

The papers below were all read, however only the findings that were deemed relevant to the study were included in the literature review. For example some studies were with older students and hence the findings were less likely to transfer into the context of the study school.

4.1 Literature from the database search

1 Baars, M., van Gog, T., de Bruin, A., & Paas, F. (2017). Effects of problem solving after worked example study on secondary school children's monitoring accuracy. Educational Psychology, 37(7), 810–834.

2 Barak, M. (2012). From 'doing' to 'doing with learning': Reflection on an effort to promote self-regulated learning in technological projects in high school. European Journal of Engineering Education, 37(1), 105–116.

3 Boote, D. N., & Beile, P. (2005). Scholars Before Researchers: On the Centrality of the Dissertation Literature Review in Research Preparation. Educational Researcher, 34(6), 3–15.

4 Brosnan, P., Erchick, D. B., & Flevares, L. (Eds.). (2010). Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Columbus, OH: The Ohio State University.

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12 Corsi, G. (2010). Studying the effects of a non-traditional instructional method in the high school science classroom. The Science Teacher, 77(7), 58-62.

13 Habók, A., & Nagy, J. (2016). In-service teachers’ perceptions of project-based learning. SpringerPlus, 5(1).

14 Hann, T. (2019). Investigating the impact of teacher practices and noncognitive factors on mathematics achievement. Research in Education, 108(1), 22-45

15 Harmer, N., & Stokes, A. (2016). “Choice may not necessarily be a good thing”: Student attitudes to autonomy in interdisciplinary project-based learning in GEES disciplines. Journal of Geography in Higher Education, 40(4), 531–545.

16 Hsu, P.-L. (2019). “It’s the magic circle”! Using cogenerative dialogues to create a safe environment to address emotional conflicts in a project-based learning science internship. Cultural Studies of Science Education. 15(1), 75-98

17 Kaldi, S., Filippatou, D., & Govaris, C. (2011). Project-based learning in primary schools: Effects on pupils’ learning and attitudes. Education 3-13, 39(1), 35–47.

18 Katz, I., & Assor, A. (2007). When Choice Motivates and When It Does Not. Educational Psychology Review, 19(4), 429–442.

19 Koparan, T. (2014). The Effect on the 8th Grade Students’ Attitude towards Statistics of PBL. European Journal of Educational Research, 3(2), 73–85

20 Lam, S., Cheng, R. W., & Ma, W. Y. K. (2009). Teacher and student intrinsic motivation in project-based learning. Instructional Science, 37(6), 565–57

21 Langer-Osuna, J. M. (2015). From Getting “Fired” to Becoming a Collaborator: A Case of the Coconstruction of Identity and Engagement in a Project-Based Mathematics Classroom. Journal of the Learning Sciences, 24(1), 53–92

22 Laughlin, P. R., & Ellis, A. L. (1986). Demonstrability and social combination processes on mathematical intellective tasks. Journal of Experimental Social Psychology, 22(3), 177–189.

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5 Appendix 5 – The classrooms and the participants

5.1 Lesson Observation 1 (LO1)

The first project to be taught after I received ethical approval was Amazon Trader. I chose to observe Mr Drew’s class as he reported feeling that the students were working reasonably well on MaPBL and was happy for me to observe. I hoped that if students were engaged in leading their own learning this would work as a “telling” case (Mitchell, 1984): for the lesson observation to be of value, students would need to be leading their own learning to some extent. Three students wanted to participate and brought in signed parental consent forms. I observed all three of these students.

Information about the lesson is in Table **5-1** and the resources used are in appendix 7.

Project Name	**Amazon Trader**		**Lesson**	**5/5**	**Year**	**10**
Gender	Females	14	Males	11	TOTAL	25
Prior attainment	High	3	Mid	18	Low	2
Other characteristics	SEN	4	Pupil premium	10	Not known	2
Lesson objective (taken from department lesson plan)	- To encourage learners to think of the mathematics of running a business. - For students to be able to perform costing calculations, best buys, exchange rate conversion, percentage increase (VAT), profit, percentage profit margin and optimisation of stock with demands.					
Level of student-led learning observed	Students were observed to be working largely independently, with students choosing the products they want to sell. Mr Drew had created an option for choice around the methods they were using, the sequencing of the methods and how they distributed the roles in the group. The two groups observed were using the same sequencing of activities, namely costing each product individually. On checking with the teacher, this method had been modelled in a previous lesson. Mr Drew intervened briefly with two whole class discussions, one to remind students of what they should be aiming to produce (a business report) and one to alert students to a common error around delivery charges. Apart from these whole class interventions, the teacher supported the individual groups.					
Student experience of mathematics	The students seemed mathematically able, calculating things such as percentages was routine for the students. However, they struggled with some of the project related concepts such as VAT or the Amazon fulfilment fee.					

Table 5-1: Information about LO1

5.2 Focus Group 1 (FG1)

Two out of the three students who had been observed in LO1 came to FG1.

5.3 Lesson Observation 2 (LO2)

The cake bake project was the next project to be taught in the department. I chose to observe this class as, similarly to Mr Drew, Mr Jafri reported thinking that the students were working reasonably well on PBL and was happy for me to observe. Five students who wanted to participate had also brought in signed parental consent forms. I observed all five of these students.

Information about the lesson in in Table **5-2** and the resources used are in appendix 8.

Project Name	**Cake Bake**		**Lesson**	**6/6**	**Year**	**9**
Gender	Females	12	Males	13	TOTAL	25
Prior attainment	High	6	Mid	20	Low	0
Other characteristics	SEN	1	Pupil premium	3	Not known	2
Lesson objective (taken from department lesson plan)	- Calculate timings of recipes - Calculate costs of recipes - Calculate profit and loss - Scale recipes up					
Level of student-led learning observed	Mr Jafri demonstrated each aspect of the project, for example changing the number of people that the cake would be suitable for. Students then completed these mathematical processes with the cakes that they had chosen. The class were given clear timings to work on each section of the project. There was some element of choice within this, for example students were able to choose which cake					

	to bake, or which criteria mattered to them. However, they were not encouraged to choose their own route through the project or develop their own methods for solving the problem.
Student experience of mathematics	The students seemed able mathematically and conceptually. Mr Jafri broke the project down into constituent parts and taught each part separately, thus meaning that the students were asked to complete a series of application of mathematics questions.

Table 5-2: Information about LO2

5.4 Interview 1

As I began to analyse the data, I became intrigued as to why Mr Jafri had not offered the students more autonomy. I invited Mr Jafri to be interviewed to discuss the lesson in more detail. During the interview I was able to probe him to understand more fully why he hadn't offered students more autonomy.

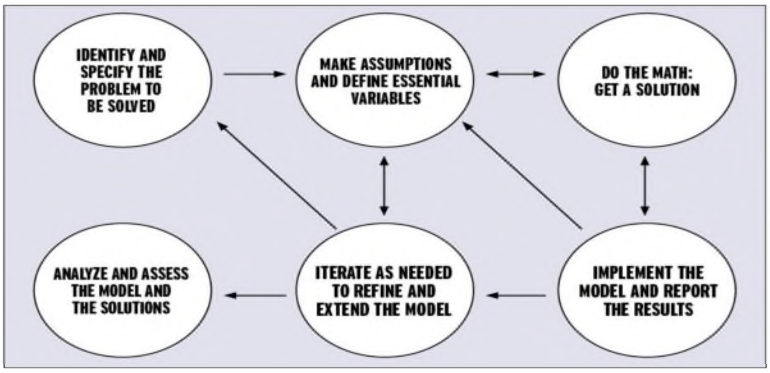
5.5 Focus Group 2 (FG2)

At the time I did not see the value in having a FG with the students from LO2. I had wanted to discuss how the students lead their own learning. I had deemed this limited in LO2. Further into analysis, I realised that even though offered autonomy was limited, students had sometimes taken autonomy and I regretted not completing this FG.

I wanted participants who had been offered the opportunity to lead their own learning, so I invited my year 11 top set. They had been taught MaPBL by another teacher in year 9 and me in year 10. Eight students out of a class of 26 participated in FG2.

5.6 Lesson Observation 3 (LO3)

To provide greater variety, I chose a lesson with students of lower prior attainment who were towards the beginning of a project. Information about the lesson is in Table **5-3** and the resources used are in appendix 9.

<table>
<tr><td>Project Name</td><td colspan="2">Family support in a micro society</td><td>Lesson</td><td>2/5</td><td>Year</td><td>10</td></tr>
<tr><td>Gender</td><td>Females</td><td>6</td><td>Males</td><td>6</td><td>TOTAL</td><td>12</td></tr>
<tr><td>Prior attainment</td><td>High</td><td>0</td><td>Mid</td><td>1</td><td>Low</td><td>7</td></tr>
<tr><td>Other characteristics</td><td>SEN</td><td>4</td><td>Pupil premium</td><td>4</td><td>Not known</td><td>4</td></tr>
<tr><td>Lesson objective (taken from department lesson plan)</td><td colspan="6">To learn how to use the modelling process as outlined in the diagram below:

</td></tr>
<tr><td>Level of student-led learning observed</td><td colspan="6">The lesson was heavily structured, and students attempted to work in small groups on specific tasks. In this lesson, students were not seen to lead their own learning in any significant way.</td></tr>
<tr><td>Student experience of mathematics</td><td colspan="6">In the first lesson of the project, prior to the observed lesson, the students had designed their own ‘families’. Mr Robinson started the lesson by explaining the overall purpose of the project: that students would need to find a way of fairly distributing income support to these families, and what the students would need to do. Mr Robinson then introduced a different modelling concept, that of distributing grades, for students to consider.

Mr Robinson had written on the board what the students needed to do and it had been discussed as a class.</td></tr>
</table>

	However, the students did not understand the task and therefore could not access the mathematical elements. This lack of understanding of what they were being asked to do made it impossible for the teacher to then go on to draw out the elements of modelling that were the aim of the lesson.

Table 5-3: Information about LO3

Again, I did not conduct a focus group with these students as I hadn't seen them lead their own learning and therefore, at this point of the research process, deemed it irrelevant. Similarly to LO2, on reflection I regretted that I had not carried out the FG.

5.7 Survey 1

The school closures meant that the first surveys were completed remotely. This added to the already significant challenges of low response rate and subsequent non-response bias often found with surveys (Roberts & Allen, 2015). To aid ease of access, and to attempt to increase the response rate (Roberts & Allen, 2015), I posted a copy of the survey home as well as putting it online. Participants who feel invested in the research, for example through personal contact, are more likely to respond (e.g. Golding, 2021), so I asked tutors, who have a relationship with the students, to discuss the research with students during routine phone calls. The school also sent out reminder text messages to parents. Despite all these measures, the response rate was still very low (13%, 47/360). The low response rate meant that the participants were unlikely to be representative: it is probable that they were some of the most motivated students during lockdown.

5.8 Survey 2

414 students in years 8 to 11 completed the survey as part of routine SSE. 151 returned a parental permission slip for their responses to the survey to be used in my research (year 8 and 9) or reported they had spoken to their parents about the research (year 10 and 11). This low rate of parental consent was disappointing but is reflective of a common challenge in the school. I made every effort within the bounds of what myself and my supervisor considered

ethical, to boost the rate of permission, including regular reminders to students and text messages to parents.

134 out of the 151 (89%) also gave consent for their responses to be linked to their background data. The contextual data about these 134 students (Table **5-4**) shows that they were largely representative of the wider school population, however the participants were slightly skewed towards female, non-school free school meal students without an SEN.

	Gender		Prior attainment			SEN		FSM
	Male	Female	Low	Mid	High	Support	EHCP	
Sample	41%	59%	18%	50%	32%	13%	2%	50%
School	48%	52%	15%	50%	35%	17%	4%	60%

Table 5-4: Contextual data about Survey2 participants

5.9 Summary

This appendix provides details about the classes I observed and the participants involved. I hope that providing description enables the reader to see if it is likely that the results of this study will be generalizable into their context.

6 Appendix 6 – Lesson observation schedule

<table>
<tr><td colspan="4">Observation
Any interpretations were marked as '?' and subject to later probing where warranted.</td></tr>
<tr><td>Name of observer</td><td></td><td>Name of teacher</td><td></td></tr>
<tr><td>Date of observation</td><td></td><td>Time</td><td></td></tr>
<tr><td>Name of project</td><td></td><td>Lesson number/how many lessons</td><td></td></tr>
<tr><td colspan="4">Notes from brief chat with teacher about where students are up to in the project</td></tr>
</table>

Codes	
Student Choice	
SC.TF	Student choice of timeframe
SC.M	Student choice of methods
SC.SA	Student choice of sequence of activities
SC.AT	Student choice of allocation of tasks
SC.O	Student choice of outcome
Collaboration	
C.DT	Collaboration – delegation of tasks
C.DI	Collaboration – discussing ideas
C.CI	Collaboration – challenging ideas
Teacher intervention	
TI.WD	Teacher intervention – whole class discussion
TI.GD	Teacher intervention – group discussion
TI.OD	Teacher intervention –one to one discussion
TI.SE	Teacher intervention –showing exemplars
TI.SA	Teacher intervention –showing assessment rubric
TI.PG	Teacher intervention – encouraging peer to peer support within group
TI.PBG	Teacher intervention – encouraging peer to peer support between groups
TI.WS	Teacher intervention –giving a written scaffold
Teacher instruction that facilitates	
TF.TF	Teacher instruction that facilitates student choice of timeframe
TF.M	Teacher instruction that facilitates student choice of methods
TF.SA	Teacher instruction that facilitates student choice of sequence of activities
TF.AT	Teacher instruction that facilitates tudent choice of allocation of tasks
TF.O	Teacher instruction that facilitates student choice of outcome

Work that illustrates	
WI.TF	Work that illustrates student choice of timeframe
WI.M	Work that illustrates Student choice of methods
WI.SA	Work that illustrates Student choice of sequence of activities
WI.AT	Work that illustrates Student choice of allocation of tasks
WI.O	Work that illustrates Student choice of outcome
Engagement	
E.H	High level of engagement
E.M	Mid level of engagement
E.L	Low level of engagement
E.OT	Off task
Level of mathematics being completed	
M.PS	Students are completing mathematical problems
M.R	Students are completing mathematical activities that appear routine for them
M.S	Students are completing simple mathematics
NM.R	Students are completing work that is not mathematical, but has relevance to the project
NM.NR	Students are completing work that is not mathematical and not intrinsic to the project
Students seeking help	
H.W	Help sort from within their group
H.A	Help sort from another groups
H.T	Help sort from the teacher
H.R	Help sort from another resource eg text book.

What level of student-led learning is being demonstrated

SLL - 1	SLL-2	SLL-3	SLL-4
Students are fully engaged in the task. They are choosing how, why and when they will complete the different activities. The work that they are doing is challenging mathematically	With some intervention the students are completing the project	With significant help and scaffold the students are completing the project.	Little evidence of student-led learning seen.

De-brief – does the teacher describe the lesson as typical?

Key Words (to form as triggers for students in focus group)

Date of lesson		Time Block	0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80 80-90 90-100
What level of SLL are students exhibiting? Student 1: Student2: Student 3:			
Student Choice			
SC.TF			
SC.M			
SC.SA			
SC.AT			
SC.O			
Engagement			
E.H			
E.M			
E.L			
E.OT			
Collaboration			
C.DT			
C.DI			
C.CI			
Work			
WI.TF			
WI.M			
WI.SA			
WI.AT			
WI.O			
Mathematics			
M.PS			
M.R			
M.S			
NM.R			
NM.NR			
Teacher intervention			

TI.WD
TI.GD
TI.OD
TI.SE
TI.SA
TI.PG
TI.PBG
TI.WS

Teacher instruction

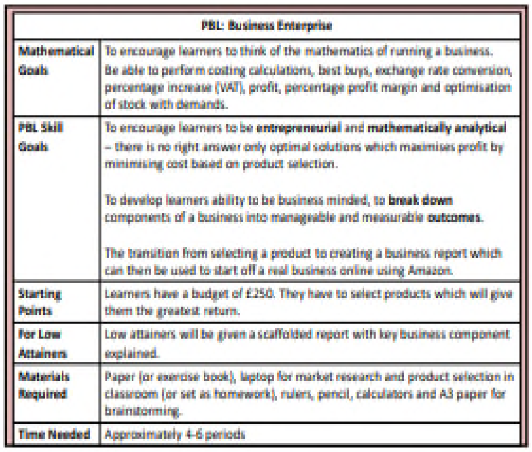
TF.TF
TF.M
TF.SA
TF.AT
TF.O

Students seeking help

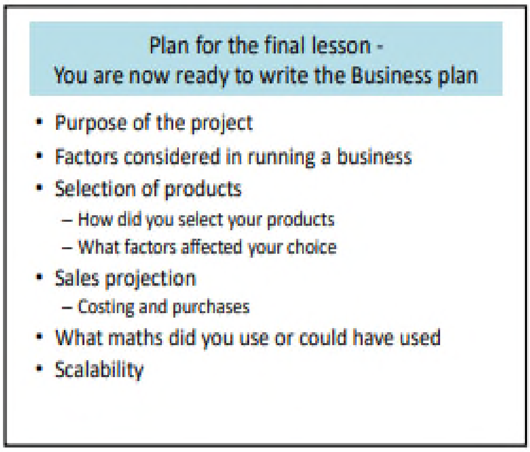
H.W
H.A
H.T
H.R

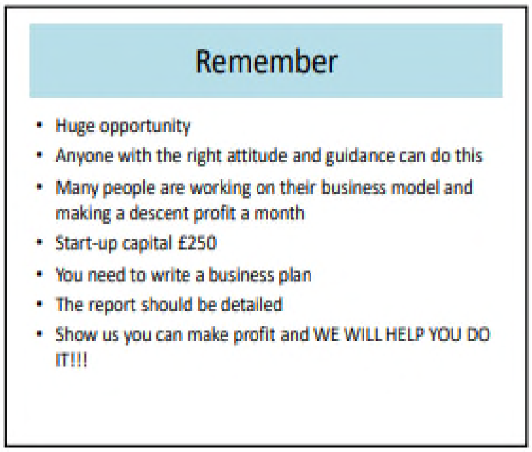
7 Appendix 7 - Resources from L01

Powerpoint slides from the shared departmental drive for the last lesson of Amazon Trader.



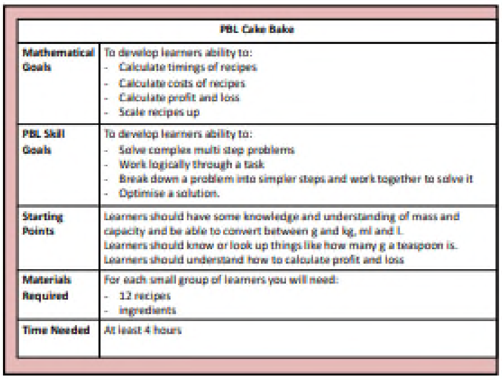
PBL: Business Enterprise	
Mathematical Goals	To encourage learners to think of the mathematics of running a business. Be able to perform costing calculations, best buys, exchange rate conversion, percentage increase (VAT), profit, percentage profit margin and optimisation of stock with demands.
PBL Skill Goals	To encourage learners to be **entrepreneurial** and **mathematically analytical** – there is no right answer only optimal solutions which maximises profit by minimising cost based on product selection. To develop learners ability to be business minded, to **break down** components of a business into manageable and measurable **outcomes**. The transition from selecting a product to creating a business report which can then be used to start off a real business online using Amazon.
Starting Points	Learners have a budget of £250. They have to select products which will give them the greatest return.
For Low Attainers	Low attainers will be given a scaffolded report with key business component explained.
Materials Required	Paper (or exercise book), laptop for market research and product selection in classroom (or set as homework), rulers, pencil, calculators and A3 paper for brainstorming.
Time Needed	Approximately 4-6 periods



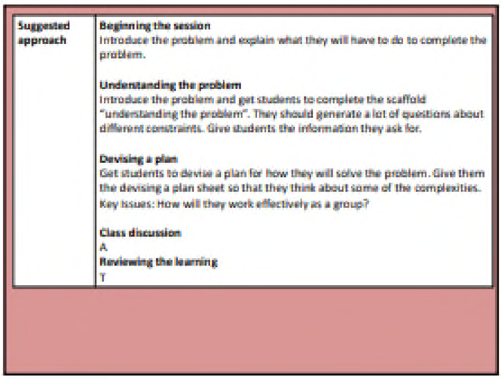


8 Appendix 8 - Resources from LO2

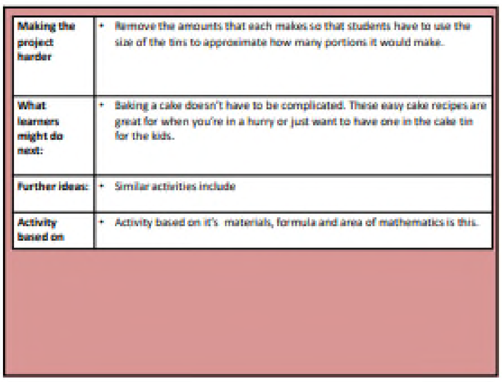
Powerpoint slides from the shared departmental drive that introduce Cake Bake.



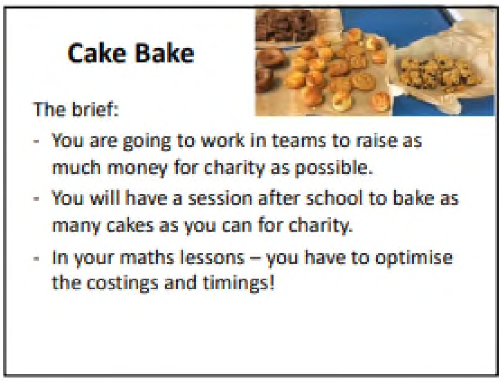
PBL Cake Bake	
Mathematical Goals	To develop learners ability to: - Calculate timings of recipes - Calculate costs of recipes - Calculate profit and loss - Scale recipes up
PBL Skill Goals	To develop learners ability to: - Solve complex multi step problems - Work logically through a task - Break down a problem into simpler steps and work together to solve it - Optimise a solution.
Starting Points	Learners should have some knowledge and understanding of mass and capacity and be able to convert between g and kg, ml and l. Learners should know or look up things like how many g a teaspoon is. Learners should understand how to calculate profit and loss
Materials Required	For each small group of learners you will need: - 12 recipes - ingredients
Time Needed	At least 4 hours

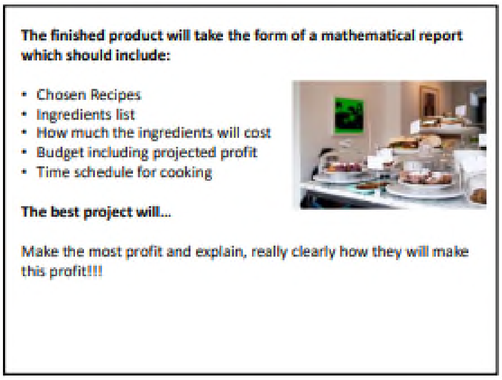


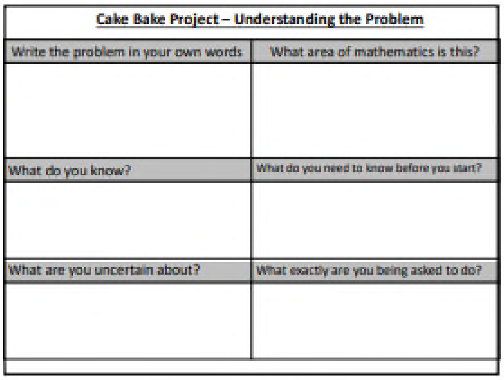
Suggested approach	**Beginning the session** Introduce the problem and explain what they will have to do to complete the problem. **Understanding the problem** Introduce the problem and get students to complete the scaffold "understanding the problem". They should generate a lot of questions about different constraints. Give students the information they ask for. **Devising a plan** Get students to devise a plan for how they will solve the problem. Give them the devising a plan sheet so that they think about some of the complexities. Key issues: How will they work effectively as a group? **Class discussion** A **Reviewing the learning** T



Making the project harder	• Remove the amounts that each makes so that students have to use the size of the tins to approximate how many portions it would make.
What learners might do next:	• Baking a cake doesn't have to be complicated. These easy cake recipes are great for when you're in a hurry or just want to have one in the cake tin for the kids.
Further ideas:	• Similar activities include
Activity based on	• Activity based on it's materials, formula and area of mathematics is this.



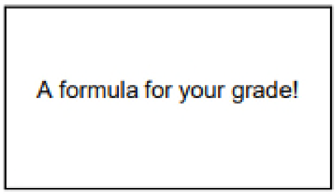


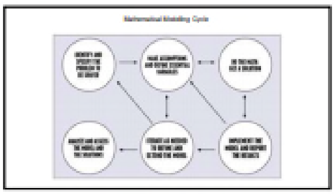


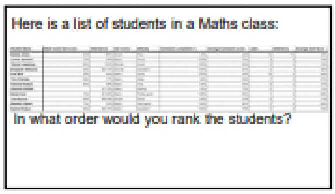
Cake Bake Project – Understanding the Problem	
Write the problem in your own words	What area of mathematics is this?
What do you know?	What do you need to know before you start?
What are you uncertain about?	What exactly are you being asked to do?

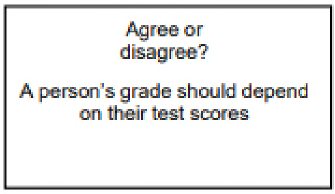
9 Appendix 9 - Resources from LO3

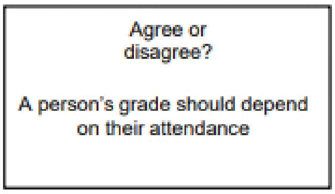
Powerpoint slides from the shared departmental drive, designed to be adapted for the second lesson of Family Support in a Model Society Project.

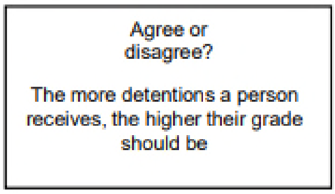


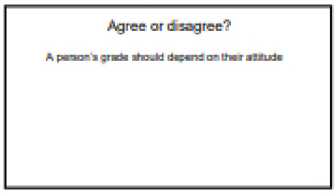


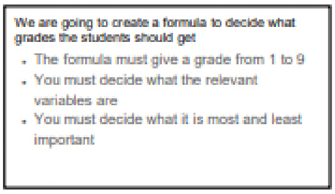


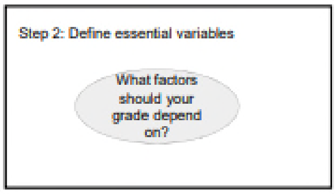












What factors do you think your AP grade should depend on?

Positive	Negative

Step 3: Do the Maths

- Having defined the variables we are going to include in our formula, it is now time to create a formula

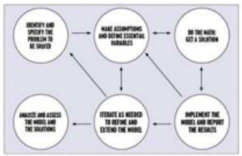
Suggestions for helping students

- If students are very stuck and not keen on having a go at this problem, try getting them to do step 3 in the quickest and most simple way possible: add together all of their positive variables and subtract all of the negative variables.
- If you have chosen not to discuss the importance of having numerical variables, they should notice this by this step
- They can then go to Step 4 and analyse the results. The first problem to tackle will be that the output is not a grade from 1-9

Step 4: Implement the model and report the results

- It is now time to look carefully at our results
- Do your results give you the type of output that you need (a number between 1 and 9)?
- Are your results fair?
- Are any variables more important than you think they should be?
- Does your Maths make sense?

Mathematical Modelling Cycle



Step 3: Do the Maths

- Now that we have seen how our model performs, we need to adjust it so that it can meet our criteria.
- Adjust your model to make sure that it gives each student a fair grade between 1 and 9

10 Appendix 10 - Draft interview schedule (Focus Group 1)

The focus groups that I am completing are for my doctoral research, I am interested in what you think about the projects and how you take responsibility for your own learning. I want you to give your own opinions and not what you think I want you to say. A lot of what we know is about what teachers think, and not about what students think, that is why it is so important you give true and honest opinions. It is fine to be critical and say you don't like things.

I will be writing up the research and presenting it at conferences to other researchers and other teachers. I may also write things about it that get published. All of your comments will be anonymous. If at any point you feel that you want to withdraw from the research, you can do this without any problem – it is important that you **want** to share your opinions. You don't have to answer any of these questions if you don't want to.

If it's ok I will record what we are saying, just so I can concentrate on what you are saying rather than having to scribble it down.

To recorder: Interview with 1/2/3 year10 students at Oaklands School on DATE

I am interested in how you lead your own learning during the project lessons. Can anyone give me an example of the projects you have completed so far?

Topic area	Question	Possible prompts
Project lessons in general	What do you think about these project lessons compared to other lessons?	Rephrase what was said and ask: What do you value about it? What do you find not a good use of your time? Can you give me an example of that?
	What do you think you are learning in project lessons that is different to other lessons?	Rephrase what was said and ask: Can anyone add to that? Does anyone agree or disagree?
	Tell me about a project lesson that you think went really well.	Rephrase what was said and ask: Why did it go well? What did you do? What did your group do? What did the teacher do? Has anyone else had a similar experience?
	Tell me about the hardest thing you have ever had to do in a project lesson.	Rephrase what they said and ask: Why was it hard? What would have made this different? Has anyone else experiences something similar?

Student-led learning in general	What does student-led learning mean to you? If you were leading your own learning – what things do you think you would be doing?	Rephrase what was said and ask What do you do when you are leading your own learning?
	Tell me about a project lesson when you feel that you were leading your own learning.	Rephrase what was said and ask: What happened in this lesson? Why did it go well? What did you do? What did your group do? Rephrase what they've said and ask: What did the teacher do? Has anyone else had a similar experience?
	How do you find working in a group?	What kind of groups work for you? What roles do you take in a group?
	What do teachers do that helps you to lead your own learning?	Rephrase what they've said and ask: Why is this helpful?
	What could your teachers do that helps you to lead your own learning?	Rephrase what they've said and ask:
Observed project lesson	Was the lesson I observed a typical project lesson?	Rephrase what they've said and ask: What about it was typical? What about it was different?
	Tell me about the most challenging part of that lesson for you	Rephrase what they've said and ask: What was it hard? What could the teacher have done to make it easier? What could other students have done?
	To student 2: In the lesson I saw you ask your team lots of questions to check what you were doing. One of the other students in your group was very disengaged. How did this impact on you? To student 1: You drove the work on your group, and your partner asked lots of questions. Was this an effective way for you to work?	can you tell me what made you do this? DO you find this an effective way to work? Is this your normal role in a group?

	In the lesson I saw your teacher talk to you before I came over. What did he say and was this helpful?	Rephrase what they've said and ask: And did that work for you, that sort of thing? Why do you think it did/didn't work?
	To student 1 – there was a point in the lesson when you got stuck at your partner wanted to ask the teacher for help. Can you explain why you didn't want to get the teacher?	Why did you want to keep working? You said: "we can think about this as we go on" did this happen?
	To student 2 – you didn't use the writing frame (table) to help you do the calculations. Why not?	
	To both students: you considered how much you were going to sell something for, how did you arrive at this choice?	How could we make the decisions become more mathematical so that you are
	The teacher at one point stopped the whole class to talk about delivery costs. What impact did this have on you?	
Round up	Is there anything else you would like me to know about leading your own learning?	

11 Appendix 11 – Table of student participants

Pseudonym	Year	Gender	SEN	Pupil premium	Prior attainment			
Tanzia	10	F	No	YES	Middle	LO1	Group v	FG1
Anjum	10	F	No	No	Middle	LO1	Group v	FG1
Ismaeel	10	M	No	No	Middle	LO1	Group w	
Syeda	9	F	No	Yes	Middle	LO2	Group x	
Rehan	9	M	No	No	Middle	LO2	Group x	
Samiha	9	F	No	No	Middle	LO2	Group y	
Fabiha	9	F	No	No	Middle	LO2	Group y	
Yusuf	9	M	No	Yes	Middle	LO2	Group Z	
Ammara	11	F	No	Yes	High			FG2
Nadman	11	M	No	Yes	High			FG2
Fahmida	11	F	No	Yes	High			FG2
Myesha	11	F	No	No	High			FG2
Shaffat	11	M	No	Yes	High			FG2
Mehdi	11	M	No	Yes	High			FG2
Anika	11	F	No	No	High			FG2
Fahiza	11	F	No	No	High			FG2

12 Appendix 12 - First draft of the survey for year 9

The initial draft was amended in the following ways:

- Too much focus on RQ1 – I needed to include more questions re RQ2 and RQ3.
- Not enough open questions – especially with lockdown, I need to capitalise on the opportunity to get further responses.
- Open questions required around students leading their own learning in other areas of the school.
- Ensure that positive and negative statements for each construct are really positive and negative.
- Questions re-ordered
- Questions re-worded to better summarise the construct.

For each statement, state if you strongly agree, agree, disagree, strongly disagree

Construct	Questions
Exam oriented	I am motivated to learn maths to improve my performance in exams
	I like it when a teacher shows us things that aren't examined
	I want to learn maths rather than learn how to pass an exam
Authentic	I like knowing how maths can be applied in 'real life'
	Sometimes I wonder why I need to learn maths
	Maths is used by a lot of people in their daily life
Structured lessons	I like working out how to solve new maths problems on my own
	I like it when our teacher first shows step by step how we have to solve a specific mathematical problems, before we complete similar exercises
	I like lessons best when they are structured.

