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# Mathematics and Cosmology in Plato's *Timaeus*

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**Abstract:** Plato used mathematics extensively in his account of the cosmos in the *Timaeus*, but as he did not use equations, but did use geometry, harmony and according to some, numerology, it has not been clear how or to what effect he used mathematics. This paper argues that the relationship between mathematics and cosmology is not atemporally evident and that Plato's use of mathematics was an open and rational possibility in his context, though that sort of use of mathematics has subsequently been superseded as science has progressed. I argue that there is a philosophically and historically meaningful space between 'primitive' or unreflective uses of mathematics and the modern conception of how mathematics relates to cosmology. Plato's use of mathematics in the *Timaeus* enabled the cosmos to be as good as it could be, allowed the demiurge a rational choice (of which planetary orbits and which atomic shapes to instantiate) and allowed *Timaeus* to give an account of the cosmos (where if the demiurge did not have such a rational choice he would not have been able to do so). I also argue that within this space it is both meaningful and important to differentiate between Pythagorean and Platonic uses of number and that we need to reject the idea of 'Pythagorean/Platonic number mysticism'. Plato's use of number in the *Timaeus* was not mystical even though it does not match modern usage.


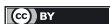
**Keywords:** Plato, mathematics, cosmology, *Timaeus*, number

Plato made extensive use of mathematics in the *Timaeus*' account of the cosmos. As Plato did not express himself in terms of equations, but employed geometry, harmony, and, according to some, numerology and associated ideas, it has been unclear how, why or to what effect he employed mathematics. In addressing these questions, I wish to keep some important methodological considerations in mind. Firstly, when we express natural laws now, we often write them in the form:

$$f(y) \propto f(x)$$

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where a function of  $y$  is proportional to a function of  $x$ , or we might write

$$f(y) = k.f(x)$$

Where  $k$  is a constant of proportionality.<sup>1</sup> This all seems so transparent and straightforward that it is easy to assume that it has always been evident that natural laws could and should be written in this manner. It has not, and it is only since the seventeenth century that they have been expressed in this way in any consistent manner.<sup>2</sup>

The second methodological issue here is what natural laws are like. As we now express natural laws in terms of equations, we slip very easily into a mathematical model of natural law. That is, we can consider natural laws to be unbreakable in a manner analogous to mathematical or geometrical laws. It is very easy to assume that natural law has always been modelled on mathematical law, but again this is not so, the seventeenth century once more being the watershed. Prior to this in the Western tradition, and in other cultures, it is common to find natural law modelled on civil or ethical law instead. So while natural law ought to be upheld, it is conceivable that it will not be, when there may consequently be a punishment for a breach of that law. So we find Heraclitus saying that:

The sun will not overstep his measure: if he does, the Erinyes, the defenders of justice, will find him out.<sup>3</sup>

In some other cultures there was more of an emphasis of a collective righting of a wrong and re-establishment of proper order than outright punishment if a natural law was breached.

These considerations are important for how we assess proposed relationships between mathematics and cosmology which do not fit into the modern equation/mathematical law model. If this model is atemporally evident, one might be led to categorise and criticise other models as examples of numerology, number mysticism, number symbolism, arithmology,<sup>4</sup> animism or anthropomorphism. They

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<sup>1</sup> The functions may be more complex, but the form of the equations is the same.

<sup>2</sup> The relevant idea of proportionality is not atemporally evident either. See Gregory (2001) pp. 5–6 on Aristotle. It is also important not to impose such equations anachronistically on Greek thinkers, cf. Unguru (1975) pp. 67–8 on not imposing algebraic formulations on Greek geometrically expressed mathematics.

<sup>3</sup> Heraclitus, Fr. 94.

<sup>4</sup> I will not generate sharply between these descriptions, as they tend to reflect difference of emphasis rather than approach, there is considerable overlap between them and historically there have been no firm boundaries between them. Arithmology is the view that the first 10 integers have particular significance and is later than the other ideas. See Delatte (1915) p. 139, Zhmud (2016) pp. 322–3, (2019) pp. 35–7.

might be thought to exhibit tendencies to magical or mystical thinking, or primitive, pre-logical, pre-scientific attitudes to cosmology.<sup>5</sup> If the current relations between mathematics and physics are not atemporally evident though, then there is another possibility. There may have been attempts to formulate a relationship between mathematics and cosmology in a rational and sophisticated manner, which appeared plausible in their historical and philosophical contexts, but have since been superseded.

A third methodological consideration I wish to keep in mind is that this space between the primitive and the modern exists and is both meaningful and interesting.<sup>6</sup> As we shall see, some commentators have denied this space and this has important consequences for how we understand Plato's natural philosophy. There has also been a tendency, in commentators both ancient and modern, to assimilate Plato's views too readily with those of the Pythagoreans where there may be interesting differentiations to be made.<sup>7</sup> In what follows, I want to examine three aspects of Plato's thinking on the relationship of mathematics and cosmology, the large scale organisation of the cosmos, the motions of the heavenly bodies and the nature of geometrical atomism. An important caveat. I do not wish to defend any non-modern approach to the relation of maths and physics or a civil/ethical law approach to natural law. It is important though that we recognise that in Plato's time, and for a considerable time after, there were other plausible, rational possibilities for the relationship of mathematics and cosmology.

## 1 |

Let us begin with the ratios of the orbits of the sun, moon and planets. *Timaeus* 35a ff. gives an account of the composition of the world soul in terms of likeness, difference and being. Once a unity of these three has been generated, the whole is divided by the demiurge:

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5 I would prefer not to use such terms as 'primitive', 'pre-logical' or 'pre-scientific' (too pejorative, too binary) but these are the terms which critics have used. See Burkert (1972) p. 335, p. 476, cf. Pedersen (1974) p. 26.

6 A verificationist or experimentalist theory of meaning may reject everything unverifiable or beyond experiment as meaningless without any differentiation, and indeed that can be seen in some histories informed by such theories of meaning.

7 I take it that Taylor's (1928) 'Pythagorean' account of the *Timaeus* has been refuted by Cornford (1937), pp. x–xi. There are others though who still find a significant Pythagorean aspect in Plato's cosmology, cf. McClain (1978), Horky (2013). I also take Kennedy (2011) on musical structure in Plato to have been refuted, see Gregory (2011).

He divided in this manner. First he took one part from the whole, then a part that was double this. The third was half as much again as the second, and three times as much as the first, the fourth twice as much as the second, the fifth three times the third, the sixth eight times the first and the seventh twenty seven times the first.<sup>8</sup>

This gives us a sequence of:

$$1 - 2 - 3 - 4 - 9 - 8 - 27.$$

The *Timaeus* treats these as two sequences,  $1 - 2 - 4 - 8$  and  $1 - 3 - 9 - 27$  and goes on to fill these intervals with the harmonic means and the arithmetic means, giving a sequence of:

$$1 - 4/3 - 3/2 - 2 - 8/3 - 3 - 4 - 9/2 - 16/3 - 6 - 8 - 9 - 27/2 - 18 - 27$$

There is then some further filling of intervals where multiplying one of the numbers in this sequence by  $9/8$  does not exceed the next number in the sequence. Thus the first part of the sequence will be:

$$1 - 9/8 - 81/64 - 4/3 - 3/2 - 27/16 - 243/128 - 2$$

These divisions have a musical significance when taken as ratios, as 2:1 can represent an octave, 3:2 a musical fifth, 4:3 a fourth, 9:8 a tone, with some semitones between.<sup>9</sup> So if we run this sequence in the key of C, we get the notes C D E F G A B C. In modern music, we use the 12 tone equal temper scale, which divides the octave into 12 with the ratio between all neighbouring semitones set equally at  $\sqrt[12]{2}$  (the 12th root of two, this giving the even temper). What Plato gives us is in modern terminology is an eight tone Pythagorean scale.<sup>10</sup> Pythagorean scales (ancient and modern) are generated from the ratios of small integers and do not have equal temper.<sup>11</sup> Pythagorean scales have the advantage of harmonies that are slightly purer, but the disadvantage that key changes within a piece and ensemble playing can be more difficult.<sup>12</sup> Philolaus had other scales which were probably closer to what Greek musicians used in practice,<sup>13</sup> This, however, is mathematically the simplest and purest of those scales, will produce good harmonies and as we will see, this suits Plato's purpose very well.

When Plato generated the more extensive sequence, why did he stop with the seventh term, 27? The scale could be generated with fewer terms and the sequence

<sup>8</sup> Plato, *Timaeus* 35b4–35c2.

<sup>9</sup> Cf. Cornford (1937) pp. 71–3, Mueller (2005) pp. 117–20.

<sup>10</sup> It is also Pythagorean in the sense of being derived from Philolaus.

<sup>11</sup> In this scale, all numbers are powers of 2 or 3. So e.g.  $243/128$  is  $3^5/2^7$ .

