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Modelling reservation-based shared autonomous vehicle services: A dynamic user equilibrium approach

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ABSTRACT

Shared Autonomous Vehicles (SAVs) are expected to be used for regular and pre-planned trips. Such trips are suitable for reservation-based services, wherein the customer needs to book for a trip in advance. Systems enabling reservation of trips can allow for better planning of routes and schedules, and if optimally designed, enable higher efficiency. The primary objective of this research is to model the effects of such a system, by formulating and solving the combined Dynamic User Equilibrium and Shared autonomous vehicle Chain Formation (DUESCF) problem. The problem is formulated as a bilevel model based on game theory, involving road users and SAV service operator. Given a situation where conventional private and shared autonomous vehicles co-exist, road users select paths and departure times to maximize a perceived utility (commonly treated as minimizing a disutility) by forming a DUE (fixed point problem), and the SAV service operator tries to maximize the performance by forming appropriate SAV chains (combinatorial problem). The final objective of this bilevel model is a traffic assignment that includes SAV chain formation, such that both road users and SAV service operator obtain optimal solutions by reaching a Nash equilibrium, where no player is better off by unilaterally changing their decisions. A solution approach, based on Iterative Optimization and Assignment (IOA) method, is proposed with path flow and SAV performance changes as convergence criteria. Furthermore, the solution approach is tested for its robustness, and a scenario analysis is carried out to evaluate the impacts of reservation-based SAV services. The results show that a ridesharing SAV system is better compared to a carsharing and a mixed system consisting of both, in terms of total system travel time, congestion levels, total vehicle kilometres travelled and vehicle requirements.

1. Introduction

Vehicles with some level of automation to assist or replace human control are called automated vehicles, and an automated vehicle with Level 5 automation is commonly termed as Autonomous vehicle (AV) or self-driving vehicle. Major technology companies are spending billions of dollars to develop AVs (e.g., Korosec, 2018; Trivedi, 2018), and hence, it can be asserted that AVs will eventually become a reality at some point in future (Brown, 2018). Although the timeframe is still uncertain, decisions of several vehicle manufacturers and related industrial partners point towards the initial deployment of AVs in shared mobility services (e.g., BMW Group, 2016; Daimler, 2017; Hawkins, 2017; LeBeau, 2018; O'Kane, 2018). The pertinent literature shows that deployment of AVs in shared mobility services, along with integration of public transport systems, could lead to a more sustainable future

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with enhanced mobility and equity (International Association of Public Transport, 2017; Narayanan et al., 2020a). Furthermore, automation enables application of smaller vehicles across a large spectrum of demand in public transport services (Tirachini and Antoniou, 2020).

Based on their operations, Shared Autonomous Vehicle (SAV) services can be divided into on-demand systems (customers can book in real time), reservation-based systems (customers must book in advance) and mixed systems (Narayanan et al., 2020b). Trips suitable for reservation-based services include regular trips like commuting trips or shopping trips, and pre-planned trips like trips to the airports or leisure activities. Reservation-based operations ensure vehicle availability, which can be a factor in increasing preference towards shared mobility services, and thus, influencing the modal shift from private to shared vehicles. In the case of reservation-based SAV services, SAV chain formation (optimization of vehicle assignment to serve multiple trip requests) can be perceived to influence the route and the departure time choices of all road users, because of the effect that they have on traffic. Conversely, the flow pattern of all road users affects the SAV chain formation, since the travel times can differ.

Therefore, the aforementioned interaction between departure time and route choice behaviour of road users and the SAV chain formation by an SAV operator is important to be studied. Besides, a traffic assignment model considering the road network and an optimization model capturing the trip chaining behaviour in reservation-based services is a necessity, when modelling this interaction. In addition, the dynamics of SAV trip chains can only be represented with a dynamic traffic assignment model, which considers the evolution of queues and congestion over time, in response to vehicle departures and arrivals. However, such a model based on Dynamic User Equilibrium (DUE) is still missing in literature, while the necessity is signified by the wide use of adapted traffic assignment methods by many public authorities around the world. Furthermore, SAV service operators could also use it to maximize their service performance by forming efficient (model-based) SAV chains.

This paper aims at formulating the combined Dynamic User Equilibrium and Shared autonomous vehicle Chain Formation (DUESCF) problem. A bilevel problem consisting of DUE and SCF models is formulated first, which is then shown to have a Nash equilibrium based on game theory. Game theory approach considers players as selfish agents with an aim to maximize their own payoff, which is often a true depiction of the real world. In the proposed game, both the players try to maximize their utilities by rationally making their decisions. Thus, a change in decision of one will encourage the other to change his/her decision, in order to reduce the costs. This would happen until Nash equilibrium is reached, wherein both the road users and the SAV operator cannot further reduce their costs by changing their decisions.

To find the solution to this Nash game, a solution approach similar to best response dynamics, wherein the DUE and the SCF models are iteratively solved until convergence, is suggested. This solution approach is tested for its robustness by adapting existing DUE and SCF models and the experiments are carried out using a toy, Braess bidirectional, Sioux Falls and Anaheim networks. Furthermore, to evaluate the impacts of reservation-based SAV services, a scenarios analysis is performed on the Sioux Falls network. Therefore, this research contributes to the pertinent body of literature by: (i) Formulating the DUESCF problem as a bilevel model based on game theory, (ii) Proposing an iterative solution method for the formulated DUESCF bilevel model, (iii) Extending the DUE model of Han et al. (2019) to enable modelling of both conventional private vehicles and SAVs, (iv) Extending the SCF model of Ma et al. (2017) to include ridesharing and (v) Evaluating the impacts of reservation-based SAV services.

The remainder of this paper is structured as follows: Literature review on existing models for evaluating reservation-based SAV services and existing DUE models is presented next (Section 2), followed by the listing of the notations used in this research work (Section 3). After that, the combined DUESCF problem is formulated as a bilevel model (Section 4.1) and proposed to be a Nash game based on game theory (Section 4.2), and then a solution approach is suggested (Section 4.3). Subsequently, the adaptation of existing DUE and SCF models for testing the formulated bilevel model and the proposed solution approach is expounded (Section 5). The applicability of Nash equilibrium holds for any DUE and SCF models, which respects the axioms mentioned in Section 4. Therefore, the DUE and SCF models adapted in this study is just an instance for the application of the formulation and hence, the formulation is described before the adaptations. Later, in Sections 6 and 7, the results of computation experiments and the scenario analysis are reported and briefed. Finally, conclusions are elucidated along with a discussion and directions for future works (Section 8).

2. Literature review

2.1. Modelling reservation-based SAV services

In the literature of reservation-based SAV services, formation of SAV chains is generally found for the assignment of vehicles to trip requests, which is a variant of vehicle routing and dial-a-ride problems. An SAV chain represents the sequence of requests served by a single SAV. An SAV chain consists of four types of trips namely dispatch, service, relocation and collection trips Ma et al. (2017). A dispatch trip is a trip from depot to the origin of a customer request and a service trip is a trip that fulfils one customer request. A trip between the destination of one customer request to the origin of another customer request is a relocation trip and a trip between destination of a customer request to depot is a collection trip. For an illustrative figure of a SAV chain, the reader is referred to Ma et al. (2017). To the authors' best knowledge, only a few studies (Bongiovanni et al., 2019; Duan et al., 2020; Levin, 2017; Ma et al., 2017; Su et al., 2020) exist in the literature that deal with vehicle assignment exclusively for reservation-based SAV services. Of these four studies, only Levin (2017) considers the underlying road network (i.e., use a network assignment model). The rest of the studies use graphs with edges that directly connect customer origins and destinations with fixed travel times.

The combinatorial nature of an SAV chain formation problem commonly raises computational time concerns. Levin (2017) incorporates SAV vehicle assignment into the system optimal DTA model of Ziliaskopoulos (2000). Though their formulation is a linear program, numerical testing on a grid network with four OD pairs shows a high computation time (35 to 45 min). He concludes that his formulation and solution method may not be tractable for realistic networks, and suggests using DUE models for route choice. Su et al. (2020) adopt the formulation of Cordeau and Laporte (2007) for forming SAV chains, and test Tabu search meta-heuristics combined with a clustering method (K-Means and K-Medoids) as a solution algorithm. The solution algorithm is tested with a New York taxi dataset, and the solution computation time is in the order of hours. Bongiovanni et al. (2019) incorporate vehicle-to-depot assignment and battery-management problem in their formulation, along with the usual vehicle assignment for the customers. Their formulation is a mixed integer problem. The problem can only be solved for instances of up to 4 vehicles and 40 customers, with a maximum computation time of 2 h. Thus, their model is not suitable for larger demands. Duan et al. (2020), Ma et al. (2017) provide computationally efficient algorithms. While Duan et al. (2020) use heuristics, Ma et al. (2017) base their solution on a linear program. Ma et al. (2017), initially, formulate an integer programming model and show that the integer program is equivalent to a linear program, by proving that the constraint matrix is totally unimodular. They test their model with a New York taxi dataset, using the CPLEX solver, and the computation time is in the order of few seconds. Thus, their model is tractable for larger demands and also provides the optimal (exact) solution.

2.2. Dynamic user equilibrium models

Dynamic traffic assignment refers to the modelling of network assignment of the time varying flows on road network, in a way that is consistent with the established traffic flow theory and travel demand theory (Friesz, 2010). A wide variety of problems fall under the category of DTA, each with different behavioural assumptions and having different capabilities in representing the traffic system (Peeta and Mahmassani, 1995). Some of the well-known DTA models include Aimsun (Barceló and Casas, 2005), Dynameq (Mahut and Florian, 2010), DynaMIT (Ben-Akiva et al., 2001, 2010), DYNASMART (Jayakrishnan et al., 1994), SUMO (Krajzewicz et al., 2012), and VISSIM (Fellendorf, 1994). DUE is a type of DTA, wherein the effective unit travel delay for users having same origin and destination and departing at the same time interval is identical and minimum (Friesz, 2010; Szeto and Wong, 2012). This formulation is a generalization of the Wardrop principle of the static user equilibrium to time-dependent case, wherein the user equilibrium conditions are obtained through a time-dependent extension of Wardrop's first principle. DUE is usually modelled for the within-day time scale based on the demand established on a day-to-day time scale (Himpe, 2016).

Many approaches have been proposed in the literature for modelling DUE. With regards to formulation, DUE models can be formulated based on optimal control theory, variational inequality, nonlinear complementarity problem, differential variational inequality, differential complementarity system and fixed-point problem. The reader is referred to Han et al. (2019) for a presentation of the different formulations. Based on the dimension of choice considered for equilibrium, DUE models are of two types, namely Route Choice model (e.g., Himpe, 2016) and Simultaneous Route and Departure Time Choice model (e.g., Han et al., 2019; Mahmassani and Herman, 1984). As the name suggests, in case of former, only route choice of the road users is considered in the formulation of equilibrium, while in the latter, both trip route and departure time choice of road users are considered. Another type of classification includes link based, path based and destination based DUE models, which is based on the type of flow variable used for equilibrium. The reader is referred to Himpe (2016) for details of the principles behind these flow variables. Depending on the type of route travel time considered for equilibrium, DUE models are of two types: 1. Reactive/instantaneous and 2. Actual/predictive (Yildirimoglu and Geroliminis, 2014). While the reactive travel time is the sum of the link travel times along the path estimated at the time of departure from origin, the predictive travel time is based on the sum of the link travel times along the path estimated at the time when drivers enter each link. Finally, based on stochasticity in the choice decision, DUE models can be divided into deterministic and stochastic DUE models (Szeto and Wong, 2012).

DUE models usually consist of two major components, namely Route (and departure time) choice model and Dynamic network loading (Han et al., 2019). The route (and departure time) choice model corresponds to the mathematical expression for equilibrium condition. The Dynamic Network Loading (DNL) component describes the spatial and temporal evolution of traffic flows on a network that is consistent with the route (and departure time) choices established in the former model. As such, the DNL component models the network performance, and this is defined using the following sub-models (Friesz, 2010; Han et al., 2019):

- **Link/Path delay** - deals with calculation of link/path delay (can be travel time or a combination of travel time and arrival penalties)
- **Flow dynamics** - represents analytical relationship between flow/speed/density and link/path traversal time
- **Flow propagation constraints** - deals with propagation of traffic through links
- **Junction dynamics and delays** - modelling of traffic at junctions

Various traffic flow propagation (link travel time) models for DUE have been found in literature and they can be broadly classified into two groups, namely 1. Delay-function models and 2. Exit-flow function models (Friesz et al., 2013; Yildirimoglu and Geroliminis, 2014). Delay-function models are based on an explicit travel delay function (e.g. link delay model, LDM). Exit-flow function models are based on explicitly modelling the underlying flow dynamics (e.g., outflow model, cell transmission model, whole link model, deterministic queuing model and mesoscopic models). DNL models need to have specific properties such as First In First out (FIFO), positivity, conservation, capacity restraint, causality, monotonicity and consistency. Readers are recommended the following works for more information about DTA and DUE models: Barceló (2010), Boyce et al. (2001), Chiu et al. (2011), Friesz (2010), Friesz and Han (2019), Garavello et al. (2016), Peeta and Ziliaskopoulos (2001), Ran and Boyce (1996), Szeto and Wong (2012), Wang et al. (2018). Out of the vast majority of works performed on DUE models, Han et al. (2019) and Himpe (2016) have been found to include open source scripts. While Han et al. (2019) has been tested for large real-world networks, Himpe (2016) has been tested only for small networks.

