Innovative-Product Sourcing: Incentives and Commitment

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We consider a buyer firm that sources the design and production of an innovative product from its suppliers. A supplier can improve its design with innovation effort and can potentially produce the design of another supplier, albeit at a switching cost added to the supplier's production cost. A sourcing mechanism allocates the design and production by considering both the design value and production cost. The connection between design value and production allocation provides a source of supplier innovation incentives. This connection may be strengthened by the buyer committing to certain rules in the sourcing mechanism before receiving suppliers' designs, in a trade-off against *ex-post* allocation efficiency. We show that the *no-commitment* mechanism that maximizes allocation efficiency reduces to either *joint sourcing* that selects a single supplier for both design and production or *separate sourcing* that always selects the best design. Interestingly, committing to either rule induces greater supplier effort than no commitment, and combining the two in an *enhanced commitment* that selects the best design and allocates its production to the same supplier incentivizes even greater effort. All three commitment mechanisms are commonly observed in practice and simple to implement. We compare these mechanisms and identify the buyer's choice. We show that separate sourcing is dominated by other mechanisms. Hence, the buyer can restrict commitments to selecting a single supplier for design and production but should be cautious about the selection criteria. An additional innovation prize does not change the choice among these mechanisms but complements strong allocation-based incentive mechanisms.

Key words: innovation; sourcing; procurement mechanism; contest; commitment

1. Introduction

Innovative products are the source of competitive advantage and engine of business growth in rapidly evolving industrial ecosystems (Loch and Kavadias 2008). To reduce time and costs as well as to expand capability and capacity, firms often rely on suppliers to provide innovative products (Eppinger and Chitkara 2006). For instance, in the automotive industry, shortened product life cycles have driven automakers to accelerate innovation by increasingly leveraging suppliers for product design in addition to production (Maurer et al. 2004). Traditional procurement mechanisms mainly focus on sourcing production for already-developed products (see Beil 2010 and Elmaghraby 2000 for reviews), ignoring the innovation process of these products. Such mechanisms aim to choose the best supplier based on metrics such as the cost of producing a developed product. Mechanisms for sourcing innovation solutions (product design), however, focus on suppliers' innovation process, aiming to maximize the value of innovation provided by suppliers (see Ales et al. 2019 for a review), abstracting away from the follow-on production process for the design. In this paper, we take a first step in understanding the sourcing mechanisms when considering both innovation and production.

Sourcing innovative products involves two tasks: sourcing innovative solutions (product design and development) and sourcing the implementation or production of these solutions. Hence, a sourcing mechanism for innovative products must specify how suppliers are selected and compensated for product design and production. Traditionally, different mechanisms are utilized to manage each task. When focusing on design alone, innovation contests emerged as an effective tool to incentivize suppliers to exert costly innovation efforts to generate better designs, by committing to rewarding the supplier that provides the best innovation value based on the buyer's (often subjective) evaluation. When focusing on production alone, procurement auctions are the go-to mechanism to extract suppliers' private cost information and award the production contract to the lowest-cost suppliers. Mechanisms for sourcing innovation and production, however, may require more sophisticated rules than such traditional mechanisms to effectively manage both activities.

The complication is two-fold. First, the supplier selection must consider both the innovation value and production cost, two attributes that are not necessarily aligned – a supplier that provides a better design may or may not have greater production efficiency. Second, the buyer does not have to use the same supplier for both activities and may transfer the design of one supplier to another for production. As a supplier that provides the design usually comes with some advantage in the production know-how, such a transfer will result in an efficiency loss in the form of a *switching cost*. Therefore, a sourcing mechanism for innovative products must consider this switching cost along with design value and production cost. These factors complicate the rules in the mechanism to select suppliers for product design and production while eliciting suppliers' private information to determine compensation.

Another consideration is when to offer the rules in the sourcing mechanism. Offering the rules after suppliers submit their designs allows the buyer to choose the best design-production combination with minimal compensation to the suppliers. Nevertheless, since suppliers' innovation efforts depend on their anticipated return from the sourcing mechanism, committing to certain rules before they invest in the designs could potentially boost suppliers' innovation efforts. Because the innovation process is usually long with highly uncertain outcomes, and the evaluation of the outcome is subjective and non-contractible (Taylor 1995, Chen et al. 2022), it is difficult for firms to write complete contracts of the sourcing mechanism contingent on all possible outcomes, before supplier innovation takes place, to fully govern the selection and trading decisions after the innovation is complete (Hart 1995). Nevertheless, firms may be able to commit to simple and practical rules upfront as part of the mechanism implemented after supplier innovation.

Some practitioners engage suppliers in competitions for design and production separately, combining procurement auctions with innovation contests. This mechanism commits to a rule that selects the best design via a contest and then selects the producer for the design via an auction; we call such a mechanism *separate sourcing*. For instance, the United States defense procurement has emphasized the use of competition in both the development and implementation of innovative products, potentially transferring the design of one supplier to be produced by another supplier.[1] In the private sector, automotive manufacturers such as Ford and Volkswagen engage suppliers in competition for R&D contracts, with the option of replacing the selected supplier with a low-cost supplier to take over the R&D results and bring the project into production (Maurer et al. 2004).

While separate sourcing offers the flexibility of supplier switching, buyers may commit to the opposite, sourcing both design and production from the same supplier; we call such a mechanism *joint sourcing*. For instance, while the European Commission has traditionally adopted separate-sourcing mechanisms in public procurement as a two-phase process, it recently introduced the Innovation Partnership procedure, in which innovation and production are sourced from a single supplier, with the innovator also providing production of the innovative solution (EC 2018). Premium automotive manufacturers such as BMW and Porsche tend to leverage supplier competition in the R&D stage and continue working with the designer for production (Maurer et al. 2004).

While separate sourcing commits to always selecting the best design (regardless of the supplier for production), and joint sourcing commits to using a single supplier for both design and production, an *enhanced-commitment* mechanism commits to a stronger rule that combines both: the supplier that provides the best design will be chosen for both design and production. For instance, NATO allies recently organized a contest that solicited suppliers such as Boeing and Lockheed Martin to develop a next-generation military aircraft where the winner is promised a hefty supply contract (CRS 2020). Similarly, the National Defense Authorization Act for the Fiscal Year 2016 allows government agencies to employ such commitments.

[1] The Competition in Contracting Act of 1984 generally governs competition in federal procurement contracting, requiring contracts be entered into after "full and open competition" while limiting the use of noncompetitive contracts (Manuel 2011). Public organizations such as the Department of Defense have the option to transfer technology from the developer to a second source, thereby enabling suppliers to compete for a production contract even without providing R&D (Riordan and Sappington 1989). The America COMPETES Reauthorization Act of 2010 further authorizes these organizations to utilize innovation contests to source technology and incentivize R&D investment.

Due to the complex interaction of innovation incentives, technology transfer, and procurement-cost efficiency, it is not obvious how exactly such committed rules will affect suppliers' innovation efforts and the overall performance of the sourcing mechanism. Thus, we analyze and compare these rules to investigate the value of commitment when sourcing innovative products. Furthermore, while these rules focus on the selection of suppliers for design and production to affect supplier innovation incentives, a buyer can directly motivate innovation effort by offering a monetary prize for the best design, as is often observed in innovation contests. Therefore, we also study the use of an innovation prize in combination with the commitment mechanisms.

We build a game-theoretic model where a buyer sources an innovative product from two potential suppliers. Suppliers exert innovation efforts to improve their stochastic design values before they privately learn about their heterogeneous cost of production. A supplier can produce another supplier's design by incurring an extra *switching cost*. The buyer's profit consists of the value of the chosen innovative product net of all payments made to suppliers. Thus, the performance of a mechanism hinges on how well it incentivizes suppliers' innovation efforts, and, how efficiently it allocates design and production to suppliers while eliciting suppliers' private cost information to maximize the net profit (hereafter, "allocation efficiency").

We first analyze a *no-commitment* mechanism where the buyer selects the suppliers for the design and production after suppliers complete their innovation process. We show that, depending on the realization of supplier innovation values, the no-commitment mechanism reduces to a *separate-sourcing* or *joint-sourcing* mechanism. In both cases, a higher innovation value improves, though in different ways, a supplier's chances of providing production and earning compensation, thereby incentivizing suppliers' innovation efforts. The switching cost strengthens this positive effect of innovation value on a supplier's payoff, thereby enhancing supplier incentives for innovation efforts.

The flexibility of the no-commitment mechanism to reduce to either separate sourcing or joint sourcing depending on supplier innovation values allows it to maximize *ex-post* allocation efficiency. Yet, we find that committing to either mechanism better incentivizes suppliers' innovation efforts. This is because both separate-sourcing and joint-sourcing commitments improve the chances of a supplier with the better design to win the production and earn compensation; the former does so when the suppliers' innovation value difference is small, and the latter does so when the difference is large. Certainly, the *enhanced-commitment* mechanism, as a combination of both commitments, provides even stronger supplier innovation incentives by guaranteeing the best design to win the production. While such commitments benefit the buyer by motivating greater suppliers' innovation efforts, they hurt allocation efficiency.

We compare the buyer profit when implementing these four mechanisms to understand whether and when commitment is beneficial. We show that when the effort cost is low and the switching cost is high, the separate-sourcing mechanism is superior to joint-sourcing and no-commitment mechanisms due to its advantage in incentivizing supplier efforts. Yet, it is dominated by the enhanced-commitment mechanism in this case, which provides an even higher effort incentive. On the opposite, when the effort cost is high, and the switching cost is low, the three commitment mechanisms are dominated by the no-commitment mechanism, which provides the best allocation efficiency. In other cases, joint sourcing achieves the best performance.

Therefore, while all the commitment mechanisms considered motivate supplier effort, our findings suggest that the buyer can commit to using a single supplier for both design and production, but the buyer should be cautious about how to select this single supplier. When the effort cost is low, and hence supplier effort is highly responsive to incentives, the buyer should adopt enhanced commitment, choosing a single supplier solely based on design value. Otherwise, when the effort and switching costs are both high, the buyer should adopt the joint-sourcing commitment, choosing a single supplier based on both the innovation value and the production cost. When the effort cost is high, and the switching cost is low, the buyer should not commit to any restriction but determine the entire design and production allocation rule after observing supplier innovation values.

While the above mechanisms indirectly incentivize supplier innovation effort through possible compensation for providing production, the buyer can also directly incentivize supplier effort by offering an innovation prize for the best design. We show that such a prize commitment does not change the relative effectiveness of the four mechanisms above, but it could provide an effective additional tool for the buyer when the effort cost is low, to complement a strong allocation-based incentive. In this case, the buyer can benefit from adopting the enhanced-commitment mechanism by promising an extra innovation prize for the best design on top of the production contract.

2. Literature review

Our work combines the sourcing of innovation and production, so it contributes to the innovation contest literature that focuses on sourcing innovation and the procurement literature that focuses on purchasing finished products.

We model the innovation stage in our paper building on the innovation-contest framework pioneered by Terwiesch and Xu (2008) who study how many participants to let in an innovation contest. Also called tournaments, such contest mechanisms award winners with pre-announced fixed prizes. Ales et al. (2017) show that it is optimal to award only the best solution in a broad set of contests, and Mihm and Schlapp (2019) analyze whether and how to provide feedback to participants. Follow-up work utilizes this framework to study different aspects of innovation contests such as whether to run contests internally or externally (Nittala et al. 2022), how to run contests with multiple attributes (Hu and Wang 2021), how to set the duration of a contest (Korpeoglu et al. 2021), whether and when to allow open entry (Ales et al. 2021), whether to allow teamwork (Candoğan et al. 2020), screening in multi-stage contests (Khorasani et al. 2023), and how to effectively run parallel contests (Körpeoğlu et al. 2022, Stouras et al. 2024). Recently Chen et al. (2022) apply this framework to procurement settings to analyze how companies can utilize contests to procure complex innovations for products possibly consisting of multiple attributes.[2]

While these works give valuable insights into how a contest organizer can source the best innovation from a group of suppliers, they abstract away from how this innovation will be implemented or assume that the supplier with the best innovation will also perform the production (i.e., enhanced commitment). Thus, we contribute to this literature by considering the sourcing of both innovation and production. Our results show that the enhanced-commitment mechanism assumed by the above papers is justified when the innovation cost is small. When the innovation cost is large, however, the buyer is better off making the decision of which supplier to choose for design and/or production after considering both the innovation values and production costs of suppliers.

Though fixed-prize innovation contests as shown in the above papers are common for sourcing innovation, a number of papers study the use of auctions in the contest mechanism. An auction allows suppliers to bid their prizes, and the buyer selects the winner based on the combination of suppliers' innovation values and prize bids. Fullerton and McAfee (1999) establishes that a first-price auction mechanism generally reduces the buyer's prize expenditure relative to a fixed-prize contest. Che and Gale (2003) analyze the optimal auction mechanism in a general form of menu prizes that can be reduced to a first-price auction or a fixed-prize contest, assuming a deterministic innovation outcome. Considering a different type of innovation technology, Schöttner (2008) shows that a contest may or may not outperform a first-price auction depending on the distribution of suppliers' innovation values. While these three papers consider only the innovation task, Che et al. (2021) further incorporate suppliers' production costs, as in our paper. They assume the innovation values are verifiable and contractible, allowing the buyer to specify contract award rules based on the innovation values to realize, *before* suppliers choose their innovation efforts. In this case, as shown in their main model assuming a single innovator (along with multiple production suppliers), supplier innovation competition is *not* essential to induce supplier effort. However, when it comes to sourcing innovation, lack of technical knowledge by the buyer or the subjective nature of the value of a design makes it difficult to specify a certain metric for evaluation that can be verifiable in court (e.g., Taylor 1995, Chen et al. 2022).[3]

[2] Some contest papers utilize other model frameworks to study problems such as how to delegate search for alternative designs (Erat and Krishnan 2012) and the optimal award scheme when heterogeneous solvers perform deterministic tasks (Stouras et al. 2022). See Segev (2020) for a review of the contest literature.

[3] Che et al. (2021) admit this verifiability issue as follows: "In practice, however, it may be difficult for a buyer to commit to a contract contingent on the value of an innovation, as this requires an evaluation procedure that can be reviewed and sanctioned by courts" (page 2167). For this reason, they have a section where they relax the verifiability assumption, albeit only for the base model where a single supplier can innovate, making the design selection moot. In our work, multiple suppliers can innovate, in which case the buyer can utilize competition among suppliers to incentivize innovation efforts.

Thus, more consistent with innovation-sourcing practice, we consider the case where the innovation value is non-verifiable (though observable by the buyer), prohibiting the parties from writing contracts contingent on the value. In this case, the buyer utilizes a relative ranking of suppliers based on innovation values, possibly combined with production costs, to determine which suppliers to work with, making supplier innovation competition an essential element to incentivize innovation efforts. Not only complicating the problem, this approach allows us to compare different practically-relevant mechanisms to source innovation and production, and generate valuable managerial insights about when to use each mechanism.

The procurement literature studies mechanisms for buyers to purchase from suppliers with unobservable characteristics (such as costs). These mechanisms typically use supplier competition or contract menus to reveal private information and extract surplus from suppliers (e.g., Laffont and Tirole 1987, Rogerson 2003, and Perry and Sákovics 2003). See Laffont and Tirole (1993) and Elmaghraby (2000) for reviews of procurement theories and sourcing mechanisms. In our paper, the buyer faces a similar problem in the procurement stage after suppliers' design values and production costs are realized. However, we consider also the innovation stage preceding the procurement stage, in which suppliers invest in innovation that affects their design values, anticipating the payoff from the procurement stage depending on the design values.

There are papers in the procurement literature studying suppliers' investment in cost reduction before they compete on costs, including Piccione and Tan (1996), Dasgupta (1990), Tan (1992), Li (2013), and Li and Wan (2017). Unlike these papers, we consider supplier investment in innovation that enhances the product value for the buyer, with exogenous and heterogeneous production costs. Therefore, the procurement mechanism must consider both the product values and production costs of suppliers. Though both a higher value and a lower cost improve a supplier's competitiveness, they impose different information and economic structures: the product value is observable by the buyer and the production cost is not. Whereas the product value is specific to the buyer's market, the production cost detracts from a supplier's profit.

A stream of papers considers the cost-reduction investment of a single incumbent supplier, who competes with entrant suppliers in a second-sourcing mechanism after the investment; see, e.g., Riordan and Sappington (1989), Stole (1994), and Rob (1986). By allowing the buyer to switch from the incumbent to an entrant supplier, a second-sourcing mechanism improves the buyer's allocation efficiency, but it hurts the incumbent's *ex-ante* incentive to invest in improvement. In our study, all suppliers can invest in improvement (of innovation values) when competing to provide the design; this feature, common in the practice of innovative-product sourcing, imposes a different incentive structure.

3. Model and Preliminaries

We build a parsimonious model that captures the main aspects of our problem while preserving tractability. A buyer (she) sources the design (innovation solution) and production of an innovative product from two symmetric risk-neutral suppliers (he) indexed $i \in \{1,2\}$ and $j = 3 - i$.

Innovation model. We follow the setup that is common in the innovation contest literature (e.g., Mihm and Schlapp 2019, Hu and Wang 2021): each supplier i exerts innovation effort $e_i \in [0,\infty)$ by incurring a cost $\psi(e_i) = ce_i^2$, where $c > 0$ determines the marginal cost of innovation effort. The value of a supplier's design depends on his effort and is also subject to uncertainty. Specifically, given effort e_i, the value of a supplier's design is $v_i = m + e_i + \xi_i$, where m is a constant that represents the base value of innovation to the buyer and ξ_i is a random shock drawn from a uniform distribution on $[-d, +d]$. Consistent with practice, the innovation value v_i depends on the subjective taste of the buyer, and a supplier's investment in innovation e_i involves complex activities that are difficult to specify in a contract. So, neither a supplier's innovation effort nor the value generated with the effort is verifiable or contractible. As a result, the parties are prohibited from writing contracts contingent on efforts or innovation values to incentivize innovation efforts.

Production model. Suppliers' production costs are *ex-ante* uncertain and are realized after the innovation stage. The uncertainty captures the variations in a supplier's internal and external environments (such as financial status, productivity, upstream suppliers, and logistics) that may occur over time when the innovation effort takes place, affecting the supplier's cost efficiency. In our base model, we assume that the production costs are independent of the design quality (innovation value), considering designs focusing on product architecture that affects product performance and function but not production costs. An extension considering the correlation between product design and production costs is presented in the Online Appendix.[4] The production cost of supplier i, c_i, is a random draw from a uniform distribution $U(\underline{c}, \bar{c})$, with $\Delta \equiv \bar{c} - \underline{c}$. Hence, the cumulative distribution function and density function of the production cost are $F(c_i) = (c_i - \underline{c})/\Delta$ and $f(c_i) = 1/\Delta$ (when $c_i \in [\underline{c}, \bar{c}]$), respectively. The prior distribution $U(\underline{c}, \bar{c})$ is common knowledge, while the realized costs c_i are private information of each supplier i. Therefore, suppliers face the same ex-ante production costs, though their ex-post costs are different. To use in the procurement mechanism design, we define a supplier's virtual production cost as follows.

Definition 1. The virtual production cost is defined as $C(c_i) = c_i + \frac{F(c_i)}{f(c_i)}$.

[4] In a general situation, a better design may require more expensive materials or special tools, leading to a higher production cost. Such a correlation can be incorporated by deducting the correlated part of production cost from the innovation value without changing the model. In §EC.4 of the Online Appendix, we model such correlation explicitly and show the robustness of our main results.

The buyer can potentially delegate a supplier's design to another supplier for production, but there is a *switching cost* of $l > 0$ for a supplier to produce the design of another supplier. This switching cost can arise because the supplier who provides the design builds know-how that makes it easier for him to implement his own design than any other supplier (see Stole 1994, Rob 1986, Riordan and Sappington 1989). This know-how advantage can come in the form of expertise in advanced technologies necessary to produce cutting-edge designs. It can also refer to intangible nuances that cannot be fully outlined in a design but require the designer and producer to work closely during the implementation process. Thus, the actual production cost of supplier i is $c_i + l$ (the virtual production cost of supplier i is $C(c_i) + l$) if the design comes from another supplier. To ensure that there is meaningful competition among suppliers when the buyer is procuring production, we assume the following:

ASSUMPTION 1. *$m - d - l > C(\bar{c})$, $l \leq 2\Delta$.*

The first part of this assumption ($m - d - l > C(\bar{c})$) ensures that the innovation generates sufficient value for the buyer (i.e., $v > l + C(\bar{c})$ for any realization of v) so that neither supplier needs to be cut off from procurement, regardless of their production cost realization (see Li 2013). It is easily satisfied when the expected base value m of an innovation to the buyer is sufficiently large. The second part of this assumption ($l \leq 2\Delta$) guarantees that supplier switching is possible, where 2Δ is the largest difference between suppliers' virtual production costs. We relax this requirement ($l \leq 2\Delta$) in extensions and proofs of our results in the Online Appendix.

Sequence of events. The buyer determines a sourcing mechanism that specifies the rules for selecting the design and producer, as well as awarding suppliers with monetary compensation. It is difficult for a buyer to commit to a complex sourcing mechanism that outlines the complete rules based on innovation values and production costs, before suppliers' uncertain and sometimes lengthy innovation process, to govern transactions in the future after the innovation conclusion (Hart 1995). Instead, the buyer can commit to some practical and easy-to-implement rules or choose to make no commitment before supplier innovation. Then, informed of the committed rules, suppliers choose their innovation effort and submit designs.[5] After observing suppliers' design values $\mathbf{v} \equiv (v_1, v_2)$, the buyer presents a sourcing mechanism subject to the pre-committed rules. Given the mechanism, suppliers bid their costs, and then the buyer allocates design and production to suppliers and compensates them. Figure 1 summarizes the sequence of events.

[5] By submitting a design, each supplier agrees to give the buyer the right to use his design in return for a chance to win the production contract. Such a policy, for instance, is not uncommon in ideation and design contests where the buyer commits to pay compensation to at least one supplier in exchange for the perpetual rights to use any submitted idea or design (cf. Ales et al. 2021).

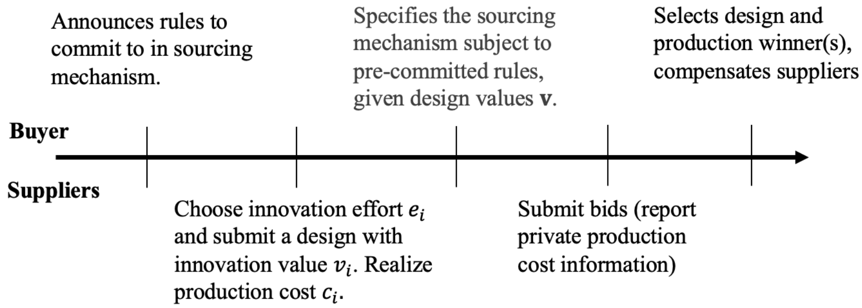


Figure 1 Timeline of the model.

Formulation of sourcing mechanism. To characterize the buyer's optimal sourcing mechanism given suppliers' innovation values $\mathbf{v}$, we adopt the optimal mechanism design framework (Myerson 1981) to extract suppliers' privately-known production costs $\mathbf{c} \equiv (c_1, c_2)$. Specifically, given $\mathbf{v}$, the mechanism specifies for each supplier $i = 1, 2$ a menu of contracts parameterized by the suppliers' (reported) production costs $\mathbf{c}$: $(x_i(\mathbf{c}), y_i(\mathbf{c}), t_i(\mathbf{c})|\mathbf{v})$, where $x_i(\mathbf{c})$ is the production-selection rule (the probability that supplier i is selected to provide production), $y_i(\mathbf{c})$ is the design-selection rule (the probability that supplier i's design is selected), and $t_i(\mathbf{c})$ the compensation rule (the payment to supplier i), all depending on $\mathbf{v}$. After the buyer announces the sourcing mechanism (menus of contracts), suppliers report their production costs, and the contracts corresponding to suppliers' reported costs from the menus are then awarded to suppliers.

As standard in optimal mechanism design (Myerson 1981), the buyer determines menus of contracts to maximize her expected profit, subject to incentive compatibility (IC) constraints ensuring that suppliers report their true production costs and individual rationality (IR) constraints ensuring that suppliers receive non-negative expected profits from participation.[6] The menus of contracts and associated variables take suppliers' innovation values $\mathbf{v}$ as inputs. For brevity, we may omit $\mathbf{v}$ from the parameters where the dependency is clear.

