On ultrasound propagation in composite laminates: advances in numerical simulation

Leandro Maio[a], Paul Fromme[b]

[a]*Department of Industrial Engineering, University of Naples Federico II, via Claudio 21, Naples, 80125, Italy*

[b] *Department of Mechanical Engineering, University College London, WC1E 7JE, UK*

Abstract

The growing use of composite materials for aerospace applications has resulted in the need for quantitative methods to analyze composite components. Ultrasonic guided waves constitute the physical approach for nondestructive evaluation (NDE) and structural health monitoring (SHM) of solid composite materials, such as carbon fiber reinforced polymer (CFRP) laminates. Ultrasound based NDE methods are commonly used in the aerospace field, but ultrasonic wave behavior can be complicated by the presence of material anisotropy, complex geometries (e.g., highly curved parts, stiffeners, and joints) and complex geometry defects. Common defects occurring in aerospace composites include delaminations, porosity, and microcracking. Computational models of ultrasonic wave propagation in CFRP composites can be extremely valuable in designing practical NDE and SHM hardware, software, and methodologies that accomplish the desired accuracy, reliability, efficiency, and coverage. Physics based simulation tools that model ultrasonic wave propagation can aid in the development of optimized inspection methods and in the interpretation of NDE data. This paper presents a review of numerical methodologies for ultrasound and guided wave simulation in fiber reinforced composite laminates summarizing the relevant works to date, different methods, and their respective applications.

Keywords: Composite, laminate, ultrasound, guided waves, numerical methods

1. Introduction

In recent decades, the aerospace, automotive, transportation, and sport industries have seen a rapid growth in the use of composite materials [1][2], such as Carbon Fiber Reinforced Plastics (CFRP). The reason of their success over conventional metals is that this class of materials can enable advanced lightweight structure designs, thanks to their higher stiffness-to-weight ratio. This advantage is a consequence of the fact that their mechanical properties, such as elastic moduli, can be tailored to be high in the directions that are expected to see high loads while remaining considerably lower in other directions. However, despite their benefits, composites pose unique challenges. There are a number of problems, not yet fully resolved, which mitigate, and sometimes cancel these benefits, causing a delay to the introduction of the technologies associated with the use of these materials [3]. Open issues include: i) the partial lack of advanced analysis tools, accepted at certification level; ii) incomplete understanding of damage tolerance; iii) variability of the production processes; iv) the long-term durability of these materials. Common types of defects occurring in composite structures include delamination damage, porosity, and micro-cracking [4][5], for which detection has always been a challenge. The identification of structural defects that can arise during manufacturing or in operation is a critical aspect that affects the development and operation phases of aircraft, from structural design, to development and certification up to operational use, including maintenance.

Practical and reliable nondestructive evaluation (NDE) and structural health monitoring (SHM) methods for the detection and quantification of such defects and damage are of key importance for certification and ensuring the safety of structures with composite parts. Currently, ultrasound techniques constitute the leading physical mechanism used for NDE [6][7][8][9][10][11][12] and SHM [13][14][15] of aerospace composite materials such as CFRP laminates. Ultrasonic guided waves are extensively exploited for damage detection in the maintenance of engineering structures, playing a dominant role for structural health monitoring [16][17]. They can be excited in thin walled flat or curved structures and propagate over long distances [18]. These properties make them well suited for the ultrasonic inspection of aircraft, pressure vessel, tanks, and pipelines. In flat plates, guided waves travel as Lamb and shear horizontal (SH) waves. Lamb waves are vertically polarized, while SH waves are horizontally polarized. Below a cutoff frequency value depending on the frequency-thickness product, only the two fundamental Lamb wave modes exist: S0, which at low frequencies is a symmetrical Lamb wave resembling longitudinal waves and A0, which at low frequencies is an anti-symmetric, flexural wave. While for metals such as aluminum and steel the cutoff frequency-thickness product of higher wave modes is often approximately 1.5 MHz mm, for composite materials the cutoff frequency also depends on the anisotropy direction and typically lies in the region of 0.5 and 1 MHz mm. In practice, due the typically high values of damping, in composites most reported guided wave investigations have been conducted well below the cutoff frequency of the higher wave modes, focusing on the A0, S0, and fundamental SH modes.

Guided wave modes are introduced to a structure by means of a bonded or contact-less source, e.g. piezoelectric transducer, MEMS device, laser, or air-coupled actuator. Along the propagation path, the wave interacts with the structural features, namely geometric discontinuities (e.g. holes, stiffeners), internal material interfaces, defects (e.g. voids, cracks, delaminations), external boundaries and the medium itself. Consequently, the acquired response carries information on the propagation path that can be extracted for damage detection and localization. For widely employed wave propagation-based damage detection techniques, linearity of the wavefield is assumed and wave–structure interaction phenomena, such as reflection, refraction, and diffraction, are studied to correlate specific signal features with possible defects. A broad review on the application of Lamb waves for damage detection in different structural components and materials may be found in [19][20][21]. However, the anisotropy, i.e. directional dependency of the mechanical properties, of composites complicates the propagation of guided waves, leading to a directional dependence of wave speed and wave skew, i.e., difference between the direction of phase and energy velocity. Multiple wave modes can exist, and their dispersive properties and mode shapes through the thickness of the medium vary, even for the same mode but at different excitation frequencies. A thorough understanding of the nature of waves in structures made of anisotropic materials is required for effective structural design and inspection using ultrasonic methods. Propagation phenomena need to be understood and quantified to allow the realization of reliable NDE or SHM inspection systems [22][23][24][25][26].

Numerical methods are often required to analyze the guided wave modes that can be excited in a given composite structure and to simulate their interaction with defects. Highly efficient numerical methods have been developed over time and today many of them are mature enough to address the complexity associated with Lamb wave propagation, i.e., multimodal nature, dispersion, and reflections from boundaries and other structural features that produce a wave field that is quite difficult to analyze. This paper aims to present a review on the state of the art of numerical methods for ultrasound propagation studies in composite laminates.

2. Numerical simulations

Several numerical tools are widely employed in the design, operation, and improvement of SHM and NDT/NDE systems based on ultrasonic waves as a means of optimizing their characteristics, as well as to enhance their ability for damage detection and localization. At the early design stage of monitoring systems, numerical simulations help to adjust the settings, identify optimal sensor positions, or examine the system's response for various damage configurations. If these operations are performed virtually, it is possible to significantly reduce the number of test samples and physical experiments, minimizing the time required for the design process and improving the performance and inspection capabilities of the monitoring system.

In order to design a robust and reliable ultrasound SHM system, the physical mechanisms governing the wave propagation have to be understood in detail, especially in advanced anisotropic and multi-layered structures. This typically requires the simulation of ultrasonic guided waves in lightweight and thin-walled composite structures. Numerical simulations represent a fast and efficient method to predict the wave field, providing an insight into mechanisms that drive the wave interaction with structural features. Therefore, numerical models are a vital part to understand the physical behavior of systems that are not accessible for direct measurements or where measurements are very costly and time-consuming.

Initially, simulation techniques were normally used for wave propagation in isotropic materials. With the recent development of new materials, ultrasonic measurement techniques are now required for the evaluation of materials that often have complex structures and anisotropic properties. In addition, the availability of fast computers with large memory capabilities has accelerated their application [27]. Nowadays, many simulation techniques have been extensively applied for the study of the interaction of wave-geometrical discontinuities, reflection and diffraction from defects inside materials, and these results have provided useful information for wave analysis and comprehension of ultrasonic phenomena observed during ultrasonic testing [28][29].

The next sections review selected numerical methods that are employed to study ultrasound propagation phenomena in composite laminates. The key ideas and the essential mathematical background are provided. However, it is beyond the scope of this contribution to provide a detailed description of each method.

3. Finite Difference Method

Ultrasonic nondestructive testing requires numerical modelling of propagation and scattering of elastic waves. Unfortunately, analytical methods are only able to handle canonical problems, whereas real life problems and geometries have to be tackled with numerical methods. Historically, one of the first and most widely utilized methods for wave propagation simulation is based on finite differences [30]. The foundations of the method were laid around the 1930s by Courant [31]. The approach was further developed during World War II and afterwards improved in the 1950s and 1960s along with the development of computing facilities.

Important progress was made for time-dependent problems [32] and by establishing time integration methods and stability criteria [33][34]. Later, fundamental contributions to the systematization of the theory were introduced [35].

The Finite-Difference Method (FDM) is a numerical method that approximates a differential equation, defined over a continuous domain, by a finite number of equations, defined only at the points of a mesh. Unlike the finite element method, discussed later, spatial meshes used in finite difference methods typically consist of regular grids based on field coordinates. The method's name is a consequence of the fact that the governing partial differential equation (PDEs) are transformed into a set of algebraic equations by approximating partial derivatives with the finite difference (FD) formulas. There is a wide variety of difference formulas that can be used for spatial and temporal derivatives. The FDM is based on the Taylor expansion of differentiable functions. For a single-variable function *f*, the Taylor expansion is given by:

$$f(a+\Delta a) = f(a) + f^{'}(a)\Delta a + ... + f^{(n)}(a)(\Delta a)^{n} + O[(\Delta a)^{n+1}] \quad (1)$$

In Eq. 1, *a* is a generic variable that may stand for the time coordinate *t* or spatial coordinates *x*, *y*, etc., and $O[(\Delta a)^{n+1}]$ is the residual error. This expansion relates the unknown value of a function for a point $a+\Delta a$ to the known function derivatives at *a* and the distance at which the value is evaluated, Δa. It may be truncated after a certain number of terms producing a so-called truncation error. Assuming that the function values are known, it is possible to use the truncation error to evaluate the function's derivative. Using the first-order expansion of Eq. 1, several expressions of the first-order derivative can be found [36][37]:

$$f^{'}(a) = \frac{f(a+\Delta a) - f(a)}{\Delta a} + O(\Delta a) \quad (2a)$$

$$f^{'}(a) = \frac{f(a) - f(a+\Delta a)}{\Delta a} + O(\Delta a) \quad (2b)$$

Approximations in the finite difference method arise from the use of truncated Taylor expansions. The approximation is said to be of order k when the truncation corresponds to errors $O[(\Delta a)^{k}]$. In other words, a k^{th}-order finite difference approximation means that for sufficiently small Δa, the approximation error is divided by α^{k} when Δa is divided by α. Accordingly, finite difference computations based on (2a) and (2b) provide first-order approximations of first-order derivatives. Higher-order approximations may be found by using more than two field values. It is out of the scope of this paper to provide details on such higher-order methods, which can be found elsewhere [38].

3.1 Finite Difference Time Domain Method

When the FDM is applied to both spatial and time derivatives, or simply in the time domain (TD), it is referred by the acronym FDTD, as opposed to the frequency domain. For wave propagation problems solved in the time domain, it provides solutions as field values given at discrete locations in space and discrete instants in time. A full account of the FDTD method can be found in [39].

Eq. 1 may be combined for different points to obtain various finite difference formulas [40][41]. Such formulas may be employed for time or space discretization. Once a given differential equation is transformed into a finite difference algebraic equation, it is implicitly assumed that both space and time have been discretized with steps Δx and Δt, respectively. The discretization consists in subdividing the domain in small pieces, called elements or cells, and discrete steps in time. In FDM it is frequently assumed that the considered domain is composed of cuboidal cells with possibly different edge lengths. Nevertheless, due to the implementation issues, it is sometimes convenient to deal with cubic cells. This means that the governing equation of a given problem is evaluated, in its discrete form, only in nodal points of a regular grid. However, difficulties arise in representing complex geometries through regular grid of points and other. In fact, the approach to domain discretization leads to a well-known "staircase" problem of curvature representation. A significant drawback of a regular mesh is the fact that the whole domain of interest is covered by cells of the same size, and cannot be refined where needed, i.e., in corners, fillets, etc., and coarse in other regions of the model, without special treatment.

An improvement over classical FDTD was achieved using velocity–stress finite differences [42], a discretization scheme proposed by Virieux, employing staggered grids both in time and space (see Fig. 1). The requirement of this scheme is that velocities only access neighboring stresses (and vice versa), providing a natural motivation for the use of what is termed 'staggered grid' FD scheme. The simulation domain is constructed from repeating subunits of grid cells. Within these cells, a given spatial location contains some, but not all types of state variables (e.g. stresses or velocities). For the velocity-stress finite difference time domain (VS-FDTD) method, velocity components are computed from the stress components considered at the neighboring half-nodes and at the previous half-step in time. Similarly, stress components are computed from the velocity components of the neighboring half-nodes considered at the previous half-time step.

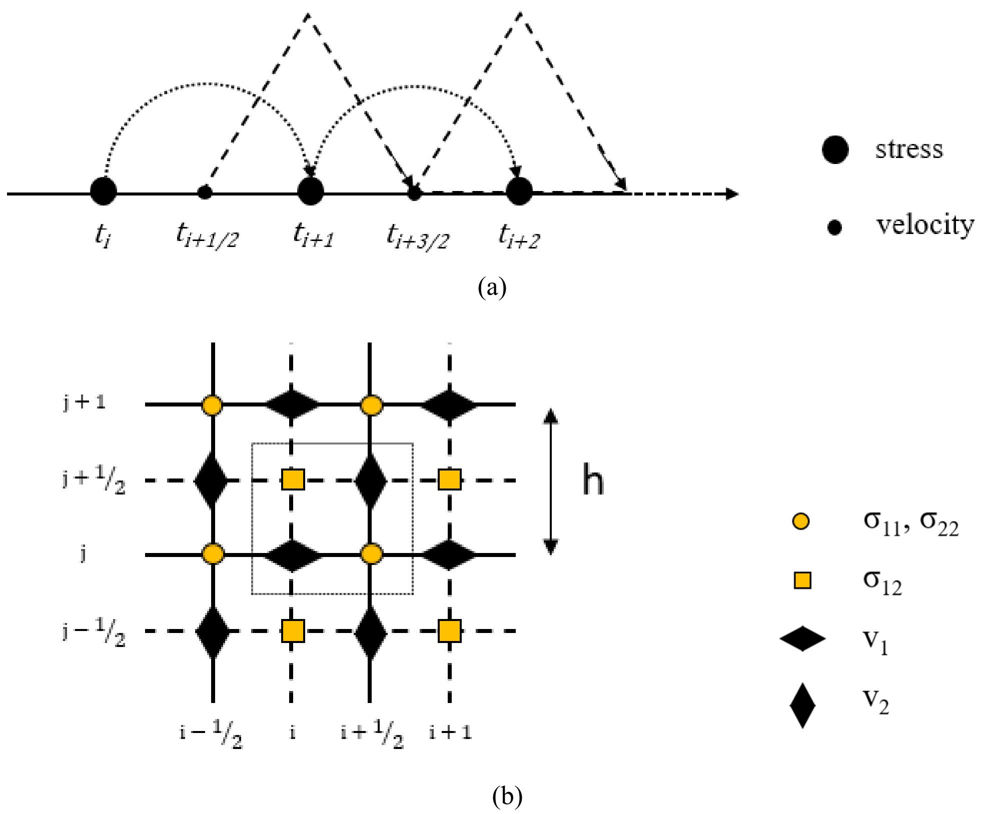


Fig. 1 – Staggered grids in time (a) and space (b) for 2D case.