2.3. Research gaps and opportunities

A model that incorporates DUE for traffic assignment to evaluate reservation-based SAV services is still missing in literature. Hence, a novel contribution of this research work is to link the DUE problem with the SCF problem, by formulating and solving the combined DUESCF problem as a bilevel model based on game theory, to achieve Nash equilibrium between roads users and SAV operator. This will enable evaluation of reservation-based SAV services with consideration of network congestion. For testing of the proposed DUESCF formulation and solution approach, the model of Han et al. (2019) is found appropriate for DUE traffic assignment, since it has already been tested for large realistic networks. For the formation of SAV chains, the model of Ma et al. (2017) is found to be most appropriate, since the resulting solution is optimal (exact solution) and the model is computationally effective for large demands.

3. Notations

The notation to be used throughout this paper is presented below, structured in three main blocks: (a) DUESCF model; (b) DUE model; and (c) SCF model. Alphabetical ordering is followed and common notation with Han et al. (2019), Ma et al. (2017) is maintained to the greatest extent possible.

DUESCF model

$\mathbf{C}$	Cost (travel time) in the network for a given $\mathbf{r}$ (corresponds to Matrix $D_p(t)$ in DUE model)
$C_c^1, \dots, C_c^n$	Cost (travel time) of individual conventional vehicle user
C_c	Total cost (travel time) of conventional vehicle users
C_e	Total SAV empty trip cost (travel time)
C_o	Total system cost (travel time) of SAV service
C_r	Total cost (travel time) of all the road users (conventional vehicle and SAV users)
$C_s^1, \dots, C_s^n$	Cost (travel time) of individual SAV user
C_s	Total cost (travel time) of SAV users
CG	Cost gap
E_1	Without loss of generality, payoff of any road user (based on user travel time)
E_2	Payoff of SAV operator (based on the total system travel time of the SAV service)
$F(r)$	Cost function, based on the DNL model used
$Fp(R, F)$	DUE problem formulated as a fixed-point system
$G(\boldsymbol{\omega}, \mathbf{r})$	Objective function of SCF problem
$g(\boldsymbol{\omega}, \mathbf{r})$	Constraints of SCF problem
PFG	Path flow gap
R	Feasible set of departure rate matrix ($\wedge$ in the adapted DUE model)
$\mathbf{r}$	Departure rate which captures both departure time choice and path choice of the road users (h in adapted DUE model); Decision of Player 1
X	Iteration count of the Iterative Optimization and Assignment (IOA) algorithm
Ω	Symbolic notation for feasible set of SAV chains
$\boldsymbol{\omega}$	SAV chain matrix (based on x_{ij} from the adapted SCF model); Decision of Player 2

DUE model

$\mathbf{G(N, A)}$	Road network (SAV network, in case of SCF model)
$\mathbf{h}$	Matrix of departure rates
$\mathbf{h}^*$	Matrix of departure rates corresponding to DUE
$h_p(t)$	Departure rate along path p at time t
K	Set of vehicle types
L	Set of expected time of arrival
P	Set of paths in the network
P^o	Set of paths originating from $o \in S$
$P_{\wedge_0}[.]$	Minimum-norm projection operator
P_{ij}	Subset of paths that connect OD pair (i, j)
P_{ijk}	Subset of paths that connect OD pair (i, j), corresponding to vehicle type k
P_{ijkl}	Subset of paths that connect OD pair (i, j), corresponding to vehicle type k and expected time of arrival l
PR	Penetration rate for SAV ridesharing service
Q_{ij}	Total demand between an OD pair $(i, j) \in W$
Q_{ijc}	Conventional private vehicle demand between an OD pair (i, j)
Q_{ije}	SAV empty trip demand between an OD pair (i, j)
Q_{ijk}	Demand between an OD pair (i, j), corresponding to vehicle type k
Q_{ijs}	SAV service demand between an OD pair (i, j)

Q_{ijsc}	SAV carsharing service demand between an OD pair (i,j)
Q_{ijsr}	SAV ridesharing service demand between an OD pair (i,j)
Q_{ij2l}	SAV demand between OD pair (i,j), corresponding to expected time of arrival l
S	Set of origins
$[t_0, t_f]$	Planning horizon starting and ending time
t	A time instant in the planning horizon $[t_0, t_f]$
v_{ij}	Dual variable
W	Set of OD pairs in the network
Y	Iteration count of the fixed-point algorithm
α	A fixed constant
β	Path flow adjustment parameter (step size) of fixed-point algorithm
Ψ	Effective delay operator (could be a combination of travel time, arrival time & road pricing)
$\mathbf{\Psi}(\mathbf{h}^*)$	Matrix of travel cost (effective delay) under departure profile $\mathbf{h}^*$
$\Psi_p(t,\mathbf{h})$	Travel cost along path p at time instant t, under departure profile $\mathbf{h}$
$\wedge$	Set of feasible path departure rate matrix
SCF model	
A	Set of all links in the network $(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)$
$A1$	Set of service links with each link representing one service trip $(i \in I)$
$A2$	Set of feasible relocation links (relocation links with travel time less than the difference between drop-off time of preceding service trip and the pick-up time of the subsequent service trip)
$A3$	Set of dispatch links
$A4$	Set of collection links
$A5$	Virtual link
c_{ij}	Cost of using a link (i,j)
D	Terminal depot node
d_{iD}	Cost for collecting an SAV from the last service trip; for A_4 links
d_{ij}	Driving cost in a relocation link; for A_2 links
d_{Oj}	Cost for dispatching an SAV to the first service trip; for A_3 links
d_r	Fleet cost per vehicle to account SAV maintenance and refuelling at the depot
FS	Available fleet size
$G(N,A)$	SAV network graph (road network, in case of DUE model)
I	Set of trip demands in a planning horizon $[t_0, t_f]$
LP	Penalty for losing a service trip
m_{ij}	Capacity of a link (i,j)
N	Set of all nodes in the network (service and depot nodes)
O	Origin depot node
p_{ij}	Parking cost when using a relocation link; for A_2 links
VO_{ij}	Vehicle occupancy in a service link; for A_1 links
x_{ij}	Number of vehicles using a link (i,j); Decision variable

4. DUESCF formulation & solution approach

In a system consisting of conventional private vehicles and SAVs, two different players are involved. On the one hand, there is a group of road users, who decide their trip departure time and travel path. On the other hand, there is an SAV operator, who takes decisions pertaining to assignment of vehicles to the SAV users. We believe that, in future, autonomous vehicles are capable of making trip decisions. Therefore, we assume that the decision of trip departure time and travel path for SAV users is taken by the SAVs, minimizing disutility of travel from the perspective of the SAV users (i.e., the SAV departure times are aligned with the ride-request arrival times). From the viewpoint of a traffic model, this does not necessitates any change, as the decisions by vehicles are based on user equilibrium. Thus, the road user group include the decision making conventional private car users and SAVs, and they all compete with each other simultaneously. The following axioms are used as the basis for this problem:

Axiom 1 (*Wardrop's User Equilibrium Principle*). No road user will choose a route that increases his/her travel cost (travel time and early/late arrival penalty).

Axiom 2 (*Cost Minimization of the SAV Operator*). No SAV operator will choose a set of inefficient SAV chains that increases his/her cost (total system travel time).

There exist interactions between the road users and the SAV operator. SAV chains (an SAV chain is a sequence of trips served by one SAV) formed by the operator can affect the traffic levels in the network, for example because of empty trips, and in turn, this might affect road users' decision regarding departure time and travel path. Similarly, if the road users change their departure time and the travel path, the operator might be required to change the SAV chain matrix, because of the change in trip departure times and the travel times in the network. The operator needs to be aware of the probable departure time and the path of each SAV user, along with the probable travel times in the paths. This information is equivalent to a path based departure rate matrix ($\mathbf{r}$) and travel time matrix ($\mathbf{C}$).

Considering the selfish nature of the road users, departure rate matrix ($\mathbf{r}$) and travel time matrix ($\mathbf{C}$) can be constructed using a DUE model. With this information, the operator can construct a trip request matrix (origin and destination of each SAV service trip, along with departure and travel time), and decide on the number of vehicles to be utilized and the sequence of trips to be served, thereby forming a matrix of SAV chains ($\boldsymbol{\omega}$). Such a matrix can be constructed using an optimization problem, which can then be used by the road users. Thus, this interaction between the road users and the SAV service operator can be formulated as a bilevel problem between two players, consisting of a DUE and an optimization model respectively (see Section 4.1). At the same time, given the nature of the players and their actions, their interaction possesses the property of Nash equilibrium (see Section 4.2) and the corresponding Nash game can be solved using a sequential solution approach (see Section 4.3).

4.1. DUESCF bilevel model

The bilevel DUESCF problem is shown in Eq. (1). This formulation has been framed to be general, without any attribution to specific DUE and SCF models, as the formulation can be applied to a combination of any type of DUE and SCF models, which does not violate Axioms 1 and 2.

$$\begin{aligned}
&\textbf{Player 1}\\
&\qquad \text{Solve } F_p(R,F) \text{ (or any other equivalent DUE formulation)}\\
&\textbf{Player 2}\\
&\qquad \min G(\boldsymbol{\omega},\mathbf{r})\\
&\qquad \text{s.t.}\\
&\qquad g(\boldsymbol{\omega},\mathbf{r}) \le 0 \;\&\; g(\boldsymbol{\omega},\mathbf{r}) = 0
\end{aligned} \tag{1}$$

where,
$G(\boldsymbol{\omega},\mathbf{r})$: Objective function of SCF problem
$g(\boldsymbol{\omega},\mathbf{r})$: Symbolic notation for the constraints of SCF problem ('≤' represents inequality constraints and '=' represents equality constraints)
F: Function mapping $\mathbf{r}$ to cost matrix
$Fp(R,F)$: Symbolic notation for a fixed-point based DUE problem
R: Feasible region of the departure rate matrix $\mathbf{r}$, for a given demand calculated based on $\boldsymbol{\omega}$
$\mathbf{r}$: Departure rate matrix (decision of player 1) $\mathbf{C}$ through DNL
$\boldsymbol{\omega}$: SAV chain matrix (decision of player 2)

For a given (exogenous) demand [demand corresponding to conventional vehicle users (Q_{ijc}) and SAV users (Q_{ijs})] and an output ($\boldsymbol{\omega}$) from SCF model, the DUE model computes the user equilibrium departure rates and path flow ($\mathbf{r}$). It should be noted that $\mathbf{r}$ is a matrix, with each column representing a time instant t and each row representing a path between different origins and destinations (set of OD pairs, W). The values in each cell of this matrix represent the flow departing in a path p at a time instant t. Furthermore, $\mathbf{r}$ represents the collective non-cooperative decision of the group of road users (Player 1). According to the departure rates and path flows, the DNL submodel of the DUE model computes the travel time values ($\mathbf{C} = F(\mathbf{r})$). $\mathbf{C}$ is also a matrix with columns representing time instants and rows representing the paths.

Based on the departure rates ($\mathbf{r}$) and the travel time values ($\mathbf{C}$), the SCF model forms SAV chains ($\boldsymbol{\omega}$). The departure rates from the DUE model are used as the basis for the construction of SAV trip requests (departure time for each SAV trip). The travel times from the DNL submodel, estimated based on the path flows, are used as the journey times for the SAV trips (both service and empty trips). The SAV service operator optimizes the total system travel time by minimizing the SCF objective function $G(\boldsymbol{\omega},\mathbf{r})$, subject to the SCF constraints ($g(\boldsymbol{\omega},\mathbf{r})$). The columns of the SAV chain matrix represent the trip requests served, while the rows represent SAVs. Therefore, the link between the two models can be summarized as follows: The DUE model uses the SAV chain matrix ($\boldsymbol{\omega}$) to find SAV empty trips and calculate total demand in the network. The SCF model uses the departure rate matrix ($\mathbf{r}$) and the travel time matrix ($\mathbf{C}$) to construct the SAV trip requests and the travel times between the nodes (depot, and origins and destinations of the SAV users) in the network.

4.2. DUESCF bilevel model as a Nash game

Because of the selfish nature of the road users and the SAV operator, a change in decision of one will encourage the other to change his/her decision, in order to reduce the costs. This would happen until a stable state (equilibrium) is reached, wherein both the road users and the SAV operator cannot further reduce their costs by changing their decisions. Such a stable state is commonly referred to as Nash equilibrium in game theory (Holt and Roth, 2004). Thus, based on game theory, the bilevel DUESCF problem can be formulated as a Nash game between the group of road users (Player 1) and the SAV service operator (Player 2), wherein both the players try to maximize their utilities by rationally making their decisions (see Fudenberg and Tirole, 1991 and Holt and Roth, 2004 for more information about game theory and Nash equilibrium). For the DUESCF formulation to be applicable, the following basic proofs are established:

Lemma 1. *The bilevel DUESCF model based on game theory is a non zero sum game*

A non zero sum game is a situation where there is a net benefit or net loss in the system based on the outcome of the game, i.e., profit of one player is not necessarily an equal loss of the other. For the DUESCF formulation, this is obvious and can be easily proved using the following equations:

$$C_c = \sum_{i=1}^{n} C_c^i \tag{2}$$

$$C_s = \sum_{i=1}^{n} C_s^i \tag{3}$$

$$C_r = C_c + C_s \tag{4}$$

$$C_o = C_s + C_e \tag{5}$$

where,
C_c^i: Cost (travel time) for a conventional vehicle user
C_s^i: Cost (travel time) for an SAV user
C_e: Total cost associated with SAV empty trips

Eq. (2) represents the total cost (travel time) of conventional vehicle users (C_c). Eq. (3) represents the total cost (travel time of) SAV users (C_s). Eq. (4) represents the summation of the above two, i.e., the total cost (travel time) of all the road users (C_r). Eq. (5) represents the total system cost (travel time) of SAV service (C_o). From Eqs. (4) and (5), one can infer that an increase in cost of the SAV operator may or may not (if the SAV operator cost increase is due to the empty trips) result in an increase in cost for road users. Furthermore, an increase in cost of conventional vehicle users does not affect the cost for the SAV operator, although an increase in cost of SAV users results in an increase in cost for the SAV operator. However, in all these scenarios, loss of one player is not an equal profit for the other. Hence, it can be concluded that the DUESCF problem formulated as a game involving roads users and SAV operator is a non-zero sum game.