Projects

At Oaklands, we have begun to introduce some projects into your maths lessons. For example the cake bake project of the maths of migration project.

For each project in turn, I want you to indicate how much you think you were able to demonstrate each of these skills. [Give options: never, occasionally, regularly, throughout, I forget, I did not complete this project, I don't want to say.]

Cake Bake - In the cake bake project you had to choose which cakes you would bake for a charity cake sale. How many batches you would make, how much they would cost, how much profit you could make for charity and whether you could bake them all in the time given.

I took responsibility for my own learning in this project.

I worked hard on this project.

Me and my team chose how to solve each of the tasks (we used our own methods)

Me and my team chose how much time to allocate to each task in this project

Me and my team chose the order in which to complete the different tasks

Me and my team chose who in the group would complete each task

Comments:

Maths of Migration In the maths of migration project, you had a large data set of 500 refugees and migrants. You complete calculations including averages and percentages and drew graphs which you turned into a poster to show people about the migration crises.

I took responsibility for my own learning in this project.

I worked hard on this project.

Me and my team chose how to solve each of the tasks (we used our own methods)

Me and my team chose how much time to allocate to each task in this project

Me and my team chose the order in which to complete the different tasks

Me and my team chose who in the group would complete each task

Comments

Learning through projects in maths (cake bake or maths of migration)

When learning through a project please say if you strongly agree, agree, disagree or strongly disagree with each statement.

Construct	Question
Understandin g 3	I am normally able to understand what to do in a project
	I don’t always know what maths to use in a project
	I think I would be able to complete projects better if the teacher spent more time at the beginning explaining it
	Sometimes projects are overwhelming because I don’t know where to start
Learning 1	I learn useful things though projects
	Projects help me develop my maths skills
	Projects help me learn maths that is relevant to my life
	I don’t like projects because they don’t help me pass exams
	The projects don't fit with the maths we do in normal lessons
Enjoyment 2	Projects are fun
	I enjoy projects because we learn about other things, such as migration
Motivation 4	I am more motivated when learning through a project
	My favourite projects are competitive
	If I like the topic of a project I will work harder
Support 5	During a project, I often need support from a teacher to know what to do next
	I would find projects easier if my teacher gave me a checklist of all the things I need to do
	I would find projects easier if we had a mid-way check-up where everyone explained to the class what they had achieved so far
	I think if we did more projects I would be better at them
Group Work	Questions around what role you assumed in a group?
Time	The teacher doesn’t always give us enough time to complete a project

13 Appendix 13 - Final Survey for year 9

Part 1 - General Questions

These first questions are all about your opinions on your maths learning in general. Please don't spend too long thinking about the answers, just go with your gut instinct.

	Strongly agree	Agree	Disagree	Strongly disagree	Not sure	I don' t want to say
I like working out how to solve new maths problems on my own						
I like having choices about how I will set about my work and what maths I will use to do that						
Sometimes I wonder why I need to learn maths						
There are often several ways to find the correct solution of a maths problem						
I like learning new ideas in maths, whether or not they're directly on the GCSE papers						
To be good at maths you have to memorise how to do things						
My main interest is in learning maths; doing well in the exam is a spin-off						
I prefer working in a pair than on my own or in a larger group						
I learn maths best through doing exercises						
I like knowing how maths can be applied in 'real life'						
I like lessons best when the teacher organises what we will do at each stage						
Mathematical thinking is used in very many jobs, even if it's not always with the same maths we learn in school, different jobs use different maths content						
I need to spend time thinking on my own in order to solve maths problems						
I like learning about different topics in maths, regardless of whether they will be useful to me in the future						

I enjoy being able to use lots of people's ideas to work with a problem						
I like it when our teacher first shows step by step how we have to solve specific mathematical problems, before we complete similar exercises						
I prefer working in a group to working by myself or in a pair						
There are lots of different things to know in maths, learning maths is all about seeing the connections between them.						
The main reason I try in maths is because it's an important exam subject						
Group work helps the learning of mathematics						

Part 2 – Considering specific projects

<u>Cake Bake -</u> In the cake bake project you had to choose which cakes you would bake for a charity cake sale. I imagine you would have considered: how many batches you would make; how much they would cost; how much profit you could make for charity; and whether you could bake them all in the time given.

Please write down three things you liked about this project

____

____

____

Please write down three things you didn't like about this project

____

____

____

	Always	Often	Occasionally	Rarely	Never	Not sure	I don' t want to say
I took responsibility for my own learning in this project.							
I worked hard on this project.							

Me and my team chose how to solve each of the tasks (we used our own methods)							
Me and my team chose how much time to allocate to each task in this project							
Me and my team chose the order in which to complete the different tasks							
Me and my team chose who in the group would complete each task							

	Significantly	Moderately	Slightly	Not at all	Not sure	I don't want to say
To what extent did the project help you understand the maths we used more deeply?						
To what extent did the project help you feel more confident about the maths we used?						

Maths of Migration - In the maths of migration project, you had a large data set of 500 refugees and migrants. I imagine you would have: completed calculations including averages and percentages and drawn graphs and infographics which you turned into a poster to show people about the migration crisis.

Please write down three things you liked about this project

____

____

____

____

Please write down three things you didn't like about this project

____

____

____

____

	Always	Often	Occasionally	Rarely	Never	Not sure	I don’t want to say
I took responsibility for my own learning in this project.							
I worked hard on this project.							
Me and my team chose how to solve each of the tasks (we used our own methods)							
Me and my team chose how much time to allocate to each task in this project							
Me and my team chose the order in which to complete the different tasks							
Me and my team chose who in the group would complete each task							

	Significantly	Moderately	Slightly	Not at all	Not sure	I don’t want to say
To what extent did the project help you understand the maths we used more deeply?						
To what extent did the project help you feel more confident about the maths we used?						

Generally in your learning (thinking about all the subjects you study) how challenging do you find it to do these things:

	Extremely challenging	Very challenging	Moderately challenging	Slightly challenging	Not at all challenging	Not Sure	I don' t want to say
Take responsibility for my own learning							
Work hard throughout a lesson							
Choose how you will solve a problem							
Allocate your time to different tasks in a problem							
Decide on an order in which to do things							
Work with others and allocate different roles to different people in the group							

For these questions, it would be helpful to know as much as you can tell me:

Do you use student-led learning skills or group work successfully elsewhere in school? If so, where, and what works well in those subjects? (Use another piece of paper if you need it.)

Part three – Learning through projects in maths

I now want to ask you some questions about learning though projects in maths in general. Please try to think about projects where you have led your own learning when you write your responses.

	Strongly agree	Agree	Disagree	Strongly disagree	Not sure	I don' t want to say
I am able to complete projects more easily if the teacher spends more time at the beginning explaining it						
The projects don't fit with the maths we do in normal lessons						
I normally take the lead when completing a project						
Projects are fun						
Something about these projects help me understand the meaning of the maths better						
When working on a project in a group, we normally all complete the same task together						
I feel confident that I am doing the right thing when I am completing a project						
I don't enjoy doing projects						
Projects help me develop the skills I need to lead my own leaning						
I don't like projects because they don't help me pass exams						
I often feel like I am wasting time when I do a project						
When working on a project I normally check each thing I do with others in my group						
The teacher doesn't always give us enough time to complete a project						
If I like the topic of a project I will work harder						
Projects help me learn maths that I feel I might use in my life						
Sometimes projects are overwhelming because I don't know where to start						
I enjoy projects because we learn about other things, such as migration						

My favourite projects are competitive						
I am normally able to understand what to do in a project I don’t always know what maths to use in a project						
Projects help me develop my maths skills						

Do you have any other comments you want to make about learning through projects in maths?

Part 4 – Last part! What helps you lead your own learning when completing a project?

It is really important for me to understand the things that teachers’ do that help and support you to lead your own learning when you are completing a project.

	Extremely helpful	Very helpful	Moderately helpful	Slightly helpful	Not at all helpful	Not Sure	I don’t want to say
The teacher working with my group when we get stuck							
Having a checklist of all the things we need to do to complete a project							
Having a mid-way check-up where each group explains to the class what they have achieved so far and what they plan to do next							
Doing projects more often							
Having more time to complete a project							
Seeing examples of what other people have done when they completed the project							
Being able to work with other people in my group							
Getting help from other people in the class							

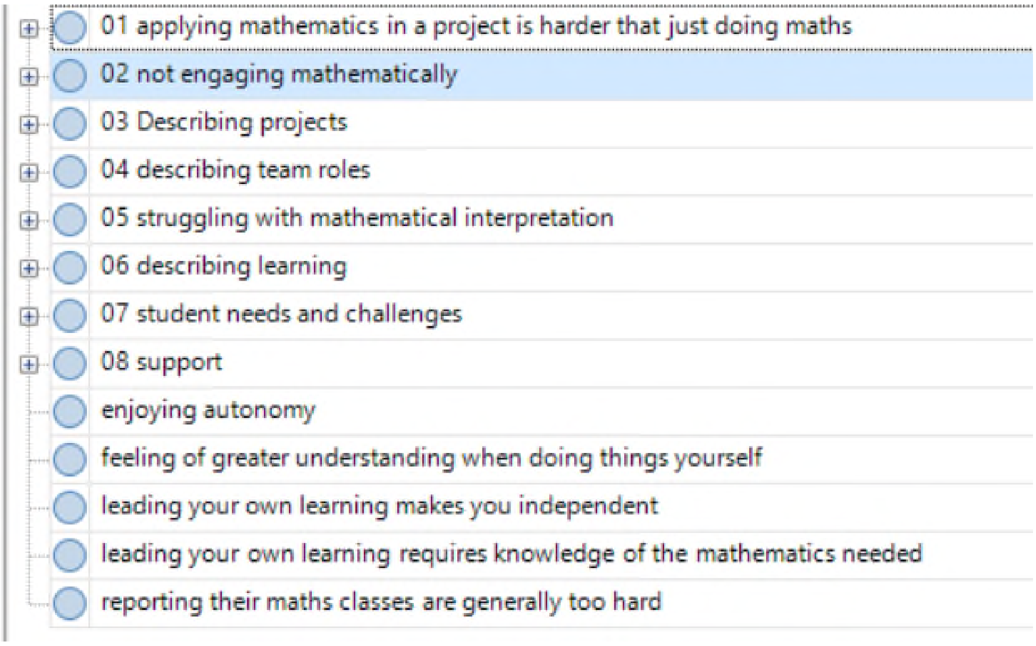
	Extremely helpful	Very helpful	Moderately helpful	Slightly helpful	Not at all helpful	Not Sure	I don't want to say
Having a whole class discussion about things people are finding challenging							
Writing a plan of what to do before starting to do it							
My teacher checking my plan of what I am going to do before I do it							
Completing starter activities that help us practice the mathematics we might use in the project							

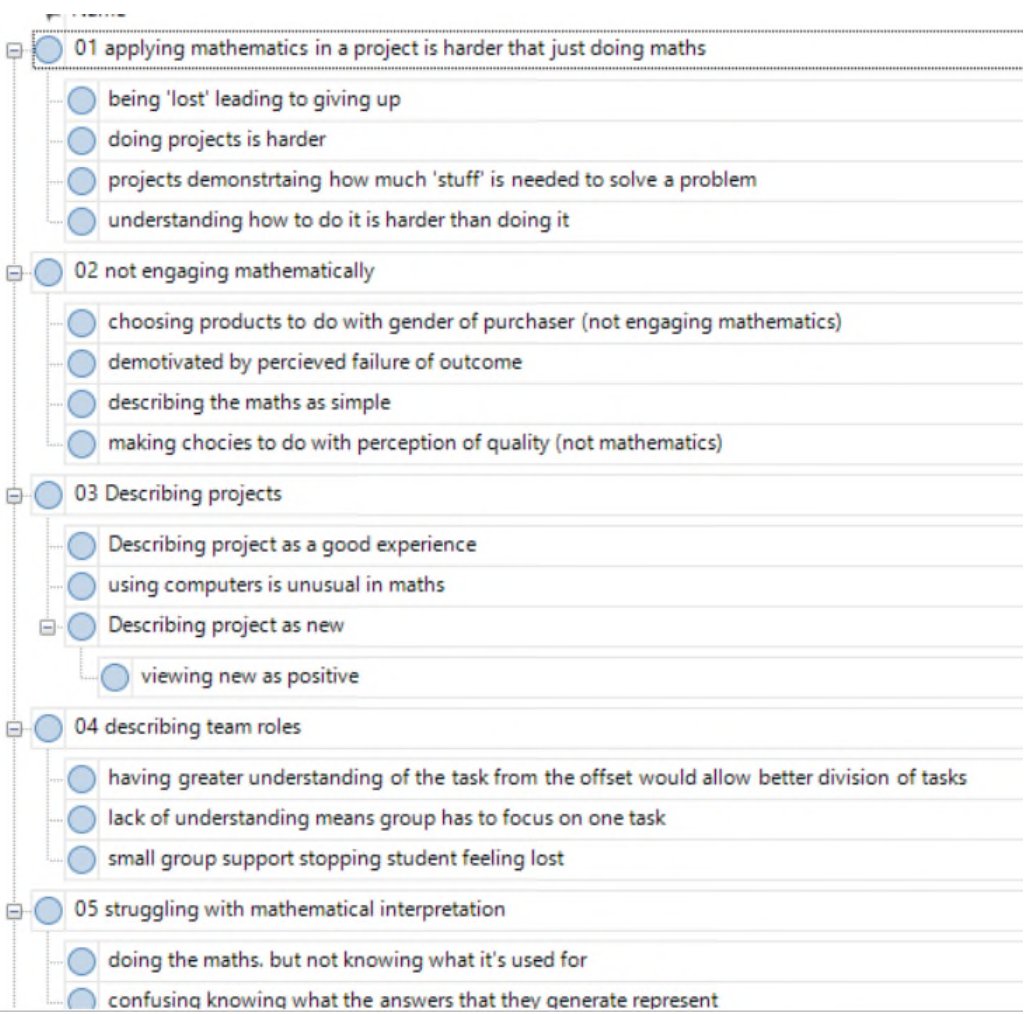
Are there any other things that you can think of that your teacher could do to help you to lead your own learning in a project?

Now all that is left to do is for you and your parent/ guardian to fill in the consent form at the end and post this back to school in the stamped addressed envelope.

14 Appendix 14 - An example of initial coding

Screen shots from the codes as of 05/01/09 showing the core codes and sub-codes at this point in the coding process.





- 06 describing learning
  - learning about business is useful
  - learning about profit
  - learning about the complexities of business
  - learning about what you need to start a business
  - gaining insight about people's jobs
- 07 student needs and challenges
  - falling back due to lack of business skills
  - feeling a need to understand everything
  - needing more teacher support
  - needing more time to do better in the project
  - needing to do more projects to be better at projects
  - not understanding terminology leading to frustration
- 08 support
  - filling in the scaffold (table) was helpful
  - starters around key topic areas helping students to know what to do
  - teacher help empowering students
  - teacher offering small group support
  - teacher showing students what to do
  - teaching peers
- enjoying autonomy

15 Appendix 15 - Development of the key conceptual codes

Initial Core Codes	**Core codes developed through engagement with literature**
Authentic Context	Goal orientation
Confidence	Self-efficacy
Making choices	Making choices
Metacognitive skills	Self-regulated learning skills
Perseverance and resilience	Resilience
Working collaboratively	Working collaboratively

16 Appendix 16 - Findings by original themes

In this appendix I report my findings from the data under the six key themes that came through from the coding: making choices, authentic contexts, metacognition, resilience, confidence and working collaboratively. Presenting all of the data under each code allows me to access the richness of the data that was collected by using a variety of complementary research tools.

The following analysis draws from: three lesson observations of students completing projects in the maths classroom (during the third observation students were primarily introduced to the topic, so the data drawn on from this lesson observation is significantly less); two focus groups of students; survey responses from 47 students in years 8, 9 and 10 at the study school which provides a greater contextualisation; and an interview with one of the teachers observed and a teacher workshop with the other two teachers observed.

The following abbreviations are used in this chapter: L0 – Lesson observation, FG – Focus Group, Int - interview

16.1 Student choice

In this section, I discuss how students were offered, and in some cases took, autonomy for different kinds of decisions and choices within the observed lessons. I explore some of the reasons that students and teachers finding it challenging for students to be given choices and the strategies that were reported to support student choice. The choices that students exercised in relation to how they worked within their teams are discussed under working collaboratively 16.6.

16.1.1 Key Findings:

- Teachers reported that the projects demanded students to make more choices and decisions than in a typical problem solving lesson, with some projects not even having one particular process to follow;
- Students didn't always exercise offered autonomy and others took autonomy for themselves; and
- Students needed support with the choices required, for example considering how much time to allocate to each task and ensuring they made appropriate mathematical choices.

16.1.2 Different projects provide different choices

Mr Robinson reported he felt the projects demanded higher autonomy than usual maths lessons, but the level of autonomy varied between projects:

> *Every… problem-solving question (in maths), even when there are five steps… there's still one right way to do it and one answer that comes at the end. But with… (Maths of Migration) and any other data project there are just so many different ways, so much freedom…, there's no real*

right way of doing things: advantages and disadvantages, rather than a particular correct answer. I really like the leading your own learning aspect in (Maths of) Migration…, there's so much there and as long as they're made aware that they can choose anything within that, then I feel like they'd feel more comfortable making their own choices.

He contrasted this with other projects which he felt offered students less choices:

With… Amazon Trader apart from picking their own products I guess there is a set way you should really do it. You need to look up the prices in a certain way in order to decide a price which is going to give you some sort of profit (Mr Robinson, TW).

Mr Drew agreed with this, adding:

It's not so structured they've got a bit of freedom to take what they want from it and to kind of experiment with it… and us not being able to say that's right or wrong or even them not being able to tell so clearly whether it's right or wrong (Mr Drew, TW).

This links to the core codes of Resilience (16.4) and self-confidence (16.5)

In summary, the teachers reported that the projects required students to make far more choices and decisions than in a typical problem solving lesson, including sometimes of core processes.

16.1.3 Different teachers offer different choices

In LO1, students had the freedom to make many of the choices themselves. It appeared that they had an overall brief (choose the products they wanted to sell and work out if they could sell them for profit, factoring exchange rate, referral cost, delivery cost and Amazon fees) but could choose how to do this. However, they did not take up all of the autonomy they had been offered. The students had a writing frame, a table, which they could fill in if they wanted to, to help structure the mathematics. Both of the groups of students being observed chose to use that. They appeared to use very similar methods, collectively going through one product at a time, working through the calculations the table required.

The students may have relied on the structure given as they didn't have the ability to manage the project without this structure, or it may have been as Mr Robinson suggested, that some students are happier doing work where they just have to repeat a process and don't have to think more deeply and make decisions:

I think there are a few kids that I teach who when you just give them something really simple to do…, they'll quite happily just go for it. Whereas as soon as they have to make any sort of decision, as soon as *they had to think a bit more and (it's) not so clear…, they kind of switch off a bit. I think some kids do prefer just not having to make any decisions (Mr Robinson, TW).*

Not all students feel like this. Two FG students (20%, 2/10), reported enjoying taking responsibility for their learning and making choices throughout the project. For example when asked what made it go well, Fahmida stated, "*Because it wasn't like you dictating the whole lesson*" (FG2).

In LO2, the lesson was structured and students were shown an example of how to complete each part of the project, before they worked independently. When asked in an interview why he had done this, the teacher reported that he felt both he and the students became anxious when he didn't structure the lesson:

I guess I was concerned that they would not engage and maybe get a bit anxious. Sometimes, they're asking a lot of questions but there's the way they ask the question, it's almost like there's a pain to it. If they're faced with something genuinely uncertain it almost it hurts them (Mr Jafri, Int).

Mr Jafri later added that he felt that students preferred having the security of knowing they were doing the right thing, even if he felt that this wasn't necessarily the point of the project or what's best for the students:

I think they find it quite scary if they really don't know what they're (doing). When they had to create the time schedule, at first I kind of got them to try and do their own and then we kind of fed back and… I analyzed some of people's time schedules and some of it was literally, start at nine and then we'll end at 11... [laughs] But then in the process of analyzing some better ones it became quite prescribed. I think they preferred having a… success criteria... but obviously it's not necessarily the best thing for them because you're almost giving away the game or giving away like an answer. Whereas I guess it's supposed to be open ended isn't it? I think that's what kids, at least at first, hate the most, not knowing what the answers supposed to look like.

However, he also reported that perhaps he didn't need to be anxious about offering students more autonomy:

There's not as much to worry about as I think there is in terms of just letting them go…. throughout my career there's been times where I've been like I've finally just let them get on with it and its gone great and I'm like, "What was I so worried about?".

Despite Mr Jafri not appearing to offer autonomy to the students, there were a number of instances in LO2 when the students seemingly attempted to 'take' autonomy. For example, Mr Jafri briefly brought the class back together and stated that they shouldn't show their methods on their final piece of work. When some of the students remonstrated, the teacher repositioned what he had said as a ‘suggestion’ allowing students to present their work in the way they choose. This could be viewed as students trying to take autonomy. Later in LO2, students ignored the fact that they were supposed to be working on the same task and instead worked on different parts of the project; this was viewed by the observer as being in order to better complete the task.

One student thought students should choose whether to complete a project:

> *I think you should discuss with the class if they want to do the project first. ….Amazon Trader…, lots of people wanted to do it …but then say other projects like the toilet water one (Splash Down) ….if they said no then it would have been a better use of time just to stay with the class work (Anika, FG1).*

It is clear from the survey responses shown in Figure 16-1 and Figure 16-2 that students varied both in their adoption of the choices made available, and in their perceptions of how challenging they found different process choices.

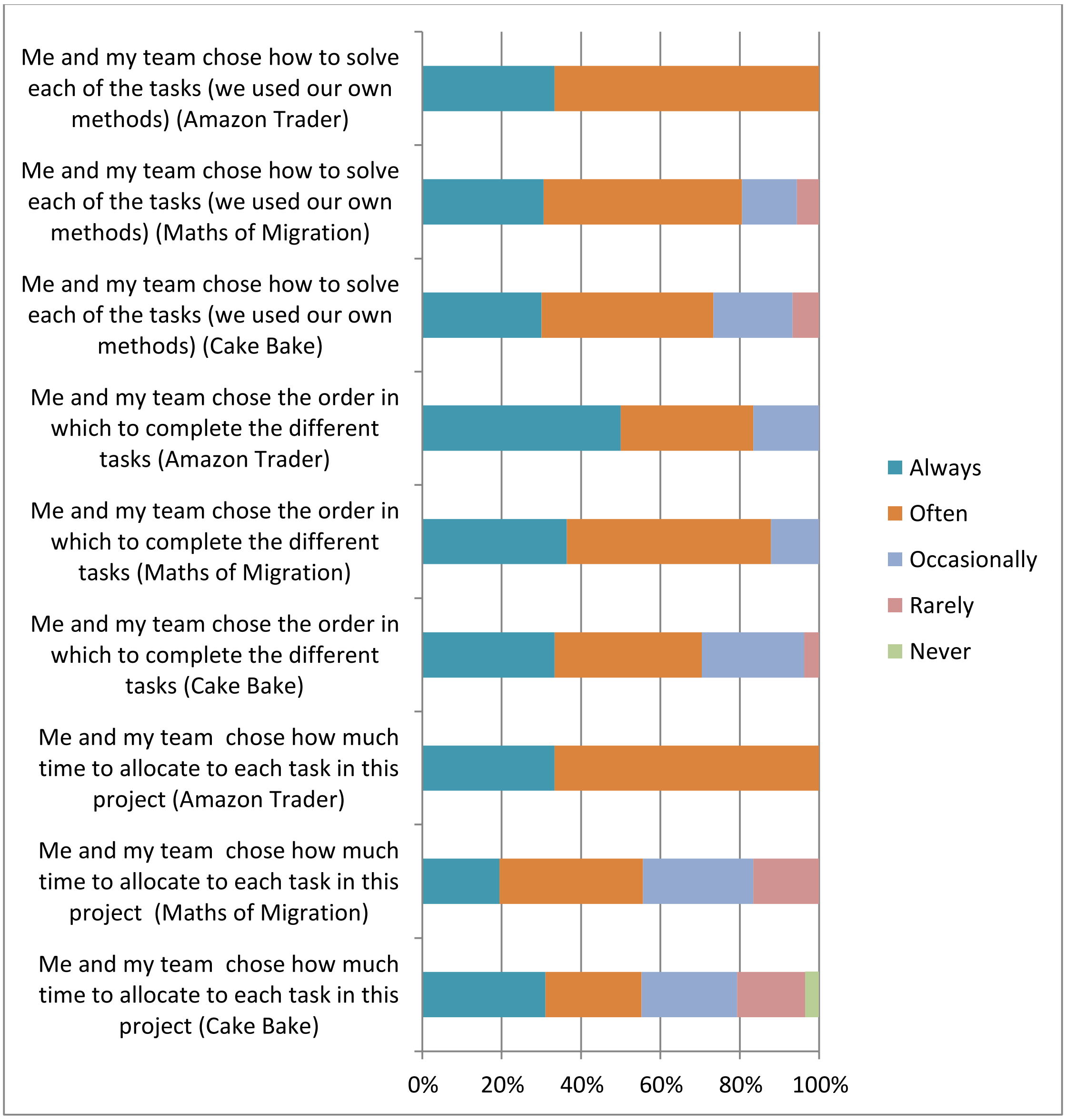


Figure 16-1: Survey responses about how often Students made different kinds of choices in different projects.

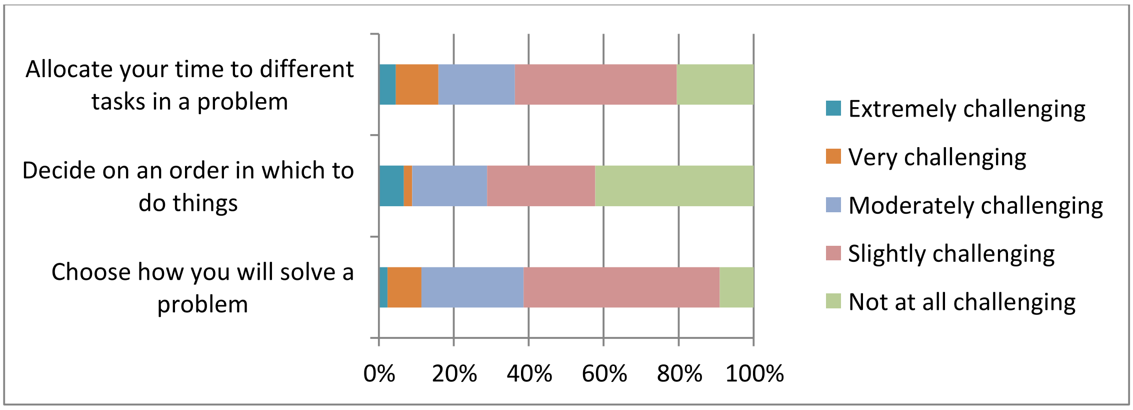


Figure 16-2: Survey responses about how challenging Students found different choices

In summary, in LO1 most of the observed students chose the more structured option offered, whereas in LO2 some students took more autonomy than had been offered. Survey responses reflected this variable.

16.1.4 Choice of timeframe

The projects are designed to give students the opportunity to manage their own time within the overall timeframe for the project. In LO1, the students had been given the task and the overall timeframe, but it appeared that they were able to choose how they allocated that time, with some reminders from the teacher. For example, reminding students when they should move onto the report write up. In conversation with the teacher after the lesson, he informed me that not all of the groups had finished the project within the given timeframe.

In LO2, the students were given a time within which each stage of the project must be completed. After this time was up, the next phase of the project was shown. In this lesson, the observed students typically appeared to do what they had been asked to, however in one instance a group ignored the teachers instructions and took ownership of their own timeframe. It was presumed that they did this to complete the task more effectively.

Some of the students in the FGs (30%, 3/10) reported finding it difficult to organise their work within the given timeframe. One student stated that managing her own time was one of the hardest elements of learning through projects:

> *It's easy to lose track of time though when you are doing the projects…. We weren't focusing on time, which was… my main issue… I'd start it, I'd be really driven to do it but the next thing you know it's the end of the lesson and I've only gotten one thing done (Fahmida, FG2).*

Another student suggested that the teacher could help support students with keeping track of time:

> *Some things took too long and they shouldn't have. So, having a set time for each part would have been a bit easier… a realistic time that you could finish that part in (Shaffat, FG2).*

However, he stated that within this timeframe, it was important that students had some autonomy as different students would progress more quickly through different parts of the project:

> *Maybe not the teacher sets out the time, but if the group, if they've got three hours to do it and you split up your time between different parts of the project well then you fill in where you put the time. So, it's to do with your needs. Some people might do one part faster than another (Shaffat, FG2).*

The survey results support the finding that students found it challenging to allocate their time to different tasks in a problem (Figure 16-2). Figure 16-2 shows that this varied across the different projects, with students stating that it was easier in Amazon Trader. This may be because these students are older (they complete Amazon Trader in year 10) or because a smaller number of year 10 completed the survey and they were likely a more self-selecting sample.

In FG1, all the students wanted more time on the projects and more projects to allow them to delve deeper into these contexts. They explained how they felt that if they had more time they would have a greater understanding of what it meant to create their own businesses:

> *(What do you think that would give you if you did it more?) It would give us more experience to actually start a business. Because if we are doing this Amazon thing and the next one is, that's on how to sell stuff on Amazon, the next one might be how to build a business. So, we already have the key things that we have to buy, acknowledge and what we are selling. So, then the other stuff, I don't know what is in Business, Services stuff, so it'll be more helpful on other subjects as well (Tanzia, FG1).*

This was supported by the survey data (Figure 16-3).

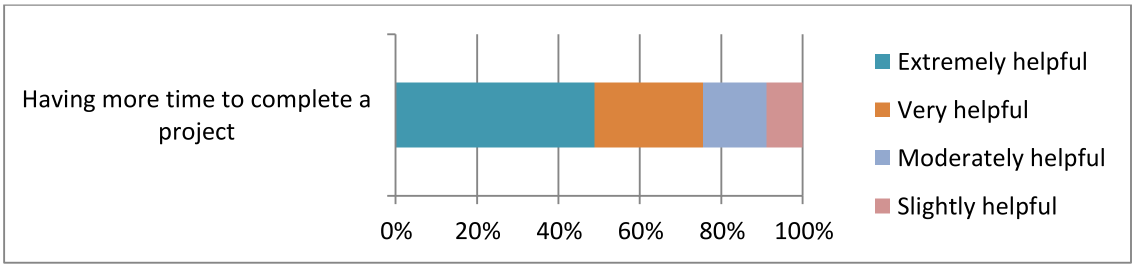


Figure 16-3: Student responses to the question "How helpful would it be to have more time to complete a project?".

16.1.5 Choice of methods

In LO1, the choices that the students were observed making seemed primarily to be about selecting the products that they wanted to sell. The purpose of authentic tasks is that sometimes such considerations are as important as the mathematics. Group W were observed making some of these choices around which was the most cost effective:

> *"Are you sure, how much money are we even getting? One of each type. 5 types. They cost more money. If we buy it we're only going to get one size" (Ismaeel, LO1).*
>
> *(In response) "We can think about this as we go on" (Student D, LO1).*

In Group V the decisions that the students were observed making appeared to be less mathematical and more often based on the products that the students' thought would sell: *"I feel like that one, for the extra £1 it looks better*" (Anjum, LO1).

There was some discussion in FG2 about whether the students felt they were able to remain focused on the mathematics during the projects. Fahmida, a student whom the projects had pushed significantly mathematically (she had learnt how to calculate product momentum correlation coefficient when completing Design a Bag in year 8) reported that she didn't always think about the mathematics:

> *It's more interactive clearly when you are doing products and stuff like that you don't really think about the maths. We don't get a lot of chances to do projects like this in maths because it's mainly just doing worksheets and textbook work. So, when we do, we kind of lose track of the actual maths. I think it's not entirely maths focused (Fahmida, FG2).*

However, Nadman reported he disagreed with this:

> *I disagree because me and Mehdi had to put in a lot of time and effort into thinking about the maths behind Amazon Trader. I think it's not just the business side of things but there's a lot of complex and interesting maths that people missed out on by not doing the project to the fullest (Nadman, FG2).*

In LO1, Group V did not appear to be using the project as a mathematical optimisation task, but were instead choosing their products based on their perception of what would sell. When asked about whether they had chosen their products based on what would be best mathematically, the students responded:

> *I don't think we really thought like that. We did think about jewellery. I thought that that price could easily go up because jewellery is more like that you could sell it for more because it might look better or… (Anjum, FG1)*
>
> *Yeah, but the trouble was that you're selling earrings for like five pounds, who's going to buy that from like a dodgy website. We already had to boost the price up because half of the income was going to Amazon and then the earrings were like two pounds. So, that's already our loss. So, we needed to make profit, so prices have to be, we have to bump the price up. So, it was a bit hard. We choose like, not cheap stuff, but it was like that stuff you can buy a market that is like affordable just there. You don't wanna spend like five pounds on just earrings (Tanzia, FG1).*

The teachers also found this:

> *Quite early on you find groups going with cakes that they know they like or cakes that they know they can make and even once you get to a point*

and show them that isn't financially the best one to go for they are quite adamant just to stay in their comfort zone and stick with what they either like or can make. But that almost is the opposite of what we're trying to get them to do from the project isn't it? (Mr Drew, TW).