<sup>12</sup> More complex chords, such as those used in blues and jazz will not sound the same.

<sup>13</sup> See Barker (2007), (2014) and Huffman (1993).

might continue to infinity. As the scale generated to seven terms exceeds what could be played musically at the time, this cannot be the criterion either. Plato has a cosmological application for this sequence. When the demiurge is setting up the cosmos and revolutions of the same and the different:

He gave the control to the revolution of the same, for this alone was undivided, while he divided the inner revolution in six places giving seven unequal circles, according to the double and triple intervals, there being three of each. The circles were ordered so as to move in opposite senses to one another, three having the same speed while four moved with speeds different from each other and the three, but moving in ratio.<sup>14</sup>

As there are only five planets in addition to the sun and moon, there is a need to go as far as the seventh term, 27, but no need to go any further. We can note two important differences with the Pythagoreans here. According to Aristotle, the Pythagorean justification for 10 heavenly bodies (Earth, Sun, Moon, five planets, central fire and counter-earth) was 10 as the perfect number, being the sum of the *tetraktys*, the first four integers ( $1 + 2 + 3 + 4 = 10$ ).<sup>15</sup> Plato's justification was astronomical (there are observed to be seven heavenly bodies),<sup>16</sup> rather than numerological.<sup>17</sup> Secondly, in the *Timaeus* and subsequent works there is no mention of any audible harmony of the heavenly bodies.<sup>18</sup> There is a harmony to the structure of the world soul, but no sound. This differs from the Pythagoreans, and from the myth of Er at *Republic* 617bc.<sup>19</sup>

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**14** Plato, *Timaeus* 36c7–36d5.

**15** See Aristotle *Metaphysics* 986a8 on the significance of 10. As Taylor (1928) p. 137 points out, it was later recognised that in the Plato sequence  $1+2+3+4+8+9 = 27$ , but Plato himself lays no stress on 27 as a significant number nor does he say anything about the sum of his sequence (which in fact is 54), so this seems a good example of Pythagorean concerns being later read into Plato. See also *De Caelo* 293a on the Pythagoreans 'doing violence to' the phenomena in order to bring them into line with their theory. For more numerological attributions (ratios of mixtures, importance of seven), see Aristotle *Metaphysics* XIV/6.

**16** Plato's alleged ban of observation from astronomy at *Republic* 530b6–c1 might be thought to run contrary to this approach. I have argued there is no such ban (Gregory (1996), (2000b) Ch. 2), but a contrast between how one does astronomy and how it ought to be used in the education of the guardians. The *Timaeus* certainly does not recognise any such ban. As (1975) p. 50 Vlastos comments, it is saturated in the language of observational astronomy, and see in particular *Timaeus* 47a ff. on eyesight and astronomy reproduced below.

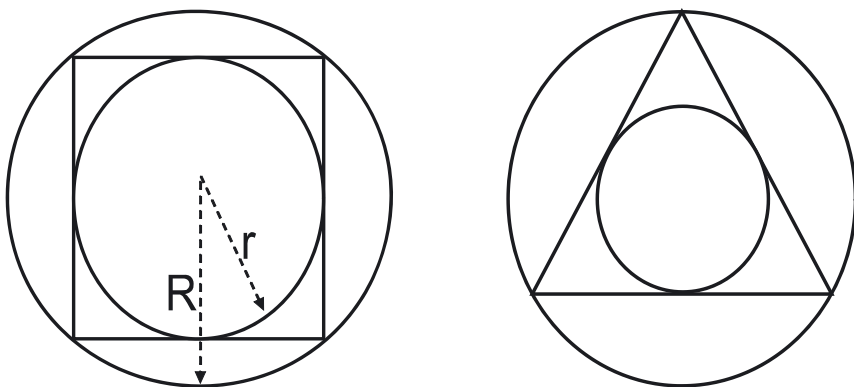
**17** It is arguable that numerology and even the *tetraktys* were not part of original Pythagoreanism but were read in by later commentators, see (Zhudmud (1999), (2016) esp. pp. 338–45, (2019) pp. 39–43). If that is true, there is still a need to distinguish Plato's views in the *Timaeus* from that sort of numerology, especially as Aristotle perceived such numerology and historically such numerology has been associated with a Platonic/Pythagorean number mysticism.

**18** See Barker (2014) p. 198.

**19** Though cf. Netz (2014) p. 150 on evidence for celestial harmony in the Pythagoreans.

## 2 II

In order to understand what Plato was doing here, it is useful to look at Johannes Kepler, the great astronomer of the late 16th and early 17th centuries. His work was pivotal in the Copernican revolution and was influenced by Plato and neo-platonism.<sup>20</sup> In *Harmonices Mundi*, Kepler attempted to derive the ratios and number of the planetary orbits from the Platonic solids, the cube, tetrahedron, octahedron, icosahedron and dodecahedron. The precise details need not concern us here, but in outline Kepler's process is this.<sup>21</sup> For each of these solids, we can imagine a sphere touching each of the surfaces on the inside, and another touching each of the vertices on the outside. It is then possible to calculate a ratio,  $r:R$  of the radius of the inner sphere and the radius of the outer sphere. This is most easily illustrated in two dimensions:



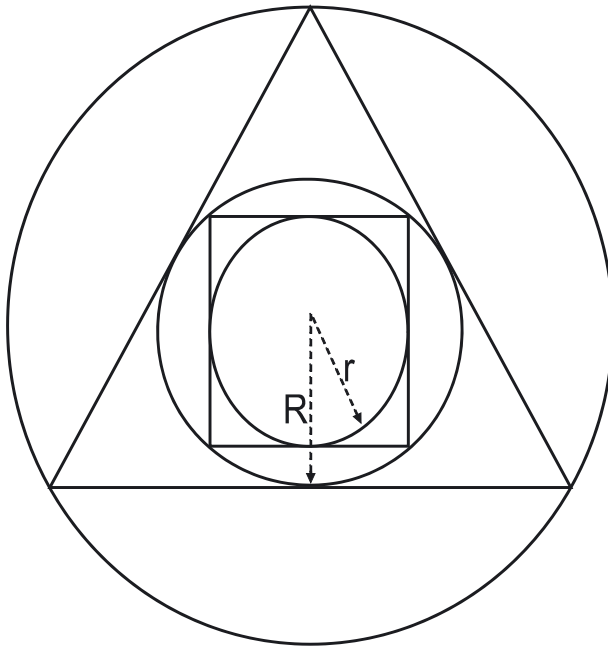
It is then possible, given some assumption about how planetary orbits nest together,<sup>22</sup> to generate ratios for the relative spacing of the orbits of the naked eye planets. This process actually gave good results in reproducing the known orbits of the planets, certainly by the standards of seventeenth century observation.<sup>23</sup>

<sup>20</sup> Field (1988) is the definitive study which recognizes the Platonic/neo-platonic leanings of Kepler.

<sup>21</sup> For more detail here, see Field (1988).

<sup>22</sup> No redundant spaces between orbits, similar to Ptolemy on epicycles touching but not overlapping.

<sup>23</sup> Field (1988).



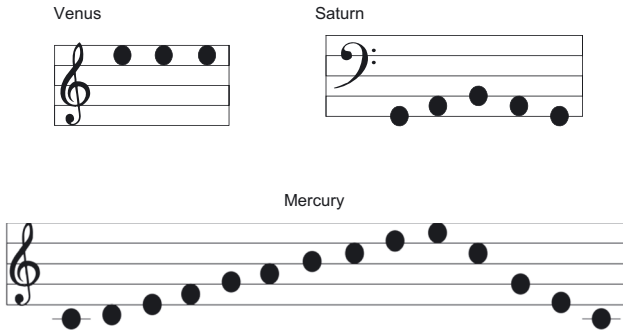
Kepler then had an explanation of why God had set up the Solar System with specific orbits. This was an issue because within the preceding Ptolemaic astronomy there was a process for spacing orbits, which depended on the assumption that planetary orbits may touch but not overlap. So put simply, the epicycle of one deferent touches the epicycle of the next giving a spacing. This became problematic again with Copernicus' advocacy of heliocentrism, and acutely problematic when Kepler rejected epicycles in favour of ellipses.<sup>24</sup> It is not the case that Kepler suddenly decided there should be a means of determining orbit sizes, he was working within a tradition where this was a standard issue. He also had an explanation for why there are a specific number of planets, as there are only a determinate number of Platonic solids to space their orbits with.