Given the menu of contracts $(x_i(\mathbf{c}), y_i(\mathbf{c}), t_i(\mathbf{c}))$, by reporting production cost $\hat{c}_i$, a supplier i with actual cost c_i receives expected utility $\hat{u}_i(\hat{c}_i, c_i) \equiv \mathbb{E}_{c_j}\left[t_i(\hat{c}_i, c_j) - x_i(\hat{c}_i, c_j)(c_i + l(1 - y_i(\hat{c}_i, c_j)))\right]$. Supplier i's expected utility from reporting true cost is $u_i(c_i) \equiv \hat{u}_i(c_i, c_i)$. Thus, the IC constraints and the IR constraints for the sourcing mechanism, respectively, are

$$u_i(c_i|\mathbf{v}) \geq \hat{u}_i(\hat{c}_i, c_i|\mathbf{v}), \quad i \in \{1, 2\}, \tag{1}$$

$$u_i(c_i|\mathbf{v}) \geq 0, \quad i \in \{1, 2\}. \tag{2}$$

[6] We assume suppliers' reservation profit is zero. Therefore, a supplier will participate in this sourcing mechanism as long as their expected profit, excluding the sunk innovation effort cost, is non-negative.

The buyer maximizes her expected profit, which can be calculated as the expected total social surplus $\sum_{i=1}^{2}\mathbb{E}_{\mathbf{c}}\left[v_i y_i(\mathbf{c}) - x_i(\mathbf{c})(c_i + l(1-y_i(\mathbf{c})))\right]$ net of the suppliers' total expected utility $\sum_{i=1}^{2}\mathbb{E}_{c_i}[u_i(c_i)]$. Therefore, the buyer's optimization problem can be written as:

$$
\begin{aligned}
\max_{(x_i(\mathbf{c}),y_i(\mathbf{c})),i\in\{1,2\}} \; & \sum_{i=1}^{2}\mathbb{E}_{\mathbf{c}}\left[v_i y_i(\mathbf{c}) - x_i(\mathbf{c})(c_i + l(1-y_i(\mathbf{c})))\right] - \sum_{i=1}^{2}\mathbb{E}_{c_i}[u_i(c_i)] \\
s.t. \quad & \sum_{i=1}^{2} x_i(\mathbf{c}) \le 1 \quad \text{and} \quad x_i(\mathbf{c}) \ge 0, \quad \forall \mathbf{c}, \\
& \sum_{i=1}^{2} y_i(\mathbf{c}) \le 1 \quad \text{and} \quad y_i(\mathbf{c}) \ge 0, \quad \forall \mathbf{c}, \\
& \sum_{i=1}^{2} x_i(\mathbf{c}) \le \sum_{i=1}^{2} y_i(\mathbf{c}), \\
& (1), (2), \text{ and } \; \textit{Commitment Constraints.}
\end{aligned}
\tag{3}
$$

The commitment constraints will depend on the specific rules that the buyer commits to before supplier innovation takes place. In the following, we first study the *no-commitment* mechanism under which the buyer specifies the full sourcing mechanism *after* suppliers invest in innovation and submit their designs, without committing to any rules that constrain the mechanism. This study leads to the consideration of several commitment mechanisms that we shall analyze later.

4. No-Commitment Mechanism

In the no-commitment mechanism, the buyer makes no commitment before supplier innovation. Using backward induction, we shall analyze the buyer's mechanism based on realized innovation values and then suppliers' effort decisions in the innovation stage. We shall denote the variables corresponding to the no-commitment mechanism by subscript N.

Mechanism Design. The buyer solves the optimization problem (3) without any commitment constraints. We summarize the optimal sourcing mechanism in the following proposition.

PROPOSITION 1. *The optimal no-commitment mechanism is described as follows:*

(i) The optimal design selection and production allocation rules for suppliers i and j are:

(a) *If* $|v_i - v_j| \ge l$, $y_i(\mathbf{c}) = \mathbf{1}_{v_i \ge v_j}$, $y_j(\mathbf{c}) = 1 - y_i(\mathbf{c})$, $x_i(\mathbf{c}) = \mathbf{1}_{C(c_i)+ly_j \le C(c_j)+ly_i}$, *and* $x_j = 1 - x_i$.

(b) *If* $|v_i - v_j| < l$, $y_i(\mathbf{c}) = x_i(\mathbf{c}) = \mathbf{1}_{v_i - C(c_i) \ge v_j - C(c_j)}$ *and* $y_j(\mathbf{c}) = x_j(\mathbf{c}) = 1 - y_i(\mathbf{c})$.

(ii) Suppliers i and j receive the following expected utility conditional on design values:

$$U_{iN} = \mathbb{E}_{c_i}[(C(c_i) - c_i)\mathbb{E}_{c_j}(x_i(\mathbf{c}))], \quad U_{jN} = \mathbb{E}_{c_j}[(C(c_j) - c_j)\mathbb{E}_{c_i}(x_j(\mathbf{c}))]. \tag{4}$$

When $v_i > v_j$, we have $U_{iN} \ge U_{jN}$, where U_{iN} is (weakly) increasing and U_{jN} is (weakly) decreasing in l.

(iii) The buyer's expected profit based on design values is

$$U_N(\mathbf{v}) = \mathbb{E}_{\mathbf{c}}[(v_i - l x_j(\mathbf{c}))y_i(\mathbf{c}) + ((v_j - l x_i(\mathbf{c})))y_j(\mathbf{c})] - \mathbb{E}_{c_i}[C(c_i)\mathbb{E}_{c_j}[x_i(\mathbf{c})]] - \mathbb{E}_{c_j}[C(c_j)\mathbb{E}_{c_i}[x_j(\mathbf{c})]]. \tag{5}$$

$U_N(\mathbf{v})$ *is increasing in* v_i *and* v_j *and (weakly) decreasing in* l.

Proposition 1(i) shows that the rule to award design and production depends on the suppliers' differences in innovation values and (virtual) production costs. When one supplier has enough advantage in innovation value to offset switching cost ($|v_i - v_j| > l$), the buyer always selects the better design provided by suppliers and gives it to the supplier with lower total costs (including virtual cost and switching cost) for production; in other words, the buyer adopts the *separate sourcing* rule. In this case, the supplier with the better design gains an advantage in production allocation due to the switching cost – the other supplier can only win production if he has a sufficiently lower production cost to offset the cost of switching. When the difference in suppliers' innovation values is smaller than the switching cost ($|v_i - v_j| < l$), it is never beneficial for the buyer to switch suppliers. In this case, the supplier who provides the highest profit margin ($v_i - C(c_i)$) is selected for both design and production, giving rise to *joint sourcing*. With this rule, again the supplier with the better design gains an advantage because the other supplier can only win if he has a sufficiently lower production cost to offset the innovation value gap. Therefore, depending on the values of the designs submitted by suppliers, the no-commitment mechanism reduces to either the separate-sourcing or joint-sourcing allocation rule.

Proposition 1(ii) characterizes suppliers' expected utility. As suppliers hold private information on their production cost, they receive information rent when providing production. As such, the supplier utility in (4) is determined by the supplier's probability of being awarded the production (i.e., $\mathbb{E}_{c_j}(x_i(\mathbf{c}))$). This probability depends on the production selection rule $x_i(\mathbf{c})$ characterized in Proposition 1(i). As $x_i(\mathbf{c})$ increases with v_i, so does supplier i's utility. Furthermore, when supplier i provides a better design than supplier j ($v_i > v_j$), supplier i intuitively obtains a higher utility than supplier j; i.e., $U_{iN} \geq U_{jN}$. In addition, since a higher switching cost l improves the chance of the supplier with a better design to also be awarded the production, a larger l increases U_{iN} while decreasing U_{jN}, enlarging a supplier's benefit from providing a better design.

Proposition 1(iii) shows, intuitively, that the buyer's expected profit U_N increases with suppliers' innovation values $\mathbf{v}$. Furthermore, given $\mathbf{v}$, a larger switching cost l lowers U_N because it inflates the effective cost of using a different supplier for production.

Recall that innovation values $\mathbf{v}$ are assessed subjectively by the buyer, so they are neither verifiable nor contractible. This raises the question of whether the buyer will truthfully reveal the innovation value of each supplier in the procurement stage. In §EC.1 of the Online Appendix, we show that the buyer does not have any incentive to manipulate suppliers' innovation values because the buyer uses innovation values only for supplier selection but not for determining the size of compensation (the latter is based solely on production costs). Thus, even though innovation value is not verifiable, the buyer is truthful in revealing it.

Supplier Innovation Effort. Using Proposition 1(ii), supplier i's expected profit given innovation efforts $\mathbf{e} = \{e_1, e_2\}$ can be written as $\pi_{iN}(\mathbf{e}) = \mathbb{E}_{\mathbf{v}}[U_{iN}(\mathbf{v})|\mathbf{e}] - \psi(e_i)$. The (pure-strategy Nash) equilibrium effort is defined as $\mathbf{e}^*_{\mathbf{N}} = \{e^*_{1N}, e^*_{2N}\}$ such that $e^*_{iN} = \arg\max_{e_i} \pi_{iN}(e_i, e^*_{jN})$ for $i \in \{1,2\}$ and $j = 3-i$. To ensure that such an equilibrium exists, we make the following mild assumption.

Assumption 2. $c > C_0 \equiv \frac{\Delta}{16d^2}$.

Assumption 2 ensures the existence of a unique pure strategy Nash equilibrium and is satisfied when the innovation uncertainty d is sufficiently high. Similar assumptions are commonly used in the innovation contest literature where supplier utility depends on the relative ranking of innovation values (e.g., Mihm and Schlapp 2019, Ales et al. 2021) as in our paper.[7]

Lemma 1. *In the no-commitment mechanism, each supplier exerts the equilibrium effort*

$$e^*_N = \begin{cases} \frac{(3\Delta - d)}{24c\Delta} & \text{if } l > 2d, \\ \frac{l(3d(4\Delta - l) - l(3\Delta - l))}{96cd^2\Delta} & \text{if } l \leq 2d, \end{cases} \tag{6}$$

which is increasing in l when $l \leq 2d$ and constant in l otherwise, and is decreasing in c.

Lemma 1 intuitively shows that a higher cost of innovation effort c always dampens the supplier's equilibrium effort e^*_N. Lemma 1 further shows that e^*_N is increasing in the switching cost $l \leq 2d$. This is because increasing l improves the utility of a supplier with the higher innovation value and reduces the utility of a supplier with the lower innovation value (see Proposition 1(ii)), and both of these effects boost the marginal return of effort for a supplier. When $l > 2d$, the buyer will always select joint sourcing because the innovation value difference in equilibrium cannot justify the switching cost. Therefore, the equilibrium effort is independent of $l > 2d$.

Buyer Profit. We shall next discuss how the buyer's *ex-ante* expected profit at the start of the innovation stage, $\Pi^*_N = \mathbb{E}_{\mathbf{v}}[U_N(\mathbf{v})|e^*_N]$, changes with the switching cost l.

Proposition 2. *Π^*_N is independent of l when $l \geq 2d$. For $l < 2d$, Π^*_N is increasing in l when $c \leq \frac{1}{4d}$, and is first decreasing and then increasing in l when $c > \frac{1}{4d}$.*

Proposition 2 shows that the impact of the switching cost l ($< 2d$) on the buyer's expected profit Π^*_N is non-trivial because l has two opposing effects on Π^*_N. A larger l increases *effort incentive* and induces suppliers to exert more effort (Lemma 1), but it also reduces *allocation efficiency* by increasing total production cost, including switching cost. Which of these effects dominates depends on the effort cost parameter c. When c is small ($\leq \frac{1}{4d}$), supplier effort is more responsive to effort incentive. In this case, l's positive effect on supplier effort outweighs its negative impact on the total production cost, so the buyer's profit increases with l. However, when c is large ($> \frac{1}{4d}$),

[7] Because the production allocation inherently depends on whether $v_i > v_j$ or not, supplier utility naturally depends on whether supplier i ranks first or second in terms of innovation values.

the opposite may happen as the positive effect of l on supplier effort is weaker. Meanwhile, the impact of higher l on the total production cost is more pronounced when supplier switching is more likely, which is the case when l is small. Therefore, increasing l, when it is small, harms allocation efficiency more than it increases effort incentive, reducing the buyer's profit. Consequently, when c is large, the buyer's expected profit Π_N^* first decreases and then increases with l.

5. The Value of Commitment

As shown in Proposition 1, the ex-post optimal no-commitment mechanism has an allocation rule reducing to separate sourcing or joint sourcing depending on the realization of suppliers' innovation values. In practice, we observe organizations committing to such rules, implementing separating-sourcing or joint-sourcing commitment mechanisms (see §1 for examples). In this section, we analyze these two commitment mechanisms (§5.1 and §5.2) and compare them with no commitment (§5.3) to understand whether and when to choose such commitments. We shall denote variables corresponding to separate-sourcing and joint-sourcing mechanisms by subscripts S and J.

5.1. Separate-Sourcing Mechanism

The separate-sourcing mechanism differs from the no-commitment mechanism by committing to choosing the best design, independent of suppliers' production costs; that is, this mechanism inserts a constraint $y_i(\mathbf{c}) = \mathbf{1}_{v_i \geq v_j}$ in the mechanism design problem (3). This mechanism has a simple implementation: it effectively runs an *innovation contest* to select the best design, followed by a *procurement auction* to select the supplier to produce this design. Proposition 3 formalizes the mechanism design for the separate-sourcing mechanism.

PROPOSITION 3. *In the optimal separate-sourcing mechanism, supplier selection rules $(x_i(\mathbf{c}), y_i(\mathbf{c})|\mathbf{v})$ and the associated compensation rule are the same as in Proposition 1(i)(a). Supplier i's expected utility $U_{iS}(\mathbf{v})$ and the buyer's expected profit $U_S(\mathbf{v})$ conditional on design values have the same expressions as in Proposition 1(ii) and (iii), respectively.*

As also stated in §4, the separate-sourcing mechanism coincides with the optimal no-commitment mechanism when two suppliers' innovation values difference is larger than the switching cost (i.e., $|v_i - v_j| \geq l$). Yet, these mechanisms differ when $|v_i - v_j| < l$, in which case the no-commitment mechanism switches to a joint-sourcing strategy, whereas the separate-sourcing mechanism sticks to selecting the best design due to the commitment beforehand. As a result, this commitment compromises *ex-post* allocation efficiency by myopically selecting the best design regardless of suppliers' production costs. In particular, when the supplier with the better design has a much higher production cost than the other supplier, with a high switching cost that prohibits supplier switching, the advantage of choosing the better design in the innovation value is dominated by the disadvantage in the design supplier's production cost.

By constraining supplier selection, this commitment affects suppliers' profits from the sourcing mechanism based on their innovation values, thereby affecting suppliers' *ex-ante* incentives for innovation efforts. Specifically, when innovating, each supplier $i \in \{1,2\}$ chooses his innovation effort to maximize his expected profit $\pi_{iS}(\mathbf{e}) = \mathbb{E}_{\mathbf{v}}[U_{iS}(\mathbf{v})|\mathbf{e}] - \psi(e_i)$. Given this formulation, Lemma 2 derives the equilibrium of suppliers' innovation effort.

LEMMA 2. *There exists a unique pure strategy Nash equilibrium in the separate-sourcing mechanism, where each supplier exerts the equilibrium effort*

$$e_S^* = \frac{l(4\Delta - l)}{32c\Delta d}, \tag{7}$$

which is increasing in the switching cost l.

As in Lemma 1, the supplier effort increases in the switching cost l. This is not surprising: As a higher switching cost increases the chance of the supplier with the better design to also win the production, it improves suppliers' return of investment in innovation, hence inducing greater effort.

Given the buyer's expected utility from the procurement auction $U_S(\mathbf{v})$, her *ex-ante* expected profit in the separate-sourcing mechanism is $\Pi_S^* = \mathbb{E}_{\mathbf{v}}[U_S(\mathbf{v})|e_S^*]$.

PROPOSITION 4. *When $c \le \frac{1}{4d}$, Π_S^* is increasing in l; and when $c > \frac{1}{4d}$, Π_S^* is first decreasing and then increasing in l.*

Proposition 4 shows the same directional effects of the switching cost l on the buyer profit as in Proposition 2 (the case when $l \le 2d$), with a similar intuition. The effects stem from the lower allocation efficiency and higher effort incentive associated with larger l. Unlike Proposition 2, this effect persists for $l > 2d$ because of the separate-sourcing commitment.

5.2. Joint-Sourcing Mechanism

The joint-sourcing mechanism commits to delegating design and production to the same supplier, by imposing a constraint $x_i(\mathbf{c}) = y_i(\mathbf{c})$ in the mechanism design problem (3). This mechanism can be implemented as a procurement auction in which suppliers are discriminated by innovation values: while suppliers bid their production costs, they are compared on the production cost adjusted by the innovation value to select the winner (see McAfee and McMillan (1989) for the design of such discriminatory auctions). Proposition 5 characterizes the joint-sourcing mechanism.

PROPOSITION 5. *In the optimal joint-sourcing mechanism, supplier selection rules $(x_i(\mathbf{c}), y_i(\mathbf{c})|\mathbf{v})$ and the associated supplier compensation rule are the same as in Proposition 1(i)(b). Supplier i's expected utility $U_{iJ}(\mathbf{v})$ and the buyer's expected profit $U_J(\mathbf{v})$ conditional on design values have the same expressions as in Proposition 1(ii) and (iii), respectively.*

As also stated in §4, the optimal joint-sourcing mechanism coincides with the optimal no-commitment mechanism corresponding to the case when the innovation value difference is smaller than the switching cost (i.e., $|v_i - v_j| < l$). Yet, these two mechanisms differ when $|v_i - v_j| \geq l$, in which case the no-commitment mechanism turns to selecting the best design and giving it to the lower-cost supplier (switching cost considered) for production. However, the joint-sourcing mechanism is bound to sourcing both design and production from a single supplier without switching, which may lead to *ex-post* allocation inefficiency when l is small. Nevertheless, this commitment will provide extra *ex-ante* incentives for suppliers to exert effort, as we shall analyze next.

When innovating, each supplier i chooses his innovation effort e_i to maximize his expected profit $\pi_{iJ}(\mathbf{e}) = \mathbb{E}_{\mathbf{v}}[U_{iJ}(\mathbf{v})] - \psi(e_i)$. Lemma 3 characterizes the symmetric pure strategy Nash equilibrium for the joint-sourcing mechanism.

LEMMA 3. *There exists a unique symmetric pure strategy Nash equilibrium in the joint-sourcing mechanism, where each supplier exerts the equilibrium effort*

$$e_J^* = \frac{(3\max\{d,\Delta\} - \min\{d,\Delta\})\Delta}{24c\max\{d,\Delta\}^2}. \tag{8}$$

Using the equilibrium effort in (8) and the buyer's expected utility $U_J(\mathbf{v})$ from the sourcing mechanism, we can calculate the buyer's *ex-ante* expected profit as $\Pi_J^* = \mathbb{E}_{\mathbf{v}}[U_J(\mathbf{v})|e_J^*]$.

5.3. Comparison results

To derive the impact of the buyer's commitment, we compare the no-commitment mechanism with joint-sourcing and separate-sourcing mechanisms. We start by comparing these mechanisms in terms of suppliers' equilibrium effort characterized in (6), (7), and (8), followed by the comparison of the buyer's expected profit. The comparison of the supplier effort is summarized in Proposition 6 and illustrated in Figure 2.

PROPOSITION 6. *The equilibrium efforts e_J^*, e_S^*, and e_N^* under joint-sourcing, separate-sourcing, and no-commitment mechanisms can be compared as follows:*

(i) $e_J^* \geq e_N^*$ and $e_S^* \geq e_N^*$.

(ii) $e_S^* - e_J^*$ is increasing in the switching cost l, and $e_S^* > e_J^*$ if and only if l is sufficiently high.

As illustrated in Figure 2, Proposition 6 (i) shows that the no-commitment mechanism yields less effort than the other two commitment mechanisms. Therefore, even though the no-commitment mechanism enjoys the flexibility of reducing to either separate or joint sourcing depending on supplier innovation values, committing to either mechanism provides a greater incentive for suppliers to invest in innovation to improve their design values. Compared to no commitment, both mechanisms enhance the advantage of the supplier of the better design in the production allocation. With the separate-sourcing commitment, that advantage is equal to the switching cost, even when the

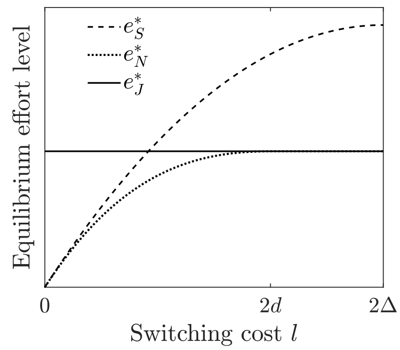


Figure 2 Equilibrium efforts under no-commitment (e_N^*), separate-sourcing (e_S^*), and joint-sourcing (e_J^*) mechanisms. Setting: $\Delta = 6, d = 4, c = 0.1$.

difference in suppliers' innovation values is too small to justify switching. With the joint-sourcing commitment, that advantage is equal to the gap of innovation values, even when it is large enough to offset the switching cost.

Proposition 6 (ii) compares efforts in the two commitment mechanisms. Because e_S^* is increasing in the switching cost l (Lemma 2) while e_J^* is independent of l (Lemma 3), $e_S^* - e_J^*$ is increasing in l and becomes positive for large l. Therefore, the separate-sourcing commitment mechanism has the most advantage over the other two mechanisms in supplier effort when the switching cost is high.

Next, in Proposition 7, we compare the buyer's profits from using each mechanism under different values of effort cost c and switching cost l. The results are illustrated in Figure 3.

Proposition 7. *There exist thresholds C_1 and C_2 ($C_1 < C_2$) such that the buyer's expected profit in the no-commitment mechanism Π_N^*, separate-sourcing mechanism Π_S^*, and the joint-sourcing mechanism Π_J^* can be compared as follows:*

(a) When $c < C_1$, there exists a threshold $l_{SJ} > 0$ increasing in c such that Π_S^* is the highest if $l > l_{SJ}$, and Π_J^* is the highest otherwise.

(b) When $c \in [C_1, C_2]$, Π_J^* is always the highest.

(c) When $c > C_2$, there exists a threshold $l_{JN} > 0$ increasing in c such that Π_J^* is the highest if $l > l_{JN}$, and Π_N^* is the highest otherwise.

Proposition 7, as illustrated in Figure 3, shows that when the effort cost parameter c is small and the switching cost l is large, the separate-sourcing mechanism dominates, when c is large and l is small, the no-commitment mechanism dominates, and in other cases, the joint-sourcing mechanism dominates. This result stems from the trade-off between supplier effort and allocation efficiency.

When the effort cost parameter c is small, supplier effort is highly responsive to provided incentives, so the mechanism that offers the best effort incentive leads to the highest buyer profit. Following Proposition 6, when the switching cost l is large, the separate-sourcing commitment is

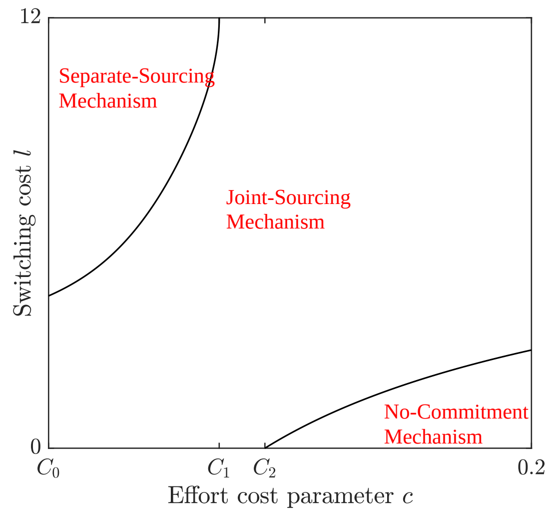


Figure 3 Buyer's optimal choice among no-commitment, joint-sourcing, and separate-sourcing mechanisms. Setting: $\Delta = 6, d = 4$. Effort cost c starts from C_0 by Assumption 2, where $C_1 = 0.09$ and $C_2 = 0.1$. The solid curves set boundaries for the optimal choice of mechanisms as indicated in the labels.

more effective than the other mechanisms at incentivizing supplier effort. Therefore, the separate-sourcing mechanism dominates the other two when the supplier effort cost is small ($c < C_1$) and the switching cost is large ($l > l_{SJ}$). When l is small, however, the joint sourcing mechanism provides a better effort incentive and hence dominates the other two.