In summary, different projects provide different levels of autonomy and choice for students. Students newer to this way of learning will require more support. This was summarised by Mr Robinson:

You need to really be really careful… about telling them what to do and that's the temptation, to tell them how to do it. I think… (a) new teacher would have to be told that the point of this project is not for you to tell them how to do it, the point is to try and get them to make decisions if they can. But that is really hard and maybe with lower (sets) you do have to give them more support and think of it as an ongoing process. Each time they do a project hopefully they start to make more decisions and start to lead it themselves (Mr Robinson, TW).

16.2 Authentic Contexts

In all the observed lessons, students worked on projects or tasks that explored authentic situations. In the first two, the output of the students' work, if it hadn't been for lock down, would have had the possibility of being turned into a reality: in Amazon Trader, the winning group would have an investment to launch their business, whilst in Cake Bake the students with the best project would have baked their cakes for a charity bake sale. In the third lesson I observed, the students discussed the factors that a teacher could use within their school to give an 'attitude grade' on a school report.

16.2.1 Key findings:

- Students reported valuing authentic contexts that allowed them to use and apply their mathematics and to learn about the authentic context itself;
- Authentic contexts that are interesting, relevant or competitive appeared to have the possibility of increasing student engagement;
- What students report perceiving to be interesting and relevant differs between students; and
- Authentic contexts were sometimes observed to create complexity and can lead to tangential conversations.

16.2.2 Valuing authentic contexts

Over half of the students in the focus group reported valuing using and applying their mathematics to solve authentic problems:

(What did you like about the project?) We had a real problem to work with (rather than working out how many chocolate bars Harry gets) (9M2, Survey).

I think it's mainly putting maths to use in the real world because in the classroom it's just worksheets and now you are going to put it into actual use, which is the main part of learning stuff (Mehdi, FG2).

It puts into perspective how you'd use maths in everyday life (Myesha, FG2).

The survey data produced similar findings (Figure 16-4).

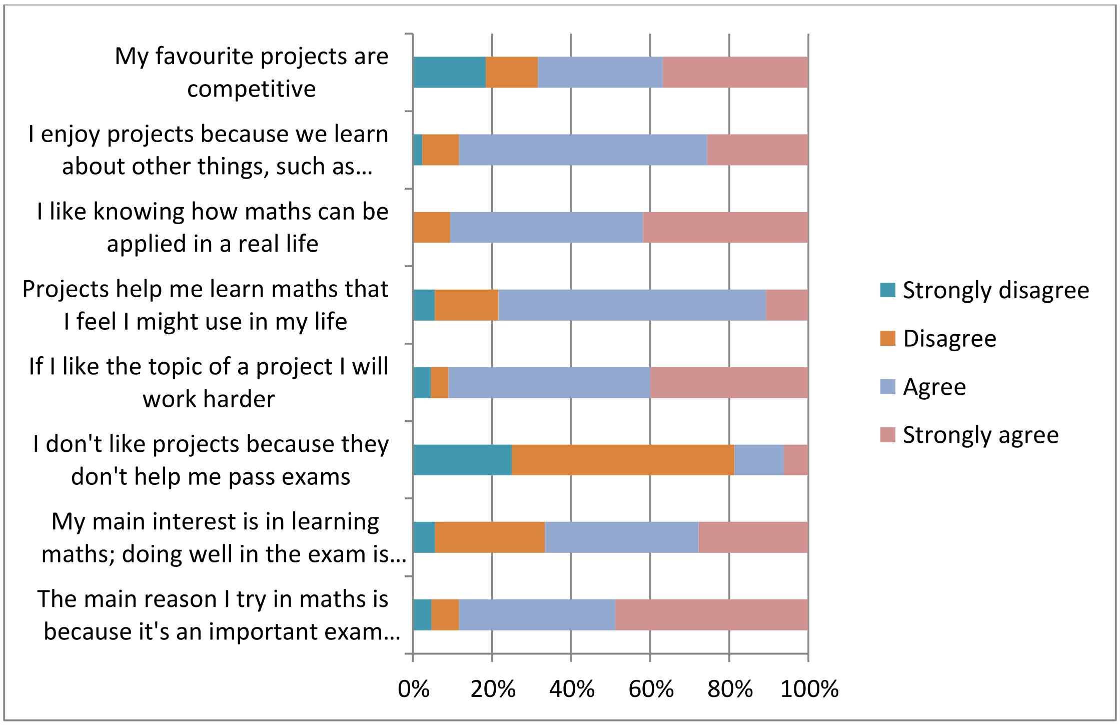


Figure 16-4: Survey responses to statements on authentic contexts.

All teachers similarly reported valuing the opportunities for students to apply mathematics in an authentic situation:

(It's a) chance to see directly how maths can be used practically and you wouldn't get that in most other lessons in such an obvious way (Mr Robinson, TW).

Just seeing maths in situations/scenarios which they wouldn't normally see maths (Mr Drew, TW).

The majority of FG students (80%, 8/10) made comments about how they valued the contextual learning in the projects, beyond mathematics:

I think the immigration project was very interesting because it showed us what other people around the world are going through when it comes to countries and conflicts and how they have to leave their countries and *travel and find safe refuge and their journeys and how dangerous it is (Ammara, FG2).*

Even Anika, who reported that she didn't think she picked up any skills from the project, found the authentic context interesting:

I think it was more interesting. I don't think it was really helpful… (for) myself, because I don't think I picked up any skills doing it, but it was interesting to find out about… immigration… and know how people are suffering and how maths can be brought into it (Anika, FG2).

These findings were supported by the survey data (Figure 16-4). Similarly the teachers felt it was interesting for them and the students to work with authentic contexts:

It shows me…(how) to work from a big page of numbers, make sense come to life from them. I think it was interesting for me as a staff member… as well as for the students (Mr Drew, TW).

In summary, students and teachers valued how authentic contexts taught the students about different contexts and gave the students the opportunity to apply their mathematics to an authentic problem.

16.2.3 Authentic contexts can increase engagement

Many students (70%, 7/10) reported working harder when the context appealed. For example, Mehdi, said Amazon Trader inspired him "to do much more" (Mehdi, FG2). This finding is supported by the survey where 91% (41/45) of students reported that if they 'like the topic' of a project they will work harder (Figure 16-4). The students attributed this increased engagement to different factors which I now detail.

Nadman seemed to imply that the freedom and autonomy given within the context led to high engagement:

(What do you mean by inspired to do much more?) After that (we) went home and started looking at different kinds of stock and we started thinking we could push this or we could sell this and we started contacting (the) seller and stuff that we learned whilst doing the project like bargaining and checking the graphs and the stock that we needs, what's in demand and that kind of stuff once we go home" (Nadman, FG2).

Nadman reported that having an interesting context seeded increased effort:

I think the migration project, they had background information on the people and there was a lesson where we could see the conflicts and route they took to come here…, that fuels your learning because you understand and it's like intrigue. So, once you have a fuel to your learning you push yourself to do more (Nadman, FG2).

Similarly, Myesha felt that studying lots of different contexts would increase motivation:

> *I think there should be more projects as well because if you're always doing class work then it's going to make you lose motivation. So, if you have more off topic kind of projects it will give you a drive (Myesha, FG2).*

Fahmida explained how she felt that a project would be more interesting if it was 'relevant' to her, she described how she liked having a context that was familiar to her:

> *Yes, I think things that are relevant to the students. Like the (Design a) Bag project, we didn't really know much about it, so it wasn't as interesting… to… some people. Amazon (Trader)… was more interesting (Fahmida, FG2).*

Similarly, Mr Robinson reported that he felt having a familiar context could work as a 'hook' into the project as it allowed students to start work on the project more easily:

> *I thought maybe it was just the initial hook of them having to choose what products they wanted and that was a challenge for them to brainstorm and… I guess it was an exciting way for them where they could talk about "what should we buy". They could all join in that conversation. Then maybe that was why the rest of the group work went well, because there was the initial time to talk about… something they all understood… so they could all feed into it (Mr Robinson, TW).*

Some students, (30%, 3/10) reported that when the projects were structured so that they were working in a group, a slight competitiveness with other groups made them more engaged in projects:

> *I feel like the whole idea of having groups made it kind of a competition. Personally, being really competitive it drove us a lot more (Myesha, FG2).*
> *In the (Design a) Bag project, the winning team gets something, and that made me want to do really well, to get the best bag (Shaffat, FG2).*
> *I'm very competitive so it's more, for me, trying to beat others, to be honest, and it is very interesting, so that is a big factor too (Ammara, FG2).*

This was not, though, universally true (Figure 16-4).

The teachers also reported that they felt that having a tangible output motivated students to work harder:

> *The motivation is a lot higher because they are going to have to present this, they need to write things clearly, they need to choose a graph… to show what they say (Mr Robinson, TW).*

Similarly Mr Drew reported:

> *When I did the solar system, a smaller project, with our classes, quite a lot of them very early were like, "Why are we doing this? This is science not maths." But then once we spoke about it more, they could see the links between maths and science. Then bringing in the art side…, they could express themselves with the drawing as well. I think they really enjoyed that. They were up out of their chairs and they were making… Quite a few students who were not normally as engaged (Mr Drew, TW).*

The survey students reported valuing having a tangible outcome in a project. When asked what they liked about Maths of Migration some students (32%, 13/41) gave an answer that included enjoying creating a poster. Similarly when asked what they disliked about Cake Bake, some students (26%, 8/30) included something about a frustration that they didn't get to cook the cakes.

As noted above, students reported that they had a higher level of engagement and perseverance in the project if: they had an interest and intrigue in the project; they deemed the project relevant to them; the context of the project had an element of competition; or they had a tangible outcome to the project.

16.2.4 The relevance to a student of an authentic context

Some of the contexts were received differently by different students. For example, all of the students who were asked about Maths of Migration were positive about the context of this project. However, Amazon Trader was viewed as useful by some students (Nadman and Mehdi), but not by others (Fahmida). A student in FG1 suggested how this particular project might appeal more to students who had an interest in business:

> *Especially if you want to become a business person, like Rahim is very into it, because he wanted to make…a business and stuff, so… he was… really into it with Ravi,… but then me on the other hand,... I don't want to start a business,… this isn't really relevant to me (Tanzia, FG1).*

This was corroborated by Fahmida in the FG2 who stated:

> *I think for the Amazon project it's only useful if you are interested in business and stuff like that. But they gave an insight into how it was regarding taxes and stuff like that and how to set your own business. I think if you want to do business that is a good route to go down, but I didn't find it that interesting because I personally don't want to do business (Fahmida, FG2).*

However, not every student reported feeling like this about Amazon Trader:

I disagree. I think the Amazon project was a good project because who I was working with as well we were able to look at prices and charts and stuff that I wouldn't really look at if it wasn't for the project because it is something, I wouldn't do on a day to day basis. Because of that I was inspired to do much more. So, I think the Amazon Trader project is a good one (Nadman, FG2).

I think it helped people get an idea of what they want to do in the future as well because Amazon Trader, people found out that they want to do something along the business line as well, so that kind of helped a bit (Myesha, FG2).

Similarly some students seemed to value different contexts. For example, Fahmida reported finding Maths of Migration useful, but not Amazon Trader:

I think some of the projects were useful. Maths of Migration, it allowed us to use maths but it was also useful, because it's about immigration, something that affects everyday life. Amazon (Trader)… I didn't really that, that much because it was really long and tedious and stuff like that. Half of us couldn't even get a good profit/it. But it was good practice for maths, so there is that (Fahmida, FG2).

As analysed above authentic contexts that are interesting, relevant and competitive appeared to have the potential to increase engagement. However, for some students, the most relevant thing to them appeared to be succeeding in their exams:

16.2.5 Preferring to learn the mathematics on the exam

In FG2, there were three students who reported not enjoying completing projects. Two of these students (20%, 2/10) explained how they equated "useful" mathematics to the mathematics that is on the GCSE exam and both of these students gave this as the reason that they did not like learning through projects:

That often [in a normal lesson] you learn the maths you can use it in real terms. Like you know what you're going to use and you learn stuff that you know will be directly useful to you… You learn all the maths and then that maths will come up on your exams so that will directly help you get the grades. If you enjoy maths for other reasons then these projects might be-- you might find them really interesting but I don't really enjoy maths that much. So, I didn't find it that enjoyable (Anika, FG2).

(Why didn't you enjoy the projects?) Because I just don't find them useful. Maths in general I don't really enjoy, so with stuff like the toilet water one (Splash Down) we had to do but to me it sounded pointless. So I wouldn't really use my time in that lesson productively. (So what do

you want to be using your time for?) The actual maths lesson, stuff that would help me in my exam (Fahiza, FG2).

This was not a commonly held opinion with only 19% (6/32) students stating that they (strongly) agreed with the statement "I don't like projects because they don't help me pass exams" (Figure 16-4). However, students who reported enjoying the projects acknowledged the importance of exams. One student explained how she felt the nature of the project should be dictated by the point of learning that students were at. Students in year 11 needed to have projects that are GCSE content focused, whereas in lower school, this wasn't important:

> *If we do projects now considering we're in year 11. I think like Anika said if we do it, then it has to be related to our GCSEs and things we don't know because then you'd be enjoying the work you do but also understanding topics you never knew also. So, it just depends on where you are and the point of time. If you are in year 11 compared to year seven it would be different (Fahmida, FG2).*

The survey results suggest that the main reason students try in mathematics is because it is an important exam subject, with 88% (38/43) agreeing, or strongly agreeing with this statement. However, 67% (24/36) (strongly) agreed with the statement that their main interest was in learning maths; doing well in the exam was a spin off (Figure 16-4). This suggests that maybe students care about both, but succeeding in exams dominates their thinking.

In summary it appears that some students' thinking has become so dominated by trying to pass exams that they report only finding maths "useful" if it is seen as helping them to pass exams. This can be a barrier for these students with engaging in projects.

16.2.6 Authentic contexts creating complexity

In LO1 and LO3, the nature of the context was observed to create challenges for the students. In LO1, the students appeared to find it challenging to understand things such as VAT or the 'fulfilment fee'; however the mathematical processes, such as calculating a percentage, seemed routine for the students. In this lesson, the teacher appeared to notice that the students were not always paying enough attention to the contextual references. At one point, he drew the class back together to highlight how the delivery cost could be per item, or could be for all of the items. This seemingly prompted the students who were being observed to check that their interpretation of the authentic context was correct.

In LO3, students were given a task where they could practice their modelling skills before the main project. In this task, they were asked to identify the factors that could be used to determine a student's 'attitude to learning' grade on a school report. They had been given a school report, which the teacher later reported he had intended for them to use as a stimulus. However, the report seemed to distract them, instead of talking about what factors the grades could be based on, they spoke about the grades that the student had received. It took a lot of support and scaffold from the teacher to help the students understand that he wanted them to generate factors that would make a grade higher or lower, and not try to understand this student's grades. At the end of the lesson, the teacher explained that he had hoped to move towards modelling in the lesson, but simply understanding the concept of what they were doing took the students much longer and required a lot more teacher support than he had anticipated.

This finding was supported by a comment from Tanzia, that the terminology of the projects was confusing:

> *The referral cost and fees and everything. And I was like, What is that? I was so confused, I was like. Okay, this is going to be hard for my tiny brain (Tanzia, FG1)*

Authentic contexts sometimes created tangential conversations, this was observed happening in over 50% of the groups (57%, 4/7). For example, in LO1, Group W started to talk about advertising campaigns, whilst in LO3 both groups discussed their own grades.

In conclusion, students reported valuing authentic contexts that allowed them to use and apply their mathematics and to learn about the authentic context itself; authentic contexts that are interesting, relevant and competitive can increase student engagement. However, what students perceive to be interesting and relevant differs between students and for some students exam success dominates their thinking. Lastly authentic contexts can create complexity for students and can lead to tangential conversations.

16.3 Metacognition

Metacognition became a core code as FG1 students reported that they found the most challenging aspects of leading their own learning during PBL to be that of conceptualising the problem and devising a plan, both metacognitive skills. Some students (40%, 4/10) reported appreciating metacognitive support from their teacher or the resources that their teacher provided and over half of the students (50%, 5/10) suggested that teachers should provide further support. In this section, I discuss the nature of the support that students and teachers reported to be effective.

16.3.1 Key Findings:

- Students reported requiring support to conceptualise the problem and to see connections between the project and their prior mathematical knowledge;
- Students reported finding it challenging to devise a plan, especially for younger students.

- Students with mathematical creativity may find it easier to devise a plan.
- Teachers can support students to devise a plan by completing a plan with the students, providing students with writing frames or showing them examples of what should aim to do.
- All students observed appeared to regularly check their work. They reported that they did this by: checking with their peers, checking how many of the tasks had been completed and checking how they were doing in relation to the goal of the project.

16.3.2 Conceptualising the problem

When asked what the most challenging thing about leading her own learning was, Tanzia replied: *"It was more of understanding of how to do it, than actually doing it"* (Tanzia, FG1). Later in FG1, Anjum reported: "S*o, I feel that the first lesson, we didn't really do anything as we didn't understand*" (Anjum, FG1). She described how she felt that having a first lesson that was focused on ensuring that the students understood, that was interactive and where students could ask lots of questions would be really helpful in aiding understanding. She described it as a lesson where "…*you can just be carrying on asking the teacher questions, quite an interactive lesson*" (Anjum, FG1). This was supported by the findings from the survey (Figure 16-5).

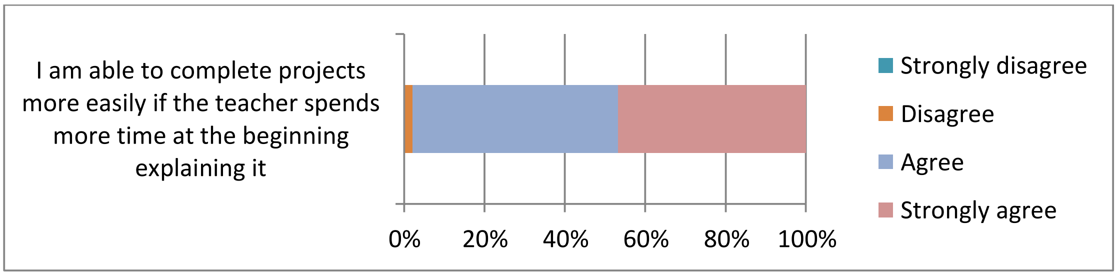


Figure 16-5: Survey responses to the statement "I am able to complete projects more easily if the teacher spends more time at the beginning explaining it."

Tanzia reported that she had felt frustrated if she didn't understand, as it could take a long time for the teacher to be able to provide help:

> *Because mostly it's one teacher then we're calling him and then the other people are calling so it's like okay, he came to us really last minute and that time I was like right, there's no point you coming now because the lesson finishes in, like, 10 minutes. We have to do this, and we haven't even started (Tanzia, FG1).*

When asked what support a teacher could give to help her in the projects she replied: *"Well for there to be more staff in the room that could help, not just one teacher. It's quite a big project"* (Tanzia, FG1). Later in this chapter under the core code of confidence, I discuss how students require enough confidence to take risks and accept that they might not fully understand everything at the start of the project.

In summary, students reported that they needed time and support in order to conceptualise a problem and understand what is required, this support included being able to ask questions and get help from a teacher.

16.3.3 Seeing connections

In FG1, students described how challenging they found it to see the connections in the mathematics that they had previously learnt and how to use this mathematics within the projects. The students implied that they understand how to complete the mathematical processes but described it is being harder to use the processes within a project:

> *(You [said] you were feeling lost…, is that usual in a maths lesson?*
> *TANZIA:Not… where the normal content that we actually learn is yeah, I learn it but it's that when it comes to projects and stuff that's,*
> *ANJUM:To put that work into project.*
> *TANZIA:Yeah.*
> *ANJUM:It's harder*
> *TANZIA:It's harder because you just get in class you are just getting that a simple equation and then we will just working out that equation*
> *ANJUM:Then we'll just work in everyday life resource.*
> *TANZIA: Yeah.*
> *ANJUM:Also, that when you learn things you are like we need to use that but then like when we did the projects and like you saw how much stuff you actually need to know to put through it.*
> *TANZIA:Yeah, we needed like,*
> *ANJUM:It's like how much percentage and stuff.*
> *TANZIA:Yeah, you need to know the percentage and then you need to also know, like, what's it called when you divide it. What was it called?*
> *(The proportion of it?)*
> *TANZIA:Is that how much that one cost? Yeah, that, yeah, portion of it.*
> *(Anjum and Tanzia, FG1)*

Both students in FG1 described finding it useful completing starter activities that got them to practice some of the skills required for the project and see the connection to what they had learnt before. Anjum explained, *"then you could get into it after*" (Anjum, FG1).

This idea was supported by the findings of the survey (Figure 16-6).

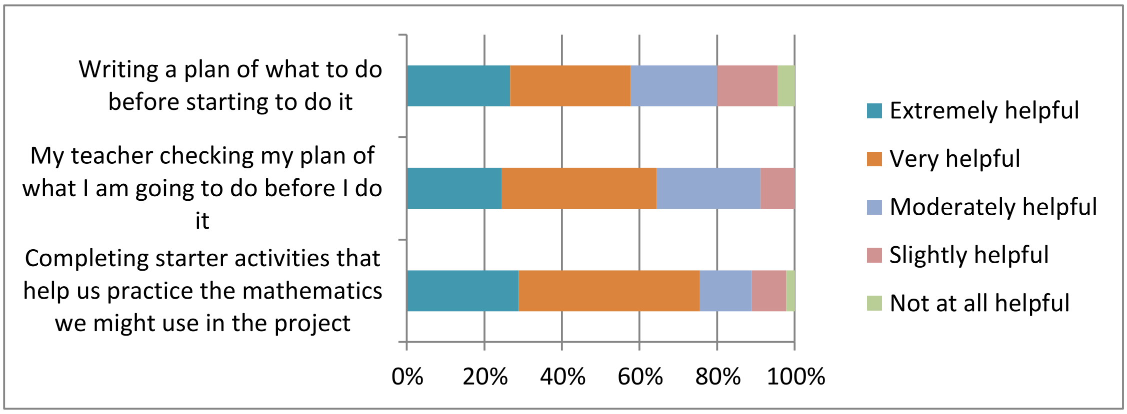


Figure 16-6: Survey responses about how helpful the students **found these strategies for leading their own learning during projects.**

In summary, students can find it challenging to see how they apply the mathematics that they have learnt previously into a project. Using starter activities that re-cap prior learning that links to the project can help students to re-activate this knowledge.

16.3.4 Devising a plan

Devising a plan was challenging for students. Many students in FG2 (62%, 5/8) stated that the teacher should provide support with this. In FG1, Tanzia explained that she had struggled with understanding in what order she should carry out different mathematical processes and in interpreting the answers that she was getting:

> *(So, what was the hardest thing about the project?) Actually understanding… what we are doing and selling? Because I was confused…I was confused of the steps, because I didn't know what to do in the steps. I knew how to… figure out how to get the Best Buy and whatnot. But it was just that I didn't know what step goes with what, and I was just confused…, because this comes out one one one and this comes out another number and I was like ok, which ones which. So, I didn't really know (Tanzia, FG1).*

One of the students reported that it was harder doing projects when they were younger as it was complex for a younger student without experience to be able to manage a larger project:

> *I think the hardest thing was in the year 8 where we did the (Design a) Bag project. We didn't have a good understanding on how to do a project like that and that was relatively large for a year eight. I thought that was hard. Me and Nadia we did the PMCC thing and we didn't know how to do it at the time so we learned that and found that difficult… I think if we did the (Design a) Bag project now it would be easier because we know how to go about it (Fahmida, FG2).*

Another student added that he felt they now had problem solving skills that mean they are much better equipped to tackle a large project; they were better at devising plans and seeing the connections within the mathematics that they know and can draw on:

> *(What skills do you think you have now that help with the projects that you didn't have then?) Problem solving like (there's) so much to get done and see the maths with it - with the maths that's involved in the project (Shaffat, FG2).*

Mr Jafri reported how he felt a lack of creativity made it difficult for students to think about ways to approach a problem:

> *I guess maybe creativity and kind of thinking outside the box. It's one of the things, because I've been… expanding the modelling project and kind of making it more general, I'm just realizing that creativity is a massive part of that. Because to be honest I thought creativity came in terms of coming up with context and stuff like that but that's the tip of the iceberg isn't it? It's more about: there's any number of ways you can go about this problem… I think… creativity is a big one. I know that (if) I was in school in a math's lesson and a solution, a problem, required some creativity I would have struggled with it. I would have really struggled... it's something I think needs to be developed (Mr Jafri, Int).*

In lesson observations a number of strategies for supporting students with devising a plan were observed. In LO2, before commencing each part of the project, the teacher modelled how to complete it with the whole class. After the teacher had finished explaining one of the sections, one of the students turned to his 'devise a plan' worksheet that he had completed earlier in the lesson to help him:

> *Students find what they had costed 'the brownie' and turn to find appropriate slide. (F) looks at his "devising a plan" sheet. (E) asks (F) to pass the costs (LO2).*

In LO1, both groups were observed to be using the writing frame which the teacher had given them to help structure their answers. It was observed that in choosing to use this, they seemed to naturally follow the methods that this writing frame suggested. When asked about what they thought of being given this table, Anjum described it as "*helpful to be honest*" (Anjum, FG1), whilst Taniza added, "*It was helpful for laying everything out... What goes where and how much you make out of it*" (Tanzia, FG2).

In LO3, the students worked together as a class unit and students were not observed devising or executing a plan.

Mr Drew suggested that students might find it easier to devise a plan if they knew what they were aiming for, for example by showing students examples of what they could create.

> *I can definitely see how it's overwhelming for them. I don't know how to attack that, I don't know if we'd want to show them a bit more, very early on… show them what we are expecting or what previous end products would look like. Maybe give… them a varied number of end products to give them… an idea (of) where they should be aiming to get to (Mr Drew, TW).*

In the survey, 80% (36/45) of students found writing a plan of what to do before starting to do it extremely, very or moderately helpful (Figure 16-6). The teachers found this result surprising, as the viewed making a plan as a key skill and they thought more students would find it very or extremely helpful. Mr Drew attributed this to the idea that maybe the plans hadn't been done very well:

> *I'm quite surprised that… only half… thought that it was (extremely or very) useful. I'd want it to be… everybody. I'd expect it to be higher than that because… if they don't really know where they are going when you are asking them to do a plan… maybe they find the plan difficult. If only half of them think it's useful maybe it's just that plan wasn't done very well (Mr Drew, TW).*

More students found teachers giving feedback on their plans helpful with 91% (41/45) reporting this as extremely, very or moderately helpful (Figure 16-6). One of the teachers also reported how making students' plans helped to inform their planning as *"it just gives you a chance when you do start the next lesson even just a reminder for yourself that you need to talk to that group about this*" (Mr Robinson, TW).

In summary, students reported finding it challenging to devise a plan, especially if it was a large and complex project. Students reported that they found it easier when they were older as they had stronger problem solving skills. One teacher reported that mathematical creativity may help students to devise a plan more easily. Other strategies that can help with devising a plan are giving students a writing frame and showing them what they are aiming for.

16.3.5 Checking

In LO1 and LO2 students in each group were observed checking their work with other students. Both groups observed in LO1 (Groups V and W) chose to work on tasks collaboratively. Whilst being observed, students in both of these groups constantly vocalised the decisions they were making and the mathematics they were doing. In the observation form, it was noted that working collaboratively in this way, seemed to give the students more confidence in what they were doing. Similar things were noted from LO2 in Groups X and Y: students often spoke their thinking out loud, although as the students were sometimes working on different things, there were instances when students appeared to ignore what the other person was saying.

The students reported different ways of checking to see that they were on track with their project. Firstly, checking in with their team mates as observed and described by Tanzia:

> *And then say to teammates oh, this is how I did it, and to check if we did it properly with the other teammates (Tanzia, FG1).*

This comment was supported by the findings from the survey (Figure 16-7).

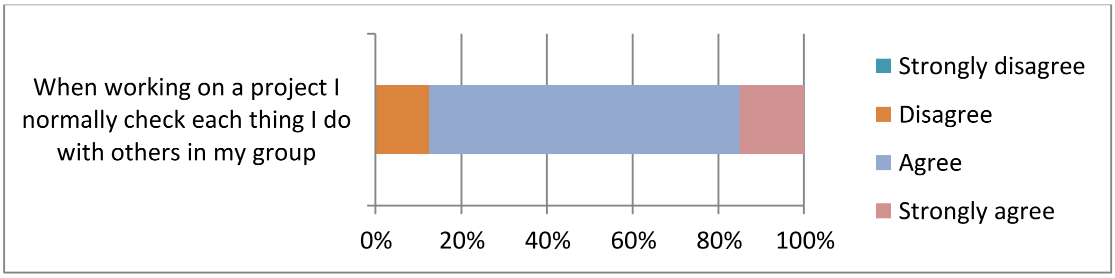


Figure 16-7: Survey responses to the statement “When working on a project, I normally check each thing I do with others in my group.”

Secondly, students reported checking their progress through a project by seeing how many of the tasks they have completed:

> *If you know you've done a lot of the things already and you've ticked off all of the tasks then you know you're doing well (Fahmida, FG2).*

Lastly, students reported checking against the final outcome required:

> *Say, for instance, Amazon Trader, if half way through you're in debt, then you're not doing well (Fahmida, FG2).*

Other strategies students reported finding helpful to check that they were on track included having a mid-way check-up as a class and a checklist. These are discussed under self-confidence, whilst they help with the meta-cognitive skill of checking, they can also develop confidence.

Mr Jafri reported that during the lessons he would try to support students with checking themselves, by simply reading to them what they had written and getting them to reflect on whether this was appropriate:

> *But in terms of the actual creative part of it was more like I let them do it… sometimes all I'd have to do was… say, "All right you read that to*

me….What does that sound like?" and then they'd realize they need to work on that… I guess I was just more… attentive to what they were doing just helping them critique themselves (Mr Jafri, Int).

When asked how he had created such good quality outcomes with a particular class, Mr Jafri reported it was the students regularly checking and reviewing work and then refining it, which created such a strong output:

I think maybe the difference was that I spent more time on it and we did them and re-did them. We did… lots of drafts…. It was always the kind of thing where actually this could be better, let's try that one again. What would you improve this time?... The only place I structured it more was I included a little bit of like stats work…, sort of prior knowledge stuff (Mr Jafri, int).

In summary, students were observed to regularly check their work with their peers. They also reported other strategies for checking their progress, including checking how many of the tasks had been completed and checking how they were doing in relation to the goal of the project. One teacher reported that it was possible to get students to self-check their work by reading aloud what they had written and asking them to consider if was correct. Checking, reviewing and re-writing were reported to help improve the outcome of the projects.

This study found that students reported struggling with the metacognitive skills of conceptualising the problem and seeing connections. In this section, I discussed some of the strategies that teachers can utilise to provide support for students such as allocating plenty of time to ensure students have understood the problem, having starter activities to activate prior knowledge, helping students to write a plan, giving students writing frames and showing students what they are aiming for with exemplars.

16.4 Resilience

Resilience is required if students are to succeed in completing projects, and is a core code for this reason. Mr Jafri described resilience as "*the number one thing, and the idea of developing and improving an answer*" (Mr Jafri, Int). As noted when discussing authentic contexts (16.2), the right context appears to be one of the factors that support students to show more resilience. Students reported further factors that affected their resilience, which will be discussed in this section.

16.4.1 Key Findings:

- Many of the students observed showed high levels of resilience when working on the projects;
- Students reported giving up if: they didn't feel they would be successful, didn't understand the mathematics, felt they had made a 'wrong' choice, or felt overwhelmed by the amount of work.

- Students were observed and reported using a number of strategies to overcome obstacles including: working collaboratively, posing questions to each other and using reference sheets; and
- Teachers felt that student resilience and supporting students to have stronger resilience were key elements to students working effectively on projects.

Most of the groups, when observed working independently from the teacher on a project, showed a strong commitment to their work (78%, 7/9 groups). This determination to work appeared to be so strong from one student that even when her team mate tried to take her off task, she remained focused, as shown in the observation of Group V:

> *Anjum exhibits a huge engagement in the task. She works solidly for the entire 10 minutes that I watch her. One of her team mates, Tanzia, is showing extreme off task behaviour and states, "You know what I want to do right now, I want to colour a picture in." Anjum simply ignores this. Tanzia later decides that she feels that they have finished and so she states "It is chill time." Again Anjum ignores her. Tanzia then says to Anjum "You really care about this don't you?" Anjum replies that she does (LO1).*

Similarly, in LO2, Fabiha was so engaged in the lesson, that she appeared not to hear what her partner asked.