Lemma 2. *Road users and SAV operator will aim to achieve a Nash equilibrium.*

To prove this, the two axioms presented before are utilized. Assume for contradiction that the road users and the SAV operator do not try to reach Nash equilibrium. By the definition of Nash equilibrium,

$$\begin{aligned} &E_1(\mathbf{r}^*, \boldsymbol{\omega}^*) \geq E_1(\mathbf{r}, \boldsymbol{\omega}^*), \forall \mathbf{r} \\ &E_2(\mathbf{r}^*, \boldsymbol{\omega}^*) \geq E_2(\mathbf{r}^*, \boldsymbol{\omega}), \forall \boldsymbol{\omega} \end{aligned} \tag{6}$$

where,
E_1: Without loss of generality, payoff of any road user (based on user travel time)
E_2: Payoff of the SAV operator (based on the total system travel time of the SAV service)
$\mathbf{r}$: Any decision of the road users (player 1)
$\mathbf{r}^*$: Decision of the road users at Nash equilibrium
$\boldsymbol{\omega}$: Any decision of the SAV operator (player 2)
$\boldsymbol{\omega}^*$: Decision of the SAV operator at Nash equilibrium

By the assumed contradiction, one of the players may end up with a lower payoff. This will violate one of the two axioms presented before. Therefore, Lemma 2 holds. This proof by contradiction is applicable to any strategic game and therefore, the DUESCF formulation is applicable to a combination of any type of DUE and SCF models, provided the two axioms are not violated.

Notes

1. In reality, there could be situations where road users may violate the principle of user equilibrium (e.g., in case of imperfect information or driving for leisure). Similar to the road users, the SAV operator may not always aim to minimize his cost, e.g., in the short term, the SAV operator may choose loss over profit to increase user demand. In such cases, the existence of Nash equilibrium does not hold.
2. The existence of a solution for Nash equilibrium depends on the type of DUE and SCF models used and their properties. For example, existence of a solution is guaranteed, if the action space is nonempty, convex and compact and the objective (cost) function is continuous and convex (Dutang, 2013).
3. Generalized Nash equilibrium is a generalization of the standard Nash equilibrium, to include the interdependency of both the objective function and the feasible set among players (Facchinei and Kanzow, 2010). In the current problem, the objective function (e.g., user equilibrium objective and the network loading function) and the feasible set (e.g., the paths between OD pairs) stays the same for both players and does not depend on the actions of the other player. Therefore, the adoption of generalized Nash equilibrium is not required and the standard Nash equilibrium is applicable.

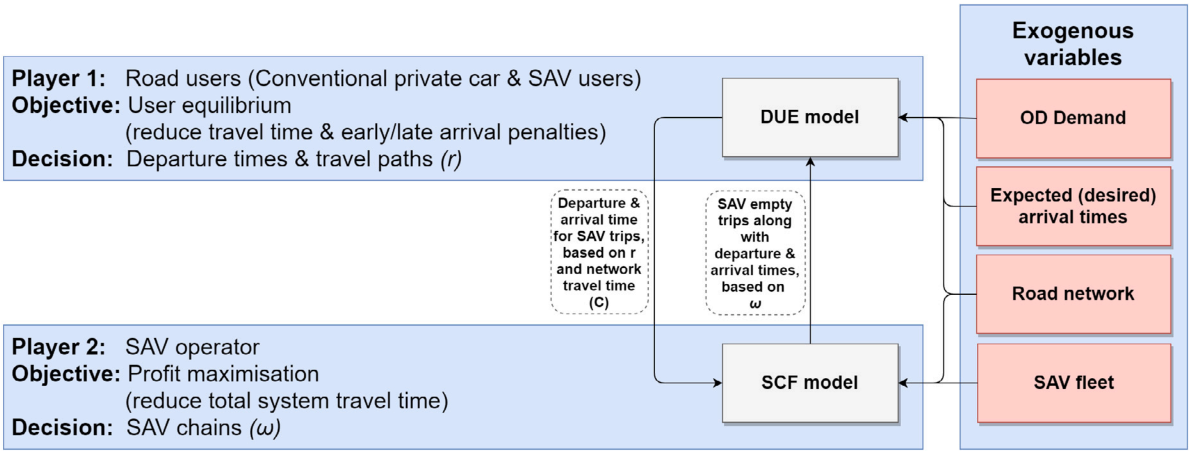

Fig. 1. Modelling schema.

An example of such a bilevel model, with the interaction between operators and transit users modelled based on game theory, is Zhou et al. (2005). The authors present a bilevel transit fare equilibrium model, wherein the decision of a group of transit operators is modelled at the upper level, while the decision of a group of passengers is modelled at the lower level. However, given the leader–follower nature, Zhou et al. (2005) model the interaction as a Stackelberg game. In the current problem, although the SAV operator's decision precedes that of the road users in reality, the road users may not be aware of the dispatch strategy of the operator, and so there is no belief on commitment. Further, the decision of taking a particular path and departing at a particular time (DUE) is usually based on the assumption that the road users have the knowledge of travel cost in the network, based on the recurrent usage of the same road network. Hence, although the act of driving through the network occurs after the operator's decision, the decision of the road users is based on previous knowledge. Under such a situation, wherein a player takes decision without knowing the decision of the other player, the game should be modelled as a simultaneous move (Nash) game, instead of sequential move (Stackelberg) game (Korzhyk et al., 2011).

The decision of Player 1, representing the group of road users, is considered to be the output from the DUE model. To avoid confusion, it should be noted that DUE by itself can be formulated based on Nash equilibrium. In this work, the DUESCF model is a level above it, wherein DUE outputs describe the decision of one of the two players (the road users). Therefore, individual road users minimize their cost against each other through the DUE model, and the individuals together as a group minimize their costs against the SAV operator through the DUESCF model. Such a consideration of collective decision of the multiple agents is not something new, as it is the basis for non cooperative games like anonymous games (Chien and Sinclair, 2011; Daskalakis and Papadimitriou, 2015; Milchtaich, 1996). An example from the transport modelling literature is (Yang et al., 2007), wherein the decisions of multiple individual road users are considered to be that of a single player. While Yang et al. (2007) differentiate the road users into two groups, we consider a single group, as both the conventional private car users and the SAV fleets aim for user equilibrium.

The final objective of the aforementioned game is to find a traffic assignment and SAV chain formation, such that both the road users and the SAV service operator obtain their solutions by reaching a Nash equilibrium. At Nash equilibrium, no player is better off by unilaterally changing their decisions (Holt and Roth, 2004). If the road users change their path flow pattern while the SAV chains are kept the same, traffic congestion may increase. On the other hand, if the SAV service operator alters the SAV chains while the path flow pattern of road users is unchanged, total system cost of the SAV service will increase or at least be equal to the previous cost. The DUESCF Nash game is summarized in Fig. 1, which will be extended later according to a specific set of DUE and SCF models adapted for the model application. As shown in the figure, besides the player characteristics described above, the exogenous inputs include OD demand, expected arrival times, road network and SAV fleet details (size and depot location).

4.3. Solution approach

In a Nash equilibrium game, no player knows the other player's strategy in advance, and hence, it is not possible to solve the problem in one step. As described in Section 4, the user equilibrium traffic conditions can vary significantly depending upon the SAV chains formed. In the same time, the SAV chains formed cannot be validated until the traffic stabilizes to user equilibrium conditions. Besides, to the best of authors' knowledge, there is no closed form solution for DUE. Therefore, the DUESCF problem is assumed to have no closed form solution, and hence, we propose a heuristic algorithm based on an Iterative Optimization and Assignment (IOA) procedure. The IOA procedure is commonly employed for solving combined traffic assignment and signal control problems (e.g., Meneguzzer, 2000; Smith and van Vuren, 1993), and the same can be adapted for solving the DUESCF problem.

The algorithm includes multiple stages, in which each player's decision is based on the other player's decision, i.e., DUE model with given demand based on SAV chains and SCF model with given departure rates and travel times based on DUE. If the algorithm converges, the obtained solution possesses the properties, such that no player has any incentive to deviate unilaterally from his/her strategy. Thus, the output from this heuristic algorithm represents a Nash equilibrium, since the players do not have advantage over the other and a stable equilibrium state is obtained, without any incentive for the players to change their decisions. However, uniqueness may not exist, as uniqueness for DUE is not proven to exist (Iryo, 2013; Trozzi et al., 2013).

In Algorithm 1, we start with the DUE problem using the initial demand ($Q_{ij} \forall (i,j) \in W$, where Q_{ij} is the demand for OD pair (i,j) and W is the set of all OD pairs), which is an exogenous input. The result (departure rate matrix ($\mathbf{r}$) and travel time matrix ($\mathbf{C}$)) from the DUE model is used to solve the SCF model. Based on the SAV chains formed, a modified demand is calculated and fed into the DUE model. A modified demand has to be calculated, since the demand corresponding to SAV empty trips (Q_{ije}) may change in every iteration until convergence. The algorithm iterates until the convergence criterion is satisfied. The heuristic algorithm can be elaborated as follows, where X represents the current iteration and W is the set of OD pairs in the network:

Algorithm 1: Iterative Optimization and Assignment (IOA) procedure

Set demand = input demand ($Q_{ij} = Q_{ijc} + Q_{ijs}, \forall (i,j) \in W$)
while *convergence criterion is not satisfied* **do**
- **Step 1**: Solve the DUE problem, $Fp(R^X, F^X)$, using a fixed-point algorithm (or other relevant algorithm in case of other type of formulation) to find departure rate matrix ($\mathbf{r}^X$) and travel time matrix ($\mathbf{C}^X$).
- **Step 2**: Given the departure rates and travel times from step 1, solve the SCF problem, $\min G(\boldsymbol{\omega}^X, \mathbf{r}^X)$, using an appropriate solver to obtain the SAV chain matrix ($\boldsymbol{\omega}^X$).
- **Step 3**: Check for convergence. If the convergence criterion is not satisfied, recalculate the demand for DUE based on the formed SAV chains ($Q_{ij} = Q_{ijc} + Q_{ijs} + Q_{ije}, \forall (i,j) \in W$; demand corresponding to SAV empty trips (Q_{ije}) will change every iteration.)

end

Note: Although the solution approach involves sequential estimations for each player (i.e., the DUE and the SCF problems), there is no commitment to a decision by any of the players and there exists an estimation iterative loop. As such, sequential estimation does not imply a sequential decision-making that would justify a Stackelberg game and the solution obtained corresponds to a Nash equilibrium solution. An example of such a solution approach, with sequential moves by the players and the moves resulting in Nash equilibrium, is the best-response dynamics (Ieong et al., 2005).

Since the DUESCF problem is defined as a simultaneous move game and there exists an estimation iterative loop in the solution algorithm, it does not matter if the algorithm is commenced with the SCF problem instead of the DUE problem. However, the initial solution of the algorithm can be different, which in turn can alter the convergence patterns. It is to be noted that Algorithm 1 defines the solution approach in a generic manner and the specific interactions and the data requirements, based on the adapted DUE and SCF models, are shown in Fig. 2. A convergence criterion that can be considered is the mutual consistency between two consecutive solutions from both DUE and SCF module, as shown below (Eq. (7)):

$$PFG < \epsilon \text{ and } CG < \epsilon \tag{7}$$

where,
CG: Change in total system cost (travel time) of the SAV service between two consecutive DUESCF iterations (Eq. (35))
PFG: Change in path flows between two consecutive DUESCF iterations (Eq. (24))

Note: No change in PFG and CG between consecutive iterations means that the players do not deviate. Such a situation wherein both the players stop deviating is a Nash equilibrium (Kleinberg and Tardos, 2006). Thus, the convergence criteria in Eq. (7) establish the convergence to Nash equilibrium.

5. Model development

The solution methods developed and tested in this work are based on a set of appropriate DUE and SCF models, which are identified from the pertinent literature (refer to Section 2.3). The identified models are extended according to the objectives of this research work, as described in below sections. Accordingly, the modelling schema shown in Fig. 1 is extended as presented in Fig. 2.

