A hierarchical approach to integrated planning of industrial gas supply chains

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Abstract

In this paper, an optimisation-based framework is proposed for integrated production and distribution planning of industrial gas supply chains. The main goal is to minimise the overall cost, which is composed of raw material, product sourced from external suppliers, production, truck, and rail-car costs, while satisfying customer demands. The overall problem is formulated as a mixed integer linear programming (MILP) model while a two-phase hierarchical solution strategy is developed to solve the resulting optimisation problem efficiently. The first phase relies on truck scheduling decisions being relaxed, whereas the second phase solves the original model at reduced space by fixing product allocation as determined by phase one. Finally, an industrial-size case study is used to illustrate the applicability and efficiency of the proposed optimisation framework.

1. Introduction

In today's global market, making optimal and efficient supply chain management decisions is a major concern in industrial gas operations. Industrial gas supply chains cover the full spectrum, starting with the production of gases and liquids and ending with the distribution of final products to customers. Typically, production management involves the planning of raw material purchases and production processes, while distribution management includes the planning of transportation for delivering the product to customers.

Historically, industrial gas supply chain problems in production and distribution activities were treated independently. First, from a production perspective, Ierapetritou et al.[1] focused on the operating scheduling of air separation plants by considering uncertainty in energy prices. Karwan and Keblis[2] developed a mixed integer programming framework to determine the optimal operation planning for industrial gas production with real-time pricing for production inputs and electricity. Zhu et al.[3] developed a multi-period non-linear mathematical model for the operation of air separation plants by considering uncertainty in demand and electricity price. Manenti and Rovaglio[4] provided the peculiarities of industrial gas manufacturing, which differ from the features in other areas. They also provided general guidelines for optimising the industrial gas supply chains which account for the raw material, power supply, market demand, and storage.

Second, from the distribution side, Raff[5] reviewed vehicle routing and scheduling problems. Ronen[6] and Desrochers et al.[7] provided a classification scheme to categorise vehicle routing and scheduling problems based on the various properties of problems. Concerning inventory routing problems, which consider vehicle routing and scheduling problems simultaneously, Campbell et al.[8] adopted existing solution methods to solve deterministic and stochastic inventory routing problems. Campbell et al.[9] provided a two-stage solution approach for inventory routing problems. The solution from the first stage yields the customer's delivery allocation and delivery amount on each day, whereas the solution from the second stage yields the detailed vehicle route and schedule to execute the delivery. Kleywegt et al.[10]

proposed a mathematical framework based on a Markov decision process. In their problem, the direct delivery case was considered in which a customer received a delivery on each route. They also introduced approximation methods to solve the problem within a reasonable computational time. Campbell and Savelsbergh [11] aimed to find the optimal delivery schedule for a route by maximising the total delivery amount. A linear time algorithm was developed to solve the problem. Dong et al. [12] developed a mixed integer programming framework to solve inventory routing problems in industrial gas supply chains by considering driver-related regulations, and this research was extended to solve large-size instances by Dong et al. [13]. In that study, they proposed a solution method based on a pre-processing algorithm and decomposition method to reduce the problem size.

Examining the gas supply chain problem as two separate problems has the advantage of reducing the complexity of the decisions. This approach can be used when production decisions have little impact on distribution decisions. [14] However, most modern industrial gas companies operate across multiple facilities where those decisions are mutually correlated. [15]

Several authors address the importance of supply chain coordination. Chandra et al. [16] investigated the value stemming from coordinating production and distribution planning. The scenario in their computational study concerned a single plant, multi-product distributed by trucks. They proposed two solution procedures. The first procedure solved the production and distribution problems within a single framework, while second solved the problems separately. These two approaches were then tested using various data and the solutions from each approach were compared. The authors found up to a 20% cost decrease using the coordination approach. Thomas and Griffin [17] reviewed the literature on the coordination of supply chain management by dividing it into three categories: buyer-vendor coordination, production-distribution coordination, and inventory-distribution coordination. From the review, they concluded that autonomous supply chain management achieves a significant saving. Erengüç et al. [18], Vidal and Goetschalckx [19] also identified the importance of jointly considering production and distribution planning.

Despite the importance of integrated supply chain management, however, only a few works have investigated industrial gas applications. Glankwamdee et al.[20] studied the optimisation model for the integrated production and distribution of industrial gases. They developed a simplified linear model, which they then extended to a minimax linear model and stochastic model to consider uncertainty in customer demand and product availability. Marchetti et al.[21] proposed a multi-period mixed integer linear programming (MILP) formulation for optimal production and distribution coordination. The production model considers multiple plants operating under different production modes by focusing on the electricity cost. With respect to the distribution model, a combined model of vehicle routing for trucks and inventory management was considered. The author analysed multiple industrial cases with different levels of coordination and found that substantial cost savings were realised when the degree of coordination was increased.

The coordinated planning and scheduling results in large-size problems with a large number of plants, intermediate storages, and customers when dealing with real-world supply chains in industrial gas companies. Subsequently, the large-size problems involve a significant number of constraints and variables which make the integrated models computationally intractable. Therefore, solution techniques along with mathematical models must be developed to address the computational limitation.[22] To handle the computational limitation, there are widely used techniques such as Lagrangian decomposition,[23–25] Benders decomposition,[26] bilevel decomposition,[27] and rolling horizon decomposition.[28]

In addition to these standard techniques, tailored solution methods applied in integrated production and distribution problems have been proposed. You et al.[29] developed an MILP model which accounts for distribution and inventory decisions on industrial gas supply chain planning simultaneously. They also proposed two approaches to solve large instances. The first one is based on a two-level decomposition method and the second is based on a continuous approximation method. These models and solution strategies were applied to industrial case studies and the results showed that they have high computational effectiveness. Zamarripa et al.[30] proposed a rolling horizon decomposition approach for a full-space optimisation problem which coordinates production and distribution decisions. In the rolling horizon approach, they investigated two aggregation strategies which rely on linear programming relaxation for the binary variables and a simplified distribution model, respectively. Furthermore, Zhang et al.[31] proposed an MILP model which includes two time grids and an iterative heuristic approach for the multi-scale production routing problem which integrates production, distribution, and inventory decisions.

However, to the best of our knowledge, none of the developed mathematical models or solution strategies provide rigorous production and distribution decisions for large-size industrial gas problems simultaneously. Marchetti et al.[21] and Zamarripa et al.[30] provided an integrated production and distribution mathematical framework, but adopted a simplified distribution model which did not provide a detailed distribution schedule for real-size instances. You et al.[29] focused on coordinated inventory and distribution planning without considering detailed production planning. The model proposed by Zhang et al.[31] captured the relatively rigorous production and distribution planning; however, it did not consider secondary storage sites (depots) and the corresponding distribution network. Additionally, none of them considered a detailed contract model for the raw material or product supply or multi-modal transportation (rail-car and truck) in their models.

The aim of this work is to develop an MILP model that incorporates the production and distribution decisions of industrial gas supply chains and a solution strategy to dealing with industrial-scale problems. In the model, we consider four novel features non-existent in recent works:[21,29–31] (i) supply contracts for raw material and product; (ii) multimodal transpiration (i.e., rail-car and truck); (iii) demand of pick-up customers; and (vi) periodic planning. The production model is formulated by considering the supply contracts and production based on multi-site plants. By contrast, rigorous scheduling for both rail-car and truck, allocation between each location, and inventory management are taken into account in the distribution model.

Furthermore, because the integrated model with this high-level of detail is hard to solve large-size problems due to the number of constraints and variables, we present an efficient solution strategy relying on a two-phase hierarchical approach that allows finding high-quality solutions within considerably reduced computational times.

The remainder of this paper is structured as follows. Section 2 describes the problem of integrated planning in industrial gas supply chains. Section 3 introduces the mathematical framework for the integrated supply chain problem and Section 4 develops the two-phase hierarchical solution strategy. The efficiency and validity of the proposed mathematical framework and solution strategy are evaluated with an industrial case study in Section 5. Finally, Section 6 concludes.

2. Problem statement

This work considers an existing CO_2 supply chain network which comprises raw material (CO_2 feedstock) suppliers, production plants, depots, third-party suppliers, and customers. Figure 1 illustrates the overall network, highlighting both the production and the distribution parts of the network.

