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Technical Note

Investigation of the ultimate particle size distribution of a carbonate sand

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Abstract

A series of ring shear tests were conducted to investigate the ultimate particle size distribution of a carbonate sand. The tests were carried out under different stress levels, on three types of specimens: 1) uniformly graded specimens made of dry natural sand 2) remoulded specimens of the crushed sand after first shearing to large strains 3) specimens made of natural sand grains but with the same grading as in (2). The first series of tests on type (1), carried out to very large strains, led to apparently stable gradings, distinct for each stress level. Only limited additional particle breakage could be induced by remoulding the specimens after shearing (type (2)) and subjecting them to more shearing. Tests on specimens created at the apparently stable gradings (type (3)) but from the intact sand particles however led to significantly greater breakage. For the three types a stable, fractal grading was achieved. Analyses of the soil particles' shape showed that the aspect ratio, sphericity and circularity reach a steady value at large strains, in parallel to reaching a stable grading. The mobilized angle of shearing resistance however was not significantly different in the different types of samples, suggesting the final grading dominates the behaviour.

Keywords: Ultimate particle size distribution; Carbonate sands; Ring shear; Breakage; Particle shape

1. Introduction

Shearing to exceptionally large strains or displacement is not uncommon in granular soils, such as along a driven pile shaft (Yang et al., 2010; Lobo-Guerrero and Vallejo, 2007; Kuwajima et al., 2009), which would make it suffer particle breakage. An accepted method to investigate particle breakage in soil is to perform ring shear tests, as very high strains can be reached. Examples of previous research include work by Hooke and Iverson (1995) or Altuhafi et al. (2017) on glacial till, Luzzani and Coop (2002) and Coop et al. (2004) on quarzitic and carbonate sand, Miao and Airey (2013) or Zhang et al. (2017) on carbonate sand and Ferreira and Coop (2002) on decomposed granite and carbonate sand. It is found that an ultimate grading, as a result of particle breakage, reached after large shear strains even under moderate stress levels in the ring shear apparatus (Lobo-Guerrero and Vallejo, 2006; Miao and Airey, 2013; Zhang et al., 2018; Wei et al., 2018), just as it can reach at very large stresses in the triaxial apparatus at moderate strain levels (Turcotte, 1986; Perfect et al., 1992; McDowell et al., 1996; Sammis et al., 1986), suggests that stress or strain alone may not be sufficient to reach an ultimate grading, but breakage is related to the amount of plastic work(Hu et al., 2011; Ovalle et al., 2015). The existence of an ultimate grading is of great significance in modeling, for example, it is found that the plastic work in crushable granular materials is related to the evolution of

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the grain size distribution (Miura and O-Hara, 1979; Daouadji et al., 2001), a proper definition of the amount of particle breakage, therefore, can be used as an energy dissipation parameter in constitutive models (Einav, 2007; Daouadji and Hicher, 2010; Liu et al., 2017; Yin et al., 2017; Hu et al., 2018; Ovalle and Hicher, 2020).

The concept of ultimate grain size distribution is however not absolute, as previous studies have indicated that the ultimate steady state at which breakage ceases has a high dependence on the initial conditions, which Lee and Farhoomand (1967) had already identified as initial grading, particle size, particle shape, and stress. Coop et al. (2004) also showed that the final grading curves reached in ring shear tests may be different for different stress levels, but at higher stress levels there was a tendency towards a fractal distribution with a fractal dimension of about 2.57. The effect of the initial grading on the susceptibility to breakage of particles during shearing was noted by Altuhafi et al. (2010) in their ring shear tests on subglacial sediment from Iceland and by Zhang et al. (2017) on carbonate sands from China. Zhang et al. (2018) also claimed that larger particles and poorly graded soils are more prone to breakage, and are closer to a fractal distribution after shearing, and it was found that the particles became more rounded after larger shearing strains. From their most recent work Ferreira and Coop (2002) concluded that a final grading may not exist for a given soil but depends on the initial particle size distribution, the stress level, initial density and speed of shearing. A factor that seems to be often overlooked however is the influence of fabric on the breakage potential and the wider soil behaviour, for example, Bandini and Coop (2011) found that soils with a similar grade size distribution, one achieved through pre-shearing and the other simply reconstituted, reach different critical state lines because in the pre-sheared samples crushing was inhibited by the presence of broken small grain pieces. Yan and Shi (2014) also showed that remoulding samples after one-dimensional compression to high stresses before re-subjecting them to large loads leads to additional breakage, in excess to what was reached in the first loading. The effect of the type of loading on the breakage has been recognised as well (e.g. Miao and Airey, 2013). The significance of the existence of an ultimate grading is particularly important for modellers, even if it is not reached necessarily during normal engineering scenarios, similar to determining a reference de-structured state for modelling structured soils (e.g. Baudet and Stallebrass, 2004). The soil grain shape and its evolution during loading are other indicators of whether a final grading exists, as Yan and Shi (2014) showed that on reaching a final limiting grading, different particle sizes have very similar shape characteristics.