The staggered grid scheme proposed by Virieux has been used for many decades in the geophysics community to model elastic wave propagation in anisotropic media. However, a study of the modelling of media with a low degree of symmetry (e.g. general anisotropic and transversely isotropic media) demonstrated that the staggered grid approach suffered from an inherent numerical instability [43].

Recently, an improved implementation of VS-FDTD has been employed for the study of ultrasonic propagation in composite laminates. It combines a semi-analytical method developed at CEA and FDTD code developed at Airbus Group Innovations (AGI) [44] for composite applications and is based on the velocity and stress variables. The numerical scheme is developed as a first-order in time system expressed in particle velocities and stresses using the elastodynamic equations (or rather equations of propagation and constitutive law) [45] that are in Cartesian coordinates and in velocity form:

$$\begin{cases} \rho \dfrac{\partial v_i}{\partial t} = \dfrac{\partial \sigma_{ij}}{\partial x_j} + f_i^v \\ \dfrac{\partial \sigma_{ij}}{\partial t} = C_{ijkl} \dfrac{\partial v_k}{\partial x_l} + f_i^\sigma \end{cases} \tag{3}$$

where v_i are the velocity components, ρ the density, f_i^v and f_i^σ represent the components of the force source vector and the moment rate tensor (neglected by AGI [44]) respectively, σ_{ij} are the stress components and C_{ijkl} the components of the stiffness tensor. Staggered grids both in time and space are adopted as discretization, similarly to Virieux's approach [42]. This scheme allows taking into account abrupt changes in impedance such as interfaces between different materials or media (solid-fluid). In order to make the numerical scheme stable, the following conservative conditions to the step size both in space and in time are imposed:

$$\Delta x \le \frac{\lambda_{\min}}{20} \tag{4}$$

$$\Delta t \le \frac{\Delta x}{c_{\min}\sqrt{n}} \tag{5}$$

where λ_{min} is the minimum wavelength in the configuration, c_{min} the minimum velocity, both deducted from the slowness curves due to the anisotropy of the composite material, while n is the dimension (2 or 3). Eq. 4 is called the Courant–Friedrichs–Levy condition. For cases with guided waves, the maximum/minimum guided wave speeds must be known in order to ensure satisfaction of Eq. 4. Finally, proper boundary conditions are implemented, such as the Neumann condition (known as stress-free or free-surface condition) for material/air interfaces, the Dirichlet condition that is the zero-velocity condition equivalent to zero displacement or the Perfectly Matched Layers condition to simulate the absorption of waves in open domains. Further information is available in [44]. Such hybrid methods have allowed to simulate the inspection of CFRP plates with complex geometries, representing a helpful tool for NDT design and performance predictions.

As stated in [47], FD schemes have several benefits, such as: (i) simple mathematics compared to other schemes; (ii) complete control over the mathematics that allow to optimize computationally the code for memory efficiency and speed on any computational hardware; (iii) ability to create parallelized code useful on multi-core architectures; (iv) no mass matrix inversion drawback since they can be coded to implement simple algebraic equations.

3.2 Lebedev Finite Differences

A second order scheme called Lebedev Finite Differences (LFD) has been recently proposed for finite difference based modeling of ultrasound in layered composite laminates for NDE applications [47].The LFD method belongs to the family of staggered grid FD schemes and was originally introduced by Lebedev [48]. The method was further developed for modeling elastic wave propagation in anisotropic media by Lisitsa and Vishnevskiy [49][50], before it reached full maturity.

The LFD equations are derived by substituting the partial derivatives in space and time of Eq. 3, expressed for anisotropic elastic media [49][51], with central difference approximations for the derivative in the *x* direction at time level *n* (equivalently for the other two derivatives) and for the time derivative at point (*ijk*) [47] (each grid point is determined by the three non-negative integral indices i, j, k corresponding to the three discrete coordinates x, y, z):

$$D_x[f]_{ijk}^n = \frac{f_{i+1jk}^n - f_{i-1jk}^n}{\Delta x} \tag{6}$$

$$D_t[f]_{ijk}^n = \frac{f_{ijk}^{n+1/2} - f_{ijk}^{n-1/2}}{\Delta t} \tag{7}$$

where *D* is the 2nd order central difference operator, $[f]_{ijk}^n \equiv f(t^n, x_i, y_j, z_k)$, Δx is the cell size along the x direction, and Δt the time step between steps *n* and *n+1*. Similar operators are used for the y and z directions. The main feature of the LFD method is that all tensor and velocity components are defined and stored at each of the corresponding grid-points along with the necessary averaged material parameters, mandatory for their computation. For the components of the velocity field v_γ with γ = x, y, z:

$$[\rho]_{ijk} D_t[v_\gamma]_{ijk}^{n+1/2} = \sum\nolimits_{\alpha=x,y,z} D_\alpha[\sigma_{\gamma\alpha}]_{ijk}^{n+1/2} \quad (i,j,k) \in \Gamma_u \tag{8}$$

where Γ_u represents the grid for velocities. For the expression of the stresses σ_I in terms of the strains ϵ_J the reduced index notation, I, J = 1, ..., 6 is employed:

$$D_t[\sigma_I]_{ijk}^n = \sum_{J=1}^{6} C_{IJ}[\varepsilon_J]_{ijk}^n \quad (i,j,k) \in \Gamma_\sigma \tag{9}$$

where the strains can be expressed in terms of velocity components and Γ_σ represents the grid for stresses. Subsequent rearrangement yields an explicit 'leapfrog' time-stepping scheme where for each subsequent time step, velocities are calculated from neighboring stress values, and vice versa. That is, the stress and velocity calculations proceed in half-time-step increments.

This grid setup is in stark contrast to other staggered grid schemes, such as the Virieux scheme or the Elastodynamic Finite Integration Technique that will be presented later (sub-section 4.1), where different tensor components are defined at different locations on the grid and the required material parameters need to be interpolated. This important difference is what makes the LFD scheme stable when modeling stress-free boundaries in monoclinic (one plane of material symmetry) and triclinic (or anisotropic) materials. Improved performance (stability and higher accuracy for the same density of nodes [49]) is achieved at the cost of using four interpenetrating staggered grids. The global grid is composed of four staggered grids [50], two of which correspond to the velocity field and two for the stress field of the body. A diagram of the LFD grid is depicted in Fig. 2, where diamonds mark the locations of the velocity grid and solid circles correspond to the stress grid. In contrast to other staggered grid schemes, relevant physical parameters are stored at all grid points to avoid the need for averaging at locations where an operation is performed but no parameters are available. For the LFD scheme, material density is stored at the velocity grid points and the stiffness matrix values (defining the layered anisotropic nature of the composite structure) are stored at stress grid points. Quintanilla and Leckey [47] demonstrated LFD is an appropriate approach for simulating ultrasound in anisotropic composite materials and that the method is stable in the presence of low-symmetry anisotropy and stress-free boundaries, unlike other FDM schemes (see EFIT sub-section 4.1). For this purpose, validation studies based on guided wave behavior in CFRP flat laminates, also in terms of propagating wavefields in multi-layered composites, were conducted and evaluations made by comparison to experimental data and dispersion curves. A later investigation by the same research group [52] focused on curved laminates. It highlighted that, when cell stair-steps (saw-tooths) discretize a curved surface, the Lebedev scheme provides two major impediments: a) mode conversion to artifact modes when repeat interactions occurred at traction free boundaries; b) the inability to apply a conservation-based scheme at bi-material interfaces. Therefore, the researchers concluded that, in the presence of that approximation, the Lebedev finite difference scheme needs additional work. Other curved implementations could be used with the scheme [53]. To overcome the drawbacks stated above, the Rotated Staggered Grid (RSG) method was proposed, demonstrating that it does not present those restrictions. In addition, a further test, run for RSG, showed that the scheme was also stable for triclinic media with traction free boundary conditions, as well as the Lebedev method [47], in contrast to EFIT which can become unstable at lateral plate edges in the same media.

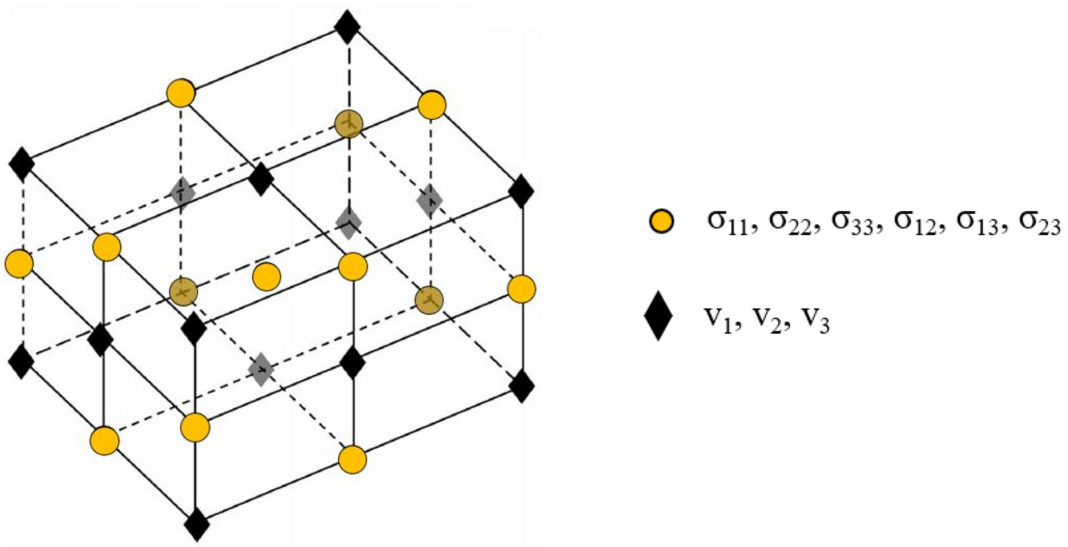


Fig. 2 – 3D Lebedev staggered grid unit cell: diamonds represent velocity locations and circles show the location of stresses.

All methods based on FDs employ the basic governing field equations in differential form [54], while the integral form is also used with the finite integration technique as an example. This leads always to a unique, stable, consistent, and convergent numerical method. Details about this technique will be discussed later.

3.3 *The Local Interaction Simulation Approach*

FDM is very convenient for wave propagation in media with homogeneous or continuously varying physical properties. However, boundaries and discontinuities between different types of media lead to approximate solutions and can produce severe errors [55][56][57]. A local interaction simulation approach (LISA) was proposed by Delsanto et al. [55] to avoid these difficulties. Although the method is formally similar to the FD approach, it does not directly use the FD equations but simulates wave propagation heuristically, i.e. directly from physical phenomena and properties [58]. Indeed, a fundamental difference between LISA and FD is that LISA solves the discrete form of the elastodynamic differential equations exactly and models the physical phenomena without using other approximations, while the FD is the solution of the partial differential equation after discretization [55][59][60]. LISA's efficiency lies in replacing the second-order spatial and temporal derivatives in the elastodynamic equilibrium equations by recursive relations based on FD transformations.

The LISA also employs the so-called Sharp Interface Model (SIM) to address the issue of discontinuities and sharp material boundaries [61], without incurring significant numerical error caused by the smearing or averaging of material properties across cell interfaces. The result is obtained by imposing continuity, or in other words perfect contacts, of displacements and stresses at interfaces and discontinuities, which distinguishes this approach from the formal FD techniques. This allows for a more physical and unambiguous treatment of interface discontinuities for different layers of material, leading to more accurate results when wave propagation problems in complex media with boundaries are studied.