When c is large, supplier effort is less sensitive to effort incentive, so allocation efficiency plays a bigger role in comparing the buyer's profit. When l is small, prohibiting supplier switching leads to significant allocation inefficiency for the joint-sourcing mechanism. Although the separate-sourcing mechanism has decent allocation efficiency in this region, it is still less efficient than the no-commitment mechanism. Thus, the no-commitment mechanism dominates the other two mechanisms when c is small ($c > C_2$) and l is low ($l < l_{JN}$). When the switching cost l is high, however, the joint-sourcing mechanism achieves a good balance – it suffers little allocation inefficiency while still enjoying a significant advantage in supplier effort incentive compared to no-commitment, hence dominating the other two mechanisms.

Our findings show that commitment to prohibiting supplier switching (joint sourcing) or to selecting the best design (separate sourcing) may improve upon the no-commitment mechanism by eliciting greater innovation effort from suppliers. To further study the power of commitment, we next analyze an *enhanced-commitment* mechanism that combines these two commitments, selecting the best design and giving the production to the same supplier.

6. Enhanced-Commitment Mechanism

In the enhanced commitment mechanism, the buyer always selects the best design, as in separate sourcing, and uses the designer for production without supplier switching, as in joint sourcing.

In other words, the enhanced commitment imposes the constraints $x_i(\mathbf{c}) = y_i(\mathbf{c}) = \mathbf{1}_{v_i \geq v_j}$ in the mechanism design problem (3). Due to this double commitment, the buyer cannot use production allocation to extract suppliers' private cost information; as a result, the buyer compensates the supplier for production based on the highest production cost, $\bar{c}$. Thus, this mechanism can be implemented by an innovation contest with a winner prize of $\bar{c}$ (e.g., Ales et al. 2021, Chen et al. 2022). We next characterize the equilibrium and buyer profit in the enhanced commitment mechanism, using the subscript H to denote its variables.

Proposition 8. *In the equilibrium outcome of the enhanced-commitment mechanism, suppliers will exert the equilibrium effort* $e_H^* = \frac{\Delta}{8cd}$*, and the buyer's expected profit is* $\Pi_H^* = e_H^* + m - \underline{c} + \frac{d}{3} - \Delta$.

Proposition 9 below shows that suppliers exert the greatest effort in the enhanced-commitment mechanism compared to the other mechanisms. This is not surprising as this mechanism combines the commitments made in the separate-sourcing and joint-sourcing mechanisms, hence making it more lucrative for a supplier to achieve the best design.

Proposition 9. *The equilibrium efforts under the enhanced-commitment, joint-sourcing, separate-sourcing, and no-commitment mechanisms satisfy* $e_H^* \geq \max\{e_J^*, e_S^*\} > e_N^*$.

Proposition 10 below adds the enhanced commitment to the comparison of the buyer's profits achieved with different mechanisms and identifies the mechanism that generates the highest profit. Figure 4 depicts the values of the effort cost c and the switching cost l under which each mechanism dominates based on the buyer's expected profit.

Proposition 10. *The buyer's expected profits (*Π_H^**,* Π_N^**,* Π_S^**,* Π_J^**) in the enhanced-commitment, no-commitment, separate-sourcing, and joint-sourcing mechanisms can be compared as follows:*

(a) When $c < C_1$*,* Π_H^* *is always the highest.*

(b) When $c \in [C_1, C_2]$*,* Π_J^* *is always the highest.*

(c) When $c > C_2$*, there exists a threshold* $l_{JN} > 0$ *increasing in* c *such that* Π_J^* *is the highest if* $l > l_{JN}$ *and* Π_N^* *is the highest otherwise.*

Recall from Proposition 7 and Figure 3 that, without considering the enhanced-commitment mechanism, the separate-sourcing, joint-sourcing, and no-commitment mechanisms dominate in region A, regions B1 and B2, and region C, respectively, as marked in Figure 4. However, when considering the enhanced-commitment mechanism, it becomes the dominant mechanism in regions A and B1, where $c < C_1$ (as defined in Proposition 7). In this area, c is small, so the effort-incentive advantage of the enhanced-commitment mechanism is large, enabling it to dominate all other mechanisms. However, when $c \geq C_1$ (regions B2 and C), its effort incentive advantage is not

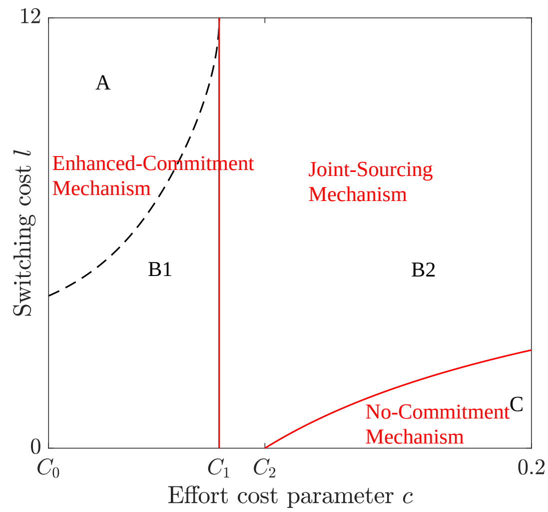


Figure 4 Buyer's optimal choice among no-commitment, joint-sourcing, separate-sourcing, and enhanced-commitment mechanisms. Setting: $\Delta = 6, d = 4$. Effort cost c starts from C_0 by Assumption 2, where $C_1 = 0.09$ and $C_2 = 0.1$. The solid line and curve set boundaries for the optimal choice of mechanisms as indicated in the labels. The dashed curve indicates the region A above which separate sourcing would dominate if not considering enhanced commitment.

significant enough to offset its allocation inefficiency. As a result, the joint-sourcing mechanism remains optimal for region B2, and the no-commitment mechanism remains optimal for region C.

As illustrated in Figure 4, Proposition 10 shows that the enhanced-commitment mechanism completely dominates the separate-sourcing mechanism in the region where the latter performs the best – when the cost coefficient c is small and switching cost l is high. This is consistent with the previous finding that in that region, separate sourcing benefits from the lower probability of supplier switching, which enhances the supplier effort incentive, and this benefit is strengthened with the commitment of no supplier switching in the enhanced-commitment mechanism.

Our findings have interesting implications. Prior studies on innovation and procurement contests often implicitly or explicitly assume that winners of a contest receive a supply contract as in our enhanced-commitment mechanism (cf. Ales et al. 2021, Chen et al. 2022). Proposition 10 shows that this approach, although simple, is not necessarily a bad choice when the cost parameter c is small. That being said, our results show that enhanced commitment is not a good idea when the innovation cost is high because the joint-sourcing mechanism or the no-commitment mechanism can provide better results in such cases due to their allocation efficiency. Thus, organizations should tread carefully when handing a production contract to the winner of an innovation contest.

In summary, we show that committing to certain rules in the sourcing mechanism can benefit the buyer by motivating *ex-ante* higher supplier innovation effort albeit at *ex-post* allocation inefficiency. Our findings point to a commitment to using a single supplier for both design and production while being cautious about how to select that single supplier. When the effort cost is low, boosting the effect of supplier effort incentive, the buyer should adopt enhanced commitment, choosing a single supplier for both design and production based on the innovation value alone. Otherwise, when the effort cost is higher and the switching cost is high, the buyer should adopt the joint-sourcing commitment, choosing a single supplier based on both the innovation value and the production cost. When the effort cost is sufficiently high and the switching cost is sufficiently low, the buyer should not commit to any restriction but determine the entire design and production allocation rule after receiving supplier innovation values. It is noteworthy that although the separate-sourcing mechanism aims to combine the best of both worlds by choosing the best design and the lowest-cost production for the design, such an approach is never optimal.

7. Innovation Prize Commitment

So far, we have found that certain commitments on the allocation rules in the sourcing mechanisms incentivize greater supplier effort in innovation. An alternative approach to directly incentivize supplier effort is to give a prize to the best design, as in an innovation contest (e.g., Ales et al. 2017, Ales et al. 2021). Thus, we next investigate the impact of committing to an innovation prize $P \geq 0$ before the innovation stage to be paid to the supplier with the best design. To ensure that the buyer's problem to optimize prize P is well defined, we assume that each supplier $i's$ effort level is constrained in $[0, \bar{e}]$ (e.g., Körpeoğlu et al. 2022), where $\bar{e} > 0$ is suppliers' effort capacity.[8]

Supplier i's expected payoff, which hinges on production allocation and innovation prize, is:

$$U_i(\mathbf{v}) = \mathrm{E}_{c_i}[(C(c_i) - c_i)\mathbb{E}_{c_j}(x_i(\mathbf{c})) + P \cdot \mathbf{1}_{v_i \geq v_j}]. \tag{9}$$

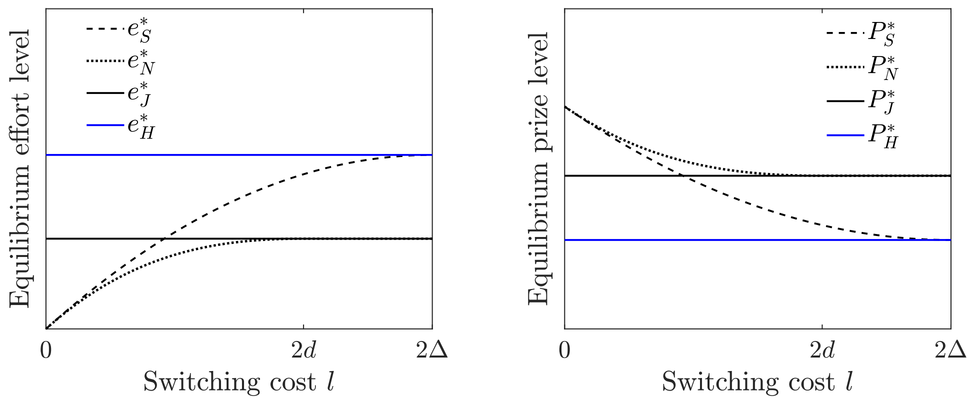
In the innovation stage, each supplier $i \in \{1, 2\}$ chooses his effort to maximize his expected profit $\pi_i(\mathbf{e}) = \mathbb{E}_{\mathbf{v}}[U_i(\mathbf{v})|\mathbf{e}] - \psi(e_i)$. To eliminate the uninteresting case where the equilibrium effort reaches $\bar{e}$ even without any prize (rendering the prize useless), we make the following assumption:

Assumption 3. $c > C_0^P \equiv \frac{\Delta}{8d\bar{e}}$.

We can derive suppliers' equilibrium efforts and the buyer's profits in each mechanism by following the same procedures in §4, §5, and §6, except with the innovation prize as an additional endogenous decision of the buyer. We relegate those derivations to the Online Appendix, while focusing on the comparisons of supplier efforts, innovation prizes, and buyer profits.

Effort and Prize Comparisons. We first compare suppliers' efforts and innovation prizes offered in each mechanism.

[8] From a practical point of view, it is reasonable to assume that suppliers have a hard capacity on the effort they exert in addition to the soft capacity imposed by their convex cost function of effort. Technically, we require this assumption because we assume that suppliers' cost function of effort is quadratic for tractability purposes. The supplier capacity becomes unnecessary when the cost of effort is more convex. Indeed, in §EC.3, we consider a cubic cost function and show that all our results hold even when relaxing suppliers' hard capacity constraints.



(a) Effort comparison. $c = 0.1$.

(b) Prize comparison. $c = 0.05$.

Figure 5 Effort and prize comparisons. Setting: $\Delta = 6, d = 4, \bar{e} = 7.5$.

PROPOSITION 11. *With prize commitment, the equilibrium efforts (e_J^*, e_S^*, e_N^*, e_H^*) and the equilibrium prizes (P_J^*, P_S^*, P_N^*, P_H^*) under joint-sourcing, separate-sourcing, no-commitment, and enhanced mechanisms can be compared as follows:*

(i) When $c \geq \frac{1}{4d}$, $P_H^* = P_J^* = P_S^* = P_N^* = 0$ and $\bar{e} > e_H^* \geq \max\{e_J^*, e_S^*\} \geq e_N^*$. $e_S^* - e_J^*$ is increasing in the switching cost l, and positive if and only if l is sufficiently high.

(ii) When $c < \frac{1}{4d}$, $e_H^* = e_J^* = e_S^* = e_N^* = \bar{e}$ and $0 < P_H^* \leq \max\{P_J^*, P_S^*\} \leq P_N^*$. $P_J^* - P_S^*$ is increasing in the switching cost l, and positive if and only if l is sufficiently high.

Proposition 11(i) shows that when c is large ($> \frac{1}{4d}$), it is not economically sensible to provide a positive innovation prize. Therefore, the comparison results coincide with the ones in Propositions 6 and 9. This case is depicted in Figure 5(a), which compares supplier efforts in the four mechanisms with a zero prize. When c is small ($< \frac{1}{4d}$), however, the buyer utilizes a positive prize in all mechanisms to boost supplier effort reaching the upper bound; see Proposition 11(ii). This case is depicted in Figure 5(b), which compares optimal prizes in the four mechanisms with supplier effort at $\bar{e}$. In this case, the mechanisms' ranking in terms of their effort incentive is reflected in the amount of reward they require to achieve suppliers' best effort $\bar{e}$. Specifically, the enhanced-commitment and no-commitment mechanisms provide the highest and least effort incentive, so they require the least and highest reward, respectively. Joint- and separate-sourcing mechanisms yield moderate effort incentives, and which one provides more effort incentive depends on the switching cost l. When l is large, the separate-sourcing mechanism provides more effort incentive so it requires less reward than the joint-sourcing mechanism. When l is small, we see the opposite.

Profit Comparisons. The comparison of the buyer's profits when using the four mechanisms along with innovation prizes is summarized in Proposition 12 and Figure 6.

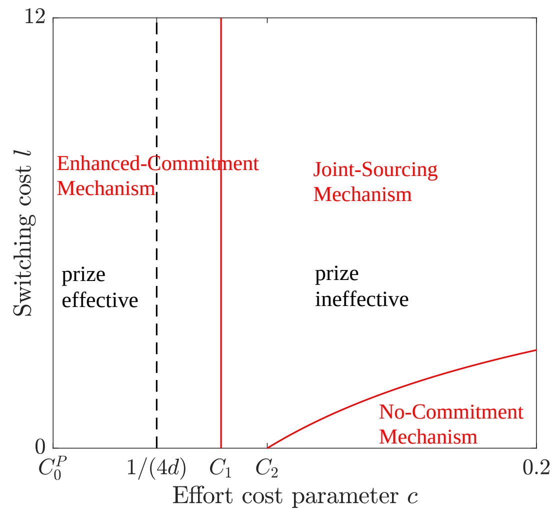


Figure 6 Profit comparisons for the four mechanisms. Setting: $\Delta = 6, d = 4, \bar{e} = 7.5$. Effort cost c starts from C_0^P by Assumptions 2 and 3, where $C_1 = 0.09$ and $C_2 = 0.1$. The dashed line separates the regions where the optimal prize is effective (strictly positive) vs. ineffective (equal to zero). The solid line and curve set boundaries for the optimal choice of mechanisms as indicated in the labels.

PROPOSITION 12. *With prize commitments, the four mechanisms follow the same comparison results in Proposition 10, with a positive prize for* $c < \frac{1}{4d}$, *where* $\frac{1}{4d} < C_1 < C_2$.

Proposition 12 states that the prize commitment does not change the sourcing mechanism of choice. This is not surprising when c is large because the buyer finds it is not cost-effective to provide a positive innovation prize. In this case, it is the compensation for production provision that provides an effective incentive for suppliers to exert innovation efforts. Therefore, production sourcing not only fulfills the needs as part of innovative product sourcing, it also serves as a strategic source of supplier innovation incentives that can be more effective than traditional innovation prizes. When c is small, the buyer offers a positive innovation prize, but the enhanced commitment mechanism still dominates other mechanisms because it requires significantly less prize to achieve the same level of effort with other mechanisms.

Our results show that when the cost of effort is small, the buyer can benefit from committing to an innovation prize on top of sourcing mechanism commitments discussed in previous sections. Therefore, the innovation prize may be considered *complementary* to the allocation-based incentives, used to further enhance the incentive provided in a strong commitment mechanism. Importantly, the addition of prize commitment does not change the best sourcing mechanism for the buyer. Therefore, when the effort cost is sufficiently small, the buyer benefits from combining enhanced commitment with an additional innovation prize, whereas when the effort cost is larger, an innovation prize is ineffective, so the buyer should focus on allocation-based incentives by choosing among the enhanced-commitment, joint-sourcing, and no-commitment mechanisms.

8. Conclusion

When sourcing innovative products, organizations need to source the design of an innovative solution and the production of this solution so as to maximize the design value and minimize the production cost, including the potential cost for supplier switching and suppliers' rent for private cost information. The sourcing mechanism must consider both the ex-post allocation efficiency in supplier selections for design and production and the ex-ante incentive effect on suppliers' innovation efforts. As suppliers receive information rent for providing production, production allocation provides a source of incentives for supplier innovation, due to the advantage a higher innovation value brings in production selection. Such a connection between innovation value and production allocation may be strengthened by the buyer committing to certain rules in the sourcing mechanism before supplier innovation. Hence production sourcing not only fulfills the need as part of innovative product sourcing but also serves as a strategic tool to motivate supplier innovation.

Whereas the ex-post optimal mechanism with no commitment reduces to either separate sourcing or joint sourcing depending on supplier innovation values, committing to either rule enhances supplier innovation incentives by improving the competitive position of the supplier with better design to win production. Combining both rules in an enhanced commitment induces even greater supplier innovation effort. Therefore, these practical and easy-to-implement commitment mechanisms may outperform no commitment despite compromising the ex-post allocation efficiency. Nevertheless, when separate sourcing provides the most advantage in supplier incentive with high switching cost, promising not to switch at all with the enhanced commitment provides only better results. Therefore, it is without a loss for buyers to commit to using a single supplier for both design and production, but buyers should be cautious of how to select this single supplier, based on the combination of innovation value and production cost via joint sourcing or based on the innovation value alone via enhanced commitment. When the supplier effort cost is relatively low, enhanced commitment should be adopted to provide the strongest supplier effort incentive, whereas when both the supplier effort cost and switching cost are relatively high, joint sourcing is favored for balancing the allocation efficiency and effort incentive. No commitment is preferred only when the supplier effort cost is high and the switching cost is low, as the former weakens its disadvantage in eliciting supplier innovation effort, and the latter strengthens its advantage in allocation efficiency.

We further reveal the value of an innovation prize as a source of incentives in addition to the allocation-based commitment mechanisms. We find that the innovation prize does not affect the choice over the allocation-based mechanisms, and it is particularly useful when supplier effort cost is sufficiently low, together with the enhanced commitment. The production allocation provides effective incentives for supplier innovation efforts even when the prize does not provide much value to the buyer. Therefore, while the innovation prize is traditionally the sole monetary tool for incentivizing innovation in the absence of production sourcing, we identify allocation-based incentives as another powerful tool in the presence of production sourcing that can be used even when the innovation prize is less effective.

Our study contributes to both the innovation contest literature and the procurement literature. The innovation contest literature has focused on the contest prize and other tools (e.g., information disclosure) to provide innovation incentives. We show that when sourcing both the design and production of innovative products, production procurement can also be used strategically to provide innovation incentives. When the innovation cost is high, a contest prize is not necessary nor economic to motivate supplier innovation, but integrating production procurement with design sourcing, with the right level of commitment to the allocation rules in the sourcing mechanism, provides effective innovation incentives in different environments. In the procurement literature, the supplier switching cost hinders the competition between incumbent and entrant suppliers, causing higher purchasing costs for a buyer. Therefore, a focus has been on sourcing decisions to increase supplier parity by reducing the switching cost. However, in innovative-product sourcing, we show that the switching cost can benefit the buyer with its positive effect on suppliers' innovation efforts. In fact, the buyer may even commit to using a single supplier for both design and production to preclude switching completely.

In our paper, we make several assumptions to ensure the tractability of the model. First, as several papers in the innovation contest literature (e.g., Nittala et al. 2022, Mihm and Schlapp 2019), we assume that the innovation effort cost is a quadratic function of the effort. We consider a more general cost function in §EC.3 of the Online Appendix, and show that our main insights still hold. Second, we assume that innovation value and production cost are independent. We relax this assumption in §EC.4, and show that all our main insights still hold. Finally, we assume that the buyer faces two potential suppliers to capture sharply the economic factors in supplier competition and switching. This setup is common in the procurement literature that considers supplier investment in improvement before competition (e.g., Stole 1994, Li 2013, Li and Wan 2017) and is also utilized by several papers in the contest literature (Bimpikis et al. 2019, Mihm and Schlapp 2019, Khorasani et al. 2024). Future work could examine a general number of suppliers and study the supply base size for different mechanisms.

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Online Appendix

EC.1. Would the Buyer Report Innovation Values Truthfully?

Take the joint-sourcing mechanism as an example and suppose that the buyer reports false innovation values $\hat{\mathbf{v}}$. To reveal suppliers' true cost information, the utility of a supplier with production cost c_i should be $u_i(c_i, \hat{\mathbf{v}}) = \int_{c_i}^{\bar{c}} \bar{x}_i(\theta, \hat{\mathbf{v}})\, d\theta$. Hence, the buyer's optimization problem at the procurement stage is:

$$\max_{x(\mathbf{c},\hat{\mathbf{v}})} U_J(\hat{\mathbf{v}}, \mathbf{v}) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} \sum_{i=1}^{2} (v_i - C(c_i))\, x_i(\mathbf{c}, \hat{\mathbf{v}})\, f(c_1)\, f(c_2)\, dc_1 dc_2.$$

We find that whatever $\hat{\mathbf{v}}$ is, the optimal selection rule is:

$$x_i(\mathbf{c}, \hat{\mathbf{v}}) = \begin{cases} 1 & \text{if } v_i - C(c_i) \geq \max(0, v_j - C(c_j)), \forall j \neq i \\ 0 & \text{otherwise.} \end{cases}$$

It shows that the optimal selection rule is based on suppliers' true innovation values. Thus, the buyer uses innovation values only for supplier selection, whereas compensation is based on production costs. The buyer has no incentive to misreport supplier innovation values.

EC.2. Detailed Results with Innovation Prize Commitment

With prize commitment, we can also derive the expressions for the equilibrium prizes (P_N^*, P_H^*, P_S^*, P_J^*) and equilibrium efforts (e_N^*, e_H^*, e_S^*, e_J^*) under no-commitment, enhanced-commitment, separate-sourcing, and joint-sourcing mechanisms. Their expressions are shown in Equations (EC.17), (EC.18), (EC.20), (EC.21), (EC.22), (EC.23), (EC.24), and (EC.25) respectively.

In terms of how the buyer's equilibrium profits Π_N^* in the no-commitment mechanism and Π_S^* in the separate-sourcing mechanism change with the switching cost l, we have the following proposition.

Proposition EC.1. *The impacts of switching cost l on Π_N^* and Π_S^* are the same as the results in Proposition 2 and 4.*

EC.3. Robustness Check for Cost Function

In our base model, consistent with several papers in the innovation contest literature (e.g., Nittala et al. 2022, Mihm and Schlapp 2019), we assume that the effort cost is quadratic in e_i. In this section, we intend to relax the quadratic cost assumption by considering a more general cost structure $\psi(e_i) = ce_i^b$ where $b > 2$ to examine the robustness of the results on how the buyer's profit compares in different sourcing mechanisms. Similar to what we do in the base model, we require the existence of symmetric equilibrium and that the effort capacity $\bar{e}$ cannot be too small. Thus, a similar assumption as Assumptions 2 and 3 needs to be made.

Assumption EC.1. $c > C_b \equiv \frac{\Delta}{4bd[2(b-1)d]^{b-1}}$, $c > C_b^P \equiv \frac{\Delta}{4bd\bar{e}^{b-1}}$.

Noting that Assumption EC.1 reduces to Assumptions 2 and 3 when $b=2$. With a more general cost function $\psi(e_i)=ce_i^b$, we can derive a supplier's effort decision by applying a similar analysis as our base model. Lemma EC.1 summarizes the equilibrium efforts associated with all mechanisms under the more general cost function.