Kepler did not finish here. Having discovered that planetary orbits are elliptical, he needed a reason why they have their specific eccentricities and why the planets have specific velocities. It is possible to express many of the properties of a planetary orbit as ratios. To take an example of one that interested Kepler, the

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<sup>24</sup> Kepler did not abandon his spacing for the orbits based on spheres inside and outside Platonic solids, but used the mean radii of the ellipses.

velocities of the planet at aphelion and perihelion can be expressed as a ratio. Musical notes too can be expressed as ratios, as we have seen. With some mathematical processing, Kepler can then produce the harmonies expressed by the planets, which can be represented:



The 'tunefulness' of the planets is an expression of the different eccentricities of their orbits. Venus has the most nearly circular orbit, and so has a monotonous tune. Mercury has the most eccentric of the orbits, and so has the most varied tune. So Kepler had a reason why the planets have specific eccentricities and velocities.

Why did Plato and Kepler approach cosmology in this manner? If we ask modern science why there are eight planets in the Solar System, with specific orbital spacings and orbital speeds, the answer is likely to be that this is largely a matter of chance. Under current models of Solar System formation there is nothing which determines the number, size, spacing or velocity of the planetary orbits.<sup>25</sup> There is the Titius-Bode rule for planetary distances in our Solar System, but this is generally recognised as a crude empirical generalisation applicable only to our Solar System rather than a fundamental law of astrophysics.<sup>26</sup> In a cosmos generated by a benevolent demiurge, however, there is nothing which is generated arbitrarily and the demiurge has a reason for all that he does.<sup>27</sup> Or put another way: what sufficient reason is there for the demiurge to use one set of orbit sizes for the planets rather than any other? Plato and Kepler recognised the need for

<sup>25</sup> This may be related to the mass and spin speed of the accretion disc which the sun and planets form from.

<sup>26</sup> It may be that there are a limited number of stable planetary orbits in the Solar System (due to perturbations and gravitational resonance between orbits) of which Titius-Bode picks out one possibility. Titius-Bode is unlikely to apply in solar systems where the star is not of the type or mass or spin speed of our sun. An analysis of other solar systems (still some years away) will help here.

<sup>27</sup> So *Timaeus* 30a: The god wished all things (πάντα) to be good as far as possible and the state of order is in all ways (πάντως) better than the previous state.

criteria here.<sup>28</sup> Possibilities for explanation open to Plato and Kepler, but closed by developments later in the seventeenth century were forms of geometrical or harmonic ordering. If the demiurge instantiates these choices, then the cosmos will be as good as it can be in this respect. We can also understand Plato's choice of this particular formulation of the musical scale from Philolaus. It is mathematically the simplest and most elegant and produces the best harmonies. There is also an important epistemological element here, in that if a God has generated the cosmos in this manner, then there is a clear role for mathematics, geometry and harmony in generating a proper understanding of the cosmos for humans. Kepler's work illustrates that even in the early 1600s the modern relation of mathematics and cosmology was not evident even to a major figure in the history of astronomy, nor was the issue of whether there is a reason for all aspects of the cosmos.<sup>29</sup>

### 3 III

Are all attempts to apply mathematics to nature which do not match what is done in modern science numerological? And does all numerology ultimately reduce to pre-logical, pre-mathematical, primitive thinking about number? Burkert's discussion in *Lore and Science in Ancient Pythagoreanism* has been influential here.<sup>30</sup> Burkert discussed numerology in several early cultures, in the early Greeks and in the Pythagoreans and characterises such views as 'pre-logical'.<sup>31</sup> He quotes anthropologist Levy-Bruhl on numbers having 'individual physiognomy, a kind of mystic atmosphere' in primitive cultures.<sup>32</sup> He then goes on to say that:

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**28** *Timaeus* 37d8 is significant here. After talking of sun, moon, Mercury and Venus, Timaeus then says 'as for the others and the reasons why he placed them there' (τὰ δ' ἄλλα οἱ δὲ, καὶ δι' αἷς αἰτίας ἰδρύσαστο).

**29** There are other subsequent episodes in the history of science where there have been accusations of numerology or the relation between number and the world has not been clear. Newton's colour theory is one, the introduction of the original periodic table by Mendeleev is another. Even today matters are less clear than one might expect, as the distinction between mathematical physics and numerology is not so easy to formulate (see Stove (1991), Chapter 7) and there are contested areas such as research into the supposed Large Number Coincidences in astrophysics.

**30** The standard English translation of *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaos und Platon*.

**31** Burkert (1972) pp. 479–480.

**32** Burkert (1972) pp. 468, quoting Levy-Bruhl (1951) p. 236.

The notion that numbers have a ‘metamathematical’ cosmic significance, and that they reveal the principle of the order of the world and of human life, is not any kind of scientific or philosophical insight, but a readily comprehensible characteristic of pre-mathematical thinking about number. Pythagorean number symbolism is therefore much older than any natural science mathematics or astronomy that Pythagoras or his pupils could be imagined to have practiced.<sup>33</sup>

This view is both philosophically and historically restrictive. Instead, between this sort of numerology and modern mathematical physics there are other possibilities for how number might relate to the world. These, quite rightly, have been rejected as science has progressed. Prior to their rejection though, they were open and rational possibilities. Here it is important to recognise that numbers have objective properties. Rejecting the idea that a numbers may have ‘individual physiognomy, a kind of mystic atmosphere’ does not mean that a number has no properties at all and so a choice of that number is based on superstition or mysticism. At the simplest level integers may be odd or even and they may be prime or perfect.<sup>34</sup> Certain combinations of integers, when applied to music, will generate a consonant scale and harmonies.<sup>35</sup> If a good and harmonious construction of the cosmos is sought then these are good numbers to choose.<sup>36</sup> If we express what Plato has to say in *Timaeus* 35b ff. in modern algebraic form, this can easily be obscured.<sup>37</sup> So earlier when I discussed Plato’s use of harmonic and arithmetic means, I did not give a modern algebraic expression of these.<sup>38</sup> Since Unguru’s seminal history of mathematics paper, it has been recognised that it is important to understand Greek mathematics in its own, often geometrical form, rather than impose modern algebraic notation on it.<sup>39</sup> Greek mathematics is not ‘algebra dressed as geometry’ or even ‘algebra dressed as harmonics’ but has its own specific form. Plato did not indulge in number mysticism here. The numbers were chosen because of their ability to generate consonance and harmony and their application is fully explained. This may not modern science but nor is this pre-logical, pre-mathematical, primitive thinking about number. Kepler explicitly denied anything mystical about his work. He said that:

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33 Burkert (1972) p. 476. Cf. Dudley (1997) p. 1, p. 5, who has number mysticism and its corollary of numerology begin with Pythagoras and the Pythagoreans.

34 Some may be ‘perfect’, when the sum of their factors equals their factors equals the number itself, e.g.  $6 = 1 + 2 + 3$ ,  $28 = 1 + 2 + 4 + 7 + 14$ .

35 Consonance and harmony can have either objective or subjective definition.

36 See Bell (1933), Betegh (2005).

37 Cf. Cornford (1937) pp. 43–52 on *Timaeus* 32ab.

38  $2ab/(a+b)$  is the harmonic mean and  $(a+b)/2$  is the arithmetic mean in modern algebraic terms.

39 Unguru (1975), pp. 5–6.

Whoever wants to nourish his mind on the mystical philosophy... will not find in my book what he is looking for.<sup>40</sup>

While Kepler's approach was superseded, it is significant that he was criticised at the time for being too concrete and too specific. In the Kepler–Fludd polemic, Robert Fludd argued that the mathematical, geometrical and harmonic relations on the heavens were more mysterious and elusive and were not amenable to such precise descriptions.<sup>41</sup>

## 4 IV

A further advantage of the approach I take here is that it is both meaningful and important to differentiate between Plato and the Pythagoreans on these issues, in way in which it would not be if all they said on these topics could be reduced to primitive thinking about number. Thus I reject terms like 'Pythagorean-Platonic number mysticism' and the reader will have noticed that I have already differentiated quite sharply between Pythagorean and Platonic application of number to the heavens. These are not minor variations in a morass of nonsense but reveal significant differences concerning the application of mathematics to cosmology. It is critical that we understand what is going on at *Timaeus* 35a ff. There can be a tendency to marginalise such passages as obscure, outdated or metaphorical or in other ways philosophically uninteresting. Alternatively, such passages have been treated as containing some hidden meaning, perhaps symbolic, perhaps mystical, perhaps revealed by further manipulation of the numbers.<sup>42</sup> The view I offer here treats the passage as philosophically interesting for what it tells us about Plato's approach to cosmology, rational choice for the demiurge and epistemology. These issues are thematically important for the *Timaeus* and we will see later how they relate to Plato's geometrical atomism and how the demiurge chooses the entities involved there. There is nothing esoteric at *Timaeus* 35a ff. in this view. The numbers are chosen because of their known relations to each other and do not carry any metaphorical or hidden meaning.<sup>43</sup> I am not offering here a general

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<sup>40</sup> Kepler, *Pro suo opera Harmonices mundi apologia* (1622), KGW 6:397.

