Influence of slow light effect on trapping force in optical tweezers

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We investigate the optical trapping of polystyrene microsphere in optical tweezers. The transverse capture gradient forces of polystyrene microsphere of different numerical aperture (NA) are theoretically and experimentally evaluated by the power spectral density (PSD) roll-off method. It is found that the trapping force of the experimental result is much larger than that of the theoretical calculation, which is calculated by electromagnetic scattering theory in the T-matrix framework. Although both theoretical and experimental data show that the transverse capture gradient force on the polystyrene microsphere increase with the increment of the numerical aperture of objective lenses, the rising tendency does not coincide. To tackle this issue, the slow light effect near the focus of a high NA focusing system is employed. Considering such an effect and introducing a modified velocity, the theoretical gradient force is recalculated, which shows good consistent with the experimental results. Our work reveals that the slow light effect in focal region of objective will affect the trapping force of optical tweezers.

OCIS Codes: ? ; ?; ?

Optical trapping, also known as optical tweezer, has become a key technique in many scientific areas, including optics [1], atomic physics [2, 3], biological science [4] and chemistry [5], since the seminal work in 1986 by Sir Ashkin et al. [6]. Optical tweezers, in their simplest configuration, are instruments based on a tightly focused laser beam that is capable to trap and manipulate a wide range of particles in its focal spot [7]. Studies showed that the trapping force depends on particle diameter, polarization direction, depth of the optical trap and numerical aperture (NA) of the objective lenses [8-10]. The optical trap strength and geometry can be controlled by adjusting polarization of the trapping beam [11]. Daniel and his colleagues have demonstrated that the finite transverse size of light beam leads to a modification of their wave vectors resulting in a change to their phase and group velocities [12]. A similar slow light phenomenon is observed near the focus in a tight focusing system [13, 14]. However, to our knowledge, the influence of slow light effect on optical trapping force has not been discussed.

Polystyrene spheres are a typical sample used in optical tweezers experiments because of their well-defined spherical shape and size [4, 15]. In this study, the non-absorbing polystyrene spheres are chosen as the sample to investigate the optical trapping force. The theoretical simulation is performed based on T-matrix method, and the experiment is implemented in a designed optical tweezer system. The comparison between numerical simulation and experimental observation reveals a significant discrepancy. One possible explanation for the discrepancy is the slow light effect in a high NA focusing system.

The optical forces acting on spherical or quasi-spherical particles depend on the particle size [16-18]. When the particle size is much larger than the wavelength of the laser beam, optical trapping forces can be calculated with the ray optics approximation [19]. If the size of the particle is much smaller than the wavelength, the dipole model is adopted [20]. In the intermediate regime, i.e. the particle size is comparable with the light wavelength, a complete wave-optical modeling of the particle-light interaction is required to calculate the trapping forces and different methods can be considered. Among many wave-optical modeling of the particle-light interaction, T-matrix method has been widely used in optical tweezers modeling for rigorously computing electromagnetic scattering by single and composite particles [21]. In this work, the diameter of the particle (2μm) is about twice of the wavelength of the laser beam (976 nm), therefore we adopt the T-matrix method to calculate the optical trapping force.

According to the vectorial Debye theory, the electric field near the focus formed by a linearly polarized Gaussian beam through a high NA objective can be expressed as [22],

$$E_{in}(r)=\frac{-ikf}{2\pi}\int_0^{\theta_{\max}}\int_0^{2\pi}\sin\theta\sqrt{\cos\theta}\,l(\theta)\exp\left[ik\hat{s}\cdot(\mathbf{r}-\mathbf{R})\right]\hat{\theta}\,d\theta\,d\phi, \tag{1}$$

where θ represents the angle of convergence, λ represents the wavelength of incident beam, $k = 2\pi/\lambda$ represents the wave-number, $\theta_{\max} = \sin^{-1}(\mathrm{NA})$ represents the maximal angle determined by the NA of the objective, and f represents the focal length of the lens. Moreover, $l(\theta)$ represents apodization at the pupil of the lens. The unit vector $\hat{s}$ is the wave vector direction of a plane wave, the vector $\mathbf{r}$ denotes the position of observation and the vector $\mathbf{R}$ denotes the center position of the sphere.