Lemma EC.1. *When $\psi(e_i)=ce_i^b$, the equilibrium effort and prize levels under joint-sourcing (J), no-commitment (N), separate-sourcing (S), and enhanced-commitment (H) mechanisms are as follows:*

Denote $\Lambda=\min\{l,2\Delta\}$. $e_S^*=(\frac{4\Delta\Lambda+8\Delta P_S^*-\Lambda^2}{16bcd\Delta})^{\frac{1}{b-1}}$, $e_H^*=e_S^*|_{l=2\Delta}$, $e_J^*=(\frac{6P_J^*\max\{d,\Delta\}+3d\Delta-\min\{d,\Delta\}^2}{12bcd\max\{d,\Delta\}})^{\frac{1}{b-1}}$, $e_N^*=(\frac{24d\Delta P_N^*+l(l(l-3\Delta)-3d(l-4\Delta))}{48bcd^2\Delta})^{\frac{1}{b-1}}$.

The optimal prizes in separate-sourcing mechanism are: $P_S^*=\min\{\max\{\hat{P}_{bS},0\},\tilde{P}_{bS}\}$, with $\tilde{P}_{bS}=2bcd\bar{e}^{b-1}-\frac{\Lambda(4\Delta-\Lambda)}{8\Delta}$, and $\hat{P}_{bS}=2^{\frac{1}{2-b}}bcd((b-1)bcd)^{\frac{b-1}{2-b}}-\frac{\Lambda(4\Delta-\Lambda)}{8\Delta}$.

The optimal prizes in enhanced-commitment mechanism are: $P_H^*=P_S^*|_{l=2\Delta}$.

The optimal prizes in joint-sourcing mechanism are: $P_J^*=\min\{\max\{\hat{P}_{bJ},0\},\tilde{P}_{bJ}\}$, with $\tilde{P}_{bJ}=2bcd\bar{e}^{b-1}-\frac{\min\{d,\Delta\}(3\max\{d,\Delta\}-\min\{d,\Delta\})}{6\max\{d,\Delta\}}$, and $\hat{P}_{bJ}=2^{\frac{1}{2-b}}bcd((b-1)bcd)^{\frac{b-1}{2-b}}-\frac{\min\{d,\Delta\}(3\max\{d,\Delta\}-\min\{d,\Delta\})}{6\max\{d,\Delta\}}$.

The optimal prizes in no-commitment mechanism are: $P_N^*=\min\{\max\{\hat{P}_{bN},0\},\tilde{P}_{bN}\}$, with $\tilde{P}_{bN}=2bcd\bar{e}^{b-1}-\frac{l(l(l-3\Delta)-3d(l-4\Delta))}{24d\Delta}$, and $\hat{P}_{bN}=2^{\frac{1}{2-b}}bcd((b-1)bcd)^{\frac{b-1}{2-b}}-\frac{l(l(l-3\Delta)-3d(l-4\Delta))}{24d\Delta}$.

Although it is possible to characterize equilibrium effort levels, it is unfortunately analytically intractable to compare the buyer's profit under a more general cost function. Thus, plugging suppliers' effort e^* into the buyer's expected profit functions, we numerically compare each mechanism and show the regions where each mechanism dominates in Figures EC.1.

As can be seen from Figure EC.1, with a more general cost function $\psi(e_i)=ce_i^b$, the enhanced-commitment mechanism still provides the strongest effort incentive for suppliers. Knowing that when c is small, the effort plays a pivotal role in the buyer's profit, so the enhanced-commitment mechanism dominates other mechanisms. On the other hand, when c is large, the innovation efforts are small so they play a smaller part in the buyer's profit as compared to the procurement cost. Thus, the allocation efficiency becomes pivotal and the no-commitment mechanism becomes dominant. To conclude, our main comparison results still hold in the case of $b>2$.

EC.4. Robustness Check for the Correlation Between Innovation Value and Production Cost

In our base model, we assume that each supplier i's innovation value v_i and production cost c_i are independent. In this extension, we relax this assumption to check the robustness of our results to the correlation between v_i and c_i. To simplify our analysis, we let $c_i\sim U[\alpha v_i+\underline{c},\alpha v_i+\overline{c}]$, where $\alpha\in[0,1]$ captures the correlation between innovation value and production cost. If supplier i is

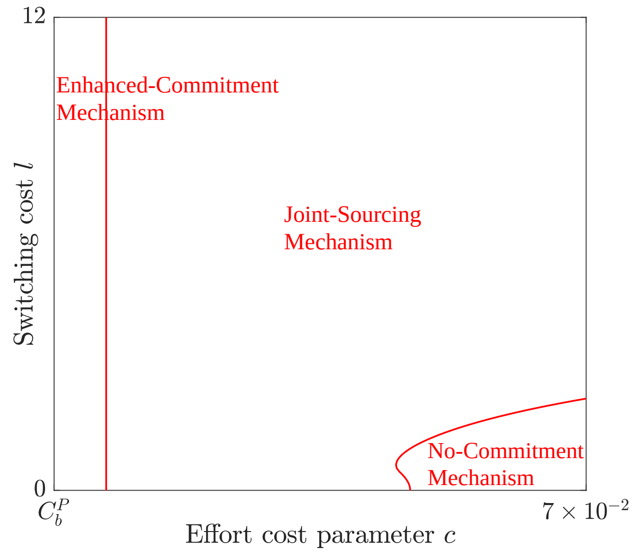

Figure EC.1 Profit comparison results. Setting: $\Delta = 6, d = 4, \bar{e} = 7.5$. Effort cost c starts from C_b^P by Assumption EC.1.

chosen to produce the design of supplier j (which is possible, for instance, under the separate-sourcing mechanism), then the production cost should be adjusted with a switch cost l such that $c_i \sim U[\alpha v_j + \underline{c} + l, \alpha v_j + \bar{c} + l]$. Since c_i is a function of innovation value $\mathbf{v}$, we can decompose c_i into two parts:

$$c_i = \tilde{c}_i + [y_i(\mathbf{c}|\mathbf{v})(\alpha v_i) + y_j(\mathbf{c}|\mathbf{v})(\alpha v_j)],$$

where the first part $\tilde{c}_i$ is the supplier specific production cost (following uniform distribution on $[\underline{c}, \bar{c}]$), whereas the second part $y_i(\mathbf{c}|\mathbf{v})(\alpha v_i) + y_j(\mathbf{c}|\mathbf{v})(\alpha v_j)$ is subject to the buyer's decision.

Given the cost distribution, similar to our base model, we can define the virtual production cost as $C_\alpha(\tilde{c}_i) = (2\tilde{c}_i - \underline{c}) + [y_i(\mathbf{c}|\mathbf{v})(\alpha v_i) + y_j(\mathbf{c}|\mathbf{v})(\alpha v_j)]$. Again similar to §3, we need the positive virtual valuation assumption to ensure each supplier is active in the procurement stage:

Assumption EC.2. $(1-\alpha)(m-d) - l > 2\bar{c} - \underline{c}$.

Applying the same analysis as our base model, under Assumptions 2 and 3 in the main model, we can derive suppliers' optimal effort levels and the buyer's expected profits under the no-commitment mechanism as in the following proposition.

Proposition EC.2. *When the innovation value and product cost are correlated with a correlation parameter α, there exists a unique symmetric pure strategy Nash Equilibrium in the no-commitment mechanism. The optimal innovation-contest prize and equilibrium effort are*

$$e_N^\alpha = \begin{cases} \bar{e} & c < \frac{1-\alpha}{4d} \\ \frac{\Delta^2+3d(-1+\alpha)\Delta}{24cd^2(-1+\alpha)} & c \geq \frac{1-\alpha}{4d}, 2\Delta \leq 2d(1-\alpha), l \geq 2\Delta \\ \frac{-d(-1+\alpha)^2-3(-1+\alpha)\Delta}{24c\Delta} & c \geq \frac{1-\alpha}{4d}, 2\Delta > 2d(1-\alpha), l \geq 2d(1-\alpha) \\ \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4l\Delta\right)}{96cd^2(1-\alpha)\Delta} & c \geq \frac{1-\alpha}{4d}, l \leq \min\{2d(1-\alpha), 2\Delta\}. \end{cases} \tag{EC.1}$$

$$P_N^\alpha = \begin{cases} 0 & c \geq \frac{1-\alpha}{4d} \\ 4cd\left(\bar{e} - \frac{\Delta^2+3d(-1+\alpha)\Delta}{24cd^2(-1+\alpha)}\right) & c < \frac{1-\alpha}{4d}, 2\Delta \leq 2d(1-\alpha), l \geq 2\Delta \\ 4cd(\bar{e} - \frac{-d(-1+\alpha)^2-3(-1+\alpha)\Delta}{24c\Delta}) & c < \frac{1-\alpha}{4d}, 2\Delta > 2d(1-\alpha), l \geq 2d(1-\alpha) \\ 4cd\left(\bar{e} - \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4l\Delta\right)}{96cd^2(1-\alpha)\Delta}\right) & c < \frac{1-\alpha}{4d}, l \leq \min\{2d(1-\alpha), 2\Delta\}. \end{cases} \tag{EC.2}$$

When $c > \frac{1-\alpha}{4d}, e_N^\alpha$ is decreasing in α; $c \leq \frac{1-\alpha}{4d}, P_N^\alpha$ is increasing in α.
The buyer's expected profit is shown in the proof of Proposition EC.2, which is decreasing in α.

It is easy to observe that as α approaches zero, the above result boils down to the one in our base model. In the presence of positive correlation between value and cost, the valuation advantage gained in the innovation stage will be partially counteracted by the potentially higher production cost. This is the reason why each supplier will exert less effort given a larger α. Furthermore, the created net value from the innovation, $v_i - c_i$ or $v_i - c_i - l$, also decreases with α, and so does the buyer's expected profit.

The following proposition repeats this analysis in the joint-sourcing and separate-sourcing mechanisms under Assumptions 2 and 3.

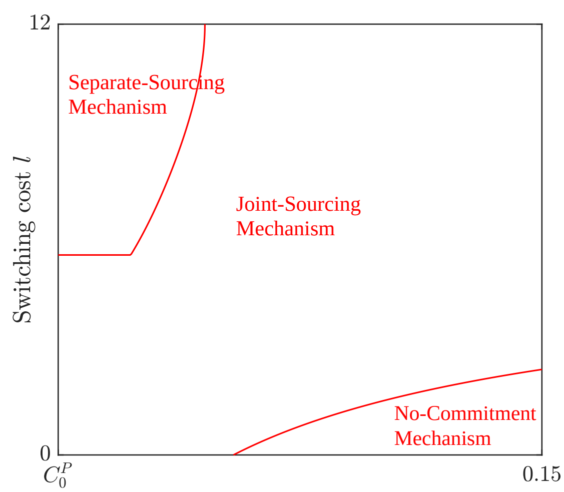

Figure EC.2 Three mechanisms profits comparison results. Setting: $\Delta = 6, d = 4, \bar{e} = 7.5, \alpha = 0.3$. Effort cost c starts from C_0^P by Assumptions 2 and 3.

Proposition EC.3. When the innovation value and product cost are correlated with a correlation parameter α,

- there exists a unique symmetric pure strategy Nash Equilibrium in the joint-sourcing mechanism. The optimal innovation-contest prize and equilibrium effort and the buyer's expected profit are the same as the no-commitment mechanism for $l > \min\{2d(1-\alpha), 2\Delta\}$ in Proposition EC.2;
- there exists a unique pure strategy Nash Equilibrium in the separate-sourcing mechanism. When $l \geq 2\Delta$, the optimal innovation-contest prize and equilibrium effort are

$$P_S^\alpha = \begin{cases} \frac{8cd\bar{e}-\Delta}{2} & c < \frac{1-\alpha}{4d} \\ 0 & c \geq \frac{1-\alpha}{4d} \end{cases}, \quad e_S^\alpha = \begin{cases} \bar{e} & c < \frac{1-\alpha}{4d} \\ \frac{\Delta}{8cd} & c \geq \frac{1-\alpha}{4d} \end{cases}. \tag{EC.3}$$

When $l < 2\Delta$, the optimal innovation-contest prize and equilibrium effort are

$$P_S^\alpha = \begin{cases} \frac{32c\Delta d\bar{e}-l(4\Delta-l)}{8\Delta} & c < \frac{1-\alpha}{4d} \\ 0 & c \geq \frac{1-\alpha}{4d} \end{cases}, \quad e_S^\alpha = \begin{cases} \bar{e} & c < \frac{1-\alpha}{4d} \\ \frac{l(4\Delta-l)}{32c\Delta d} & c \geq \frac{1-\alpha}{4d} \end{cases}. \tag{EC.4}$$

The buyer's expected profit is

$$\Pi_S^\alpha = \begin{cases} (1-\alpha)\left(\frac{d}{3}+e_S^\alpha+m\right) - \frac{\Delta}{2} - P_S^* & l \geq 2\Delta \\ (1-\alpha)\left(\frac{d}{3}+e_S^\alpha+m\right) - \frac{\Delta}{6} - \frac{l^3}{24\Delta^2} + \frac{l^2}{4\Delta} - \frac{l}{2} - P_S^* & l < 2\Delta. \end{cases}$$

Comparing the buyer's expected profits associated with no-commitment, joint-sourcing, and separate-sourcing mechanisms, we replicate the results in §5.3: the separate-sourcing mechanism dominates when the switching cost l is high and the effort cost parameter c is low; the no-commitment mechanism dominates when l is low and c is high; and the joint-sourcing mechanism dominates in other regions. Numerical results are shown in Figure EC.2.

EC.5. Proofs

Proof of Proposition 1. The buyer's optimization problem in the no-commitment mechanism is:

$$\begin{aligned} \max_{x(\mathbf{c}|\mathbf{v}),y(\mathbf{c}|\mathbf{v}),t(\mathbf{c}|\mathbf{v})} \; & \mathbb{E}_{\mathbf{c}}\left[\sum_{i=1}^{2}(v_i-c_i)y_i(\mathbf{c}|\mathbf{v})x_i(\mathbf{c}|\mathbf{v}) + \sum_{i=1}^{2}(v_i-c_j-l)y_i(\mathbf{c}|\mathbf{v})x_j(\mathbf{c}|\mathbf{v})\right] - \sum_{i=1}^{2}E_{c_i}[u_i(c_i,\mathbf{v})] \\ s.t. \; & \sum_{i=1}^{2}x_i(\mathbf{c}|\mathbf{v}) \leq 1 \quad \text{and} \quad x_i \geq 0, \quad \forall \mathbf{c}, \forall \mathbf{v}, \\ & (IC), (IR). \end{aligned} \tag{EC.5}$$

Note that the expected utility of supplier i who has true cost c_i and reports his cost as $\hat{c}_i$ as:

$$u_i(\hat{c}_i, c_i, \mathbf{v}) = \mathbb{E}_{c_j}\left[t_i(\hat{c}_i, c_j|\mathbf{v}) - x_i(\hat{c}_i, c_j|\mathbf{v})(c_i + l(1-y_i(\hat{c}_i, c_j|\mathbf{v})))\right].$$

Hence, $u_i(c_i, c_i, \mathbf{v})$ (presented as $u_i(c_i, \mathbf{v})$ for simplicity) is supplier i's expected utility when he chooses to report his cost information truthfully. The IC constraint requires that reporting true cost information generates the highest expected utility for each supplier i, i.e., $c_i = \arg\max_{\hat{c}_i} u_i(\hat{c}_i, c_i, \mathbf{v})$.

Based on the Envelope Theorem, we have:

$$\frac{\partial u_i(c_i,\mathbf{v})}{\partial c_i} = \frac{\partial u_i(\hat{c}_i,c_i,\mathbf{v})}{\partial c_i}|_{\hat{c}_i=c_i} = -E_{c_j}[x_i(c_i,c_j|\mathbf{v})] \Rightarrow u_i(c_i,\mathbf{v}) = u_i(\underline{c}) - \int_{\underline{c}}^{c_i} \bar{x}_i(\theta|\mathbf{v})d\theta, \tag{EC.6}$$

where $\bar{x}_i\left(c_i|\mathbf{v}\right) \equiv E_{\mathbf{c}\setminus c_i}\left[x_i\left(\mathbf{c}|\mathbf{v}\right)\right]$. Because $u_i(c_i,\mathbf{v})$ is a decreasing function as implied by (EC.6) and based on the set of IR constraints, we have $u_i(\bar{c},\mathbf{v}) = 0$. Therefore, we can get $u_i(\underline{c})$ and $u_i(c_i,\mathbf{v})$ as follows:

$$u_i(\bar{c},\mathbf{v}) = u_i(\underline{c}) - \int_{\underline{c}}^{\bar{c}} \bar{x}_i(\theta|\mathbf{v})d\theta = 0 \Rightarrow u_i(\underline{c}) = \int_{\underline{c}}^{\bar{c}} \bar{x}_i(\theta|\mathbf{v})d\theta$$

$$\Rightarrow u_i(c_i,\mathbf{v}) = \int_{c_i}^{\bar{c}} \bar{x}_i(\theta|\mathbf{v})d\theta.$$

Also, with $u_i(c_i,\mathbf{v}) = \mathbb{E}_{c_j}[t_i(c_i,c_j|\mathbf{v}) - x_i(c_i,c_j|\mathbf{v})(c_i + l(1-y_i(c_i,c_j|\mathbf{v})))] = \int_{c_i}^{\bar{c}} \bar{x}_i(\theta|\mathbf{v})d\theta$, we have $\bar{t}_i(c_i|\mathbf{v}) \equiv E_{\mathbf{c}\setminus c_i}\left[t_i\left(\mathbf{c}|\mathbf{v}\right)\right] = \int_{c_i}^{\bar{c}} \bar{x}_i(\theta|\mathbf{v})d\theta + E_{\mathbf{c}\setminus c_i}\left[x_i(\mathbf{c}|\mathbf{v})(c_i + l(1-y_i(\mathbf{c}|\mathbf{v})))\right]$. Hence, using IC and IR constraints, we obtain the expected utility of each supplier $i \in \{1,2\}$ with production cost c_i is

$$u_i\left(c_i,\mathbf{v}\right) = \int_{c_i}^{\bar{c}} \bar{x}_i\left(\theta|\mathbf{v}\right)d\theta, \tag{EC.7}$$

where $\bar{x}_i\left(c_i|\mathbf{v}\right) \equiv E_{\mathbf{c}\setminus c_i}\left[x_i\left(\mathbf{c}|\mathbf{v}\right)\right]$ is the expected probability that supplier i wins the buyer's contract given the other supplier's cost. Using (EC.7), we have

$$E_{c_i}[u_i(c_i,\mathbf{v})] = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} F(c_i)x_i(\mathbf{c}|\mathbf{v})f(c_j)dc_j dc_i. \tag{EC.8}$$

By definition 1, $C\left(c_i\right) = c_i + \frac{F(c_i)}{f(c_i)}$, supplier's utility shown in Proposition 1(ii) can be easily obtained.

Plugging Equation (EC.8) to Equation (EC.5), the buyer's optimization problem becomes:

$$\max_{x(\mathbf{c}|\mathbf{v}),y(\mathbf{c}|\mathbf{v})} \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} \sum_{i=1}^{2} \left[(v_i - c_i)y_i(\mathbf{c}|\mathbf{v}) + (v_j - c_i - l)y_j(\mathbf{c}|\mathbf{v}) - \frac{F(c_i)}{f(c_i)}\right] x_i(\mathbf{c}|\mathbf{v})f(c_i)f(c_j)dc_i dc_j.$$

Then we can derive the optimal design selection and production selection rule by solving the pointwise optimization problem given $\mathbf{c},\mathbf{v}$, i.e.,

$$\max_{x(\mathbf{c}|\mathbf{v}),y(\mathbf{c}|\mathbf{v})} \sum_{i=1}^{2} \left[(v_i - c_i)y_i(\mathbf{c}|\mathbf{v}) + (v_j - c_i - l)y_j(\mathbf{c}|\mathbf{v}) - \frac{F(c_i)}{f(c_i)}\right] x_i(\mathbf{c}|\mathbf{v}).$$

There exist four possible solutions for the above pointwise optimization problem:

1. $y_i(\mathbf{c}|\mathbf{v}) = 1, x_i(\mathbf{c}|\mathbf{v}) = 1$, then buyer's ex-post profit is $v_i - C(c_i)$;
2. $y_i(\mathbf{c}|\mathbf{v}) = 1, x_i(\mathbf{c}|\mathbf{v}) = 0$, then buyer's ex-post profit is $v_i - C(c_j) - l$;
3. $y_i(\mathbf{c}|\mathbf{v}) = 0, x_i(\mathbf{c}|\mathbf{v}) = 1$, then buyer's ex-post profit is $v_j - C(c_i) - l$;
4. $y_i(\mathbf{c}|\mathbf{v}) = 0, x_i(\mathbf{c}|\mathbf{v}) = 0$, then buyer's ex-post profit is $v_j - C(c_j)$.

By comparing the buyer's ex-post profits in the above four solutions given $\mathbf{c},\mathbf{v}$, we can get the optimal design and production selection rules in Proposition 1(i).

When $v_i - v_j > l$, supplier i and $j's$ utilities can be expanded as:

$$U_{iN}(\mathbf{v}) = \int_{\underline{c}}^{\bar{c}} F\left(c_i\right)\left(1 - F\left(c_i - \frac{l}{2}\right)\right)dc_i,$$

$$U_{jN}(\mathbf{v}) = \int_{\underline{c}}^{\bar{c}} F\left(c_j\right)\left(1 - F\left(c_j + \frac{l}{2}\right)\right)dc_j.$$

When $0 < v_i - v_j < l$, supplier i and $j's$ utilities can be expanded as:

$$U_{iN}(\mathbf{v}) = \int_{\underline{c}}^{\overline{c}} F(c_i)\left(1 - F\left(c_i - \frac{v_i - v_j}{2}\right)\right) dc_i,$$

$$U_{jN}(\mathbf{v}) = \int_{\underline{c}}^{\overline{c}} F(c_j)\left(1 - F\left(c_j - \frac{v_j - v_i}{2}\right)\right) dc_j.$$

It is easy to see that when $v_i > v_j$, we have $U_{iN} \geq U_{jN}$, where U_{jN} is (weakly) decreasing and U_{iN} is (weakly) increasing in l.

And the buyer's expected utility is $U_N(\mathbf{v}) = \mathbb{E}_{\mathbf{c}}[(v_i - lx_j(\mathbf{c}))y_i(\mathbf{c}) + ((v_j - lx_i(\mathbf{c})))y_j(\mathbf{c})] - \mathbb{E}_{c_i}[C(c_i)\mathbb{E}_{c_j}[x_i(\mathbf{c})]] - \mathbb{E}_{c_j}[C(c_j)\mathbb{E}_{c_i}[x_j(\mathbf{c})]]$. It is obvious that the buyer's expected utility $U_N(\mathbf{v})$ increases with v_i, v_j, but decreases with l. ■

Proof of Lemma 1. We prove this lemma with a more complete version by incorporating innovation prize commitment. The proof without prize commitment follows a similar manner by letting $\bar{e} \to \infty$ in supplier equilibrium effort derivation.

By definition, $\pi_{iN}(\mathbf{e}) = \mathbb{E}_{\mathbf{v}}[U_{iN}(\mathbf{v})|\mathbf{e}] - \psi(e_i)$. To derive the supplier's equilibrium effort, We employ the following four steps.

Step 1: Calculate the allocation rule. Following the proof of Proposition 1, depending on the relative magnitudes of l and $|v_i - v_j|$, we get the optimal allocation rule as:

1. If $-l \leq v_i - v_j \leq l$, we have
$$x_i(\mathbf{c}|\mathbf{v}) = \begin{cases} 1 & \text{if } v_i - v_j > 2(c_i - c_j) \\ 0 & \text{otherwise.} \end{cases}$$
2. If $v_i - v_j > l$, we have
$$x_i(\mathbf{c}) = \begin{cases} 1 & \text{if } c_j \geq c_i - \frac{l}{2} \\ 0 & \text{otherwise.} \end{cases}$$
3. If $v_i - v_j < -l$, we have
$$x_i(\mathbf{c}) = \begin{cases} 1 & \text{if } c_j \geq c_i + \frac{l}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Step 2: Calculate supplier's expected profit in the procurement stage. With the optimal selection rule, we can derive each supplier's expected utility in the procurement stage:

1. When $-l \leq v_i - v_j \leq l$, we get
$$U_{iN}(\mathbf{v}) = \int_{\underline{c}}^{\overline{c}} \frac{c_i - \underline{c}}{\Delta} \Pr(c_j > c_i - \frac{(v_i - v_j)}{2}) dc_i$$
$$= \begin{cases} \frac{\Delta}{2} & v_i - v_j \geq 2\Delta \\ \frac{8\Delta^3 + 12\Delta^2(v_i - v_j) - (v_i - v_j)^3}{48\Delta^2} & 0 \leq v_i - v_j < 2\Delta \\ \frac{(2\Delta + (v_i - v_j))^3}{48\Delta^2} & -2\Delta < v_i - v_j < 0 \\ 0 & v_i - v_j \leq -2\Delta. \end{cases}$$

2. When $v_i - v_j > l$, we get

$$\begin{aligned} U_{iN}(\mathbf{v}) &= \int_{\underline{c}}^{\overline{c}} \frac{c_i - \underline{c}}{\Delta} \Pr(c_j \geq c_i - \frac{l}{2}) dc_i \\ &= \begin{cases} \frac{\Delta}{2} & \text{if } l \geq 2\Delta \\ \frac{1}{48\Delta^2}\left(8\Delta^3 + 12\Delta^2 l - l^3\right) & \text{if } l < 2\Delta. \end{cases} \end{aligned}$$

3. When $v_i - v_j \leq -l$, we get

$$\begin{aligned} U_{iN}(\mathbf{v}) &= \int_{\underline{c}}^{\overline{c}} \frac{c_i + l - \underline{c}}{\Delta} \Pr(c_j \geq c_i + \frac{l}{2}) dc_i \\ &= \begin{cases} 0 & \text{if } l \geq 2\Delta \\ \frac{1}{48\Delta^2}(2\Delta - l)^3 & \text{if } l < 2\Delta. \end{cases} \end{aligned}$$

Step 3: Derive and analyze the equilibrium prize and effort. To calculate suppliers' equilibrium effort in the no-commitment mechanism, we need to take expectations over ξ_i and ξ_j to compute supplier expected utility. We have the following cases, depending on the relative magnitudes of $2d$ and 2Δ.