<sup>41</sup> See Vickers (1984).

<sup>42</sup> Cf. Brumbaugh (1954) pp. 3–4 on humanist scholars ignoring such passages and neo-platonists seeking some hidden enlightenment in them, with modern scholars following suit.

<sup>43</sup> I offer no further calculation or matrices (see (Brumbaugh (1954) pp. 227 ff.)). One thing computer modelling of ancient materials has shown us is that it is possible, from virtually any set of numbers or quantities, to generate some interesting mathematical relationships. Whether the author always meant or was aware of those relationships is another issue, which I am quite sceptical on.

theory on how to approach the mathematical passages in Plato, as their nature and intent are quite diverse.<sup>44</sup> I also take it as an advantage of this interpretation that we can make sense of this passage without reference to considerations relating to number in Plato's supposed 'unwritten doctrine'.<sup>45</sup> This passage though can be understood as an attempt to apply number to the cosmos in a historically and philosophically interesting manner.

It is also important not to treat the 'Pythagoreans' and 'Pythagoreanism' as undifferentiated wholes, so let me qualify what I have said. There were many Pythagoreans taking different views and Pythagoreanism was a broad church of different views and approaches. One important theme among Pythagoreans and Pythagoreanism though is, as Horky puts it, an 'assimilation of natural objects and numbers'.<sup>46</sup> That too I take to be a historically and philosophically interesting project even if it failed. Certainly it is not the approach of modern science, but neither is it pre-scientific or pre-philosophical in the manner of early folk numerology. We also need to view Aristotle's testimony on the Pythagoreans with due care when he tells us the thinking behind Pythagorean cosmology,<sup>47</sup> or attributes numerological views in *Metaphysics* XIV/6. The strength of the contrast between Plato and individual Pythagoreans on the use of number may vary, but it does exist and it is significant.

## 5 V

I have argued extensively elsewhere that the motions of the sun, moon and five planets in the *Timaeus* are perfectly regular and are amenable to precise mathematical analysis so I will be brief here, before moving on to why the planets move like this.<sup>48</sup> At 34a Timaeus tells us that the universe revolves uniformly and has no trace of any wandering motion. At 40b Timaeus distinguishes between 'the stars which do not wander' (ὅσ' ἀπλανῆ τῶν ἄστρον) (40b4), and the planets 'which turn and as such wander' (τὰ δὲ τρεπόμενα καὶ πλάνην τοιαύτην ἴσχοντα) (40b6).<sup>49</sup>

<sup>44</sup> So I do not think all mathematical passages in Plato describe diagrams (Brumbaugh (1954) p. 3) or that the mathematics is metaphorical (Brumbaugh (1954) p. 6), cf. McClain (1978)

<sup>45</sup> Cf. Zhmud (2012) pp. 415–52, (2016) p. 322, pp. 334–5 (2019) p. 32.

<sup>46</sup> Horky (2013) p. 131, also see his Ch. 1. See also Horky (2013) p. 165 on whether Plato saw the Pythagoreans in an undifferentiated manner.

<sup>47</sup> See here Zhmud (1989), Netz (2014).

<sup>48</sup> See Gregory (2000a), Ch. 4, (2000b), (2003).

<sup>49</sup> 'Wandering' is an issue here as Burkert comments that in the *Republic*, Plato: "Was still of the opinion that real exactitude was impossible in the physical world, so that the true astronomer should not depend on sense perception but busy himself with purely ideal magnitudes and movements. In the *Timaeus*, too, Plato speaks without hesitation of the 'wandering' of the planets" Burkert (1972) p. 324, cf. Heath (1913) p. 171, Mittelstrass (1962) pp. 130 ff., Owen (1953) pp. 325 ff.

It is important here that all other motions are taken away.<sup>50</sup> If the universe and the fixed stars have regular motion, there cannot be any metaphysical reason why the rest of the heavenly bodies cannot move in a regular manner as well. In terms of the compromise between reason and necessity, the compromise here is that the universe and the fixed stars move, not that they move in an irregular manner.<sup>51</sup> There is also a hierarchy here. The universe revolves in one place, the fixed stars revolve and have translational motion, the planets revolve and have two translational motions, but none of these motions needs to be irregular. At 39c *Timaeus* tells us that ‘The wanderings of these stars constitute time (χρόνον ὄντα τὰς τούτων πλάνας).’ If the wanderings of the planets constitute time, and these wanderings are irregular, then time will be irregular. There is no need for time to be irregular in the *Timaeus*. All that is needed for a distinction between time and eternity is that time flows while eternity stands still.<sup>52</sup> That time moves ‘according to number’ (*Timaeus* 37d, 38a, 38c) would also suggest it flows regularly. At 39d *Timaeus* tells us of the ‘Great Year’, the time taken for all the heavenly bodies to return to the same positions. This is a calculable amount of time, so the motions of the planets cannot be irregular. Finally, at *Timaeus* 47a ff. we are told that:

God devised and gave to us vision in order that we might observe the rational revolutions of the heavens and use them against the revolutions of thought that are in us, which are like them, though those are clear and ours confused, and by learning thoroughly and partaking in calculations correct according to nature (λογισμῶν κατὰ φύσιν ὀρθότητος), by imitation of the entirely unwandering (πάντως ἀπλανεῖς) revolutions of God we might stabilise the wandering (πεπλανημένους) revolutions in ourselves.

Two very important things come out of this passage. Firstly, the motions of the heavens (‘revolutions of god’) are entirely unwandering. Secondly, we can make ‘calculations correct according to nature’ concerning the heavens.<sup>53</sup>

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<sup>50</sup> *Timaeus* 40b3, cf. 34a, 43b.

<sup>51</sup> See e.g. *Timaeus* 47e.

<sup>52</sup> One might also contrast the regular flow of time in the cosmos with the irregularity of the pre-cosmos chaos and eternity. All that is required is that time moves, not that it moves irregularly.

<sup>53</sup> Mueller (2005) 102–3 has commented that: ‘In the *Timaeus* mathematics is related to the divine because the demiurge uses mathematics in fashioning the world; in the *Republic* mathematics is related to the divine because knowledge of it is an important step on the pathway to knowing forms.’ This may simply be a difference in emphasis, the *Timaeus* more interested in cosmogony, the *Republic* more interested in epistemology. Number does play an epistemological role in the *Timaeus* (e.g. 47ab), while at *Republic* VII a demiurge has brought together (*sunestanaí*) the heavens in the best manner and that involves proportions.

## 6 VI

One objection to entirely regular motion for the heavens in Plato can be based on the ‘contrary ability’ (ἐναντίαν... δύναμιν) of *Timaeus* 38d. We are told that Venus and Mercury are placed in circles with speeds equal to that of the sun, but due to this contrary ability the Sun, Mercury and Venus overtake and are overtaken by each other.<sup>54</sup> This overtaking and being overtaken cannot be generated by Plato’s combination of two regular circular motions. However, this is not the only, or even the most serious phenomenon that Plato was aware of that the model of the *Timaeus* cannot account for. As the sun, moon and planets are all permanently in the same plane with the Earth, then there will be a lunar eclipse every full moon, and there will be a solar eclipse every new moon.<sup>55</sup> These eclipses will always be of the same type.<sup>56</sup> There will be occultations (the technical term for one object obscuring another) of each of the planets by the moon roughly once a month.<sup>57</sup> There will be occultations of the outer planets by the inner planets far too often. There will also be far too frequent occultations of the set of stars which lie in the plane of the moon and planets, while other stars within the zodiac will not undergo occultations at all. Clearly this is a major problem, as Plato cannot have failed to be aware at least that eclipses do not occur each month. Plato was aware of retrogression, when planets as seen from Earth appear to reverse their motion relative to the fixed stars. That would certainly be the most natural reading of ἐπανακυκλήσεις at *Timaeus* 40c.<sup>58</sup> So too Plato was aware of the phenomenon whereby Mercury and Venus periodically disappear and re-appear again, which is the best interpretation of κατακαλύπτονται καὶ πάλιν ἀναφαινόμενα at *Timaeus* 40c.<sup>59</sup> The Babylonians considered retrogression and this behaviour of Mercury and Venus to

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**54** There is a common misconception that Plato is concerned with the retrograde motion of Mercury and Venus here. As they orbit the sun, sometimes they will precede it (so Venus will be seen as the ‘morning star’, just before sunrise) and sometimes follow it (so Venus will be seen as the ‘evening star’, just after sunset. Thus Venus, Mercury and the sun overtake and are overtaken by each other. As inferior planets (orbits less than that of the Earth) Mercury and Venus are always seen close to the sun (a phenomenon known as bounded elongation). See e.g. Cornford (1937) pp. 135–7 and Gregory (2003).