LISA dates back to the early 1990s [55], mostly as a one-dimensional implementation. The approach was then used to study the wave propagation characteristics through the thickness of a single orthotropic layer plate structure [62] and Lamb wave interactions with a hole in a composite plate [63]. More than a decade later, it was applied to study wave propagation in isotropic plates and their interaction with damage with a two-dimensional model [56][57]. Only afterwards LISA was extended to three-dimensional problems in isotropic and unidirectional orthotropic plates [58]. Orthotropic plates used in these studies are comprised of either single layers [58] or multiple plies having the same properties and orientation. They are discretized with uniform cell sizes [62]. Recent work by Nadella and Cesnik [64] has extended and assessed LISA's capabilities to model 3D multi-layered orthotropic structures with arbitrary lamination angles and non-uniform cell aspect ratios. For curved geometries finer discretization of the Cartesian grid is required to avoid the so-called "staircase" problem and short time steps have to be employed to fulfill the Courant stability criterion,

LISA can be considered as a specialized instance of FD formulas for spatial derivatives and an explicit central difference approach for time derivatives [65]. It follows a top–down modeling strategy in which an overarching set of governing differential equations is discretized to yield local iteration formulas. For a 2-D medium, LISA discretizes the governing equations using a uniform grid of rectangular cells. Material properties are assigned to cells, while displacements are assigned to nodes. A given unit cell has a unique material property, so it cannot span across different layers. While the material properties are constant within each cell, they are allowed to vary across cells, which enables the modeling of multilayer anisotropic and heterogeneous structures. Indeed, in most of the practical applications, lay-ups with different orientation are stacked to construct the required composite structure. To address this issue, in-plane rotation of an orthotropic medium is considered in the formulation. Therefore, stiffness matrix rotations are necessary in order to appropriately represent the ply level properties due to a given layup [66]. The LISA iteration equations are derived for a grid point located at the intersection of four adjacent material cells (with possibly different material properties), as depicted in Fig. 3, using the classical elastodynamic wave equation in displacement form, which can be stated as:

$$\rho \frac{\partial^2 \boldsymbol{X}}{\partial t^2} = \nabla \cdot \boldsymbol{\sigma} \tag{10}$$

where ρ represents the mass density, $\boldsymbol{X}=[u,v]^T$ is the particle displacement vector in 2D space, $\boldsymbol{\sigma}$ denotes the stress tensor and t time. Assuming small strains, the infinitesimal strain tensor can be expressed as follows:

$$\varepsilon_{ijkl} = \frac{1}{2}\left(\frac{\partial X_k}{\partial D_l} + \frac{\partial X_l}{\partial D_k}\right) \tag{11}$$

where X_k indicates the k^{th} component of the particle displacement vector and D_k represents the k^{th} spatial direction. The constitutive relation for composite materials completes the equations set:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{12}$$

where σ_{ij} are the stress components and C_{ijkl} the components of the stiffness tensor.

The equilibrium equation (Eq. 10), combined with geometric (Eq. 11), and constitutive relationship (Eq. 12), fully describes the wave propagation via particle displacements. The iteration formulas for a grid point are then formed by a specific averaging scheme. For guided wave propagation in bounded media, Eqs. 10 to 12 are supplemented by a set of boundary conditions. For the Lamb waves of interest, stress-free boundary conditions for two parallel surfaces are considered such that:

$$\sigma_{ij} n_j = 0 \quad at \;\; y = \pm h \quad i = 1,2 \tag{13}$$

where n_j is the outward pointing normal in the j^{th} direction and the layer enclosed by the two surfaces has thickness 2h. The set of Eqs. 10 to 13 constitutes the Rayleigh-Lamb problem and governs wave propagation for plate structures. For a full mathematical discussion on the method, references [65][67] are suggested.

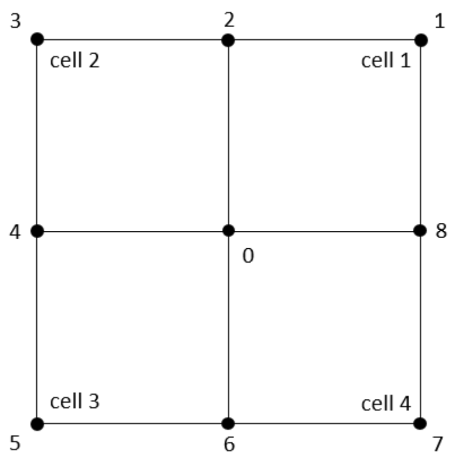


Fig. 3 - Cell and node numbering scheme for the 2-D LISA. Nodes (0–8) are located in the corners of material cells (1–4) [65].

The LISA model has been extended, integrating the electromechanical behavior of a linear piezoelectric material [68] in the elastodynamic equations to simulate the propagation of guided waves generated by piezoceramic transducers in composite structures with damage in different layers [69]. The approach has been demonstrated to be capable of accurately modeling the behavior of ultrasound in composite laminates, representing an effective tool for damage identification, localization, and sensor placement optimization. Recent progress has concerned the LISA hybridization [70] with other numerical methods for the study of wave propagation in complex composite structures, including anisotropic damping effects on guided wave propagation in composite plates [71]. Shen et al. [72][73] have explored the use of the local interaction simulation approach to investigate the nonlinear interactions between guided waves and structural damage. The same research team parallelized LISA using compute unified device architecture (CUDA) running on graphical processing unit (GPU), achieving high computational efficiency [71][74][75] and demonstrating the good multi-core computing capability of the approach. Additional and detailed information on LISA are available in [16].

4. Finite Integration Technique

The Finite Integration Technique, for short FIT, was first proposed by Weiland more than 40 years ago [76], as a method for the simulation of electromagnetic fields and of various coupled problems. The key idea was to use the discretization of the integral, rather than the differential form of Maxwell's equations. This early intuition proved to be correct and to have numerous theoretical, algorithmic, and numerical advantages. A comprehensive review about the FIT application to compute acoustic, electromagnetic, elastodynamic, and coupled wave fields is available in [77]. In the following, its application to wave propagation problems is discussed.

4.1 Elastodynamic Finite Integration Technique

Fellinger et al. [76] used Weiland's idea for the governing equations of ultrasonic waves in solids [78], and developed a numerical procedure called the Elastodynamic Finite Integration Technique (EFIT) [79][80][81]. EFIT is a grid based numerical time-domain method, using a velocity-stress formalism, similarly to VS-FDTD and easily handles different boundary conditions essential to model ultrasonic wave propagation [82]. It can be considered as a special variant of the VS-FDTD method, which is the staggered-grid FD approach discussed previously. The EFIT approach has also been employed by several researchers for NDE applications [83][84][85] and especially by Leckey et al. [86][87][88] for NDE ultrasound modeling in composite plates. They developed and parallelized, using the Message Passing Interface (MPI), a 3D anisotropic elastodynamic finite integration code for ultrasonic waves in CFRP structures. The studies demonstrated the EFIT's effectiveness by comparisons between simulations of guided waves in undamaged anisotropic composite plates and both experimental laser Doppler vibrometer (LDV) wavefield data and dispersion curves. In addition, they proved that realistic damage types and geometries can be incorporated into the EFIT code, potentially allowing for a cost-effective method of investigating damage detection techniques for composites In another investigation [89], EFIT simulations were employed to investigate the wave interaction with multilayer delamination damage in CFRP aerospace composites. They found changes in the trapped energy at the composite surface when additional delaminations exist through the composite thickness.

The major advantage of EFIT is represented by the straightforward equations that can be easily implemented using any programming language, allowing control over all mathematical and computational operations. Moreover, the equations can be easily parallelized for use into multicore computers or computing clusters, enabling the development of an extremely memory-efficient code [89]. Nevertheless, a downside about the use of a custom code is that it may not be sufficiently flexible or user-friendly as commercial simulation software and requires validation.

The governing mathematical equations of elastic waves in general anisotropic media are simply the Cauchy equation of motion and deformation rate. These equations are given in integral form for a finite volume V with surface S by [90]:

$$\frac{\partial}{\partial t}\int_V \rho v_i dV = \int_S T_{ij} n_j dS + \int_V f_i dV \tag{13}$$

$$\frac{\partial}{\partial t}\int_V s_{ijkl} T_{kl} dV = \int_S \frac{1}{2}(v_i n_j + v_j n_i) dS \tag{14}$$

where $\boldsymbol{v}$ is the particle velocity vector, $\boldsymbol{T}$ is the stress second rank tensor, ρ is the density, $\boldsymbol{n}$ is the outward normal vector on the surface S , and $\boldsymbol{f}$ is the body force vector. In Eq. 14, $\boldsymbol{s}$ is the compliance tensor of rank four. The inverse of $\boldsymbol{s}$ is the stiffness tensor $\boldsymbol{c}$, namely $\boldsymbol{s}^{-1} = \boldsymbol{c}$. It is possible to express Eq. 14 by the stiffness tensor $\boldsymbol{c}$ of rank four:

$$\frac{\partial}{\partial t}\int_V T_{kl} dV = \int_S \frac{1}{2} c_{klij}(v_i n_j + v_j n_i) dS = \int_S c_{klij} v_i n_j dS \tag{15}$$

The whole mathematic formulation of EFIT is available in the work by Halkjaer [85]. When particular elements of the stiffness matrix (c_{klij}) of a simulated material are equal to zero, the anisotropic EFIT equations naturally reduce to the corresponding type of anisotropy (e.g. transversely isotropic, orthotropic, etc.). The Courant–Friedrichs–Levy condition in Eq. 4 determines the numerically stable time step size, and the spatial step sizes must meet the requirement shown in Eq. 5.

The accuracy and computational requirements of custom EFIT code has been compared versus FE commercial software packages for simple laminate problems [91]. EFIT has evidenced superior accuracy over two commercial finite-element analysis packages, ABAQUS and COMSOL, in the delamination case, allowing to capture the essential physics involved in the propagation of guided waves in pristine CFRP laminates and the scattering of such waves by delamination defects. However, employing the anisotropic EFIT scheme discussed in [86], local instabilities have been observed at the locations of stress-free boundaries when monoclinic and triclinic plates are modelled [92]. As described by Strikwerda, the source of the instability is likely related to the numerical scheme [93]. However, in accordance with the study on VS-FDTD [43], with which EFIT shares the same formalism, it is noted that no instabilities are observed with EFIT for hexagonal (e.g., transversely isotropic) material cases [86] in accordance with the study of Igel et al. [94] for anisotropic wave propagation.

5. Finite Element Method

The finite element method (FEM), also known as finite element analysis (FEA), is the most popular numerical calculation approach in engineering and science for solving differential equations by utilization of subdomains, called elements that discretize a structure. The first ideas about FEM date back to the nineteenth century, when Kirsch proposed the first concept of substitution of a general solution to a continuum body by a set of primitive geometrical shapes with assumed solution types [95]. In detail, he derived the fundamental equations of elasticity from a system of springs approximating a small cube. However, the intense development of the method began in the 1940s in the field of structural engineering [96][97], when lattices of line (one-dimensional) elements (bars and beams) were defined and then employed for the solution of stresses in continuous solids. In 1943, Courant [98] proposed the variational approach to obtain the solution of stresses and introduced shape functions (or piecewise interpolation function) over triangular sub-regions making up the whole region as a method to obtain approximate numerical solutions. Turner et al. [99] derived stiffness matrices for truss elements, beam elements, and two-dimensional triangular and rectangular elements in plane stress and outlined the procedure commonly known as the direct stiffness method for obtaining the total structure stiffness matrix. It represented the first treatment of two-dimensional elements. This work, along with the development of high-speed digital computers in the early 1950s, determined further development of finite element stiffness equations that were expressed in matrix notation. The term "finite element" was coined, for the first time, by Clough [100] in 1960 when both triangular and rectangular elements were used for plane stress analysis. From the early 1960s to the present, enormous advances have been made in the exploitation of the finite element method to solve complicated engineering problems. Today, it has become an industry standard to solve practical engineering problems [101].

FEM is based on the principle of virtual work:

$$\int \{\delta\varepsilon\}^T \{\sigma\} dV = \int \{\delta \boldsymbol{u}\}^T \{\boldsymbol{P}\} dV + \int \{\delta \boldsymbol{u}\}^T \{\Phi\} dS \tag{16}$$

where $\{\varepsilon\}$ is the strain tensor, $\{\sigma\}$ the stress tensor, $\{\boldsymbol{u}\}$ the displacement vector, $\{\boldsymbol{P}\}$ body force in volume V, $\{\Phi\}$ the surface traction on surface S, δ a functional derivative.

To numerically solve a problem by FEM, the structure under study is dived into discrete elements with nodes. These discrete elements compose a mesh that represents the investigated domain. It is assumed that the solution type is known in the elements. After a structure is meshed into a number of finite elements, all of the loading functions are applied to the nodes of each element instead of on the surface or in the volume of the structure. In addition, FEM only calculates field variables on the nodes. Therefore, an interpolation function is needed to determine values of all the variables inside the elements. This interpolation function is called "shape function". FEM can provide accurate and computationally efficient solutions that may be improved by refining the elements used in the study.

As stated in the reviews on the use of ultrasonic waves for structural health monitoring purposes by Su et al. [102], Willberg et al. [103] and Mitra et al. [104], nowadays, FEM is the most used numerical approach for solving problems of ultrasound propagation in composite plates. In general, there are two possible FEM-based solutions for ultrasound propagation in laminated structures: (a) large scale 3D models that account for the stack of plies using a mesh of solid elements over the volume; and (b) plate elements based on specific theories, which reduce the size of the numerical problem. As a matter of fact, FEA of laminated structures should be classified into three approaches [105][106]: (i) equivalent single layer (ESL); (ii) layer-wise (LW); (iii) continuum-based three dimensional (3D) elasticity. This classification is valid also for the structural theories of multi-layered materials.

In the ESL approach, the multilayered structure is substituted with an equivalent anisotropic single layer or rather the laminate is homogenized into one layer with properties equivalent to the multilayered structure. In this way, the 3D physical problem is collapsed into a 2D mathematical problem [106]. The number of DOFs is thus independent from the number of layers constituting the laminate. Furthermore, ESL description is easily embedded into standard element formulations by simply declaring a composite cross-section. Thus, a good compromise between efficiency and accuracy is often reached.