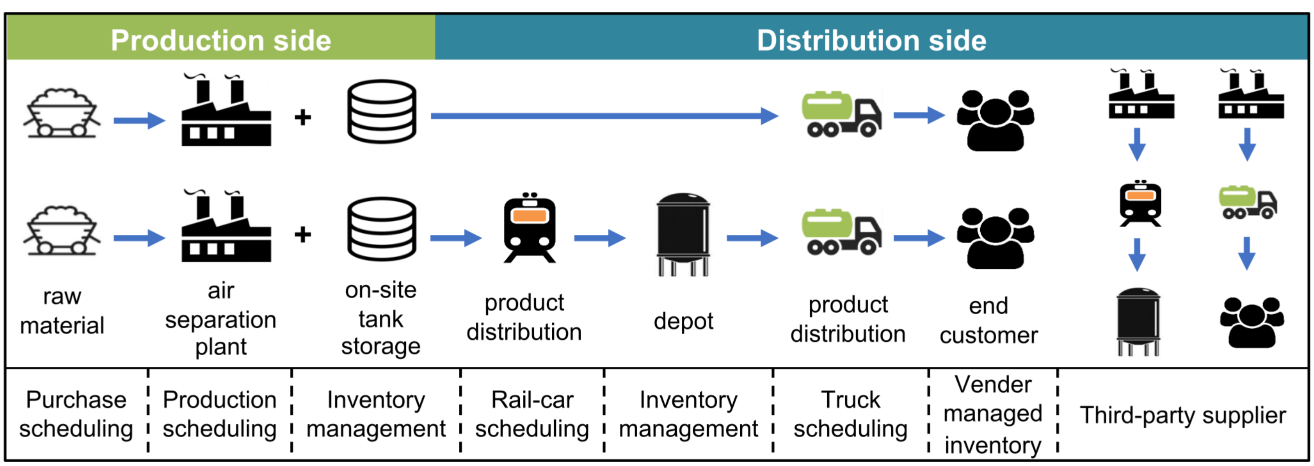

Figure 1: Schematic of the industrial CO_2 supply chain network

Within the production part network, each plant purchases the raw material from the external suppliers under its individual sourcing contacts. First, the purchased raw material is transformed into a high purity CO_2 product; then, the product is stored in an on-site storage tank at each plant. The CO_2 product can also be sourced from third-party suppliers, but this is only considered when the plants cannot satisfy customer demand or when distribution from the plants to the customer is restricted by any constraints related to transportation.

The distribution between locations to satisfy customer demand is guaranteed via rail-cars or trucks. The rail-cars distribute the CO_2 product from the plants/third parties to the depots using the existing rail infrastructure, while the trucks travel from the plants, depots, and/or third parties to the customers. All the rail-cars and trucks must return to their origin after distributing the product to the depots and customers. Table 1 summarises the possible connections and transportation modes between locations.

Table 1: Transportation modes between locations

	destination			
origin	plant	depot	third party	customer
plant	no	rail-car	no	truck
depot	no	no	no	truck
third party	no	rail-car	no	truck

The customers in this problem are classified into vendor-managed inventory (VMI) and pick-up customers. The product inventories of VMI customers are operated under the VMI paradigm, which is necessary to decide the time and amount of product delivery based on their consumptions. Given the requirement to manage individual customers' inventories within certain set levels, the decision is to whether deliver products to a customer on any day, and if the product is to be delivered on that day, what amount needs to be trucked. By contrast, pick-up customers directly collect their demands at prespecified plants or depots; therefore, no distribution decisions are considered.

Lastly, this problem also considers periodic planning, which means that the set of optimal decisions over a given time horizon could be repeated in the next time horizon.

Overall, the problem can be stated as follows:

Given:

- For each raw material supplier: plant allocation, maximum supply available, contract type, and related costs;
- For each plant: location, production capacity, and corresponding costs;
- For each depot: location;
- For each third party: location, maximum supply available, contract type, and related costs;
- For each customer: location and consumption of product;
- For each inventory of plants, depots, and customers: upper and lower bounds of the capacity level, and
- For transportation: possible connections and transportation modes between locations, lead time, loading capacity, maximum quantity, and transfer unit cost.

Determine:

- Production mode, production amount, and raw material purchasing amount at each plant for each time period;
- Purchasing schedule of product sourced from each third-party supplier;
- Customer allocation to each plant/depot/third-party supplier;
- Depot allocation to each plant/third-party supplier, and
- Product amount and time to be delivered by rail-car and truck.

So as to:

Minimise the overall cost of CO_2 supply chain planning.

3. Mathematical framework

The overall problem is formulated as an MILP model which integrates supply contracts, production scheduling, truck and rail-car scheduling, and inventory management under the VMI paradigm. The indices, sets, parameters, and variables are listed in the nomenclature section at the end of this paper.

3.1 Objective function

The proposed model aims to minimise the total operating cost of the CO_2 supply chain:

$$min\ TC^{total} \tag{1}$$

$$TC^{total} = TC^{raw} + TC^{prod} + TC^{st} + TC^{os} + TC^{rail} + TC^{truck} \tag{2}$$

where TC^{raw} refers to the raw material cost, TC^{prod} is the production cost, TC^{st} is the plant start-up cost, TC^{os} is the pure CO_2 purchasing cost sourced from third parties, and TC^{rail} and TC^{truck} refer to the transportation cost by rail-car and truck, respectively.

Raw material cost. The production plants produce pure CO_2 by transforming, purifying, and liquefying the raw CO_2 supplied by external sources. The raw material purchasing cost is charged by considering the discount contract type. Under the discount contract, the price is reduced when the total cumulative purchasing amount over a time horizon exceeds the break points, as illustrated in Figure 2.

The mathematical formulation of the discount contract follows the model published by Park et al.[32] In their formulation, the selection of the cost region is represented by a binary variable which apart from depending on the cost region depends on the time, since the discount is applied in every period. In our problem, the discount is applied by considering the total purchasing amount, which allows us to reformulate the contract model by considering

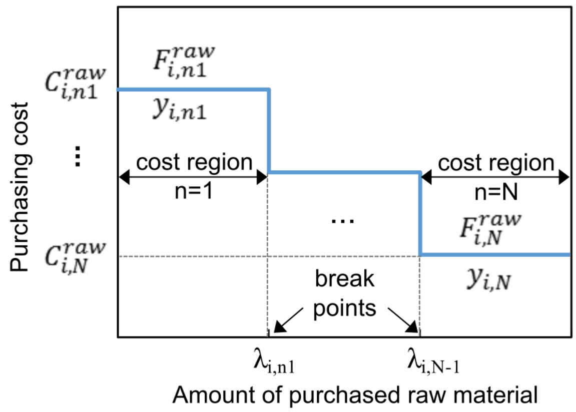

Figure 2: Representation of the discount contract cost model

a non-time-dependent binary variable as follows:

$$TC^{raw} = \sum_{i} \sum_{n} C_{in}^{raw} F_{in} \tag{3}$$

$$\sum_{n} F_{in} = \sum_{t} R_{it} \qquad \forall i \tag{4}$$

$$(\lambda_{in} - \lambda_{i,n-1} \mid_{n>1}) \; y_{i,n+1} \leq F_{in} \leq (\lambda_{in} - \lambda_{i,n-1} \mid_{n>1}) \; y_{in} \qquad \forall i, n < N \tag{5}$$

$$y_{in} \geq y_{i,n+1} \qquad \forall i, n < N \tag{6}$$

$$F_{in} \leq M \; y_{in} \qquad \forall i, n = N \tag{7}$$

Eq. 3 calculates the total cost of the raw material which depends on the purchased amount in cost region n (F_{in}) and the corresponding cost (C_{in}^{raw}). Eq. 4 indicates that the sum of amount of the raw material corresponding to each cost region is equal to the sum of the purchased amount over the time horizon. Eq. 5 determines the amount corresponding to each cost region; namely, y_{in} is equal to 1 when the total purchased amount $\sum_t R_{it}$ exceeds the break point $\lambda_{i,n-1}$. Eq. 6 is a logical relationship which avoids the selection of intermediate cost regions and Eq. 7 restricts the amount in the last cost region (n=N); in other words, F_{in} has to be lower than the big M value. For more comprehensive understanding of the reformulated contract model, assume one break point, which results in two cost regions (N=2). If the total purchased amount at plant i over the time horizon ($\sum_t R_{it}$) exceeds the break point $\lambda_{i,n1}$, the binary variable $y_{i,n2}$ is equal to 1 and consequently, $y_{i,n1}$ is also 1 according to Eq. 6. The corresponding amount in the first cost region ($F_{i,n1}$) is equal to the break point ($\lambda_{i,n1}$) based on the Eq. 5 and the binary variables, and this is purchased at cost $C^{raw}_{i,n1}$. Finally, the second amount ($F_{i,n2}$) which exceeds the break point ($\lambda_{i,n1}$) can be any value less than the big M parameter; however, it must be equal to $\sum_t R_{it} - F_{i,n1}$ based on Eq. 4.