The data presented in this paper bring further evidence that the distributions of particle shapes and particle sizes should be considered together when identifying whether a grading is final. They also show how the fabric, where fabric refers to both the arrangement of particles and their shapes, plays a large part in reaching a uniform grading. Ring shear tests were first performed on carbonate sand from an initial uniform grading, to strains large enough to reach an ultimate, fractal grain size distribution. Ring shear tests were then carried out on reconstituted specimens with that fractal grading, some specimens simply made by remoulding the pre-sheared samples, and other specimens created by combining relevant sizes of the intact particles. The distribution of sand particle shapes and their evolution with shearing were mapped using a laser scanner.

2. Material and procedures

2.1. Material

The biogenic carbonate sand from the South China Sea, mainly composed of broken shell fragments was used in this study (See Fig. 1). The sand was first sieved into uniformly sized samples($d_{small} = 0 \sim 0.063$ mm, $0.063 \sim 0.15$ mm, $0.15 \sim 0.212$ mm, $0.212 \sim 0.3$ mm, $0.3 \sim 0.425$ mm and $0.425 \sim 0.6$ mm for the smaller particles; and $d_{large} = 0.6 \sim 1.18$ mm for the large particles)). A first series of ring shear tests was carried out on specimens made of the larger size range ($d_{large} = 0.6 \sim 1.18$ mm), prepared at the initial void ratio $v_0 = 2.150$ by pluviation and gentle tamping. Several tests at different strains on the specimens were performed. With the increase of the shear strains, no change in the grading of the specimen means that the breakage ceases. After the breakage ceases, the crushed samples were sieved again. Tests were then performed on specimens prepared at the stable grading achieved in the pre-sheared samples, with one series using the sieved, crushed grains to create the specimens and another series using the intact grains obtained from the original sieving. The purpose was to investigate the effect of fabric, including the shape of particles, on the sand breakage potential. Those specimens were prepared loose using pluviation methods with dry carbonated sands.

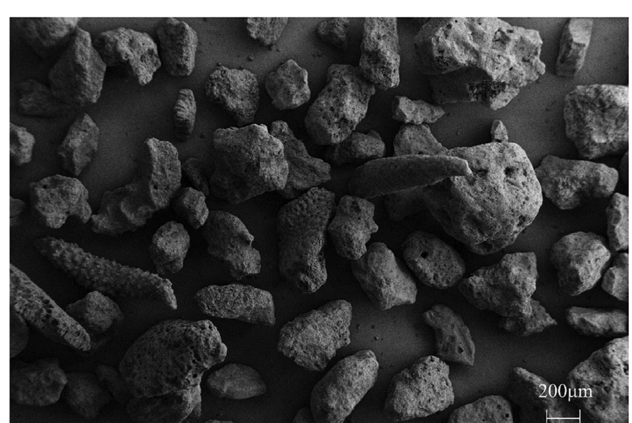

Fig. 1. SEM image of the biogenic carbonate sand.

2.2. Shear tests

The tests were conducted in a Bromhead ring shear apparatus (Bromhead, 1979), with an inner diameter of 70 mm, an outside diameter of 100 mm, and an initial height of 5 mm. The sample height being as small as 5 mm in the ring shear tests had the advantage of having the whole sample in an effective shear zone, but it is may not particularly ideal as a result of the ratio sample height/d_{max} slightly less than 5. Details of the tests are given in Table 1. Normal stresses σ of 370 kPa (samples RS370_1-7) and 615 kPa (samples RS615_1-8) were applied on the uniformly graded specimens. The specimens prepared at the ultimate grading, which was achieved under 615 kPa, were also tested under a normal stress of 615 kPa. Samples are called RSC for crushed particles and RSI for intact particles. To achieve a stable grading very large displacements were sought, reaching shear strains in excess of 100,000%, where the shear strain is defined as: $\gamma = \frac{\delta v}{H_0}$ with δ_v the shear displacement and H_0 the sample height at the state of shearing. The evolution of the particle size and shape distributions was mapped by repeating similar tests from strain 0 to different strain levels up to the maxima of 110,000% and 170,000% for 615 kPa and 370 kPa normal stress respectively. The shearing rate used was 2.2 mm/min.