5.1. DUE model

For the DUE, we base our solution on the fixed point algorithm developed by Han et al. (2019), to solve a Simultaneous Route and Departure Time Choice DUE model (Section 2.2). The DUE model is formulated as a fixed-point problem as shown below:

$$\mathbf{h}^* = P_{\wedge_0}[\mathbf{h}^* - \alpha \boldsymbol{\Psi}(\mathbf{h}^*)] \tag{8}$$

$$\wedge = \left\{ \mathbf{h} \geq 0 : \sum_{p \in P_{ij}} \int_{t_0}^{t_f} h_p(t)dt = Q_{ij} \quad \forall (i,j) \in W \right\} \tag{9}$$

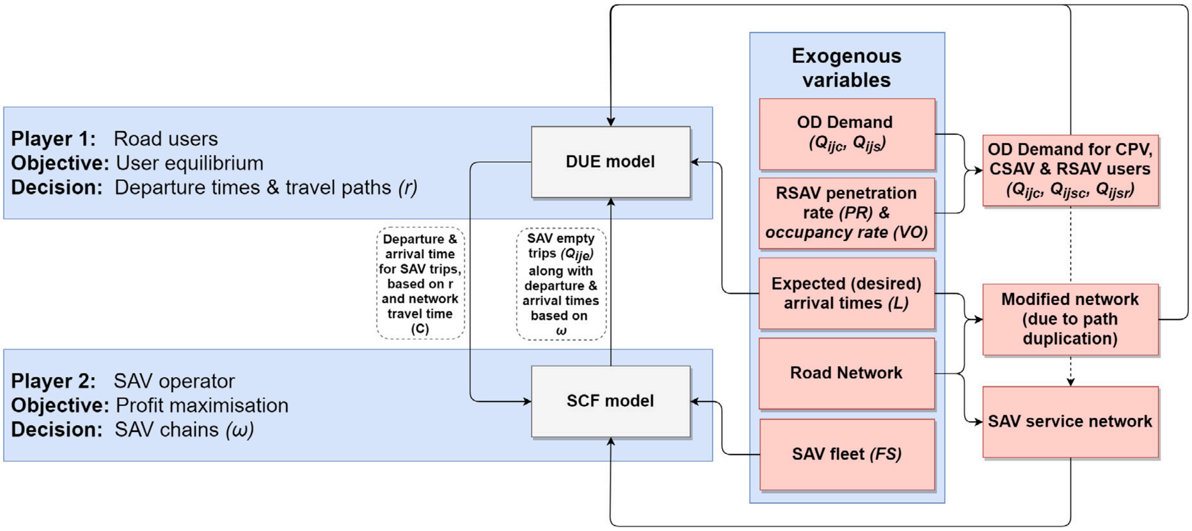

Fig. 2. Extended modelling schema. Note — CPV: Conventional Private Vehicle; CSAV: Car-sharing Shared Autonomous Vehicle; RSAV: Ridesharing Shared Autonomous Vehicle.

where,
h: Matrix of departure rates
$\mathbf{h}^*$: Matrix of departure rates corresponding to DUE
$h_p(t)$: Departure rate along path p at time instant t
$[P_{ij}]$: Set of paths between an OD pair (i, j)
$P_{\wedge_0}[.]$: Minimum-norm projection operator [map that takes a variable closer to the counterpart in a manifold; refer to Abatzoglou (1978), Friesz (2010) for more details]
Q_{ij}: Total demand between an OD pair (i, j)
$[t_0, t_f]$: Planning horizon starting and ending time
W: Set of OD pairs in the network
α: A fixed constant
$\boldsymbol{\Psi}(\mathbf{h}^*)$: Matrix of effective delay under departure profile $\mathbf{h}^*$
$\wedge$: Set of feasible departure rate matrix (R in Eq. (1))

Eq. (8) represents the fixed-point restatement of the optimal control theory based DUE problem (see Friesz, 2010 for more info). The expression for the set of feasible departure rate matrix is shown in Eq. (9). As this model only considers one vehicle class, we extend it to a multi-class model. In order to model conventional private vehicles and SAVs, without loss of generality, we use two identical path sets (P_{ij}; the set of paths that connect each OD pair), one per vehicle class. This approach allows the straightforward extraction of path and departure time choice of the trips belonging to the respective classes. The identical path sets are based on the same underlying links, and hence, both private vehicles and SAVs share the same road space. An index, $k \in K$ (described in Eq. (10)), has been introduced for the path set of each vehicle class. Therefore, the total set of paths for an OD pair (i, j) is as shown in Eq. (11).

$$k = \begin{cases} 1, & \text{for conventional private vehicles} \\ 2, & \text{for SAVs} \end{cases} \tag{10}$$

$$P_{ij} = P_{ij1} \cup P_{ij2} \quad \forall (i,j) \in W \tag{11}$$

In a similar fashion, the DUE model of Han et al. (2019) caters for only one set of demand and expected time of arrival per OD pair. Although the road users travelling between an origin and destination may have a single expected time of arrival, the expected time of arrivals for SAV empty trips (dispatch, relocation and collection trips) will be different. This necessitates the inclusion of multiple sets of demand and expected time of arrival per OD pair. In order to overcome the limitation, again we create identical path sets for vehicle class $k = 2$ (SAVs). The number of additional path sets depend on the number of different expected time of arrival per OD pair. As done for the previous case, a new index, $l \in L$ (described in Eq. (12)), has been introduced to represent each path set corresponding to one set of demand and expected time of arrival. Therefore, the total SAV paths for an OD pair (i, j) is as shown in Eq. (13).

$$l = \begin{cases} 1, & \text{for SAV service trips} \\ 2, \ldots, n, & \text{for SAV empty trips} \end{cases} \tag{12}$$

$$P_{ij2} = P_{ijk1} \cup P_{ijk2} \dots \cup P_{ijkl} \tag{13}$$

Consequently, the total demand for an OD pair (i, j) is

$$Q_{ij} = \sum_{K=1}^{2} Q_{ijk} \tag{14}$$

Furthermore,

$$Q_{ij1} = Q_{ijc} \tag{15}$$

$$Q_{ij2} = \sum_{l=1}^{n} Q_{ij2l} \tag{16}$$

$$Q_{ij21} = Q_{ijs} \tag{17}$$

$$\sum_{l=2}^{n} Q_{ij2l} = Q_{ije} \tag{18}$$

where,
Q_{ijc}: Demand corresponding to conventional private vehicle trips
Q_{ije}: Demand corresponding to SAV empty trips
Q_{ijs}: Demand corresponding to SAV service trips

Based on the variational representation of the Lighthill–Whitham–Richards link model and triangular fundamental diagram, Han et al. (2019) formulated DNL as a Differential Algebraic Equation (DAE) system. A complete presentation of the model is outside the scope of this paper. However, the most relevant equations are included in Appendix. For a full coverage, the reader is referred to Han et al. (2019) and the works cited therein. The following solution approach is utilized for solving the aforementioned DUE model:

Algorithm 2: DUE solution approach

Initialization - Select a suitable value for the constant α such that $\alpha > 0$ and an initial departure rate matrix $\mathbf{h}^0 \in \wedge$
while *convergence criterion is not satisfied* **do**

Step 1: **DNL** - Using the departure rate matrix $\mathbf{h}^Y \in \wedge$, execute the dynamic network loading procedure to compute the effective path delays $\Psi_p(t, \mathbf{h}^Y)\ \forall\ t \in [t_0, t_f]$ and $p \in P$

Step 2: **Fixed-point update** - Solve the following equation for the dual variable v_{ij} for every OD pair $(i, j) \in W$ using a standard root-finding algorithm:

$$\sum_{p \in P_{ij}} \int_{t_0}^{t_f} \left[h_p^Y(t) - \alpha \Psi_p(t, \mathbf{h}^Y) + v_{ij} \right]_+ dt = Q_{ij} \tag{19}$$

where $[v_{ij}]_+ \doteq max\{0, x\}$ assures non-negativity. Compute $h_p^{Y+1}(t)$ using the following equation:

$$h_p^{Y+1}(t) = \left[h_p^Y(t) - \alpha \Psi_p(t, \mathbf{h}^Y) + v_{ij} \right]_+ \quad \forall t \in [t_0, t_f] \text{ and } p \in P_{ij} \tag{20}$$

Step 3: If the convergence criterion in Eq. (21) is not satisfied, adjust the value of $h_p^{Y+1}(t)$ based on Eq. (22)

$$\frac{\| \mathbf{h}^{Y+1} - \mathbf{h}^Y \|^2}{\| \mathbf{h}^Y \|^2} \leq \epsilon \tag{21}$$

$$h_p^{Y+1}(t) = \beta h_p^Y(t) + (1-\beta) h_p^{Y+1}(t) \quad \forall t \in [t_0, t_f] \text{ and } p \in P_{ij} \tag{22}$$

where,

$$\beta = \left(\frac{1}{1+Y} \right)^{0.9} \tag{23}$$

end

---

where,
P: Set of paths in the network
Q_{ij}: Total demand between an OD pair (i, j)
$[t_0, t_f]$: Planning horizon starting and ending time
W: Set of OD pairs in the network

Y: Current DUE iteration
$\wedge$: Set of feasible departure rate matrix (R in the DUESCF problem)

The convergence criterion for the DUESCF model, corresponding to the DUE model, is calculated as follows:

$$PFG = \frac{\| \mathbf{h}^X - \mathbf{h}^{X-1} \|^2}{\| \mathbf{h}^{X-1} \|^2} \tag{24}$$

where h^X is the departure rate matrix obtained from the DUE model in the current iteration and h^{X-1} is the one from previous iteration. PFG stands for Path Flow Gap.

5.2. SCF model

SAVs, unlike the conventional sharing services, can relocate themselves to any customers' location without human drivers, due to their self-driving capabilities. Thus, constraints due to driver involvement could be negated in the formulation of the SCF model and the objective is to form optimal SAV chains for efficiently serving service requests. Ma et al. (2017) have formulated such a model for SAVs, without driver constraints and for a detailed information about the formulation and the SAV network construction, kindly refer their paper. As mentioned in Section 2.1, an SAV chain represents the sequence of requests served by a single SAV. In the formulation of Ma et al. (2017), each trip is represented by a link, i.e., a dispatch trip will be represented by one dispatch link, a service trip by one service link and so on. To make required fleet size an implicit variable of the formulation, a virtual link, which represents flow of SAVs from a depot to the same depot, with null cost is introduced. Vehicles that do not serve any customer request will use the virtual link. The SAV network will consists of a number of dispatch, service, relocation and collection links and one virtual link, and each of these will have an associated decision variable. The travel times in all the links will depend on the network travel times from the DUE model. While the departure time for the service trips will be based on the DUE model, the departure time for empty trips will be based on the schedule of the service trips. As proven by Ma et al. (2017), the SCF model, which has been treated long as an integer programming problem, is equivalent to a linear problem.

The model of Ma et al. (2017) is formulated only for SAV carsharing services, and in order to extend it to include ridesharing, we employ a pre-processing technique. Users with the same OD pair and expected time of arrival are considered to be eligible for ridesharing, which is usually called OD ridesharing (Narayanan et al., 2020b). Thus, the difference between the carsharing service and OD ridesharing service is the following: while only one customer request is served at a time by an SAV in the case of the former, multiple requests are served in the case of latter. The ridesharing penetration rate and the vehicle occupancy rate (represents the average number of service requests served per ridesharing SAV trip) are taken as inputs. The ridesharing demand is calculated by multiplying the SAV demand with the penetration rate of the ridesharing SAV service, as shown in Eq. (25). Furthermore, the number of ridesharing trips to be carried out is calculated by dividing the ridesharing demand with the vehicle occupancy rate.

$$\begin{aligned} Q_{ijsr} &= [PR(Q_{ijs})]/VO_{ij} \\ Q_{ijsc} &= Q_{ijs} - Q_{ijsr} \end{aligned} \tag{25}$$

This leads to the further modification of Eq. (17) as follows:

$$Q_{ij21} = Q_{ijsr} + Q_{ijsc} \tag{26}$$

where,
PR: Penetration rate for SAV ridesharing service
Q_{ijs}: SAV service demand between an OD pair ij
Q_{ijsc}: Carsharing SAV service demand between an OD pair ij
Q_{ijsr}: Ridesharing SAV service demand between an OD pair ij
VO_{ij}: Vehicle occupancy rate for ridesharing trips between an OD pair ij

Note: The ij subscript in Eqs. (25) and (26) and other DUE related equations corresponds to an OD pair, while in SCF related equations (Eqs. (27)–(34)), it corresponds to a node pair of any service (A_1), relocation (A_2), dispatch (A_3), collection (A_4) or virtual (A_5) link.

As an example, consider a total demand of 50 trips for SAV services (Q_{ijs}). Ridesharing service penetration rate is 10% with vehicle occupancy of 5. Then, the number of ridesharing trips (Q_{ijsr}) will be $0.1 * 50/5 = 1$. The resulting SAV service demand (Q_{ij21}) will be $(0.9 * 50) + 1 = 46$. If the given vehicle occupancy is 3, then the number of ridesharing trips will be 2 trips, with one trip consisting of 3 customers and the other with 2 customers. The resulting SAV service demand in this case (given vehicle occupancy of 3) will be 45+2=47 trips. When modelling SAV trips in the DUE algorithm, the trips are not differentiated as ridesharing and car-sharing trips. When creating the request matrix for the SCF model, the SAV trips are randomly assigned as ridesharing and carsharing trips.

To ensure priority for servicing the high occupancy trips, when the available SAVs are inadequate, the penalty for losing a service trip ($-LP$ in Eq. (33)) is multiplied by the occupancy rate. With the above implemented changes, the formulation of the adapted linear problem becomes:

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{27}$$

Subject to

$$x_{ij} \leq m_{ij} \quad \forall (i,j) \in A \tag{28}$$

$$\sum_{j \in N \setminus \{D\}} x_{ji} = \sum_{j \in N \setminus \{O\}} x_{ij} \quad \forall i \in N \setminus \{O, D\} \tag{29}$$

$$\sum_{j \in N \setminus \{O\}} x_{ij} = FS, i = O \tag{30}$$

$$\sum_{j \in N \setminus \{D\}} x_{ji} = FS, i = D \tag{31}$$

$$x_{ij} \geq 0 \quad \forall (i,j) \in A \tag{32}$$

and

$$c_{ij} = \begin{cases} -VO_{ij} * LP, & \text{if (i,j) } \in A_1 \ (service\ links) \\ d_{ij} + p_{ij}, & \text{if (i,j) } \in A_2 \ (relocation\ links) \\ d_r + d_{Oj}, & \text{if (i,j) } \in A_3 \ (dispatch\ links) \\ d_{iD}, & \text{if (i,j) } \in A_4 \ (collection\ links) \\ 0, & \text{if (i,j) } \in A_5 \ (virtual\ link) \end{cases} \tag{33}$$

$$m_{ij} = \begin{cases} 1, & \text{if (i,j) } \in A \setminus A_5 \\ FS, & \text{if (i,j) } \in A_5 \end{cases} \tag{34}$$

where,
A: Set of all links in the SCF network ($A1 \cup A2 \cup A3 \cup A4 \cup A5$)
c_{ij}: Cost of using a link $(i,j) \in A$
d_{iD}: Cost for collecting an SAV from the last service trip (for $A4$ links)
d_{ij}: Driving cost in a relocation link (for $A2$ links)
d_{Oj}: Cost for dispatching an SAV to the first service trip (for $A3$ links)
d_r: Fleet cost per vehicle to account SAV maintenance and refuelling at the depot
FS: Available SAV fleet size
LP: Penalty for losing a service trip
m_{ij}: Capacity of a link $(i,j) \in A$
N: Set of all nodes in the network (service trip and depot nodes)
O, D: Origin and terminal depot node
p_{ij}: Parking cost when using a relocation link (for $A2$ links)
VO_{ij}: Vehicle occupancy in a service link (for A1 links)
x_{ij}: Number of vehicles using a link $(i,j) \in A$; (decision variable)

Note: The SAV empty trip demand between an OD pair (i,j), i.e., Q_{ije}, is based on $x_{ij} \in \{A2, A3, A4\}$, i.e., decision variables corresponding to relocation, dispatch and collection trips. The SAV service network is constructed based on the original road network and the SAV service demand. As such, there exists a mapping between the two different networks. To convert nodes from the SAV service network to the original road network, this mapping is used.