Production cost. The production cost is calculated based on the production mode and the production amount as follows:

$$TC^{prod} = \sum_i \sum_t \mu_i P_i^{max} w_{it} + \sum_i \sum_t \nu_i P_{it} \tag{8}$$

The first term of Eq. 8 shows the fixed cost of production for each time period, while the second term represents the production cost, which is linear to production amount P_{it}. Here, μ_i is the unit fixed cost and ν_i is the unit production cost. w_{it} is a binary variable coded 1 when plant i produces products at time t (on-mode).

Plant start-up cost. There is a cost of transitioning from the off-mode to the on-mode when each plant starts to operate. This plant start-up cost is taken into account in the total cost and is given by:

$$TC^{st} = \sum_i \sum_t C_i^{st} u_{it} \tag{9}$$

where C_i^{st} is the start-up cost of each plant and u_{it} is the binary variable coded 1 when the production mode switches from the off-mode to the on-mode.

Purchasing cost from third parties. The product CO_2 supply can be sourced from third-party suppliers either when the produced amount at each plant cannot meet customer demand or when it is more cost effective than producing at each plant. The third-party supply also relies on the business contract. The discount contract type is regarded as the same as the contract model for the raw material supply:

$$TC^{os} = \sum_m \sum_n C_{mn}^{os} S_{mn} \tag{10}$$

$$\sum_n S_{mn} = \sum_t O_{mt} \qquad \forall m \tag{11}$$

$$(\lambda_{mn} - \lambda_{m,n-1} \,|_{n>1})\, z_{m,n+1} \le S_{mn} \le (\lambda_{mn} - \lambda_{m,n-1} \,|_{n>1})\, z_{mn} \qquad \forall i, n < N \tag{12}$$

$$S_{mn} \le M\ z_{mn} \qquad \forall m, n = N \tag{13}$$

$$z_{mn} \ge z_{m,n+1} \qquad \forall i, n < N \tag{14}$$

where O_{mt} is the amount of product purchased from the third party m in time period t, S_{mn} is the amount corresponding to cost region n, and z_{mn} is the binary variable which is equal to 1 when the amount of product purchased over the time horizon, $\sum_t O_{mt}$, is in cost region n.

Transportation cost. The total rail-car cost is calculated as follows:

$$TC^{rail} = \sum_i \sum_{j \in J_i} \sum_t C_{ij}^{rail} NR_{ijt} + \sum_m \sum_{j \in J_m} \sum_t C_{mj}^{rail} NR_{mjt} \tag{15}$$

Here, the sets J_i and J_m denote the depots pre-allocated to each plant and third-party supplier by considering the existing rail infrastructure. Here, C_{ij}^{rail} and C_{mj}^{rail} are the transportation costs between the locations for each rail-car, while NR_{ijt} and NR_{mjt} are the integer variables, which are the number of rail-cars used to transfer the product from the plant and third party to the depot, respectively

$$\begin{aligned} TC^{truck} = & \sum_i \sum_{k \in K_i} \sum_v \sum_t C^{truck}\ L_{ik}\ A_{ikvt} \\ & + \sum_j \sum_{k \in K_j} \sum_v \sum_t C^{truck}\ L_{jk}\ A_{jkvt} \\ & + \sum_m \sum_{k \in K_m} \sum_v \sum_t C^{truck}\ L_{mk}\ A_{mkvt} \end{aligned} \tag{16}$$

Eq. 16 calculates the total truck cost, and each term represents the transportation cost of trucks which depart the plants, depots, and third-party suppliers, respectively. Similar to the sets presented in Eq. 15, K_i, K_j, and K_m represent the VMI customers initially allocated to each plant, depot, and third party by their geographical location. The total truck cost depends on the unit transfer cost per distance (C^{truck}), round-trip distance between each location (L_{ik}, L_{jk}, and L_{mk}), and number of trips each truck performs (A_{ikvt}, A_{jkvt}, and A_{mkvt}) during a period t.

3.2 Constraints

Production constraints. As mentioned earlier, each plant is operated based on two modes, the on- and off-modes. If a plant decides to produce the product (on-mode), its production cannot exceed certain limits. In other words, the production during each time period is bounded between the minimum and maximum production capacities:

$$P_i^{min} w_{it} \leq P_{it} \leq P_i^{max} w_{it} \qquad \forall i, t \tag{17}$$

where w_{it} is the binary variable used to select the operation mode. w_{it} is equal to 1 when plant i is under the on-mode at time period t.

$$w_{it} - w_{i,t-1} \mid_{t>1} \leq u_{it} \qquad \forall i, t \tag{18}$$

Constraint 18 is the logical constraint for the plant start-up. This enforces that the binary variable u_{it} appearing on the right-hand side is equal to 1 when each plant's operation mode switches from the off-mode ($w_{i,t-1} = 0$) to the on-mode ($w_{it} = 1$).

The amount of the raw material purchased at each plant and in each time period must not exceed the maximum limit:

$$R_{it} \leq R_i^{max} \qquad \forall i, t \tag{19}$$

where R_{it} denotes the amount of the raw material purchased at each plant in time period t, and R_i^{max} is the maximum amount which each plant can buy in each period.

The relationship between the amount of the raw material and production amount is given as:

$$R_{it} = \alpha_i P_{it} \qquad \forall i, t \tag{20}$$

Here, α_i is the given coefficient to relate the production amount and the raw material amount.

Third-party supplier constraints.

$$O_{mt} \leq O_m^{max} \qquad \forall m, t \tag{21}$$

$$O_{mt} = \sum_{j \in J_m} Q_{mjt}^{RC} + \sum_{k \in K_m} Q_{mkt}^{TR} \qquad \forall m, t \tag{22}$$

Constraint 21 limits the amount of product purchased from each third-party supplier in each time period (O_{mt}) by the given parameter (O_m^{max}) and constraint 22 states that the amount purchased from third-party supplier m in time period t (O_{mt}) must be equal to the total amount delivered from the third-party supplier to any depots by rail-cars ($\sum_{j \in J_m} Q_{mjt}^{RC}$) and to any customers by trucks ($\sum_{k \in K_m} Q_{mkt}^{TR}$) in that time period.

Truck constraints. Considering the truck schedule, it is assumed that each truck must return to its origin after visiting the customer:

$$\sum_{k \in K_i} \theta_{ik} \, A_{ikvt} \leq \Delta_t \qquad \forall i, v, t \tag{23}$$

$$\sum_{k \in K_j} \theta_{jk} \, A_{jkvt} \leq \Delta_t \qquad \forall j, v, t \tag{24}$$

$$\sum_{k \in K_m} \theta_{mk} \, A_{mkvt} \leq \Delta_t \qquad \forall m, v, t \tag{25}$$

Constraints 23–25 set the travel time limitation for each truck. The total travel time of truck v during the time period t from each source to the customers cannot be greater than the length of each time period Δ_t, where θ_{ik}, θ_{jk}, and θ_{mk} are the round-trip times between each location; these are formed as a proportion of the time period. A_{ikvt}, A_{jkvt}, and A_{mkvt} are the integer variables which denote the number of round-trips that truck v performs between each source and each customer in a time period.

$$Q^{TR}_{ikt} \leq \sum_{v} A_{ikvt} \;\; Cap^{truck}_{i} \qquad \forall i, k \in K_i, t \tag{26}$$

$$Q^{TR}_{jkt} \leq \sum_{v} A_{jkvt} \;\; Cap^{truck}_{j} \qquad \forall j, k \in K_j, t \tag{27}$$

$$Q^{TR}_{mkt} \leq \sum_{v} A_{mkvt} \;\; Cap^{truck}_{m} \qquad \forall m, k \in K_m, t \tag{28}$$

Constraints 26-28 pose the delivery amount based on the number of deliveries to each customer and the truck capacity. The variables Q^{TR}_{ikt}, Q^{TR}_{jkt}, and Q^{TR}_{mkt} indicate the delivery amount from plant i, depot j, and third party m to each customer k by any trucks stationed at each plant, depot, and third party, respectively. Thus, the variables must be less than summation of the number of deliveries multiplied by the truck capacity over truck v.