Based on the order of displacement approximation in the thickness direction, the ESL approach breaks down into several theories. The ESL theories, otherwise known as Single Layer Theories (SLT), are based on the plane-stress assumption to reduce a 3-D continuum problem to a 2-D problem. They include: classical laminated plate theory (CLPT), first-order shear deformation laminated theory (FSDT), second-order shear deformation laminated theory (SSDLT), third-order shear deformation laminated theory (TSDLT), parabolic shear deformation laminated theory (PSDLT), trigonometric shear deformation laminated theory (THDT), hyperbolic shear deformation laminated theory (HSDLT), and higher order shear deformation laminated theory (HSDT). The simplest among them is the CLPT [107], which is an extension of the classical plate theory (CPT) developed by Kirchhoff [108] and Love [109] (also called Kirchhoff-Love plate theory). In CLPT, composite laminates are plates or thin-shell structures whose stiffness properties may be found by integration of the in-plane stress in the normal direction to the laminates surface, e.g. for thin orthotropic (that is, with principal properties in orthogonal directions) plates [110]. In the hierarchy of ESL theories, the next one is the FSDT of plates, which is an extension of the First Order Theory developed by Reissner [111] and Mindlin [112] (also called Reissner-Mindlin plate theory) to study transverse waves along plates. FSDT implies a linear displacement variation through the plate thickness. FSDT based methods are developed for calculating improved transverse shear stresses in laminated composite plates [113] and to accommodate higher frequencies [114]. CLPT and FSDT are available in most standard FEM tools [115][116]. Unfortunately, these ESL theories are limited for the analysis of thick composite laminates. HSDT theories have been defined to overcome this drawback. They have been developed based on the same assumptions as the CLPT and FSDT, but without the assumption that a straight line normal to the mid-plane before deformation remains straight and normal to the plane even after deformation. The HSDT based approaches for calculating dynamic characteristics and shear stresses in relatively thick composite laminates have been presented for dynamic analysis [117][118], for free vibration analysis [119], and for Lamb wave propagation [120].

However, given a generic laminate, the mechanical properties mismatch among the layers leads to a change in slope of the in-plane displacement distribution at layer interfaces, due to the elasticity continuity condition on transverse shear stresses. Therefore, even if the ESL approach is computationally attractive, it provides poor local and global response predictions when employed in the analysis of thick and highly heterogeneous multi-layered composite and sandwich structures [121].

An alternative to ESL theories is the layer-wise (LW) approach, also known as discrete layer theory (DLT). It is a kind of quasi 3D theory as the laminate is modelled layer by layer through the thickness using an ESL theory [122][123]. Thus, the kinematic assumptions are made for each lamina. This ensures accurate responses, allowing it to capture the intralaminar deformations with higher resolution but at the expense of computational cost, that increases with the number of layers (extra DOF associated to each layer). The numerical efforts required makes the LW models not suitable for the analysis of laminated structures of practical applications, wherein a substantial number of layers are used. The layer-wise laminate approaches include also zig-zag theory introduced by Di Sciuva [124][125][126], that enriched, by a through-thickness piecewise linear contribution denoted as "zigzag", the ESL theory, allowing to model moderately thick multilayered anisotropic shells and plates.

ESL and LW kinematics have been tested and compared in a study [127] adopting the FSDT. The investigation has demonstrated that LW solutions are able to provide a 3D resolution in the computation of wave propagation, while higher-order ESL solutions are able to show LW-like levels of accuracy as the number of plies increases.

However, the computational effort required to obtain the dispersion relation of composite laminates using the ESL and the layer-wise theory can be very large. This has resulted into the development of various analytical and numerical approaches for modelling the dispersion and wave propagation properties of periodic layered structures. Such approaches are reviewed in the next paragraphs.

In addition to ESL and LW, the 3D elasticity theory [128], the mixed (hybrid) plate theories [129], unified theories [130], and advanced shear deformation theories represent other methods for the study of laminates. A comprehensive review about the laminated composite plate theories, including ESL and LW ones, is provided by Maji et al. [131].

The solid element modelling approach based on elasticity theory is the most complete. It assumes a full 3D displacement field. Therefore, it is capable of capturing the entirety of wave motion types in the waveguide under investigation in a very accurate and efficient manner [132]. In addition, the 3D approach is very useful when a geometrical representation of a laminate is required, compared to the equivalent single-layer theory that provides just a sectional representation of a multi-layered plate. In fact, each layer is discretized using solid elements [133][134]. This technique is recommended especially when the transverse shear deformation effect is not negligible, because it can accurately predict local effects. Using a tridimensional approach, Zhao et al. [135] verified the dispersion results obtained by the finite element method and studied the directivity characteristics of Lamb waves numerically in laminates. Ng et al. used a 3D FE method to simulate guided waves in a quasi-isotropic laminate [136]. They modelled each individual layer as a quasi-isotropic laminate, using the assumption of homogeneous orthotropic material properties for each ply. Wang et al. [137] computed equivalent 3D quasi-isotropic laminate properties using different approaches [138][139] based on long wave assumptions but employed 3D elements to reduce the computational effort. Leckey et al. conducted benchmark studies about four different commercial FE simulation tools for 3D modelling of guided ultrasonic waves in CFRP laminates [140]. A comparative evaluation was made among the numerical simulations, experimental laser Doppler vibrometry data, and theoretical dispersion curves. The accuracy of numerical results and the respective computational performance of the four different simulation tools were evaluated. An accurate comparison between 3D and FSDT approaches along with experimental results was presented by Maio et al. [141][142]. The investigation, focused on fundamental modes of guided waves propagating in symmetric and nonsymmetric composite laminates, has highlighted the limits of the first-order shear deformation theories in ultrasonic applications for composite structures.

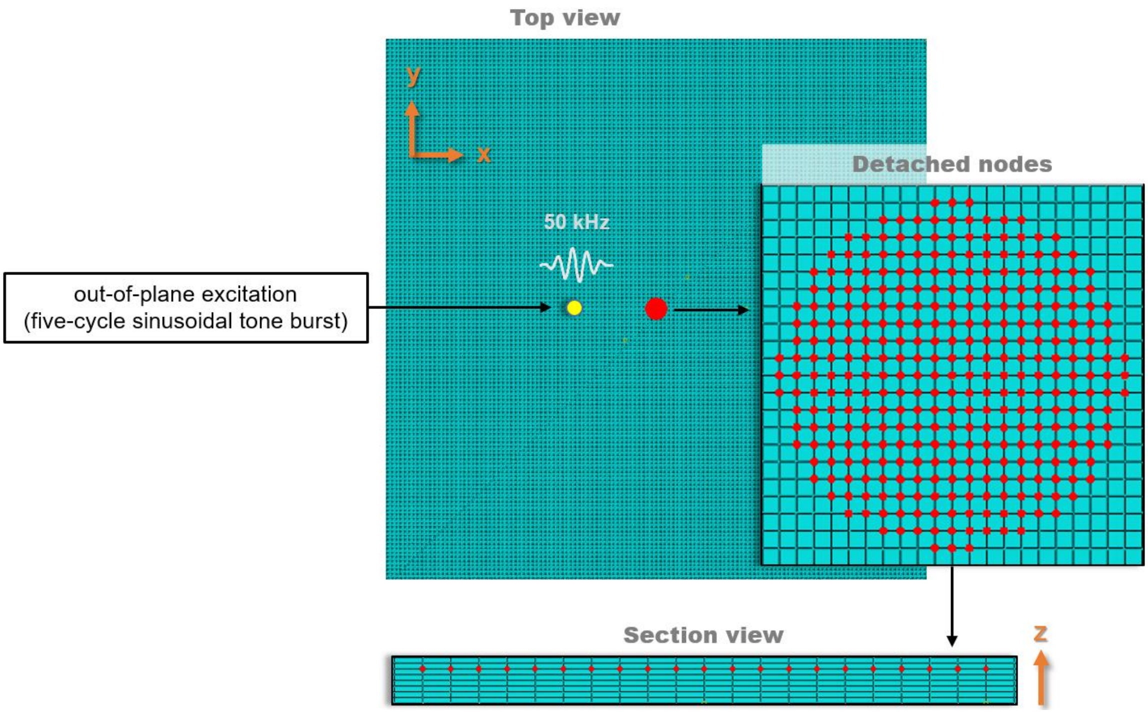
However, the 3D approach introduces a large number of DOFs that depends on the number of layers constituting the laminate. Furthermore, dynamic explicit analysis, often used to simulate wave propagation, requires attention to the spatial and temporal discretization. The minimum wavelength should be large compared to the element dimension and the time step must be less than the wave propagation time across a single element, similar to the FDM stability criteria. Thus, the numerical problem quickly becomes computationally expensive. To improve computational efficiency, the ESL approach is useful to obtain solutions of Lamb waves in composites. Numerical research focusing on the interaction between ultrasound and laminate damage, such as delamination or matrix cracking [143], requires the use of the layer-wise approach [144] or a 3D finite element model [145]. In the literature, different numerical studies about material discontinuities, such as damage, have been carried out to investigate the wave scattering from a delamination, modelling the structure (see Fig. 4) both in 2D [146][147][148][149][150][151] and 3D [152][153][154][155][156][157][158][159][160][161] or from other types of defects [162][163]. More detailed analysis of the scattered wave signal for indications of specific damage types, e.g. mode conversion, or localized inspection using a different NDT technique, e.g. ultrasonic inspection, would typically be required to distinguish between different damage types.

The solid element modelling approach, similar to 3D elasticity theory, and the shell element approach, based on the equivalent single layer theories, are compared numerically in [164] to analyze the propagation of guided waves in composite laminates for delamination detection. While highly accurate in predicting local effects, 3D models are computationally expensive, especially for modelling an entire laminated structure.

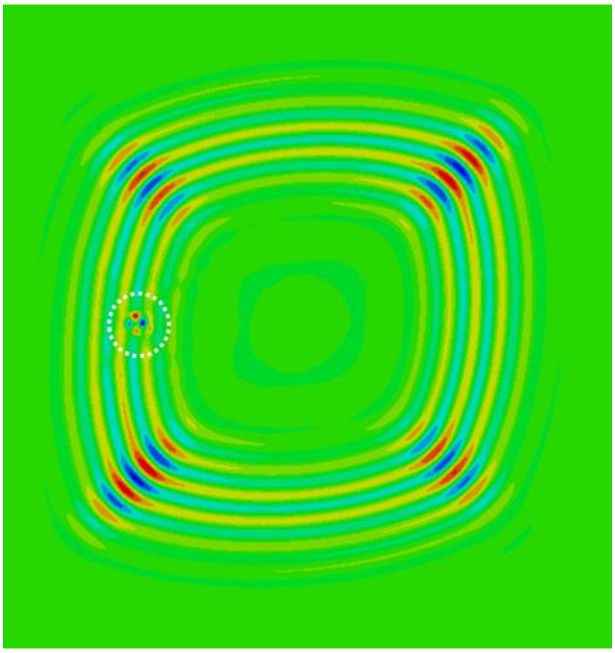
As discussed above, the FE technique is the most commonly reported modelling technique, often adopted to validate and guide experimental studies, as surrogate experimental results to evaluate the efficacy of different post-processing techniques. However, for ultrasonic applications, in spite of the numerous papers available in literature, initially the advancement of FEM was relatively slow due to the limitations in commercial code for the solution of wave propagation problems. The reasons of this is that a conventional finite element model of a laminated structure containing detailed information of each lamina may become computationally prohibitive to comply with the severe accuracy requirements for high frequency and short wavelength ultrasonic waves. Indeed, at high frequencies, the wavelength of the ultrasonic waves is very small, and the size of the elements must be chosen so that the propagating waves are spatially resolved [165]. Therefore, the number of elements needed to solve an ultrasonic problem becomes very large. Explicit time marching and implicit solution schemes can be employed, for which absorbing layers remove unwanted boundary reflections and allow smaller, more computationally efficient models to be used [166][167][168].

However, the trade-off to the computational cost is the advantage in handling structures with complex geometry and the availability of several commercial FE packages and open-source programs using the graphics processing units (GPU) [169]. Nowadays, advances in computational power and numerical optimization routines have enabled the application of rigorous simulation and optimization techniques to large scale problems and as a consequence, the FEM is becoming more popular for solving ultrasonic problems.

FEM may be combined with analytical analysis (e.g. the semi-analytical finite element method [170]) or other numerical methods to achieve optimum solutions, such as the LISA framework, already discussed in the dedicated section [71]. Finite element-based modeling techniques are discussed in the following.



(a)



(b)

Fig. 4 – (a) 3D FE model of an eight-layer quasi-isotropic composite plate, 600 mm x 600 mm, with circular delamination (radius 10 mm); (b) view of displacement wave field (50 kHz), showing interaction between guided wave and delamination (delamination inside dotted white circle).

5.1 Semi-Analytical Finite Element Method

Due to very short wavelengths in the ultrasonic regime, a high resolution in both spatial and temporal domains is required, becoming computationally intensive. Over the years different methodologies have been developed to overcome this issue, the foremost being the semi-analytical finite element method (SAFE) [171][172]. This method requires just the discretization of the cross-section of the investigated waveguide, being very efficient computationally. In detail, the material variation along the plate thickness direction is described using a finite element approach, while in the wave propagation direction analytical, complex-valued exponential functions are used [173]. Therefore, among the existing approaches, the SAFE approach is a suitable candidate for modeling guided wave propagation in composite plates, given the variation in material properties through the thickness direction that characterize them. SAFE can be regarded as a method to obtain normal mode solutions of the plate at each frequency. This method exploits the benefits of numerical and analytical approaches and retains well-known advantages from purely analytical methodologies, such as the infinite dimensions of the structure. In addition, it can be applied for arbitrary cross-sections [172].

The SAFE mathematical framework is presented briefly for the case of an infinitely wide plate (waveguide) that can be composed of anisotropic viscoelastic materials, immersed in vacuum (see Fig. 5). However, the formulation is applicable to arbitrary cross-sections. The cross-section lies in the y–z plane while the wave propagates along direction x with wavenumber ξ and frequency ω.