The objective function of this problem is the total cost of the SAV service (Eq. (27)). Eq. (28) is the standard link capacity constraint. Flow conservation at the intermediate nodes (nodes other than depot nodes) is satisfied by Eq. (29). While the SAVs serving customer requests depart from the depot through dispatch links and return to depot through collection links, idle SAVs depart from and return to the depot through the virtual link, i.e., all the SAVs should depart from the depot and return to the depot. This condition is ensured by Eqs. (30) and (31). Eq. (32) states that the value for the decision variable has to be positive. Ma et al. (2017) originally formulated the problem with the decision variable as an integer variable. Then they reformulated the problem (which is presented in Eqs. (27) - (34)) with the decision variable being a continuous positive variable, since the constraint matrix is totally unimodular and the value obtained will, in any case, be an integer. For more details on the formulation, the reader is referred to Ma et al. (2017).

As shown in Eq. (33), relocation links have a sum of driving cost (for relocation) and parking cost (SAVs have to be parked either at the destination of one customer or the origin of the subsequent customer, when the travel time between those two nodes is less than the time available between the drop-off and the pick-up). It is to be noted that any relocation link should follow the principle of reachability, which can be defined as follows: the required travel time on the relocation link should be less than the difference between the end time of a service trip and the start time of another service trip, between which the relocation occurs. The cost associated with a dispatch link is d_{Oj}, along with a fleet cost d_r, and the cost associated with a collection link is d_{iD}. A penalty cost, LP, is attached to the service links. Usage of the virtual link does not incur any cost. Finally, the capacity of the links is defined in Eq. (34).

Each service trip will have an associated vehicle occupancy value (VO_{ij}). Since vehicle occupancy is an input variable (which means that the variable can be treated as a constant in the formulation), inclusion of it in the SCF model does not make the formulation non-linear. On the other hand, inclusion of it enforces the model to assign vehicles for the high occupancy trips first (because of the high penalty for losing such trips), and then for the single occupancy carsharing trips. With regards to solution for this formulation, any linear program can be used. Finally, the convergence criterion for the DUESCF model, corresponding to the SCF model, is calculated as follows:

$$CG = \left(\frac{\mid C_o^X - C_o^{X-1} \mid}{C_o^{X-1}} \right) 100 \tag{35}$$

where, C_o is the total system cost (travel time) of SAV service [sum of cost of service trips (C_s) and cost of empty trips (C_e)] and X represents the current DUESCF iteration. CG stands for Cost Gap. Note: Since a penalty value is used for the service trips in the formulation, the objective function of the above model does not represent the true total system cost of SAV service. C_o is calculated based on the travel time matrix from the DUE model ($\mathbf{C}$) and the SAV chains from the SCF model ($\boldsymbol{\omega}$).

5.3. Solution existence, uniqueness and global optimality

The DUE and the SCF models used in this paper are the adapted versions of Han et al. (2019) and Ma et al. (2017), respectively. The adaptations performed in this study do not affect the general structure of the original models and therefore, the existence and uniqueness properties of the original models are also applicable to the adapted versions. With regards to the DUE model, the existence of the simultaneous route and departure choice dynamic user equilibrium and the possibility to obtain an unique flow profile is shown in Han et al. (2013, 2019). Concerning the SCF model, it has a unimodular constraint matrix. Integer linear programs with (totally) unimodular constraint matrices have an integral polyhedron as the convex hull of its feasible solutions (Wagler, 2014). Therefore, the existence of a solution is also guaranteed for the SCF model.

With respect to global optimality, the solution algorithm implemented in this study belongs to the category of heuristics. Therefore, convergence of the algorithm to a global solution cannot be guaranteed. The convergence criteria only guarantee that both players, i.e., the group of road users and the SAV operator, do not have any incentive to unilaterally change their decision within a small perturbation value ϵ. Hence, a local optimal point is obtained. In the next section, numerical experiments are conducted to evaluate the empirical convergence and ascertain the benefit of using the proposed formulation and algorithm.

6. Model experiments and applications

In this section, we run experiments that are constructed to demonstrate the properties of the proposed solution algorithm. The focus is on the verification of the optimality of the solution, and the examination of the model convergence and running time. The algorithm is coded in MATLAB. The tests on toy, Braess bidirectional and Sioux Falls networks are carried out using a personal computer running on an Intel core i7-8700 processor with 6 cores and 16 GB RAM, with partial parallelization of some of the functions. For Anaheim, a HPC server with 40 cores and 170 GB RAM is used, with the same (semi-parallelized) code. The following basic assumptions are introduced for all the experiments:

1. The effective delay in the DUE model, Ψ, is a linear combination of travel time and arrival time penalties, as shown in Eq. (36). The term $T_A - T_E$ represents the difference between actual and expected arrival time

$$\Psi_p(t, \mathbf{h}) = D_p(t, \mathbf{h}) + (T_E - T_A)^{2\gamma} \tag{36}$$

where, γ is the effective delay parameter, and is set as 0.8 for early arrival trips and 1.2 for late arrival trips. $\Psi_p(t, \mathbf{h})$ is the effective delay along path p at time instant t, under departure profile $\mathbf{h}$, and $D_p(t, \mathbf{h})$ is the travel time in path p at time instant t, under departure profile $\mathbf{h}$
2. A single planning horizon is considered $[t_0, t_f]$, with one expected arrival time considered for all the travellers travelling between an OD pair.
3. Vehicle characteristics do not differ. Although the dynamics of SAVs are not considered as being different from the conventional vehicles, as mentioned in Section 5.2, the constraints due to the driver involvement are negated in the formulation of the SCF model, and thus, the operational side of the SAV service is different from the conventional shared services.
4. SAV service is assumed to be operating with a single depot.
5. A constant vehicle occupancy is defined for all vehicle trips, i.e., VO_{ij} = a constant (e.g., 3).
6. Zero fleet cost is assumed for SAV dispatch, i.e., $d_r = 0$, and the cost associated with the dispatch and collection link will be the travel time in the respective links.
7. A value larger than the maximum path travel time in a network is applied as penalty for losing a customer request (LP).
8. The cost associated with the relocation links will be the travel time in the corresponding links. Parking is not explicitly modelled in this study, essentially assuming that there will always be a vacant cost-free parking spot, i.e., $p_{ij} = 0$.

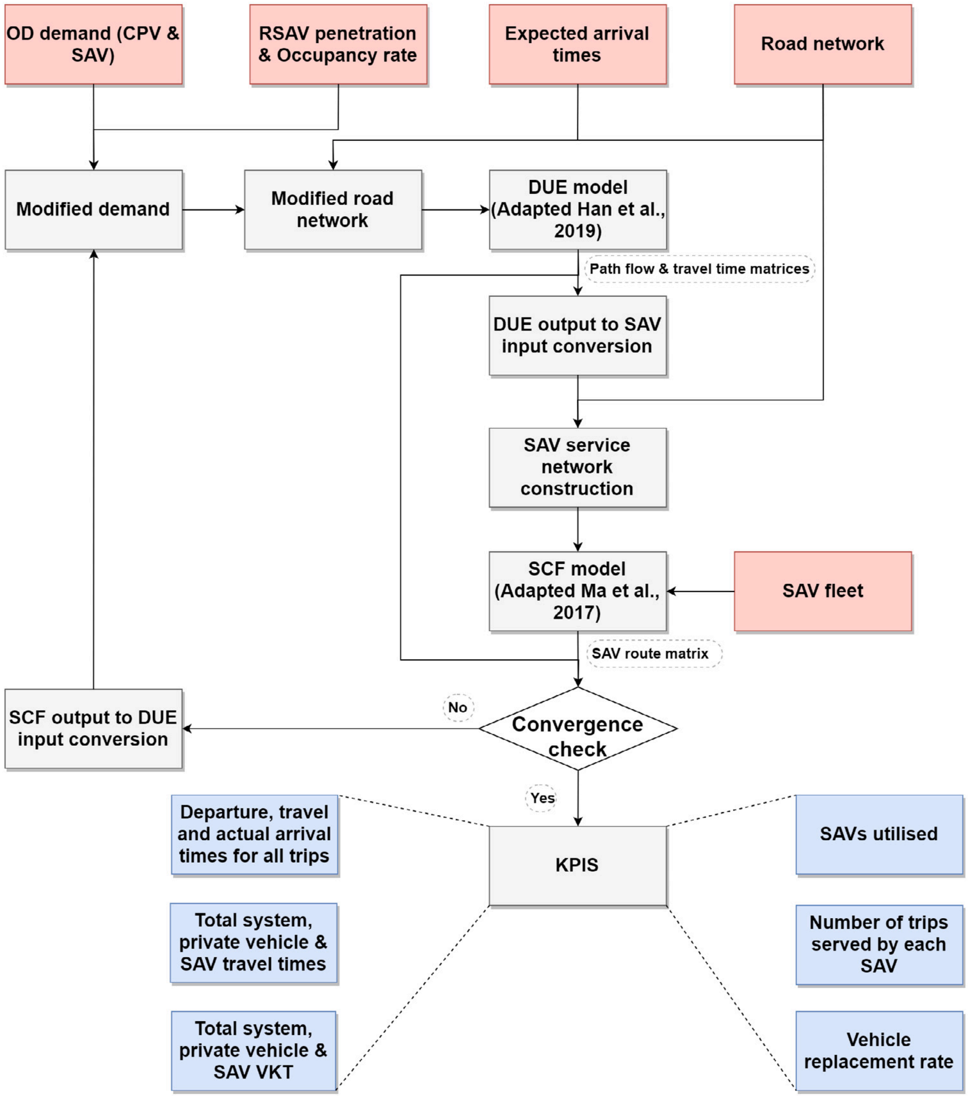

Fig. 3. Modelling flowchart for the experiments. Note — CPV: Conventional Private Vehicle; RSAV: Ridesharing SAV; KPI — Key Performance Indicator; VKT — vehicle Kilometres Travelled.

The extended modelling schema, presented in Fig. 2, is further developed as a detailed flowchart (Fig. 3), which forms the basis for the experiments. As shown in the figure, for each experiment, the following items are taken as input: (i) OD demand for conventional private vehicles and SAVs, (ii) Penetration and occupancy rate (the average number of service requests served per trip) for ridesharing SAVs, (iii) Expected arrival times associated with the OD demand (desired arrival times is needed because of the departure time choice component), (iv) Road network and (v) SAV fleet size and the depot location. In the initial iteration, the actual OD demand will be modified according to the penetration and the occupancy rate for ridesharing SAV service, as described in Section 5.2. In subsequent iterations, modified demand will also include the SAV empty trips. Furthermore, due to path duplication (refer to Section 5.1, a modified road network is generated. Afterwards, the DUE model is run to obtain path flow and travel time matrices, followed by creation of SAV service requests, which consists of details, such as request number, origin and destination nodes, and departure and arrival times. Then, the abstract SAV service network is constructed and the SCF model is run. Later, convergence check is performed. The entire procedure is iterated till convergence is achieved. Thereafter, Key Performance Indicators (KPIs) are calculated. Some of the possible KPIs include (i) Departure, travel and arrivals times for all the individual trips, (ii) Total travel time corresponding to the entire system, private vehicles and SAVs, (iii) Total Vehicle Kilometres Travelled (VKT) corresponding to the entire system, private vehicles and SAVs, (iv) Number of SAVs utilized, (v) Number of trips served by each SAV and (vi) Vehicle replacement rate (the ratio between trip demand served by the SAVs and the number of SAVs utilized).

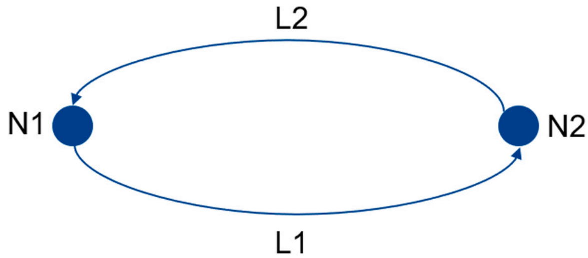

Fig. 4. 2-node toy network.

6.1. 2-node toy network

A simple network, shown in Fig. 4, is used to verify the optimality of the solution, i.e., chain forming capability of the proposed algorithm, along with the consideration for the required number of vehicles, trip travel times and arrival times. The simple network consists of two nodes (N1 and N2) and two links (L1 and L2). Both links have a capacity of 1800 veh/h and are of 8 km length. The free flow travel time in the links is 12 min. Demand of 100 trips is assumed between N1 and N2, and another 100 trips between N2 and N1. All 200 trips have to be served by SAVs. The expected time of arrival for the former trips is the first hour from the beginning of the planning horizon (t_0) and the expected time of arrival for the latter trips is the fourth hour. A fleet of 200 homogeneous SAVs are available for service during the planning horizon of 300 min (5 h). Depot is assumed to have the capacity to serve the available vehicles and be located at node N1. DUE is modelled with a time step size of 2 min.