Rail-car constraints. The maximum number of rail-cars can be used simultaneously in a time period is given by:

$$\sum_{i}\sum_{j\in J_i}\sum_{t'=0}^{2\tau_{ij}-1} NR_{ij\pi(t-t')} + \sum_{m}\sum_{j\in J_m}\sum_{t'=0}^{2\tau_{mj}-1} NR_{mj\pi(t-t')} \leq NR^{max} \qquad \forall t \tag{29}$$

where τ_{ij} and τ_{mj} are the transportation time between the locations based on one-way trip. Constraint 29 implies that if rail-cars leave plant i or third parity m for depot j, the rail-cars are not available for other depots during its round-trip. Here, $\pi(\cdot)$ is the wrap-around time operator. The concept of wrap-around was considered by Shah et al.[33] and Papageorgiou and Pantelides[34]. This wrap-around operator allows that execution of tasks starting within periods in the planning horizon extends across its boundaries into the next planning horizon.

The wrap-around operator is defined as follows:

$$\begin{aligned} &\pi(t) = t && if \;\; t \geq 1, \\ &\pi(t) = \pi(t+T) && if \;\; t \leq 0 \end{aligned} \tag{30}$$

where T is the number of time period. For example, if $T = 30$, then $\pi(-2) = \pi(-2+30) = \pi(28)$.

The amount of product transported by rail-cars in each period is bounded by the capacity of one rail-car (Cap^{rail}) multiplied by the number of rail-cars used for each delivery (NR_{ijt} and NR_{mjt}):

$$Q^{RC}_{ijt} \leq Cap^{rail}\, NR_{ijt} \qquad \forall i, j \in J_i, t \tag{31}$$

$$Q^{RC}_{mjt} \leq Cap^{rail}\, NR_{mjt} \qquad \forall m, j \in J_m, t \tag{32}$$

Inventory mass balance.

$$I_{it} = I_{i,t-1} \mid_{t>1} + I_i^{ini} \mid_{t=1} + P_{it} - \sum_{j \in J_i} Q_{ijt}^{RC} - \sum_{k \in K_i} Q_{ikt}^{TR} - \sum_{k \in K_i^P} DP_{ik} \qquad \forall i, t \tag{33}$$

$$\begin{aligned} I_{jt} = I_{j,t-1} \mid_{t>1} + I_j^{ini} \mid_{t=1} &+ \sum_{i:j \in J_i} Q_{ij,\pi(t-\tau_{ij})}^{RC} + \sum_{m:j \in J_m} Q_{mj,\pi(t-\tau_{mj})}^{RC} \\ &- \sum_{k \in K_j} Q_{jkt}^{TR} - \sum_{k \in K_j^P} DP_{jk} \qquad \forall j, t \end{aligned} \tag{34}$$

$$\begin{aligned} I_{kt} = I_{k,t-1} \mid_{t>1} + I_k^{ini} \mid_{t=1} &+ \sum_{i:k \in K_i} Q_{ikt}^{TR} + \sum_{j:k \in K_j} Q_{jkt}^{TR} \\ &+ \sum_{m:k \in K_m} Q_{mkt}^{TR} - D_k \qquad \forall i, j, k \notin K_i^P \cup K_j^P, t \end{aligned} \tag{35}$$

Constraints 33–35 correspond to the mass balance of the product inventory at each plant, depot, and customer, respectively. where, I_{it}, I_{jt}, and I_{kt} are the inventory levels of the plant, depot, and customer at the current time period; I_i^{ini}, I_j^{ini}, and I_k^{ini} are the given initial inventory levels of the plant, depot, and customer; DP_{ik} and DP_{jk} are the pick-up customer consumption in each time period, which are collected at the plant and depot; and D_k is the consumption of VMI customer k in each period. Here, the wrap-around operator presented in constraint 30 is also adopted in the variables which are the product amounts delivered by rail-cars ($Q_{ij,\pi(t-\tau_{ij})}^{RC}$ and $Q_{mj,\pi(t-\tau_{mj})}^{RC}$). It is important to note that $t - \tau_{ij}$ and $t - \tau_{mj}$ may take a zero or negative value corresponding to the product being arrived at depot j in the current planning horizon by deliveries that actually started during the previous one.

To derive the periodic scheduling, each inventory level at the end of the time horizon has to be equal to the given initial level:

$$I_{it} = I_i^{ini} \qquad \forall i, t = T \tag{36}$$

$$I_{jt} = I_j^{ini} \qquad \forall j, t = T \tag{37}$$

$$I_{kt} = I_k^{ini} \qquad \forall i, j, k \notin K_i^P \cup K_j^P, t = T \tag{38}$$

The inventory level in each time period is bounded by the minimum and maximum capacities:

$$I_i^{min} \leq I_{it} \leq I_i^{max} \qquad \forall i, t \tag{39}$$

$$I_j^{min} \leq I_{jt} \leq I_j^{max} \qquad \forall j, t \tag{40}$$

$$I_k^{min} \leq I_{kt} \leq I_k^{max} \qquad \forall i, j, k \notin K_i^P \cup K_j^P, t \tag{41}$$

Multiple sourcing constraints. There is a multiple sourcing constraint on the number of plants, depots and third-parties that each customer can be supplied the product over a time horizon, and this is given by:

$$\sum_{i:k \in K_i} x_{ik} + \sum_{j:k \in K_j} x_{jk} + \sum_{m:k \in K_m} x_{mk} \leq NS \qquad \forall i, j, k \notin K_i^P \cup K_j^P \tag{42}$$

Constraint 42 indicates that each customer demand can be fulfilled by at most NS sources among the plants, depots, and third-party suppliers. The binary variables x_{ik}, x_{jk}, and x_{mk} represent the selection of sources for customer k.

$$Q_{ikt}^{TR} \leq M x_{ik} \qquad \forall i, k \in K_i, t \tag{43}$$

$$Q_{jkt}^{TR} \leq M x_{jk} \qquad \forall j, k \in K_j, t \tag{44}$$

$$Q_{mkt}^{TR} \leq M x_{mk} \qquad \forall m, k \in K_m, t \tag{45}$$

Constraints 43-45 impose that each customer can be served the product from the plant, depot, or third party only when the sources are selected to be supplied.

4. Hierarchical solution strategy

The proposed mathematical framework for the multiple integrated decisions taken on the production and distribution planning of industrial gas supply chains requires an extensive computational cost, and could even be intractable when dealing with large-scale problems. For instance, the problem taking into account over 30 of plants/depots/third-parties and over 700 customers results in 250,740 equations, 138,275 continuous variables, and 632,239 discrete variables. Therefore, an efficient two-phase solution strategy on a hierarchical approach is presented in this section.

The first phase of the proposed solution strategy focuses on finding the optimal allocation of depots into plants and third parties and allocation of customers into plants, depots, and third parties. In this phase, the decisions on the exact truck scheduling are disregarded. The truck cost is estimated, and all the integer variables associated with the trucks are relaxed into continuous variables. Section 4.1 describes the relaxed MILP model used in this phase. In the second phase, the problem is solved using the original MILP model in Section 3, but the model size is reduced by fixing the optimal allocation result obtained in the previous phase.

Throughout the paper, the original model which considers the full-space problem is referred to as M^S, the relaxed MILP model used in the first phase as M^R, and the reduced MILP model as M^H.