Not every student who was observed showed resilience. When discussing a student's lack of resilience, Mr Jafri reported that he felt that student was very out of his comfort zone and that this is why he gave up during the lesson:

> *(I've moved where he sits three times), not because he's badly behaved, it's just trying to find someone that… he's going to work well with, that kind of fires him up... you've seen his book, he's really messy in his book but … he's (often) answered a lot of questions…when it's like straightforward… he's fine…, so I guess PBL is way out of his comfort zone (Mr Jafri, Int).*

In summary, most of the students showed strong resilience during projects, however this was not always the case. I will now go onto explore the factors that can lead to lower students resilience:

16.4.2 What triggers a break down in resilience?

The students reported a number of situations where they were likely to have low resilience and give up. Tanzia explained how she lost motivation as she didn't think that her project would be successful. Even though it is likely that she would never launch her business, the idea of her business idea being viable was seemingly important to her:

> *For me I was arguing with her… "no one's gonna buy it because it's like five pounds and there's no point of doing this project anymore…." and I was just… looking at the negative side of it (Tanzia, FG1).*

Tanzia also reported that she gave up if she felt she could not understand the mathematics. During Amazon Trader, Tanzia reported that whilst one of her team mates had understood the work, she had not. This led to her ultimately failing to try:

> *She was really into it and… got the hang of it and… I gave up at that point. I said, "right, you can do all the work, I'm not doing anything anymore" (Tanzia, FG1).*

Mr Robinson reported that in the projects, some choices had more consequences than others and that students would lack resilience when they felt that they had made a wrong choice:

> *I don't think I've ever done a project where kids are really confidently making decisions all of the way through. In the migration (project), when they choose their own line of inquiry…, that maybe goes smoothly because there is less of a repercussion if they make a (mistake)… it doesn't matter really what they choose… as long as it's something they are interested in. Whereas later on… they have to start thinking about which graph (they) should… choose…, some graphs can work and some graphs can't work. So, I think those sorts of decisions when it is clear that they've chosen the wrong thing, maybe that's the point where they sort of retreat a bit. I think some decisions they make are easier and they are more comfortable with and some they are less. You have to really sort of push them to talk about it with you and that's the only way they make that decision (Mr Robinson, TW).*

Tanzia described how her motivation came and went throughout the project. When she felt confident in what she was doing mathematically, she reported feeling more motivated. However she reported that sometimes she became overwhelmed by the amount of work she had to do and this led to her giving up:

> *Because I didn't know anything about Best Buy until Sir actually taught me. And then I was like, oh, this is how you do it. Then I was telling Anjum, and…she was working it out and then she got stuck so I helped. And then that's when I was a little bit motivated, because that's when the first time we actually doing it, I… (felt) it might be fun. But then when it came to all the other papers…, I was like, this is a lot of work, you have to put in a lot of brains into this, and that's not for me (Tanzia, FG1).*

One of the teachers reported that he felt that it was often the students who showed least motivation in classes generally who also showed less resilience when learning through projects:

But generally speaking, the people who are really engaged in a normal lesson would be the ones who are really engaged in the project... The ones who struggle to get motivated in normal lessons they're the ones who will be sitting back in a group… They're quite happy to be led (Mr Drew, TW).

In this study, students reported giving up if they didn't feel they would be successful, didn't understand the mathematics or felt overwhelmed by the amount of work they had to do. I now consider strategies to help ensure students demonstrate strong resilience:

16.4.3 Supporting student resilience

Students described a number of strategies that they used to overcome barriers. One of the strategies identified by both students and teachers was getting help from the teacher. Tanzia explained how a pivotal moment in her engagement was when the teacher came to her group and talked them through an example:

He gave us examples of one of the items and… that helped us because that's what we had to do… for our products, as well. So, I feel like that helped. Because if he didn't teach us that, I'll be so lost, like I don't know what I'm doing. We… have to keep on like, try different methods. And then me, it's just that if I tried different methods, and it still doesn't work. I'll give up. Now I'll be like, I don't want to do it anymore (Tanzia, FG1).

Similarly, Ammara reported how when someone helps and supports you, this not only helps you overcome that obstacle, but also encourages you to try harder:

If you got confused…, having a figure to tell you, "Oh you can move on by doing this. You can overcome this by doing this" it will encourage you further and you want to achieve your goal (Ammara, FG2).

This finding is supported by the survey (*Figure 16-8*).

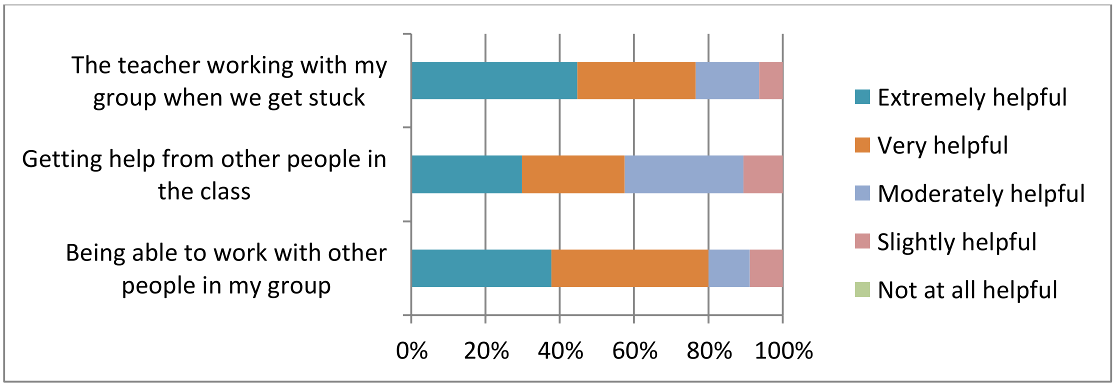


Figure 16-8: Survey responses about how helpful students find particular strategies for leading their own learning during projects.

The teachers highlighted how the student's attitude to help was really important:

Some of them might just put their hand up and say, "I can't do it, what do I do?..." like they've already given up. Then… other kids… ask a question like, "how do I know whether this" or "which one of these things should I do first". That means they're starting to think about (it). They are starting to structure things and then you can have… (a) conversation and hopefully they come up with the decision on their own. But they seem genuinely interested in how to solve the problem or how to reach the end point. Rather than the person… who just wants you to do it for them (Mr Robinson, TW).

The teachers reported that they felt it important to support students who appeared to need it rather than waiting for the students to come to them:

And you need to give them a little nudge like, "Come on what do you think?" and, "What would you do in this situation? (Mr Drew, TW).

Another strategy for supporting students to overcome barriers was that of using a resource that was given by the teacher. During LO1, students were observed using the reference sheets to help them when they got stuck. At one point Group W struggled to work out the answer together. Student D tried to get the teacher, but the teacher didn't come over, so they referred to the reference sheets they have been given to help them.

Most groups in LO1 and LO2 (80%, 4/5 groups) were observed working collaboratively to overcome barriers. When Group W were observed seemingly hitting difficulties, for example, being unsure of what things meant or how to calculate the things required, they worked together to figure out the problem. They did this by posing questions to each other which they then tried to answer.

These findings are supported by the survey data (Figure 16-8).

In summary, students were observed or reported using a number of strategies to overcome obstacles including: working collaboratively, posing questions to each other, and using reference sheets. I now consider how we can help develop resilience in students:

16.4.4 How can we help students develop resilience?

Mr Jafri explained that he thought that the series of single lesson problem solving tasks that teachers did with students at the beginning of year 9, before embarking on longer projects were challenging, but developed students' resilience:

"They found those (the single lesson problem solving tasks) tough all year, but… that's why I think they help… They properly build resilience and that kind of thing... It was one where basically they had to keep improving on an answer or it was like they found out one way of doing it

but there were loads of others and it was like "no go on, get to the next one." "What's the next way, what's another way we can do it?"… Just the sheer amount of struggle with them… but that's how I kind of knew that it was good: it was working (Mr Jafri, int).

The study found that resilience is a key skill for students during projects. In this section, I discussed the factors that can lead to a break down in resilience: students feeling they wouldn't be successful, students not understanding the mathematics or feeling overwhelmed by the amount of work they had to do. I explored the strategies that can be used to support students: working collaboratively, posing questions to each other and using reference sheets. Lastly I considered a possible way of developing students' resilience through using regular shorter problem solving tasks.

16.5 Self-confidence

Self-confidence is a core code as students reported being more able to take control and lead their own learning when they were confident that they knew what to do:

We kind of took more control because Sir came at first and explained it to us (DG, FG1).

In this way, self-confidence links to choice (16.1.2): students that seemed to have more confidence took more control; and teachers reported that students require more confidence when there isn't a definitive 'right answer'. Self-confidence is also linked to context (16.2) and working collaboratively (16.6): both have the potential to increase student confidence.

16.5.1 Key findings:

- Projects require students to take risks and make decisions which necessitates self-confidence;
- Student confidence during projects can be supported by: outline plans/ checklists, students presenting their work, midway check-ups and students working collaboratively; and
- Projects can develop student confidence.

Teachers reported thinking it was important that students felt confident enough during projects to take risks. Mr Drew, with agreement from Mr Robinson reported:

Some students might find it a bit overwhelming because it's something they have no idea about so (they need) some of that confidence to put (them)self in the danger zone… and just see what happens (Mr Drew, TW).

Similarly, when discussing the autonomy he offered students in LO2, Mr Jafri reported that he structured the lessons as he felt that some students in his class were afraid of getting things wrong:

> *[sighs] probably lots of accumulative bad experiences with getting things wrong… a real fear of getting things wrong. Maybe.. an association of… a sense of progress with… marks and scores and tests and things like that… Maybe early on in education not getting that safe opportunity to go wrong.*

The idea that some students lacked confidence is supported by students in FG1 and FG2 (20%, 2/10) who stated that they disliked being confused and not knowing what to do, even though this can be an important part of the problem solving process. Similarly in the survey, 78% (31/40) (strongly) agreed that they sometimes found projects overwhelming because they didn't know what to do at the start. This is juxtaposed with 81% (30/37) agreeing or strongly agreeing that they feel confident that they are doing the right thing when they are completing a project (Figure 16-9). These results may indicate that students struggle at the beginning of a project, but feel more confident when they have commenced working. Further, some of the strategies that I discuss below may have helped support students' confidence as they progressed through the projects.

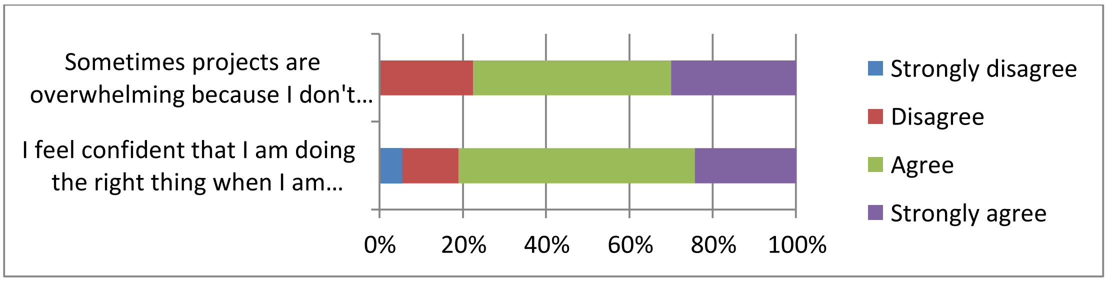


Figure 16-9: Survey responses to statements on confidence.

As analysed under metacognition (16.3), many students felt having an outline or checklist would support them and give them confidence about what was expected from them:

> *I think many times when we were doing the project I felt really confused about what we were supposed to do, so I think it would have been good if we had an outline of what we needed to get done, but not really detailed, just an overall what we need to get done (Anika, FG2).*

One student explained how she would have liked an outline with "*boxes to tick and it gives you an exact plan to follow, so you know where you're going and you won't get confused as you would just like starting from scratch*" (Ammara, FG2).

Whilst no student in FG2 disagreed with this idea, one student said "*I think it should just be vague. Just show what you are supposed to do but not exactly so then it has some sort of independence to it for the students*" (Fahmida, FG2) with Amara adding, " *and creativity*" (Ammara, FG2). These students were both largely positive about the projects and seemingly wanted to feel reassurance that they were doing what was expected of them, without losing the freedom and the autonomy that the projects provided.

Nadman highlighted how having to give a presentation on their work gave them increased confidence in their own knowledge:

> *I think it built [my] confidence because we had to speak in front of a panel and [this] made me realize how we have to be on the ball and know everything, and the ins and outs of what (the) project really is. If come in there with a brief idea and didn't actually know what the project is truly about…, (I would) be lost (Nadman, FG2).*

Shaffat, when asked how he would know if the project was going well, suggested a midway check-up to compare how you are doing against others and to help reassure that you that you are where you should be:

> *Do like a check-up in the middle of the project to see how everyone is doing. Everyone gives feedback on what they have done so far and how it's gone and compare and see where you should be (Shaffat, FG2).*

The concept of a midway check-up being helpful is reasonably well supported by the survey data (Figure 16-10).

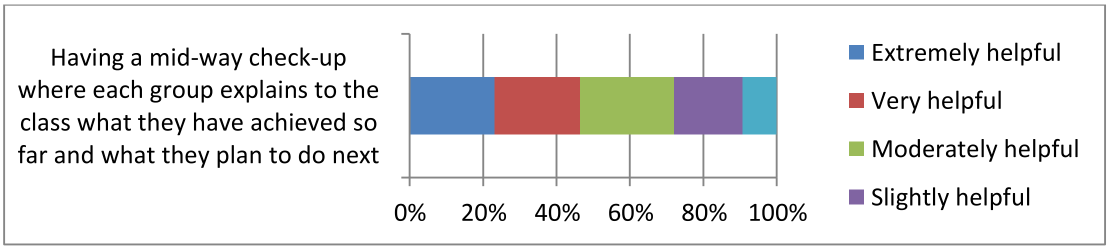


Figure 16-10: Survey responses about how helpful the Students found this strategy for leading their own learning during projects.

Students seemed to gain in confidence from working together. Tanzia reported how, if she was working without her partners support, she might give up:

> *If you give us another project to do, and it's like 'Best Buy' as well…, I wouldn't know because I didn't get enough from the project that we did and she did so she'll be like, "Okay, I'll do this". But if I do something else, and I don't know how to do it, I'll sit there and be like "I don't know what I'm doing" (Tanzia, FG1).*

Two of the teachers reported that through completing projects they hoped students would develop the confidence to take risks and work independently:

> *I think some independence…, having that skill of just having a go at something when you're not sure what the outcomes going to be. I think working on something, (where) they don't know where it's going to lead*

and sometimes lead down avenues which… (aren't) successful and trying to get the confidence to do that - I think is massive (Mr Drew, TW).

I feel like it could hopefully… make them a more proactive person and less of an ostrich. When they are given a problem… then having the confidence of having done that before in maths lessons… maybe (they would have the confidence) to get started and just dive into it rather than… letting it fester (Mr Robinson. TW).

The survey data showed that the majority of students felt projects went someway to helping them feel more confident about the mathematics they used (Figure 16-11).

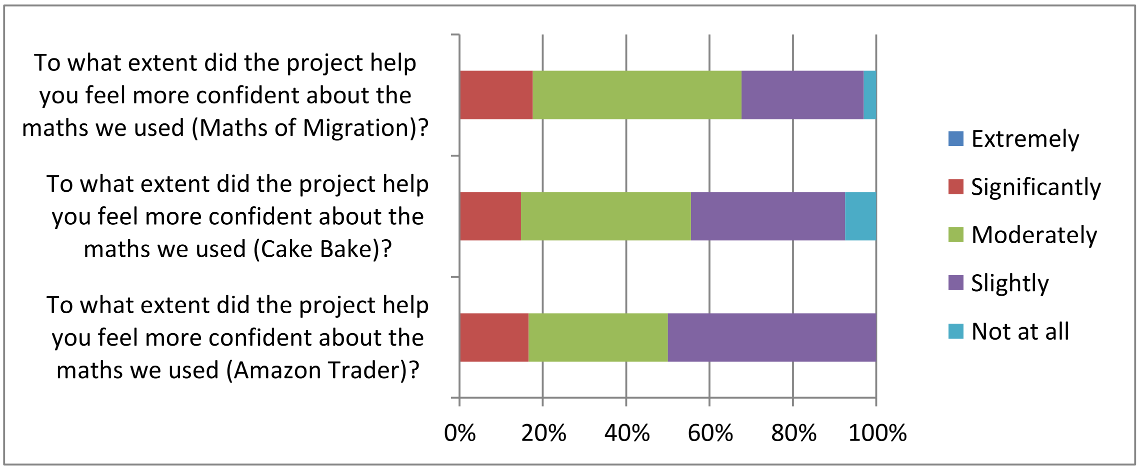


Figure 16-11: Survey responses to the question "To what extent did the project help you feel more confident about the maths we used?"

The study found that students require self-confidence in order to lead their own learning to make decisions that they can't be sure are correct and to take risks. Student confidence during projects seemed to be developed by giving students an outline plan, asking them to present their work, having midway check-ups and working collaboratively. The majority of students also reported that projects can develop their confidence with the mathematics they used.

16.6 Students Working Collaboratively

Working collaboratively was a key theme in FG1 and FG2 with almost all students (90%, 9/10) commenting about working with others.

16.6.1 Key findings:

- Students reported that effective student collaboration requires: everyone in the group to work hard, the students to motivate each other and to have good communication;
- Students value working collaboratively;
- Students appeared to find distributing work challenging; and
- Group roles may support effective collaboration.

16.6.2 Effective Collaboration

Students were observed to be working well together for the majority of the time in 4/5 groups from LO1 and LO2 (80%). However, in Group Z, one of the group members appeared to be largely off task. In the lesson observations, students were observed showing effective collaboration including: questioning each other (Groups W and V), challenging each other's thinking (Group W), seeking reassurance from each other (Group Y) and working together to collaboratively solve a problem (Groups V, W, Y, Z).

One of the students in FG2 highlighted how having an effective group was an important part of successfully working on a project. In an effective group, you have "*people that are motivating, they encourage you to do the work*" (Myesha, FG2). This idea was supported by Mr Drew's comments, when describing a class that he felt worked well collaboratively he reported how he thought that there were students in that class who would push other members of their group:

> *This was with the year nines who were really keen on things like that. I'm sure with a different group of students it wouldn't have gone that way. But there are some groups of quite intelligent kids and quite a lot of loud forthcoming kids, the sort of the ones who would take over the groups (Mr Drew, TW).*

One student stated that to be a good team member in an effective group it was important that you contributed to the team effort:

> *Not being lazy, that's important because you've got [to] feel… bothered and want to do your part in it. You can't be lazy and sit back and watch other people take on more work than you. You've got to work for your place (Ammara, FG2).*

This was added to by Myesha who stated that for a group to be performing well, everyone would be doing something to ensure that the work would be completed:

> *You know what Ammara said, that little checklist that's really vague, how everyone is doing something to get it done (Myesha, FG2).*

Two of the students in FG2 reported how important communication was within a group. "*If you don't have a really good group that communicate well then you can't get anything done*" (Anika, FG2). Nadman explained that you needed to communicate effectively with the people in your group as this would help you to overcome challenges together:

> *Because normally you know who you can speak to more freely and if you have someone that you don't really know you might not communicate with them as much. If you run into a barrier no one is going to overcome it and (it) kind of comes to a halt, no one really does anything (Nadman, FG2).*

All of the comments in focus groups were positive about group work. This was supported by the survey data with 95% of students (38/40) (strongly) agreeing with the statement that they think that group work helps the learning of mathematics (Figure 16-12). In the survey, when asked to give three things that they liked about completing projects, over 50% of students reported liking things linked to working collaboratively for one or more of the projects, for example working in a group or with a partner (51%, 24/47).

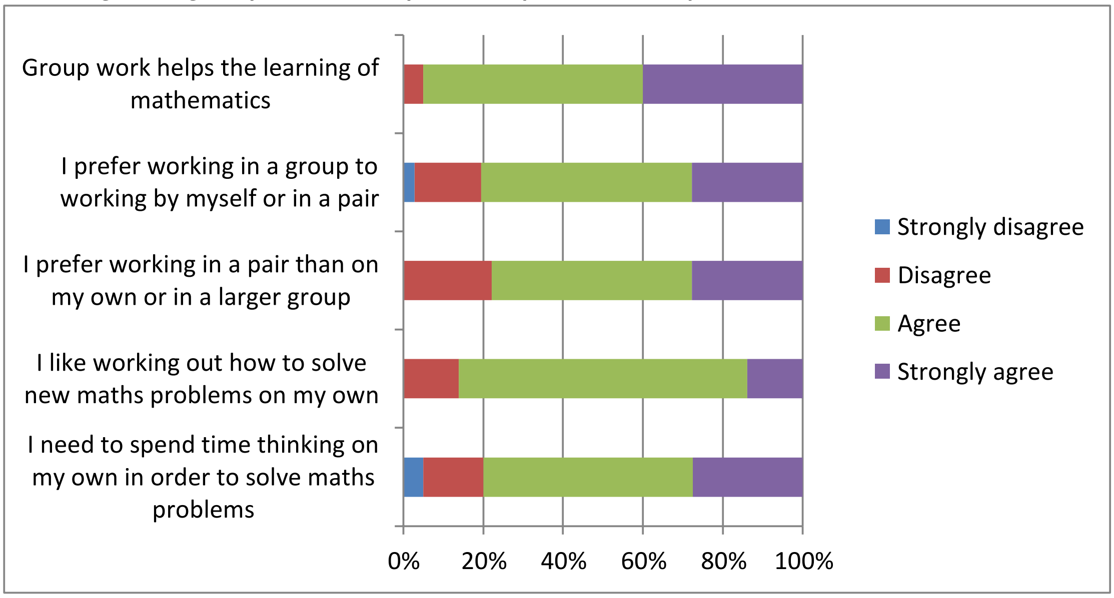


Figure 16-12: Survey responses to questions on group work.

The survey produced inconclusive results about whether students preferred to work in a group or a pair (Figure 16-12). This unclear result is in part due to a limitation with the wording of the questions, as it could be that both responses included the notion that pair or group work was preferable to working on their own.

Students reported appreciating working on their own with 80% (32/40) agreeing or strongly agreeing the need to spend time thinking on their own in order to solve maths problems and 86% (31/36) agreeing or strongly agreeing that they like to work out how to solve new maths problems on their own (Figure 16-12). Whilst this may initially seem contradictory, it may be that the students value having time to think on their own as part of their collaborative work.

Some of the students reported the reasons that they value group work in the survey. Two students explained that they liked to "*share ideas*" (81 and 9F13, survey). Another student explained how working collaboratively "*not only helped my partner, but also myself*" (8M1, survey). One of the students reported that they found working collaboratively the most helpful way to work in all classes:

> *I find group-work… the most helpful system around all classes and subjects because you can see other people's point of view (91, survey).*

The teachers interviewed as part of the teacher workshop also reported viewing students working collaboratively as a positive aspect of the projects. For example, Mr Drew reported that he felt that working collaboratively helped to increase students' employability skills:

> *A lot of time when we think about doing team activities within a group it might just be doing a puzzle or some sort or just working through a problem together. But this is, as you say,… the closest to real life teamwork that they might experience in a work environment, they get (Mr Drew, TW).*

Mr Drew also said that he felt that some classes were better at working collaboratively than others, but that even when students found it challenging to work collaboratively, it was still worthwhile because of the wider gain:

> *It's not the same experience for everybody. From the teachers and the students 'perspective, I think, it's quite different. But worthwhile nonetheless (Mr Drew, TW).*

In the survey, when asked to give three things that they liked about completing projects, 19% (9/47) of students reported disliking something to do with working collaboratively. These comments were about "how the group was bad" (9M4, survey), "how people weren't trying" (survey, 9F1) and to do with a fair distribution or recognition of the work:

> *Often the teachers wouldn't know who took the most responsibility in the project (basically if somebody was doing more work than others). This didn't happen in my group though (8M1, survey)*

Three of these nine students who gave one of the reasons for disliking the project to be linked to group work, also gave something they liked about the project as group work. For example one student wrote:

> *I liked how there was a lot of group work this allowed us to share ideas. I… didn't enjoy how I worked with my partner (9F13).*

In summary, students reported that effective student collaboration requires every student in their group to work hard, the students to motivate each other and to have good communication. Strong collaboration was viewed by students as being positive as it allowed them to share ideas and help each other. Teachers also reported on the wider benefits such as employability skills. Whilst the students valued working collaboratively, many of them found it challenging to distribute work throughout their group, which will be discussed in the next section:

16.6.3 The challenges of distributing the work

For Group V and Group W in LO1, the distribution of tasks appeared limited: students seemed typically to work together on the same task. They did, however, contribute in different ways to the completion of the task, as discussed in the next section on group roles. In LO2 (Groups X and Y) the students were observed distributing the tasks: students would work on different things simultaneously. In LO3, students were given short discussion based tasks which they worked on collaboratively in their groups.

This was also reported by one of the students who said:

> *I remember in Amazon Trader and my group would all research for the products together on Alibaba and all figure out how tax worked and stuff like that. So, it wasn't really like separate work (Fahmida, FG2).*

One student described how she felt that when they were doing something new and challenging it was better to work together: if they worked on separate things then they might not know how to complete the task and therefore wouldn't be able to get on with the work:

> *We really need experience for the stuff what we're gonna do because if you give us like another project to do, and it's like Best Buy as well, me I wouldn't know because I didn't get enough from the project that we did and she did so she'll be like, Okay, I'll do this. But if I do something else, and I don't know how to do it, I'll sit there and be like I don't know what I'm doing. And if we come together, and I don't have anything she'll be like,ok, now I have to help you and we'll be losing time. So it's just… (Tanzia, FG1).*

Anjum described how having a stronger understanding of what to do to complete the project would allow them to distribute the work more easily:

> *I think that would be easier. If we had like a whole first lesson of like everything that we'll have to do and then we could, so we already had an understanding and that we could divide it by like our group members. I think that would be easier and then come back and see that how it helped them and what we all did individually. Then as a team of three we can fix anything or do anything (Anjum, FG1).*

One of the students reported that if they had an effective group, they felt that they could distribute the work between the members of the group and trust that everyone would get things done:

> *I think if you have a good group as well, it's really important, because if you have a good group you can split the work equally and you know that they are going to get their bits done (Anika, FG2).*

This links to the sub theme of students wanting support with choices about the timeframe: if students don't distribute the work, it will be harder to get things completed on time. Myesha described how in her group they realised that in order to complete the project on time, they would have to distribute the work:

No, we were kind of forced to complete it even if it was really hard. Everyone had to split the work and it had to be rushed. So, it wasn't properly done (Myesha, FG2).

Mr Robinson reported that he often found that students worked on their own doing their own thing rather than distributing the work between them:

I think all the other times, [apart from Amazon Trader,] things have just happened so they [the students] just end up working on their own and doing their own project and that's maybe why I feel like I need to improve my project lessons a bit (Mr Robinson, TW).

The results from the survey questions to do with distributing work could be viewed as being unclear. When asked how often they chose who in the group would complete each task, 100% (6/6) of students reported often or always on Amazon Trader , 76% (22/29) on Cake Bake and 84% (31/37) on Maths of Migration (Figure 16-13). However, when asked 'when working on a project in a group, we normally all complete the same task together', 71% of students (27/38) reported that they (strongly) agreed (Figure 16-14). The teachers in the FG speculated that this discrepancy could come from students thinking that *"working together is… how they should attack this problem"* (Mr Drew, TW). Equally, it could be that students simply chose to all work on the same task together, which as highlighted above, they reported sometimes working effectively.

Students were observed and reported often finding it challenging to distribute the work between the members of their group. They may have found that they were more able to get the project completed on time if they were able to do this more effectively. However they also identified situations where they reported finding it helpful to work collaboratively on one task.

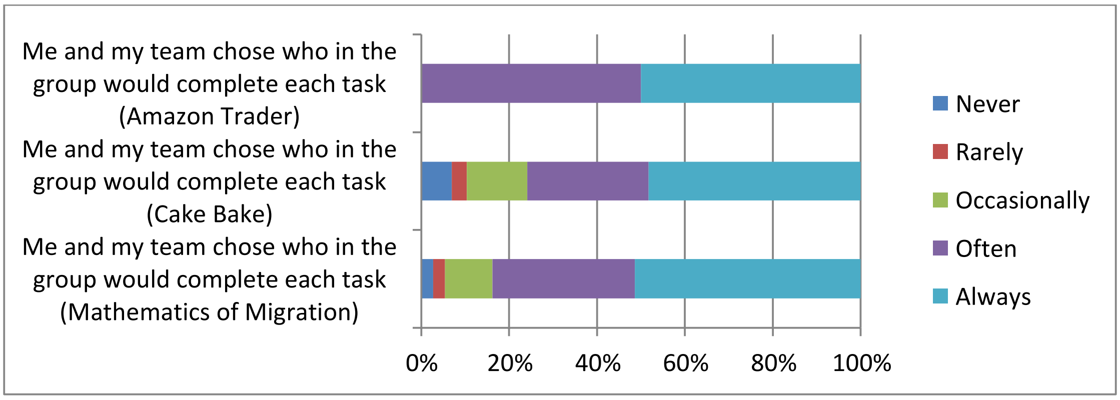


Figure 16-13: Survey responses to the question "Me and my team chose who in the group would choose each task?"

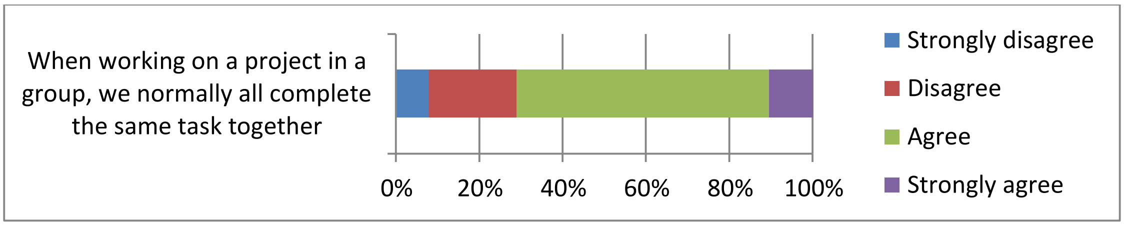


Figure 16-14: Survey responses to the statement "When working on a project in a group, we normally complete the same task together."

16.7 Group Roles

The students had not been given formal roles in any of the lessons observed. However, in the first lesson, students appeared to take on different roles within the group. In the observation of Group W in LO1, Ismaeel completed all of the calculations and wrote everything down. However, his partner constantly asked him questions, which he responded to, and in this way, his partner helped to lead the thinking. In Group V, again one student, Anjum, took on the role of doing all the calculations. However, in this group, she also drove the direction that their project went in. She was the one who posed constant questions and articulated her thought processes. It was her team mates who had to answer these questions and completed the calculations she requested.

Some students (40%, 4/10) reported taking on different roles within their teams quite naturally. Similarly, the results from the survey found that the majority of students only found not at all, or slightly challenging to work with others and allocate roles (Figure 16-15).

The students had not been given formal roles in any of the lessons observed. However, often students appeared to take on different roles within the group. For example, in LO1, Ismaeel completed all the calculations and wrote everything down. However, his partner constantly asked him questions, which he responded to, and in this way, his partner helped to drive the thinking. Some students in the FGs (4/10) reported taking on different roles within their teams quite naturally. Myesha explained: "*We basically just saw what everyone was strong with, what they thought they could do.*"

Then we just went from there" (Myesha, FG2).

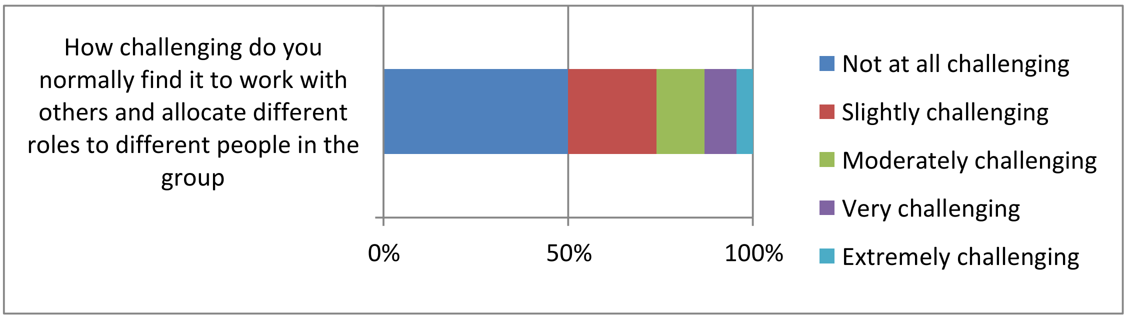


Figure 16-15 : Survey responses to the question "How challenging do you normally find it to work with others and allocate different roles to different people in the group?"

One of the teachers reported observing a similar phenomenon: someone in the group took on a leadership role and distributed the tasks:

> *I think quite often you get someone who leads the group whether it's decided by the group or decided by the individual. But they'll often need someone to be the driving force behind it and quite often there is someone who gives out jobs to people because they see it's too much to do on their own and so they end up prioritizing who can do what (Mr Drew, TW).*

Interestingly over 50% of students in the survey reported that they (strongly) agreed that they normally took the lead when completing a project (Figure 16-16). This question had the highest number of "not sure" responses (38%, 18/47).

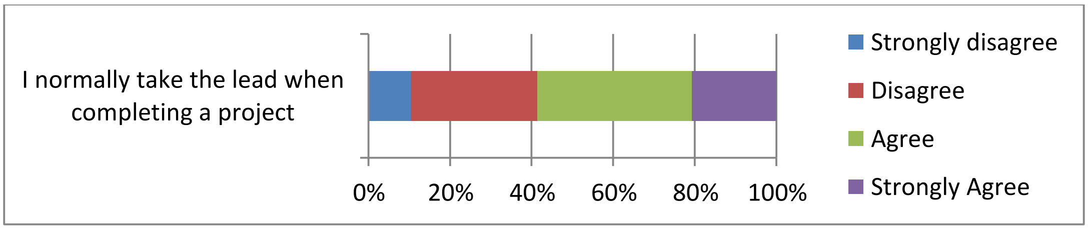


Figure 16-16: Survey responses to the statement "I normally take the lead when completing a project."