Case 1: $d \geq \Delta$. In this case, we can get the expressions for e_N^* for $l \geq 2\Delta$ and $l < 2\Delta$ respectively.

When $l \geq 2\Delta$, given supplier j' effort e_j, supplier i 's best-response effort is determined by solving the following problem (W.L.O.G. assume that $e_j \geq e_i$):

$$\begin{aligned} \max_{e_i \in [0, \bar{e}]} \pi_{iN}(\mathbf{e}) = &\int_{-2\Delta + e_j - e_i}^{0} \frac{1}{48\triangle^2}\left(2\Delta + e_i - e_j + \epsilon\right)^3 \frac{2d + \epsilon}{4d^2} d\epsilon \\ &+ \int_{0}^{e_j - e_i} \frac{1}{48\triangle^2}\left(2\Delta + e_i - e_j + \epsilon\right)^3 \frac{2d - \epsilon}{4d^2} d\epsilon \\ &+ \int_{e_j - e_i}^{2\Delta + e_j - e_i} \left(\frac{1}{48\triangle^2}\left(8\triangle^3 + 12\triangle^2\left(e_i - e_j + \epsilon\right) - \left(e_i - e_j + \epsilon\right)^3\right) + P\right) \frac{2d - \epsilon}{4d^2} d\epsilon \\ &+ \int_{2\Delta + e_j - e_i}^{2d} \left(\frac{\Delta}{2} + P\right) \frac{2d - \epsilon}{4d^2} d\epsilon - \psi\left(e_i\right), \end{aligned} \tag{EC.9}$$

where $\epsilon = (v_i - e_i) - (v_j - e_j) = \xi_i - \xi_j$. Because $\xi_i \sim U[-d, +d]$, we can easily obtain the cumulative distribution function $H(\epsilon)$ and the probability density function $h(\epsilon)$ of ϵ as follows:

$$H(\epsilon) = \begin{cases} 1 & \text{if } 2d < \epsilon \\ \frac{(2d+\epsilon)^2 - 2\epsilon^2}{8d^2} & \text{if } 0 < \epsilon \leq 2d \\ \frac{(2d+\epsilon)^2}{8d^2} & \text{if } -2d \leq \epsilon \leq 0 \\ 0 & \text{if } \epsilon < -2d, \end{cases} \quad \text{and } h(\epsilon) = \begin{cases} 0 & \text{if } 2d < \epsilon \\ \frac{2d-\epsilon}{4d^2} & \text{if } 0 < \epsilon \leq 2d \\ \frac{2d+\epsilon}{4d^2} & \text{if } -2d \leq \epsilon \leq 0 \\ 0 & \text{if } \epsilon < -2d. \end{cases} \tag{EC.10}$$

Note that we only focus on solving a symmetric equilibrium. Thus, in the sense of symmetric equilibrium, we can solve the above optimization problems by KKT conditions.

The Lagrange function can be expressed as:

$$\mathbf{L}(e_i, \lambda, \mu) = \pi(e_i | e_j) - \lambda(e_i - \bar{e}) - \mu(0 - e_i).$$

Taking the first derivative of $\mathbf{L}(e_i,\lambda,\mu)$ with respect to e_i, and letting $e_j=e_i$, we can get the stationary condition:

$$\frac{(3d-\Delta)\Delta+6dP}{12d^2}-\lambda+\mu-2ce_i=0.$$

Also we have primal feasibility condition: $e_i-\bar{e}\le 0$, $-e_i\le 0$; dual feasibility condition: $\lambda\ge 0,\mu\ge 0$; and complementary slackness condition: $\lambda(e_i-\bar{e})+\mu(-e_i)=0$. To maximize the Lagrange function $\mathbf{L}(e_i,\lambda,\mu)$ under the above constraints, we can get: if $e_N(P)=\bar{e}$, then $\bar{e}\le\frac{(3d-\Delta)\Delta+6dP}{24cd^2}$; if $e_N(P)=\frac{(3d-\Delta)\Delta+6dP}{24cd^2}$, then $\bar{e}>\frac{(3d-\Delta)\Delta+6dP}{24cd^2}$. Therefore, if the symmetric equilibrium $e_N(P)$ exists, it is either $\bar{e}$ or $\frac{(3d-\Delta)\Delta+6dP}{24cd^2}$.

Moreover, taking the second-order derivative of $\pi_i(\mathbf{e})$ with respect to e_i, and letting $e_j=e_i$, we can get $-2c+\frac{\Delta}{24d^2}+\frac{P}{4d^2}$. Thus, the expression for $e_N(P)$ when $-2c+\frac{\Delta}{24d^2}+\frac{P}{4d^2}<0$ is:

$$e_N(P)=\begin{cases}\frac{(3d-\Delta)\Delta+6dP}{24cd^2} & P<4cd(\bar{e}-\frac{(3d-\Delta)\Delta}{24cd^2})\\ \bar{e} & P\ge 4cd(\bar{e}-\frac{(3d-\Delta)\Delta}{24cd^2}).\end{cases} \tag{EC.11}$$

When $-2c+\frac{\Delta}{24d^2}+\frac{P}{4d^2}\ge 0$, if the symmetric equilibrium $e_N(P)$ exists, then $e_N(P)=\bar{e}$. Since it attains the same effort $\bar{e}$ by incurring a larger prize P compared with above, it suffices to consider $-2c+\frac{\Delta}{24d^2}+\frac{P}{4d^2}<0$ only.

When $l<2\Delta$, supplier i 's optimization problem is as follows (W.L.O.G. assume that $e_j\ge e_i$):

$$\begin{aligned}\max_{e_i\in[0,\bar{e}]}\pi_{iN}(\mathbf{e})=-ce_i^2+\int_{-e_i+e_j+l}^{2d}(\frac{8\Delta^3-l^3+12\Delta^2 l}{48\Delta^2}+P)\frac{2d-\epsilon}{4d^2}d\epsilon+\\ \int_{-2d}^{-e_i+e_j-l}\frac{(2\Delta-l)^3}{(48\Delta^2)}\frac{2d+\epsilon}{4d^2}d\epsilon+\int_0^{e_j-e_i}\frac{(2d-\epsilon)\left(2\Delta+e_i-e_j+\epsilon\right)^3}{(4d^2)\,(48\Delta^2)}d\epsilon+\\ \int_{-e_i+e_j-l}^{0}\frac{(2d+\epsilon)\left(2\Delta+e_i-e_j+\epsilon\right)^3}{(4d^2)\,(48\Delta^2)}d\epsilon+\\ \int_{e_j-e_i}^{-e_i+e_j+l}(\frac{8\Delta^3+12\Delta^2\left(e_i-e_j+\epsilon\right)-\left(e_i-e_j+\epsilon\right)^3}{48\Delta^2}+P)\frac{2d-\epsilon}{4d^2}d\epsilon.\end{aligned} \tag{EC.12}$$

Similarly, by applying KKT conditions to solve the optimization problem, we can get:
if $e_N(P)=\bar{e}$, then $\bar{e}\le\frac{l(l(l-3\Delta)-3d(l-4\Delta))+24d\Delta P}{96cd^2\Delta}$; if $e_N(P)=\frac{l(l(l-3\Delta)-3d(l-4\Delta))+24d\Delta P}{96cd^2\Delta}$, then $\bar{e}>\frac{l(l(l-3\Delta)-3d(l-4\Delta))+24d\Delta P}{96cd^2\Delta}$.

Taking the second derivative of $\pi_i(\mathbf{e})$ with respect to e_i, and letting $e_j=e_i$, we can get: $-2c-\frac{l^3}{96d^2\Delta^2}+\frac{l^2}{32d^2\Delta}+\frac{P}{4d^2}$. Thus, the expression for $e_S(P)$ when $-2c-\frac{l^3}{96d^2\Delta^2}+\frac{l^2}{32d^2\Delta}+\frac{P}{4d^2}<0$ is:

$$e_N(P)=\begin{cases}\frac{l(l(l-3\Delta)-3d(l-4\Delta))}{96cd^2\Delta}+\frac{P}{4cd} & P<4cd(\bar{e}-\frac{l(l(l-3\Delta)-3d(l-4\Delta))}{96cd^2\Delta}))\\ \bar{e} & P\ge 4cd(\bar{e}-\frac{l(l(l-3\Delta)-3d(l-4\Delta))}{96cd^2\Delta})).\end{cases} \tag{EC.13}$$

When $-2c-\frac{l^3}{96d^2\Delta^2}+\frac{l^2}{32d^2\Delta}+\frac{P}{4d^2}\ge 0$, if the symmetric equilibrium $e_N(P)$ exists, then $e_N(P)=\bar{e}$. Since it attains the same effort $\bar{e}$ by incurring a larger prize P compared with the second term in Equation (EC.13), it suffices to consider $-2c-\frac{l^3}{96d^2\Delta^2}+\frac{l^2}{32d^2\Delta}+\frac{P}{4d^2}<0$ only.

Case 2: $d<\Delta$. we can get the expressions for e_N^* for $l\geq 2d$ and $l<2d$ respectively.

When $l\geq 2d$, given supplier j 's effort e_j, supplier i 's best response should solve:

$$\begin{aligned}\max_{e_i\in[0,\bar{e}]}\pi_{iN}(\mathbf{e})=&\int_{-2d}^{0}\left(\frac{1}{48\triangle^2}\left(2\triangle+e_i-e_j+\epsilon\right)^3\right)\frac{2d+\epsilon}{4d^2}d\epsilon\\&+\int_{0}^{e_j-e_i}\left(\frac{1}{48\triangle^2}\left(2\triangle+e_i-e_j+\epsilon\right)^3\right)\frac{2d-\epsilon}{4d^2}d\epsilon\\+\int_{e_j-e_i}^{2d}&\left(\frac{1}{48\triangle^2}\left(8\triangle^3+12\triangle^2\left(e_i-e_j+\epsilon\right)-\left(e_i-e_j+\epsilon\right)^3\right)+P\right)\frac{2d-\epsilon}{4d^2}d\epsilon-\psi\left(e_i\right).\end{aligned}\tag{EC.14}$$

Similarly, by applying KKT conditions to solve the optimization problem, we can get: if $e_N(P)=\bar{e}$, then $\bar{e}\leq\frac{-d^2+3d\Delta+6\Delta P}{24cd\Delta}$; if $e_N(P)=\frac{-d^2+3d\Delta+6\Delta P}{24cd\Delta}$, then $\bar{e}>\frac{-d^2+3d\Delta+6\Delta P}{24cd\Delta}$.

Moreover, taking the second-order derivative of $\pi_i(\mathbf{e})$ with respect to e_i, and letting $e_j=e_i$, we can get: $-2c-\frac{d}{12\Delta^2}+\frac{1}{8\Delta}+\frac{P}{4d^2}$.

Thus, the expression for $e_S(P)$ when $-2c-\frac{d}{12\Delta^2}+\frac{1}{8\Delta}+\frac{P}{4d^2}<0$ is:

$$e_N(P)=\begin{cases}\frac{-d^2+3d\Delta+6\Delta P}{24cd\Delta} & P<4cd(\bar{e}-\frac{3\Delta-d}{24c\Delta})\\ \bar{e} & P\geq 4cd(\bar{e}-\frac{3\Delta-d}{24c\Delta}).\end{cases}\tag{EC.15}$$

When $-2c-\frac{d}{12\Delta^2}+\frac{1}{8\Delta}+\frac{P}{4d^2}\geq 0$, if the symmetric equilibrium $e_N(P)$ exists, then $e_N(P)=\bar{e}$. Since it attains the same effort $\bar{e}$ by incurring a larger prize P compared with the second term in Equation (EC.15), it suffices to consider $-2c-\frac{d}{12\Delta^2}+\frac{1}{8\Delta}+\frac{P}{4d^2}<0$ only.

When $l<2d$, supplier i has the same form of optimization problem as the case when $d\geq\Delta$ and $l<2\Delta$, implying the same expression for $e_S(P)$ (Equation (EC.13)).

Step 4: Calculate the buyer's expected profit in the no-commitment mechanism. Given the innovation value $\mathbf{v}$, the buyer's payoff in the procurement stage depends on the relative magnitudes of $2d$ and 2Δ.

When $l\leq\min\{2d,2\Delta\}$, to derive the buyer's expected profit Π_N^* in the innovation stage, we should calculate $U_N(\mathbf{v})$ first by considering the following cases.

Case 1: $\epsilon=\xi_i-\xi_j>l$**.** The buyer's expected utility in the procurement stage is: $U_N(\xi_i,\xi_j|e_i=e_j=e_N(P))=\int_{\underline{c}+\frac{l}{2}}^{\bar{c}}\left(1-\frac{-\underline{c}+c_i-\frac{l}{2}}{\Delta}\right)\left(\underline{c}-2c_i+e_N(P)+m+\epsilon+\xi_j\right)\frac{1}{\Delta}dc_i+\int_{\underline{c}}^{\underline{c}+\frac{l}{2}}\left(\underline{c}-2c_i+e_N(P)+m+\epsilon+\xi_j\right)\frac{1}{\Delta}dc_i+\int_{\underline{c}}^{\bar{c}-\frac{l}{2}}\frac{\left(1-\frac{-\underline{c}+c_j+\frac{l}{2}}{\Delta}\right)\left(\underline{c}-2c_j+e_N(P)+m-l+\xi_i\right)}{\Delta}dc_j=-\underline{c}+e_N(P)+m-\frac{2\Delta}{3}-\frac{l^3-6\Delta l^2+12\Delta^2 l}{24\Delta^2}+\xi_i.$

Case 2: $0<\epsilon\leq l$**.** The buyer's expected utility in the procurement stage is: $U_N(\xi_i,\xi_j|e_i=e_j=e_N(P))=\int_{\underline{c}+\frac{\epsilon}{2}}^{\bar{c}}\left(1-\frac{-\underline{c}+c_i-\frac{\epsilon}{2}}{\Delta}\right)\left(\underline{c}-2c_i+e_N(P)+m+\epsilon+\xi_j\right)\frac{1}{\Delta}dc_i+\int_{\underline{c}}^{\underline{c}+\frac{\epsilon}{2}}\left(\underline{c}-2c_i+e_N(P)+m+\epsilon+\xi_j\right)\frac{1}{\Delta}dc_i+\int_{\underline{c}}^{\bar{c}-\frac{\epsilon}{2}}\left(1-\frac{-\underline{c}+c_j+\frac{\epsilon}{2}}{\Delta}\right)\left(\underline{c}-2c_j+e_N(P)+m-\epsilon+\xi_i\right)\frac{1}{\Delta}dc_j=-\underline{c}-\frac{2\Delta}{3}+e_N(P)+m-\frac{\epsilon^3}{24\Delta^2}+\frac{\epsilon^2}{4\Delta}+\frac{\xi_i+\xi_j}{2}.$

Similarly to the above cases, we can derive U_N in the case of $-l < \epsilon \le 0$ and $\epsilon \le -l$. We summarize the buyer's expected utility in each case in the following equation:

$$U_N(\xi_i,\xi_j|e_i=e_j=e_N(P)) = \begin{cases} e_N(P)+m-\underline{c}-\frac{2\Delta}{3}-\frac{l^3-6\Delta l^2+12\Delta^2 l}{24\Delta^2}+\xi_i & \epsilon > l \\ e_N(P)+m-\underline{c}-\frac{2\Delta}{3}-\frac{\epsilon^3}{24\Delta^2}+\frac{\epsilon^2}{4\Delta}+\frac{\xi_i+\xi_j}{2} & 0<\epsilon\le l \\ e_N(P)+m-\underline{c}-\frac{2\Delta}{3}+\frac{\epsilon^3}{24\Delta^2}+\frac{\epsilon^2}{4\Delta}+\frac{\xi_i+\xi_j}{2} & -l<\epsilon\le 0 \\ e_N(P)+m-\underline{c}-\frac{2\Delta}{3}-\frac{l^3-6\Delta l^2+12\Delta^2 l}{24\Delta^2}+\xi_j & \epsilon\le -l. \end{cases} \tag{EC.16}$$

Taking the expectation of U_N over ξ_i and ξ_j and considering the innovation prize P, we can get the buyer's expected profit in the innovation stage.

$$\begin{aligned}\Pi_N^* &= e_N(P)+m-\underline{c}-\frac{2\Delta}{3} \\ &+\int_{-d}^{d-l}\left(\int_{l+\xi_j}^{d}(\xi_i-\frac{-6\Delta l^2+l^3+12\Delta^2 l}{24\Delta^2})\frac{1}{4d^2}d\xi_i\right)d\xi_j+\int_{l-d}^{d}\left(\int_{-d}^{\xi_j-l}(\xi_j-\frac{-6\Delta l^2+l^3+12\Delta^2 l}{24\Delta^2})\frac{1}{4d^2}d\xi_i\right)d\xi_j \\ &+\int_{-d}^{l-d}\left(\int_{-d}^{\xi_j}\frac{\frac{(\xi_i-\xi_j)^3}{24\Delta^2}+\frac{(\xi_i-\xi_j)^2}{4\Delta}+\frac{\xi_i+\xi_j}{2}}{4d^2}d\xi_i\right)d\xi_j+\int_{l-d}^{d}\left(\int_{\xi_j-l}^{\xi_j}\frac{\frac{(\xi_i-\xi_j)^3}{24\Delta^2}+\frac{(\xi_i-\xi_j)^2}{4\Delta}+\frac{\xi_i+\xi_j}{2}}{4d^2}d\xi_i\right)d\xi_j \\ &+\int_{d-l}^{d}\left(\int_{\xi_j}^{d}\frac{-\frac{(\xi_i-\xi_j)^3}{24\Delta^2}+\frac{(\xi_i-\xi_j)^2}{4\Delta}+\frac{\xi_i+\xi_j}{2}}{4d^2}d\xi_i\right)d\xi_j+\int_{-d}^{d-l}\left(\int_{\xi_j}^{l+\xi_j}\frac{-\frac{(\xi_i-\xi_j)^3}{24\Delta^2}+\frac{(\xi_i-\xi_j)^2}{4\Delta}+\frac{\xi_i+\xi_j}{2}}{4d^2}d\xi_i\right)d\xi_j-P.\end{aligned}$$

Thus, when $l \le \min\{2d, 2\Delta\}$, we have

$\Pi_N(e_N(P)) = e_N(P)-P+m-\underline{c}-\frac{2\Delta}{3}+\frac{5\Delta l^2\left(24d^2-16dl+3l^2\right)+l^3\left(-20d^2+15dl-3l^2\right)-20\Delta^2(l-2d)^3}{480d^2\Delta^2}$.

When $l > \min\{2d, 2\Delta\}$, by similar analysis, we have

$\Pi_N(e_N(P)) = e_N(P)-P+m-\underline{c}-\frac{2\Delta}{3}+\frac{5\Delta l^2\left(24d^2-16dl+3l^2\right)+l^3\left(-20d^2+15dl-3l^2\right)-20\Delta^2(l-2d)^3}{480d^2\Delta^2}\Big|_{l=\min\{2d,2\Delta\}}$.

Considering $e_N(P)$ for above cases, optimizing over $P \ge 0$, we can get:

$$P_N^* = \begin{cases} 0 & \text{if } c \ge \frac{1}{4d} \\ 4cd(\bar{e}-\frac{(3d-\Delta)\Delta}{24cd^2}) & \text{if } l > \min\{2d,2\Delta\}, d\ge\Delta, c<\frac{1}{4d} \\ 4cd(\bar{e}-\frac{3\Delta-d}{24c\Delta}) & \text{if } l > \min\{2d,2\Delta\}, d<\Delta, c<\frac{1}{4d} \\ 4cd(\bar{e}-\frac{l(3d(4\Delta-l)-l(3\Delta-l))}{96cd^2\Delta}) & \text{if } l\le\min\{2d,2\Delta\}, c<\frac{1}{4d}. \end{cases} \tag{EC.17}$$

Plugging the optimal prize expression P_N^* into $e_N(P)$, we have:

$$e_N^* = \begin{cases} \bar{e} & \text{if } c<\frac{1}{4d} \\ \frac{(3d-\Delta)\Delta}{24cd^2} & \text{if } l>\min\{2d,2\Delta\}, d\ge\Delta, c\ge\frac{1}{4d} \\ \frac{(3\Delta-d)}{24c\Delta} & \text{if } l>\min\{2d,2\Delta\}, d<\Delta, c\ge\frac{1}{4d} \\ \frac{l(3d(4\Delta-l)-l(3\Delta-l))}{96cd^2\Delta} & \text{if } l\le\min\{2d,2\Delta\}, c\ge\frac{1}{4d}. \end{cases} \tag{EC.18}$$

■

Proof of Proposition 2. We aim to analyze the impact of the switching cost l on the buyer's expected profit Π_N^*. When $l \ge \min\{2d, 2\Delta\}$, Π_N^* does not change with l.

When $l < \min\{2d, 2\Delta\}$,

$$\begin{aligned}\Pi_N^* =& \frac{l(3d(4\Delta-l)-l(3\Delta-l))}{96cd^2\Delta}+m-\underline{c}-\frac{2\Delta}{3} \\ &+\frac{5\Delta l^2\left(24d^2-16dl+3l^2\right)+l^3\left(-20d^2+15dl-3l^2\right)-20\Delta^2(l-2d)^3}{480d^2\Delta^2}.\end{aligned}$$

Taking the derivative of Π_N^* on l, we have $\frac{\partial \Pi_N^*}{\partial l} = -\frac{(2d-l)(l-2\Delta)(c(2d-l)(l-2\Delta)+\Delta)}{32cd^2\Delta^2}$. The sign of $\frac{\partial \Pi_N^*}{\partial l}$ depends on the sign of $c(2d-l)(l-2\Delta)+\Delta$. It is easy to check that when $4cd \le 1$, we have $\frac{\partial \Pi_N^*}{\partial l} \ge 0$, which implies that when $4cd \le 1$, we have Π_N^* is increasing in $l \in [0, \min\{2d, 2\Delta\}]$. When $4cd > 1$, $\frac{\partial \Pi_N^*}{\partial l}$ is first smaller than 0 and then greater than 0 for $l \in [0, \min\{2d, 2\Delta\}]$. That is to say, when $4cd > 1, \Pi_N^*$ is first decreasing then increasing in l for $l \in [0, \min\{2d, 2\Delta\}]$. ■

Proof of Proposition 3. Similar to the proof of Proposition 1. ■

Proof of Lemma 2. In the proof, we take the prize commitment into account. The proof without innovation prize follows similar procedures by relaxing the upper bound of supplier effort ($\bar{e} \to \infty$). Without loss of generality, assume that $v_i > v_j$ ex post, by Proposition 1, we have

$$U_{iS} = \begin{cases} \frac{\Delta}{2} & \text{if } l \ge 2\Delta \\ \frac{1}{48\Delta^2}(8\Delta^3 + 12\Delta^2 l - l^3) & \text{if } l < 2\Delta, \end{cases} \text{ and } U_{jS} = \begin{cases} 0 & \text{if } l \ge 2\Delta \\ \frac{1}{48\Delta^2}(2\Delta - l)^3 & \text{if } l < 2\Delta. \end{cases}$$

We next derive supplies' equilibrium effort by considering the following two cases.