4.1 Relaxed MILP model (M^R)

The mathematical framework of the first phase encompasses exactly the same equations as the original MILP model except for the equations related to the rigorous truck scheduling (Eq. 16 and constraints 23-28). Therefore, only the relaxed formulation for trucks is stated in this section.

$$
\begin{aligned}
TC^{truck} = & \sum_{i} \sum_{k \in K_i} \sum_{t} \frac{Q^{TR}_{ikt}}{Cap^{truck}_{i}} \, L_{ik} \, C^{truck} \\
& + \sum_{j} \sum_{k \in K_j} \sum_{t} \frac{Q^{TR}_{jkt}}{Cap^{truck}_{j}} \, L_{jk} \, C^{truck} \\
& + \sum_{m} \sum_{k \in K_m} \sum_{t} \frac{Q^{TR}_{mkt}}{Cap^{truck}_{m}} \, L_{mk} \, C^{truck}
\end{aligned}
\tag{46}
$$

Eq. 46 is the relaxed formulation for calculating the estimated transfer cost of trucks departing plant i, depot j, and third party m. Since each vehicle can perform multiple deliveries to each customer, the information was represented using the integer variables (A_{ikvt}, A_{jkvt}, and A_{mkvt}) in the original MILP model. Here, the integer variables are relaxed to continuous variables by adding the delivery amount/truck capacity ratio ($Q^{TR}_{ikt}/Cap^{truck}_{i}$, $Q^{TR}_{jkt}/Cap^{truck}_{j}$, and $Q^{TR}_{mkt}/Cap^{truck}_{m}$). This ratio approximates the number of multiple deliveries to the customer.

$$\sum_{k\in K_i} \frac{Q^{TR}_{ikt}}{Cap^{truck}_i}\theta_{ik} \leq NT^{max}_i \Delta_t \qquad \forall i,t \tag{47}$$

$$\sum_{k\in K_j} \frac{Q^{TR}_{jkt}}{Cap^{truck}_j}\theta_{jk} \leq NT^{max}_j \Delta_t \qquad \forall j,t \tag{48}$$

$$\sum_{k\in K_m} \frac{Q^{TR}_{mkt}}{Cap^{truck}_m}\theta_{mk} \leq NT^{max}_m \Delta_t \qquad \forall m,t \tag{49}$$

Constraints 47-49 are the relaxed formulation used to limit both the total amount of the product transferred and the total travel time of trucks during each period. Here, the terms appearing on the left-hand side of constraints 47-49, namely, the summation of the delivery amount/capacity ratio multiplied by the duration of the round-trip to the customers, calculate the total travel time completed by trucks during the period. The total travel time must be bounded by the length of each time period (Δ_t) and number of trucks available at each plant, depot, and third party (NT^{max}_i, NT^{max}_j, and NT^{max}_m). These constraints also imply that the total delivery amount is restricted by truck capacity and the number of available trucks.

As a result, by introducing the delivery amount/capacity ratio, not only can all the integer variables be relaxed, but also the constraints for both capacity and time limitations can be combined.

4.2 Reduced MILP model (M^H)

As aforementioned, the mathematical framework presented in Section 3 is used in the second phase. However, the model size is reduced, as it solves the problem by considering only the optimal depot and customer allocations gained from the first-phase process. Additionally, the equations for the multiple sourcing constraints (constraints 42-45) and related binary variables (x_{ik}, x_{jk}, and x_{mk}) can be disregarded since the optimal allocation result is determined by taking account of the constraints. The procedure for fixing the optimal allocation to reduce the model size is carried out by updating K_i, K_j, and K_m, which represent the initial customer allocation to the plant, depot, and third party, respectively and J_i and J_m, which are the depots pre-allocated to the plant and third party. Algorithm 1 describes the detailed procedure used to update these sets and Figure 3 presents the flow diagram of the overall solution procedure.

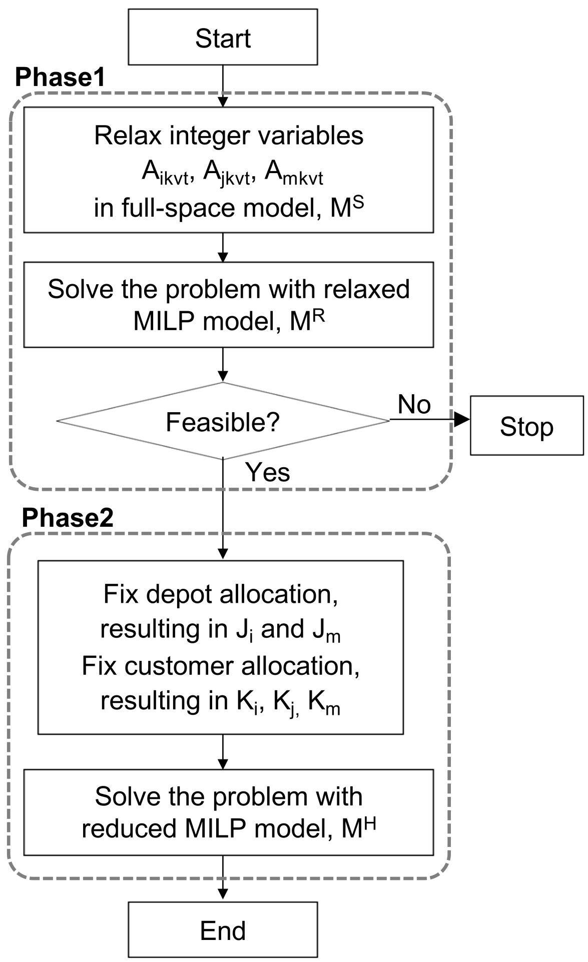

Figure 3: Flow diagram for the proposed hierarchical solution strategy

Algorithm 1: Updating procedure for the allocation sets

Solve the relaxed MILP model (M^R), and obtain the optimal values of Q_{ijt}^{RC}, Q_{mjt}^{RC}, Q_{ikt}^{TR}, Q_{jkt}^{TR}, and Q_{mkt}^{TR}

The values are denoted as Q_{ijt}^{RC*}, Q_{mjt}^{RC*}, and Q_{ikt}^{TR*}, Q_{jkt}^{TR*}, Q_{mkt}^{TR*}, respectively

for *Sets for depot allocation* **do**

  Input: Q_{ijt}^{RC*}, Q_{mjt}^{RC*}

  Output: J_i, J_m

  $J_i = \emptyset$, $J_m = \emptyset$

  if *total delivery amount from plant i to depot j,* $\sum_t Q_{ijt}^{RC*} > 0$ **then**

    add j to J_i

  if *total delivery amount from third party m to depot j,* $\sum_t Q_{mjt}^{RC*} > 0$ **then**

    add j to J_m

for *Sets for customer allocation* **do**

  Input: Q_{ikt}^{TR*}, Q_{jkt}^{TR*}, Q_{mkt}^{TR*}

  Output: K_i, K_j, K_m

  $K_i = \emptyset$, $K_j = \emptyset$, $K_m = \emptyset$

  if *total delivery amount from plant i to customer k,* $\sum_t Q_{ikt}^{TR*} > 0$ **then**

    add k to K_i

  if *total delivery amount from depot j to customer k,* $\sum_t Q_{jkt}^{TR*} > 0$ **then**

    add k to K_j

  if *total delivery amount from third party m to customer k,* $\sum_t Q_{mkt}^{TR*} > 0$ **then**

    add m to K_m

5. Case study

To demonstrate the applicability of the proposed mathematical framework and developed solution strategy, a case study of the industrial gas supply chain network in the United States is conducted. Most of the input data are unavailable for confidentiality reasons.
The existing industrial gas supply chain network consists of:

- Over 30 plants, depots, and third-party suppliers;
- Over 750 VMI and pick-up customers;
- Over 300 rail-cars, and
- Over 100 trucks.

The problem considers a one-month planning horizon and the time discretisation is one day. The developed MILP model and solution strategy were implemented in GAMS 26.1 software and solved using Gurobi 8.1.0 on an Intel 3.60 GHz, 16 GB RAM computer.

Table 2 shows the problem size, optimal objective value, and computational performance under both the full-space and the hierarchical approaches.

Table 2: Optimal solution and computational performance

	full-space approach	hierarchical approach	
model	M^S	M^R	M^H
equations	252,229	137,266	54,127
continuous variables	138,275	138,275	48,976
discrete variables	632,239	5,389	155,221
total cost (M$)	5.78	5.27	5.80
optimality gap (%)	5	1	5
CPU time	79 hr	239 s	4 hr

The full-space model M^S includes 632,239 discrete variables and requires 79 hours to reach an optimal solution with a 5% optimality gap. By contrast, the CPU time can be reduced significantly when the problem is solved using the hierarchical approach. The relaxed model M^R in the first phase is solved in 239 seconds, as all the integer variables associated with the truck scheduling are relaxed into continuous variables, resulting in only 5,389 discrete variables. In the second phase, the original problem is reduced to 155,221 discrete variables by fixing the optimal allocation result gained from the relaxed model M^R.