One student, who stated he preferred lessons with more structure, suggested that it might have been good to have pre-allocated roles for students:

> *When you have a group, everyone is trying to do something different and it's hard to get organized from the beginning. In the future if we were doing projects, if we were given a certain role, I think it would be easier to figure out ourselves (Shaffat, FG2).*

However, this didn't appear to have much traction with other members of the focus group, when asked if anyone else thought it would be helpful to have them, the only student to answer was Fahmida who reported that as the students were working in small groups of only 2 or 3, they weren't necessary:

> *We weren't in big groups though; we were in groups of two or three so it wasn't that hard. It was more like distributing the work you do properly…. We never really assigned roles as well in the groups so that kind of altered how we did it too (Fahmida, FG2).*

However, this student was one of the students who said that she didn't typically distribute the work in the teams she was in.

This idea of group roles appeared to resonate with the teachers. Mr Robinson stated he wanted develop his practice by allocating group roles:

> *One thing I want to maybe try is someone to do a group one just gives specific roles for all the members in the group so that at least they know what their role is within the group and that might make things a little less overwhelming, maybe. It might give them an idea about what they need to do first (Mr Robinson, TW).*

Mr Drew added to this explaining that as a department they could choose four different group roles that students could take on:

> *Even if we just say there is going to be four people per group, if… then as a [department] team we decided what… we think four different job tasks should be for four different people and give them the opportunity to take one on. That is then structuring it a bit more isn't it? Do we want to structure it that much or do we want them to make that decision? Maybe they do need that, they have to make decisions (Mr Drew, TW).*

He seemed worried that this was perhaps providing too much structure; however Mr Robinson refuted this explaining how this is what would happen in the workplace.

> *That's what they would do in a workplace, right? There would be specific roles for people… or one member of the group would allocate different roles. It would be… good if there was a project manager where one of them has to make more decisions and oversee things and allocate jobs out and things like that. Then maybe if over a few different projects you swap that around so that everyone has a chance to do it (Mr Robinson, TW).*

During the projects some students naturally took on different roles, however, it may have helped students to work together more effectively if they were given group roles.

17 Appendix 17 – Vignettes

Here I present further data as four vignettes, each of a student or students who seemed to demonstrate different beliefs. I chose to use vignettes as I felt that they would help me highlight particular findings (Ely, 1997 and Spalding, 2007) and to portray what I had learnt from my interpretation of the data (Ely, 1997). I used the vignettes to demonstrate my emergent inductive conjecture around attitudes: that a student's self-efficacy, vision of mathematics, and goal orientation appeared to mediate that student's attitude towards PBL. As Langer (2016) highlights in their discussion around vignettes, this writing is a situated act within the interpretive process. A vignette provides one account of the truth: it is a mediated account, constructed by myself through the selection and interpretation of the data.

To try to ensure that the vignettes provided a trustworthy account, for each account I drew from the data: the student's attitude, their reported or observed beliefs, emotions and behaviours, alongside their reported challenges and perceptions on strategies that they felt supported them. To increase the trustworthiness, as suggested by Spalding et al. (2007) I included quotes. Miles (1990) suggests that the vignettes should be created through an iterative interaction between the participant and the researcher, whilst Langer (2016) suggest using member checking. I wrote the vignettes too late for this to happen in this study, however I did show the participants earlier interpretations of their responses. I received feedback from one of the participants who agreed with what I had written. This suggests that she felt it was a fair summary. The Covid-19 pandemic stopped me completing further member checking.

In vignette 1, I present data from Tanzia who appeared to have low self-efficacy and a relational vision of mathematics; in vignette 2, from Anika, a student who appeared to have a high self-efficacy, instrumental understandings of mathematics and a strong exam or performance orientation; in vignette 3, from Shaffat, a student who appeared to have an instrumental vision of mathematics and a mastery goal. Both Anika and Shaffat shared some similarities in their attitude towards PBL. Lastly, in vignette 4, I present data from Nadman and Fahmida, students who appeared to show a high self-efficacy, relational understanding mathematics and mastery goal orientation.

There were no students in the study whom I deemed to have a low self-efficacy and instrumental vision of mathematics, although from my broader knowledge of education, I am sure that they exist within the school. I assume that such students did not have a motivation to take part: they did not see the value in PBL, nor did they believe in themselves as mathematicians and for these reasons, I presume, they did not feel that they wanted to contribute to the study.

17.1 Vignette 1

'I'm interested, but I don't think I can do it' - Perceived low self-efficacy and relational understanding of mathematics.

Tanzia's (year 10 student, in LO1 and FG1) account of learning through MaPBL was largely positive. However, she experienced passing, in the moment emotions such as frustration. These led to her having a low resilience and exhibiting off task behaviour. The challenges Tanzia reported when studying through PBL were: drawing connections between prior mathematical knowledge and the mathematics in the project; understanding the order that she needed to carry out mathematical processes; understanding how to interpret the answers she was getting; approaching the project mathematically and the terminology of the projects. Tanzia appeared to most value personalised support from the teacher. She made use of all other support offered.

I deemed Tanzia to have low self-efficacy and a relational understanding of mathematics. She reported how she thought it was important to understand mathematically and not just complete mathematical processes:

> *It was more… understanding… how to do it, than actually doing it. Because if you don't understand it, and you're just doing the maths, it's just like, okay, I'm doing this, but I don't even know what it's used for.*

Beliefs

Tanzia reported: valuing learning about authentic contexts, feeling that some projects might appeal more to some students than others, and that learning through projects was harder that typical classwork. She reported:

> *I have learned how people actually start their business and that what items they need… you have to think of delivery…, postage…, how much money is going to Amazon.*

> *Especially if you want to become a business person, like Abdullahi is very into it, because he wanted to make…a business…, I don't want to start a business,… this isn't really relevant to me.*
>
> *It's harder because… in class you are just getting a simple equation and then we… [are] just working out that equation*

Emotions

Tanzia reported enjoying learning through PBL. She described it as *"new… a good experience".* However, she appeared to have significant anxiety about whether she had understood the mathematics and would be successful: she frequently wanted reassurance from her peers or the teacher. This in turn impacted on her behaviour. She reported:

> *Because mostly it's one teacher then we're calling him and then… other people are calling, he came to us really last minute and… I was like right, there's no point you coming now because the lesson finishes in, like, 10 minutes.*

Behaviours

Tanzia was observed to struggle to remain on task. She described how her motivation fluctuated throughout the project. She reported that she gave up if she felt she could not understand the mathematics, didn't think that her project would be successful, or felt over overwhelmed by the amount of work she had to do. When she felt confident in what she was doing mathematically, she reported feeling more motivated.

> *Tanzia, is showing extreme off task behaviour and states, "You know what I want to do right now, I want to colour a picture in." Anjum simply ignores this. Tanzia later decides that she feels that they have finished and so she states "It is chill time" (LO1).*
>
> She was really into it and… got the hang of it and… I gave up at that point. I said, "right, you can do all the work, I'm not doing anything anymore."
>
> *For me I was arguing with her… "no one's gonna buy it because it's like five pounds and there's no point of doing this project anymore…." and I was just… looking at the negative side of it.*
>
> *I didn't know anything about Best Buy until Sir actually taught me… Then I was telling Anjum, and…she was working it out and then she got stuck so I helped… That's when I was a little bit motivated, because that's when the first time we actually doing it, I… [felt] it might be fun. But then when it came to all the other papers…, I was like, this is a lot of work, you have to put in a lot of brains into this, and that's not for me.*

One of the teachers reported that he felt that it was often the students who showed least motivation in classes generally who also showed less resilience when learning through projects:

> *But generally speaking, the people who are really engaged in a normal lesson would be the ones who are really engaged in the project... The ones who struggle to get motivated in normal lessons they're the ones who will be sitting back in a group… They're quite happy to be led (Mr Drew, TW).*

Challenges

When asked about the most challenging aspect of driving her own learning, Tanzia replied: *"It was more of understanding of how to do it, than actually doing it"*. Tanzia described how challenging she found it to: see the connections between the mathematics that they had previously learnt and how to use this mathematics within the projects; understand in what order she should carry out different mathematical processes; and interpret the answers that she was getting (this was a common finding, with students in FG2 (5/8) stating that the teacher should provide related support). She didn't always base her decisions on mathematics, but on her perceptions of what was best and sometimes she found the terminology of the projects confusing. She reported:

> *(You [said] you were feeling lost…, is that usual in a maths lesson?) TANZIA: Not… where the normal content that we actually learn is yeah, I learn it but it's that when it comes to projects and stuff that's,*
> *ANJUM: To put that work into a project… It's harder. (So, what was the hardest thing about the project?) Actually understanding… what we are doing and selling?... I knew how to… figure out how to get the Best Buy and whatnot. But it was just that I didn't know what step goes with what, and I was just confused…, because this comes out one one one and this comes out another number and I was like ok, which one's which.*

> *(did you choose your products based on what would be best mathematically?) We chose, not cheap stuff, but it was that stuff you can buy a market that is like affordable just there. You don't wanna spend like five pounds on just earrings.*

> *The referral cost and fees and everything... I was so confused. Okay, this is going to be hard for my tiny brain.*

Strategies for support

The support that Tanzia reported as most helpful was personalised support from the teacher. Tanzia also utilised the writing frame that both groups in LO1 were observed using. This was given to them by the teacher to help them structure their answers. Taniza explained, "*It was helpful for laying everything out... What goes where and how much you make out of it.*" She also reported:

> *He gave us examples of one of the items and… that helped us because that's what we had to do… for our products, as well. So, I feel like that helped. Because if he didn't teach us that, I'll be so lost, like I don't know what I'm doing. We… have to keep on like, try different methods. And then me, it's just that if I tried different methods, and it still doesn't work. I'll give up.*
>
> *(What support could a teacher give you?) Well for there to be more staff in the room that could help, not just one teacher. It's quite a big project."*

The teachers in the workshop highlighted how the student's attitude to personalised support was really important:

> *Some of them might just put their hand up and say, "I can't do it, what do I do?..." like they've already given up. Then… other kids… ask a question like, "how do I know whether this" or "which one of these things should I do first". That means they're starting to think about (it). They are starting to structure things and then you can have… (a) conversation and hopefully they come up with the decision on their own. But they seem genuinely interested in how to solve the problem or how to reach the end point. Rather than the person… who just wants you to do it for them (Mr Robinson, TW).*

The teachers reported that they felt it important to support students who appeared to need it rather than waiting for the students to come to them:

> *And you need to give them a little nudge like, "Come on what do you think?" and, "What would you do in this situation? (Mr Drew, TW).*

17.2 Vignette 2

'Maths is only useful if it helps me pass my exams'– perceived high self-efficacy, instrumental understanding of mathematics and performance orientation.

Anika (year 11 student, attended FG2), in her account, reported she did not enjoy learning through PBL. Her main reasons for not enjoying the projects were that she did not feel that they helped her to gain a knowledge or understanding of the mathematics on the exam. She did, however, acknowledge that the contexts of some of the projects meant that they were interesting and that some students in her class may enjoy the projects in a way that she did not. Anika did not report on her own behaviour during lessons, however, another student in the focus group implied that perhaps she did not work to her fullest during PBL. Anika reported experiencing challenges in knowing what to do during PBL and described feeling confusion. She suggested that to help with this, teachers could provide an outline of what the students needed to do. She also reported that it would be challenging if you did not have a group who communicated well. However, she felt that a good group would provide support in completing the work.

Anika, demonstrated a very clear exam orientation stating that she considered mathematics 'useful' if it is the mathematics that "*will come up on your exams so that will directly help you get the grades."* She did not seem to see any use in mathematics beyond this. She also demonstrated an instrumental understanding of mathematics. For example, she described how she didn't develop *"skills"* and her summary of a project where students had to develop their own line of enquiry and explore it through a large data set appeared very instrumental: "*we had to get data on certain groups of people and we had to put that into a poster."* I viewed Anika as having high self-efficacy.

One of the challenges with using a grounded approach is that when interviewing, I did not realise that I would need to probe students to understand their goal orientation. From Anika's comments, I assume that she has a performance goal orientation; however I cannot be sure of whether she has a mastery orientation too.

Beliefs

Anika reported that she did not feel that learning through projects was useful, she felt *"they were just so random, like the toilet water one, it was so out the blue and it didn't really help me with maths." As* highlighted above, she reported only perceiving mathematics as useful if she thought would directly help her to pass her GCSE exam.

She did, however, report finding one of the Maths of Migration projects interesting:

> *I don't think it was really helpful in terms of myself because I don't think I picked up any skills doing it, but it was interesting to find out about the immigration stuff and know how people are suffering and how maths can be brought into it.*

Emotions

The main emotions that Anika reported experiencing when learning through PBL were a lack of enjoyment: "*I think that I personally didn't really enjoy the projects*" and confusion: "*I think many times when we were doing the project I felt really confused about what we were supposed to do."* The reason she gave for not enjoying them was described under the section on beliefs: she didn't feel that they helped her to learn the mathematics that she would need on the exam. She had some suggestions for how a teacher could support her with her feelings of confusion, which are discussed below. However, confusion is not necessarily a negative emotion.

Behaviours

Anika did not comment on her own behaviour during PBL. However, when Anika made a comment about not feeling that she had learnt much mathematically from the projects, Nadman responded:

> *I disagree because me and Yusef had to put in a lot of time and effort into thinking about the maths behind the Amazon project. I think it's not just the business side of things but there's a lot of complex and interesting maths that people missed out on by not doing the project to the fullest (Nadman, FG2).*

I interpret this statement as implying that perhaps Anika did not engage as fully as she could have done whilst completing the project.

Challenges

Anika reported she found it challenging it you don't have a 'good' group as this would hinder your ability to work effectively: "*If you don't have a really good group that communicate well then you can't get anything done.*" As analysed in the section on emotions, Anika reported feeling confused and not knowing what to do. She explained that she felt this more strongly in some projects than others:

> *With the migration one we knew that we had to get data on certain groups of people and we had to put that into a poster. So, we knew what we were doing, but with the bag we didn't really know what we needed to do or how we were supposed to do it.*

Strategies for support

Anika suggested that the teacher could provide students with a written outline of what they needed to get done in the project and reported how important it is to have an effective group in order to distribute the work:

> *So I think it would have been good if we had an outline of what we needed to get done, but not really detailed just an overall what we need to get done.*

> *I think if you have a good group as well, it's really important, because if you have a good group you can split the work equally and you know that they are going to get their bits done.*

17.3 Vignette 3

'This isn't the best way to learn maths' – perceived high self-efficacy and strong performance goal orientation

Shaffat's account (year 11 student, attended FG2) of learning through PBL was largely negative. He reported he did not enjoy learning through projects as he felt he learnt better in a more traditional classroom. He reported experiencing challenges in being organised when learning through PBL and ensuring that the work was finished in a timely manner. The strategies for support that he suggested were often connected to supporting his organisation during PBL, such as a checklist with suggested timings and having group roles. He also reported that he felt that a midway check-up and strong problem-solving skills, especially the ability to understand which mathematics to use, would aid students during PBL. Despite not enjoying PBL, he seemed to respect the autonomy that this style of learning provided.

Shaffat appeared to demonstrate a strong performance goal orientation. For example, he wanted a midway check-up so he could *"compare"* how he was getting on relative to his peers. He also appeared to want to outperform his peers*: "the winning team gets something, and that made me want to do really well."* From the data it was difficult to ascertain whether he had an instrumental or relational understanding of mathematics, however it was clear that he valued his learning being structured. I also viewed him as having a high self-efficacy.

Beliefs

Shaffat appeared to believe that mathematics was better learnt in a more traditional classroom environment rather than through PBL. He reported that he preferred the structure of normal lessons:

> *I like maths a lot but… I see more value in the classwork. I felt like I learn better in a classroom environment than doing a project. That is just for me, I know other people find it different… I like the structure more.*

Emotions

Shaffat clearly reported that he disliked the projects, “*I just didn't like doing the projects. It just felt like I could be doing something else to learn more than do the project.”*

Challenges

Shaffat reported that he found it challenging to complete the work in time: “*Some things took too long and they shouldn't have.”* As data in the next section suggests, he also appeared to find it challenging to be organised during PBL.

Strategies for support

Shaffat reported on three different strategies that he felt might be supportive, particularly with his organisation: group roles; a checklist with timings on it; a mid-way check-up; and better developed problem solving skills. He reported that:

> *When you have a group, everyone is trying to do something different and it's hard to get organized from the beginning. In the future if we were doing projects, if we were given a certain role, I think it would be easier to figure out ourselves.*
>
> *So it's a bit more organized… Having a set time for each part would have been a bit easier because a realistic time that you could finish that part in. That would have been more helpful.*
>
> *Maybe not the teacher sets out the time, but if the group, if they've got three hours to do it and you split up your time between different parts of the project well then you fill in where you put the time. So, it's to do with your needs. Some people might do one part faster than another.*
>
> *Do a checkup in the middle of the project to see how everyone is doing. Everyone gives feedback on what they have done so far and how it's gone and compare and see where you should be.*
>
> *(What skills do you have now that you didn't have when first completing PBL). Problem solving, like there's so much to get done and see the maths with it. With the maths that's involved in the project.*

17.4 Vignette 4

'Let me do this!' – Perceived high self-efficacy, relational understanding of mathematics and mastery goal orientation.

Nadman and Fahmida's (year 11 students, attended FG2) accounts of learning through PBL were positive. They reported valuing the wider learning and found the context of some of the projects interesting. Both reported that they felt that the projects helped them mathematically. Nadman particularly, reported working harder during PBL. Fahmida experienced challenges in managing her own time properly, understanding the project well enough to distribute the work between her team members and focusing on the mathematics. Fahmida reported that support such as a checklist may help students; however she wanted it to be vague to allow the students independence. Nadman appreciated the support that he got from his peers and being able to work collaboratively.

Both Nadman and Fahmida appeared to demonstrate high self-efficacy, a relational understanding of mathematics and mastery goal orientations.

Whilst these students are viewed as having a mastery goal orientation, it does not mean that they didn't also want to succeed in their exams, just that they saw other reasons for learning mathematics also. For example, Fahmida explained how "*if we do projects now, considering we're in year 11… then it has to be related to our GCSEs and things we don't know because then you'd be enjoying the work you do but also understanding topics you never knew also."*

Beliefs

Fahmida reported that she thought that the projects were useful. However, unlike Anika who appeared to view 'useful' as a synonym for ensuring exam success, Fahmida defined useful as something that helped students to use their mathematics in an everyday context:

> *I think some of the projects were useful. The immigration thing it allowed us to use maths but it was also useful about immigration or something that affects everyday life. The Amazon project I didn't really that, that much because it was really long and tedious and stuff like that. Half of us couldn't even get a good profit out of it. But it was good practice for maths*

Fahmida was clear that she felt some projects were more 'useful' than others. This theme ran through many of the students' responses.

Nadman also reported that he valued the wider learning gained through PBL:

> *I think the Amazon project was a good project because [of] who I was working with, as well [as the fact that] we were able to look at prices and charts and stuff that I wouldn't really look at if it wasn't for the project (Nadman, FG2).*

Both Nadman and Fahmida reported perceiving that PBL helped their mathematics. Nadman described how there was "*lots of complex and interesting maths*" (Nadman, FG2), whilst Fahmida reported that it was *"good practice for maths" (Fahmida, FG2).*

Nadman also reported that having a tangible output to the project helped to build his confidence:

> *I think it build confidence because we had to speak in front of a panel and made me realize how we have to be on the ball and know everything and the ins and outs of what your project really is. If come in there with a brief idea and not actually know what your project is truly about you can be lost (Nadman, FG2).*

Fahmida also reported valuing the autonomy that she had in completing the projects:

> *Because it wasn't like you dictating the whole lesson, it was us with the actual laptops. So, we were engaging with the data and we were discussing so it wasn't like regular maths lessons (Fahmida, FG2).*

Behaviours

Nadman reported that the projects inspired him to work harder:

> *Because of that I was inspired to do much more… After that we went home and started looking at different kinds of stock and we start thinking we could push this or we could sell this and we started contacting seller and stuff that we learned whilst doing the project like bargaining and checking the graphs and the stock that we needs, what's in demand and that kind of stuff once we go home (Nadman, FG2).*

Challenges

Fahmida reported that: managing her own time was one of the hardest elements of learning through projects; she didn't always feel she had a strong enough understanding of the project to be able to distribute the work between the members of her group; and how it could be challenging to keep focused on the mathematics. She reported:

> *It’s easy to lose track of time though when you are doing the projects…. We weren't focusing on time, which was… my main issue… I'd start it, I'd be really driven to do it but the next thing you know it’s the end of the lesson and I've only gotten one thing done.*
>
> *We weren't in big groups though; we were in groups of two or three so it wasn't that hard. It was more like distributing the work you do properly.*
>
> *It’s more interactive clearly when you are doing projects and stuff like that you don't really think about the math. We don't get a lot of chances to do projects like this in maths because it’s mainly just doing worksheets and textbook work. So, when we do, we kind of lose track of the actual maths. I think it’s not entirely maths focused.*

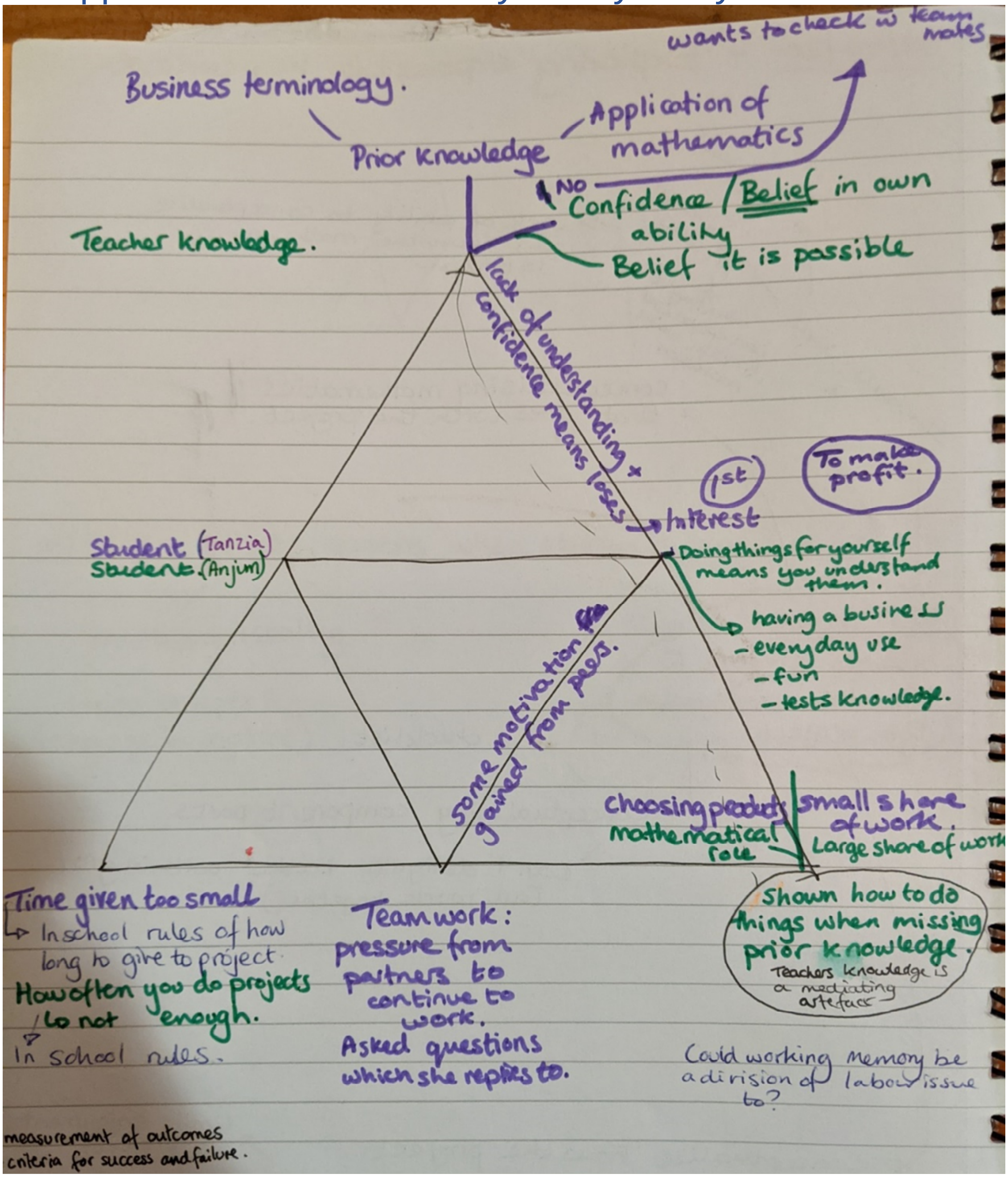
Strategies for support

Similarly to Anika, Nadman reported valuing working with the other people in his group and found that this helped him to overcome barriers. However, he also highlighted how important it was to feel comfortable with the members of the group and have effective communication. He reported:

> *I think it’s if they’re overcoming the barriers because if they're going to [do] complex maths or complex ideas they will run into a barrier and the only way they can prove that they are better than it, is if they overcome the barrier and if they can't then-- well if they're a good group they will be able to pass and achieve what they were meant to achieve.*
>
> *Because normally you know who you can speak to more freely and if you have someone that you don't really know you might not communicate with them as much. If you run into a barrier no one is going to overcome it and (it) kind of comes to a halt, no one really does anything.*

In keeping with her valuing the autonomy that PBL provided, when other students suggested that the teacher provide the students with a checklist, Fahmida reported that she wanted it to be “*vague*” and *“just show what you are supposed to do, but not exactly so then it has some sort of independence to it for the students” (Fahmida, FG2).*

18 Appendix 18 - Initial activity theory analysis



19 Appendix 19 – Survey 2 cognitive interviews

The second survey was piloted with a range of responses from colleagues at the study school, students at the study school and EdD peers. They were all asked to give feedback if there was anything that they didn't understand. Students and colleagues reported that they found the survey easy to use and they had understood it. One or two amendments were suggested, one of which was taken on board and can be seen in appendix 20.

Two cognitive interviews were also conducted. I discussed each question in the survey, except for the ones on MaPBL (year 7 had not done any MaPBL) with two different students to elicit their understanding of each question. Based on their responses, I changed one of the questions in consultation with my supervisor. I then completed a further 2 cognitive interviews around the new question to ensure that it made sense to the students. The question is highlighted below.

	Student A 29/04/21	Student B 29/04/21
Mathematical self-efficacy batch		
I look forward to my mathematics class	Enjoy maths class	Feel about maths
I feel tense doing mathematics problems	Nervous hard problems	Pressurised
I get good grades in mathematics	Unsure what "grades" he is getting	How well you do
I learn quickly in mathematics	New subjects are easy to do	How quickly you learn
I feel helpless doing mathematics problems. (CHANGED)	If you have a hard problem and there is no one around to help you would feel helpless.	Feel like you don't get any help
Goal orientation batch		

It is important for me to do well compared to others in my mathematics class	Far behind everyone doing well with the Q they are doing.	Compared to other students
It is important for me to take time to understand new ideas in mathematics.	New topics hard, takes a while for you to get good at it	Go over everything to understand it
It is important for me to I understand all of my mathematics lessons.	In future you will be tested on what you learnt	It's saying you only need maths for exams, but It can help me later -
I only need to learn mathematics for my exams.	You only need one main thing in maths, for exams. You need algebra for GCSEs.	I think everyone learns at different paces
My goal in mathematics is to get a better grade than most of the other students.	Try to get the best and be better than everyone else	
I want to learn as much as possible from my mathematics lessons.	Learn as much as possible	I want to learn a lot
Vision of mathematics batch		
In mathematics it is impossible to do a problem unless you've first been taught how to do one like it.	If covering something new when you were absent, it would be impossible to do the question	I think sometimes you can take one look and it's not that hard
I usually try to understand the reasoning behind all of the rules I use in mathematics.	Certain rule e.g. algebra, rule or some wouldn't work you need to understand the reason of the rule.	I think this is how much you understand the work
Learning mathematics mainly involves memorising procedures and formulas.	If you are learning, you need to memorise for future tests	I think this is what people say about maths

Doing mathematics consists mainly of using rules.	If you are able to use a rule or a sum in maths, it's more important to know how it works.	I think using rules of formula is more important than understanding how it works
Mathematics involves relating many different ideas.	In a class there are many different ideas, everyone thinks differently.	I think you can use different ideas. Like you take ideas from English in maths (like reading questions)
When I learn something new in mathematics I often continue exploring and developing it on my own.	If you learn something basic you could learn things that were similar	I think you can add to your own knowledge
Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works.	Maths has a lot of rules, rules more important than understanding how to do it	I think sometimes you don't need rules you can use your own knowledge
Getting the right answer is the most important part of mathematics.	Lots of working out, used all rules, got answer wrong, doesn't matter.	I think maths is about making mistakes

Second cognitive interview of new question.

I feel powerless doing mathematics problems.	If you don't understand it.	When you can't get the answer to the question.

20 Appendix 20 – Surveys considered when designing Survey 2

Questions used

Theme	Source of questions	Questions	Decisions/ thoughts
Mathematical Self-Efficacy	Trends in International Mathematics and Science Study (TIMSS) 2019: National report for England Research report	4-point scale from 'Agree a lot' to 'Disagree a lot'. 1) I usually do well in mathematics 2) Mathematics is more difficult for me than for many of my classmates 3) Mathematics is not one of my strengths 4) I learn things quickly in mathematics 5) Mathematics makes me nervous (no equivalent science statement) 6) I am good at working out difficult mathematics problems 7) My teacher tells me I am good at mathematics 8) Mathematics is harder for me than any other subject 9) Mathematics makes me confused	USE ONE OR THE OTHER. The second batch has less questions and hence likely to be more tightly aligned, and I must be very careful as I don't have a large sample size. TIMSS questions are more robust and have been tested more fully and therefore may have greater reliability. Using these would also mean I can place students in national cohort - but effect of lockdown is likely to be significant. TIMSS questions are titled mathematical confidence. They seem -possibly more like self-concept than self-efficacy. Use Blotnicky et al.'s questions.
	Blotnicky, K.A., Franz-Odendaal, T., French, F. et al. A study of the correlation between STEM career knowledge, mathematics self-efficacy, career interests, and career activities on the likelihood of pursuing a STEM career among middle school students. IJ STEM Ed 5, 22 (2018).	On a scale ranging from (1) Strongly disagree to (5) Strongly agree: 1. I get good grades in mathematics; 2. I learn quickly in mathematics; 3. I look forward to my mathematics class; 4. I feel tense doing mathematics problems; 5. I feel helpless doing mathematics problems.	

	https://doi.org/10.1186/s40594-018-0118-3		
Achievement goal orientation	Elliot, A. J., & McGregor, H. A. (2001). A 2 9 2 achievement goal framework. Journal of Personality and Social Psychology, 80, 501–519.	1. It is only useful for me to learn the mathematics that is on my exams. [Replaced original question "*It is important for me to do better than other students*".] 2. It is important for me to do well compared to others in my mathematics class. 3. My goal in mathematics is to get a better grade than most of the other students. 4. I want to learn as much as possible from my mathematics lessons. 5. It is important for me to understand what I'm taught in maths lessons as thoroughly as possible. [Replaced original question "*It is important for me to understand the content of the mathematics lessons as thoroughly as possible*". Content identified by another teacher doing the pilot survey as harde to understand for this age group] 6. It is important for me to take time to understand new ideas in mathematics. [Replaced original question *"I desire to completely master the material presented in this class".* Content identified by another teacher doing the pilot survey as harder to understand]	These questions typically have a comparable level of detail/ granularity as my qualitative data. My research hasn't considered mastery avoidance or performance avoidance. Remove these questions. The questions are more orientated towards a performance goal orientation than an exam goal orientation. Perhaps because performativity wasn't as strong an issue when this was written. Exams are gatekeeper qualifications. I view exam goal orientation as being a subset, of performance goal orientation. There would be greater construct validity if one or two questions were about exams instead.