During the tests, a small amount of soil was lost from the gap between the top platen and the sample (see Fig. 2), making it difficult to measure volume changes. The effect of this soil loss on the ultimate particle size distribution will be discussed later.

2.3. Particle size and shape analysis

The size and shape of sand particles were measured by a Morphologi G3 laser scanning instrument which can capture two-dimensional images of the sand grains and calculate various size and shape parameters, such as the aspect ratio or convexity, based on the dimensions, area and perimeter of the 2D images. Although the magnifications available allow measurements in the range 0.5–3000 μm, only particles with sizes greater than 63 μm were measured because, for the smaller grains, even the higher magnification could not guarantee the accuracy of the shape quantification. Particle size and particle shape distributions could be determined for each tested sample.

3. Results and discussion

3.1. Evolution of particle size distribution of uniformly graded specimen

Fig. 3(a) and 4(a) show the evolution of the grain size distributions of the uniformly graded sand when subjected to a vertical stress of 370 kPa and 615 kPa respectively. The compression barely affected the initial grading, while significant crushing is visible from shear strains of 1,000%. In both sets of tests the curves start converging for shear

Table 1
Details of the tests conducted.

Test	σ during shearing: kPa	Initial Grading: mm	Final γ: %	Note
RS0	370	0.6–1.18	0	Morphologi G3
RS370_1			1000	
RS370_2			5000	
RS370_3			12,000	
RS370_4			25,000	
RS370_5			50,000	
RS370_6			110,000	
RS370_7			170,000	Morphologi G3
RS615_1	615		1000	
RS615_2			2500	
RS615_3			5000	Morphologi G3
RS615_4			10,000	
RS615_5			20,000	Morphologi G3
RS615_6			35,000	
RS615_7			50,000	Morphologi G3
RS615_8			110,000	Morphologi G3
RSC615_1		Fractal dimension	10,000	
RSC615_2			30,000	
RSC615_3			50,000	Morphologi G3
RSI615_0			0	Morphologi G3
RSI615_1			10,000	
RSI615_2			30,000	
RSI615_3			50,000	
RSI615_4			70,000	
RSI615_5			120,000	Morphologi G3

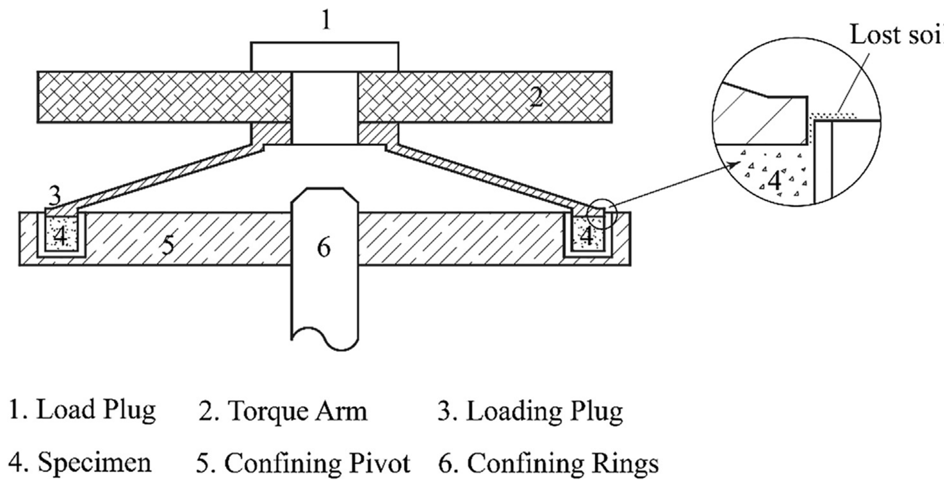

Fig. 2. Details of the ring shear cell.