Table 3 presents the number of binary variables and integer variables associated with the rail-car and truck scheduling. In the full-space problem, 660 and 626,850 integer variables are involved in the rail-car and truck scheduling, respectively. These values are reduced to 180 and 153,915 by fixing the optimal allocation results from the relaxed model, which allows us to solve this large-scale problem in a reasonable time. This proves that the proposed hierarchical approach has benefit in reducing the number of discrete variables, which enables the model to handle a large-scale problem by reducing computational time dramatically.

Table 3: The number of binary and integer variables involved in model M^S and M^H

	M^S	M^H
binary variables	4,729	1,126
integer variables (rail-car scheduling)	660	180
integer variables (truck scheduling)	626,850	153,915
total	632,239	155,221

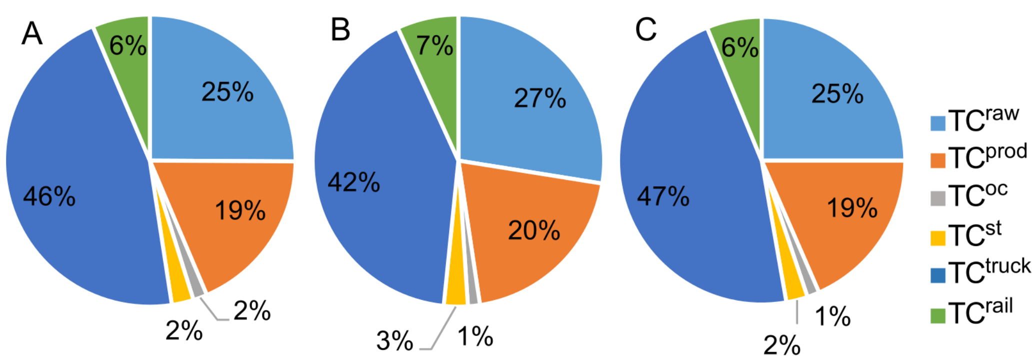

Figure 4: Percentage of the breakdown cost obtained from (A) model M^S; (B) model M^R and (C) model M^H

Table 4: Optimal breakdown cost

model	full-space approach M^S	hierarchical approach M^R	M^H
TC^{raw} (M\$)	1.45	1.45	1.45
TC^{prod} (M\$)	1.07	1.06	1.07
TC^{st} (M\$)	0.09	0.07	0.08
TC^{oc} (M\$)	0.14	0.14	0.14
TC^{truck} (M\$)	2.66	2.19	2.70
TC^{rail} (M\$)	0.37	0.36	0.36
TC^{total} (M\$)	5.78	5.27	5.80

Figure 4 provides the breakdown of the total costs gained from the full-space model M^S, relaxed model M^R, and reduced model M^H to compare the solutions quantitatively. In the figure, each breakdown cost obtained from the models accounts for almost the same percentage. According to the result from model M^H, the truck cost represents the highest percentage of the total cost (47%), followed by the raw material cost (25%) and production cost (19%). The purchasing cost of the CO_2 product from third-party suppliers and plant start-up cost show the lowest percentages of the total cost (2% and 1%, respectively). Looking at the breakdown costs in Table 4, model M^S and model M^H have similar optimal solutions, showing only a 0.3% difference in the total cost.

It is noticeable that the relaxed model M^R used in the first phase also has the capability of predicting optimal costs (see Figure 4 and Table 4). The most affected one is the truck cost, which is estimated by relaxing the associated integer variables. However, the other costs yield similar solutions with a 3% difference at most.

The ability of the relaxed model to find the optimal allocation is depicted in Figure 5, which compares the customer demand allocated to plants, depots, and third parties over the time horizon. Total customer demand is scaled for confidentiality reasons.

Overall, the results show that the proposed approach can solve the problem without a significant degradation of the solution quality. Since the full-space model M^S and reduced model M^H provide similar optimal solutions, only the result from the reduced model M^H is

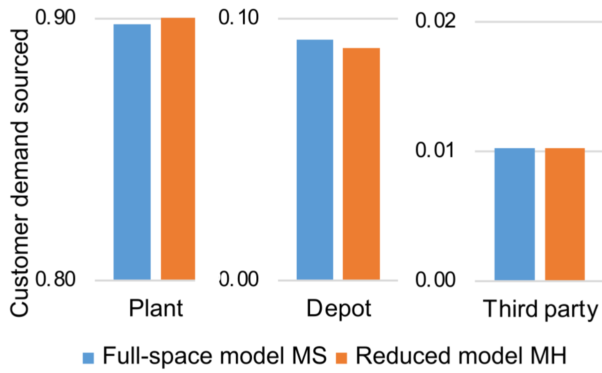

Figure 5: Result of the customer demand allocation

presented in the remainder of the paper.

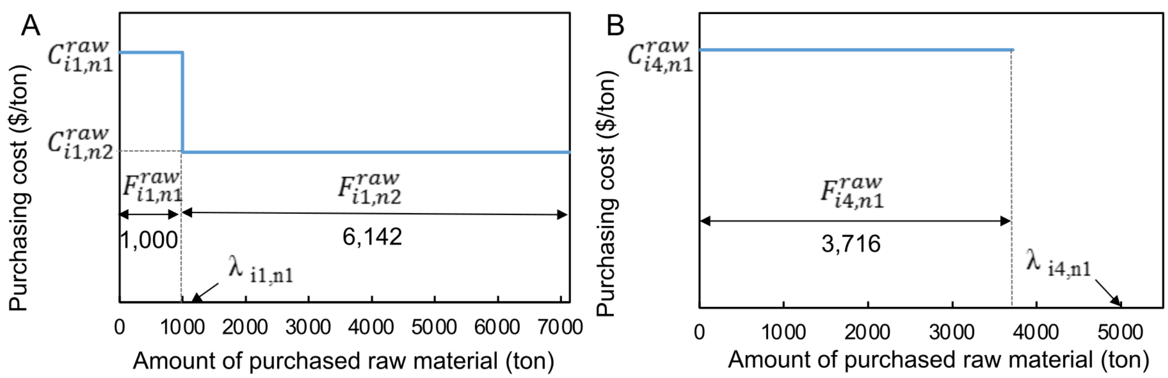

Figure 6: Result of the discount contract model (A) plant $i1$ and (B) plant $i4$

Figure 6 displays the result from the discount contract model for plant $i1$ and plant $i4$. The contract model for plant $i1$ has one break point ($\lambda_{i1,n1}$=1,000) and two cost regions ($N = 2$). The total amount of plant $i1$ purchased over the time horizon is 7,142 tons. As shown in the figure, 1,000 tons of the raw material, corresponding to the cost region $n1$, are bought at the original cost ($C^{raw}_{i1,n1}$) and 6,142 tons of the raw material in the cost region $n2$ are purchased at the reduced cost ($C^{raw}_{i1,n2}$). On the contrary, the total amount of product purchased by plant $i4$ is 3,716 tons, all purchased at the original cost ($C^{raw}_{i4,n1}$), as the total purchased amount does not exceed the break point ($\lambda_{i4,n1}$).

Figure 7 illustrates the optimal production schedule. The coloured cell represents that the plant is under the on-mode and the value represents the production utilisation rate in each period, which is defined as the production amount divided by the maximum production

Plant	Time period (day)																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
i1		99	98	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100							
i4			70	100	70	100	100	100	100	70	70	96	70	70	100	100	100	100	100	100										
i5				70	70	70	70	87	70	100	70	100	100	70	100															
i10			77	100	100	100	100	100	100	88	100						86	100	100	100	100	100	100	100	100					
i16					100	100	99	100	100	100	100	100					100	100												
i17				89	87	100	100	100	100	100	100			100	100	70	100	100	100	88	100									100
i18	100	100	100	100	100	81	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Utilisation rate (%)

Figure 7: Optimal production schedule

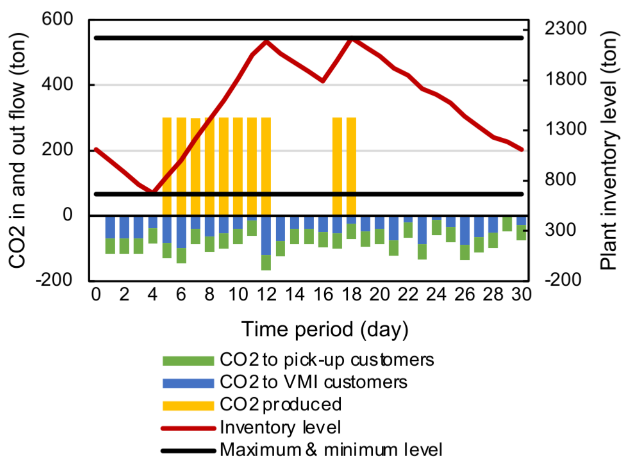

Figure 8: Inventory profile for plant $i16$

capacity. As observed, when the plant decides to operate, it tends to maintain the operation mode as much as possible to avoid the plant start-up cost. This trend can be captured more clearly with the inventory profile of the plant (see Figure 8). Plant $i16$ starts to produce the CO_2 product after its inventory hits the minimum level on day 4 and keeps the operation mode until it meets the physical constraint of the tank capacity on day 12.