**55** As far as I am aware the only modern work to discuss this is Taylor (1928) pp. 236–237.

**56** Identical eclipses in the sense of the linear alignment of the sun, moon and Earth giving identical total/partial eclipses and in the sense of the relative distances of sun and moon giving either a complete or an annular eclipse each time.

**57** Aristotle at least sees these as important – see *De Caelo* 292a1–8 on the moon occulting Mars, and the following comment that the Egyptians and Babylonians have records of similar events.

**58** Cf. *Republic* 617b2.

**59** As Mercury and Venus orbit the sun, at times their angular separation from the sun becomes so small that they disappear in the sun’s rays.

be regular and calculable, generating algorithms to predict them.<sup>60</sup> Plato continued this passage by talking of these phenomena ‘sending fears and signs of things to come to those incapable of calculation’, which at least suggests that these phenomena are in principle susceptible to calculation and are therefore regular, particularly taken in conjunction with *Timaeus* 47c on ‘calculations correct according to nature’.

## 7 VII

Did Plato believe that the model he gave in the *Timaeus* was adequate to explain all the phenomena he was aware of?<sup>61</sup> One can ask a similar question of many other Greek thinkers. There is a very interesting passage in Simplicius, who says:

The unrolling spheres of Eudoxus’ school do not save the phenomena, not only those that were found later, but also those known before and recognised by them.<sup>62</sup>

The three phenomena Simplicius cited are that Venus and Mars vary in apparent brightness, that the moon varies in apparent size and that solar eclipses vary in type relating to the apparent size of the moon.<sup>63</sup> These cannot be accounted for on the known schemes of Plato, Eudoxus or Calippus. I have argued elsewhere that there is a parallel problem in Anaximander.<sup>64</sup> On the evidence we have, the sun would have exactly the same passage across the sky every day for Anaximander.<sup>65</sup> One can speculate on further motions for the sun which would bring the theory more in line with observations. However, this would breach the symmetry of Anaximander’s cosmos necessary for his argument that the Earth is stationary as it has no reason to move in any particular direction.<sup>66</sup> There is then a tension

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**60** Note here that Aristotle had access to Egyptian and Babylonian records and believed they show no change in the heavens over many generations (*De Caelo* 270b) and that they record occultations for all the planets (*De Caelo* 292a). The author of the *Epinomis* is aware of Egyptian and Syrian records at 986e6–987a7 which he praised. *Timaeus* 22d ff. praises the Egyptians for having the most ancient and comprehensive records, *Laws* 967b ff. is full of praise for those who have accurately studied the heavens.

**61** *Timaeus* 38d8 ff. would suggest not – to give a full account of the stars other than sun, moon, Mercury and Venus would be too much for this account, but could be done later.

**62** Simplicius in *De Caelo* 504.17 ff.

**63** Depending on its distance from the Earth, the moon sometimes covers the entire sun, while sometimes a small rim of the sun is visible around the moon. The latter is an annular eclipse.

**64** See Gregory (2016) Ch. 9.

**65** Stobaeus I, 26, 1a (=Aetius, II, 25, 1) on Anaximander has the circle of the sun ‘angled’ but that angle does not change.

**66** Cf. Couprie (1995), (2009), (2011), (2018).

between the best cosmological reconstruction and the best astronomical reconstruction, as there is with Plato. Here we need to be careful of two things. Firstly, whatever we moderns may feel about the relation of astronomy and cosmology, it is the views of Anaximander and Plato here that are key. Secondly, we should not be swayed by the fact that most accounts of the ancients on the heavens have been written by historians of astronomy and so have favoured the best astronomical reconstruction. I have argued elsewhere that we should treat Anaximander's model as primarily cosmological.<sup>67</sup> We should do the same with Plato. So we accept that the two RCM per planet model of the *Timaeus* as a prototype, open to development. It is strong in cosmological principle but weaker in astronomy. It is a prototype of models which employ combinations of RCM and employ motions for the Sun, Moon and planets offset to those of the fixed stars. Although the model of the *Timaeus* is relatively weak in terms of astronomy, it is a significant advance on the model which Plato gives in the *Republic*, which does not offset the motion of the planets.<sup>68</sup> The tenor of the *Timaeus* is much more towards strong, teleological, cosmological principles rather than accounting for the details of astronomy. Secondly, if we take it that the *Timaeus* model can account for all of the phenomena Plato was aware of, this would utterly wreck any circular motion. It is not a matter of an occasional deviation from RCM for retrograde motion, but permanent deviation to account for issues of celestial latitude (eclipses, when planets pass each other, etc.).<sup>69</sup> Thirdly, this seems to be the way that Plato was taken in antiquity, the cosmological principles being held sacrosanct while more sophisticated application was made of them to generate a better fit with observation.<sup>70</sup> One advantage of this view is that then Simplicius' twice made claim (*in De Caelo* 488.18 ff., 492.31 ff.) that Plato set a problem of which circular, regular and ordered

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<sup>67</sup> Gregory (2016) Ch. 9.

<sup>68</sup> See Gregory (2003).

<sup>69</sup> Does RCM in the *Timaeus* constitute a change from *Republic* 530ab where 'the proportions (συνμετρίαν) of night to day, of these to month, of month to year, and of the other stars to these and each other' undergo change (deviation, quite possibly change for the worse, παραλλάττειν)? Yes, but there is a change in Plato anyway as *Laws* 822a says that 'The usual opinion concerning the sun, moon, and other planets, that they at some time wander (πλανᾶται), is not the case; precisely the opposite is true. For each of these bodies always travel on one path, and not many, although this may not seem so.' Cf. *Laws* 967ac on calculation.

<sup>70</sup> I reject the idea that Eudoxus was an instrumentalist, while Plato was a realist, or alternatively put one was engaged in a mathematical rather than a cosmological project, the other vice versa. I do not think it appropriate to categorise ancient thinkers in terms of a modern philosophy of science debate and even if we do it is by no means clear that Eudoxus was an instrumentalist or engaged in a purely mathematical project. See Gregory (2003).

(ἐνγκύκλιον καὶ ὁμαλὲς καὶ τεταγμένον) motions will match the heavens makes considerable sense.<sup>71</sup>

## 8 VIII

If the heavens move in a perfectly regular manner, why do they do so? For Plato, regular and orderly behaviour requires further explanation. In the *Laws* he puts it like this:

Those who engaged in these matters accurately would not have been able to use such wonderfully accurate calculations if these entities did not have souls.<sup>72</sup>

It is worth looking closely at what sort of entities Plato postulates here, what they do and what they are supposed to explain. The *Timaeus* describes the cosmos in these terms:

Thus in accordance with the likely account it is necessary to say that this living, ensouled intelligent cosmos has in truth come about through the foresight of god.<sup>73</sup>

The *Timaeus* states that the cosmos does not have many of the things we would usually associate with an animal (or even, within many religious traditions, with a god). From *Timaeus* 33b onwards we are told that this 'animal' is perfectly spherical, has neither eyes nor ears as there is nothing external to see or hear, there is no air for it to breathe, no need of any organ to receive food or to excrete the remains, as it is entirely self-sufficient and nothing comes in or goes out. As it needs neither hands to defend itself nor feet to stand on, and has no need of legs or feet to propel itself, it has no limbs, and at 34b we are told it is a god. To say the least, this is a somewhat strange animal, and certainly could not be considered to be an anthropomorphic god. If the cosmos is like an animal, then we have an analogue for the integrity and the internal organisation of the cosmos, and the ongoing motion and order of the cosmos. A typical ancient contrast here is between the body with life, self-motion and mind and what happens to the body after death.<sup>74</sup> What does this god do? All it does is revolve uniformly in one place.<sup>75</sup> This

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<sup>71</sup> See Vlastos (1975) App. L & M.

<sup>72</sup> Plato, *Laws* 967b.

<sup>73</sup> Plato, *Timaeus* 30b.

<sup>74</sup> See e.g. Plutarch *On the Phenomenon of a Face in the Moon's Circle* 926d ff.

<sup>75</sup> Plato, *Timaeus* 34a.

is so for the fixed stars and the planets as well. The stars are spherical and intelligent like the cosmos, are divine and are living creatures.<sup>76</sup> As with the cosmos, they have motions befitting their intelligence, and so we are told in the *Timaeus* that:

To each of these he gave two motions, one being uniform and in the same place, always thinking the same thoughts concerning the same things, the other being a forward motion obeying the revolution of the same and similar. With regard to the other five motions, they were motionless and still, in order that each might attain the greatest possible perfection.<sup>77</sup>

The essential point here then is that the celestial bodies have no freedom of action (or no desire to deviate from regular circular motion). They have the intelligence to carry out their assigned duties, and not to do anything else. It is their intelligence which explains their regular and orderly behaviour. That they move, and do so without any external compulsion, is explained by their having souls. So in the *Laws* we find that:

Athenian Stranger - If we should see that this motion (self-generating motion) had come about in something composed of Earth, water or fire, whether separate or in combination, what should we say was the state of that thing?