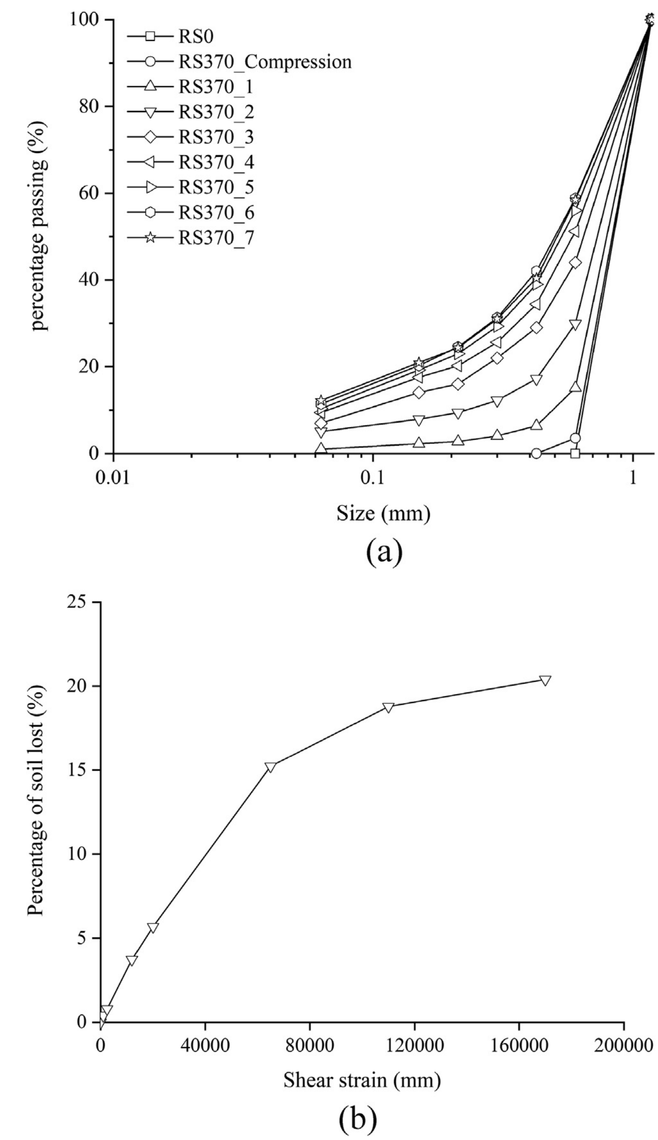

Fig. 3. Evolution of particle size distribution and soils lost for the tests with σ of 370 kPa: (a) particle size distribution; (b) percentage of soil lost.

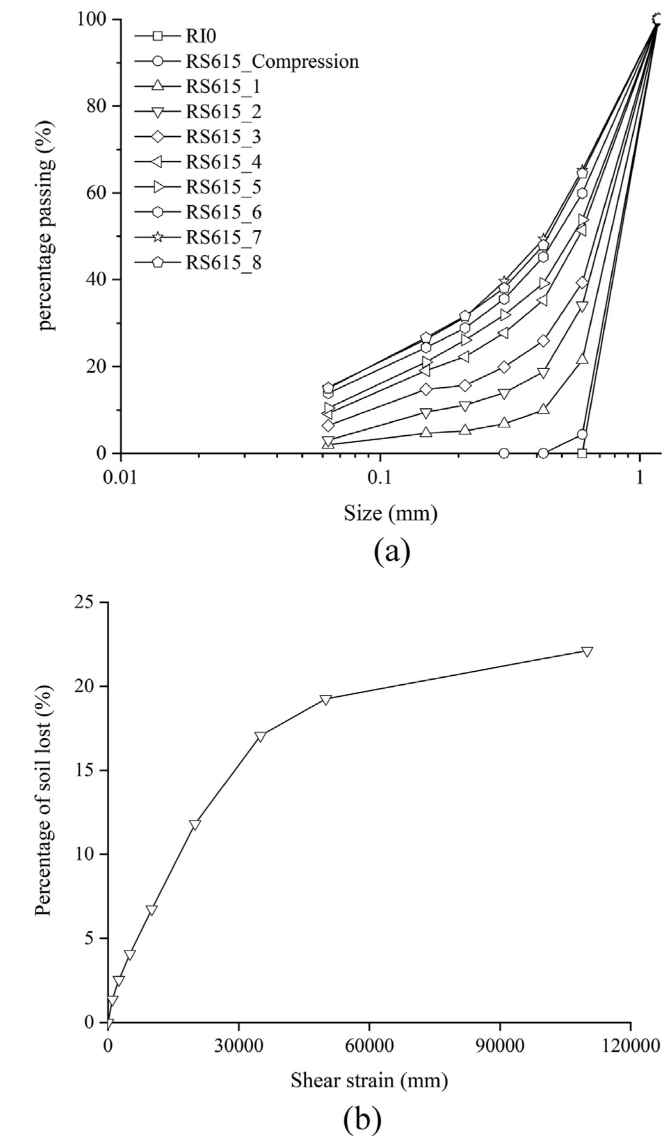

Fig. 4. Evolution of particle size distribution and soils lost for the tests with σ of 615 kPa: (a) particle size distribution; (b) percentage of soil lost.