Figure 9 shows the Gantt chart for part of the optimal rail-car schedule. The solid cells represent a one-way trip, whereas the vertical striped cells represent a return trip. As can be seen, the proposed model captures the wrap-around effect spanning the rail-car schedule in successive planning time horizons. For instance, rail-cars which arrive at depot $j2$ on day 3 depart plant $i15$ on day 30 in the previous month. Similarly, those that depart plant $i15$ on day 30 return to the origin on day 7 of the next month.

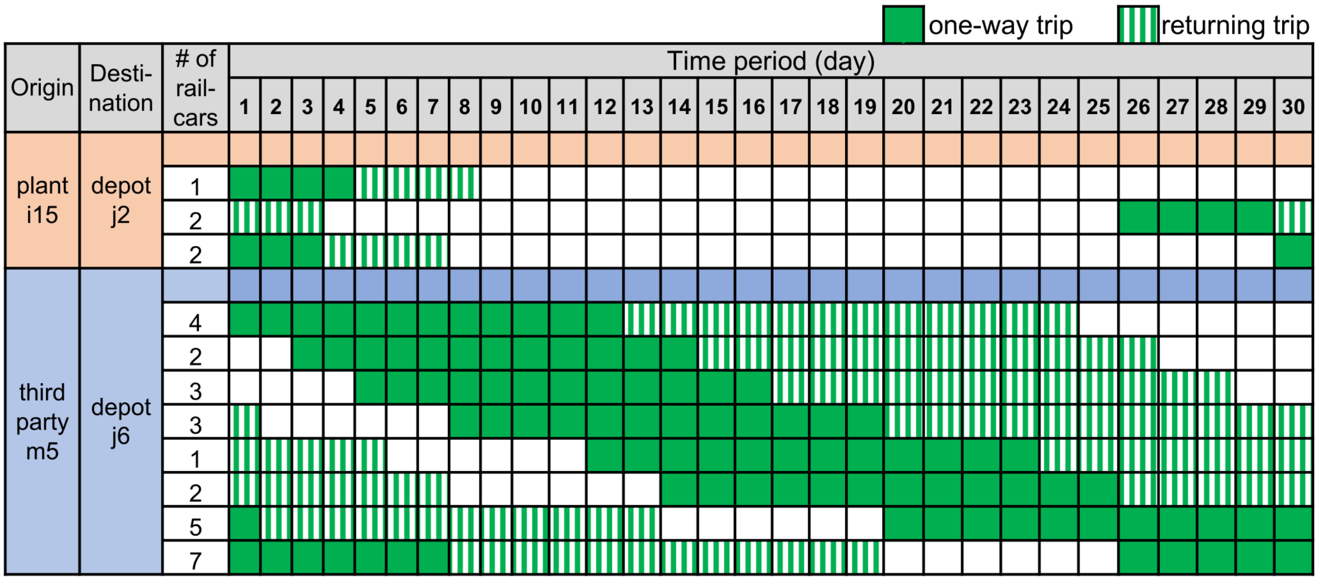

Figure 9: Gantt chart for the rail-car schedule

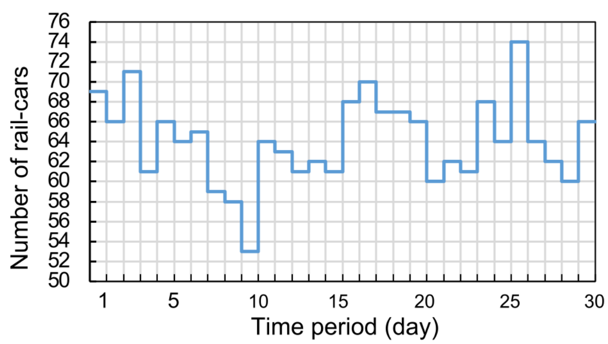

Figure 10: Rail-car resource usage profile

Figure 10 provides the number of rail-cars in transition during each period. The maximum number of rail-cars required for the distribution in each period is 74, while the maximum available number of rail-cars is over 300.

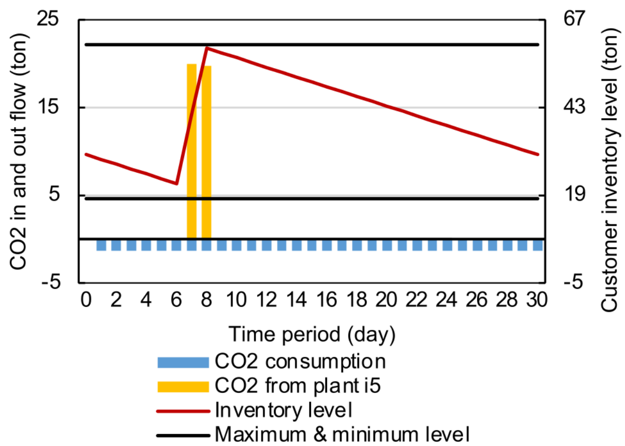

Figure 11: Inventory profile for customer $k242$

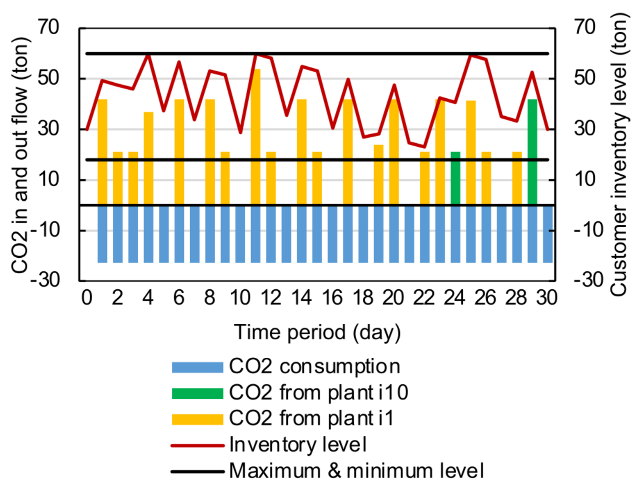

Figure 12: Inventory profile for customer $k246$

Finally, Figures 11 and 12 provide the inventory profiles for two VMI customers, $k242$ and $k246$, which have the same tank capacity but different product consumptions. Information on their consumptions, tank capacities, minimum levels, and product amounts fed into the tanks are included in the figures. As can be observed, the frequency and replenishment amount fulfilled by trucks differ depending on their consumptions. Because the consumption of customer $k242$ is smaller than its tank capacity, it needs only two replenishments over a given time horizon, while customer $k246$ needs 21 replenishments owing to its high consumption.

Testing the proposed hierarchical solution strategy

To illustrate the applicability and efficiency of the proposed hierarchical solution strategy, we solve the six additional realistic instances, with 80 trucks, over 30 plants/depots/third parties, 750 customers, and planning horizons of 1-6 weeks, which are derived from the original case study presented above. The optimality gap is set to 5% for the model (M^S) in the full-space approach and set to 1% and 5% for the relaxed and reduced models (M^R and M^H) in the hierarchical approach, respectively. Additionally, the computation time is limited to 150,000 s. The results and problem sizes of instances are reported in Table 5.