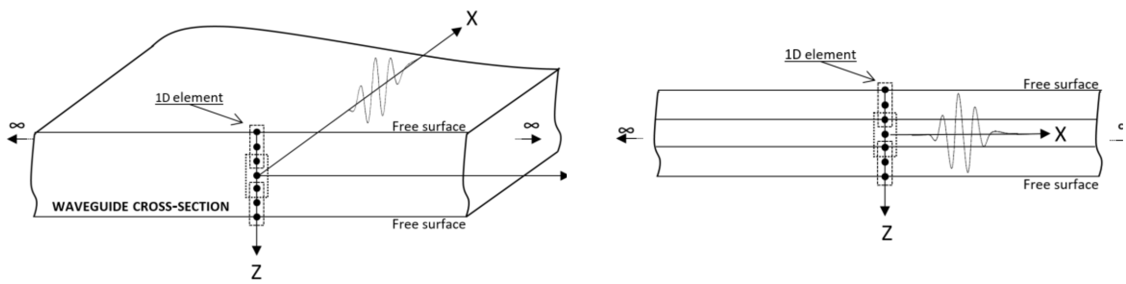
Equations of motion for the cross-section are formulated by inserting the kinetic and potential energies into Hamilton's equation:

$$\delta H = \int_{t_1}^{t_2} \delta(\Phi - K)dt = 0 \tag{17}$$

where Φ is the strain energy and K is the kinetic energy that can be expressed respectively in function of the strain tensor, through constitutive relation, and the displacement field of the waveguide.

Assuming harmonic wave propagation along the propagation direction, x, and employing spatial functions to describe its amplitude in the cross-sectional plane y–z, the displacement field is:

$$\boldsymbol{u}(x,y,z) = \begin{bmatrix} u_x(x,y,z,t) \\ u_y(x,y,z,t) \\ u_z(x,y,z,t) \end{bmatrix} = \begin{bmatrix} U_x(y,z) \\ U_y(y,z) \\ U_z(y,z) \end{bmatrix} e^{i(\xi x - \omega t)} \tag{18}$$



(a) (b)

Fig. 5 - (a) General waveguide cross section and wave propagation along x axis. Discretization is needed only in the waveguide cross-section herein represented by mono-dimensional three-node elements (delimited by dashed line rectangles). (b) SAFE model of an infinite layered plate.

Once the waveguide's cross-sectional domain is represented by a system of finite elements, the displacement expressions in Eq. 18 can be written in the discretized version, over the element domain, in terms of the shape functions and the unknown nodal displacements. Applying Hamilton's principle, summing up the contributions from every element as in the standard FE method, the SAFE governing equation for the plate without external force is obtained as:

$$[\mathbf{K}_1 + i\xi\mathbf{K}_2 + \xi^2\mathbf{K}_3 - \omega^2\mathbf{M}]_M \,\mathbf{U} = \mathbf{0} \tag{19}$$

where the subscript M is the number of total degrees of freedom (dof) of the system, $\mathbf{K}_1$ $\mathbf{K}_2$ $\mathbf{K}_3$ are the global stiffness matrices, $\mathbf{M}$ is the global mass matrix, $\mathbf{U}$ is the vector of global displacements at particular circular frequency ω. Such homogeneous general wave equation can be transformed in a standard eigenvalue problem in ω(ξ), once the imaginary unit is eliminated by matrix manipulation:

$$[\mathbf{K}_1 + \xi\mathbf{K}_2 + \xi^2\mathbf{K}_3 - \omega^2\mathbf{M}]_M \,\mathbf{U} = \mathbf{0} \tag{20}$$

Eigenvalues and eigenvectors obtained from the eigensystem of Eq. 20 correspond to wave numbers and the displacement distribution on the cross-section, respectively. Eigenvalues of the system are the values of ξ which make the equation singular. These eigenvalues are frequency dependent (ω), giving place to the dispersion relation. In other words, phase velocity dispersion curves may be drawn by obtaining the wave numbers (ξ) at different frequency steps. By solving the homogeneous system for each of these eigenvalues, it is possible to find their corresponding eigenvectors that give the displacement profile within the plate for every mode [173]. Therefore, the wave propagation can be simulated by collecting the displacement data for all frequency steps in the frequency bandwidth [174].

For plane waveguides, only one-dimensional (1D) elements are required. Each material layer in the plate is represented by at least one 1D element (Fig. 5). 2D elements, however, are required for modelling complex, 3D waveguides [172].

An important issue with the SAFE theory comes from the resulting wave modes and their distinction from each other, since the eigen solutions include all modes such as propagating, evanescent and non-propagating wave modes [175]. This distinction is crucial for obtaining dispersion curves and selection of desired wave modes or mode control. This problem was studied on a hollow cylinder structure with viscoelastic coatings [176] and solved for guided wave dispersion curves and attenuation characteristics considering both axisymmetric and flexural wave modes. Mode distinction was made by developing an algorithm using the orthogonality relation between modes. Another SAFE study applied this method for a multilayered structure with de-coupled circumferential Lamb and shear wave modes [177].

The SAFE approach is mainly used to obtain dispersion curves of isotropic and composite plates [178], representing a valid alternative to most common methods such as Transfer Matrix Method [179] and the Global Matrix Method [180]. It was demonstrated for the first time in 1973 for waveguides of arbitrary cross-section by Aalami [181]. Dispersive solutions were obtained for the propagating modes only (i.e. real wavenumbers). The same technique was used a decade later to calculate both propagating modes and non-propagating, evanescent modes (complex wavenumbers) in laminated orthotropic cylinders by Huang and Dong [182]. In this latter work, displacements were represented as trigonometric functions in the circumferential and axial directions while the cylinder was modeled with finite elements in the radial direction. A hybrid method was discussed later for the analysis of Lamb wave reflection at a crack at the fixed edge of a composite plate by combining finite element formulations, in a bounded interior region of the plate, with a wave function expansion representation, in an unbounded exterior region [183]. Laminated composite waveguides were studied by SAFE methods [184], especially by Datta et al. [185][186]. The semi-analytical approach with an analytical field applied in the longitudinal direction and layer-wise displacement field employed in transverse direction was discussed by Chitnis et al. [187] to propose a simple numerical technique, able to produce accurate results in comparison with the available analytical solution and also to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis. The focus of previous SAFE modeling was obtaining propagating and evanescent modes in undamped waveguides. Bartoli et al. [172] extended the technique to account for viscoelastic material damping, introducing complex stiffness matrices, in waveguides of arbitrary cross-sections. However, as in conventional FEM applications, convergence has to be considered to guarantee the simulation accuracy [187]. The influence of external loads and a piezoelectric excitation can also be included in the SAFE formulation. For two-dimensional problems this is discussed in [188][189] and three-dimensional (3D) cases are considered in [190][191]. Simulations of the time-transient response of the plate due to an external force and to a piezoelectric wafer active sensor (PWAS) excitation have been investigated using the SAFE approach in two-dimensional [192] and three-dimensional cases [191][193][194]. A semi-analytical technique is presented also by Mei and Giurgiutiu [195] to model guided wave excitation and propagation in damped composite plates. The simulations by SAFE solve the wave propagation problem for plates with infinite (in-plane) dimensions, implicitly assumed in the SAFE method formulation. Additional effects on the propagation of elastic guided waves, i.e., reflections and transmissions from damages, actuators or boundaries make the wave propagation behavior more complex and must be added separately, as stated by Duczek et al. [196].

In these cases, the SAFE method can be combined with classical methods, such as the FEM and other computational approaches, as demonstrated by Ahmad et al. [197][198]. Finally, it is worth to underline that the SAFE method is computed in the frequency domain and thus, in order to recover time signals, it is necessary to perform a Fourier transform, while in real experiments the signals are acquired in time. This can become costly in practice, if the frequency domain sampling is dense (typically in broad-range excitation signals) or when it is necessary to accurately represent the propagation phenomena close to cut-off frequencies.

5.2 Wave and Finite Element Method

Many real engineering systems exhibit characteristics that repeat periodically in either one, two or three dimensions and that can be exploited for simplifications when their dynamic behavior is under study. Some examples are regularly supported composite plates or shells, flat or curved panels (e.g. stringer stiffened panels), pipes, etc. This kind of periodic structures can be considered as an assemblage of identical elements, called cells or periods, which are coupled to each other by identical junctions. The dynamic behavior can thus be predicted through the analysis of a single cell [199].

Indeed, periodic structural components can be considered as uniform waveguides [200][201]. They are uniform in one direction, along the axis x of the waveguide, so that the cross-section of the waveguide has the same physical and geometric properties at all points along the waveguide’s axis. The cross-section might be zero-,one-, or two-dimensional with the motion over the cross-section being a function of one, two or three coordinates that represent respectively the case of rod or beam, cross-section of a laminated solid and extruded section.

Early work about the wave propagation in periodic structures is listed in [202]. The basic idea of studies available in literature [200][201] is that the propagation of waves in a periodic structure can be obtained from propagation constants λ, which depend on the frequency, or by the transfer matrix **T**, each one relating the displacements **q** and the forces **f** on both sides of the periodic element by the relationship:

$$\begin{bmatrix} \boldsymbol{q}_R \\ -\boldsymbol{f}_R \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \boldsymbol{q}_L \\ -\boldsymbol{f}_L \end{bmatrix} = \lambda \begin{bmatrix} \boldsymbol{q}_L \\ \boldsymbol{f}_L \end{bmatrix} \tag{21}$$

where the subscripts L and R denote the left- and right-hand sides of the periodic element, respectively. The propagation constants λ are the eigenvalues of a transfer matrix $\boldsymbol{T}$. Following this idea, an estimate of the wave motion along a uniform waveguide is carried out by considering a transfer matrix to relate the displacements and the forces on both left and right hand–sides of a section of the waveguide [200][201]. To better understand this approach, a structural waveguide whose properties are constant along the waveguide axis x, is considered. A cell of axial length Δ is cut from the structure as indicated in Fig. 6. The cell is meshed, as in conventional FEA, with the only additional constraint being that the nodes, DOFs, and their ordering on the left- and right-hand sides of the section are identical [200]. Normally the length Δ will be equal to the length of one finite element in the x direction. The choice of Δ should then be made, taking into account the same criteria as in conventional applications of finite element analysis. For example, the length must not be too large compared to the shortest wavelength in the structure or the accuracy of the model suffers. Typically, the element should be about one-tenth of this wavelength or shorter. However, if Δ is chosen extremely small compared to the shortest wavelength of interest, the accuracy suffers because of numerical problems that can arise due to machine rounding errors. When the discretization is made, the discrete dynamic equation of a cell obtained from the FE model at a frequency ω is given by:

$$(\mathbf{K} + \mathrm{j}\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{q} = \mathbf{f} \tag{22}$$

where **K**, **M**, and **C** are respectively the stiffness, mass and damping matrices, **f** is the loading vector and **q** the vector of the displacement degrees of freedom. Introducing the dynamic stiffness matrix $\mathbf{D} = (\mathbf{K} + \mathrm{j}\omega\mathbf{C} - \omega^2\mathbf{M})$, it is possible to express, after a certain number of operations on **D** (partitioning and rearrangement mainly), the transfer matrix **T** (relating the nodal degrees of freedom and nodal forces on the two sides of the section) in terms of the dynamic stiffness matrix for a single cell.

Thus, it is evident that one of the key concepts is the matrix **D**. It may be considered a combination of mass, damping, and stiffness matrix for the conventional FEM. In addition, the dynamic stiffness matrix is frequency dependent. Once periodicity conditions are applied, the wave propagation is described by the eigenvalues and eigenvectors of **T**. In this way, dispersion curves and wave modes can be obtained and many of the problems encountered in the classical FEM are eliminated, e.g. mesh refinement for high frequency wave components.

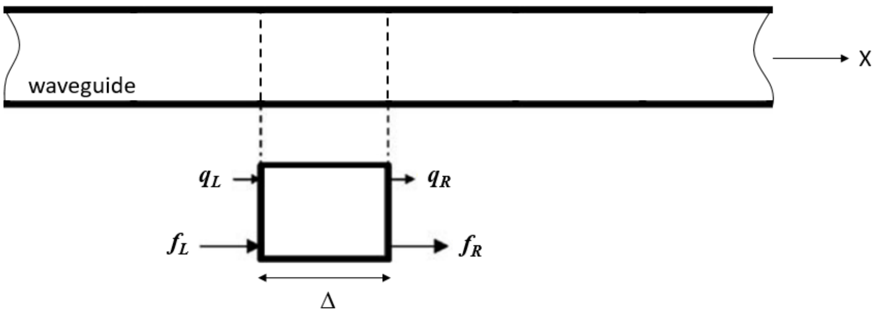


Fig. 6 - Cell or period cut from waveguide (e.g. section of plate).

Compared with the standard FE approach, where the whole waveguide is meshed, the computational cost of the WFE approach is very low, because only a small section of the waveguide has to be meshed. Moreover, another great advantage is that it does not require the creation of FE elements when a new structure is studied, but standard FE packages can be used to obtain the mass and stiffness matrices of one period of the structure.

When the transfer matrix T in Eq. 21 is formed not analytically (possible only for simple cases) but from the dynamic stiffness matrix by FE (generally by standard FE software), discretizing the cross-section of the periodic structure, and then an eigenvalue problem is obtained applying periodicity condition, the approach is called the wave and finite element or waveguide-finite element (WFE) method [200][201][203]. It is evident that the WFE approach couples the periodic structure theory [199] to FEM. However, it is not so clear to establish the method's origins [204][205][206][207], but its current name appeared in the work of Mace et al. [200] and it was successively used by other authors. In [200] an application of the WFE method to wave propagation in a laminated plate using the FE software is shown. WFE has been extended immediately to two-dimensional structures, including composites [208]. It has been applied to a variety of structural situations, such as stiffened structures or forced boundary conditions thanks to Renno and Mace [209][210], for structural identification and wave interaction with localized structural defects by Chronopoulos et al. [211][212], multiscale wave propagation [213] and wave steering in composites [214]. Finally, there have been efforts recently to develop a full transient simulation method based on WFE [215][216], addressing ultrasonic guided waves [217]. WFE can be considered as another semi-analytical method useful to bypass FEM inadequacy in dynamic analyses at medium-high frequencies [218].