<table>
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Table 1
Study 1: Factor Loadings for Achievement Goals
<table>
<tr><th rowspan="2">Achievement goal item</th><th colspan="4">Factor</th></tr>
<tr><th>Performance approach</th><th>Mastery avoidance</th><th>Mastery approach</th><th>Performance avoidance</th></tr>
<tr><td>1. It is important for me to do better than other students.</td><td>.93 (.97)</td><td></td><td></td><td></td></tr>
<tr><td>2. It is important for me to do well compared to others in this class.</td><td>.89 (.90)</td><td></td><td></td><td></td></tr>
<tr><td>3. My goal in this class is to get a better grade than most of the other students.</td><td>.89 (.91)</td><td></td><td></td><td></td></tr>
<tr><td>4. I worry that I may not learn all that I possibly could in this class.</td><td></td><td>.90 (.93)</td><td></td><td></td></tr>
<tr><td>5. Sometimes I'm afraid that I may not understand the content of this class as thoroughly as I'd like.</td><td></td><td>.86 (.88)</td><td></td><td></td></tr>
<tr><td>6. I am often concerned that I may not learn all that there is to learn in this class.</td><td></td><td>.84 (.85)</td><td></td><td></td></tr>
<tr><td>7. I want to learn as much as possible from this class.</td><td></td><td></td><td>.91 (.93)</td><td></td></tr>
<tr><td>8. It is important for me to understand the content of this course as thoroughly as possible.</td><td></td><td></td><td>.90 (.93)</td><td></td></tr>
<tr><td>9. I desire to completely master the material presented in this class.</td><td></td><td></td><td>.80 (.78)</td><td></td></tr>
<tr><td>10. I just want to avoid doing poorly in this class.</td><td></td><td></td><td></td><td>.87 (.90)</td></tr>
<tr><td>11. My goal in this class is to avoid performing poorly.</td><td></td><td></td><td></td><td>.85 (.88)</td></tr>
<tr><td>12. My fear of performing poorly in this class is often what motivates me.</td><td></td><td></td><td></td><td>.74 (.70)</td></tr>
</table>
Note. $N = 180$. All factor loadings $> .35$ are presented in the table. Factor loadings were obtained using principal components extraction with orthogonal and oblique (in parentheses) rotation
</td>
<td>To use the questions as they stand has limitations in relation to my data and what I am trying to find out.

Amend one performance goal orientation question so there are two questions about exams.</td>
</tr>
<tr>
<td>Vision of Mathematics (instrumental to relational)</td>
<td>Yackel 1984 in Quillen, M. A. (1998). Relationships among prospective elementary teachers' beliefs about mathematics, mathematics content knowledge, and previous mathematics course experiences (Doctoral dissertation, Virginia Tech).

And yackel 1984 in Cifarelli, V., Goodson-Espy, T., & Chae, J.</td>
<td>Yackel has not published the survey anywhere.

1. Doing mathematics consists mainly of using rules. [I]
2. Learning mathematics mainly involves memorizing procedures and formulas. [I]
3. Mathematics involves relating many different ideas. [R]
4. Getting the right answer is the most important part of mathematics. [I]
5. In mathematics it is impossible to do a problem unless you've first been taught how to do one like it. [I]

8. When I learn something new in mathematics I often continue exploring and developing it on my own. [R]
9. I usually try to understand the reasoning behind all of the rules I use in mathematics. [R]</td>
<td>Based directly on instrumental and relations conceptualisations of Skemp (1976).

One limitation is that it was designed for older students and may not work as well with younger students.

The inter-item correlation for four items, 13, 15, 16, and 19 was found to be low and did not fit well into the scale psychometrically.

Remove the four items.</td>
</tr>
</table>

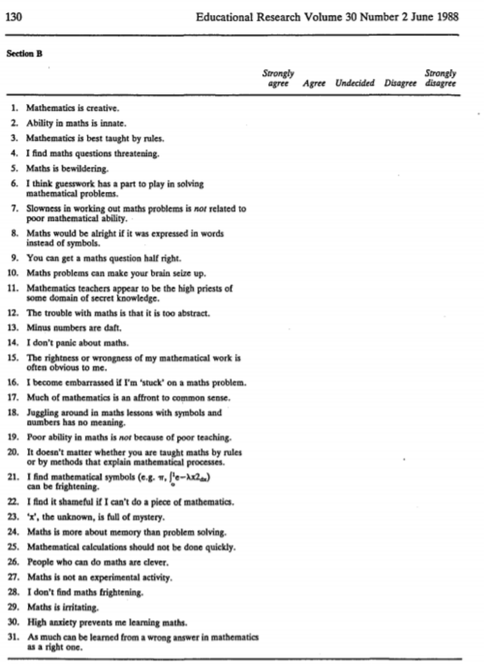
	L. (2010). Associations of students' beliefs with self-regulated problem solving in college algebra. *Journal of Advanced Academics*, *21*(2), 204-232. Yackel has not published the survey anywhere.	10. Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works. [R] **Questions from the batch not used:** 6. One reason learning mathematics is so much work is that you need to learn a different method for each new class of problems. [I] 7. Getting good grades in mathematics is more of a motivation than is the satisfaction of learning the mathematics content. [remove - overlap with goal orientation] 11. A common difficulty with taking quizzes and exams in mathematics is that if you forget relevant formulas and rules you are lost. [I] 12. It is difficult to talk about mathematical ideas because all you can really do is explain how to do specific problems. [I] [remove - not aligned with comments students made] *13. Solving mathematics problems frequently involves exploration.* [remove as found to be less reliable] 14. Most mathematics problems are best solved by deciding on the type of problem and then using a previously learned solution for that type problem. [I] *15. I forget most of the mathematics I learn in a course soon after the course is over* [remove as found to be less reliable] *16. Mathematics consists of many unrelated topics* [remove as found to be less reliable] 17. Mathematics is a rigid, uncreative subject. [I]	The Cronbach alpha for the revised 16-item document has been found to be .89 in Quillen's study Another study with 139 participants had Cronbach alpha .73. Cannot be sure if this was the whole survey or just the 16 questions used by Qullien. As I need to keep a low number of questions, I will pick a subset of the questions. I will choose questions most aligned with what the participants are saying and that the question will make sense to a 14 year old (eg computation).

		18. In mathematics there is always a rule to follow. [I] *19. I get frustrated if I don't understand what I am studying in mathematics* [remove as found to be less reliable] 20. The most important part of mathematics is computation. [I] Yackel (1984) [remove as students may not understand]	
Opinions on PBL	Based on conclusions on students' attitudes towards PBL from quals.	1. Projects are an effective way to learn mathematics (belief +) 2. Projects don't help me learn the mathematics I need for my exams (belief -) 3. I enjoy learning more independently in projects. 4. I enjoy learning through projects (emotion +) 5. I often feel confused when learning through projects (emotion -) 6. I work harder in projects than I do in 'normal' mathematics lessons (behaviour +) 7. I give up more often during projects than 'normal' lessons (behaviour -)	

Other question batches that were considered.

Enjoyment of matheatics	Pupils responded to the following statements using a 4-point scale from 'Agree a lot' to Disagree a lot'. 1) I enjoy learning mathematics/science 2) I wish I did not have to study mathematics 3) Mathematics is boring 4) I learn many interesting things in mathematics 5) I like mathematics 6) I like any schoolwork that involves numbers 7) I like to solve mathematics problems 8) I look forward to mathematics lessons 9) Mathematics is one of my favourite subjects	Trends in International Mathematics and Science Study (TIMSS) 2019: National report for England Research report	Not the right concept for my study.
Value Mathematics	pupils responded to the following statements using a 4-point scale from 'Agree a lot' to 'Disagree a lot'. 1) I think learning mathematics will help me in my daily life 2) I need mathematics to learn other school subjects 3) I need to do well in mathematics to get into the university of my choice 4) I need to do well in mathematics to get the job I want 5) I would like a job that involves using mathematics 6) It is important to learn about mathematics to get ahead in the world 7) Learning mathematics will give me more job opportunities when I am an adult 8) My parents think that it is important that I do well in mathematics	Trends in International Mathematics and Science Study (TIMSS) 2019: National report for England Research report	All phrased positively

	9) It is important to do well in mathematics		
Achievement Goal	Instructions: The following statements represent types of goals that you may or may not have for this class. Circle a number to indicate how true each statement is of you. All of your responses will be kept anonymous and confidential. There are no right or wrong responses, so please be open and honest. 1 not true of me — 2 — 3 slightly true of me — 4 moderately true of me — 5 — 6 very true of me — 7 extremely true of me Task-approach goal items To get a lot of questions right on the exams in this class. To know the right answers to the questions on the exams in this class. To answer a lot of questions correctly on the exams in this class. Task-avoidance goal items To avoid incorrect answers on the exams in this class. To avoid getting a lot of questions wrong on the exams in this class. To avoid missing a lot of questions on the exams in this class. Self-approach goal items To perform better on the exams in this class than I have done in the past on these types of exams. To do well on the exams in this class relative to how well I have done in the past on such exams. To do better on the exams in this class than I typically do in this type of situation. Self-avoidance goal items To avoid doing worse on the exams in this class than I normally do on these types of exams. To avoid performing poorly on the exams in this class compared to my typical level of performance. To avoid doing worse on the exams in this class than I have done on prior exams of this type. Other-approach goal items To outperform other students on the exams in this class. To do well compared to others in the class on the exams. To do better than my classmates on the exams in this class. Other-avoidance goal items To avoid doing worse than other students on the exams in this class. To avoid doing poorly in comparison to others on the exams in this class. To avoid performing poorly relative to my fellow students on the exams in this class.	Elliot, A. J., Murayama, K., & Pekrun, R. (2011). A 3× 2 achievement goal model. *Journal of educational psychology*, *103*(3), 632.	In this model - mastery is separated into task and self. |My research does not use this level of detail.

Vision of mathematics	 130 Educational Research Volume 30 Number 2 June 1988 **Section B** *Strongly agree* / *Agree* / *Undecided* / *Disagree* / *Strongly disagree* 1. Mathematics is creative. 2. Ability in maths is innate. 3. Mathematics is best taught by rules. 4. I find maths questions threatening. 5. Maths is bewildering. 6. I think guesswork has a part to play in solving mathematical problems. 7. Slowness in working out maths problems is *not* related to poor mathematical ability. 8. Maths would be alright if it was expressed in words instead of symbols. 9. You can get a maths question half right. 10. Maths problems can make your brain seize up. 11. Mathematics teachers appear to be the high priests of some domain of secret knowledge. 12. The trouble with maths is that it is too abstract. 13. Minus numbers are daft. 14. I don't panic about maths. 15. The rightness or wrongness of my mathematical work is often obvious to me. 16. I become embarrassed if I'm 'stuck' on a maths problem. 17. Much of mathematics is an affront to common sense. 18. Juggling around in maths lessons with symbols and numbers has no meaning. 19. Poor ability in maths is *not* because of poor teaching. 20. It doesn't matter whether you are taught maths by rules or by methods that explain mathematical processes. 21. I find mathematical symbols (e.g. π, $\int_0^1 e-\lambda x2_{dx}$) can be frightening. 22. I find it shameful if I can't do a piece of mathematics. 23. 'x', the unknown, is full of mystery. 24. Maths is more about memory than problem solving. 25. Mathematical calculations should not be done quickly. 26. People who can do maths are clever. 27. Maths is not an experimental activity. 28. I don't find maths frightening. 29. Maths is irritating. 30. High anxiety prevents me learning maths. 31. As much can be learned from a wrong answer in mathematics as a right one.	'Why we didn't like mathematics, and why we can't do it' Dennis Quilter & Eon Harper To cite this article: Dennis Quilter & Eon Harper (1988) 'Why we didn't like mathematics, and why we can't do it', Educational Research, 30:2, 121-134, DOI: 10.1080/0013188880300206 To link to this article: https://doi.org/10.1080/0013188880300206	This survey was designed to find undergraduate students with a "negative" attitude towards mathematics. It doesn't do what I want - but could form a basis to take questions from if I can't find anything better.

<table>
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<td>Behaviourist vs constructivist approach to learning</td>
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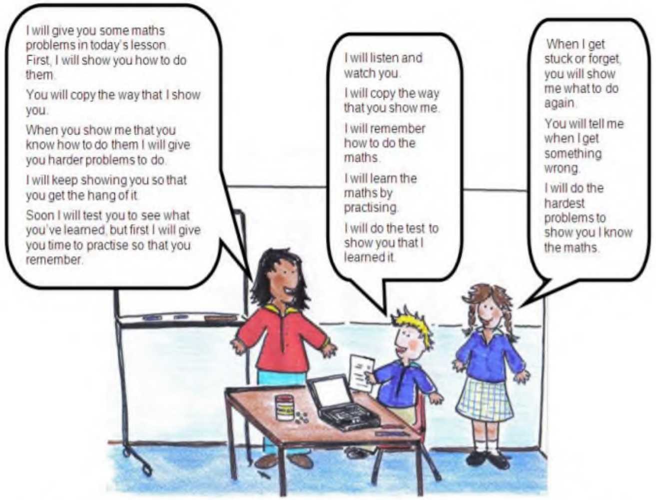

Figure 1. The concept cartoon (Cartoon A) that depicts a behaviourist approach to learning

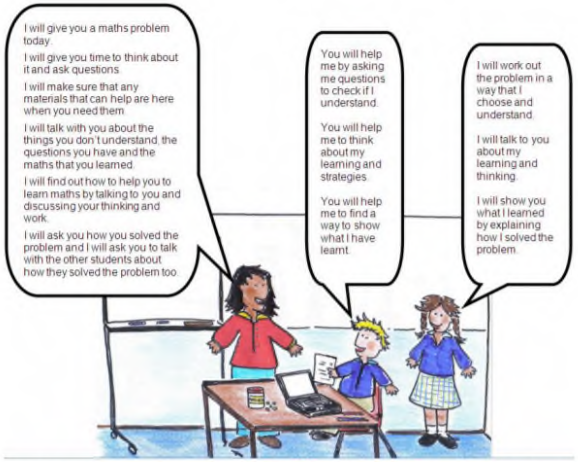

Figure 2. The concept cartoon (Cartoon B) that depicts a constructivist approach to learning.
</td>
<td>Sexton, M. (2010). Using Concept Cartoons to Access Student Beliefs about Preferred Approaches to Mathematics Learning and Teaching. Mathematics Education Research Group of Australasia.</td>
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<table>
<tr><td rowspan="9">Traditional</td><td>E2. In a mathematics lesson, there should be more emphasis on the practising phase.</td><td>3.48±0.9</td></tr>
<tr><td>E6. In mathematics teaching, one has to practice much above all.</td><td>3.93±0.9</td></tr>
<tr><td>E13. The learning of central computing techniques must be stressed.</td><td>4.14±0.8</td></tr>
<tr><td>E14. Pupils should get the right answer when solving tasks.</td><td>2.72±1.0</td></tr>
<tr><td>E16. A pupil need not necessarily understand each reasoning and procedure.</td><td>2.26±1.1</td></tr>
<tr><td>E17. Routine tasks should be solved where the use of the known procedure will surely lead to the result.</td><td>2.93±0.9</td></tr>
<tr><td>E19. Mathematical knowledge, such as facts and results, should be taught.</td><td>2.51±0.9</td></tr>
<tr><td>E23. In assessment, the presented solutions of the tasks should be taken into account.</td><td>3.75±1.0</td></tr>
<tr><td>Average</td><td>3.22±0.</td></tr>
<tr><td rowspan="9">Formal</td><td>E1. One has to pay attention to the exact use of language.</td><td>4.43±0.8</td></tr>
<tr><td>E4. Working with exact proof forms an essential objective of mathematics teaching.</td><td>3.31±1.0</td></tr>
<tr><td>E7. The proof of the Pythagorean theorem has to be worked in a mathematics lesson.</td><td>3.51±1</td></tr>
<tr><td>E8. The irrationality of the number $\sqrt{2}$ has to be proved.</td><td>2.62±1</td></tr>
<tr><td>E11. In particular, the use of mathematical symbols should be practised.</td><td>3.42±0.9</td></tr>
<tr><td>E12. In teaching, one should proceed systematically above all.</td><td>4.54±0.6</td></tr>
<tr><td>E18. Abstraction practice should be stressed in mathematics.</td><td>2.89±0.8</td></tr>
<tr><td>E20. Mathematics teaching should emphasise logical reasoning.</td><td>4.56±0.6</td></tr>
<tr><td>Average</td><td>3.66±0.</td></tr>
<tr><td rowspan="11">Constructivist</td><td>E3. Mathematics has to be taught as an open system that will develop via hypotheses and cul-de-sacs.</td><td>3.57±0.9</td></tr>
<tr><td>E5. Sometimes teaching should be realised as project-oriented, and prerequisites for it should be created.</td><td>4.06±0.9</td></tr>
<tr><td>E9. In mathematics teaching, learning games should be used.</td><td>4.05±0.8</td></tr>
<tr><td>E10. As often as possible, pupils should work using concrete materials.</td><td>4.17±0.8</td></tr>
<tr><td>E15. The teacher should try to get pupils involved in an intensive teaching discussion.</td><td>3.95±0.8</td></tr>
<tr><td>E21. Pupils should develop as many different ways as possible of finding solutions, and in teaching they should be discussed.</td><td>4.32±0.8</td></tr>
<tr><td>E22. Pupils should formulate tasks and questions themselves, and then work on them.</td><td>3.98±0.8</td></tr>
<tr><td>E24. The teacher should deal with tasks in which pupils have to think first and for which it is not enough to merely use calculation procedures.</td><td>4.23±0.7</td></tr>
<tr><td>E25. Pupils should learn how the mathematics can be used in everyday life.</td><td>4.17±0.8</td></tr>
<tr><td>E26. Students don't need to drill mathematical routines that can be done by computer</td><td>3.03±1.2</td></tr>
<tr><td>Average</td><td>3.95 ±1.</td></tr>
</table>
</td>
<td>Šapkova</td>
<td>Survey for teachers. Trad, formal and constructivist.</td>
</tr>
</table>

21 Appendix 21 – Survey 2

Survey used for year 10 and year 11. The year 8 and 9 survey had different questions around consent at the end.

All responses were given using a 5 part Likert scale: strongly agree, agree, neither agree nor disagree, disagree, strongly disagree.

I look forward to my mathematics class.
I feel tense doing mathematical problems.
I get good grades in mathematics.
I learn quickly in mathematics.
I feel powerless doing mathematics problems.
It is important for me to do well compared to others in my mathematics class.
It is important to me to take time to understand new ideas in mathematics.
It is important to me that I understand all of my mathematics lessons.
I only need to learn mathematics for my exams.
My goal in mathematics is to get a better grade than most of the other students.
I want to learn as much as possible from my mathematics lessons.
In mathematics it is impossible to do a problem unless you've first been taught how to do one like it.
I usually try to understand the reasoning behind all of the rules I use in mathematics.
Learning mathematics mainly involves memorising procedures and formulas.
Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works.
Mathematics involves relating many different ideas.
When I learn something new in mathematics I often continue exploring and developing it on my own.
Doing mathematics consists mainly of using rules.
Getting the right answer is the most important part of mathematics.
Projects are an effective way to learn mathematics.
I enjoy learning through projects.
I often feel confused when learning through projects.
I work harder in projects that I do in 'normal' maths lessons.
Projects don't help me learn the mathematics I need for my exams.
I give up more often during projects than 'normal' lessons.
I enjoy learning more independently in projects.
Are you happy for Miss Barnecutt to use your responses for her research?
Are you happy that your responses to the survey are linked to information such as gender, SEN, FSM and prior attainment?
Have you spoken to your parent/ carer about this?

22 Appendix 22 - Quantitative data findings

22.1 Mathematical self-efficacy

I interpret the results on mathematical self-efficacy as suggesting that typically the sample students had a moderate mathematical self-efficacy. This conclusion is based on the interpretations of Blotnicky et al. (2018) whose instrument I adapted for this study. Using reverse scoring where required, they interpreted students who scored mainly 4s and 5s as having high mathematical self-efficacy, and students who scored mainly 1 through 3 as being less confident and comfortable. The 5% trimmed mean of the participants in this study was 3.26 for mathematical self-efficacy, suggesting they had a moderate mathematical self-efficacy.

The one response that did not appear to fit with the responses to the other questions was "I feel tense doing mathematical problems". The students reported a mean (not reversed) of 3.19 for this question, and this seems inconsistent with the responses to the other questions: 3.32 for looking forward to their maths class, 3.24 for thinking they get good grades in mathematics, 3.5 for learning quickly in mathematics and 2.71 for feeling powerless when doing mathematical problems.

A principal component analysis was conducted on the 5 items that were used to measure self-efficacy to test for internal reliability. The Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis, *KMO* = .75 ("middling" according to Kaiser & Rice, 1974), and all *KMO* values for individual items were greater than .66 which is well above the acceptable limit of .5 (Kaiser & Rice, 1974). An initial analysis was run to obtain eigenvectors for each factor in the data. As expected, only one factor had an eigenvalue over Kaiser's criterion of 1 and explained 46% of the variance. This means that this one factor can explain 46% of the variance. The scree plot showed a clear point of inflection that also justifies retaining just one factor, in other words, that he five items are essentially measuring just one construct. As only one factor was extracted, the solution was not rotated. The subscale showed a reasonable level of reliability with Cronbach α = .7. The Cronbach α with each item removed ranged from .6 to .7 suggesting that no item would cause a substantial change to alpha if it was removed, and none would make the scale more reliable if it were removed.

When checking for outliers, 2 outliers were found. These cases were checked to see if there could have been an error. It was decided that these were genuine, if somewhat polarised responses and therefore they were maintained in the data set, however the 5% trimmed mean was used. Both a histogram and the corresponding P-P plot were used as advised by Field (2013) to check for normality. These can be seen below and suggest that it is fair to assume normality for this data set. Skewness (-.393) and kurtosis (.896) were both between -2 and +2 the values which are considered acceptable to show normal univariate distribution by George and Mallery (2010). This is important since the data need to be well-modelled by a normal distribution as t – tests and ANOVAs require a normally distributed sample population. I also tested for homoscedasticity in the data. The homoscedasticity of variance shows that the variances in the different groups that I am testing are similar. I checked for homoscedasticity by plotting the standard residuals against standardised predicted values (*zpred vs. zresid*) as suggested by Field (2013). This graph can be seen below.

I created a subscale for mean mathematical self-efficacy, that was an average of the self-efficacy variable. I used this to interpret whether student's mathematical self-efficacy seemed to vary with any of the contextual variables. There was no significant effect for FSM, [$t(132) = -.739$, $p = 0.461$], SEN, [$t(132) = .651$, $p = .516$], between low prior attainment and middle prior attainment [$t(90) = -.207$, $p = 0.837$] or middle prior attainment and high prior attainment [$t(74) = .175$, $p = 0.861$]. However, the 54 male participants ($M = 3.4$, $SD = .66$) compared to the 79 female participants ($M = 3.14$, $SD = 0.59$) reported significantly higher self-efficacy, [$t(131) = 2.15$, $p = .03$] and the 34 students in key stage 3 ($M = 3.6$, $SD = .54$) compared to than the 100 KS4 students ($M = 3.1$, $SD = 0.64$) reported significantly higher self-efficacy, [$t(132) = -3.63$, $p < .001$].

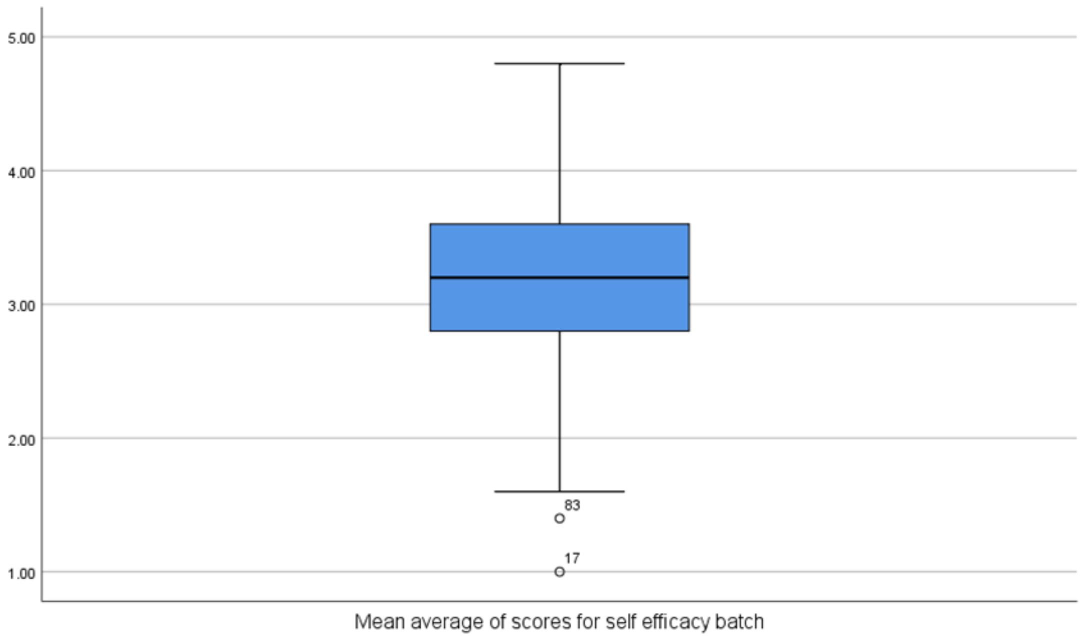
Self-efficacy had a weak correlation with relational vision of mathematics (significant at the 0.01 level (2-tailed) $r = .314$ and a weak correlation with the mastery orientation subscale (significant at the 0.01 level (2-tailed)) $r = .231$.

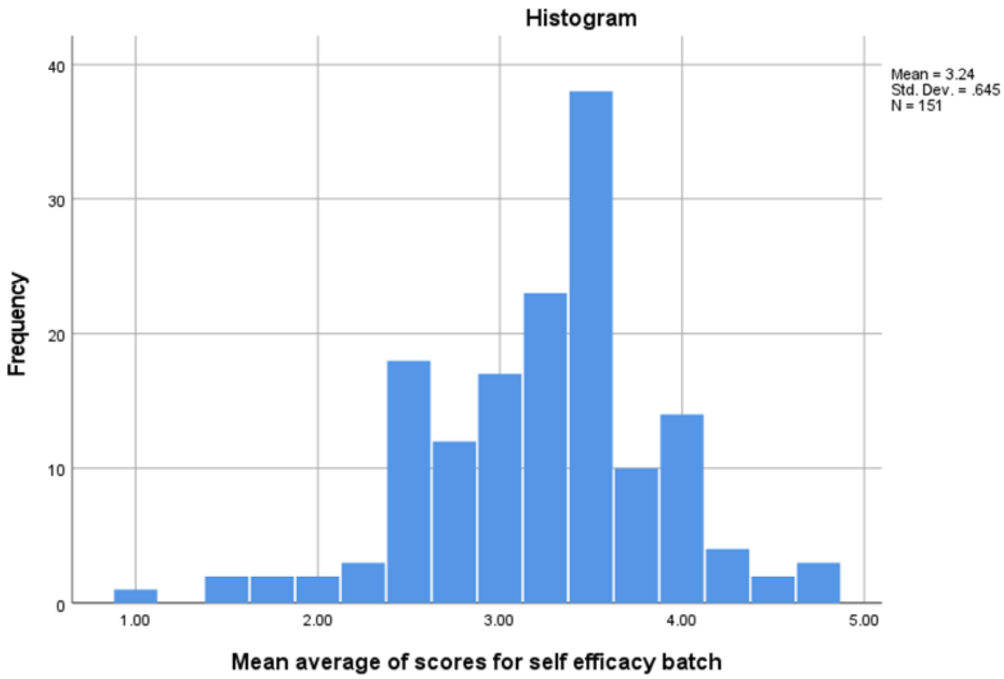
Self-efficacy correlated to the following statements (significant at the 0.05 level (2-tailed)):

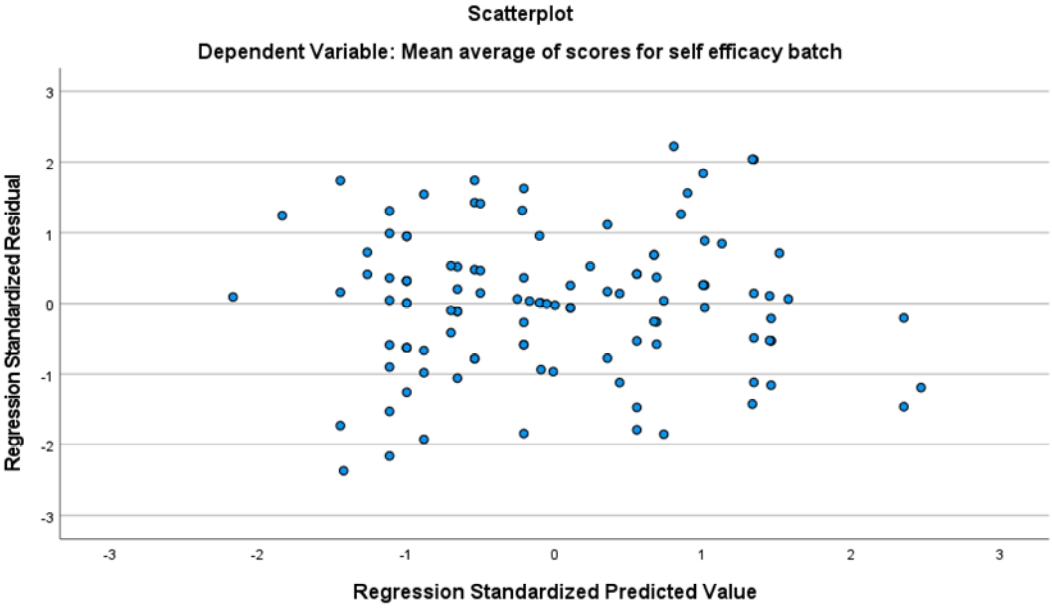
- I enjoy learning through projects (r = .19)
- I work harder in projects that I do in 'normal' maths lessons (r = -.19)
- I only need to learn mathematics for my exams (r = - .18)

Descriptive statistics for questions on mathematical self-efficacy			
	N	Mean	Std. Deviation
I look forward to my mathematics class.	151	3.32	1.062
I feel tense doing mathematical problems (RS).	152	2.81	.926
I get good grades in mathematics.	152	3.24	.975
I learn quickly in mathematics.	152	3.50	.928
I feel powerless doing mathematics problems (RS).	152	3.29	1.001
Valid N (listwise)	151		

Descriptive statistics for mean average of self-efficacy question batch				
			Statistic	Std. Error
Mean average of scores for self-efficacy batch	Mean		3.2447	.05247
	95% Confidence Interval for Mean	Lower Bound	3.1410	
		Upper Bound	3.3484	
	5% Trimmed Mean		3.2600	
	Median		3.2000	
	Variance		.416	
	Std. Deviation		.64478	
	Minimum		1.00	
	Maximum		4.80	
	Range		3.80	
	Interquartile Range		.80	
	Skewness		-.393	.197
	Kurtosis		.896	.392


5.00
4.00
3.00
2.00
1.00
83
17
Mean average of scores for self efficacy batch


Histogram
Mean = 3.24
Std. Dev. = .645
N = 151
Frequency
40
30
20
10
0
1.00
2.00
3.00
4.00
5.00
Mean average of scores for self efficacy batch



22.2 Vision of mathematics

This batch of questions was designed to probe a student's vision of mathematics. The questions were a reduced set of the questions developed by Yackel ((1984) cited in Quillen, 2004). Yackel based her work on the conceptualisation of Skemp: that a student's vision of mathematics can be viewed on a continuum from instrumental to relational. Therefore, I would expect that if a student was answering questions that suggest they have a relational understanding of mathematics positively, they would answer questions that suggest that they have an instrumental understanding of mathematics negatively and vice versa. However, the mean average response to every question was positive, with students, on average, giving a more positive response to the questions that suggest that they have a relational vision of mathematics than to those that suggest they have an instrumental vision of mathematics.

Participants who respond on the same side of neutral on a Likert scale for both reversed and non-reverse items are defined by Swain et al. (2008) as giving a 'mis response'. They argue that this is not due to respondent inattention or acquiescence, but instead is due to the complexity and the cognitive load of the reverse worded question. In this particular example, the questions weren't worded in a complex way, nor did they need participants to negate an answer Therefore in this particular instance, I think that it is student's acquiescence that has led to 'mis response'. Baumgartner et al. (2001) highlight how response style can be enough to threaten the validity in data. Their research is in the field of market research perspective, I think that it is likely that this could be even more true in a school setting where students try to do what is required of them and which is highly possible within the dominant sub-culture of the study school.

A principal component analysis was conducted on the 8 items that were used to measure vision of mathematics. One item was removed from the factor analysis as it had weak correlation (< 0.20) with each other variable (I was looking for at least some correlations > 0.3 as suggested by Field (2013)) and had an anti-image correlation of 0.496 less than the criterion suggested by Field (2013) of 0.5.

A principal factor analysis of the remaining 7 variables had the Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis, *KMO* = .65 ('mediocre' according to Kaiser & Rice, 1974), and all *KMO* values for individual items were greater than .59 which is above the acceptable limit of .5 (Kaiser & Rice, 1974). An initial analysis was run to obtain eigenvectors for each factor in the data. Two factors had an eigenvalue over Kaiser's criterion of 1 and explained 53% of the variance. The scree plot showed a clear point of inflection that also justifies retaining just two factors. The table below shows the factor loading after rotation. The items that cluster on the same factor suggest that factor 1 represents an instrumental understanding of mathematics and factor 2 represents a relational understanding of mathematics. I have interpreted the first two factors as being distinct due to a “method factor”: the variance can be attributed to student’s acquiescence or ‘mis response’, rather than a part of the construct that the method represents.

Rotated Factor Matrix[a]

	Factor	
	1	2
Doing mathematics consists mainly of using rules (RS).	.664	
Learning mathematics mainly involves memorising procedures and formulas (RS).	.567	
Getting the right answer is the most important part of mathematics (RS).	.528	
Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works (RS).	.452	
I usually try to understand the reasoning behind all of the rules I use in mathematics.		.701
When I learn something new in mathematics I often continue exploring and developing it on my own.		.687
Mathematics involves relating many different ideas.		.458

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.[a]
a. Rotation converged in 3 iterations.