The test results show that the convergence value corresponding to both the DUE (24) and the SCF (35) models converge to 0, thus indicating that both the road users and the SAV operator have obtained their optimal solutions and do not have an incentive to change their decisions. This verifies the ability of the model to reach a Nash equilibrium. The output from the model shows that the total number of vehicles utilized is 100, which is the optimal number of vehicles required. The travel time experienced in all the trips is 12 min. The actual arrival time for the N1-N2 trips ranges from hour 0.8 (46th minute from t_0) to 1.16 (70th minute), and for N2-N1 trips, the value ranges between hour 3.8 (228th minute) to 4.16 (250th minute). Distribution of departures does not create a large gap between the actual and expected time of arrivals, and as expected, the actual arrival times are close to the expected time of arrival.

The route matrix shows that each vehicle serves two trips, one N1-N2 trip and one N2-N1 trip. Thus, each vehicle travels from N1 to N2 to serve one N1-N2 trip, parks in N2 and then travels from N2 to N1 to serve one N2-N1 trip. There is no extra dispatch or collection trip, since the vehicles start from and end at N1. Also, no relocation trip is found. The results show that an optimal solution for the DUESCF problem can be obtained using the proposed solution algorithm. According to the formulation (Eq. (1)), $\mathbf{r}$ is the departure rate matrix. In the current case, $\mathbf{r}$ will have positive values for row 1 (corresponds to Path 1, which contains link $L1$) and columns 17 to 29, and for row 2 (corresponds to Path 2, which contains link $L2$) and columns 108 to 119. Rest of the columns will have a value of zero. The columns with positive values correspond to the departure times of the road users. Each cell of $\mathbf{C}$, which is the travel time matrix obtained from the DNL sub-model, contains the free flow travel time. $\boldsymbol{\omega}$, which is the SAV chain matrix, will have 100 rows, representing the 100 SAVs utilized, and 2 columns, representing the N1-N2 trips and N2-N1 trips.

6.2. Braess bidirectional network

The Braess network (Braess, 1968; Lin and Lo, 2009; Transportation networks for research, 2020) has been modified to include bidirectional links as shown in Fig. 5. The modified Braess bidirectional network consists of four nodes (N1 to N4) and ten links (L1 to L10). All the links have a capacity of 1800veh/h, and are of 7.2 km length. The free flow travel time in the links is 6 min. The depot is assumed to have the capacity to serve the available vehicles and be located at node N1. DUE is modelled with a time step size of 2 min. Two different cases of demand levels are tested: (1) OD pairs 1–2 and 2–1 having a demand of 100 trips each and the rest having zero demand; (2) All the OD pairs having a demand of 100 each.

6.2.1. Case 1: OD pairs 1–2 and 2–1 having a demand of 100 trips each and the rest having zero demand

Similar to the 2-node toy network, this case is evaluated to establish the optimality of the solution. The expected time of arrival is hour 2.5 (from t_0) for OD pair 1–2 and hour 3.5 for OD pair 2–1. A fleet of 200 homogeneous SAVs are assumed to be available for service during the planning horizon of 300 min (5 h).

Similar to the 2-N toy network, the convergence values corresponding to both DUE (24) and SCF (35) models converge to 0, showing the establishment of a Nash equilibrium. The output from the model shows that the total number of vehicles required is 100, which is indeed the optimal number of vehicles required. The actual arrival time for the OD $1-2$ trips ranges from hour 2.22 (132^{nd} minute from t_0) to 2.8 (168th minute), and for the OD $2-1$ trips, the value ranges between hour 3.2 (192^{nd} minute) to 3.8 (228th minute). The travel time value ranges between 6 min and 12 min, with majority having 6 min (85%). The trips with travel time values greater than 6 min are usually those trips that have arrival time close to 2.5 (OD $1-2$ trips) and 3.5 (OD $2-1$ trips), which shows the trade-off between travel time and arrival time.

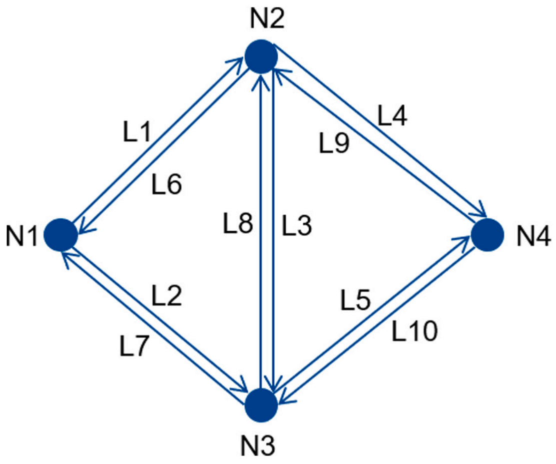

Fig. 5. Braess bidirectional network.

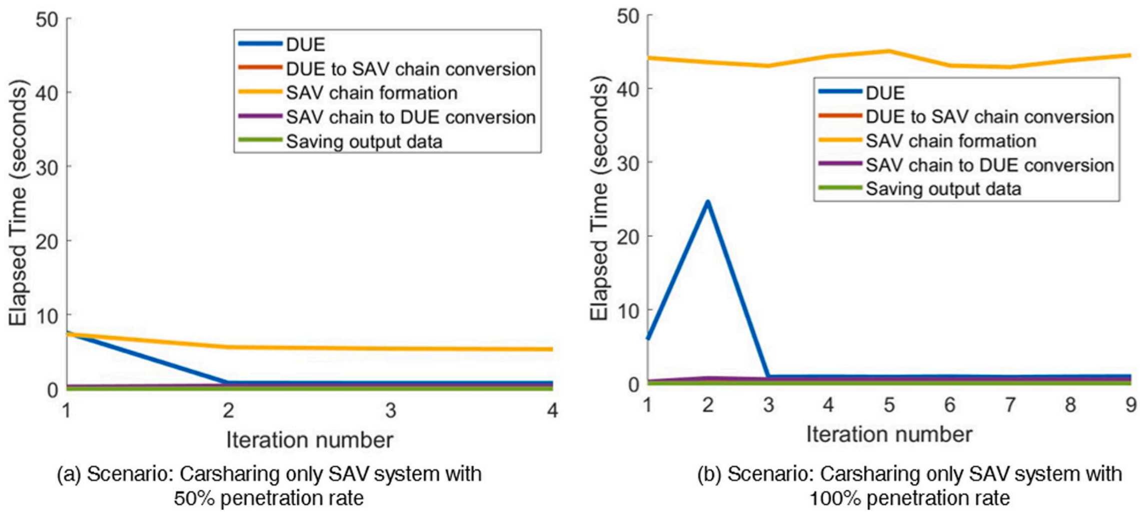

Fig. 6. Elapsed time for two sample scenarios.

Analysing the route matrix, it is found that each vehicle serves two trips, one OD 1 – 2 trip and one OD 2 – 1 trip. Thus, each vehicle travels from N1 to N2 to serve one OD 1 – 2 trip, parks in N2 and then travels from N2 to N1 to serve one OD 2 – 1 trip. There is no extra dispatch or collection trip, since the vehicles start from and end at N1. Also, no relocation trip is found. The results again show that an optimal solution can be obtained using the proposed solution approach.

6.2.2. Case 2: All the OD pairs having a demand of 100 each

The objective behind this case is to evaluate the robustness and ensure the stability of the proposed algorithm. The evaluation is based on the ability of the algorithm to model different scenarios, and convergence is used as the evaluation criteria. Computation times will also be examined in this case.

Expected time of arrival for the trips ranges from hour 2.5 (from t_0) to hour 4.0. A fleet of 1200 homogeneous SAVs are available for service during the planning horizon of 300 min (5 h). By varying the SAV service penetration rate (0% to 100% in steps of 10%), ridesharing penetration rate (0% to 100% in steps of 10%) and vehicle occupancy (2 to 5 in steps of 1), 411 scenarios are tested and the model is able to run all the scenarios without any issue. This proves the robustness of the proposed solution approach to run different scenarios. In most of the cases, the algorithm converged to a value of 0 within 10 iterations.

The computation time for modelling each scenario ranges from few seconds to around 5 min. As observed in Fig. 6a, the elapsed time for DUE computation reduces steadily along the iteration of the bilevel model, and this is due to the decrease in the number of required DUE iterations. However, in Fig. 6b, the elapsed time for DUE computation increases initially, and then decreases. SAV empty trips are added only in the second iteration, and hence, the required DUE iterations increase. Therefore, an increase in the elapsed time for DUE computation. This increase and then decrease trend is not observed in Fig. 6a, since the added empty trips

Table 1
Summary of the results from the experiments.

Network	Size	Objective	Result
2-node toy	2 nodes & 2 links	(A) Convergence (B) Solution Optimality	Convergence to 0 within 5 iterations Number of SAVs: 100; Travel time: 12 min
Braess bidirectional (C1)	4 nodes & 10 links	(A) Convergence (B) Solution optimality	Convergence to 0 within 5 iterations Number of SAVs: 100; Travel time: 6 – 12 min
Braess bidirectional (C2)	4 nodes & 10 links	(A) Convergence (B) Running time	Convergence to 0 within 10 iterations for most of the scenarios, while for the rest, convergence to a value of less than 0.0001 Few seconds to 5 min
Sioux Falls	24 nodes & 76 links	(A) Convergence B) Running time	Convergence within 24 iterations 30 min for 24 iterations
Anaheim	416 nodes & 914 links	(A) Convergence (B) Running time	Convergence within 24 iterations 1 day for 24 iterations

Note: The tests on toy, Braess bidirectional and Sioux Falls networks are carried out in a PC consisting of Intel core i7-8700 processor with 6 cores and 16GB RAM, with partial parallelization of some of the functions. For Anaheim, a HPC server with 40 cores and 170GB RAM is used.

do not cause substantial increase in the required DUE iterations. With regards to the elapsed time for the SAV chain formation, it stays almost constant throughout the iteration process, and the same can be observed in Figs. 6a and 6b. The SAV chain formation computation time depends majorly on the number of SAV service requests, which is a constant throughout the bilevel iteration process. Hence, the elapsed time is almost constant. As the SAV service penetration increases, the elapsed time for SAV chain formation increases, and a comparison of Fig. 6a and Fig. 6b clearly shows the increase in elapsed time for SAV chain formation. This increase in elapsed time is due to the time required for forming the constraints. As the number of service requests increases due to the increase in the penetration rate, the number of possible relocations gets almost squared (n*n-1 to be exact), and this in turn increases the number of constraints. The increase in the number of constraints, naturally, increases the time required for forming the constraints, and hence, the elapsed time for the SAV chain formation increases.

6.3. Empirical investigation of convergence

Besides the tests in Sections 6.1 and 6.2, an empirical investigation of the convergence for three different networks of varying complexity is carried out in this section. The three different networks are Braess bidirectional (Section 6.2), Sioux Falls and Anaheim.

The Sioux Falls network (LeBlanc et al., 1975; Transportation networks for research, 2020) consists of 24 nodes and 76 links. Paths between all nodes are generated using a K-shortest paths algorithm, and the total number of paths generated is 716. Demand data is based on the data from the study of Han et al. (2019). The total demand in the whole network is 16,896 trips, with the majority of the OD pairs (528 out of 552 OD pairs) having a demand of 32 trips (it is to be note that when running the DUE model, because of the departure time choice, this demand will get temporally distributed in a non-uniform way, thereby resulting in a dynamic demand pattern). The rest of the OD pairs have a zero demand. A 10% penetration rate is assumed for the SAV service, and the system type is assumed to be carsharing. The expected time of arrival ranges from hour 3.0 to hour 4.0. A fleet of 1690 homogeneous SAVs (corresponds to the SAV service demand) is assumed to be available for the service.

The Anaheim sketch network is composed of 416 nodes, 914 links and 30,719 paths. The demand is based on the data from the study of Han et al. (2019), wherein the original data from Transportation Networks for Research Core Team (2020) is adapted. The total demand in the whole network is 28,569 trips, with the minimum demand between an OD pair being 1 trip and the maximum being 172 trips. A 10% penetration rate is assumed for the SAV service, and the system type is assumed to be carsharing. The expected time of arrival ranges from hour 2.0 to hour 4.0. A fleet of 2857 homogeneous SAVs (corresponds to the SAV service demand) is assumed to be available for the service.

The Planning horizon is considered to be 360 min (6 h), for both Sioux Falls and Anaheim. The depot is assumed to have the capacity to serve the available vehicles and be located at node 1. DUE is modelled with a time step size of 2 min. A comparison of the convergence plots of the three networks (Fig. 7) shows that the algorithm exhibit satisfactory empirical convergence within limited number of iterations. There could be local fluctuations for larger networks. The algorithm may not converge to zero for larger networks and therefore, different convergence thresholds may be required for different networks, with higher values for larger networks.

The results from the experiments is summarized in Table 1. As presented in the table, the proposed IOA algorithm can converge to a value of 0 within a short time for small networks. However, the convergence and the computation time may increase with an increase in network size, along with a necessity of a large convergence threshold.