Cleinias – You are asking me is whether we should speak of this thing as being alive?

AS – Yes.

Cleinias – Emphatically so.<sup>78</sup>

So too at *Laws* 897b:

If the whole course and motion of the heavens and everything that are in them are similar in nature to the motion and revolution and calculations of intelligence, and work in a related manner, then clearly we must say that the best soul has charge of the whole cosmos and leads it along these selfsame courses.<sup>79</sup>

Although the cosmos and the heavenly bodies have life, soul, divinity and intelligence, they have these in a highly circumscribed and attenuated manner, from a

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<sup>76</sup> Plato, *Timaeus* 40a–b.

<sup>77</sup> Plato, *Timaeus* 40a.

<sup>78</sup> Plato, *Laws* 895c.

<sup>79</sup> Plato, *Laws* 897b. Cf. *Epinomis* 982b on intelligence and regularity.

modern point of view.<sup>80</sup> If this is describable as vitalism, it is a highly depersonalised one where the attributes of animate beings that are required for the description of the cosmos have been carefully sorted from those that are not. This allows us to distinguish quite sharply between Plato and the mythological and magical traditions. The key matter here is that these souls/gods are not capricious. They always act for the best so will always act in the same manner. There is nothing unpredictable or irregular about their behaviour.<sup>81</sup> Athene might well choose to hold back the dawn (*Odyssey* XXIII, 242) just as Aphrodite chooses for love to break Paris' helmet strap and envelop him in a mist to save him (*Iliad* III/369 ff.) or the gods choose from honour to prevent decay to the bodies of Patroclus and Achilles (*Iliad* XIX, 21 ff., XXIII/184 ff., XXIV/18 ff.) or Zeus in anger chooses to gather clouds and hurl thunderbolts (Homer, *passim*), but the celestial intelligences of Plato will not deviate.

Plato's conception of natural law here is one that is modelled very much on a civil or ethical conception rather than a mathematical model. The heavenly bodies have intelligence, understand what they ought to do for the best and being good souls carry that out. This is a strong conception of natural law for while it is conceivable that the heavenly bodies will deviate from the intelligent course, due to their nature they will not in fact do so.<sup>82</sup> We might compare here the fate of the cosmos in the *Timaeus*. As the cosmos was generated, it is dissoluble, but as the demiurge is good, this will not in fact happen and the cosmos will continue indefinitely. We can also see why Plato does not formulate the mathematics of the motions of the heavens in terms of proportionality laws. There is no question here, for instance, of there being a force impressed and there being a resulting action in proportion to the size of that force, or any question of an expenditure of energy or fuel.<sup>83</sup> The heavenly bodies simply manage their own motions, either a single RCM or combinations of RCMs. Here I must disagree with Burkert, who says that:

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**80** Both Plato and Aristotle would hold that these entities are fully alive, intelligent, etc. but would want to distinguish them from terrestrial living, intelligent entities.

**81** The *Laws* is also adamant that prayer or other offerings will be of no avail in trying to change the minds of the gods.

**82** See also Plato, *Laws* X 896a ff. on the goodness and regularity of the heavens, and *Gorgias* 503d ff. on virtue, order and consistency

**83** Important if the heavens are to be eternal.

Plato thought it an inescapable conclusion that the orderly motion of the stars is due to their having souls; it is a voluntary, chosen order. Here sophisticated Greek science harks back to the pre-scientific way of thinking and comes to rest in it.<sup>84</sup>

Again the question is whether there is something between 'pre-scientific' thinking and the approach of modern science. Plato's view is not a pre-scientific animism or vitalism. This is in a sense a voluntary order but good souls will always choose to uphold that order as the demiurge will always choose not to dissolve the cosmos. That gives a very strong backing to the regularities of the heavens, quite different to the caprice of the gods. There is an interesting reversal here. In the modern world, we take machines to be the exemplars of regularity, with humans deviating from that and have done from around the seventeenth century and the rise of the mechanical philosophy. Plato took intelligence and in particular divine intelligence to be the exemplar of regularity. In an age where machines were prone to rapid wear and breakdown and before mechanical clocks, clockwork being the key analogy for the mechanical philosophy, this was a reasonable option.

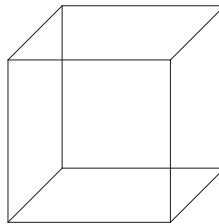
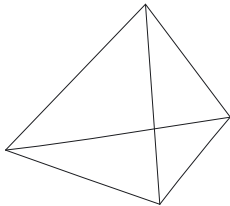
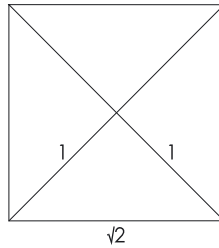
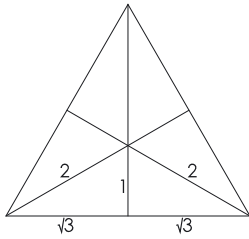
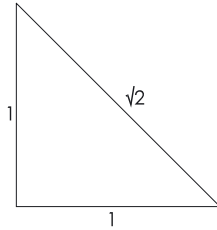
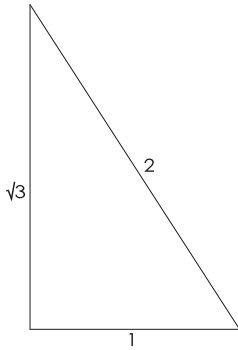
It is also worth considering here that within a geocentric scheme, where all the motions of the heavens are real and not apparent, it is impossible to conceive of a force that will make the planets behave as they do (generate real retrograde loops, etc.). The ancient alternatives are simply Aristotle's natural circular motion for the aether or Plato's intelligent planets. If the heavens are driven around by a vortex, that may give the basic motion of the stars but will fail on the more complex motions of sun, moon and planets. It is also worth noting that ancient Greek arguments for the centrality and stability of the Earth were sophisticated and plausible, well beyond any primitive prejudice about the nature of the Earth.

## 9 IX

Finally, some aspects of Plato's matter theory where there are some interesting parallels with his treatment of the heavens on issues of perfect instantiation, reasons for choice and epistemology. Plato hypothesised two ultimate triangles, 1, 1,  $\sqrt{2}$  and 1,  $\sqrt{3}$ , 2. These triangles formed up into more complex shapes, two of the former generating a square which is  $1 \times 1$  with the  $\sqrt{2}$  edges touching, and six of the latter generating an equilateral triangle with sides which are  $2\sqrt{3}$ . The equilateral triangles can then form tetrahedra (fire), octahedra (air) or eikosahedra (water), while the squares can form cubes (Earth).

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<sup>84</sup> Burkert (1972) p. 335, Cf. Pedersen (1974) p. 26 'In many ways, the Timaios reflects an animistic and anthropomorphic philosophy foreign to the Ionian philosophers' fundamental attitude, although there are a few similar ideas.'



Are the instantiations of these primary triangles perfect? They are, for several reasons. Firstly, we are given specific dimensions for these triangles and there is nothing to suggest these dimensions are not attained. Secondly, *Timaeus* 50d4–51b2 lays great emphasis on the characterlessness of the receptacle, so that it does not distort what is in it. There is no metaphysical reason why the simple triangles should

be imperfect or why they should deteriorate.<sup>85</sup> Thirdly, any imperfection in the triangles would rapidly cause misfits in both the complex two dimensional shapes and the three dimensional figures as well, but we have no mention of any such misfit. Fourthly, Aristotle is highly critical of geometrical atomism and if there were this fault in the triangles one would expect him to criticise it, but he does not.<sup>86</sup> It is also significant that Aristotle links together Leucippus and Democritus in interesting ways. According to him, those who advocate indivisible magnitudes are Leucippus and Democritus, who suppose them to be bodies, and Plato in the *Timaeus* who supposes them to be surfaces.<sup>87</sup>

After the demiurge generates the simple triangles, I see no evidence or any suggestion even that he generates any more simple triangles or undertakes any repairs to them. The key passage here is *Timaeus* 53b4–5, where the demiurge:

οὕτω δὴ τότε πεφυκότα ταῦτα πρῶτον διεσχηματίσατο  
εἶδεσί τε καὶ ἀριθμοῖς

First generated these things shaping them

with forms and numbers.

There seems to be no constraint on the application of number here (as with the astronomy) and the passage continues that the demiurge made them as fair and good as possible. As the subsequent account makes clear, especially 55b, it is the simple triangles that are being referred to here. The perfect participle *pephukota* would strongly incline one to believe that this was a singular past action.