Case 1: $l \ge 2\Delta$**.** In this case, decoupling design and production is inefficient. Hence, the supplier with the best design will also be allocated the production contract. Thus, supplier i 's optimization problem given supplier j 's effort e_j, is as follows (W.L.O.G. assume that $e_j \ge e_i$):

$$\max_{e_i \in [0, \bar{e}]} \pi_i(e_i \mid e_j) = \int_{e_j - e_i}^{2d} \left(P + \frac{\Delta}{2}\right) \frac{2d - \epsilon}{4d^2} d\epsilon - \psi(e_i).$$

Noting that given the value of e_j, $\pi_i(e_i \mid e_j)$ is a quadratic function of e_i with symmetry axis $\frac{(P+\frac{\Delta}{2})(e_j - 2d)}{P+\frac{\Delta}{2}-8cd^2}$ and the second derivative $\frac{P+\frac{\Delta}{2}}{4d^2} - 2c$.

Under Assumption 2 ($8cd^2 - \frac{\Delta}{2} > 0$), we first discuss the case that $P < 8cd^2 - \frac{\Delta}{2}$.

(1) When $\bar{e} \le \frac{(P+\frac{\Delta}{2})(e_j - 2d)}{P+\frac{\Delta}{2}-8cd^2}$, $\pi_i(e_i \mid e_j)$ is strictly increasing in $[0, \bar{e}]$, which implies $e_i^* = \bar{e}$. Since we only focus on symmetric equilibrium (e_j^* also equals $\bar{e}$), we have $e_S(P) = \bar{e}$ when $\bar{e} \le \frac{2P+\Delta}{8cd}$.

(2) When $\bar{e} > \frac{(P+\frac{\Delta}{2})(e_j - 2d)}{P+\frac{\Delta}{2}-8cd^2}$, $\pi_i(e_i \mid e_j)$ is first increasing and then decreasing in $[0, \bar{e}]$, which implies $e_i^* = \frac{(P+\frac{\Delta}{2})(e_j - 2d)}{P+\frac{\Delta}{2}-8cd^2}$. Since we only focus on symmetric equilibrium (e_j^* also equals e_i^*), we have $e_S(P) = \frac{2P+\Delta}{8cd}$ when $\bar{e} > \frac{2P+\Delta}{8cd}$.

Next, we consider the case that $P \ge 8cd^2 - \frac{\Delta}{2}$.

In this case, the second derivative $\frac{P+\frac{\Delta}{2}}{4d^2} - 2c \ge 0$, which implies that the maximization of $\pi_i(e_i \mid e_j)$ can only be taken at the boundaries. Noting that the symmetry axis of $\pi_i(e_i \mid e_j)$ is $\frac{(P+\frac{\Delta}{2})(e_j - 2d)}{P+\frac{\Delta}{2}-8cd^2}$. Thus, if the maximization is taken at $e_i^* = 0$, we must have $e_j^* \ge 2d$, indicating that $e_S(P) = 0$ can never be a symmetric equilibrium. When $\bar{e} > 2\frac{(P+\frac{\Delta}{2})(e_j - 2d)}{P+\frac{\Delta}{2}-8cd^2}$, we have $e_i^* = \bar{e}$. In a symmetric equilibrium, e_j also equals $\bar{e}$, which implies that such equilibrium exists when $\bar{e} < \frac{8dP+4d\Delta}{16cd^2+2P+\Delta}$. Thus, when $l \ge 2\Delta$ and $P < 8cd^2 - \frac{\Delta}{2}$, each supplier exerts the equilibrium effort

$$e_S(P) = \begin{cases} \frac{2P+\Delta}{8cd} & P \le \frac{8cd\bar{e}-\Delta}{2} \\ \bar{e} & P > \frac{8cd\bar{e}-\Delta}{2}. \end{cases}$$

And when $l \geq 2\Delta$, $P \geq 8cd^2 - \frac{\Delta}{2}$, and $\bar{e} < \frac{8dP+4d\Delta}{16cd^2+2P+\Delta}$, we have $e_S(P) = \bar{e}$.

Case 2: $l < 2\Delta$. In this case, the supplier with the best design may not be selected to produce his product, and hence supplier i 's optimization problem, given e_j, is as follows (W.L.O.G. assume that $e_j \geq e_i$):

$$\max_{e_i \in [0,\bar{e}]} \pi_i(\mathbf{e}) = \int_{e_j-e_i}^{2d} \left(P + \frac{1}{48\triangle^2}\left(8\triangle^3 + 12\triangle^2 l - l^3\right)\right) \frac{2d-\epsilon}{4d^2} d\epsilon + \int_0^{e_j-e_i} \frac{1}{48\triangle^2}(2\triangle - l)^3 \frac{2d-\epsilon}{4d^2} d\epsilon$$
$$+ \int_{-2d}^{0} \frac{1}{48\triangle^2}(2\triangle - l)^3 \frac{2d+\epsilon}{4d^2} d\epsilon - \psi(e_i).$$

Noting that given the value of e_j, $\pi_i(e_i \mid e_j)$ is a quadratic function of e_i with symmetry axis $\frac{(e_j-2d)(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{(P-\frac{l^2}{8\Delta}+\frac{l}{2})-8cd^2}$ and the second derivative $\frac{1}{4d^2}(P - \frac{l^2}{8\Delta} + \frac{l}{2}) - 2c$.

Under Assumption 2, we have that $\frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2 > 0$ is satisfied. We first discuss the case that $P < \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$.

(1) When $\bar{e} \leq \frac{(e_j-2d)(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{(P-\frac{l^2}{8\Delta}+\frac{l}{2})-8cd^2}$, $\pi_i(e_i \mid e_j)$ is strictly increasing in $[0,\bar{e}]$, which implies $e_i^* = \bar{e}$. Since we only focus on symmetric equilibrium, we get that when $\bar{e} \leq \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d}$, $e_S(P) = \bar{e}$.

(2) When $\bar{e} > \frac{(e_j-2d)(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{(P-\frac{l^2}{8\Delta}+\frac{l}{2})-8cd^2}$, $\pi_i(e_i \mid e_j)$ is first increasing and then decreasing in $[0,\bar{e}]$, which implies $e_i^* = \frac{(e_j-2d)(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{(P-\frac{l^2}{8\Delta}+\frac{l}{2})-8cd^2}$. Since we only focus on symmetric equilibrium, we get that when $\bar{e} > \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d}$, $e_S(P) = \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d}$.

Next, we consider the case that $P \geq \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$.

In this case, the second derivative $\frac{1}{4d^2}(P - \frac{l^2}{8\Delta} + \frac{l}{2}) - 2c \geq 0$, which implies that the maximization of $\pi_i(e_i \mid e_j)$ can only be taken at the boundaries. Noting that the symmetry axis of $\pi_i(e_i \mid e_j)$ is $\frac{(e_j-2d)(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{(P-\frac{l^2}{8\Delta}+\frac{l}{2})-8cd^2}$. Thus, if the maximization is taken at $e_i^* = 0$, we must have $e_j^* \geq 2d$, indicating that $e_S(P) = 0$ can never be a symmetric equilibrium. Also, we get that when $\bar{e} > 2\frac{(e_j-2d)(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{(P-\frac{l^2}{8\Delta}+\frac{l}{2})-8cd^2}$, we have $e_i^* = \bar{e}$. In a symmetric equilibrium, e_j^* also equals $\bar{e}$, which implies that such equilibrium exists when $\bar{e} < \frac{4d(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{P-\frac{l^2}{8\Delta}+\frac{l}{2}+8cd^2}$. Thus, when $l < 2\Delta$ and $P < \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$, each supplier exerts the equilibrium effort

$$e_S(P) = \begin{cases} \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d} & P \leq \frac{32c\Delta d\bar{e} - l(4\Delta-l)}{8\Delta} \\ \bar{e} & P > \frac{32c\Delta d\bar{e} - l(4\Delta-l)}{8\Delta}. \end{cases} \tag{EC.19}$$

And when $l < 2\Delta$, $P \geq \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$, and $\bar{e} < \frac{4d(P-\frac{l^2}{8\Delta}+\frac{l}{2})}{P-\frac{l^2}{8\Delta}+\frac{l}{2}+8cd^2}$, we have $e_S(P) = \bar{e}$.

To derive the optimal prize and the corresponding supplier effort, we first need to calculate the buyer's expected profit in the separate-souring mechanism under the cases of $l \geq 2\Delta$ and $l < 2\Delta$. If $l \geq 2\Delta$, the supplier who wins in the innovation stage will be the design producer. Without loss of generality, assume that $v_i > v_j$.

Accordingly, the expression of U_S is

$$U_S(\mathbf{v}) = \int_{\underline{c}}^{\bar{c}} (v_i - 2c_i + \underline{c})\, f(c_i)\, dc_i = m + e_S(P) + \max\{\xi_i, \xi_j\} - \underline{c} - \Delta.$$

Taking expectation over ξ_i and ξ_j, buyer's expected profit in the innovation stage is:

$$\Pi_S(e_S(P)) = \mathbb{E}[U_S(\mathbf{v} \mid e_S(P))] - P = e_S(P) + m - \underline{c} + \frac{d}{3} - \Delta - P.$$

To derive the optimal winning prize P_S^* and the equilibrium effort e_S^*, we first concentrate on $P < 8cd^2 - \frac{\Delta}{2}$.

(1) Given that $0 \le P \le \frac{8cd\bar{e}-\Delta}{2}$ (i.e., $\bar{e} \ge \frac{2P+\Delta}{8cd}$), $e_S(P) = \frac{2P+\Delta}{8cd}$. In this case, through solving the optimization problem $\max_P \Pi_S(e_S(P))$, we derive the optimal prize

$$P_S^* = \begin{cases} \frac{8cd\bar{e}-\Delta}{2} & c < \frac{1}{4d} \\ 0 & c \ge \frac{1}{4d}. \end{cases}$$

(2) Given that $\frac{8cd\bar{e}-\Delta}{2} < P < 8cd^2 - \frac{\Delta}{2}$ (i.e., $\bar{e} < \frac{2P+\Delta}{8cd}$), $e_S(P) = \bar{e}$. In this case, under Assumptions 2 and 3, $\Pi_S(e_S(P))$ is dominated by $\Pi_S(e_S(P))$ in (1).

For $P \ge 8cd^2 - \frac{\Delta}{2}$, since it attains the same effort $\bar{e}$ by incurring a larger prize P, it suffices to consider $P < 8cd^2 - \frac{\Delta}{2}$ only.

Thus, in the separate-sourcing mechanism, when $l \ge 2\Delta$, the expression for P_S^* is:

$$P_S^* = \begin{cases} \frac{8cd\bar{e}-\Delta}{2} & c < \frac{1}{4d} \\ 0 & c \ge \frac{1}{4d}. \end{cases} \tag{EC.20}$$

Plug the expression for P_S^* into $e_S(P)$, we can get the expression for e_S^*:

$$e_S^* = \begin{cases} \bar{e} & c < \frac{1}{4d} \\ \frac{\Delta}{8cd} & c \ge \frac{1}{4d}. \end{cases} \tag{EC.21}$$

Note that when $l \ge 2\Delta$, the above P_S^* and e_S^* also equal to the enhanced-commitment equilibrium prize and effort P_H^* and e_H^*, respectively.

If $l < 2\Delta$, the supplier who wins in the innovation stage cannot ensure that he will be selected to produce his own design. Without loss of generality, we assume that $v_i > v_j$. Hence, the expression of U_S is

$$\begin{aligned} U_S(\mathbf{v}) = & \underbrace{\int_{\underline{c}+\frac{l}{2}}^{\bar{c}} (v_i - 2c_i + \underline{c}) \left(1 - \frac{c_i - \frac{l}{2} - \underline{c}}{\Delta}\right) f(c_i)\, dc_i + \int_{\underline{c}}^{\underline{c}+\frac{l}{2}} (v_i - 2c_i + \underline{c})\, f(c_i)\, dc_i}_{\text{Part (1)}} \\ & + \underbrace{\int_{\underline{c}}^{\bar{c}-\frac{l}{2}} (v_i - 2c_j + \underline{c} - l) \left(1 - \frac{c_j + \frac{l}{2} - \underline{c}}{\Delta}\right) f(c_j)\, dc_j}_{\text{Part (2)}} \\ & = e_S(P) + m + \max\{\xi_i, \xi_j\} - \underline{c} + \frac{l^2}{4\Delta} - \frac{l^3}{24\Delta^2} - \frac{l}{2} - \frac{2\Delta}{3}. \end{aligned}$$

Taking expectation over ξ_i and ξ_j, the buyer's ex-ante expected profit at the start of the innovation stage is:

$$\Pi_S\left(e_S(P)\right) = \mathbb{E}\left[U_S\left(\mathbf{v} \mid e_S(P)\right)\right] - P = e_S(P) + m - \underline{c} + \frac{d}{3} + \frac{l^2}{4\Delta} - \frac{l^3}{24\Delta^2} - \frac{l}{2} - \frac{2\Delta}{3} - P.$$

To derive the optimal winning prize P_S^* and the equilibrium effort e_S^*, we first concentrate on $P < \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$.

(1) Given that $0 \le P \le \frac{32c\Delta d\bar{e} - l(4\Delta - l)}{8\Delta}$ (i.e., $\bar{e} \ge \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d}$), we get $e_S(P) = \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d}$ (recall Equation (EC.19)). In this case, through solving the optimization problem $\max_P \Pi_S\left(e_S(P)\right)$, we derive the optimal prize

$$P_S^* = \begin{cases} \frac{32c\Delta d\bar{e} - l(4\Delta-l)}{8\Delta} & c < \frac{1}{4d} \\ 0 & c \ge \frac{1}{4d}. \end{cases}$$

(2) Given that $\frac{32c\Delta d\bar{e} - l(4\Delta-l)}{8\Delta} < P < \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$ (i.e., $\bar{e} < \frac{l(4\Delta-l)+8\Delta P}{32c\Delta d}$), we get $e_S(P) = \bar{e}$. In this case, under Assumptions 2 and 3, $\Pi_S\left(e_S(P)\right)$ is dominated by $\Pi_S\left(e_S(P)\right)$ in (1).

For $P \ge \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$, since it attains the same effort $\bar{e}$ by incurring a larger prize P, it suffices to consider $P < \frac{l^2}{8\Delta} - \frac{l}{2} + 8cd^2$ only.

Thus, in the separate-sourcing mechanism, when $l < 2\Delta$, the expression for P_S^* is:

$$P_S^* = \begin{cases} \frac{32c\Delta d\bar{e} - l(4\Delta-l)}{8\Delta} & c < \frac{1}{4d} \\ 0 & c \ge \frac{1}{4d}. \end{cases} \tag{EC.22}$$

Plug the expression for P_S^* into $e_S(P)$, we can get the expression for e_S^*:

$$e_S^* = \begin{cases} \bar{e} & c < \frac{1}{4d} \\ \frac{l(4\Delta-l)}{32c\Delta d} & c \ge \frac{1}{4d}. \end{cases} \tag{EC.23}$$

■

Proof of Proposition 4. Taking the derivative of $\Pi_S\left(e_S^*\right)$ on l, we have

$$\frac{\partial \Pi_S\left(e_S^*\right)}{\partial l} = \frac{(2\Delta - l)(2cd(l-2\Delta)+\Delta)}{16cd\Delta^2}.$$

The sign of the above derivative is equivalent to the sign of $\Delta(1-4cd) + 2cdl$.

When $4cd \le 1$, we have $\frac{\partial \Pi_S\left(e_S^*\right)}{\partial l} \ge 0 \Rightarrow \Pi_S^*$ is increasing in l.

When $4cd > 1$, we have $\Delta(1-4cd)+2cdl$ is positive only when $l > \frac{(4cd-1)2\Delta}{4cd}$; otherwise, it is negative. Thus, Π_S^* is first decreasing and then increasing in l. ■

Proof of Proposition 5. Similar to the proof of Proposition 1. ■

Proof of Lemma 3. See the proof of Lemma 1 for $l > \min\{2d, 2\Delta\}$. For the joint sourcing mechanism, the expressions for P_J^* and e_J^* are:

$$P_J^* = \begin{cases} 0 & \text{if } c \ge \frac{1}{4d} \\ 4cd\left(\bar{e} - \frac{(3\max\{d,\Delta\} - \min\{d,\Delta\})\Delta}{24c\max\{d,\Delta\}^2}\right) & \text{if } c < \frac{1}{4d}. \end{cases} \tag{EC.24}$$

$$e_J^* = \begin{cases} \bar{e} & \text{if } c < \frac{1}{4d} \\ \frac{(3\max\{d,\Delta\} - \min\{d,\Delta\})\Delta}{24c\max\{d,\Delta\}^2} & \text{if } c \geq \frac{1}{4d}. \end{cases} \tag{EC.25}$$

It is easy to show that e_J^* degenerates to $\frac{(3\max\{d,\Delta\}-\min\{d,\Delta\})\Delta}{24c\max\{d,\Delta\}^2}$ in Lemma 3 in the absence of prize commitment. ■

Proof of Proposition 6. Let $l_1 = 2\left(\Delta - \sqrt{\frac{d^2-3d\Delta+3\Delta^2}{3}}\right)$, $l_2 = 2\left(\Delta - \sqrt{\frac{\Delta^3}{3d}}\right)$.
Using Lemmas 2 and 3, we can compare the equilibrium effort levels e_J^* and e_S^* in the joint-sourcing and separate-sourcing mechanisms as follows:

1. When $l \geq 2\Delta, e_S^* > e_J^*$.
2. When $l < 2\Delta$ and $\Delta \geq d$, we have $e_S^* > e_J^*$ if and only if $l > l_1$, since l_1 satisfies $e_S^* - e_J^* = 0$, and e_S^* is increasing in l.
3. When $l < 2\Delta$ and $\Delta < d$, we have $e_S^* > e_J^*$ if and only if $l > l_2$, since l_2 satisfies $e_S^* - e_J^* = 0$, and e_S^* is increasing in l.

Combining the above results, we conclude that when $l > \tilde{l}$, where

$$\tilde{l} = \begin{cases} l_1 & \Delta \geq d, \\ l_2 & d > \Delta, \end{cases}$$

the separate-sourcing mechanism generates a higher effort than the joint-sourcing mechanism. As for the equilibrium effort level e_N^* in the no-commitment mechanism, knowing that e_N^* is increasing in l and $e_N^*|_{l=\min\{2d,2\Delta\}} = e_J^*$, we can easily obtain that $e_J^* \geq e_N^*$. Also, we have

$$e_N^* = \frac{l(4\Delta - l)}{32cd\Delta} \underbrace{-\frac{l^2(3\Delta - l)}{96cd^2\Delta}}_{\leq 0} \leq \frac{l(4\Delta - l)}{32cd\Delta} = e_S^*.$$

■

Proof of Proposition 7. We formalize the proof of this proposition with innovation prize commitment incorporated. The proof without prize commitment follows a similar manner. The comparison of the buyer's profits under the separate-sourcing, joint-sourcing, and no-commitment mechanisms can be decomposed into the following steps.

1. The pairwise comparison between the separate-sourcing and joint-sourcing mechanisms (see Proposition EC.4).
2. The pairwise comparison between the separate-sourcing and no-commitment mechanisms (see Proposition EC.5).
3. The pairwise comparison between the joint-sourcing and no-commitment mechanisms (see Proposition EC.6).
4. Compare these three mechanisms together.

PROPOSITION EC.4. There exist thresholds C_{SJ}^1 and C_{SJ}^2, where $C_{SJ}^1 < C_{SJ}^2$, such that the buyer's expected profit in the separate-sourcing mechanism Π_S^* and that in the joint-sourcing mechanism Π_J^* can be compared as follows:

(a) When $c > C_{SJ}^2$, there exists a threshold $l_{SJ} > 0$ such that $\Pi_S^* > \Pi_J^*$ if and only if $l < l_{SJ}$;

(b) When $c \in [C_{SJ}^1, C_{SJ}^2]$, we have $\Pi_J^* > \Pi_S^*$;

(c) When $c < C_{SJ}^1$, there exists a threshold $l_{SJ} > 0$ such that $\Pi_S^* > \Pi_J^*$ if and only if $l > l_{SJ}$. The threshold l_{SJ} is defined by $G(l_{SJ}) = 0$, with $G(l) \equiv \min\{4cd, 1\}(\frac{l(-l+4\Delta)}{32cd\Delta} - \frac{(-\min\{\Delta,d\}+3\max\{\Delta,d\})\Delta}{24c\max\{\Delta,d\}^2}) + \frac{\min\{\Delta,d\}}{3} - \frac{l}{2} - \frac{l^3}{24\Delta^2} + \frac{\min\{\Delta,d\}^2(\min\{\Delta,d\}-5\max\{\Delta,d\})}{30\max\{\Delta,d\}^2} + \frac{l^2}{4\Delta}$.

Proof of Proposition EC.4. To formally compare the buyer's expected profits in separate-sourcing and joint-sourcing mechanisms, we need to consider two cases: (1) $d < \Delta$ and (2) $d \geq \Delta$.

Case 1: $d < \Delta$. The profit difference of the two mechanisms is

$$\Pi_S^* - \Pi_J^* = \underbrace{\min\{4cd, 1\}(\frac{l(4\Delta - l)}{32c\Delta d} - \frac{3\Delta - d}{24c\Delta})}_{\text{increasing in } l} + \underbrace{\frac{d}{3} - \frac{l}{2} - \frac{l^3}{24\Delta^2} + \frac{d^2(d-5\Delta)}{30\Delta^2} + \frac{l^2}{4\Delta}}_{\text{decreasing in } l}.$$

Denote $G_1(l) = \frac{d}{3} - \frac{l}{2} - \frac{l^3}{24\Delta^2} + \frac{d^2(d-5\Delta)}{30\Delta^2} + \frac{l^2}{4\Delta} - \frac{3\Delta-d}{24c\Delta} + \frac{l(4\Delta-l)}{32cd\Delta}$. When $4cd > 1$, we have $\Pi_S^* - \Pi_J^* = G_1(l)$. Differentiating $G_1(l)$, we obtain that when $4cd > 1, G_1(l)$ is first decreasing and then increasing in l with a turning point at $l = \Delta\left(2 - \frac{1}{2cd}\right)$. Next, we examine whether $G_1(l=0)$ and $G_1(l=2\Delta)$ are positive or not. $G_1(l=0) = -\frac{1}{8c} + \frac{d}{3} + \frac{d^3}{30\Delta^2} + \frac{d}{24c\Delta} - \frac{d^2}{6\Delta} = \frac{5(d-3\Delta)\Delta + 4cd\left(d^2-5d\Delta+10\Delta^2\right)}{120c\Delta^2}$. Hence, we have

$$G_1(l=0)\begin{cases} \geq 0 & \text{if } 4c \geq \frac{5(3\Delta-d)\Delta}{\left(d^2-5d\Delta+10\Delta^2\right)d} \\ < 0 & \text{if } 4c < \frac{5(3\Delta-d)\Delta}{\left(d^2-5d\Delta+10\Delta^2\right)d}. \end{cases}$$

And $G_1(l=2\Delta) = \frac{5\Delta\left(d^2-3d\Delta+3\Delta^2\right)+4cd\left(d^3-5d^2\Delta+10d\Delta^2-10\Delta^3\right)}{120cd\Delta^2}$. Hence, we have

$$G_1(l=2\Delta)\begin{cases} \geq 0 & \text{if } 4c \leq \frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d} \\ < 0 & \text{if } 4c > \frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d}. \end{cases}$$

Comparing the above thresholds, we get that $\frac{1}{d} < \frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d} < \frac{5(3\Delta-d)\Delta}{\left(d^2-5d\Delta+10\Delta^2\right)d}$.