As the mean average score for relational understanding was higher than that of instrumental, I decided to move forwards only considering this factor, as I felt it would be less influenced by ‘mis response’. I am aware that acquiescence has the potential to make constructs appear to correlate more strongly (Podsakoff et al., 2003) and was mindful of this in the interpretation.

The subscale of the mean average score for relational understanding showed a reasonable level of reliability with Cronbach α = .7. The Cronbach α with each item removed ranged from .6 to .7 suggesting that no item would cause a substantial change to alpha if it was removed, and none would make the scale more reliable if it were removed.

The subscale appeared from the histogram and the PP test (seen below) to have a normal distribution. This is supported by the statistics: it has a mean of 3.59, standard deviation of 0.703, skewness of -.16 and kurtosis of 0.04. From observations of the box plot and histogram, there did not appear to be any outliers. The plot of *zpred vs zresid* below suggests homoscedasticity of variance.

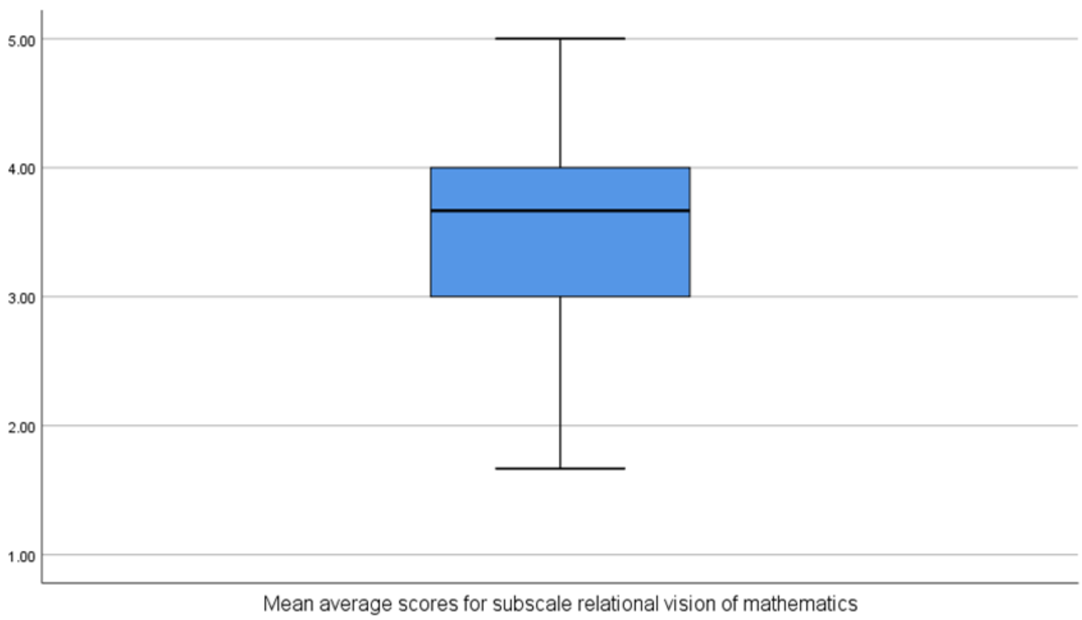
There was no significant effect for SEN [$t(132) = .671$, $p = .504$], key stage [$t(132) = .017$, $p = .987$], FSM [$t(132) = -.134$, $p = .893$] or gender [$t(131) = -1.49$, $p = .140$].

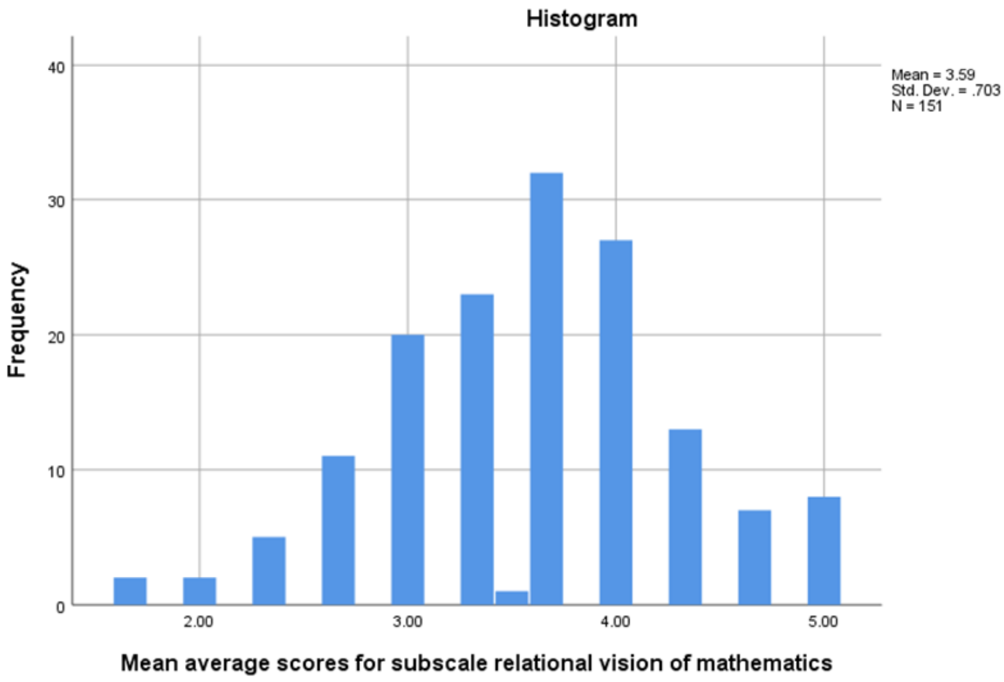
The 20 participants who had low prior attainment ($M = 3.85$, $SD = .63$) compared to the 92 participants who had middle or high prior attainment ($M = 3.44$, $SD = .68$) demonstrated significantly higher relational vision of mathematics scores [$t(110) = 2.46$, $p = .02$]. This result surprised me as often I presume that students with a lower prior attainment are more likely to have a more instrumental vision of mathematics.

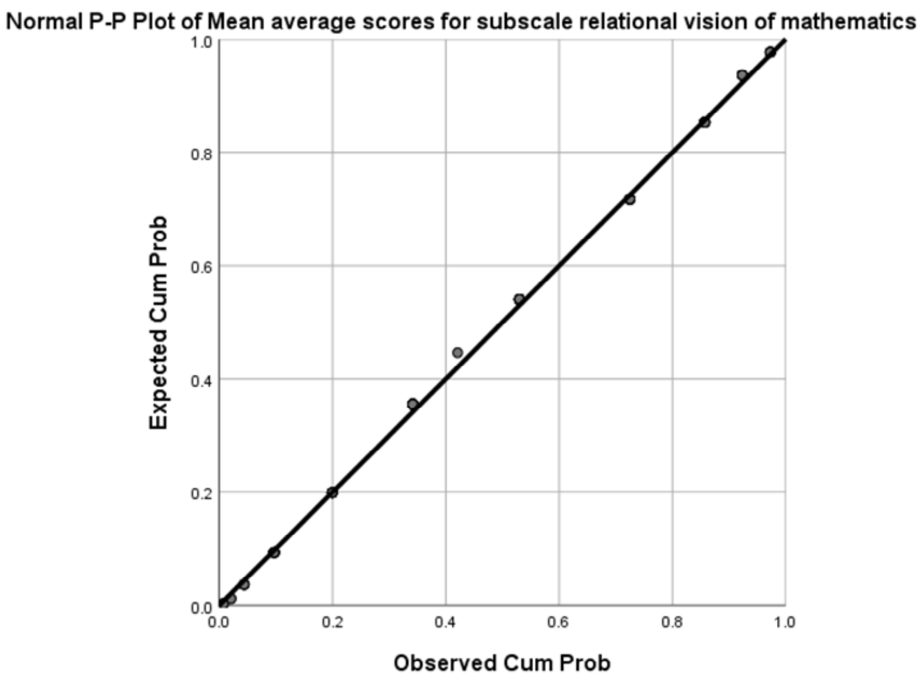
Descriptive statistics for items in the vision of mathematics question batch

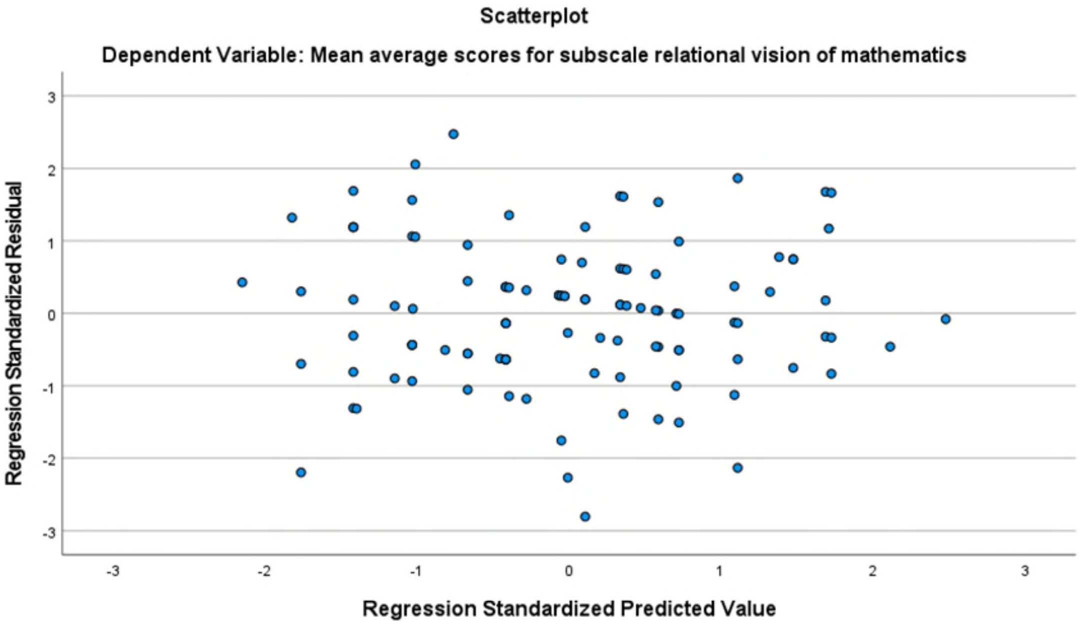
	N	Mean	Std. Deviation
In mathematics it is impossible to do a problem unless you've first been taught how to do one like it (RS).	152	2.79	1.059
I usually try to understand the reasoning behind all of the rules I use in mathematics.	150	3.81	.900
Learning mathematics mainly involves memorising procedures and formulas (RS).	151	2.36	.983
Being able to successfully use a rule or formula in mathematics is more important to me than understanding why and how it works (RS).	151	2.78	1.006
Mathematics involves relating many different ideas.	150	3.88	.785
When I learn something new in mathematics I often continue exploring and developing it on my own.	151	3.07	1.056
Doing mathematics consists mainly of using rules (RS).	149	2.48	.905
Getting the right answer is the most important part of mathematics (RS).	150	2.80	1.198
Valid N (listwise)	143		

Descriptive statistics for mean average scores for subscale relational vision of mathematics				
			Statistic	Std. Error
Mean average scores for subscale relational vision of mathematics	Mean		3.5949	.05721
	95% Confidence Interval for Mean	Lower Bound	3.4819	
		Upper Bound	3.7080	
	5% Trimmed Mean		3.6017	
	Median		3.6667	
	Variance		.494	
	Std. Deviation		.70304	
	Minimum		1.67	
	Maximum		5.00	
	Range		3.33	
	Interquartile Range		1.00	
	Skewness		-.157	.197
	Kurtosis		.036	.392




Histogram
Mean = 3.59
Std. Dev. = .703
N = 151
Frequency
40
30
20
10
0
2.00
3.00
4.00
5.00
Mean average scores for subscale relational vision of mathematics


Normal P-P Plot of Mean average scores for subscale relational vision of mathematics
Expected Cum Prob
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Observed Cum Prob



22.3 Goal orientation

Similarly to the questions on vision of mathematics, many of the reverse questions on goal orientation were answered positively. This can be viewed as students 'mis responding' due to acquiescence however, it could also be viewed as being due to or partially due to the fact that a performance goal orientation and a mastery goal orientation are not viewed theoretically as being on a continuum: students can have high mastery and high-performance goal orientations. One of the questions (I only need to learn mathematics for my exams) had did have an average score on the disagree side of neutral (M = 2.73) before being reverse scored. It also has the largest standard deviation of any item (SD = 1.24).

A principal component analysis was conducted on the 6 items that were used to measure goal orientation. The items loaded onto two factors, one factor of the questions that suggested a mastery orientation and one factor that suggested a performance orientation. The "method factor" (the variance can be attributed to student's acquiescence, rather than a part of the construct that the method represents) appeared to have an impact on these factors. However, one item loads with a positive correlation on one factor and a negative correlation on another factor, so I will use both constructs.

One item was removed from the factor analysis and the principal factor analysis re-done with 5 items, as this item did not load on either factor. The factor analysis of the remaining variables had the Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis, *KMO* = .67 ('mediocre' according to Kaiser & Rice, 1974), and all KMO values for individual items were greater than .61 which is above the acceptable limit of .5 (Kaiser & Rice, 1974). An initial analysis was run to obtain eigenvectors for each factor in the data. Two factors had an eigenvalue over Kaiser's criterion of 1 and explained 68% of the variance. The scree plot showed a clear point of inflection that also justified retaining two factors. The factor loading after rotation is below. The items that cluster on the same factor suggest that factor 1 represents a mastery goal for mathematics and factor 2 represents a performance goal for mathematics.

The mastery orientation subscale showed a reasonable level of reliability with Cronbach α = .68. The Cronbach α with each item removed ranged from .49 to .69 suggesting that no item would cause a substantial change to alpha if it was removed, and none would make the scale more reliable if it were removed. The performance orientation subscale showed a negative average covariance among items which violates reliability assumptions. The Cronbach's α = -.12. For this reason, this subscale was no longer used as part of the interpretation.

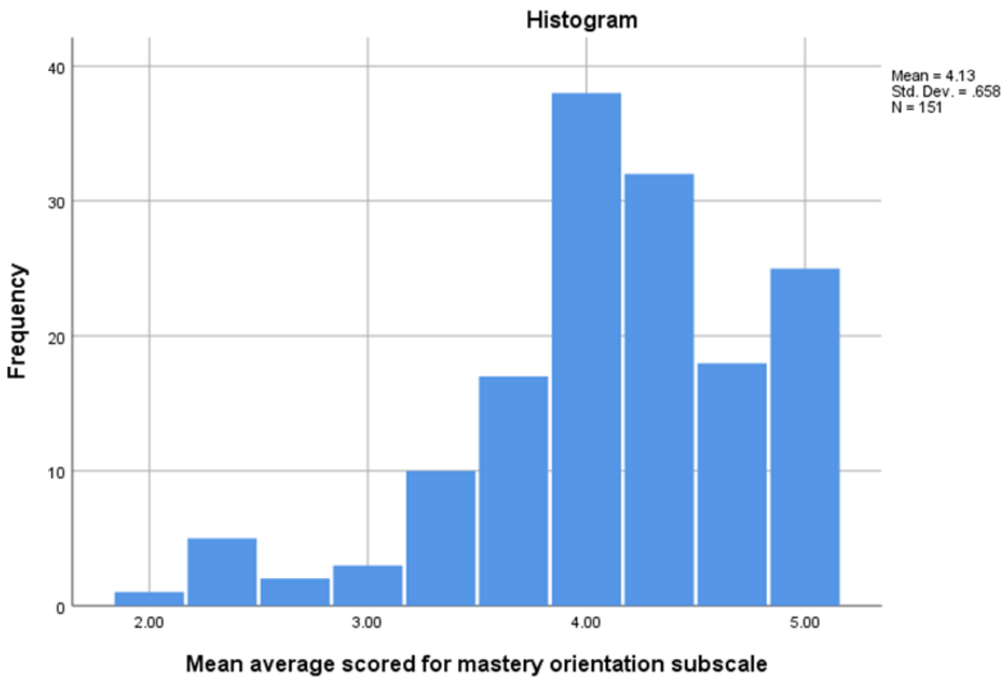
The mastery orientation subscale had mean of 4.13 and smallest standard deviation of 0.66. The subscale appears reasonably normally distributed, with a skewness of -0.86 and a kurtosis of 0.92. The P-Plot shown below suggests that the data is normally distributed. The box plot (shown below) showed a number of outliers to the data set. On exploration, two of these seemed to be due to students who typically gave polarised opinions and a further two seemed to have a negative response bias – they had low scores for the other subscales too. It was decided that there was no reason to exclude these data but that care should be taken when considering how these could affect analyses. The plot of *zpred vs zresid* below suggests homoscedasticity of variance.

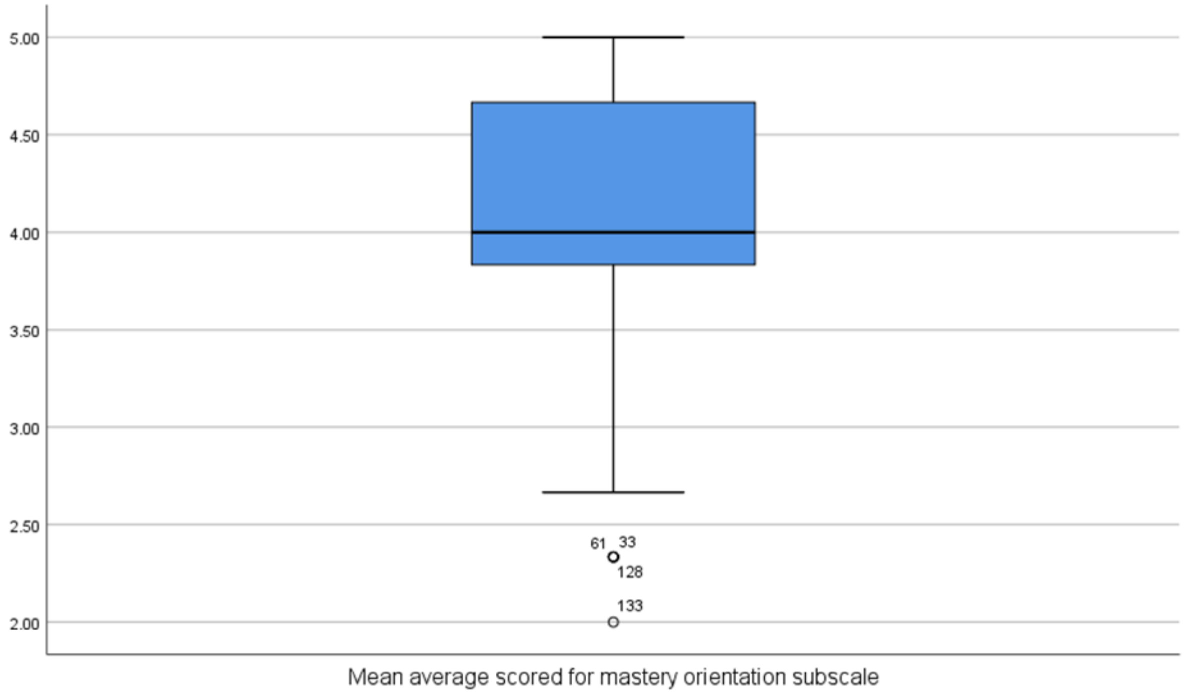
Rotated Factor Matrix[a]		
	Factor	
	1	2
My goal in mathematics is to get a better grade than most of the other students (RS).		.770
I want to learn as much as possible from my mathematics lessons.	.434	-.434
It is important to me that I understand all of my mathematics lessons.	.798	
It is important to me to take time to understand new ideas in mathematics.	.654	
It is important for me to do well compared to others in my mathematics class (RS).		.562
Extraction Method: Principal Axis Factoring. Rotation Method: Varimax with Kaiser Normalization.		
a. Rotation converged in 3 iterations.		

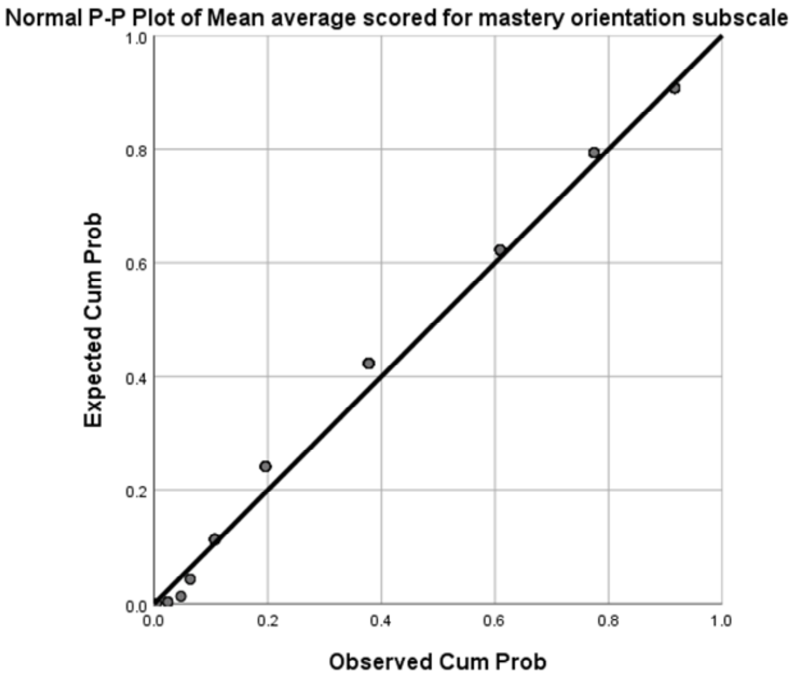
There was no significant effect on the mean average scored for the mastery orientation subscale for SEN [$t(132) = .086$, $p = .931$], for low versus middle prior attainment [$t(74) = 1.86$, $p = .065$], or middle versus high prior attainment [$t(90) = 0.28$, $p = .78$], key stage [$t(132) = -0.26$, $p = .79$], gender [$t(131) = 0.015$, $p = 0.88$], or FSM [$t(132) = 0.019$, $p = .85$].

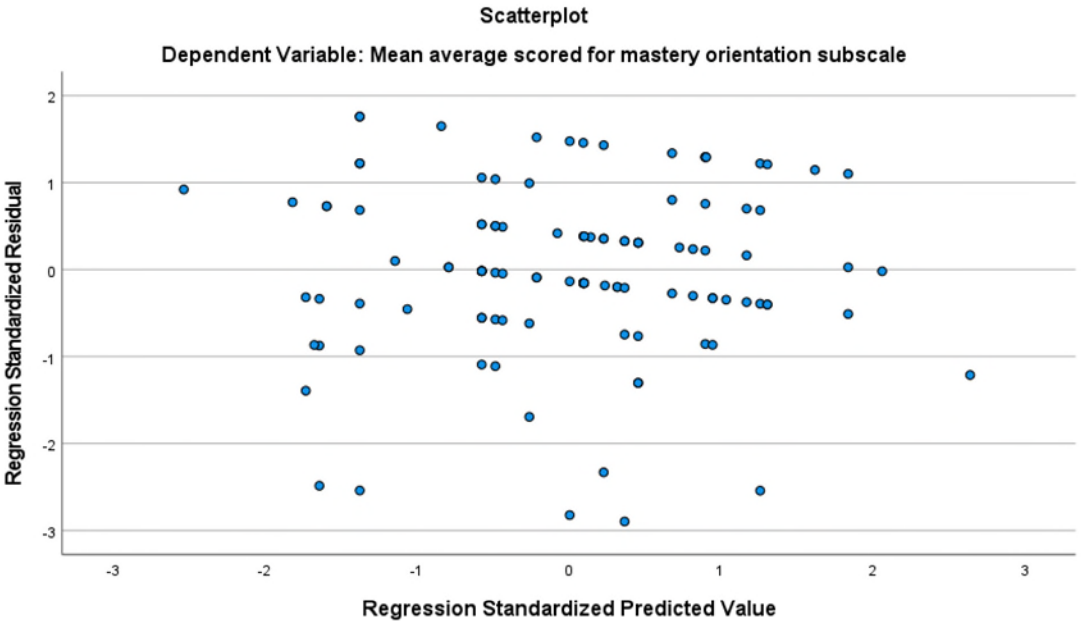
Descriptive statistics for questions on goal orientation			
	N	Mean	Std. Deviation
It is important for me to do well compared to others in my mathematics class (RS).	152	2.24	.970
It is important to me to take time to understand new ideas in mathematics.	152	3.96	.883
It is important to me that I understand all of my mathematics lessons.	152	4.23	.809
I only need to learn mathematics for my exams (RS).	151	3.27	1.238
My goal in mathematics is to get a better grade than most of the other students (RS).	149	2.70	1.100
I want to learn as much as possible from my mathematics lessons.	152	4.19	.812
Valid N (listwise)	148		

Descriptive statistics for mean average scored for mastery orientation subscale				
			Statistic	Std. Error
Mean average scored for mastery orientation subscale	Mean		4.1280	.05351
	95% Confidence Interval for Mean	Lower Bound	4.0223	
		Upper Bound	4.2338	
	5% Trimmed Mean		4.1779	
	Median		4.0000	
	Variance		.432	
	Std. Deviation		.65756	
	Minimum		2.00	
	Maximum		5.00	
	Range		3.00	
	Interquartile Range		1.00	
	Skewness		-.860	.197
	Kurtosis		.916	.392




5.00
4.50
4.00
3.50
3.00
2.50
2.00
61
33
128
133
Mean average scored for mastery orientation subscale


Normal P-P Plot of Mean average scored for mastery orientation subscale
Expected Cum Prob
Observed Cum Prob
0.0
0.2
0.4
0.6
0.8
1.0



22.4 Attitude towards projects

The overarching attitude to projects was positive. With students tending towards agreeing that they enjoyed learning mathematics (*M* =3.71, *SD* = 1.06) and that projects are an effective way to learn mathematics (*M* = 3.64, *SD* = 1.02) and a slightly positive response to I work harder in projects than I do on 'normal' maths lessons (*M* = 3.25, *SD* = 1.08). Whilst there was a slightly negative response to the statements: I often feel confused when learning through projects (*M* = 2.9, *SD* = 1.00) and I often give up more often during projects than 'normal' lessons (*M* = 2.75, *SD* = 1.00). Students seemed largely neutral about whether projects help them learn the mathematics they need for exams (*M* = 2.99, *SD* = 1.06) and whether they enjoy learning more independently in projects (*M* = 3.09, *SD* = 1.10).

A principal component analysis was conducted on the 6 items that were used to measure students' attitude to projects in mathematics to test for internal reliability. The Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis, *KMO* = .72 ('middling' according to Kaiser & Rice, 1974), ('middling' according to Kaiser and Rice, 1974), and all *KMO* values for individual items were greater than .67 which is well above the acceptable limit of .5 (Kaiser & Rice, 1974). An initial analysis was run to obtain eigenvectors for each factor in the data. As expected, two factors had an eigenvalue over Kaiser's criterion of 1 and explained 65% of the variance. The scree plot showed points of inflection that justified retaining one or two factors. Two factors were extracted, the rotated factor matrix can be seen below. It is very probable that one of the reasons for two factors is a method factor as, despite the fact that students did not always respond on the positive side of neutral to this question, all of the positively framed items load on one factor and the rest on the other. Factor one shows a positive attitude to projects, whilst factor two shows a negative attitude to projects.

Rotated Factor Matrix[a]

	Factor	
	1	2
Projects are an effective way to learn mathematics.	.879	
I enjoy learning through projects.	.722	
I work harder in projects that I do in 'normal' maths lessons.	.511	
Projects don't help me learn the mathematics I need for my exams (RS).		.775
I give up more often during projects than 'normal' lessons (RS).		.634
I often feel confused when learning through projects (RS).		.485

Extraction Method: Principal Axis Factoring.

Rotation Method: Varimax with Kaiser Normalization.[a]

a. Rotation converged in 3 iterations.

The negative attitude to projects subscale showed a reasonable level of reliability with Cronbach α = .69. The Cronbach α with each item removed ranged from .4 to .7 suggesting that no item would cause a substantial change to alpha if it was removed, and none would make the scale more reliable if it were removed.

The mean average score for the negative attitude to projects subscale scale was 3.1 suggesting that typically students were neutral about these statements. The standard deviation was .80. The P-Plot and the histogram suggest that the subscale is normally distributed, with a skewness of -.65 and kurtosis of .43. There were four outliers to the data. On checking these data, it was decided to include them all within the analysis. The plot of z*pred vs zresid* below suggests homoscedasticity of variance.

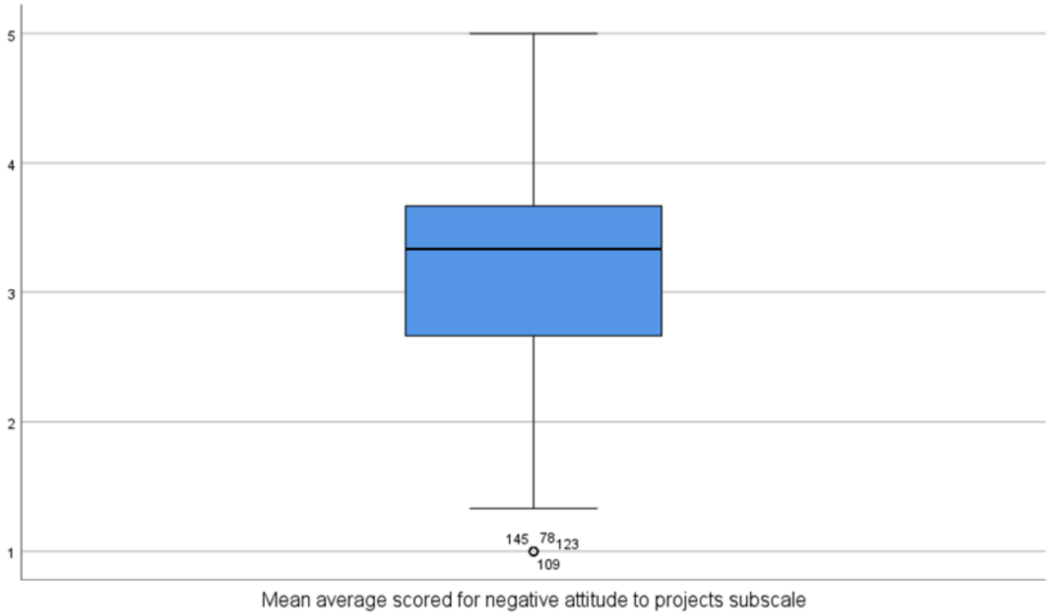
The positive attitude to projects subscale showed a reasonable level of reliability with Cronbach α = .77. The Cronbach α with each item removed ranged from .58 to .81 suggesting that no item would cause a substantial change to alpha if it was removed, and none would make the scale significantly more reliable if it were removed.

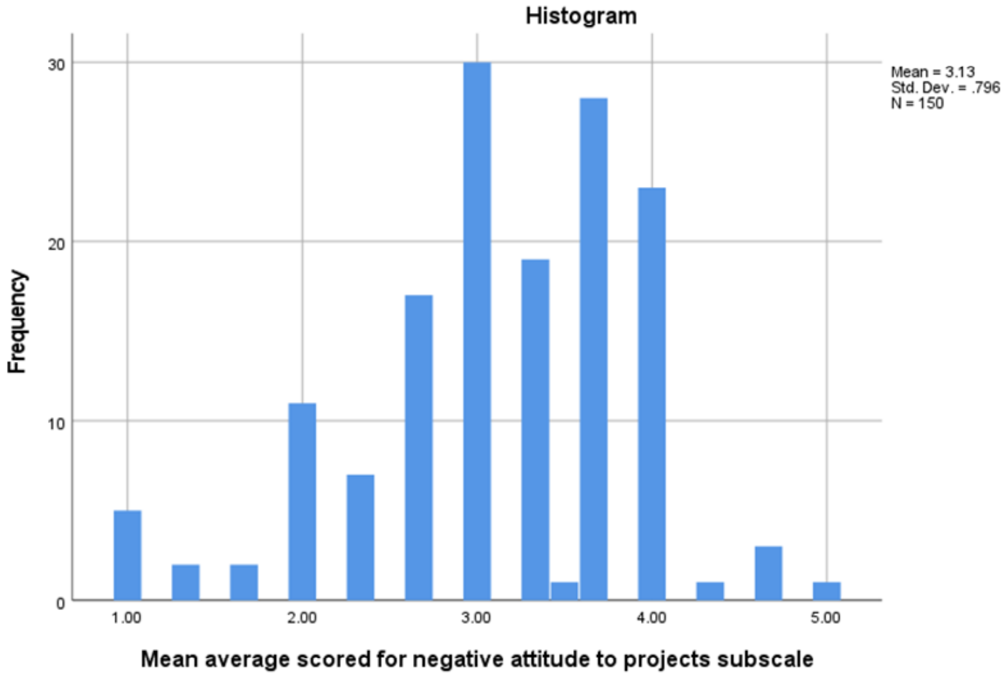
The mean average score for the positive attitude to projects scale was 3.53 suggesting that typically students were positive about the projects. The standard deviation was .87. The subscale appeared to be normally distributed the histogram, with a skewness of -.47 and kurtosis of .09. There were three outliers to the data. On checking these data, it was decided to include them all within the analysis. The plot of *zpred vs zresid* below suggests homoscedasticity of variance.