5.3 Scaled Boundary Finite Element Method

As an alternative to the previously mentioned numerical approaches, the scaled boundary finite element method (SBFEM) was proposed by Wolf and Song in the 1990s [219][220][221][222] and evolved rapidly over the last years. It is related to the boundary element method (BEM), which has been used for wave propagation in isotropic [223], anisotropic [224] and sandwich structures [225], although the modeling of wave propagation problems in anisotropic materials is not trivial using BEM. The SBFEM is a semi-analytical approach to solving partial differential equations, in which a finite element approximation is deployed for the domain's boundary, while analytical solutions are sought to describe the behavior in the interior of the domain. This method originated from concepts to model unbounded domains using FE technique [226]. Its basic idea is to only discretize the boundary of a computational domain and translate the resulting mesh along a scaling direction to describe the complete geometry. When dealing with a two-dimensional domain, a one-dimensional boundary is discretized (Fig. 7), while the general three-dimensional case requires the discretization of a two-dimensional surface. The main steps are: (i) carrying out a coordinate transformation, from Cartesian (x,y) to scaled boundary coordinates (η,ξ), where η denotes a parametrization of the domain's boundary while ξ is a linear parameter that equals 1 at the boundary and 0 at the origin or scaling center, which is usually inside the domain; (ii) applying the method of weighted residuals or the virtual work principle to obtain a weak form of the governing equation with respect to the coordinate η [227].

The relationship between coordinates is:

$$x = \xi \mathbf{N}(\eta)\mathbf{x}$$
$$y = \xi \mathbf{N}(\eta)\mathbf{y} \tag{23}$$

where **x**, **y** are vectors of nodal coordinates and **N**(η) are shape functions of isoparametric elements used to interpolate the boundary. They describe an arbitrary point in the interior of the domain. Hence, it is possible to obtain a point inside the domain by scaling the boundary in the direction of the scaling center by multiplication with the coordinate x. The name of the Scaled Boundary Finite Element Method is derived from this geometric interpretation [228]. Hence, applying the method of weighted residuals or the virtual work principle in the η direction, the PDE that is to be solved on a domain is transformed into a set of ODEs. The solution u(x,y) that represents a displacement (or temperature) field is approximated as:

$$u(x(\eta,\xi), y(\eta,\xi)) = \mathbf{N}(\eta)\mathbf{u}(\xi) \tag{24}$$

where $\mathbf{N}(\eta)$ is a set of shape functions in one spatial coordinate and **u**(ξ) are the corresponding "nodal" solutions which are a priori unknown functions of the second spatial coordinate.

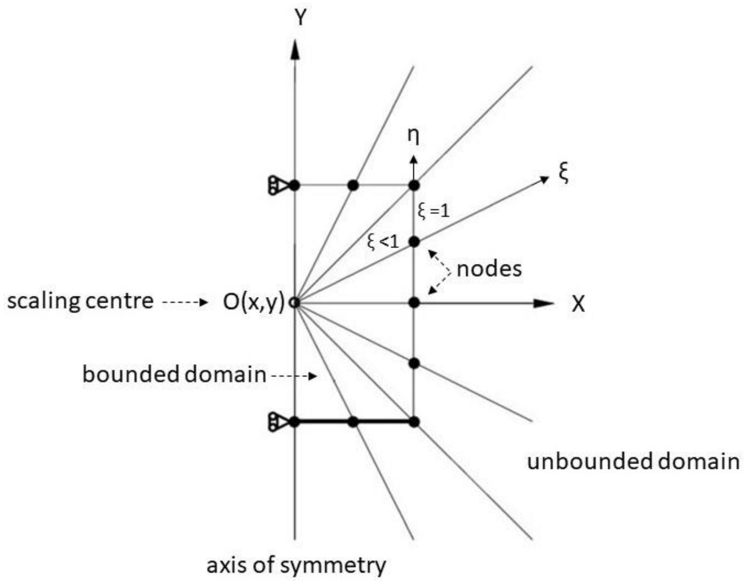


Fig. 7 – Example of spatial discretization for a bounded domain in SBFEM.

In many applications, such as elastostatics [221] and elastodynamics [229] even at high frequency [230], this method drastically reduces computational costs compared with the traditional Finite Element Method. Particularly, SBFEM is advantageous when modeling cracked structures because the side-faces of a simple crack do not require discretization and the stress-singularity can be treated in a proper way. The first attempt to apply this method to the modeling of ultrasonic wave propagation is discussed by Gravenkamp [227]. Thereafter, it has been used for the computation of mode shapes and dispersion curves [228] and for the simulation of elastic guided waves interacting with notches, adhesive joints, delaminations and inclined edges in plate structures [231][232]. In a recent investigation, it has been employed to analyze wave propagation in a plate consisting of an isotropic aluminum layer bonded as a hybrid to an anisotropic carbon fiber reinforced plastics layer [233].

The SBFEM is highly efficient for geometries with a constant cross-section. Large but highly regular structures can be described effectively by discretizing the boundary only. The discretization of cracks is another major advantage of this method, useful in NDT and SHM applications where the interaction of ultrasonic waves with cracks is of interest. In addition, the combination of the previously mentioned features allows for a fast re-meshing in the SBFEM, compared to standard FEM approaches.

5.4 Strip Element Method

The strip element method (SEM) was proposed initially by Kausel et al. [234][235] for solids of isotropic materials, and later by Liu and Achenbach [236][237] for anisotropic solids based on the dynamic finite strip method developed by Cheung [238]. It is a semi-analytic approach for stress analysis of solids and structures, closely related to the SAFE method [171] and the sub-layers technique by Dong and Nelson [239][240].

SEM discretizes the problem domain in one or two directions. Polynomial shape functions are then used in these directions together with the weak forms of system equation to produce a set of dimension-reduced spatial differential equations. These differential equations are afterwards solved analytically. Therefore, the dimension of the final discretized system equations is reduced by one order.

As an example of the SEM application, a two-dimensional problem of wave propagation in a laminated plate is considered in Fig. 8. The excitation and resulting wave fields are independent of y. The plate, consisting of N anisotropic layers, is divided into N strip elements in the thickness direction (z direction). The thickness, the elastic coefficient matrix, the fiber orientation, and the density of the n_{th} element are defined by h_n, $(C_{ij})_n$, ρ_n, and Φ_n respectively. The wave propagates in the x direction. The equation of motion within an element is given in matrix form as:

$$\rho \ddot{\boldsymbol{U}} - \boldsymbol{L}^T \boldsymbol{\sigma} = 0 \tag{25}$$

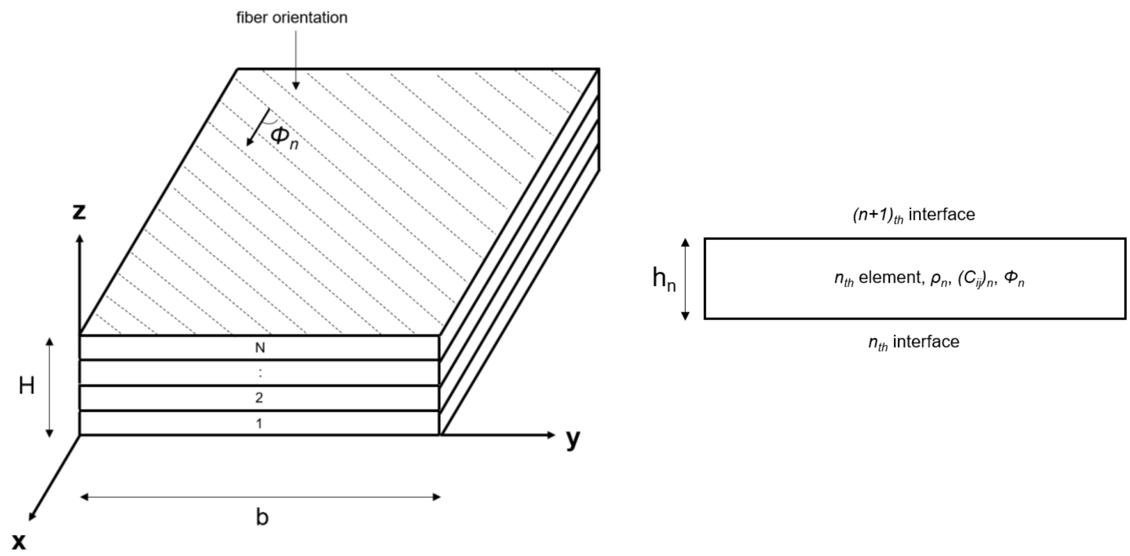
where $\boldsymbol{U}$ is the displacement vector, the dots indicates differentiations with respect to time, “T” stands for the transposed matrix and $\boldsymbol{L}$ is a differential operator matrix:

$$\boldsymbol{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \tag{26}$$

In each strip element, the displacement vector $\boldsymbol{U}_e^T(x,z) = \{u(x,z)\, v(x,z)\, w(x,z)\}$ can be expressed as a product of an interpolation function or shape function, $\boldsymbol{N}$, and the displacement function $\boldsymbol{V}_e$:

$$\boldsymbol{U}_e(x,z) = \boldsymbol{N}(z)\boldsymbol{V}_e(x)\exp(i\omega t) \tag{27}$$

where $i = \sqrt{-1}$, the variables t and ω are time and angular frequency, respectively, and $\boldsymbol{V}_e^T = \{\boldsymbol{V}_L^T \boldsymbol{V}_M^T \boldsymbol{V}_U^T\}$ with $\boldsymbol{V}_L$, $\boldsymbol{V}_M$, $\boldsymbol{V}_U$, functions of x, denoting the displacement amplitude vectors on the lower, middle and upper node lines of the strip element, respectively.



(a) (b)

Fig. 8 – Laminated plate discretized in the thickness direction (a); isolated strip element (b).

By applying the principle of virtual work to each strip element, a set of approximate governing differential equations can be derived for the displacement on the node lines. Assembling all elements and applying boundary conditions on the horizontal boundaries, a set of second-order approximate system governing differential equations for the whole plate can be obtained as:

$$\boldsymbol{q} = \left[-\boldsymbol{A}_{2t}\frac{\partial^2 \boldsymbol{V}}{\partial x^2} + \boldsymbol{A}_{1t}\frac{\partial \boldsymbol{V}}{\partial x} + \boldsymbol{A}_{0t}\boldsymbol{V} - \omega^2 \boldsymbol{M}_t \boldsymbol{V} \right] \tag{28}$$

where q is the external force vector acting on the node lines, and $\boldsymbol{A}_{it}$ (i = 0, 1, 2), $\boldsymbol{M}_t$ and the vector $\boldsymbol{V}$ can be obtained by assembling the corresponding matrices $\boldsymbol{A}_i$, $\boldsymbol{M}$ and vector $\boldsymbol{V}_e$ of adjacent elements (similar to FEM). The set of approximate governing differential equations can be solved analytically. The unknown constants in the complementary solution can be determined using boundary conditions on the vertical boundaries of the plate. The matrices $\boldsymbol{A}$ and $\boldsymbol{M}$ for an element can be found in [236] for monoclinic materials, and in [241] for general anisotropic materials for 2-D and 3-D problems. Further details and the mathematical description can be found in the monograph by Liu et al. [237]. Due to its semi-analytic nature, it is applicable for problems of arbitrary boundary conditions including infinite boundary conditions. It has been adopted to simulate the stress wave propagation in a composite laminate [242] and to analyze scattering by cracks [243] and flaws [244] in laminated plates. In addition, the coupling of SEM with FEM has also been proposed by Liu [245]. This fusion of approaches has allowed the analysis of elastic wave interaction with cracks and inclusions in laminates. In such a combination, the FEM is used for small domains of complex geometry and the SEM is employed for bulky domains of regular geometry. For multilayered composites, no validation with experimental data, but verification against another numerical approach (semi-numerical method) has been carried out [243][244].

6. Distributed Point Source Method

The distributed point source method (DPSM) is a mesh-free modeling technique, based on the concept of Green's function [246], for computing a field in a solution domain. It can be employed for different types of physical problems, both 2D and 3D, including magnetic, acoustic, electrostatic, and electromagnetic fields. DPSM does not require the discretization of the whole problem geometry. This can be computationally advantageous for those applications where localized features have to be studied in a large solution domain. According to the method, first a collection of points, referred as source and target points, is distributed over the domain under study, based on the problem description and solution requirements. The method assumes two types of source points: active and passive [247]. Active sources are used to discretize the wave actuators, transducers, and any other form of energy sources that sends energy into the material. Crack edges are also considered as internal sources (in term of acoustic emission) inside a material. In general, active sources are distributed at the interfaces or boundaries where initial conditions and boundary conditions are available. Passive sources are used to discretize the problem boundaries and interfaces that may act as wave sources during the wave energy interactions, or where matching interface conditions or homogeneous boundary conditions can be specified. In fact, any material interface generates reflected and transmitted ultrasonic fields. Therefore, it can be replaced by two layers of sources distributed on its two sides: one layer to generate the reflected field and the second layer to generate the transmitted field.

Fig. 9 presents schematically an NDE problem setup consisting of a transducer placed in a fluid (e.g. water) to inspect a submerged specimen [247]. When an ultrasonic pulse excites the actuator, the active sources at the transducer are illuminated. This active illumination causes the passive sources at the material interface to become ultrasonically illuminated. When the solution (e.g. displacements and stresses) at a specific point of the domain is sought, that point of interest is called target point. The solution at a target point can be expressed as the superposition of the effect of all individual source points (synthesized field) on that target point. This applies both to active and passive sources. The effect of an individual source point on a target point is obtained by the Green's function. Therefore, a successful implementation of DPSM entails an effective evaluation of the Green's function between many pairs of source and target points.

In some cases, analytical solutions are available for the Green's function. For acoustic wave propagation in a fluid or an isotropic solid, this function is available as a closed-form analytical expression. In those cases, the DPSM method can be considered a semi-analytical method in which the solution building block, i.e. the Green's function, is readily available as an algebraic expression. For more complicated problems, such as wave propagation in an anisotropic solid or defect detection in composite materials, the Green's function needs to be evaluated numerically, as well as the total solution of the DPSM model. However, some analytical processes can be performed on the Green's function before its numerical computation, and the method can still be considered semi-analytical, compared to a pure numerical method that starts directly from the discretization of the governing equations.