Table 5: Computational results of the full-space and hierarchical approaches

instance	approach	equations	continuous variables	discrete variables	CPU^b s	optimality gap (%)	total cost (M$)
1-week	full-space	62,561	35,810	98,631	373	4.97	1.58
	hierarchical^b	12,848	15,267	22,845	86	3.02	1.60
2-week	full-space	119,793	66,995	181,791	1,475	4.99	2.80
	hierarchical^b	24,769	26,361	45,728	169	4.91	2.83
3-week	full-space	177,025	98,180	264,951	10,439	3.40	4.07
	hierarchical^b	36,718	37,501	68,779	3,930	3.50	4.13
4-week	full-space	234,257	129,365	348,111	13,389	4.95	5.39
	hierarchical^b	48,709	48,664	91,438	4,305	2.67	5.37
5-week	full-space	291,489	160,550	431,271	15,000	-^{c}	-^{c}
	hierarchical^b	60,637	59,757	113,726	12,063	5.00	6.78
6-week	full-space	348,721	191,735	514,431	15,000	-^{c}	-^{c}
	hierarchical^b	73,405	71,792	140,074	14,268	5.00	8.07

^{a} reduced problem in the second phase of the hierarchical approach
^{b} total CPU time for each approach
^{c} a feasible solution has not been found within the time limitation (15,000 s)

For the 1- and 2-week instances, the proposed hierarchical approach can find the optimal solutions that are almost the same as the solutions from the full-space model within only 86 s and 169 s, respectively. In the 3- and 4-week cases, we can observe that the CPU time has been increased exponentially for both full-space and hierarchical approaches. However, the proposed approach can still find the solutions within significantly reduced time (4,305 s), while the full-space model takes more than 37,500 s to find the solutions. It should be noticed that despite the solutions by the full-space approach are slightly better than the solutions from the hierarchical approach, the differences between the solutions are within only 1.5%. These results trend becomes more apparent when dealing with very large problems. For the 5- and 6-week instances, the full-space model cannot even find any feasible solution in the time limit, while the proposed approach can provide optimal solutions that have less than 5% optimality gap.

This comparison allows us to demonstrate the efficiency and necessity of the proposed solution strategy, especially when solving large-size problems.

6. Concluding remarks

This paper presented an MILP model that considers supply contracts, production, inventory management, and rail-car and truck scheduling simultaneously. Further, a two-stage hierarchical solution strategy was also developed to handle the complexity of solving the integrated model. In the first stage, the problem was solved using a relaxed MILP model in which the integer variables associated with the trucks were relaxed into continuous variables. In the second stage, the problem size of the original full-space MILP model was reduced by fixing the optimal allocation result obtained in the first stage.

The validity and efficiency of the proposed mathematical framework and strategy were tested on a real-world industrial gas supply chain problems. The results showed that the developed MILP model can incorporate multiple production and distribution decisions. Furthermore, it was verified that the proposed approach can find good quality solutions with modest computational effort.

Further work could be directed at considering other solution techniques (e.g. rolling horizon strategy, bilevel decomposition, Benders decomposition, and dual problem) to enhance the computational efficiency of the proposed solution strategy for large-size industrial problems.

Nomenclature

Indices

i production plant

j depot

k customer

m third-party supplier

n cost region

t time period

v truck

Sets

J_i set of depots allocated to plant i

J_m set of depots allocated to third parties m

K_i^P set of pick-up customers allocated to plant i

K_j^P set of pick-up customers allocated to depot j

K_i set of VMI customers allocated to plant i

K_j set of VMI customers allocated to depot j

K_m set of VMI customers allocated to third party m

Parameters

α_i coefficient relates amount of raw material and product produced by plant i

Δ_t length of time period t, day

λ_{in} amount of raw material corresponding to plant i and cost region n, ton

μ_i unit fixed production cost of plant i, \$/ton

ν_i unit production cost of plant i, \$/ton

σ_{mn} amount of product corresponding to third party m and cost region n, ton

τ_{ij} transportation time of rail-cars between plant i and depot j based on one-way trip, day

τ_{ij} transportation time of rail-cars between third party m and depot j based on one-way trip, day

θ_{ik} transportation time of trucks between plant i and customer k based on round-trip, day

θ_{jk} transportation time of trucks between depot j and customer k based on round-trip, day

θ_{mk} transportation time of trucks between third party m and customer k based on round-trip, day

C^{truck} unit transportation cost for trucks, \$/mile

C^{rail}_{ij} transportation cost for each rail-car between plant i and depot j, \$

C^{raw}_{in} unit cost for raw material purchased by plant i corresponding to cost region n, \$/ton

C^{st}_{i} plant start-up cost of plant i, \$

C^{rail}_{mj} transportation cost for each rail-car between third party m and depot j, \$

C^{os}_{mn} unit cost for product purchased from third party m corresponding to cost region n, \$/ton

Cap^{rail} loading capacity for each rail-car, ton

Cap_i^{truck} loading capacity for each truck stationed at plant i, ton

Cap_j^{truck} loading capacity for each truck stationed at depot j, ton

Cap_m^{truck} loading capacity for each truck stationed at third party m, ton

D_k product consumption of VMI customer k in each time period, ton

DP_{ik} product consumption at plant i of pick-up customer k in each time period, ton

DP_{jk} product consumption at depot j of pick-up customer k in each time period, ton

I_i^{inv} initial inventory level at plant i, ton

I_j^{inv} initial inventory level at depot j, ton

I_k^{inv} initial inventory level at customer k, ton

I_i^{max}, I_i^{min} maximum and minimum inventory levels at plant i, ton

I_j^{max}, I_j^{min} maximum and minimum inventory levels at depot j, ton

I_k^{max}, I_k^{min} maximum and minimum inventory levels at customer k, ton

L_{ik} round-trip distance between plant i and customer k, mile

L_{jk} round-trip distance between depot j and customer k, mile

L_{mk} round-trip distance between third-party m and customer k, mile

M big number

NR^{max} maximum available rail-cars

NS maximum number of sources each customer can be served over a time horizon

NT_i^{max} maximum available trucks at plant i

NT_j^{max} maximum available trucks at depot j

NT_m^{max} maximum available trucks at third party m

O_m^{max} maximum availability of product at third party m in each time period, ton

P_i^{max}, P_i^{min} maximum and minimum production capacities of plant i in each time period, ton

R_i^{max} maximum availability of raw material for plant i in each time period, ton

T number of time period

Integer Variables

A_{ikvt} number of round-trips between plant i and customer k made by truck v during time period t

A_{jkvt} number of round-trips between depot j and customer k made by truck v during time period t

A_{mkvt} number of round-trips between third party m and customer k made by truck v during time period t

NR_{ijt} number of rail-cars departing plant i to depot j in time period t

NR_{mjt} number of rail-cars departing third-party m to depot j in time period t

Binary Variables

u_{it} 1 if production mode of plant i is switched from the off-mode to the on-mode at time period t; 0, otherwise

w_{it} 1 if production mode of plant i is the on-mode in time period t; 0, otherwise

x_{ik} 1 if customer k is served the product from plant i; 0, otherwise

x_{jk} 1 if customer k is served the product from depot j; 0, otherwise

x_{mk} 1 if customer k is served the product from third party m; 0, otherwise

y_{in} 1 if total amount of raw material purchased by plant i is in cost region n; 0, otherwise

z_{mn} 1 if total amount of product purchased from third party m is in cost region n; 0, otherwise

Continuous Variables

F_{in} amount of raw material purchased by plant i corresponding to cost region n, ton

I_{it} inventory level of plant i at time period t, ton

I_{jt} inventory level of depot j at time period t, ton

I_{kt} inventory level of customer k at time period t, ton

O_{mt} product amount purchased from third party m during time period t, ton

P_{it} production amount at plant i during time period t, ton

Q^{RC}_{ijt} product amount delivered from plant i to depot j by rail-cars during time period t, ton

Q^{RC}_{mjt} product amount delivered from third party m to depot j by rail-cars during time period t, ton

Q^{TR}_{ikt} product amount delivered from plant i to customer k by trucks during time period t, ton

Q^{TR}_{jkt} product amount delivered from depot j to customer k by trucks during time period t, ton

Q^{TR}_{mkt} product amount delivered from third party m to customer k by trucks during time period t, ton

R_{it} amount of raw material purchased by plant i during time period t, ton

S_{mn} amount of product purchased from third party m corresponding to cost region n, ton

TC^{os} total product cost purchased from third parties, \$

TC^{prod} total production cost, \$

TC^{rail} total transportation cost by rail-cars, \$

TC^{raw} total raw material cost purchased by plants, \$

TC^{st} total plant start-up cost, \$

TC^{total} total operating cost, \$

TC^{truck} total transportation cost by trucks, \$

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