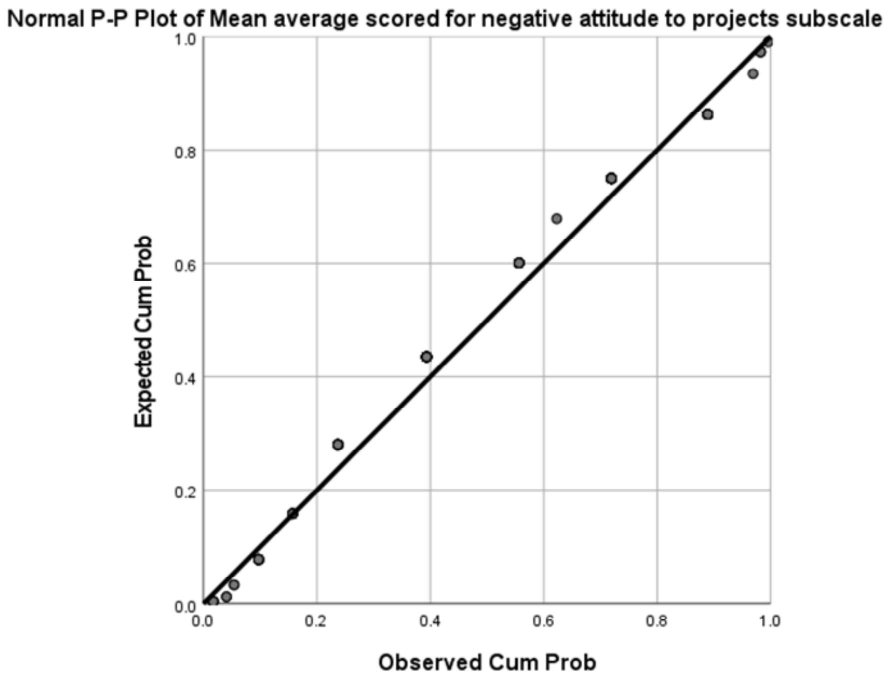
None of the background contextual variables had a significant effect on students' attitudes to projects, positive or negative.

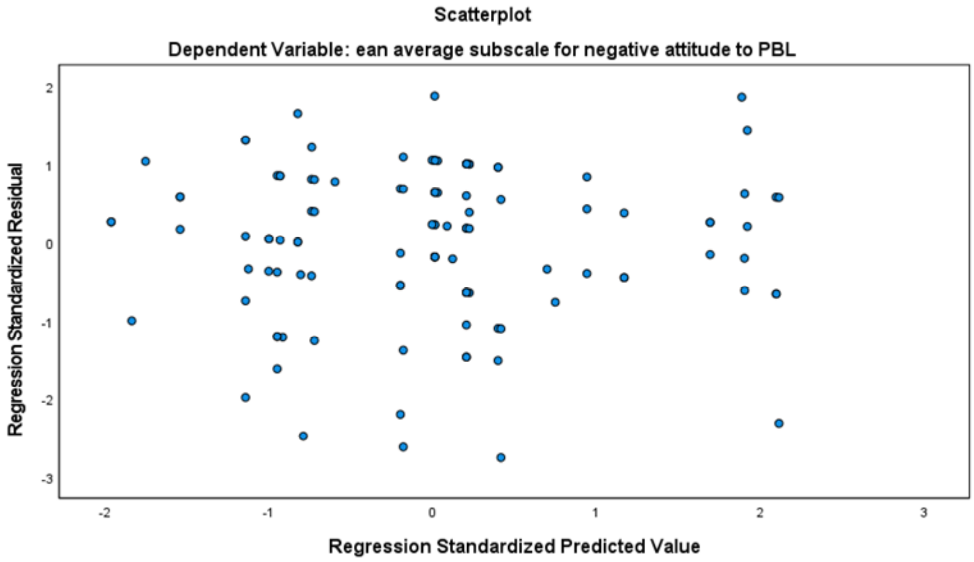
Descriptive statistics for questions on projects

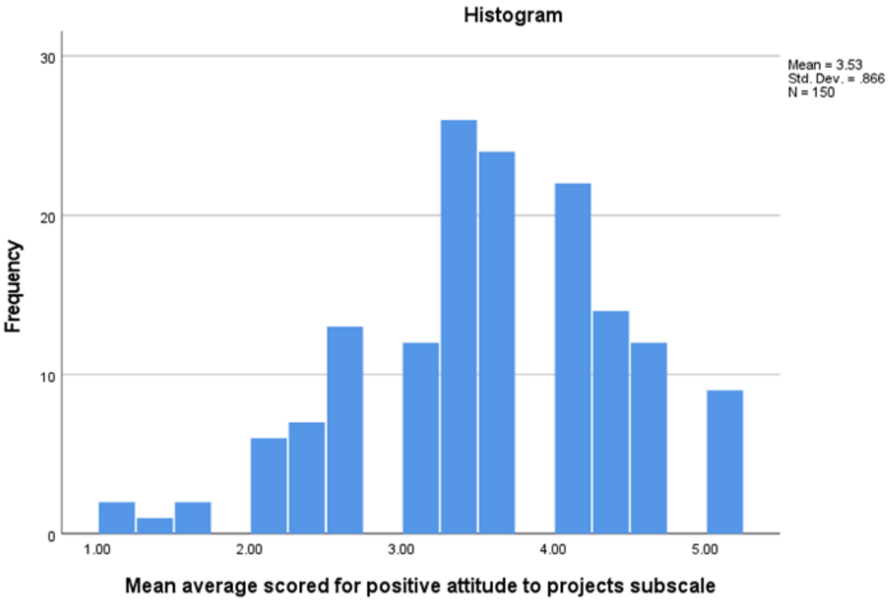
	N	Mean	Std. Deviation
Projects are an effective way to learn mathematics.	151	3.64	1.016
I enjoy learning through projects.	151	3.71	1.056
I often feel confused when learning through projects (RS).	150	3.10	.995
I work harder in projects that I do in 'normal' maths lessons.	148	3.25	1.075
Projects don't help me learn the mathematics I need for my exams (RS).	149	3.01	1.056
I give up more often during projects than 'normal' lessons (RS).	150	3.25	.998
I enjoy learning more independently in projects.	152	3.09	1.100
Valid N (listwise)	145		

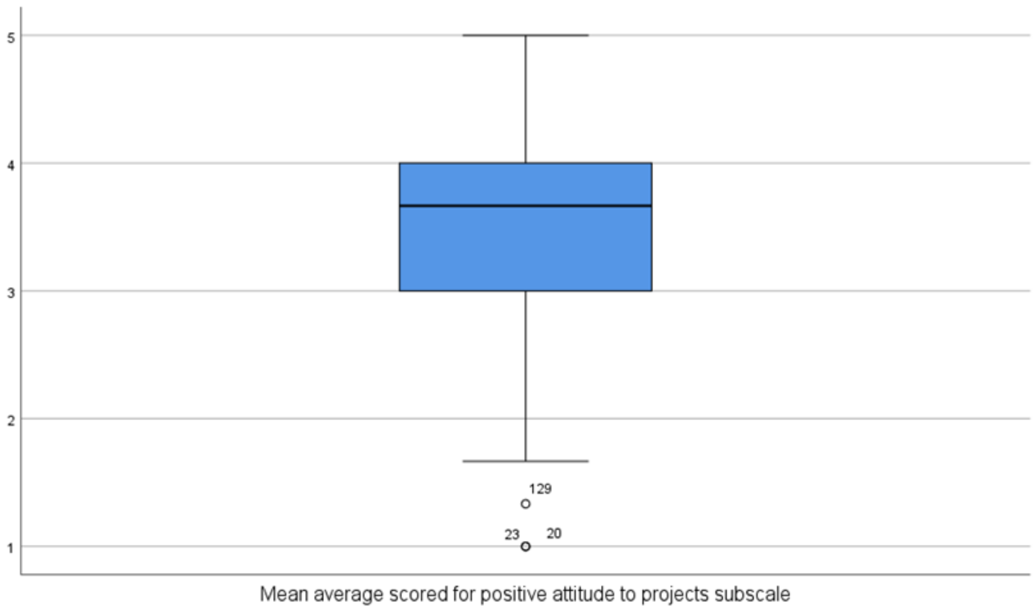

5
4
3
2
1
145
78
123
109
Mean average scored for negative attitude to projects subscale

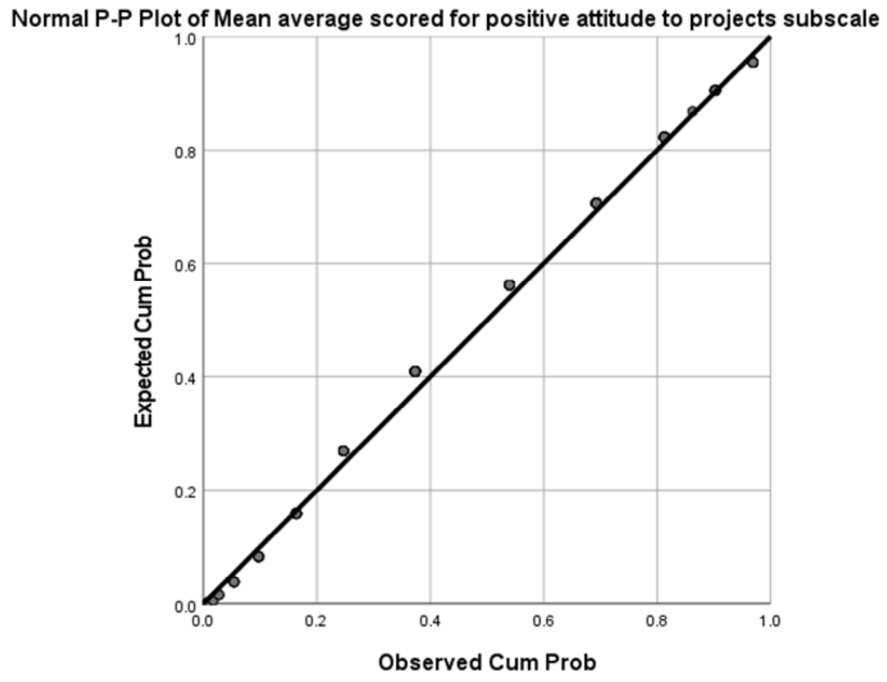

Histogram
Mean = 3.13
Std. Dev. = .796
N = 150
Frequency
30
20
10
0
1.00
2.00
3.00
4.00
5.00
Mean average scored for negative attitude to projects subscale

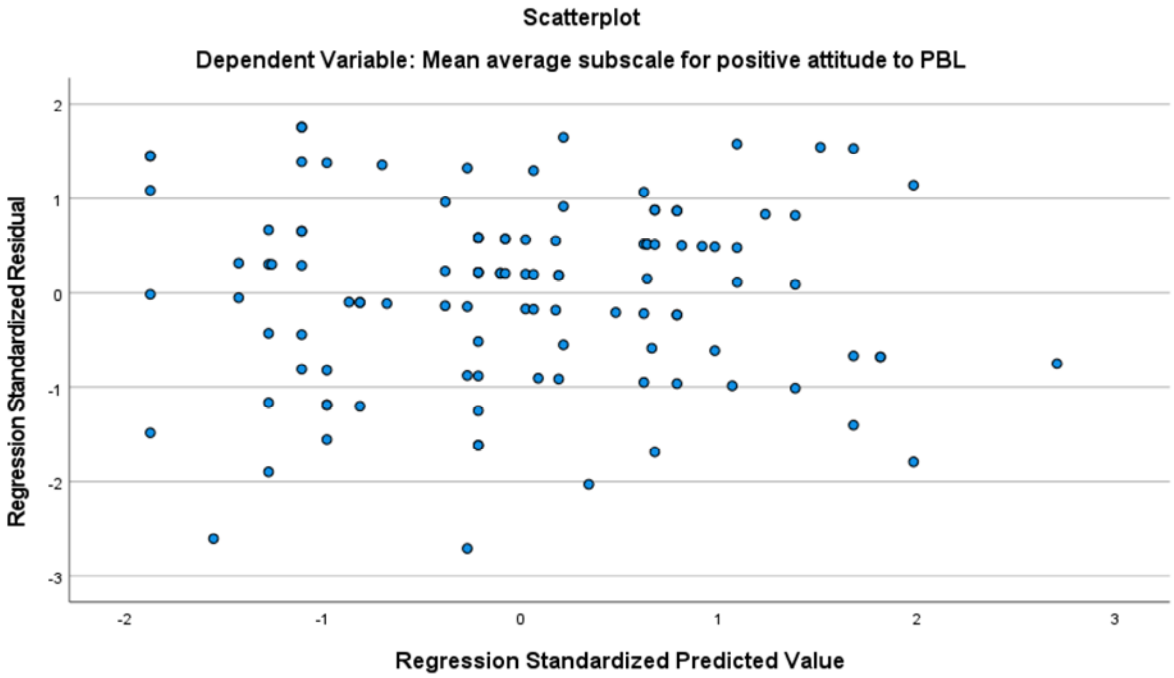

Normal P-P Plot of Mean average scored for negative attitude to projects subscale
Expected Cum Prob
Observed Cum Prob
0.0
0.2
0.4
0.6
0.8
1.0


Scatterplot
Dependent Variable: ean average subscale for negative attitude to PBL
Regression Standardized Residual
Regression Standardized Predicted Value
-3
-2
-1
0
1
2
3


Histogram
Mean = 3.53
Std. Dev. = .866
N = 150
Frequency
0
10
20
30
1.00
2.00
3.00
4.00
5.00
Mean average scored for positive attitude to projects subscale


1
2
3
4
5
129
23
20
Mean average scored for positive attitude to projects subscale





22.5 Autonomy in projects

Data from the first phase of the study suggested that students valued autonomy. Even students who did not typically value learning through MaPBL still defended their right to have some autonomy. However, the responses to the survey question about whether students enjoyed learning more independently in projects did not confirm this. Students did not appear to have strong feelings about valuing learning independently (*M* = 3.09, *SD* = 1.10). This could have been due to how they interpreted the question.

The responses appeared to be normally distributed and have homogeneity of variance (see graphs below). There were no outliers to the data.

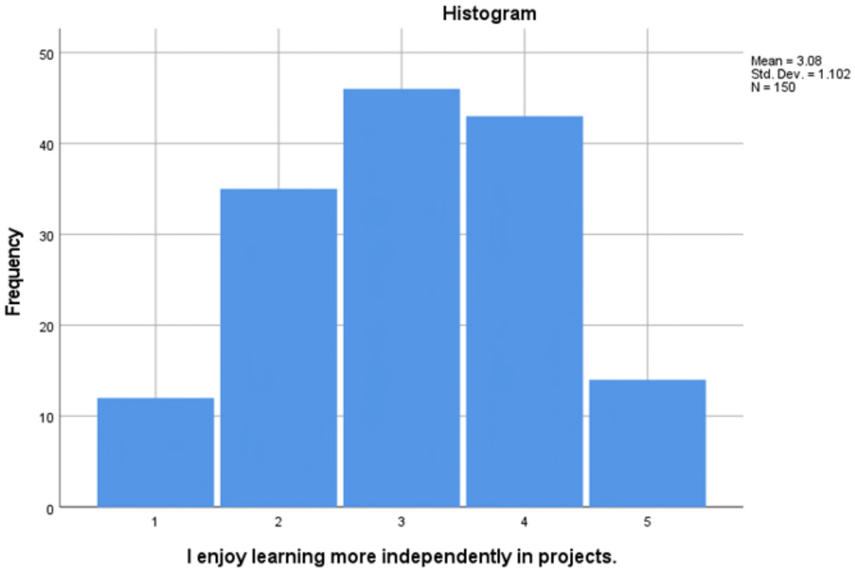
The 79 female participants (*M* = 3.3, *SD* = 1.0) compared to the 54 male participants (*M* = 2.69, *SD* = 1.2) reported significantly more positive opinions about enjoying learning more independently in projects [*t*(131) = 3.3, *p* = .001].

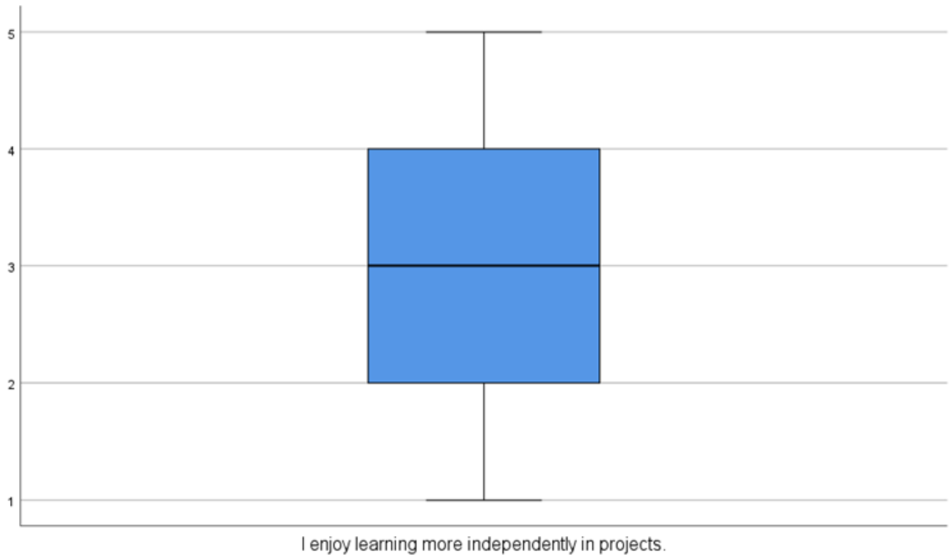
The 66 FSM participants (*M* = 3.3, *SD* = 1.0) compared to the 68 Non FSM participants (*M* = 2.85, *SD* = 1.1) reported significantly more positive opinions about enjoying learning more independently in projects [*t*(132) = -2.13, *p* = .035].

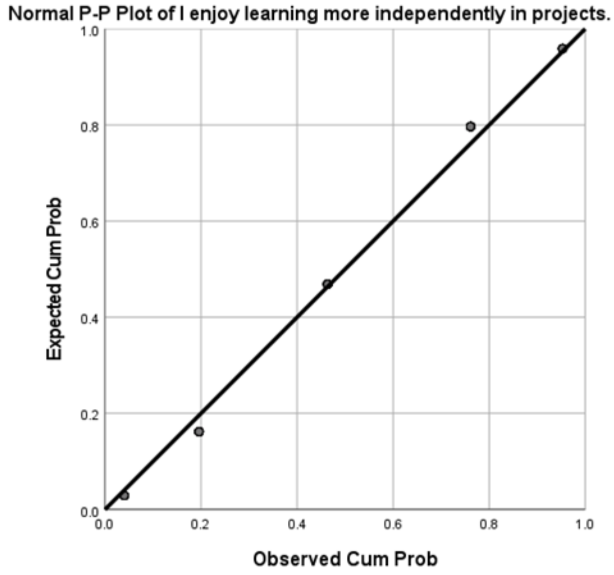
Students reported enjoyment of learning more independently in projects showed a very slight, but significant correlation with the relational vision of mathematics subscale (*r* =.20) and with three of the questions that are part of this subscale.

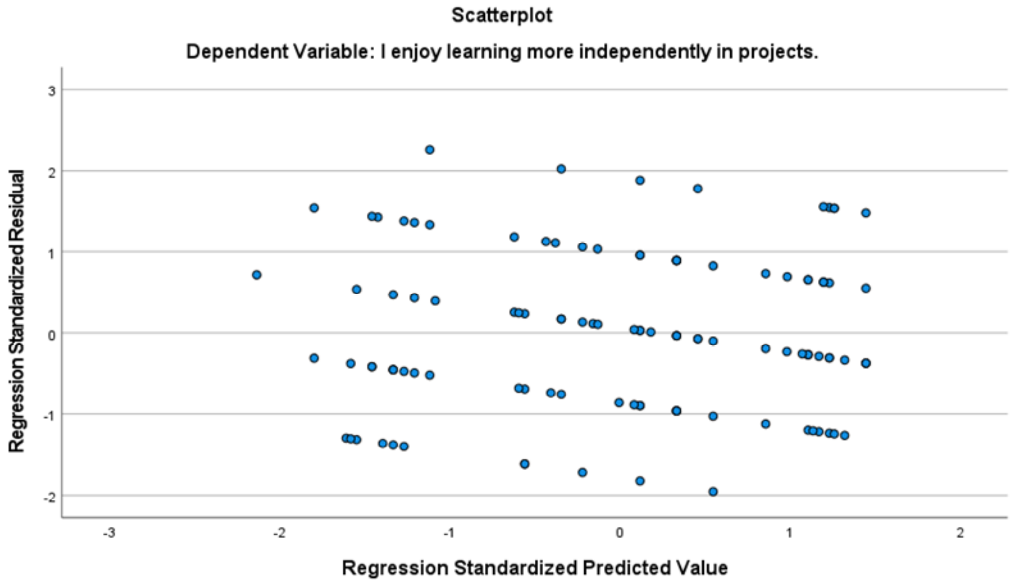
It showed a low, but significant correlation with one of the mathematical self-efficacy statements “I feel powerless doing mathematics problems” (*r* = 0.19). A possible interpretation of this is that students who are more likely to feel powerless are empowered by the independent aspect of the problems.

It showed a low, but significant correlation with two of the statements that indicate a more negative attitude to learning through projects: “Projects don't help me learn the mathematics I need for my exams” and I give up more often during projects than 'normal' lessons.


Histogram
Mean = 3.08
Std. Dev. = 1.102
N = 150
Frequency
50
40
30
20
10
0
1
2
3
4
5
I enjoy learning more independently in projects.


5
4
3
2
1
I enjoy learning more independently in projects.





23 Appendix 23 – Rigor and trustworthiness

Traditionally evaluation of qualitative studies was based on the scientific inquiry paradigm (Lincoln & Guba, 1986), with specific attention paid both to validity, the extent to which the phenomena has been accurately depicted, and reliability, the consistency of results. Some researchers have argued that the terms validity and reliability are not appropriate for qualitative or mixed methods studies and have come up with replacement terms. I adopt the position of Long and Johnson (2000) who argue that the concepts of reliability and validity are both applicable to qualitative research studies, there is no need for different terminology as essentially, the concepts are unchanged. What is required, however, is an appropriate means of addressing these criteria.

Next, I define these criteria. I follow this with a discussion on the means that I used to try to ensure validity and reliability, and the impact that the Covid-19 pandemic had on the methodology and thus my ability to ensure rigor.

- **Internal validity or credibility**: the extent to which the findings of a study represent the experiences of the participants: whether the account is believable, trustworthy, and appropriate.
- **External validity or transferability**: the extent to which the findings may be transferred from this study to another context. To establish how widely the theories generated in this study will be able to be applied, it is important to identify the similarities and differences that exist between this context and others.
- **Reliability or dependability**: the extent to which the same results are achieved if you carried out the same data collection at a different time; the replicability or consistency of the findings. Reliability in qualitative studies can be challenging to demonstrate (Long & Johnson, 2000).
- **Objectivity or confirmability**: the extent to which the findings of a study are truthful. In using a constructivist grounded approach, I did not seek to create an objective account. In fact, I don't view this as being possible. As Gasson argues, "we need to substitute reflexive self-awareness for objectivity" (2004, p. 90). The interpretation that I generate from this study will be **an** interpretation of the data, grounded in time and context, and influenced by me as the researcher. However, it is still important that the conclusions I reach are grounded in the data and dependant on the participants and conditions of the study, rather than me as the researcher.

23.1 Means of assessing reliability and validity

Table 23-1 highlights the means that were suggested in the literature (Amin et al., 2020; Lincoln & Guba, 1986; Long & Johnson, 2000) that I had hoped to use to assess reliability and validity in this study, which I discuss below. ✖ indicates a means I had aimed to use, but cannot claim to have used fully in this study.

	Internal validity or credibility	External validity or transferability	Reliability or dependability	Objectivity or confirmability
Reflexivity	✓	✓	✓	✓
Insider research	✓			
Peer debriefing	✓			
Member checking	✗			
Thick contextual description		✓		
Audit Trail			✓	✓
Triangulation	✗			✗
Persistent observation and prolonged engagement	✗			

Table 23-1: Means used to assess reliability and validity of the study.

23.1.1 Reflexivity

Reflexivity “provides researchers with means to deal with the inherent influence that the researcher brings to this type of investigation” (Amin et al., 2020, p. 7). It involves the researcher engaging in the processes of self-awareness and self-criticism, reflecting on their own beliefs similarly to the way they reflect on the beliefs of the participants. I kept a research journal throughout the process to try to develop my own practice of reflexivity; I documented assumptions, interpretations, decisions, and personal reactions to try to understand my own underlying motivations for choosing a line of enquiry or a particular interpretation.

23.1.2 Thick contextual description

Thick contextual description needs to be provided to the reader so they can establish whether the results of this study will transfer to another. This description needs to include information about the setting, the participants and should include quotations (Amin et al., 2020). It needs to go beyond facts and surface appearance and provide a narrative for the context. This description is given in Ch1, 5, 6 and 7.

23.1.3 Audit trail

An audit trail needs to pay particular attention to the decisions that a researcher makes, especially around sampling (Long & Johnson, 2000) and the analysis of the data. These decisions are detailed in Ch 5-7 and in the appendices. Having a detailed audit trail allows the reader to understand how my interpretations and values as a researcher helped to shape the research. In making this explicit, it enables the reader to understand the process that I've utilised in collecting, analysing, and interpreting the data. This allows the reader to ascertain the dependability of the study (Amin et al., 2020).

23.1.4 Peer debriefing

Long and Johnson (2000) suggest three types of peer debriefing, where someone uninvolved helps to probe a researcher's thinking. The first is discussing the emerging findings throughout the research process. As a doctoral student, I did this regularly in supervision and with my EdD peers. The second is to present and defend methods and findings at a conference. I presented an early draft of part of Ch6 at the BSRLM new researchers' day. The third is to share findings and implications with interested groups. This I have done formally as part of the study with the teacher workshop and informally, by sharing my findings with my different networks. Engaging in these processes helped me to explore alternate interpretations and to respond to critical challenge. As Long and Johnson (2000) highlight, it also helped me to continue to search for meaning and patterns beyond those I initially saw.

23.1.5 Member checking

I initially wanted to complete member checking with the FG participants, however it proved challenging to get everyone back together and research suggests that often the dynamic will then have changed (Barbour, 2005).

Instead, I asked participants to individually validate a summary of the FG as suggested by Morgan et al. (1998). As we were in lockdown, I posted the summaries to the students and put them on our online platform, only one student returned her summary. She made some marginal additions and reported finding the summary an accurate interpretation. This limited member checking suggests credibility (Lincoln and Guba (1986).

23.1.6 Triangulation

I intended to triangulate methods and theoretical perspectives. To triangulate methods, I aimed to collect data from different sources, to generate greater breadth and depth of the data and to allow me a deeper understanding of the phenomena (Flick, 2018). I didn't view triangulation as pursuing an 'objective' truth as perceived by some positivist researchers, but rather as a way of using different complementary methods to help construct a deep and full understanding of the participants' experiences and perceptions. The Covid-19 pandemic limited my data collection and therefore triangulation of methods was much more limited than originally envisaged.

I used two theoretical lenses for my interpretation. They helped me to view the data in different ways and to consider alternative interpretations, thus strengthening my study. However, the limited data meant that the interpretations cannot be viewed as more than exploratory.

23.1.7 Persistent observation and prolonged engagement

Persistent observation and prolonged engagement lead to an increase in trust between the participants and the researcher, creating less chance for miscommunication or misinformation on the part of the participants (Lincoln & Guba, 1986) and leads to greater sensitivity on the part of the researcher (Long & Johnson, 2000). Engaging with participants over a longer period allows a researcher time to think about the topic, establish meaning, and test out tentative emergent theories. It gives the researcher more time to focus on the elements most relevant to the research questions and explore them in detail (Lincoln & Guba, 1986).

I was an insider researcher; the challenges and ethical considerations of this are discussed below. As previous head of department, I co-constructed, with the teachers, our working definition for MaPBL and student-led learning. I started the study from a position of prolonged engagement: I was already familiar with the terminology and concepts surrounding PBL and the department.

The Covid-19 pandemic meant that the number of LOs and FGs were significantly reduced. Whilst I consider that I started from a position of prolonged engagement and was aiming for persistent observation and prolonged engagement, I do not claim to have achieved these in the study.

24 Appendix 24 – Ethical considerations

24.1 Informed voluntary consent and the right to withdraw

Informed voluntary consent of young people presents additional challenges, including ensuring consent is freely given and students feel comfortable to exercise their right to withdraw. I spoke to the students about the nature and specifics of the research. I asked some students to paraphrase what was said to ensure a good understanding of what was involved and the risks to them (Heath et al., 2004). I gained direct consent, rather than using an 'assumed consent' from gatekeepers such as parents or teachers (Robson, 2011).

The power imbalance between the students and myself, a senior member of staff and also the researcher, has the potential to limit students' ability to freely offer consent (Roberts & Allen, 2015). Except for my own class, I asked interested students to return the consent form to their class teacher. One student who was happy to be observed did not later attend FG1. Roberts and Allen (ibid) highlight how students can struggle to withdraw part way through. I viewed this as him exercising his right to only participate in parts of the research and, although he may have wanted to continue to be part of the research and had simply forgotten, to alleviate any pressure on him, I did not ask him why he had not attended. Throughout, I reminded participants that they had the right not to answer questions.

Sometimes parents, as gatekeepers, block student involvement in research (Robson, 2011). This can be a challenge at the study school, so I offered to speak to parents, if students wanted to be involved, to allay their concerns. No student requested this.

As this study followed a grounded approach, and due to the impact of Covid-19, the methodology shifted from my initial plan. With the teachers, I used 'process consent', a concept developed in nursing (e.g. Dewing, 2007) and recommended for use in education by Roberts and Allen (2015). In this process, I, as the researcher, discussed with the participants any change in direction of the research and the potential risks involved before mutually agreeing to proceed.

24.2 Pseudonymisation

All the participants have pseudonyms. The key is saved in a remote access drive, that only I have access to, on the UCL network (with Advanced Encryption Standard 256 bit encryption). I tried to limit the background information shared about the participants and the school. I deem it unlikely that the students will be identifiable: there are over 120 students in a year and few characteristics beyond prior attainment band, gender and set are reported. However, it is likely that anyone who knows me and reads the study will be able to identify the school and possibly the teachers; I made all the teacher participants aware of this.

24.3 Data protection and the processing of personal data

As far as possible I held the data in a form that will not be attributable to the individuals who provided them, as discussed under pseudonymisation. I had to be particularly careful about how I processed the data as some of the data were personal. For example, some of the data generated about participants attitudes towards PBL could be viewed as constituting their philosophical beliefs.

Audio recordings of the FGs and teacher workshop were held on a password protected mobile device for up to 24 hours. The data were transferred to a transcriber in the UK via www.upwork.com. All access to the Upwork site is encrypted using industry-standard transport layer security technology (TLS). The transcriber signed a contract which specified that the data must be treated confidentially and after transcription should be destroyed by the transcriber. The handwritten notes I made were typed up and saved on the UCL network, before being destroyed.

All data will be stored for 10 years in an electronic format, in line with UCL guidance.

24.4 Insider researcher

As a senior leader in the study school, I was known by nearly all the students and may have benefited from having a rapport with some of them (Kawulich, 2005). Students were used to me completing LOs. However, my presence in the classroom will have influenced what happened. I discussed whether a lesson was typical both with the teacher at the end of each LO and in FG1. I am aware that in the LOs, some students demonstrated higher than typical levels of engagement. I view my presence as more likely to have altered the level of students' engagement with the path they had chosen rather than altering the extent to which the students were leading their own learning. This increased engagement may have been due to my position of authority in the school, or perhaps in volunteering to be part of the project, students wanted to show how committed they were.

Whilst I did not observe entirely typical lessons, the LOs were designed to be a stimulus for the FGs. That some students had higher levels of engagement than usual also helps with the telling case – what challenges do students experience leading their own learning, even when they are engaged and trying to do their best.

How you are viewed by the participants will determine what they tell you (Charmaz, 2014). Further, in FGs, an overemphasis can be placed on consensus (Barbour, 2005) and there is the possibility of false findings due to group pressure to conform (Morgan & Krueger, 1998). To establish trust, I chose a school meeting room, where student voice activities usually take place. To try to ensure students gave an authentic account of their lived experiences, I regularly reminded students about the nature of the research and how I valued their views, there was no right answer.

24.5 Costs and benefits to participants

Participants gave up their free time, I tried to minimize this by having a clear interview schedule, focused on the emergent concepts. They may have felt an increased pressure having someone observe their lessons. To help mitigate this, I explained the observations were not judgemental.

The student participants hopefully benefited from considering how they lead their own learning, reflecting on how they learn (Coulson & Harvey, 2013), and feeling that their voice is heard (Cook-Sather, 2006). The teacher participants will have gained from deep reflection on their practice with an experienced other (Baker & Johnson, 1998) and the department, as a professional community, developed from conversations around teachers' practice (Peng, 2007). Furthermore, the research informed some of the departmental support and training as well as future curriculum direction.

It is important to consider those who volunteer to participate, but who are not then selected to be participants. All the students and teachers who volunteered for a specific aspect of the research were invited to participate.

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