The incident and scattered fields can be expanded in vector spherical wave function (VSWF) in the T-matrix method as

$$E_{in}(r)=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[a_{mn}M_{mn}^{1}(kr)+b_{mn}N_{mn}^{1}(kr)\right],$$

(2)

$$E_{sca}(r)=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[e_{mn}M_{mn}^{3}(kr)+f_{mn}N_{mn}^{3}(kr)\right],$$

(3)

where the expansion coefficient a_{mn}, b_{mn} of incident field can be related by the T-matrix. The $M(x)$ and $N(x)$ represent different vector sphere harmonic function, and the superscript 1 and 3 are the type of vector sphere harmonic function [23].

The term $\sqrt{\cos\theta}\,l(\theta)\exp(ik\hat{s}\cdot\mathbf{r})\hat{\theta}$ in Eq. (1) represents a plane wave, and the light field near the focus can be expressed as the superposition of a series of plane waves. Moreover, this term can be expanded as

$$\sqrt{\cos\theta}\,l(\theta)\exp(ik\hat{s}\cdot r)\hat{\theta}=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[A_{mn}M_{mn}^{1}(kr)+B_{mn}N_{mn}^{1}(kr)\right],$$

(4)

with

$$\begin{bmatrix}A_{mn}\\B_{mn}\end{bmatrix}=\left(-4i^{n+1}\right)\pi\sqrt{\cos\theta}\,l(\theta)\begin{bmatrix}m\prod_{n}^{m}(\theta)e^{-im\phi}\\\tau_{n}^{m}(\theta)e^{-im\phi}\end{bmatrix},$$

(5)

where $\prod_{n}^{m}(\theta)$ and $\tau_{n}^{m}(\theta)$ are Legendre function. Substituting Eq. (1) into Eqs. (2) and (3), the expansion coefficient of the incident field can be calculated as

$$a_{mn}=-8\pi\left(mi^{n+1-m}e^{-im\phi_0}\right)\times\int_{0}^{\alpha}\sqrt{\cos\theta}\,l(\theta)\prod_{n}^{3}(\theta)J_{m}(k\rho_0\sin\theta)\exp(-ikz_0\cos\theta)\sin\theta d\theta, \quad (6)$$

$$b_{mn}=-8\pi\left(mi^{n+1-m}e^{-im\phi_0}\right)\times\int_{0}^{\alpha}\sqrt{\cos\theta}\,l(\theta)\tau_{n}^{m}(\theta)J_{m}(k\rho_0\sin\theta)\exp(-ikz_0\cos\theta)\sin\theta d\theta, \quad (7)$$

where (ρ_0,ϕ_0,z_0) denote the position of the center of the particle [22].

The expansion coefficients of incident and scattered fields must be linear and connected to each other by the transition matrix [23],

$$e_{mn}=\sum_{n'=1}^{\infty}\sum_{m'=-n'}^{n'}\left(T_{mnm'n'}^{11}a_{m'n'}+T_{mnm'n'}^{12}b_{m'n'}\right), \quad (8)$$

$$f_{mn}=\sum_{n'=1}^{\infty}\sum_{m'=-n'}^{n'}\left(T_{mnm'n'}^{21}a_{m'n'}+T_{mnm'n'}^{22}b_{m'n'}\right), \quad (9)$$

For calculating and memorizing conveniently, these equations can be written in matrix form as,

$$\begin{bmatrix}e_{mn}\\f_{mn}\end{bmatrix}=[T]\begin{bmatrix}a_{m'n'}\\b_{m'n'}\end{bmatrix}=\begin{bmatrix}T_{mnm'n'}^{11}&T_{mnm'n'}^{12}\\T_{mnm'n'}^{21}&T_{mnm'n'}^{22}\end{bmatrix}\begin{bmatrix}a_{m'n'}\\b_{m'n'}\end{bmatrix}, \quad (10)$$

where $[T]$ is the T-matrix [23], which only depends on the parameter of trapped particle, such as the particle's shape, size and refractive index. According to the expansion coefficients of incident and scattered fields, we can obtain the formulae about the z-components of field [24]. The axial trapping efficiency Q is,

$$\begin{aligned}Q_z=&\frac{2}{\sum_{n=1}^{\infty}\sum_{m=-n}^{n}|a_{nm}|^2+|b_{nm}|^2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{m}{n(n+1)}\\&\times\mathrm{Im}(a_{nm}^{*}b_{nm}-e_{nm}^{*}f_{nm})\\&-\frac{1}{n+1}\left[\frac{n(n+2)(n-m+1)(n+m+1)}{(2n+1)(2n+3)}\right]^{\frac{1}{2}}\\&\times\mathrm{Im}(a_{nm}a_{n+1,m}^{*}+b_{nm}b_{n+1,m}^{*}-e_{nm}e_{n+1,m}^{*}-f_{nm}f_{n+1,m}^{*}),\end{aligned} \quad (11)$$

where Im represents imaginary part and asterisk $*$ denotes the complex conjugate.