Case 2: $d \geq \Delta$. The profit difference between the two mechanisms is

$$\Pi_S^* - \Pi_J^* = \underbrace{\min\{4cd, 1\}(\frac{l(4\Delta - l)}{32c\Delta d} - \frac{(3d-\Delta)\Delta}{24cd^2})}_{\text{increasing in } l} \underbrace{- \frac{l}{2} - \frac{l^3}{24\Delta^2} + \frac{l^2}{4\Delta} + \frac{\Delta}{3} - \frac{\Delta^2}{6d} + \frac{\Delta^3}{30d^2}}_{\text{decreasing in } l}.$$

Denote $G_2(l) = -\frac{l}{2} - \frac{l^3}{24\Delta^2} + \frac{l^2}{4\Delta} + \frac{\Delta}{3} - \frac{(3d-\Delta)\Delta}{24cd^2} - \frac{\Delta^2}{6d} + \frac{\Delta^3}{30d^2} + \frac{l(4\Delta-l)}{32cd\Delta}$. When $4cd > 1$, we have $\Pi_S^* - \Pi_J^* = G_2(l)$. Taking the derivative of $G_2(l)$, we get that when $4cd > 1, G_2(l)$ is first decreasing and then increasing in l with a turning point at $l = \Delta\left(2 - \frac{1}{2cd}\right)$. Next, we examine whether $G_2(l=0)$ and

$G_2(l=2\Delta)$ are positive or not. $G_2(l=0)=\frac{\Delta\left(40cd^2-5d(3+4c\Delta)+\Delta(5+4c\Delta)\right)}{120cd^2}=\frac{4c\left(10d^2+\Delta^2-5d\Delta\right)+5\Delta-15d}{120cd^2}$. Hence, we have

$$G_2(l=0)\begin{cases}\geq 0 & \text{if } 4c\geq\frac{15d-5\Delta}{10d^2-5d\Delta+\Delta^2}\\ <0 & \text{if } 4c<\frac{15d-5\Delta}{10d^2-5d\Delta+\Delta^2}.\end{cases}$$

And $G_2(l=2\Delta)=\frac{\Delta^2(5+4c(-5d+\Delta))}{120cd^2}$. Hence, we have

$$G_2(l=2\Delta)\begin{cases}\geq 0 & \text{if } 4c\leq\frac{5}{5d-\Delta}\\ <0 & \text{if } 4c>\frac{5}{5d-\Delta}.\end{cases}$$

Comparing the above thresholds, we can easily get that $\frac{1}{d}<\frac{5}{5d-\Delta}<\frac{15d-5\Delta}{10d^2-5d\Delta+\Delta^2}$.

Combining the above two cases, we obtain the following comparison results about the buyer's ex-ante profits Π_S^* and Π_J^* in the separate-sourcing and joint-sourcing mechanisms:

1. When $d\leq\Delta$,
   (a) If $4c>\frac{5(3\Delta-d)\Delta}{\left(d^2-5d\Delta+10\Delta^2\right)d}$, there exists a threshold l_{SJ} (that solves $G_1(l_{SJ})=0$) such that $\Pi_S^*>\Pi_J^*$ if and only if $l<l_{SJ}$.
   (b) If $\frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d}\leq 4c\leq\frac{5(3\Delta-d)\Delta}{\left(d^2-5d\Delta+10\Delta^2\right)d}$, the joint-sourcing mechanism always generates a higher profit for the buyer, i.e., $\Pi_J^*>\Pi_S^*$.
   (c) If $\frac{1}{d}\leq 4c<\frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d}$, there exists a threshold l_{SJ} (where $G_1(l_{SJ})=0$) such that when $l>l_{SJ}$, the separate-sourcing mechanism generates a higher profit for the buyer (i.e., $\Pi_S^*>\Pi_J^*$), and vice versa.
   (d) If $4c<\frac{1}{d}$, $\Pi_S^*-\Pi_J^*=4cd(\frac{l(4\Delta-l)}{32c\Delta d}-\frac{3\Delta-d}{24c\Delta})+G_1(l)$. It is always increasing in l. Under Assumption 2, it is easy to verify that $\Pi_S^*-\Pi_J^*|_{l=2\Delta}=\frac{1}{30}(-5d+\frac{d^3}{\Delta^2}+5\Delta)>0$. And $\Pi_S^*-\Pi_J^*|_{l=0}=\frac{1}{30}d\left(-5+\frac{d^2}{\Delta^2}\right)<0$. Then there exists a threshold l_{SJ} (where $G_1(l_{SJ})=0$) such that $\Pi_S^*>\Pi_J^*$ if and only if $l>l_{SJ}$.
2. When $d>\Delta$,
   (a) If $4c>\frac{15d-5\Delta}{10d^2-5d\Delta+\Delta^2}$, there exists a threshold l_{SJ} (where $G_2(l_{SJ})=0$) such that when $l<l_{SJ}$, the separate-sourcing mechanism generates a higher profit for the buyer (i.e., $\Pi_S^*>\Pi_J^*$), and vice versa.
   (b) If $\frac{5}{5d-\Delta}\leq 4c\leq\frac{15d-5\Delta}{10d^2-5d\Delta+\Delta^2}$, the joint-sourcing mechanism always generates a higher profit for the buyer, i.e., $\Pi_J^*>\Pi_S^*$.
   (c) If $\frac{1}{d}\leq 4c<\frac{5}{5d-\Delta}$, there exists a threshold l_{SJ} (where $G_2(l_{SJ})=0$) such that when $l>l_{SJ}$, the separate-sourcing mechanism generates a higher profit for the buyer (i.e., $\Pi_S^*>\Pi_J^*$), and vice versa.
   (d) If $4c<\frac{1}{d}$, $\Pi_S^*-\Pi_J^*=4cd(\frac{l(4\Delta-l)}{32c\Delta d}-\frac{(3d-\Delta)\Delta}{24cd^2})+G_2(l)$. It is always increasing in l. Under Assumption 2, it is easy to verify that $\Pi_S^*-\Pi_J^*|_{l=2\Delta}=\frac{\Delta^3}{30d^2}>0$. And $\Pi_S^*-\Pi_J^*|_{l=0}=\frac{1}{30}\Delta\left(-5+\frac{\Delta^2}{d^2}\right)<0$. Thus, there exists a threshold l_{SJ} (where $G_2(l_{SJ})=0$) such that $\Pi_S^*>\Pi_J^*$ if and only if $l>l_{SJ}$.

Combining the above two cases, we have that there exist thresholds $C_{SJ}^1 \equiv \frac{5\left(d^2-3\Delta\min\{d,\Delta\}+3\Delta^2\right)\Delta}{4\left(-d^2\min\{d,\Delta\}+5d^2\max\{\Delta,d\}-10\Delta^2\min\{d,\Delta\}+10\Delta^3\right)\min\{d,\Delta\}}$ and $C_{SJ}^2 \equiv \frac{5(3\max\{\Delta,d\}-\min\{\Delta,d\})\Delta}{4\left(\min\{\Delta,d\}^2-5d\Delta+10\max\{\Delta,d\}^2\right)\min\{\Delta,d\}}$, where $C_{SJ}^1 < C_{SJ}^2$, such that proposition EC.4 holds.

The following proposition characterizes the profits comparison between the no-commitment and the separate-sourcing mechanisms. Since the expression for Π_N^* degenerates to Π_J^* when $l \geq \min\{2d, 2\Delta\}$. Thus, we focus on the comparison between the no-commitment and the separate-sourcing mechanisms when $l < \min\{2d, 2\Delta\}$.

PROPOSITION EC.5. When $d < \Delta$, the buyer's expected profits Π_N^* and Π_S^* under the no-commitment and the separate-sourcing mechanisms satisfy the following:

1. When $c \leq \frac{1}{4d}$, if $\frac{d(15d\Delta-9d^2-5\Delta^2)}{30\Delta^2} \leq 0$, we always have $\Pi_S^* \leq \Pi_N^*$ for $l < 2d$; if $\frac{d(15d\Delta-9d^2-5\Delta^2)}{30\Delta^2} > 0$, there exists a $l_{SN} > 0$ (where $\Pi_S^* - \Pi_N^* = 0$) such that $\Pi_S^* > \Pi_N^*$ if and only if $l > l_{SN}$;.
2. When $c \in (\frac{1}{4d}, max\{\frac{1}{4d}, \frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)}\}]$, there exists a threshold l_{SN} (where $\Pi_S^* - \Pi_N^* = 0$) such that $\Pi_S^* > \Pi_N^*$ if and only if $l > l_{SN}$;
3. When $c > max\{\frac{1}{4d}, \frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)}\}$, $\Pi_S^* < \Pi_N^*$.

When $d \geq \Delta$, the buyer's expected profits Π_N^* and Π_S^* under the no-commitment and the separate-sourcing mechanisms satisfy the following:

1. When $c < \frac{5}{4(5d-\Delta)}$, there exists a threshold l_{SN} (where $\Pi_S^* - \Pi_N^* = 0$) such that $\Pi_S^* > \Pi_N^*$ if and only if $l > l_{SN}$;
2. When $c > \frac{5}{4(5d-\Delta)}, \Pi_S^* < \Pi_N^*$.

Proof of Proposition EC.5. Comparing the buyer's expected profits in separate-sourcing and no-commitment mechanisms, we obtain

$$\Pi_S^* - \Pi_N^* = \min\{4cd, 1\}\left(\frac{l(4\Delta-l)}{32c\Delta d} - \frac{l(3d(4\Delta-l)-l(3\Delta-l))}{96cd^2\Delta}\right) + \frac{l^2\left(3l^2(-5d+l)+5(16d-3l)l\Delta+20(-6d+l)\Delta^2\right)}{480d^2\Delta^2}.$$

(a) When $\Delta > d$ and $4cd > 1$, the-first order condition yields:
$$\frac{\partial \Pi_S^* - \Pi_N^*}{\partial l} = \frac{l(l-2\Delta)(-c(4d-l)(l-2\Delta)-\Delta)}{32cd^2\Delta^2}.$$
Thus, $\Pi_S^* - \Pi_N^*$ is first decreasing and then increasing in l, and the minimum value is achieved at $l = 2d + \Delta - \sqrt{\frac{\Delta+c(\Delta-2d)^2}{c}}$; by letting $\Pi_S^* - \Pi_N^*|_{l=2d} = 0$, we get $c = \frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)}$.

(1) If $\frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)} > \frac{1}{4d}$, when $\frac{1}{4d} < c \leq \frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)}$, there exists a threshold $l_{SN} > 0$ (where $\Pi_S^* - \Pi_N^* = 0$) such that $\Pi_S^* > \Pi_N^*$ if and only if $l > l_{SN}$. When $c > \frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)}$, we always have $\Pi_S^* < \Pi_N^*$.

(2) If $\frac{5\Delta(3\Delta-2d)}{4d\left(9d^2-25d\Delta+20\Delta^2\right)} < \frac{1}{4d}$, when $c > \frac{1}{4d}$, we always have $\Pi_S^* < \Pi_N^*$.

(b) When $\Delta > d$ and $4cd \le 1$, the-first order condition yields:

$$\frac{\partial \Pi_S^* - \Pi_N^*}{\partial l} = \frac{l(l-2\Delta)(l(l-2\Delta)+4d(-l+\Delta))}{32d^2\Delta^2}.$$

When $c \le \frac{1}{4d}$, for $l \in (0, 2d]$, $\Pi_S^* - \Pi_N^*$ is first decreasing in $l \in (0, 2d+\Delta-\sqrt{4d^2+\Delta^2})$, then increasing in $l \in [2d+\Delta-\sqrt{4d^2+\Delta^2}, 2d]$. Moreover, $\Pi_S^* - \Pi_N^*|_{l=0} = 0$, and $\Pi_S^* - \Pi_N^*|_{l=2d} = \frac{d(15d\Delta-9d^2-5\Delta^2)}{30\Delta^2}$.

We have either of the following two cases depending on the sign of $\Pi_S^* - \Pi_N^*|_{l=2d}$:

(1) If $\Pi_S^* - \Pi_N^*|_{l=2d} > 0$, there exists a $l_{SN} > 0$ (where $\Pi_S^* - \Pi_N^* = 0$) such that $\Pi_S^* > \Pi_N^*$ if and only if $l > l_{SN}$.

(2) If $\Pi_S^* - \Pi_N^*|_{l=2d} \le 0$, we always have $\Pi_S^* \le \Pi_N^*$.

(c) When $\Delta \le d$, and $4cd > 1$, $\Pi_S^* - \Pi_N^*$ is first decreasing and then increasing in l and the minimum value is achieved at $l = 2d+\Delta-\sqrt{\frac{\Delta+c(\Delta-2d)^2}{c}}$. By letting $\Pi_S^* - \Pi_N^*|_{l=2\Delta} = 0$, we can get $c = \frac{5}{4(5d-\Delta)} = C_{SJ}^2$.

(d) When $\Delta \le d$ and $4cd \le 1, \Pi_S^* - \Pi_N^*|_{l=2\Delta} > 0$. And for $l \in (0, 2\Delta]$, $\Pi_S^* - \Pi_N^*$ is first decreasing in $l \in (0, 2d+\Delta-\sqrt{4d^2+\Delta^2})$, then increasing in $l \in [2d+\Delta-\sqrt{4d^2+\Delta^2}, 2\Delta]$. There exists a $l_{SN} > 0$ (where $\Pi_S^* - \Pi_N^* = 0$) such that $\Pi_S^* > \Pi_N^*$ if and only if $l > l_{SN}$.

Next, we compare the buyer's profits between no-commitment and the joint-sourcing mechanisms.

Proposition EC.6. There exists a threshold C_{JN} such that the buyer's expected profits Π_N^* and Π_J^* under the no-commitment and the joint-sourcing mechanisms satisfy the following:

1. When $c \le C_{JN}$, $\Pi_J^* > \Pi_N^*$;
2. When $c > C_{JN}$, there exists a threshold l_{JN} (where $\Pi_J^* - \Pi_N^* = 0$) such that $\Pi_J^* > \Pi_N^*$ if and only if $l > l_{JN}$.

Proof of Proposition EC.6. To compare the buyer's profits in joint-sourcing and no-commitment mechanisms, we need to consider $\Delta > d$ and $\Delta \le d$. When $\Delta > d$, we have

$$\Pi_J^* - \Pi_N^* = \min\{4cd, 1\}\left(\frac{3\Delta-d}{24c\Delta} - \frac{l(3d(4\Delta-l)-l(3\Delta-l))}{96cd^2\Delta}\right) - \frac{(2d-l)^3\,(2d^2+3dl+3l^2-5(2d+3l)\Delta+20\Delta^2)}{480d^2\Delta^2}).$$

(a) When $4cd \ge 1$, the first-order condition is

$$\frac{\partial \Pi_J^* - \Pi_N^*}{\partial l} = \frac{(2d-l)(l-2\Delta)(c(2d-l)(l-2\Delta)+\Delta)}{32cd^2\Delta^2}.$$

We can obtain that $\Pi_J^* - \Pi_N^*$ is first increasing and then decreasing in $l \in [0, 2d]$. The maximal value is achieved at $l = \left(d+\Delta-\sqrt{(d-\Delta)^2+\frac{\Delta}{c}}\right)$. Since $\Pi_J^* - \Pi_N^*|_{l=2d} = 0$, we only need to check the sign of $\Pi_J^* - \Pi_N^*|_{l=0}$. It is easy to see that when $c < \frac{15\Delta^2-5d\Delta}{4d^3-20d^2\Delta+40d\Delta^2}$, we always have $\Pi_J^* > \Pi_N^*$. Otherwise, there exists a threshold l_{JN} (where $\Pi_J^* - \Pi_N^* = 0$) such that $\Pi_J^* > \Pi_N^*$ if and only if $l > l_{JN}$.

(b) When $4cd < 1$, the first-order condition is

$$\frac{\partial \Pi_J^* - \Pi_N^*}{\partial l} = \frac{(2d-l)l(l-2\Delta)(2d+2\Delta-l)}{32cd^2\Delta^2}.$$

Note that $\frac{\partial \Pi_J^* - \Pi_N^*}{\partial l}$ is always smaller than 0 for $l \in [0, 2d]$, implying that $\Pi_J^* - \Pi_N^*$ is decreasing in $l \in [0, 2d]$. And $\Pi_J^* - \Pi_N^*|_{l=2d} = 0$, which implies that $\Pi_J^* > \Pi_N^*$ for $c < \frac{1}{4d}$.

Applying a similar analysis for the case where $\Delta \le d$, we obtain part (ii) of the proposition where

$$C_{JN} = \begin{cases} \frac{15d-5\Delta}{40d^2-20d\Delta+4\Delta^2} & \Delta \le d \\ \frac{15\Delta^2-5d\Delta}{4d^3-20d^2\Delta+40d\Delta^2} & \Delta > d. \end{cases}$$

Having conducted pairwise comparison with the buyer's profits under these three mechanisms, we can get the mutual comparison result in Proposition 7 by combining the results in Proposition EC. 4, Proposition EC. 5, and Proposition EC. 6 together. Note that by Proposition EC. 6, when $c < C_{JN}$, the no-commitment mechanism is always dominated by the joint-sourcing mechanism. Thus, when $c < C_{JN}$ (Notice that $C_{JN} = C_{SJ}^2$ in Proposition EC. 4), it suffices to compare the joint-sourcing and separate-sourcing mechanisms. When $c \ge C_{JN}$, it is easy to see that the separate-sourcing mechanism is always dominated by the no-commitment mechanism. It suffices to compare the joint-sourcing and no-commitment mechanisms in this case.

To summarize, there exist thresholds $C_1 = C_{SJ}^1$ and $C_2 = C_{SJ}^2 = C_{JN}$, where $C_1 < C_2$, such that the buyer's expected profit in the no-commitment mechanism Π_N^*, separate-sourcing mechanism Π_S^* and that in the joint-sourcing mechanism Π_J^* can be compared as follows:

(a) When $c > C_2$, there exists a threshold $l_{JN} > 0$ (where $\Pi_J^* - \Pi_N^* = 0$) such that Π_J^* dominates if and only if $l > l_{JN}$; Π_N^* dominates if and only if $l \le l_{JN}$;

(b) When $c \in [C_1, C_2]$, Π_J^* always dominates;

(c) When $c < C_1$, there exists a threshold $l_{SJ} > 0$ (where $\Pi_J^* - \Pi_S^* = 0$), such that Π_S^* dominates if and only if $l > l_{SJ}$; Π_J^* dominates if and only if $l \le l_{SJ}$.

It is easy to show that both l_{SJ} and l_{JN} are increasing in c. ■

Proof of Proposition 8. Note that the equilibrium in the enhanced-commitment mechanism is the same as that under the separate-sourcing mechanism when $l > 2\Delta$. Thus, the result follows from the proofs of Proposition 1 and Lemma 2. ■

Proof of Proposition 9. The proof follows from Proposition 6 and the equilibrium efforts comparisons for joint-sourcing, separate-sourcing, and enhanced-commitment mechanisms. ■

Proof of Proposition 10 and Proposition 12. We show the proof of Proposition 12. The proof of Proposition 10 is similar. To prove Proposition 12, we first need to prove the following Proposition

EC.7. In other words, we compare the buyer's profit under the enhanced-commitment mechanism with profits under the joint-sourcing and separate-sourcing mechanisms.

PROPOSITION EC.7. (i) The buyer's expected profit Π_H^* under the enhanced-commitment mechanism and expected profit Π_S^* under the separate-sourcing mechanism satisfy the following:

1. When $c \le \frac{3}{8d}$, $\Pi_S^* \le \Pi_H^*$;
2. When $c > \frac{3}{8d}$, there exists a threshold $(2-\frac{3}{4cd})\Delta$ such that if and only if $l > (2-\frac{3}{4cd})\Delta$, $\Pi_H^* > \Pi_S^*$.

(ii) The buyer's expected profit Π_H^* under the enhanced-commitment mechanism and expected profit Π_J^* under the joint-sourcing mechanism satisfy the following:

1. When $c < C_1$, $\Pi_H^* > \Pi_J^*$;
2. When $c \ge C_1$, $\Pi_H^* \le \Pi_J^*$.

Proof of Proposition EC.7. (i) First, we compare the buyer's expected profits in separate-sourcing and enhanced-commitment mechanisms.

$$\Pi_S^* - \Pi_H^* = \min\{4cd, 1\}(\frac{l(4\Delta - l)}{32c\Delta d} - \frac{\Delta}{8cd}) + \frac{l^2}{4\Delta} - \frac{l^3}{24\Delta^2} - \frac{l}{2} + \frac{\Delta}{3}.$$

(a) When $4cd \ge 1$, $\frac{\partial \Pi_S^* - \Pi_H^*}{\partial l} = -\frac{(l-2\Delta)(2cd(l-2\Delta)+\Delta)}{16cd\Delta^2}$. Thus, $\Pi_S^* - \Pi_H^*$ is first decreasing and then increasing. Note that $\Pi_S^*|_{l=2\Delta} = \Pi_H^*$ and $\Pi_S^* - \Pi_H^*|_{l=0} = \frac{1}{24}\left(8 - \frac{3}{cd}\right)\Delta$ (which is negative when $c < \frac{3}{8d}$).

(b) When $4cd < 1$, $\frac{\partial \Pi_S^* - \Pi_H^*}{\partial l} = \frac{l}{8\Delta^2}(2\Delta - l) \ge 0$. Thus, $\Pi_S^* - \Pi_H^*$ is increasing in l. Note that $\Pi_S^* - \Pi_H^*|_{l=2\Delta} = 0$; thus, in this case, we always have $\Pi_S^* \le \Pi_H^*$ regardless of l.

Combining the above two cases, we conclude that 1) when $1-4cd \ge 0$, or when $1-4cd < 0$ and $c \le \frac{3}{8d}, \Pi_H^* \ge \Pi_S^*$; and 2) when $1-4cd < 0$ and $c > \frac{3}{8d}$, there exists a threshold $\left(2-\frac{3}{4cd}\right)\Delta$ such that $\Pi_H^* > \Pi_S^*$ if and only if $l > \left(2-\frac{3}{4cd}\right)\Delta$.

(ii) Next, we compare the enhanced-commitment mechanism with the joint-sourcing mechanism by considering cases of $d < \Delta$ and $d \ge \Delta$.

Case 1: $d < \Delta$.

$$\Pi_H^* - \Pi_J^* = \min\{4cd, 1\}(\frac{\Delta}{8cd} - \frac{3\Delta - d}{24c\Delta}) + \frac{d}{3} - \frac{\Delta}{3} + \frac{d^2(d-5\Delta)}{30\Delta^2}.$$

(a) When $4cd > 1$, we have $\frac{\partial \Pi_H^* - \Pi_J^*}{\partial c} < 0$ and $\Pi_H^* - \Pi_J^*|_{c=\frac{1}{4d}} > 0$. Thus, there exists a threshold $\frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{4\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d}$ such that $\Pi_J^* > \Pi_H^*$ if and only if $c > \frac{5\left(d^2-3d\Delta+3\Delta^2\right)\Delta}{4\left(-d^3+5d^2\Delta-10d\Delta^2+10\Delta^3\right)d}$.

(b) When $4cd \le 1$, we always have $\Pi_H^* > \Pi_J^*$.

Case 2: $d \ge \Delta$. Applying similar analysis as Case 1 , we have that $\Pi_J^* > \Pi_H^*$ if and only if $c > \frac{5}{4(5d-\Delta)}$. Combining cases 1 and 2, we find that the threshold is exactly C_{SJ}^1 shown in Proposition EC.4.

Having compared the enhanced-commitment mechanism with joint-sourcing and separate-sourcing mechanisms, the results in Proposition 10 and Proposition 12 are easily obtained by incorporating the three mechanisms comparisons results (Proposition 7). ■

Proof of Proposition 11. The four mechanism's equilibrium prizes and efforts are shown in Equation (EC.17), (EC.18), (EC.20), (EC.21), (EC.22), (EC.23), (EC.24), and (EC.25) respectively. And recall that we denote $l_1 = 2\left(\Delta - \sqrt{\frac{d^2-3d\Delta+3\Delta^2}{3}}\right)$, $l_2 = 2\left(\Delta - \sqrt{\frac{\Delta^3}{3d}}\right)$ in the proof of Proposition 6. When $4cd \geq 1$, the comparisons degenerate to the proof of Proposition 6.

When $4cd < 1$, it is easy to see that efforts in all four mechanisms attain $\bar{e}$, and the order of equilibrium prizes follows $0 < P_H^* \leq \max\{P_J^*, P_S^*\} \leq P_N^*$ by definitions of equilibrium prizes for four mechanisms.

As for how $P_J^* - P_S^*$ varies with l, we take $\Delta > d$ as an example. $P_J^* - P_S^* = 4cd(\bar{e} - \frac{3\Delta-d}{24c\Delta}) - 4cd(\bar{e} - \frac{l(4\Delta-l)}{8\Delta}) = 4cd(\frac{l(4\Delta-l)}{8\Delta} - \frac{3\Delta-d}{24c\Delta})$, which is increasing in l. And it is positive if and only if l is sufficiently high. ■

Proof of Proposition EC.1. Similar to the proof of Proposition 2 and Proposition 4. ■

Proof of Lemma EC.1. Here, we only present of the proof of the joint-souring mechanism when $d \geq \Delta$. The proofs of other mechanisms with a general cost function follow similar procedures.