If there is ongoing deterioration of the simple triangles but no repair of them, then the cosmos must ultimately, with due passage of time, be in danger of dissolution, and retrogression back to the chaotic pre-cosmic state. This is precisely what the *Timaeus* is adamant cannot happen though, so there cannot be any deterioration of the simple triangles. The cosmos can have a perpetual existence, the heavens can move in an orderly manner perpetually and the simple triangles can also be perfect

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<sup>85</sup> Miller (2003) p. 173 says that ‘Plato conceives of triangles as physical things that are subject to the imperfections to which all physical things are subject.’ This might apply in *Republic* VII, but we have perfectly spherical astronomical bodies in perfect orbits in the *Timaeus*. There are passages in the *Timaeus* where triangles are said to be imperfect or to deteriorate, but those can all be read as referring to complex triangles which can be imperfect or change without the two simple triangles from which they are composed being imperfect or changing, see below and Gregory 2000 Ch. 7.

<sup>86</sup> See Aristotle *De Caelo* III/7, *Physics* IV/2, *De Generatione et Corruptione* I/1, I/2 and II/1.

<sup>87</sup> Aristotle, *De Generatione et Corruptione* 315b25 ff. See also 325b25 ff. where indivisibles for the early atomists are solids which are unlimited as to shape, while for Plato they are surfaces and limited in number.

and changeless. At *Timaeus* 42d, the demiurge retires from making things, turning over further work to the demi-gods, so it is inconceivable that the demiurge makes more simple triangles or repairs them.<sup>88</sup> A further consideration on these lines is that if the simple triangles can deteriorate, gradually being worn down or cut up, some of one type of the triangles might change into the other type of triangle, thus allowing transmutation of Earth to air/fire/water or vice versa which again is specifically excluded. Both Plato and Aristotle were capable of thought experiments on deterioration over long periods. Plato does so in the myth of the *Politicus*, where gradual deterioration is thought to lead to an ‘endless sea of unlikeness’<sup>89</sup> and Aristotle does so with his argument that the four terrestrial elements would become completely dissociated over time due to their natural motion if they were not affected by the motions of the heavens.<sup>90</sup>

Does *trigōna* indicate simple triangles or triangles which are complexes of simple triangles? At 81b5–9 the *Timaeus* begins the discussion of ageing by saying that:

When the organisation (*sustasis*) of the whole animal is new, the triangles (*trigōna*) which constitute the atoms being ‘fresh off the stocks’ (*ek druochōn*), they have strong joins between one another (*pros allela*).

As the demiurge generates the simple triangles, but the demi-gods organise animals together, *trigōna* here must mean complex triangles, and note the issue is bonds between triangles. *Timaeus* 89c is also significant:

For plainly in the beginning the triangles (*trigōna*) of each animal are organised (*sunistatai*) with the power to last only up to a certain time.

Again, this must be complex triangles and it is the bonding and organisation of these complex triangles which will deteriorate, not the simple triangles. In both cases, it is significant there are different verbs to the generation of the simple triangles at 53b. A further consideration here is that surely all simple triangles were generated at once by the demiurge. Therefore any relational temporal reference to *trigōna*, such as those which are older or newer at 81c,<sup>91</sup> or those fresh off the stocks at 81b must refer to complex triangles and any talk of dissolution must be about the dissolution of complex triangles and not simple triangles. All the simple triangles

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<sup>88</sup> See also *Timaeus* 69c which reprises the handover to the demigods.

<sup>89</sup> Plato, *Politicus* 273d9.

<sup>90</sup> Aristotle, *On Generation and Corruption* book II chapters 10 and 11, and in *Meteorology* book I chapters 1–3.

<sup>91</sup> A small translation note – *Timaeus* 81c6 *chala* I take to mean loosen rather than ‘blunted’, so complex triangles come apart without damage to the simple triangles.

date from one action of the demiurge. At 53d *Timaeus* postulates two sorts of triangle as the *archai* of the elements, and at 55b and 57c they are *stoicheia*, letters or ultimate elements, in contrast to the complex triangles.<sup>92</sup> It is highly unlikely that Plato would consider either an *archē* or a *stoicheion* to be either imperfect or to undergo substantial change.

There is no need for any imperfection or change in the simple triangles. The theory of geometrical atomism and its explanation of all the phenomena work perfectly well without either. The bonds between simple triangles may change, but not the simple triangles themselves. I would emphasise here that there is still a perceptual flux and the attendant philosophical issues. However, there is an underlying stability of the simple triangles. Again in parallel with the astronomy, *Laws* 822a says that although the planets may appear to wander, in fact they do not, as there is an underlying invariance. The *Timaeus* has perfect RCM and perfect simple triangles.

## 10 X

There is an important contrast between Plato's geometrical atomism and the Pythagoreans on the nature of matter. According to Aristotle the Pythagoreans treated physical entities as in some way constituted out of number.<sup>93</sup> So the Pythagoreans had an arithmetical approach to matter, while Plato, placing the emphasis on shape, had a geometrical account of matter. Number is important in describing matter through geometry, but entities are not constituted out of number. I add the usual caveats about the diversity of Pythagorean thought and caution about the testimony of Aristotle which may affect the strength of contrast here in individual cases, but again there are contrasts and they are significant.

There is also an important contrast with Leucippus and Democritus, who clearly thought of their atoms in terms of shapes. They do not seem to have applied mathematics or geometry to these shapes though, and Simplicius tells us that:

Leucippus supposed there to be an infinite number of atoms that are always in motion and have an infinite number of shapes on the grounds that nothing is such rather than such (*dia to meden mallon toiouton e toiouton einai*).<sup>94</sup>

<sup>92</sup> See Plato *Theaetetus* 202e ff., *Politicus* 277e ff., *Philebus* 18b ff., for more on stoichea and the letters/syllable analogy in later Plato.

<sup>93</sup> See e.g. Aristotle *Metaphysics* 987b11–13, 1083b17–19. See also Horky (2013) p 131.

<sup>94</sup> Simplicius *Physics* 28, 8.

It is with geometrical atomism that we see the choices the demiurge has to make in their starkest form. At 53d4–54a6 *Timaeus* says that:

This we hypothesise as the principle of fire and of the other bodies... but the principles of these which are higher are known only to God and whoever is friendly to him. It is necessary to give an account of the nature of the four best (κάλλιστα) bodies, different to each other, with some able to be produced out of the others by dissolution... We must be eager then to bring together the best (κάλλει) four types of body, and to state that we have adequately grasped the nature of these bodies. Of the two triangles the isosceles has one nature, the scalene an unlimited number. Of this unlimited number we must select the best (κάλλιστον), if we intend to begin in the proper manner. If someone has singled out anything better (κάλλιον) for the construction of these bodies, his victory will be that of a friend rather than an enemy. We shall pass over the many (τῶν πολλῶν) and postulate the best (κάλλιστον) triangles.<sup>95</sup>

Without the geometrisation of matter in this sense, it is hard to see how the demiurge would have sufficient reason for his choice of the shape of the basic particles. It is interesting to note here how many times Plato refers to ‘the best’ in this passage. Just as numbers have objective properties, so do two dimensional and three dimensional shapes. They can have more or less symmetry and there are a finite number of convex Platonic solids, that is solids with all of their faces the same shape and size.<sup>96</sup> There are shapes which can be combined to make those face and shapes which cannot. If symmetry is thought to be good, then instantiating these shapes will help to bring about a cosmos that is as good as it can be.

We do not get any sort of mathematical chemistry out of geometrical atomism, however. Vlastos has suggested that the *Timaeus* account gives the basis for equations such as:<sup>97</sup>

$$1W \leftrightarrow 5F$$

where we read that 1 unit of water may transform into 5 units of fire, and vice versa, and we can understand this as:

$$20t = (5 \times 4t)$$

where we read that the 20 triangles of one icosahedron of water may disassemble and reassemble as five units of the tetrahedron of fire. It is quite possible to understand many chemical processes, in a qualitative fashion, using geometrical atomism. So as Vlastos notes, we might see the above reaction as part of what is

<sup>95</sup> Cf. Plato, *Timaeus* 53b ‘Firstly marking them out into shapes by means of forms and numbers (πρώτον διεσχημαίσατο εἶδεσί τε καὶ ἀριθμοῖς), god constructed (συνιστάναι) them, so far as he could, to be as fair and good as possible (δυνατὸν ὡς κάλλιστα ἄριστά).’

<sup>96</sup> On symmetry in geometrical atomism, see Lloyd 2006, 2007, 2010.