7. Scenario analysis

Scenario analysis is carried out for the Sioux Falls network, since the network is well-known and also has lower computation time and resource consumption. The scenario analysis is based on the demand data utilized in Section 6.3. DUE is modelled with a convergence threshold of 0.001 and a time step size of 2 min. The following six scenarios, apart from the base scenario, are analysed:

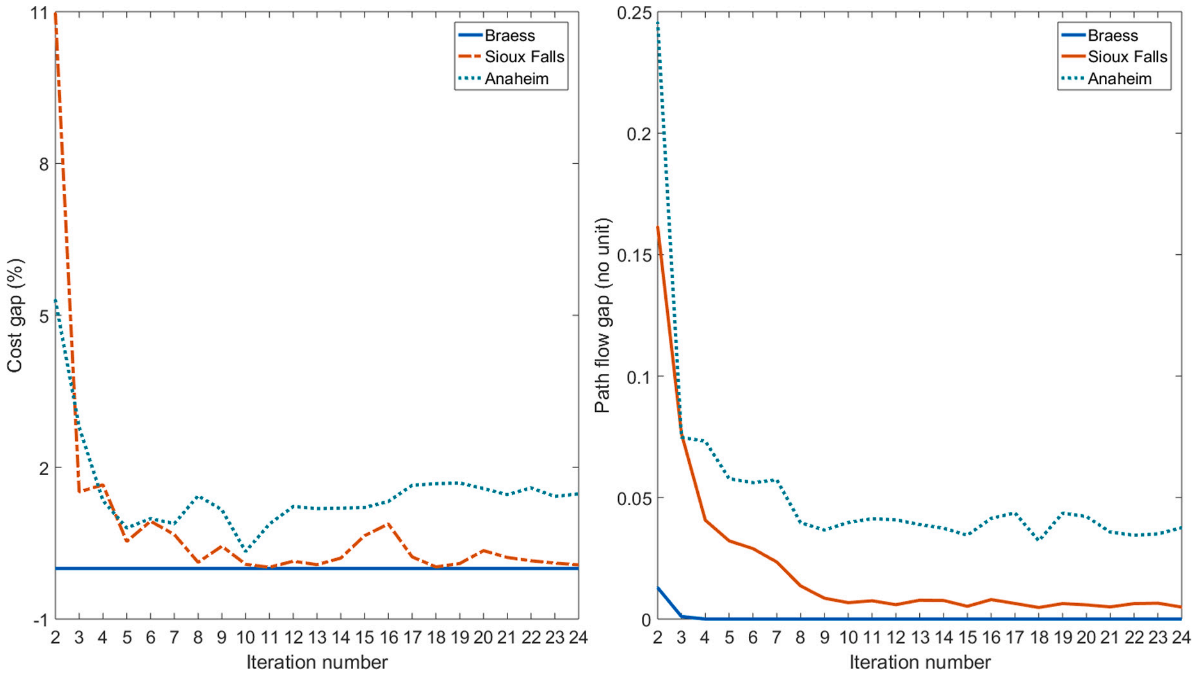

Fig. 7. Convergence for carsharing only SAV system with 10% penetration rate. Note: Calculation of convergence parameters involves difference between two consecutive iterations and hence, no value exists for first iteration.

- **Base scenario**: 0% SAV penetration rate (only conventional private cars are used)
- **Scenario 1 (S1)**: Carsharing only SAV system with 10% penetration rate
- **Scenario 2 (S2)**: Ridesharing only SAV system with 10% penetration rate & vehicle occupancy of 3
- **Scenario 3 (S3)**: Carsharing only SAV system with 20% penetration rate
- **Scenario 4 (S4)**: Ridesharing only SAV system with 20% penetration rate & vehicle occupancy of 2
- **Scenario 5 (S5)**: Ridesharing only SAV system with 20% penetration rate & vehicle occupancy of 3
- **Scenario 6 (S6)**: SAV system with 20% penetration rate - 50% carsharing % 50% ridesharing (vehicle occupancy of 3)

The scenarios have been chosen in such a way to include carsharing only SAV systems, ridesharing only SAV systems with two different vehicle occupancy rates and a mixed SAV system consisting of both carsharing and ridesharing. This selection enables studying the effect of carsharing, ridesharing and vehicle occupancy on the total system travel time, traffic congestion level and vehicle requirements. Furthermore, based on Bansal and Kockelman (2018), Cambridge Systematics (2016), Moreno et al. (2018), Trommer et al. (2016), Walker and Johnson (2016), the penetration rate for SAV services in the next decade will be less than or around 20%. Hence, only 10% and 20% penetration rates for SAV system are considered for the scenarios. The scenarios are compared in terms of actual trip arrival times, total system travel time, mean of individual trip times and vehicle requirements. Values of different variables from each scenario are presented in Table 2.

7.1. Total system travel time

As shown in Table 2, the total system travel time is highest in the case of Scenario 3, larger than the base scenario, which can be attributed to SAV empty trips. The total system travel time value from Scenarios 1 and 6 is also higher than the base scenario. Scenario 5 has the lowest total system travel time, and this shows that a ridesharing system with higher vehicle occupancy rate is better compared to a carsharing or a mixed SAV system. In terms of percentages for change in total system travel time, as can be seen in Fig. 8, Scenarios 1, 3 and 6 have an increase of 17.8%, 22.5% and 12.5% compared to base scenario, while Scenarios 2, 4 and 5 have a decrease of 11.6%, 2.8% and 15.2% respectively. SAV empty rides account for 2% to 16% of the total system travel time.

7.2. Individual trip time

With regards to the mean of individual trip times, again Scenarios 1, 3 and 6 have poor performance, with a value of 36, 34 and 35 min respectively, around 7.7%, 1.5% and 5.1% higher than the base scenario. Scenarios 2 and 5 have lower values when compared to other scenarios, and this again proves that a ridesharing system with higher vehicle occupancy rate is better than both

Table 2
Scenario analysis results.

Scenarios	Base	1	2	3	4	5	6
Total demand	16896	16896	16896	16896	16896	16896	16896
Demand for SAV services	–	1584	1584	3168	3168	3168	3168
No. of private vehicle trips	16896	15312	15312	13728	13728	13728	13728
No. of SAV service trips	–	1584	528	3168	1584	1056	2112
Actual arrival time (planning horizon hour)	1.1 to 5.6	1.1 to 5.1	1.3 to 5.6	1.0 to 6.8	1.4 to 5.63	1.5 to 5.4	1.4 to 5.1
Mean of actual arrival time (planning horizon hour)	3.4	3.4	3.4	3.3	3.4	3.4	3.4
Travel time related							
Total system travel time (hr)	9434	11110	8343	11562	9170	8001	10610
Total private vehicle travel time (hr)	9434	9210	7868	7929	7373	7015	8037
Total SAV travel time (hr)	–	1900	475	3633	1797	986	2573
Total SAV empty trip travel time (hr)	–	940	205	1868	880	461	1289
Mean of individual trip time (min)	33.5	36.1	30.8	34.0	32.5	30.6	35.2
Distance related							
Total VKT (km)	191662	222634	187954	244622	203916	186091	220199
Total private vehicle VKT (km)	191662	173782	173542	155813	155653	155463	155768
Total SAV VKT (km)	–	48852	14412	88809	48263	30628	64431
Total SAV empty VKT (km)	–	30892	8441	52900	30327	18666	40489
Vehicle related							
Total No. of vehicles used	16896	16040	15509	14954	14475	13925	14704
No. of SAVs used	–	728	197	1226	747	433	976
No. of trips per SAV vehicle	–	1 to 5	1 to 6	1 to 6	1 to 5	1 to 6	1 to 6
Vehicle replacement	–	2.2	8.0	2.6	4.2	7.3	3.2

Note:
• There is a trade-off between arrival and travel time in the DUE model, due to the departure time choice. Hence, in scenarios, wherein there is a higher possibility of increased congestion, there can be a greater dispersion of the actual arrival time.
• The change in total system travel time is majorly impacted by the number of SAV service trips and empty trips, besides the congestion levels in the network.

a carsharing and a mixed SAV system. The reduction in the number of service trips as well as the decrease in the number of empty trips could be attributed for this. The mean of individual trip times can be considered as a proxy for traffic congestion, and the model results are in line with the results related to traffic congestion from the review of Narayanan et al. (2020b), i.e., carsharing system is found to increase congestion, while on the other hand, ridesharing system reduces congestion.

7.3. Vehicle kilometres travelled

Scenarios 2 and 5 outperforms rest of the scenarios in terms of Vehicle Kilometres Travelled (VKT), showing a reduction in total VKT compared to the base scenario. Total VKT increases in the rest of the scenarios, as can be seen in Fig. 8. Scenarios 1, 3, 4 and 6 result in an increase of 16%, 28%, 6% and 15% respectively. Thus, it can be ascertained that a ridesharing system with higher vehicle occupancy (Scenarios 2 and 5 consist of trips with vehicle occupancy of 3) is required to reduce total VKT. SAV empty rides accounts for 4% to 22% of the total VKT.

7.4. Vehicle requirements

In the case of the base scenario, it is assumed that the number of vehicles used is equal to the number of trips. In the case of the SAV scenarios, the number of trips served by a single vehicle range from 1 to 6 trips in Scenarios 2, 3, 5 and 6, and in the case of Scenarios 1 and 4, the range is 1 to 5 trips. Looking into the vehicle replacement values, which refer to the ratio between trip demand served by the SAVs and the number of SAVs utilized in a scenario, it is obvious that the SAV services have the potential to reduce vehicle requirements, as can be seen in Fig. 8. Further, a ridesharing system has a high vehicle replacement rate compared to a carsharing system (based on a comparison of Scenarios 1–2 and Scenarios 3-4-5). Based on a comparison of Scenarios 3 and 4, it is certain that the vehicle replacement increases as vehicle occupancy increases. By comparing Scenarios 3, 4 and 6, one can confirm that a mixed system requires more vehicles than a ridesharing SAV system.

Finally, based on the above results, a ridesharing SAV system has lower total system travel time, traffic congestion levels, total VKT and vehicle requirements, compared to the base scenario, as well as other scenarios with carsharing and mixed SAV system. Further, the benefits obtained increase as vehicle occupancy increases in the case of ridesharing SAV system. Scenarios with

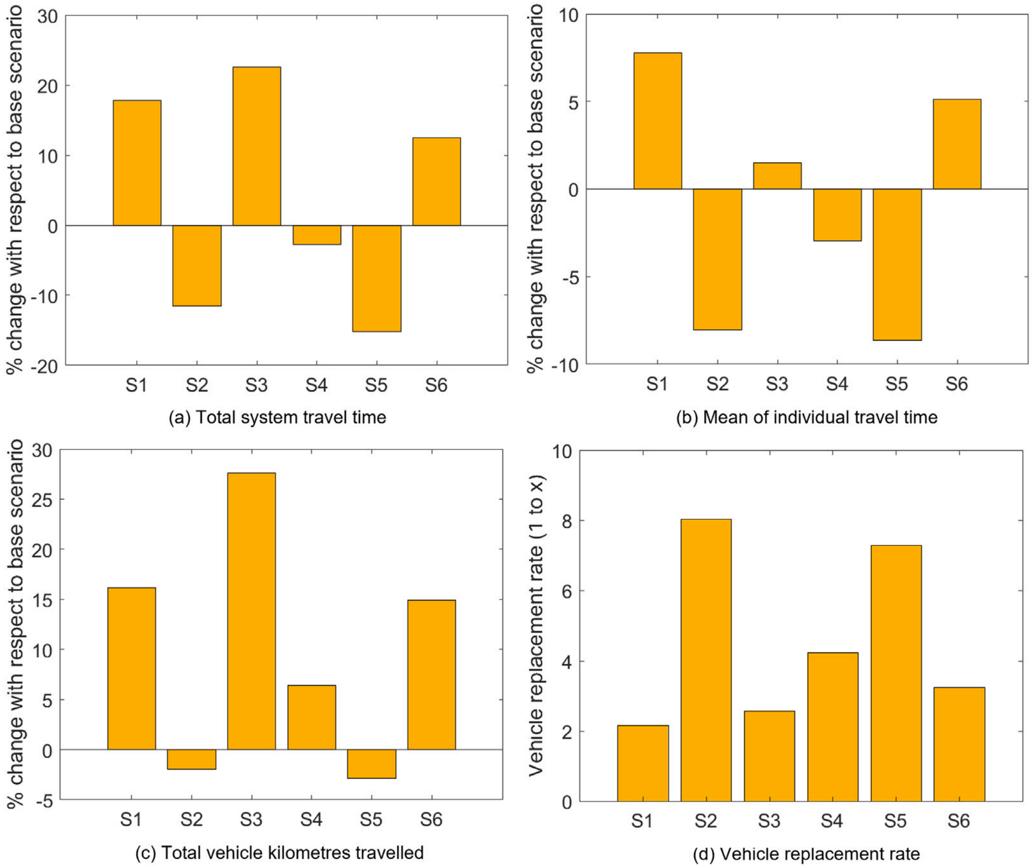

Fig. 8. Comparison of the scenarios tested to the base scenario.

carsharing SAV system and mixed SAV system show lower performance compared to the base scenario in terms of total system travel time and traffic congestion levels, though perform well in terms of vehicle requirements. All these show that a ridesharing SAV system is better among the three and can lead to a better future. Implementation of carsharing SAV system or mixed SAV system can lead to a less sustainable future.

8. Conclusions and future works

This research contributes to the field of modelling reservation-based SAV services by formulating the Dynamic User Equilibrium with SAV Chain Formation (DUESCF) problem as a bilevel model based on game theory, and proposing a solution approach based on an iterative procedure to solve the formulated model. In order to test the proposed solution algorithm, existing DUE and SCF models from the literature are utilized after required modifications. The DUE model considers not only route choice, but also departure time choice. Modifications in the DUE model enable modelling of both SAV services and conventional private vehicles in a single system, and modifications in the SCF model enable inclusion of ridesharing systems. The DUE model was solved using a fixed point algorithm and the SAV chain formation model using a linear program. The bilevel model is solved using an Iterative Optimization and Assignment (IOA) method, which is commonly used in solving combined traffic assignment and control problems.