The axial force F_z can be calculated as [25],

$$F_z=Q_z n_1 P/c,$$

(12)

where P is the laser power at the sample and c is the speed of light in vacuum, n_1 is the refractive index of the medium in which the particle is immersed. The x and y components of optical trapping force can be calculated by a 90°-rotation of the coordinate system of the z-component [26].

In simulation, we choose the horizontally polarized light with wavelength of 976nm and power of 29mw at the sample. The refractive index of the medium is 1.33. The sample's diameter is 2μm and refractive index is 1.6.

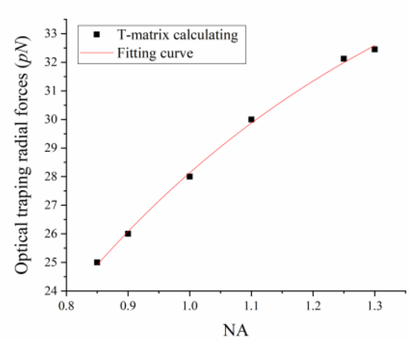

Fig. 1. The fitting curve of optical trapping radial forces versus NA. The wavelength of light is 976nm. The other parameters are chosen as P = 29mw, n_1 = 1.33, c = 3×10^8 m/s. The sample is polystyrene particle whose diameter is 2μm and refractive index is 1.6.

In Fig. 1, the trend of optical trapping radial forces increases with the increasing NA of objective. The result of the simulation coincides with the theory presented in Ref. [27].

The optical trapping force in optical tweezer can be experimentally estimated by stiffness coefficient, which is a proportionality coefficient connecting the force acting on the particle and the displacement of the particle from the center of the trap. Modeled by Hooke's Law, the optical force of trapped particles can be expressed as [28],

$$F = -k_i x_i,$$
(13)

where F is the optical forces, k_i is the trap stiffness coefficient and x_i is the displacement.

To demonstrate the influence of NA on the optical trapping forces, an experiment that reflects the force changing tendency is implemented. The experimental configuration is shown in Fig. 2(a). The laser source is a single-mode laser diode with a maximum output power of $P=340mW$ and wavelength of 976 nm. The light is converted to the horizontally polarized light by the linear polarizer (LP) and expanded by a pair of lens (L1 and L2). The horizontally polarized light is reflected by the dichroic mirror and focused by objectives with NA of 0.85, 0.9, 1.25, and 1.3. The sample is polystyrene particle with diameter of 2μm and refractive index of 1.6. The laser is focused on the sample solution by the objective lens. The sample will be trapped near the focus when the gradient force is larger than scattering and absorption forces. At the conjugate back-focal plane of the condenser lens, the quadrant position detector (QPD) collects the experiment dates, which are send to OTKBFM-CAL (Thorlabs Products) for obtaining the stiffness and position calibration. Then, to provide illumination for CCD, light-emitting diodes shine the sample through the condenser.

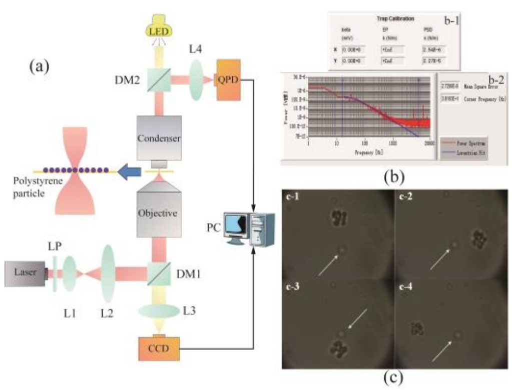

Fig. 2. Schematic of experiment setup and measurements in the experiment. (a) Schematic of experiment setup. L-lens, LP-linear polarizer, DM-dichroic mirror, QPD-quadrant position detector. (b-1) The stiffness coefficient k are 2.54pN/m and 2.27pN/m, respectively. (b-2) PSD as a function of frequency for finding the roll-off frequency f_c, being 3.8193×10 Hz. (c) a captured particle (the arrow indicates the captured particle; the others are uncaptured particle as a reference).