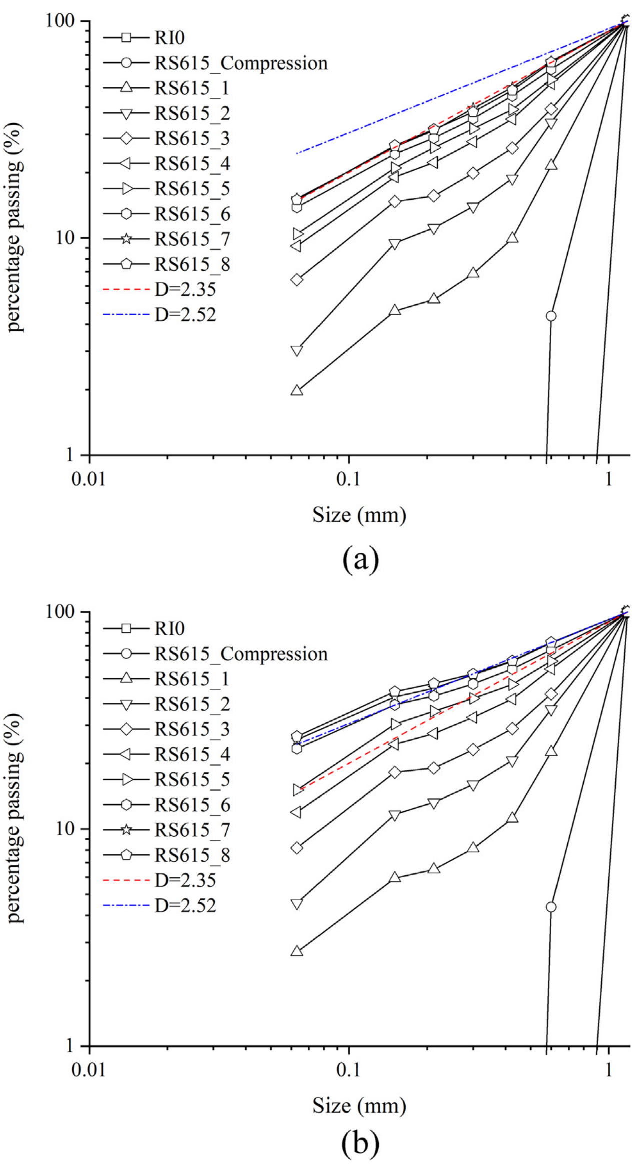

Fig. 5. Evolution of particle size distribution for the tests with σ of 615 kPa: (a) without lost particles; (b) with lost particles.