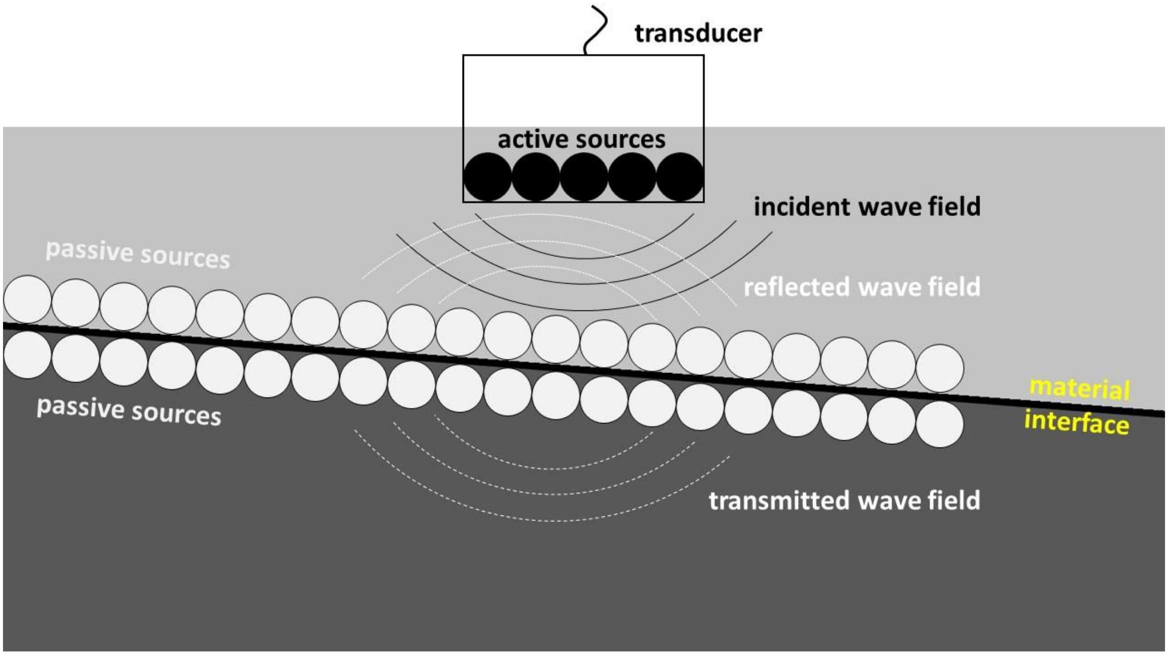


Fig. 9 – Specimen submerged in water and inspected by a transducer. The actuator and interface are idealized as distributed source points.

The DPSM was proposed by Placko and Kundu [248] to model ultrasonic fields in a homogeneous fluid medium and isotropic solid media. It was extended to simulate the ultrasonic fields generated in both fluid and solid media when a transducer is placed in the fluid half-space [249]. For anisotropic media, the evaluation of Green's function is more complicated and needs to be done numerically. Considering the prohibitive computational cost of evaluating the function numerically for a large number of points, a technique called "windowing" was suggested by Fooladi and Kundu [250] to reduce considerably the number of evaluations of Green's function. The resulting approach has been applied both to a problem of ultrasonic waves in an anisotropic plate immersed in a fluid [250] and to evaluate the scattering of elastic waves by a circular cavity in an anisotropic half-space [251]. However, a comparison with traditional numerical approaches or experimental data has not been provided. To improve the computational efficiency, a Symmetry Informed Sequential Mapping of Anisotropic Green's function (SISMAG) was introduced with DPSM by Shrestha and Banerjee [252]. The effectiveness of this approach was demonstrated by virtual NDE experiments of anisotropic plates immersed in a fluid, with normal and angle incidence of the ultrasonic wave, simulated using a circular transducer. Wave fields inside both the fluid and the solid media were calculated and verified with the physical understanding of the wave propagation.

DPSM allows to avoid limitations of various techniques [253] such as incorrect modeling of critical reflection phenomena, spurious reflection of high frequency waves at the multi-scale interfaces, inability to adapt to the change in interface curvature, and the necessity of far field approximation. However, its application to anisotropic laminated structures requires further investigations.

7. Spectral Methods

Although many computational techniques are available to solve various types of problems considered in SHM applications [219], it is very difficult to designate one universal numerical method for such problems. Therefore, it is very important to choose the method or the approach that is suitable and efficient for the purpose. For instance, there are two major approaches to time dependent problems. The first is to analyze a system in the time domain, the second one is to decompose a system into a number of frequency components, solve the problem in the frequency domain, and, if necessary, transform the solution back to the time domain. The advantage of this last approach is that it allows reducing the size of spatial discretization. Based on these considerations, for the problems of wave propagation modelling, it is conceivable that the most popular methods are the spectral ones. In the most general definition, the spectral methods involve defining the solution to a problem as an abridged series of known functions of separate variables [253]. According to Ham et al. [255], they can be classified into three basic approaches: spectral method [256][257][258][259], spectral element methods [260][261][262][263][264] and spectral finite element method (SFEM) [265][267], although these approaches are frequently confused.

Despite of different names, in all three cases higher-order polynomials or harmonic functions are used in the solution space [255]. This is the primary difference between spectral approaches and the FE method. The choice of an appropriate spectral method should be based on both accuracy and efficiency. In order to be useful, a spectral method should be performed to provide results of greater accuracy than conventional difference methods of a similar spatial resolution. The choice of convenient spectral representation depends also on the kind of boundary conditions in the problem.

The spectral method, introduced in the 1970s by Kreiss et al. [256], Orszag [257][258] and Fornberg [259] amongst others, can be used to obtain numerical solutions very close to exact solutions because harmonic functions are used as basis functions and the solutions of wave equations are essentially harmonic functions. Nevertheless, it is difficult to use this method for domains with complicated geometry, as often happens in practice. The reason is that the method uses global basis functions. Hence, in the analysis of solids and structures, the method has found limited practical use. Employed to solve elastic wave problems in seismology, it seems to have been largely ignored for a long time as a means of solving elastic wave problems in acoustic engineering.

A natural extension of the previously mentioned approach is the spectral element method. The spectral element method chooses, as opposed to the previous approach, a high degree piecewise polynomial basis functions, also achieving a very high order of accuracy. Such polynomials are usually orthogonal Chebyshev polynomials or very high order Legendre polynomials over non-uniformly spaced nodes. The computational error decreases exponentially with the order of the approximating polynomial. Therefore, a fast convergence of solution to the exact solution is realized with fewer degrees of freedom of the structure in comparison with FEM. The spectral element method can be divided in two basic approaches. The first one has been popularized by Doyle [260] and is named frequency domain spectral element method (FDSEM). The second approach, called time domain spectral element method (TDSEM), was originally introduced by Patera in 1984 for the solution of laminar flows [261] and utilized later for solving elastic wave propagation problems [262][263].

The spectral element method achieves low numerical dispersion with respect to standard finite element methods and can be very effective in explicit time integration but does not lend itself to modelling complex structures and to the hierarchical increase in the displacement interpolations [264].

In general, the key advantages of the spectral element methods over the conventional FEM are [16]: (i) the high accuracy due to the exact form of the shape functions; (ii) reduction of number of degrees of freedom as one element provides a very accurate solution for a regular part of the domain; (iii) relatively low computational cost, given the resolution offered; (iv) effective for frequency-dependent problems, since it is formulated in the frequency domain; (v) easy formulation of absorbing boundary conditions; (vi) locking-free method; explicit availability of the system transfer functions. Additional details about TDSEM and FDSEM are available in [265].

The spectral finite element method has been developed and is used effectively to solve certain wave propagation problems [266][267]. Its applications are discussed in the next paragraph. However, this method uses a transformation of the governing wave equations to the frequency domain, solution in the frequency domain, and back transformation to the time domain. The method is overall an expensive procedure and difficult to extend to general nonlinear analysis.

7.1 Spectral element method

One of the highly considered methods to model wave propagation is the spectral element method in the time domain [261][268]. TDSEM is similar to the popular isoparametric polynomial finite element method. The main difference is that it uses higher-order shape functions, namely Lagrange polynomials, and a large number of nodes in a single element. Nodal coordinate distributions in an element are chosen as roots of orthogonal polynomials:

$$\begin{cases} (1-\xi^2)-P_N^{'}(\xi)=0 \\ (1-\eta^2)-P_N^{'}(\eta)=0 \end{cases} \tag{24}$$

where $\xi,\eta \in [1,1]$ and where $P_N^{'}$ is the first derivative of the *N*th order Legendre polynomial P_N. As an alternative, Chebyshev polynomials are also used. In this way, the nodes of the element can be specified in the local coordinate system of the element. For instance, adopting the fifth-order Legendre polynomial, 36 nodes can be specified in the local coordinate system of the element ξη, as depicted in Fig. 10 [269].

A set of shape functions can be built on the specified nodes by Lagrange interpolation function to approximate the geometry of the element in the global coordinate system xy and to approximate the transverse and the angular displacements within the element. Further details concerning the mathematical formulation are available in [269]. Nodes of spectral element (interpolations points) coincide with quadrature points used in numerical integration. Gauss-Lobatto nodal distributions and Gauss-Lobatto-Legendre quadrature rules are adopted in the method leading to a diagonal mass matrix.

The process of building and solving a numerical model using TDSEM is similar to FEM [270]. First, the analysed structure is discretized in a finite number of elements (called spectral elements), identified by a number of characteristic points or nodes located at their edges through which the elements are connected. The number of nodes of an element identifies the useful shape functions (aforementioned polynomials) to describe the distribution of the physical quantities within the spectral finite elements, depending on their node values. The ordinary or differential equations describing the analysed physical phenomenon are then transformed to equations of the spectral finite element method. This transformation may be a weak formulation of the method, applying a weighted residual method, or a strong formulation, where there the method of minimising the variation functional is applied. Such equations, being the problem description, are composed at the level of individual elements and are called local equations, whereas the mentioned transformations correspond to the characteristic matrices of the elements, which are derived. Afterwards, the element matrices are aggregated to form the global characteristic matrices. Once the boundary conditions have been implemented, it is possible to start the solution process that leads to obtaining the values of the physical quantities in nodes of individual elements.

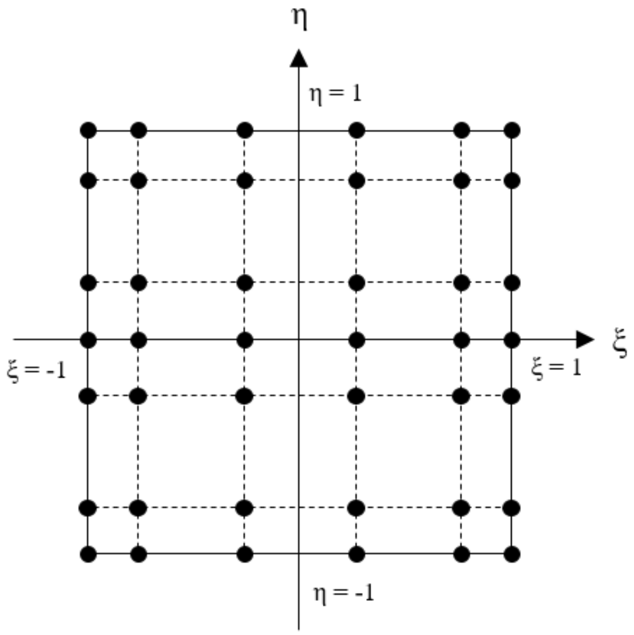


Fig. 10 – A 2D spectral finite element with 36-nodes in the local coordinate system [269].

The method is suitable for wave propagation modelling in composite materials and has the same flexibility as the conventional finite element method, but it is much more computationally efficient. Efficiency comes mostly from the diagonality of mass matrix (no matrix system to invert) that reduces computational cost and storage requirement. It lends itself very well to high performance computing on parallel computers, as demonstrated by Kudela et al. [271][272][273]. Finally, the spectral element method makes it possible to use coarser meshes, typically comprising a single spectral element per wavelength [274].

Gopalakrishnanetal et al. [266][267] focused on the SEM in composites extensively as well as Ostachowicz et al. [268]. The problem of the numerical simulation of the propagation of transverse elastic waves corresponding to the A0 mode of Lamb waves in a composite plate has been solved by Kudela et al. [269] through the TDSEM. Mindlin's theory for plates is employed to express the displacement field within the element. The investigation has shown how the velocities of transverse elastic waves in composite materials depend on the orientation and the relative volume fraction of the reinforcement. Rekatsinas et al. [275] introduced in TDSEM formulation initially a high order lamination theory, in order to accurately simulate symmetric and anti-symmetric Lamb wave propagation in composite laminated strips. Thereafter, the layerwise theory was integrated for improved laminate modelling accuracy [276][277]. A three-dimensional spectral element has been developed by Lonkar [278] (Fig. 11) to provide simulations of the transient coupled electromechanical wave response for thick laminated composite strips with piezoelectric actuators and sensors. Li et al. [279] also developed a three-dimensional spectral element method for the analysis of Lamb wave propagation in composite laminates containing a delamination.

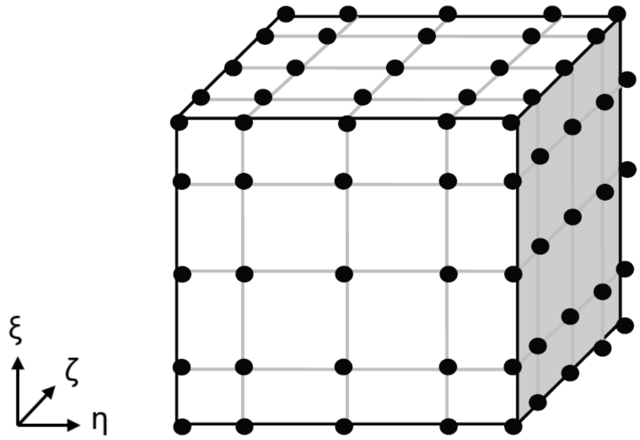


Fig. 11 – A 3D solid spectral element of order 4x4x4: the element in the physical domain is mapped to a parent (ξ,η,ζ) domain, [-1,1]x[-1,1]x[-1,1].