Using the OTKBFM-CAL (Thorlabs) software, the average stiffness coefficients k_i along x-axle are measured as 0.22pN/um, 0.24pN/um, 3.46pN/um and 5.61pN/um for objective lens with NA of 0.85, 0.9, 1.25 and 1.3 respectively. When the particle's position is determined, the change of the stiffness coefficient can reflect the force. Obviously, same with the T-matrix simulation, the stiffness coefficients increase with the increasing NA of the objective.

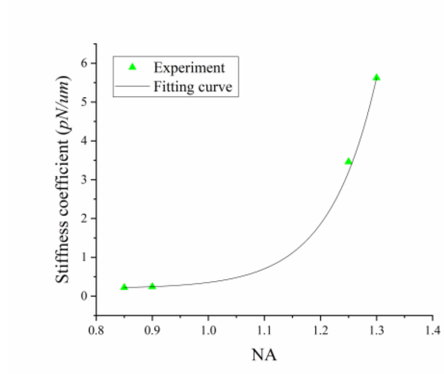

Fig. 3. The fitting curve of stiffness coefficient versus NA in the trap.

A significant jump is observed in the tendency of stiffness coefficient as the NA increases from 0.9 to 1.25. For NA increases from 0.85 to 0.9, the stiffness coefficient shows a moderate increment, which is consistent well with the theoretical prediction by T-matrix. While a sharp increment of stiffness coefficient is observed as NA increases from 0.9 to 1.3, deviating from the theoretical prediction by T-matrix.

One possible explanation of the deviation is the velocity change near the focus of a high NA focusing system. In simulation, the trapping force is calculated by the T-matrix method shown in Eq. (12), the power of laser and the refractive index of the medium are fixed during the experiment, and c denotes the speed of light in the free space. It was demonstrated in Refs. [12-14] that the speed of light focused by the high NA objective slows down, indicating a delay occurs near the focus. The delay is a quadratic function of the ratio of beam size and focal length, which is defined by the NA of optical system.

For studying the relation of NA and velocity, we can establish the link between NA and radio of beam size and focal length from the definition of numerical aperture angle. The equation can be written as,

$$NA = n / \sqrt{1 + (f / \omega)^2}. \tag{14}$$

In Ref. [29], the velocity of light is lower than the speed in the free space and relates to the size of beam and focal length of lens for focused Gausses beam light,

$$v_{eff} = c(1 - \omega^2 / 4f^2), \tag{15}$$

where c is the speed of light in free space. By substituting the ω / f with NA, Eq.(15) can be rewritten as follow,

$$v_{eff} = c\left[1 - \frac{1}{4}\frac{1}{\left(\frac{n}{NA}\right)^2 - 1}\right]. \tag{16}$$

As seen by the Eq. (16), the velocity decreases accordingly with the increasing NA.

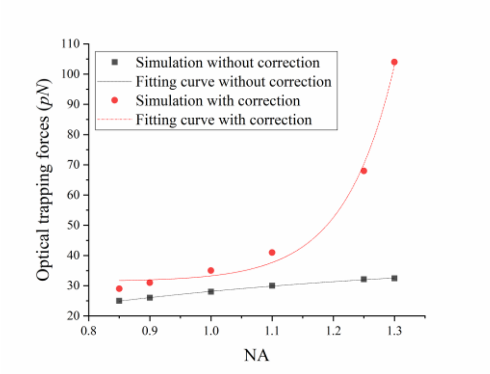

Fig. 4. The fitting curve of optical trapping forces affected by NA. The only difference of the parameters in black color and red color is the velocity. Here the velocity is modified according to the slow light effect.

Considering the slow-light effect, a modified velocity is necessary. Accordingly, the optical force is theoretically recalculated by T-matrix. The simulations with modified velocity are now consistent well with the experimental results, indicating that the trapping force may depends on the velocity. The increment of the force may be explained by energy accumulation because of the slow propagation velocity of light near the focus [30].

In conclusion, based on the T-matrix method, we calculated the optical trapping forces and find that the force increases with increasing NA of objective lens. We also implemented the experiment with a designed optical tweezer. The experimental observation deviates that of theoretical prediction, especially for objective with a large NA. To explain such a phenomenon, we propose hypothesis that the slower velocity near the focus have an effort on the force the modified velocity, the trend of simulation is consistent well with the result of experiment, confirming our assumption. The results are significant for the application of the optical tweezer in domains of cellular and molecular biological and medical research.

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