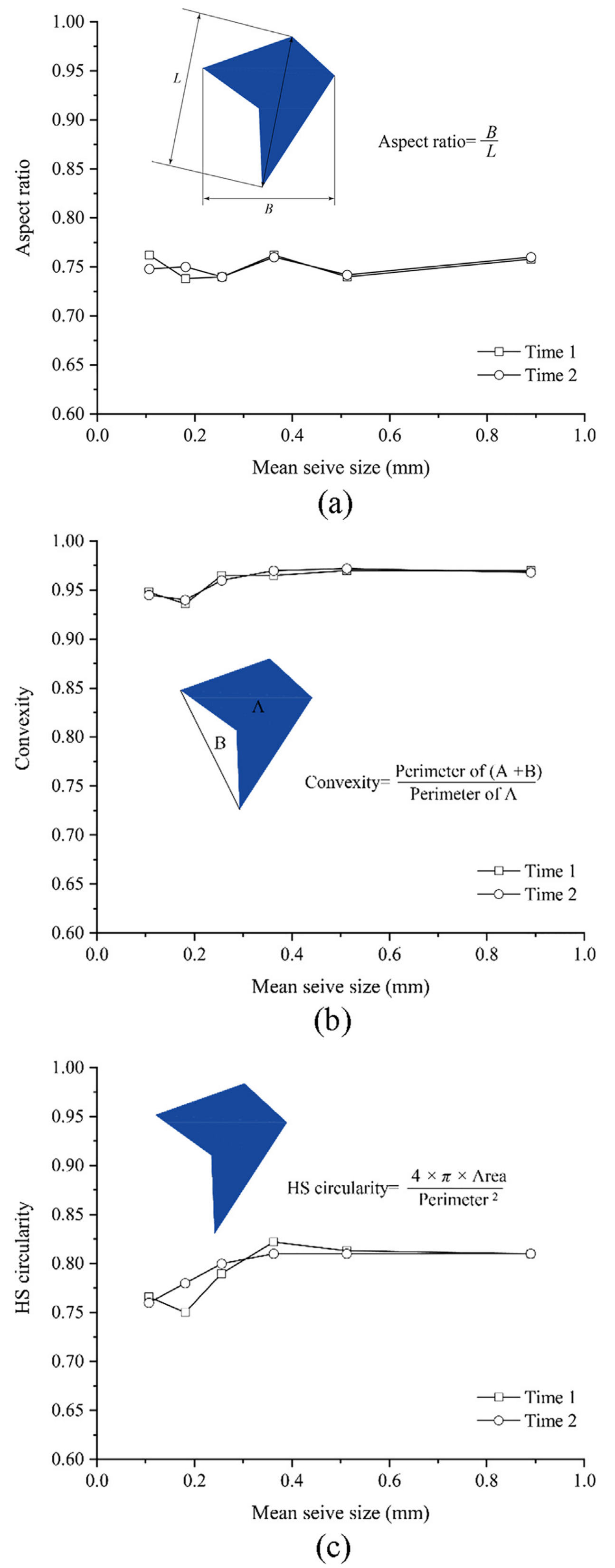

Fig. 6. Repeated measurement of particle shape factors of the same carbonate sands sample taken from the natural sand: (a) Aspect ratio; (b) Convexity; (c) HS circularity.

strains larger than 20,000–25,000%, reaching a stable grading at very large strains between 100,000 and 170,000%. A small amount of sand, mostly fines smaller than 0.15 mm resulting from the crushing, was lost during the tests. The cumulative percentages of soil loss at each shear strain level are plotted in Figs. 3(b) and 4(b). The rate of sand loss was regular during the shearing process, slowing down as less breakage occurred and the grading tended to become stable. A stable grading was reached in spite of the loss of fines, suggesting that the arrangement of particles within the samples RS370_7 and RS615_8 did not vary anymore.

The fractal dimension, D, is commonly used to characterise the ultimate grading soils which is thought to show self-similar characteristics. The definition of fractal dimension is from Einav (2007). Einav's definition is mass-based, and takes a limiting particle size into account: hence it has

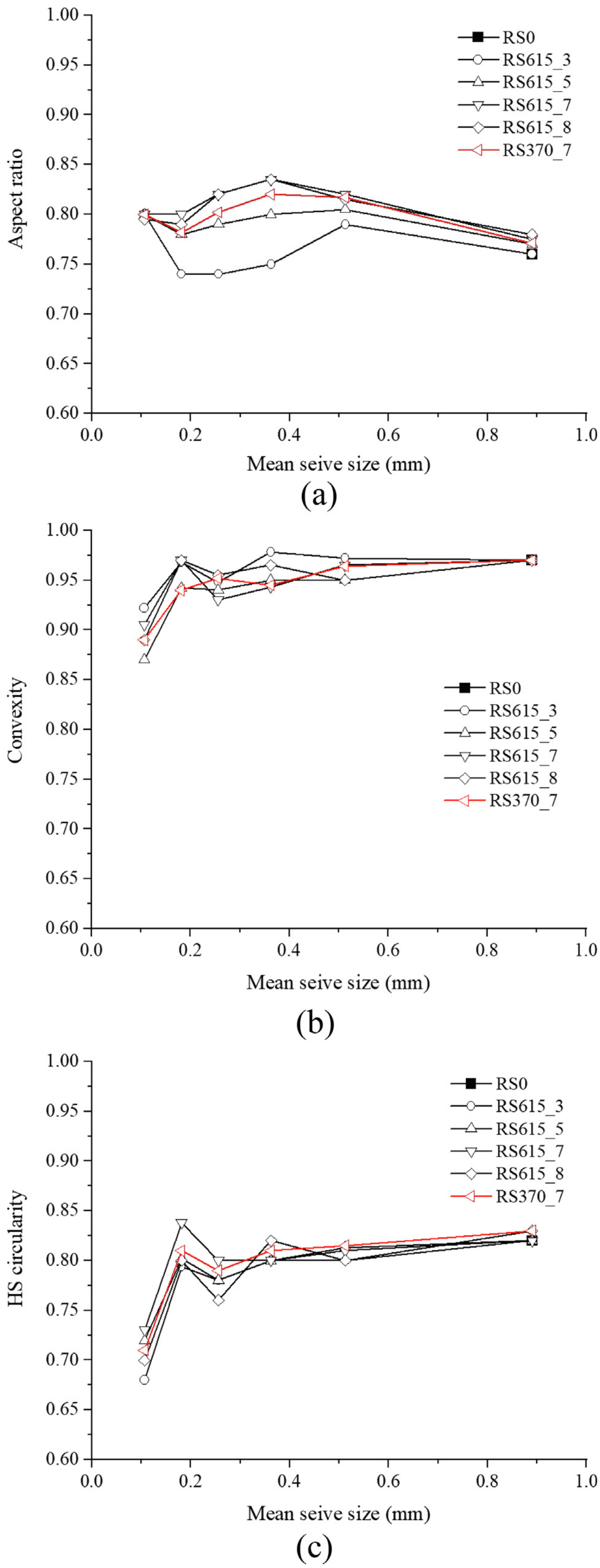

Fig. 7. Evolution of particle shape factors during shearing: (a) Aspect ratio; (b) Convexity; (c) HS circularity.