Given supplier j' effort e_j, supplier i 's best-response effort is determined by solving the following problem (W.L.O.G. assume that $e_j \geq e_i$):

$$\begin{aligned}
\max_{e_i \in [0,\bar{e}]} \pi_i(\mathbf{e}) = \mathbb{E}_{\mathbf{v}}\left[U_{iJ}(\mathbf{v} \mid e_i, e_j)\right] - \psi(e_i) &= \int_{-2\Delta+e_j-e_i}^{0} \frac{1}{48\triangle^2}(2\Delta + e_i - e_j + \epsilon)^3 \frac{2d+\epsilon}{4d^2} d\epsilon \\
&+ \int_0^{e_j-e_i} \frac{1}{48\triangle^2}(2\Delta + e_i - e_j + \epsilon)^3 \frac{2d-\epsilon}{4d^2} d\epsilon \\
&+ \int_{e_j-e_i}^{2\Delta+e_j-e_i} (\frac{1}{48\triangle^2}\left(8\triangle^3 + 12\triangle^2(e_i - e_j + \epsilon) - (e_i - e_j + \epsilon)^3\right) + P)\frac{2d-\epsilon}{4d^2} d\epsilon \\
&+ \int_{2\Delta+e_j-e_i}^{2d} (\frac{\Delta}{2} + P)\frac{2d-\epsilon}{4d^2} d\epsilon - ce_i^b.
\end{aligned}$$

Taking the second derivative of $\pi_i(\mathbf{e})$ with respect to e_i, and letting $e_j = e_i$, we get: $\frac{6P+\Delta}{24d^2} - (b-1)bce_i^{-2+b}$. By applying similar approaches as the quadratic effort case, the expression for $e_J(P)$ when $\frac{6P+\Delta}{24d^2} - (b-1)bce_i^{-2+b} < 0$ is:

$$e_J(P) = \begin{cases} \left(\frac{\Delta(3d-\Delta)+6dP}{12bcd^2}\right)^{\frac{1}{b-1}} & P < 2bcd\bar{e}^{b-1} - \frac{\Delta(3d-\Delta)}{6d} \\ \bar{e} & P \geq 2bcd\bar{e}^{b-1} - \frac{\Delta(3d-\Delta)}{6d}. \end{cases} \tag{EC.26}$$

When $\frac{6P+\Delta}{24d^2} - (b-1)bce_i^{-2+b} \geq 0$, if the symmetric equilibrium $e_N(P)$ exists, then $e_N(P) = \bar{e}$. Since it attains the same effort $\bar{e}$ by incurring a larger prize P compared with the second term in Equation (EC.26), it suffices to consider $\frac{6P+\Delta}{24d^2} - (b-1)bce_i^{-2+b} < 0$ only.

Denote $\tilde{P}_{bJ} = 2bcd\bar{e}^{b-1} - \frac{\Delta(3d-\Delta)}{6d}$, and $\hat{P}_{bJ} = 2^{\frac{1}{2-b}} bcd((b-1)bcd)^{\frac{b-1}{2-b}} - \frac{\Delta(3d-\Delta)}{6d}$. Optimizing over buyer's expected profit, we can get the optimal prizes in joint-sourcing mechanism: $P_J^* = \min\{\max\{\hat{P}_{bJ}, 0\}, \tilde{P}_{bJ}\}$. Plugging in P_J^* into $e_J(P)$, we can derive the expression for the optimal prize. ■

Proof of Proposition EC.2. Recall that we define $\tilde{c}_i = c_i - (y_i(\mathbf{c}|\mathbf{v})(\alpha v_i) + y_j(\mathbf{c}|\mathbf{v})(\alpha v_j))$ in Section EC.4. Similarly, we have $\tilde{c}_j = c_j - (y_i(\mathbf{c}|\mathbf{v})(\alpha v_i) + y_j(\mathbf{c}|\mathbf{v})(\alpha v_j))$. After normalization, $\tilde{c}_i$ and $\tilde{c}_j$ follow uniform distribution on $[\underline{c}, \overline{c}]$. To derive suppliers' equilibrium effort and the buyer's expected profit in the no-commitment mechanism, the buyer's ex-post profit is:

1. $y_i(\mathbf{c}|\mathbf{v}) = 1, x_i(\mathbf{c}|\mathbf{v}) = 1$, then buyer's ex-post profit is $v_i - (2\tilde{c}_i - \underline{c}) - \alpha v_i$;
2. $y_i(\mathbf{c}|\mathbf{v}) = 1, x_i(\mathbf{c}|\mathbf{v}) = 0$, then buyer's ex-post profit is $v_i - (2\tilde{c}_j - \underline{c}) - \alpha v_i - l$;
3. $y_i(\mathbf{c}|\mathbf{v}) = 0, x_i(\mathbf{c}|\mathbf{v}) = 1$, then buyer's ex-post profit is $v_j - (2\tilde{c}_i - \underline{c}) - \alpha v_j - l$;
4. $y_i(\mathbf{c}|\mathbf{v}) = 0, x_i(\mathbf{c}|\mathbf{v}) = 0$, then buyer's ex-post profit is $v_j - (2\tilde{c}_j - \underline{c}) - \alpha v_j$.

We apply the following four steps.

Step 1: Calculate the allocation rule. Applying a similar analysis as our base model, we get the optimal allocation rule as follows:

1. If $-\frac{l}{1-\alpha} \le v_i - v_j \le \frac{l}{1-\alpha}$, we have
$$x_i(\tilde{\mathbf{c}}|\mathbf{v}) = \begin{cases} 1 & \text{if } v_i - v_j > \frac{2}{1-\alpha}(\tilde{c}_i - \tilde{c}_j) \\ 0 & \text{otherwise.} \end{cases}$$
2. If $v_i - v_j > \frac{l}{1-\alpha}$, we have
$$x_i(\tilde{\mathbf{c}}) = \begin{cases} 1 & \text{if } \tilde{c}_j \ge \tilde{c}_i - \frac{l}{2} \\ 0 & \text{otherwise.} \end{cases}$$
3. If $v_i - v_j < -\frac{l}{1-\alpha}$, we have
$$x_i(\tilde{\mathbf{c}}) = \begin{cases} 1 & \text{if } \tilde{c}_j \ge \tilde{c}_i + \frac{l}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Step 2: Calculate supplier's expected profit in the procurement stage. With the optimal selection rule, we can derive each supplier's expected utility in the procurement stage:

1. When $-\frac{l}{1-\alpha} \le v_i - v_j \le \frac{l}{1-\alpha}$, we get
$$\begin{aligned} U_{iN}(\mathbf{v}) &= \int_{\underline{c}}^{\overline{c}} \frac{\tilde{c}_i - \underline{c}}{\Delta} \Pr(\tilde{c}_j > \tilde{c}_i - \frac{(1-\alpha)(v_i - v_j)}{2}) d\tilde{c}_i \\ &= \begin{cases} \frac{\Delta}{2} & v_i - v_j \ge \frac{2}{1-\alpha}\Delta \\ \frac{8\Delta^3 + 12(1-\alpha)\Delta^2\left(v_i - v_j\right) - (1-\alpha)^3\left(v_i - v_j\right)^3}{48\Delta^2} & 0 \le v_i - v_j < \frac{2}{1-\alpha}\Delta \\ \frac{\left(2\Delta + (1-\alpha)(v_i - v_j)\right)^3}{48\Delta^2} & -\frac{2}{1-\alpha}\Delta < v_i - v_j < 0 \\ 0 & v_i - v_j \le -\frac{2}{1-\alpha}\Delta. \end{cases} \end{aligned}$$

2. When $v_i - v_j > \frac{l}{1-\alpha}$, we get

$$
\begin{aligned}
U_{iN}(\mathbf{v}) &= \int_{\underline{c}}^{\overline{c}} \frac{\tilde{c}_i - \underline{c}}{\Delta} \Pr(\tilde{c}_j \geq \tilde{c}_i - \frac{l}{2}) dc_i \\
&= \begin{cases} \frac{\Delta}{2} & \text{if } l \geq 2\Delta \\ \frac{1}{48\Delta^2}\left(8\Delta^3 + 12\Delta^2 l - l^3\right) & \text{if } l < 2\Delta. \end{cases}
\end{aligned}
$$

3. When $v_i - v_j \leq -\frac{l}{1-\alpha}$, we get

$$
\begin{aligned}
U_{iN}(\mathbf{v}) &= \int_{\underline{c}}^{\overline{c}} \frac{\tilde{c}_i - \underline{c} + l}{\Delta} \Pr(\tilde{c}_j \geq \tilde{c}_i + \frac{l}{2}) dc_i \\
&= \begin{cases} 0 & \text{if } l \geq 2\Delta \\ \frac{1}{48\Delta^2}(2\Delta - l)^3 & \text{if } l < 2\Delta. \end{cases}
\end{aligned}
$$

Step 3: Derive and analyze the equilibrium prize and effort. To calculate suppliers' equilibrium effort in the no-commitment mechanism, when $2d(1-\alpha) < 2\Delta$ and $l \leq 2d(1-\alpha)$, supplier i 's best-response effort is determined by solving the following problem:

$$
\begin{aligned}
&\max_{e_i \in [0, \bar{e}]} \pi_{iN}(\mathbf{e}) = -ce_i^2 + \int_{-e_i+e_j+\frac{l}{1-\alpha}}^{2d} \left(\frac{8\Delta^3 - l^3 + 12\Delta^2 l}{48\Delta^2} + P\right) \frac{2d-\epsilon}{4d^2} d\epsilon \\
&+ \int_{-2d}^{-e_i+e_j-\frac{l}{1-\alpha}} \frac{(2\Delta - l)^3}{(48\Delta^2)} \frac{2d+\epsilon}{4d^2} d\epsilon + \int_0^{e_j-e_i} \frac{(2d-\epsilon)\left(2\Delta + (1-\alpha)(e_i - e_j + \epsilon)\right)^3}{(4d^2)(48\Delta^2)} d\epsilon \\
&+ \int_{-e_i+e_j-\frac{l}{1-\alpha}}^{0} \frac{(2d+\epsilon)\left(2\Delta + (1-\alpha)(e_i - e_j + \epsilon)\right)^3}{(4d^2)(48\Delta^2)} d\epsilon \\
&+ \int_{e_j-e_i}^{-e_i+e_j+\frac{l}{1-\alpha}} \left(\frac{8\Delta^3 + 12(1-\alpha)\Delta^2(e_i - e_j + \epsilon) + (1-\alpha)^3\left(-(e_i - e_j + \epsilon)^3\right)}{48\Delta^2} + P\right) \frac{2d-\epsilon}{4d^2} d\epsilon.
\end{aligned}
$$

Following similar procedures in the main model, we can derive the expressions for $e_N^\alpha(P)$ given the innovation prize P.

$$
e_N^\alpha(P) = \begin{cases} \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4(l+2P)\Delta\right)}{96cd^2(1-\alpha)\Delta} & P \leq 4cd\left(\bar{e} - \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4l\Delta\right)}{96cd^2(1-\alpha)\Delta}\right) \\ \bar{e} & P > 4cd\left(\bar{e} - \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4l\Delta\right)}{96cd^2(1-\alpha)\Delta}\right) \end{cases}.
$$

When $2d(1-\alpha) < 2\Delta$ and $l > 2d(1-\alpha)$, we have

$$
e_N^\alpha(P) = \begin{cases} \frac{-d^2(-1+\alpha)^2+6P\Delta-3d(-1+\alpha)\Delta}{24cd\Delta} & P \leq 4cd\left(\bar{e} - \frac{-d^2(-1+\alpha)^2-3d(-1+\alpha)\Delta}{24cd\Delta}\right) \\ \bar{e} & P > 4cd\left(\bar{e} - \frac{-d^2(-1+\alpha)^2-3d(-1+\alpha)\Delta}{24cd\Delta}\right) \end{cases}.
$$

Similar arguments apply when $2d(1-\alpha) \geq 2\Delta$. To summarize, we have

- When $l \leq \min\{2d(1-\alpha), 2\Delta\}$,

$$
e_N^\alpha(P) = \begin{cases} \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4(l+2P)\Delta\right)}{96cd^2(1-\alpha)\Delta} & P \leq 4cd\left(\bar{e} - \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4l\Delta\right)}{96cd^2(1-\alpha)\Delta}\right) \\ \bar{e} & P > 4cd\left(\bar{e} - \frac{l^2(l-3\Delta)+3d(-1+\alpha)\left(l^2-4l\Delta\right)}{96cd^2(1-\alpha)\Delta}\right) \end{cases}.
$$

- When $2d(1-\alpha) < 2\Delta$ and $l > \min\{2d(1-\alpha), 2\Delta\}$,

$$e_N^{\alpha}(P) = \begin{cases} \frac{-d^2(-1+\alpha)^2+6P\Delta-3d(-1+\alpha)\Delta}{24cd\Delta} & P \leq 4cd\left(\bar{e} - \frac{-d^2(-1+\alpha)^2-3d(-1+\alpha)\Delta}{24cd\Delta}\right) \\ \bar{e} & P > 4cd\left(\bar{e} - \frac{-d^2(-1+\alpha)^2-3d(-1+\alpha)\Delta}{24cd\Delta}\right) \end{cases}.$$

- When $2d(1-\alpha) \geq 2\Delta$ and $l > \min\{2d(1-\alpha), 2\Delta\}$,

$$e_N^{\alpha}(P) = \begin{cases} \frac{\Delta^2+3d(-1+\alpha)(2P+\Delta)}{24cd^2(-1+\alpha)} & P \leq 4cd\left(\bar{e} - \frac{\Delta^2+3d(-1+\alpha)\Delta}{24cd^2(-1+\alpha)}\right) \\ \bar{e} & P > 4cd\left(\bar{e} - \frac{\Delta^2+3d(-1+\alpha)\Delta}{24cd^2(-1+\alpha)}\right) \end{cases}.$$

Step 4: Calculate the buyer's expected profit in the no-commitment mechanism. Given the innovation value $\mathbf{v}$, the buyer's payoff in the procurement stage depends on the relative magnitude of $2d(1-\alpha)$ and 2Δ. When $l \leq \min\{2d(1-\alpha), 2\Delta\}$, we have the following four cases:

Case 1: $\epsilon = \xi_i - \xi_j > \frac{l}{1-\alpha}$**.** The buyer's expected utility in the procurement stage is: $U_N(\xi_i, \xi_j | e_i = e_j = e_N(P)) = (1-\alpha)(e_N^{\alpha}(P) + m) - \underline{c} - \frac{2\Delta}{3} - \frac{l^3 - 6\Delta l^2 + 12\Delta^2 l}{24\Delta^2} + (1-\alpha)\xi_i$.

Case 2: $0 < \epsilon \leq \frac{l}{1-\alpha}$**.** The buyer's expected utility in the procurement stage is:

$$U_N(\xi_i, \xi_j | e_i = e_j = e_N(P)) = \frac{1}{24}\left(-4\Delta + 12(-1+\alpha)\epsilon + \frac{6(-1+\alpha)^2\epsilon^2}{\Delta} + \frac{(-1+\alpha)^3\epsilon^3}{\Delta^2} - 24(-1+\alpha)\xi_i\right).$$

Case 3: $\epsilon = \xi_i - \xi_j < -\frac{l}{1-\alpha}$**.** The buyer's expected utility in the procurement stage is: $U_N(\xi_i, \xi_j | e_i = e_j = e_N(P)) = (1-\alpha)(e_N^{\alpha}(P) + m) - \underline{c} - \frac{2\Delta}{3} - \frac{l^3 - 6\Delta l^2 + 12\Delta^2 l}{24\Delta^2} + (1-\alpha)\xi_j$.

Case 4: $-\frac{l}{1-\alpha} \leq \epsilon < 0$**.** The buyer's expected utility in the procurement stage is:

$$U_N(\xi_i, \xi_j | e_i = e_j = e_N(P)) = \frac{1}{24}\left(-4\Delta + 12(-1+\alpha)\epsilon + \frac{6(-1+\alpha)^2\epsilon^2}{\Delta} - \frac{(-1+\alpha)^3\epsilon^3}{\Delta^2} - 24(-1+\alpha)\xi_i\right).$$

Taking the expectation of U_N over ξ_i and ξ_j and considering the innovation prize P, we can get the buyer's expected profit in the innovation stage.

$$\begin{aligned}
\Pi_N(e_N^{\alpha}(P)) &= (1-\alpha)(e_N^{\alpha}(P) + m) - \underline{c} - \frac{2\Delta}{3} - P \\
&+ \int_{-d}^{d-\frac{l}{1-\alpha}} \left(\int_{\frac{l}{1-\alpha}+\xi_j}^{d} (-\frac{l^3 - 6\Delta l^2 + 12\Delta^2 l}{24\Delta^2} + (1-\alpha)\xi_i) \frac{1}{4d^2} d\xi_i \right) d\xi_j \\
&+ \int_{\frac{l}{1-\alpha}-d}^{d} \left(\int_{-d}^{\xi_j - \frac{l}{1-\alpha}} (-\frac{l^3 - 6\Delta l^2 + 12\Delta^2 l}{24\Delta^2} + (1-\alpha)\xi_j) \frac{1}{4d^2} d\xi_i \right) d\xi_j \\
&+ \int_{-d}^{\frac{l}{1-\alpha}-d} \left(\int_{-d}^{\xi_j} (\frac{1}{24}\left(12(-1+\alpha)\epsilon + \frac{6(-1+\alpha)^2\epsilon^2}{\Delta} - \frac{(-1+\alpha)^3\epsilon^3}{\Delta^2} - 24(-1+\alpha)\xi_i\right)) \frac{1}{4d^2} d\xi_i \right) d\xi_j \\
&+ \int_{\frac{l}{1-\alpha}-d}^{d} \left(\int_{\xi_j - \frac{l}{1-\alpha}}^{\xi_j} (\frac{1}{24}\left(12(-1+\alpha)\epsilon + \frac{6(-1+\alpha)^2\epsilon^2}{\Delta} - \frac{(-1+\alpha)^3\epsilon^3}{\Delta^2} - 24(-1+\alpha)\xi_i\right)) \frac{1}{4d^2} d\xi_i \right) d\xi_j \\
&+ \int_{d-\frac{l}{1-\alpha}}^{d} \left(\int_{\xi_j}^{d} (\frac{1}{24}\left(12(-1+\alpha)\epsilon + \frac{6(-1+\alpha)^2\epsilon^2}{\Delta} + \frac{(-1+\alpha)^3\epsilon^3}{\Delta^2} - 24(-1+\alpha)\xi_i\right)) \frac{1}{4d^2} d\xi_i \right) d\xi_j \\
&+ \int_{-d}^{d-\frac{l}{1-\alpha}} \left(\int_{\xi_j}^{\frac{l}{1-\alpha}+\xi_j} (\frac{1}{24}\left(12(-1+\alpha)\epsilon + \frac{6(-1+\alpha)^2\epsilon^2}{\Delta} + \frac{(-1+\alpha)^3\epsilon^3}{\Delta^2} - 24(-1+\alpha)\xi_i\right)) \frac{1}{4d^2} d\xi_i \right) d\xi_j \\
&= (1-\alpha)(e_N^{\alpha}(P) + m) - \underline{c} - \frac{2\Delta}{3} - P + \frac{-160d^3(-1+\alpha)^3\Delta^2 + l^3\left(-3l^2 + 15l\Delta - 20\Delta^2\right)}{480d^2(-1+\alpha)^2\Delta^2} \\
&+ \frac{-20d^2l(-1+\alpha)^2\left(l^2 - 6l\Delta + 12\Delta^2\right) - 5dl^2(-1+\alpha)\left(3l^2 - 16l\Delta + 24\Delta^2\right)}{480d^2(-1+\alpha)^2\Delta^2}.
\end{aligned}$$

The case when $l > \min\{2d(1-\alpha), 2\Delta\}$ follows similar analysis. To summarize, we have the following results.

- When $l \le \min\{2d(1-\alpha), 2\Delta\}$, $\Pi_N(e_N^\alpha(P)) = (1-\alpha)(e_N^\alpha(P)+m) - \underline{c} - \frac{2\Delta}{3} - P + \frac{-160d^3(-1+\alpha)^3\Delta^2 + l^3\left(-3l^2+15l\Delta-20\Delta^2\right) - 20d^2l(-1+\alpha)^2\left(l^2-6l\Delta+12\Delta^2\right) - 5dl^2(-1+\alpha)\left(3l^2-16l\Delta+24\Delta^2\right)}{480d^2(-1+\alpha)^2\Delta^2}$.
- When $2\Delta \le 2d(1-\alpha)$ and $l \ge 2\Delta$,
  $\Pi_N(e_N^\alpha(P)) = (1-\alpha)(e_N^\alpha(P)+m) - \underline{c} - \frac{2\Delta}{3} - P + \frac{1}{30}\left(-10d(-1+\alpha) - \frac{\Delta^3}{d^2(-1+\alpha)^2} + 5\Delta\left(-2+\frac{\Delta}{d-d\alpha}\right)\right)$.
- When $2\Delta > 2d(1-\alpha)$ and $l \ge 2d(1-\alpha)$,
  $\Pi_N(e_N^\alpha(P)) = (1-\alpha)(e_N^\alpha(P)+m) - \underline{c} - \frac{2\Delta}{3} - P + \frac{d^2(-1+\alpha)^2(d(-1+\alpha)+5\Delta)}{30\Delta^2}$.

Combining the results in Step 3 and Step 4 and optimizing over P, we can obtain the equilibrium prize and effort shown in Proposition EC.2. And the buyer's equilibrium profit is $\Pi_N(e_N^\alpha(P^*))$.

As for the monotonicity of e_N^α and Π_N^α with α, we take $c > \frac{1-\alpha}{4d}$, $2\Delta \le 2d(1-\alpha)$, and $l \ge 2\Delta$ as an example. Taking the derivative of the equilibrium effort on α, we have

$$\frac{\partial e_N^\alpha}{\partial \alpha} = -\frac{\Delta^2}{24cd^2(-1+\alpha)^2} < 0.$$

Taking the derivative of the equilibrium profit on α, we have

$$\frac{\partial \Pi_N^\alpha}{\partial \alpha} = -\frac{40d^3 - \frac{8\Delta^3}{(-1+\alpha)^3} + 5d\Delta\left(\frac{3}{c} - \frac{4\Delta}{(-1+\alpha)^2}\right)}{120d^2} < 0.$$

Thus, both e_N^α and Π_N^α are decreasing in α.

■

Proof of Proposition EC.3. The proof for the joint-sourcing mechanism is similar to the proof of Proposition EC.2 when $|v_i - v_j| \le \frac{l}{1-\alpha}$. The proof for the separate-sourcing mechanism is similar to the proof of Proposition EC.2 when $|v_i - v_j| > \frac{l}{1-\alpha}$, in which case the buyer selects the design with a higher value.

When $l \ge 2\Delta$, the optimal innovation-contest prize and equilibrium effort are

$$P_S^\alpha = \begin{cases} \frac{8cd\bar{e}-\Delta}{2} & c < \frac{1-\alpha}{4d} \\ 0 & c \ge \frac{1-\alpha}{4d}, \end{cases} \quad e_S^\alpha = \begin{cases} \bar{e} & c < \frac{1-\alpha}{4d} \\ \frac{\Delta}{8cd} & c \ge \frac{1-\alpha}{4d}. \end{cases} \tag{EC.27}$$

When $l < 2\Delta$, the optimal innovation-contest prize and equilibrium effort are

$$P_S^\alpha = \begin{cases} \frac{32c\Delta d\bar{e}-l(4\Delta-l)}{8\Delta} & c < \frac{1-\alpha}{4d} \\ 0 & c \ge \frac{1-\alpha}{4d}, \end{cases} \quad e_S^* = \begin{cases} \bar{e} & c < \frac{1-\alpha}{4d} \\ \frac{l(4\Delta-l)}{32c\Delta d} & c \ge \frac{1-\alpha}{4d}. \end{cases} \tag{EC.28}$$

The buyer's equilibrium profit is: $\Pi_S^\alpha = \begin{cases} (1-\alpha)\left(\frac{d}{3}+e_S^\alpha+m\right) - \frac{\Delta}{2} - P_S^\alpha & l \ge 2\Delta \\ (1-\alpha)\left(\frac{d}{3}+e_S^\alpha+m\right) - \frac{\Delta}{6} - \frac{l^3}{24\Delta^2} + \frac{l^2}{4\Delta} - \frac{l}{2} - P_S^\alpha & l < 2\Delta. \end{cases}$

■