Numerical analysis to study the S0/A0 elastic wave mode conversion at discontinuities in glass fiber reinforced polymer (GFRP) plates have been performed by Wandowski et al. [280]. The investigation has shown that based on effects of S0/A0 mode conversion it is possible to detect the damage and determine its shape and size. The spectral-element method in the time-domain has allowed addressing problems of ultrasonic wave propagation through immersed damaged structures [274]. Sridhar et al. [281] simulated wave scattering from a delamination in an anisotropic medium and showed the computational efficiency of TDSEM. Munian et al. [282] studied the wave interaction with delaminations using TDSEM simulation and quantified the defect in terms of delamination length and thickness position in a laminate. In addition, the method has demonstrated its capability to simulate wave propagation in curved structures and has proven its usefulness in investigating the effect of frequency and curvature on guided wave response [283]. Spectral element modelling of wave propagation in anisotropic shell-structures including different types of damage has been also addressed [284]. Many applications of the TDSEM have regarded composite beams and a summary about them is provided by Nanda [285].

Nevertheless, as reported in [280], it should be emphasized here that despite the significant progress of SEM in the time domain it is still challenging to model high frequency multimodal guided waves in thin composite plates achieving high accuracy.

The frequency domain spectral element method proposed by Doyle [260] is another widely used approach in the dynamic analysis of structures. It is based on the use of Fourier analysis and the fast Fourier transform algorithm. The method is very similar to the technique of FEM as far as the assembly and the solution of the equation of motion is concerned. However, the major difference is that in general the spectral element formulation begins with transforming the governing partial differential equations of motion from the time domain into the frequency domain by using the Fourier transform. This operation allows converting the governing partial differential equations to a set of ordinary differential equations for each discretized frequency. The frequency-domain ordinary differential equations are then solved, and the wave solutions are used to derive frequency-dependent dynamic shape functions. The dynamic stiffness matrix called the "spectral element matrix" is finally formulated by using the dynamic shape functions in an analogous way to another approach known as Dynamic Stiffness Method (DSM) [286]. The dynamic stiffness matrix relates the Fourier transformed nodal forces to the corresponding transformed nodal displacements [104]. The calculated frequency domain responses can be afterwards transformed back to the time domain using inverse fast Fourier transformation (IFFT) [269].

This technique is well suited to simple 1-D and 2-D waveguides [266][287][288][289][290][291][292][293][294][295], but becomes difficult to use for complex geometries. In addition, some difficulties arise from the periodic nature of FFT when 2D or 3D problems are analyzed [269]. However, the application of the FFT-based spectral element method allowed the study of wave propagation phenomena in a laminated composite plate [296]. A comparative study of the FDSEM with the LISA approach can be found in Lee et al. [297]. The main limitation of the FDSEM is the difficulty of modelling finite length structures due to the inherent problem of "wraparound" associated with the finite domain Fourier transform. This drawback was circumvented using Daubechies compactly supported wavelet as the basis function instead of the Fourier transform [104].

The concept of wavelet was introduced by Morlet in 1982 [298] and it was originally used for signal processing. With the rapid development and application in many areas, wavelet techniques have become the second most significant breakthrough after the Fourier transform. The Fourier transform is global and provides a description of the overall regularity of signals, but it is not well adapted for finding the location and the spatial distribution of singularities. This is a major motivation for studying the wavelet transform in mathematics and in applied domains [299]. Thanks to this technique for the numerical treatment of partial differential equations [300], Mitra and Gopalakrishnan presented the 2-D wavelet transform based spectral finite element method, also called wavelet spectral finite element method (WSFEM), to overcome the "wraparound" problem present in FDSEM. WSFEM allows modelling accurately 2-D plate structures of finite dimensions [301][302][303]. This is due to the use of compactly supported Daubechies scaling functions [304] as basis for approximation of the time and spatial dimensions. However, the WSFEM plate formulation of Mitra and Gopalakrishnan is based on the classical laminated plate theory [107]. The CLPT based formulations exclude transverse shear deformation and rotary inertia resulting in significant errors for wave motion analysis at high frequencies, especially for composite laminates which have relatively low transverse shear modulus [305][306]. Wave propagation in composite laminates based on the first order shear deformation theory (FSDT) [107], which accounts for transverse shear and rotary inertia, yields accurate results comparable with 3-D elasticity solutions and experiments for wave motion at high frequencies [305][306]. FSDT based 2-D WSFE model for shear deformable composite laminates have been developed by Samaratunga et al. [307]. A transverse crack having an arbitrary length and depth was modeled in the wavenumber–frequency domain by introducing bending flexibility of the plate along the crack edge. Numerical examples have shown excellent agreement with conventional finite element simulations using shear flexible elements in the commercial finite element software Abaqus [308] in terms of both interaction with the crack and propagating wavefield. An analogous approach was used by the same authors for damage detection purposes in laminated composite plates [309] and stiffened composite panels [310]. Wavelet spectral finite element results were validated with conventional finite element simulations performed in Abaqus. In addition to transient response predictions, the usefulness of a wavelet spectral finite element–based skin–stiffener model was shown in a structural health monitoring application for detecting skin–stiffener debonding and transverse surface cracks. Khalilia et al. [311] implemented a user subroutine UEL (User-Defined Element) in Abaqus based on WSFEM for wave propagation analysis in 2-D composite structures. The WSFEM is founded on the FSDT. Numerical examples, namely an undamaged plate, impacted plate, plate with ply drop, folded plate and plate with stiffener demonstrated that the wave motions predicted by the developed UEL correlate well with standard Abaqus simulations. Zuo et al. [312] discussed an application of the wavelet finite element method adopting the B-spline wavelet to investigate static and free vibration problems of laminated composite plates. The same authors proposed a parallel implementation of this method for laminated composite plates GPU using CUDA [313]. Such implementation on GPU resulted in an acceleration of 146 times compared with the same wave motion problem executed on central processing units (CPU). The validity and accuracy of the proposed parallel implementation has been demonstrated by comparing with the conventional finite element method. The comparison has highlighted that the computation time can be reduced from hours to minutes.

As discussed above, the WSFEM can be easily applied to damage detection and has a strong advantage of mesh refinement: only very few elements are needed for accurate analysis. However, the WSFEM must still satisfy the strict requirements of time steps because step-by-step integration in the time domain is adopted, a higher computational cost has to be expected.

8. Conclusions

The growing use of composite materials for aerospace applications, also thanks to recent developments of advanced manufacturing techniques such as automated fiber placement, has resulted in a need for quantitative nondestructive evaluation and structural health monitoring methods appropriate for characterizing damage in composite components. Ultrasound based NDE methods are commonly used in the aerospace field, but ultrasonic wave behavior can be complicated by the presence of material anisotropy, complex geometries, and complex geometry defect types. Physics based simulation tools that model ultrasonic energy propagation can aid in the development of optimized inspection methods and in the interpretation of NDE data. Indeed, the optimization of these tools for composites has the potential to reduce both individual part inspection time and overall certification time for composite parts and structures. Inspection guidance based on simulation establishes increased confidence in the veracity of inspection results in addition to time reductions.

This manuscript aims to provide a review of the available literature on numerical methods to simulate ultrasonic and guided wave propagation in composite laminates. Guided ultrasonic waves can travel over large distances in (flat or curved) thin-walled structures and are widely used for structural health monitoring of multi-layered systems. Considerable advancements have been made in the field of numerical approaches for composite structures that include the development of suitable modelling schemes. This paper describes the background and summary of the early work as well as the most recent contributions. Numerical methods such as finite difference, finite integration, finite element, and spectral element methods can incorporate detailed composite material properties and complex damage morphologies. An ultrasound simulation code based on a finite difference mathematical foundation has several benefits such as: mathematics rather straightforward in comparison with other schemes; optimization in computational terms for memory efficiency and speed; ease of parallelization to leverage computing cluster and/or many-integrated-core architectures. However, boundaries and discontinuities between different types of media lead to approximate solutions and can produce severe errors. A local interaction simulation approach, LISA, allows avoiding these difficulties. Although the method is formally similar to the FD approach, it does not use directly any FD equations but simulates wave propagation heuristically, i.e. directly from physical phenomena and properties. The method involves implementation of fairly straightforward equations and lends itself to parallelization for computational efficiency. The SIM approach used with LISA, imposing continuity, or in other words perfect contacts, of displacements and stresses at interfaces and discontinuities, makes the approach well suited for modelling waves in inhomogeneous, complex media with complex boundaries. However, to date, LISA has not been demonstrated for triclinic anisotropy but only to include monoclinic anisotropy. The methods based on finite differences use the differential form of the governing field equations, while the finite integration technique is based on the integral forms of the same equations. This means that for a fixed mesh that includes an inherent spatial discretization error, no additional equation discretization error is introduced when passing from the continuous to the discrete form, leading to reduced inherent error. One popular FIT algorithm is the elastodynamic finite integration technique. This numerical scheme has been used for NDE ultrasound modelling in transversely isotropic composites. The EFIT is a grid-based numerical time domain technique and easily treats different boundary conditions, essential to model ultrasonic wave propagation in heterogeneous materials. The EFIT equations are practical to parallelize for use on computing clusters and multicore machines, thus allowing for the implementation of larger, memory-efficient 3D simulations than may be viable with current commercial FE packages. The adaptability of custom EFIT code, in terms of simplicity of incorporating complex damage, and the potential for directly altering the physics-based equations and boundary conditions, make it a desirable and cost-effective route for further development. FIT and FDM are both more widely implemented than FEM mainly due to the relative simplicity of their programming. The Finite Element method is formulated using unstructured grids, which gives it greater versatility for complex geometries but requires advanced knowledge of mesh generation in order to implement properly. FEM requires far more computational resources than FDM and FIT for relatively simple geometric cases. However, the typical strategy for handling complex geometry in FDM and FIT is to use a finer mesh, either globally or locally. Due to this limitation, FEM can be the more computationally efficient method for complex geometries. FEM is also generally superior to FDM when material interfaces are involved, but FIT can easily account for continuity conditions at these interfaces. The primary advantage of the FE method is that there are numerous commercial FE codes available, thus eliminating the need to develop actual code. These commercial FE codes have the additional advantages of being very user friendly, and providing sophisticated pre- and post-processing options. For non-destructive testing purposes, the FE models must be able to accurately represent ultrasonic waves with frequencies in the kHz/MHz range. The high frequencies are associated with short wavelengths and require both high time resolution (small time increments between calculated solution points) and high spatial resolution (small elements, in the mm range or below). An additional complication of this small element size is that a large number of elements (and thus a large system of equations) is needed to model a realistic component. Alternative FEM-based numerical procedures for simulating guided wave propagation in composites are the semi-analytical finite element method and wave finite element method. These methods are attractive as they avoid some of the discretization problems found in the FEM and are tailored to the needs of guided wave simulations. In SAFE, only the cross section of the waveguide is modelled, reducing the dimensions of the simulation model to one (for plates) or two (for other general waveguides). The elements used have a prescribed displacement of complex exponential functions that is associated with the wavelength. By applying this element in the equation of motion, an eigenvalue problem is obtained which is solved to give the displacement profile within the plate. However, a potential disadvantage of this method is that the necessary elements are not available as standard in commercial FEM packages and need to be developed by the user. The WFE method has the advantage that it can be developed from existing FE packages. This allows the available element library and grid generation procedures to be used for the modelling of various types of waveguide structures. The convenient connection with standard finite element solvers also provides a rapid solution of the wave-defect interaction problem. In addition, it can be extended to two or three-dimensional problems. Furthermore, unlike the spectral finite element method, it does not require the governing equation for the exact or approximate wave solutions. Nevertheless, the method faces some drawbacks due to the loss of computational efficiency when cross-sectional discretizations are required. Specific applications like the non-destructive evaluation of composite materials in a fluid by ultrasound may benefit from the distributed point source method, a modelling technique based on the superposition of fundamental solutions corresponding to individual pairs of points. The fundamental solution, or Green's function, between a pair of points serves as the building block for DPSM. For an ideal fluid or a homogeneous, isotropic solid, the elastodynamic Green's function is available as closed form algebraic expressions For anisotropic solids, the set of governing equations are considerably more complex and the elastodynamic Green's function needs to be evaluated numerically. Various spectral methods have been also proposed for the analysis of elastic wave propagation in complex media. Two basic approaches can be distinguished: the frequency domain spectral element method and the time domain spectral element method. The main advantages over the conventional finite element method are the high accuracy using a lower number of degrees of freedom for wave propagation problems. The FDSFEM leads to significant computational efficiency in comparison with FEM, especially in modelling anisotropic and inhomogeneous structures with damage. Its main limitation is the difficulty of modelling finite length structures due to the inherent problem of "wraparound" associated with the finite domain Fourier transform. This drawback is circumvented by the wavelet spectral finite element method, but this must satisfy the time step requirements as step-by-step integration in the time domain is adopted. The TDSFEM is extremely efficient in simulating guided wave propagation in computational solid and fluid mechanics. Nevertheless, it is still challenging to model high frequency multimodal guided waves in thin composite plates while achieving high accuracy. Despite the fact that there are many possibilities of computational techniques employed to solve various types of problems considered in SHM or NDE applications, it is very difficult to indicate one universal numerical method for such problems. The primary factors defining the suitability of a simulation scheme are the computational cost along with efficiency to handle the complexities of the structure and damage under investigation. Each specific numerical method has its advantages and disadvantages, making it useful for certain applications, also depending on the available expertise in numerical computation. Therefore, it is important to choose a method that is suitable and efficient both for the purpose and the user.

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