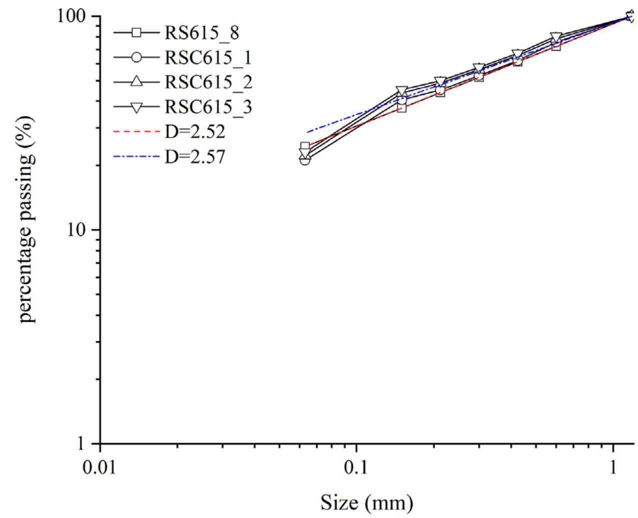

Fig. 8. Evolution of particle size distribution for the tests on crushed sands.

a more convenient form for the analysis of empirical data. A particle-size distribution with a fractal dimension is defined as

$$F(d) = \frac{M_d(\Delta \leqslant d)}{M_T} = \frac{d^{3-D} - d_m^{3-D}}{d_M^{3-D} - d_m^{3-D}}$$

where $M_d(\Delta \leqslant d)$ is the mass smaller than or equal to a reference particle size, M_T is the total sample mass, d is the reference particle size, d_M is the largest particle size, and d_m is the smallest particle size. Independent of the initial distribution, most granular soils tend to $D \approx 2.5$ under compression alone (Turcotte, 1986; Perfect et al., 1992; McDowell et al., 1996). Coop et al. (2004) determined that Dogs' Bay carbonate sands sheared to large shear strains under 650–930 kPa normal stress reached a fractal grading of dimension $D = 2{\cdot}57$. In this study, the fractal dimension of the stable gradings was determined from the slope of the mass cumulative grain size distribution, which is equal to 3 – D. After shearing under 615 kPa, the stable grading had a fractal dimension of 2.35 (Fig. 5(a)), although it could be argued that a multiple fractal would represent the grading curve better, as was proposed by Miao and Airey (2013). Note that mono-fractal means that the whole gradation is described by a fractal dimension, while multiple-fractal means that the gradation of each section of particle size is described by a fractal dimension. By accounting for the lost particles, which were carefully retrieved at the end of testing, in the mass particle size distribution, the grading curve tends to be mono-fractal, with a dimension of 2.52 (Fig. 5(b)). Similarly, a stable grading was achieved in tests carried out under 370 kPa normal stress, with a fractal dimension of 2.25, which increased to 2.47 when the lost particles were included. It is interesting to note that the loss