In order to ascertain the efficiency and the robustness of the proposed algorithm, computational experiments are performed. Computational experiments on a 2-node toy network and the Braess bidirectional network (Case 1) prove that the model converges to a Nash equilibrium. The SAV chains formed in the two above-mentioned computational experiments show absence of empty trips and that each of the 100 vehicles utilized serve 2 trips, forming an optimal solution. This is possible due to the fact that the SCF model uses linear program, which is an exact optimization method. Looking into the actual arrival times, they are distributed close to the expected time of arrivals. Tests related to Case 2 of the Braess bidirectional network show that the model has the capacity to evaluate multiple scenarios. With regards to computation time, the elapsed time for running the DUE model reduces steadily along the iteration of the bilevel model due to the decrease in required number of DUE iterations. However, elapsed time for forming SAV chains stays almost constant throughout the bilevel iteration process. This is due to the fact that the computation time for SAV chain formation depends majorly on the number of SAV service requests, which is a constant throughout the bilevel iteration process. It is also found that the elapsed time for SAV chain formation increases as the SAV service penetration increases, and this is due to the increase in the number of constraints to be formed for optimization. A comparison of the convergence plots of the Braess, Sioux Falls and Anaheim networks shows that the algorithm exhibit satisfactory empirical convergence within limited number of iterations.

Besides the methodological contributions with regards to formulating and solving the DUESCF model, the scenario analysis on the Sioux Falls network offers a comprehensive evaluation of SAV impacts on traffic. A reservation-based ridesharing SAV system can perform significantly better than a reservation-based carsharing SAV system or a reservation-based mixed SAV system. In fact, the reservation-based carsharing SAV system and mixed SAV system perform poorer than the base scenario, wherein conventional private cars are used. It should be noted that the current results from the model are based on lower penetration rates for SAV services and scenarios with higher SAV penetration rates, wherein empty SAV rides may be significantly higher, are to be tested to ensure that the direction of impact is not altered. To conclude, the proposed model can be used by transport modellers and planners to evaluate the impacts caused by reservation-based SAV services and by SAV service operators to maximize their service performance by forming efficient SAV chains.

On the limitations of this research, the adapted DUE model does not differentiate the characteristics of conventional and autonomous vehicles. Furthermore, maintenance, parking and other similar costs are not included in the SAV chain formation model. Specifically, the inclusion of parking cost may help to understand the impact of parking behaviour on the network congestion levels (e.g., for conventional ridesharing services, under certain situations, an increase in network congestion levels due to cruising to avoid parking cost can be observed in Beojone and Geroliminis, 2021). Future research could also consider the addition of road pricing to the effective delay operator in the DUE model, inclusion of private autonomous vehicle users, implementation of dynamic SAV services, introduction of ridesharing en-route, variation of total demand (road user trips) in the network, usage of alternative DUE models. The convergence of the proposed solution algorithm is empirically tested in this study. When using alternative DUE or SCF models, the convergence might be affected by the stochasticity of the respective model, and this needs to be investigated.

The solution algorithm implemented in this study belongs to the category of heuristics and it is not guaranteed that the result obtained is global optimal. Other solution approaches can be tested and the existence of global optimality can be verified. Another future research can be to examine the competition among multiple SAV service operators, where a Nash game between them could also be formulated. The output of this formulation, which is the collective decision of the multiple SAV operators, can be considered as the decision matrix of Player two. Yet another interesting future research is to explore a situation wherein an SAV operator aims at maximizing social welfare, rather than being a profit maximizer. In such a case, the SCF algorithm has to be adapted, i.e., the objective of the SCF algorithm will be different. Under certain conditions (i.e., both of the players try to maximize their utility, be it for welfare or profit), the interaction between the road users and the SCF operator can still be defined as a Nash equilibrium.

To include dynamic SAV services, the following strategy can be utilized: As demand for reservation-based service will be known beforehand, SAV chains shall be formed first. Then, by applying iteration for the time instants, demand for dynamic services can be dynamically fed into the model based on the time instant. Extra vehicles available after allotting vehicles for the reservation-based services can be used for dynamic services, and vehicles can be assigned based on a heuristic technique. Additional future research direction include the coupling of a behavioural demand model (e.g., Tsiamasiotis et al., 2021). The theoretical potential for ridesharing may not be realized in the real world. To capture this, inclusion of a behavioural demand model is beneficial. Such a demand model can also be used to encapsulate the impact of increase in supply on demand. Finally, as observed in computational experiments, the elapsed time for SAV chain formation stays almost constant throughout the bilevel model iteration process. Clustering trips before forming SAV chains and solving the SCF problem for the clustered trips in parallel, as done in Su et al. (2020), can reduce time for forming SAV chains. Alternatively, time for formation of relocation links and constraints can be reduced by using marginal simulation technique. This technique is implemented in Himpe (2016) for the modelling of DUE.

CRediT authorship contribution statement

Santhanakrishnan Narayanan: Conceptualisation, Data collection, Methodology, Formal analysis, Interpretation of results, Writing – original draft, Writing – review & editing, Visualisation. **Emmanouil Chaniotakis:** Conceptualisation, Methodology, Interpretation of results, Writing – review & editing. **Constantinos Antoniou:** Conceptualisation, Interpretation of results, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. DUE model

A.1. Notations (only for the equations presented in the Appendix)

$\mathbf{A^J(t)}$	Flow distribution matrix of junction J at time instant t
$[a_i, b_i]$	Entrance and exit of link i
C_i	Capacity of link i
$D_i(t)$	Demand in link i at time instant t
$D_o(t)$	Demand at origin $o \in S$ at time instant t
$D_p(t, \mathbf{h})$	Travel time in path $p \in P$ at time instant t, under departure profile $\mathbf{h}$
$f_i^{in}(t)$	Inflow of link i
$f_i^{out}(t)$	Outflow of link i
$\mathbf{h}$	Matrix of departure rates Departure rate along path p at time t
l_i	Length of link i
M	A large arbitrary number, e.g. larger than the flow capacity of link j
m_J	No. of incoming links to a junction J
$N_i^{up}(t)$	Cumulative link entering count for link i at time instant t
$N_i^{dn}(t)$	Cumulative link exiting count for link i at time instant t
n_J	No. of outgoing links from a junction J
P	Set of paths in the network
P^o	Set of paths originating from $o \in S$
$q_o(t)$	Volume of point queue at the origin node $o \in S$ at time instant t
S	Set of origins
$S_i(t)$	Supply of link i at time instant t
$[t_0, t_f]$	Planning horizon starting and ending time
t	A time instant in the planning horizon $[t_0, t_f]$
v_i	Speed of forward-propagating waves in link i
w_i	Speed of backward-propagating waves in link i
$\alpha_{ij}(t)$	Turning ratios of cars discharged from link i that enter downstream link j, calculated at time instant t
$\Theta(.)$	Symbolic representation for a junction model
ρ_i^{jam}	Jam density of link i
$\rho_i(t, x)$	Traffic density in link i at time instant t and location x
$\mu_i^p(t, x)$	Percentage of flow on link i that belongs to path p at time instant t and location x (link entrance and exit); Path disaggregation variable
$\tau_i(t)$	Entry time for link i corresponding to exit time t
$\lambda_i(t)$	Exit time for link i corresponding to entry time t

A.2. Equations

$$\frac{d}{dt}q_o(t) = \sum_{p\in P^o} h_p(t) - min\{D_o(t), S_j(t)\} \tag{A.1}$$

$$D_o(t) = \begin{cases} M, & \text{if } q_o(t) > 0 \\ \sum_{p\in P^o} h_p(t), & \text{if } q_o(t) = 0 \end{cases} \tag{A.2}$$

$$D_i(t) = \begin{cases} f_i^{in}\left(t - \frac{l_i}{v_i}\right), & \text{if } N_i^{up}\left(t - \frac{l_i}{v_i}\right) = N_i^{dn}(t) \\ C_i, & \text{if } N_i^{up}\left(t - \frac{l_i}{v_i}\right) > N_i^{dn}(t) \end{cases} \tag{A.3}$$

$$S_i(t) = \begin{cases} f_i^{out}\left(t - \frac{l_i}{v_i}\right), & \text{if } N_i^{up}(t) = N_i^{dn}\left(t - \frac{l_i}{w_i}\right) + \rho_i^{jam} l_i \\ C_i, & \text{if } N_i^{up}(t) < N_i^{dn}\left(t - \frac{l_i}{w_i}\right) + \rho_i^{jam} l_i \end{cases} \tag{A.4}$$

$$N_i^{dn}(t) = N_i^{up}(\tau_i(t)), \quad N_i^{up}(t) = N_i^{dn}(\lambda_i(t)) \tag{A.5}$$

$$\left([f_i^{out}(t+)]_{i=1,\dots,m_J}, [f_j^{in}(t+)]_{j=1,\dots,n_J}\right) = \Theta\left([D_i(t)]_{i=1,\dots,m_J}, [S_j(t)]_{j=1,\dots,n_J}; \mathbf{A^J(t)}\right), \forall J \subset S \tag{A.6}$$

$$\mathbf{A^J(t)} = \alpha_{ij}(t), \forall\{i,j\} \in J; \quad \alpha_{ij}(t) = \sum_{p\ni i,j} \mu_i^p(\tau_i(t), a_i) \tag{A.7}$$

$$\mu_j^p(t,a_j) = \frac{f_i^{out}(t)\mu_i^p(\tau_i(t),a_i)}{f_i^{in}(t)} \quad \forall\, p \text{ s.t. } \{i,j\} \subset p \tag{A.8}$$

$$\sum_{i=1}^{m_J} f_i(\rho_i(t,b_i)) = \sum_{j=1}^{n_J} f_j(\rho_j(t,a_j)) \quad \forall\, t \in [t_0,t_f],\ \forall J \subset S \tag{A.9}$$

$$f_i(\rho_i(t,b_i)) \le D_i(\rho_i(t,b_i-)),\quad f_j(\rho_j(t,a_j)) \le S_j(\rho_j(t,a_j+))\ \ \forall\, i \in \{1,\dots,m_J\},\ j \in \{1,\dots,n_J\},\ J \subset S \tag{A.10}$$

$$\frac{d}{dt}N_i^{up}(t) = f_i^{in}(t),\quad \frac{d}{dt}N_i^{dn}(t) = f_i^{out}(t) \tag{A.11}$$

$$D_p(t,\mathbf{h}) = \lambda_o \circ \lambda_1 \circ \lambda_2 \dots \circ \lambda_k(t) - t, p = 1,2,\dots,k \tag{A.12}$$

Eqs. (A.1) and (A.2) are related to the point-queue model, which represents the dynamics at the origin nodes. If the departure rate from an origin node exceeds the flow capacity of the first link, a queue is formed. Eq. (A.1) caters for the change in volume of point-queue per time instant at an origin [$q_o(t)$] in the form of the difference between the flow entering the queue [$\sum_{p\in P^o} h_p(t)$] and the flow exiting the queue [$D_o(t)$]. Based on variational formulation (Lax–Hopf formula), link supply [$S_i(t)$] and demand [$D_i(t)$] (which controls the flow that can enter and exit a link) is given by Eqs. (A.3) and (A.4), wherein f_i^{in} is the inflow of link i, f_i^{out} is the link outflow, C_i is the link capacity, l_i is the link length, v_i is the speed of forward-propagating waves, w_i is the speed of backward-propagating waves, and ρ_i^{jam} is the jam density. The relationship between cumulative link entering [$N_i^{up}(t)$] and exiting [$N_i^{down}(t)$] counts is shown in Eq. (A.5), where $\tau_i(t)$ represents the entry time for link i corresponding to exit time t and $\lambda_i(t)$ represents the exit time for link i corresponding to entry time t. Eq. (A.6) represents a junction model for modelling the dynamics at the junctions (nodes with incoming and outgoing links), whose inputs will be $D_i(t)$ (corresponds to demand from incoming links), $S_j(t)$ (corresponds to supply of outgoing links) and $\mathbf{A}^{\mathbf{J(t)}}$. The outputs will be outflows from the incoming links and inflows to the outgoing links. The flow distribution matrix [$\mathbf{A}^{\mathbf{J(t)}}$] required for the junction model is formed using Eq. (A.7) and the path disaggregation variable [$\mu_j^p(t,a_j)$], which is necessary for forming the flow distribution matrix, is calculated using Eq. (A.8). α_{ij} is the turning ratios of cars discharged from link i which enter downstream link j. The junction model should satisfy the flow conservation constraint (A.9) and the demand–supply constraints (A.10). Eq. (A.9) means that the total flow through a junction is conserved and Eq. (A.10) ensures the physical feasibility of the flows through a junction. $[a_i,b_i]$ is the entrance and exit of link i. The change in cumulative link entering count per time instant for link i is equal to the inflow to the link i, and the cumulative link exiting count is equal to the outflow from the link i, as shown in Eq. (A.11). Finally, for a path expressed as $p = 1,2,\dots,k$, Eq. (A.12) gives the path travel time corresponding to the departure time t with departure rate $\mathbf{h}$, wherein λ is the link exit time and the symbol }∘′ denotes the composition of two functions. To summarize, Eqs. (A.1) and (A.2) relate to the queueing at the origin nodes (source model), Eqs. (A.3)–(A.5) relate to the link dynamics (link model), and Eqs. (A.6), (A.8), (A.7), (A.9) and (A.10) relate to the junction dynamics (junction model). While Eq. (A.11) is used to calculate the cumulative link entering and exiting counts for subsequent time instants, Eq. (A.12) is used to calculate path travel times at time instant t. The above presented DAE system is time-discretized and solved in a forward fashion.

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