<sup>97</sup> See Vlastos (1975) pp. 70 ff. and appendix N.

going on when oil (water mixed with fire) is burnt.<sup>98</sup> However, I do not see any evidence that Plato used his matter theory as a basis for a mathematical treatment of chemistry in this sense, or that he had any intention of doing so. He was not setting up the basis for chemical equations, but allowing the demiurge sufficient reason for his choice of particles. Plato to some extent does explain the properties of the elements in terms of the geometrical structure of their particles, as fire cuts because of the sharp angles of the tetrahedron. There is no use though of a chemical equation. Indeed, it is a very long time until chemistry begins to use equations, arguably not until Anton Lavoisier in the eighteenth century.<sup>99</sup> One can find chemical recipes before this time (take three measures of  $x$ , one of  $y$ , one of  $z$ , mix and heat, etc.) but not equations where a strict balance of chemical constituents between the original materials and the products.<sup>100</sup>

It would be wrong to see the seventeenth century as solely reviving the atomism of Leucippus and Democritus to the exclusion of Plato's geometrical atomism. This is historically important, as one of the great claims for the atomism of Democritus, Leucippus, Epicurus and Lucretius is that it inspired the great revival in atomism in the seventeenth century, and that this was important in the fight against scholasticism. That the seventeenth century felt the need to cure atomism of atheism was not merely a religious predilection. The philosophical problems with presocratic atomism, of why atoms should have certain shapes and combine in certain ways, which Plato addressed with teleology and the demiourgos, were now addressed with a Christian deity. So we can find Boyle saying that:

The provident demiourgos wisely suited the fabric of the parts to the uses that were to be made of them.<sup>101</sup>

So too there was a considerable debate between those who advocated a plenum of particles and those who favoured atoms and a void. It is notable that those who favoured the plenum and rejected the idea of action at a distance, such as

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**98** Vlastos (1975) pp. 71. Cf. Brumbaugh (1954) p. 248.

**99** Lavoisier is most prominent in laying the foundations for a treatment of chemistry in terms of equations, though he is building on the work of other eighteenth century chemists. It is not until later though, perhaps with figures like Dalton, that we find something that would be recognised as a proper chemical equation.

**100** We have to wait even later for a thermodynamic balance in chemical equations.

**101** Boyle, *A Disquisition about the Final Causes of Natural Things*, Works vol. 5 p. 409. Similarly, we can find Newton saying that: 'It seems probable to me that God in the beginning formed matter into solid, massy, hard, impenetrable movable particles, of such sizes and figures and with such other properties and in such proportion in space, as most conduced to the end for which he formed them.' Newton, *Optics* IV 260.

Descartes, adopted a similar solution to Plato for the awkward cases of gravity and magnetism. *Timaeus* 80c argues that the 'attractive' powers of electricity and magnetism are not due to any action at a distance, but can be explained by the fact that there is no void and the atoms jostle each other and move to their own region. Descartes, while he used vortices to explain gravitational effects did so without a void, and used screw shaped particles moving among smaller particles to explain magnetism.<sup>102</sup> Descartes also gave a strongly mathematical treatment of the constitution of his ultimate particles, and although this would lead us too far astray, one can also argue that Descartes' identification of extension and matter has its ultimate roots in the receptacle of the *Timaeus*.

## 11 XI

If number mysticism and associated ideas were not part of Plato's intent in the *Timaeus*, when did such interpretations come about? The *Laws*, as we have seen, strongly supports the *Timaeus* on the heavens, number and regular motion. Certainly a number mystical view does not originate in the *Epinomis*, which in places leans heavily on the *Timaeus*.<sup>103</sup> Passages on the heavens at 978a–979b, on the elements at 981b–982d, on the role of mind/intelligence in orderly behaviour at 982c–984d and the role of mathematics in epistemology at 991d–992a all follow the *Timaeus* and have no trace of any number mysticism. It is also interesting to note that ps-Timaeus Locrus' *On the Nature of the Cosmos and the Soul*, probably 1<sup>st</sup> century BCE, shows no sign of any number mysticism view.

It can be argued that number mystical views of the *Timaeus* began with Speusippus and Xenocrates in the Academy.<sup>104</sup> However, evidence is very sparse and Speusippus' *On Pythagorean Numbers* discusses the number 10 as a perfect number solely in terms of the mathematical properties of 10, with no mysticism or symbolism. Number mystical views in general became popular in the last few centuries BCE, and Zhmud has argued that arithmology, a privileging of the first 10 integers, came about during this period.<sup>105</sup> It was also a period which saw spurious attributions to Pythagoras, Pythagoreans and Plato and to some extent an

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**102** Descartes, *Principia Philosophia* IV/133 ff.

**103** If the *Epinomis* was not by Plato (possibly authored or edited by Philip of Opus?) I take it to be by someone close to Plato and well versed in Plato's thought, attempting to imitate Plato.

**104** So Burkert (1972), in a more nuanced manner Zhmud (2016) p. 322, (2019) pp. 32–5.

**105** Zhmud (2016) pp. 321 ff., (2019) 32 ff., cf. Robbins (1920), (1921).

assimilation of the views and supposed views of Pythagoras and Plato.<sup>106</sup> One must strongly suspect that it is from here the number mystical interpretation of the *Timaeus* originated. It may also be the case that some fundamental ideas in Pythagoreanism which we consider to be numerological, such as the *tetraktys*, date from this period rather than pre-Plato Pythagoreanism.<sup>107</sup> It is clearly too large, complex and contentious a task for this paper to sort all of the evidence here, and undertake the delicate task of unpicking all this information on Pythagoras, the Pythagoreans and Plato. However, it is important to suggest a plausible post-*Timaeus* origin for the number mystical interpretation in antiquity and for the more general idea of Platonic-Pythagorean number mysticism. Modern approaches to the *Timaeus* have been hampered both by these issues with the evidence and by a narrow historiography which has rejected anything not matching the modern approach to the relation of mathematics and physics as number mystical.

## 12 XII

Plato's *Timaeus* did not mathematise cosmology in the same manner or for the same reasons as modern science. However, it would also be quite wrong to attribute numerology, number mysticism or primitive attitudes to cosmology to it. Above all, Plato needed to give the demiurge sufficient reasons for the choices he makes about the organisation of the cosmos, whether on the macro scale issues of the distribution of the planets and their motions or the micro scale issue of the shapes of the fundamental particles. This requirement is generated by the supposition that there is nothing arbitrary about the organisation imposed on the primordial chaos by a benevolent demiurge. When these choices are instantiated, they are part of what makes the cosmos as good as it can be. They are also part of what allowed Plato to generate an account of the cosmos where otherwise choices would be underdetermined. The foundational issue of the choice of mathematical formulation for the musical scale can also be seen in these terms. From the various possible scales, Plato chooses the mathematically simplest, most elegant and most harmonious.

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**106** One interesting example here is the alleged discovery of the relation of numbers and musical notes from hearing a smith's hammers on various pieces of metal (see Bell (1946) 101–2). This tale is simply impossible as the note produced by the metal is not related in the required way to its weight. There is also a much printed woodcut suggesting four ways in which Pythagoras could have come to the note/number relations, all of which are impossible!

**107** So Zhmud (2016) p. 341, (2019) pp. 39–43.

Plato's model of natural law for the heavenly bodies was very much a civil/ethical one, but interestingly a civil/ethical law that will not be broken. If the souls of the planets are entirely good, then they will always do the right thing. Plato had no need for the proportionality relations familiar to modern science here. There is no question of quantities of one factor (e.g. force) acting on quantities of another (e.g. mass) and producing an effect in due proportion to those factors. Rather, it is a question of how a good soul will manage its own motion.

It is important to recognise that there is some interesting ground between how modern science treats these issues and primitive or mystical approaches. Numbers and shapes do have objective properties on which to base a rational choice. Plato's account of the motions of the heavens is not a primitive animism but addressed a real issue of regular motion for the heavens which did not have an easy solution in antiquity. This has important implications for not only how we see Plato's account of the natural world but also for many other ancient thinkers who tried to explain the regularity of the cosmos through similar means.

It is critical to realise that questions such as how mathematics applies to the cosmos, how we think of laws, whether there are arbitrary aspects to the cosmos, and whether intelligence is a better model for regularities than mechanism did not have atemporally evident answers. Important scientists, much later than Plato, did not give modern answers to these questions. As Lloyd has commented, it is important not to assimilate Plato to Archimedes or Galileo.<sup>108</sup> It is also important not to collapse Plato into the Pythagoreans or indeed the Pythagoreans into primitive numerology. As we have seen there were many significant differences and it is not adequate to talk of 'Pythagorean-Platonic number mysticism'. That we can make such a separation is important for the debate about how Pythagorean Plato's account of the natural world is. It is also important in showing that we can, and indeed should make discriminations between the ways that the ancients used number when that use does not match modern usage. Plato pursued what to him would have been open possibilities for the mathematical structure of the cosmos, possibilities which were only closed off with the scientific revolution of the seventeenth century. There was nothing irrational, mystical or unsophisticated about those possibilities or their pursuit in that context.

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**108** Lloyd (1991) p. 351.

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