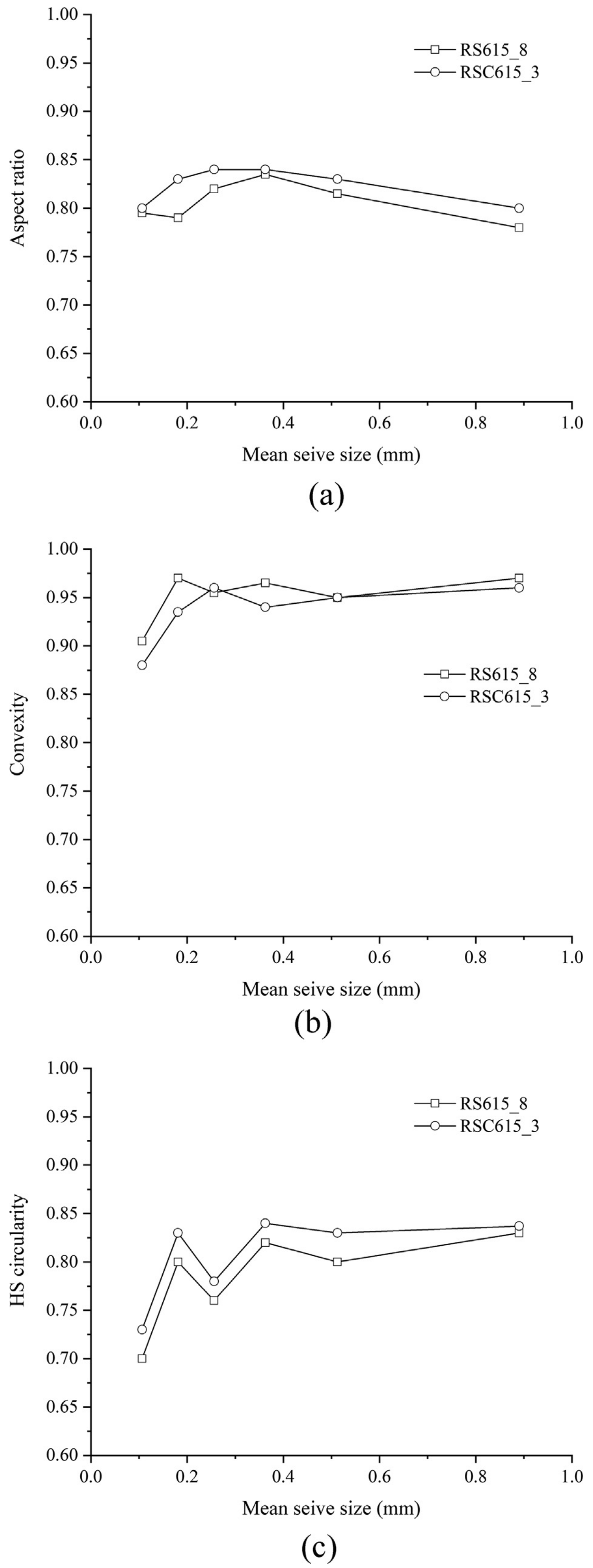

Fig. 9. Evolution of particle shape factors during shearing for the tests on crushed sands: (a) Aspect ratio; (b) Convexity; (c) HS circularity.

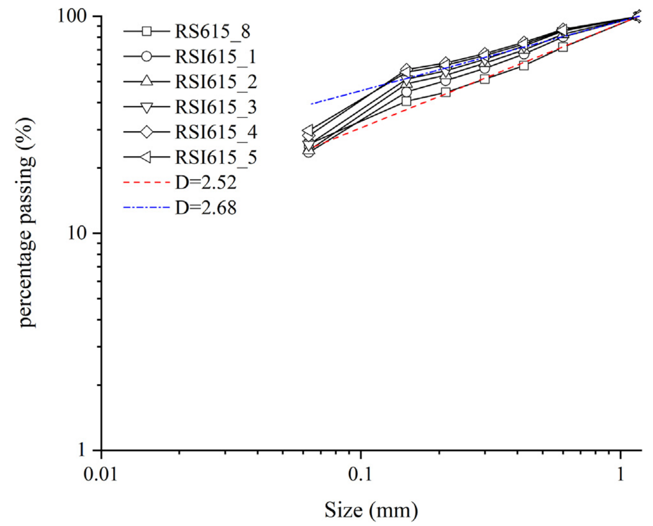

Fig. 10. Evolution of particle size distribution for the tests on intact natural sands.

of fines did not prevent the soil from reaching a stable grading and a stable strength. Adding the fines to the ultimate particle size distributions had for effect to shift the ultimate gradings upwards and slightly increase the fractal dimensions. However, it is not clear what grading would be reached without the loss of fines during the shearing. To continue conducting the follow-up tests to study the evolution of particle size distribution of well-graded specimens, the ultimate particle size distribution with the lost fines was chosen in this paper.

3.2. Evolution of particle shape during shearing

Three shape factors, the aspect ratio, convexity and circularity, were chosen to describe the particle morphology. They were calculated directly by the Morphologi G3 built-in software, according to the following definitions, which may defer slightly from other definitions seen in the literature. Here aspect ratio is defined as the width (B) divided by the length (L), where B is the longest of the possible lines between two points on the perimeter of the particle, and L is the longest of the possible lines between two points on the perimeter projected. Convexity is a measurement of the surface roundness of a particle, calculated by dividing the convex hull perimeter by the actual particle perimeter. High sensitivity circularity (HS circularity) was calculated, as the square of the ratio of the perimeter of a circle with the same area as the particle divided by the perimeter of the actual particle image, the squaring achieving higher accuracy by sensitising the parameter to very subtle variations in the area-perimeter relationship.

The shape analyses were done separately for each size fraction, by randomly selecting about 100 particles in each

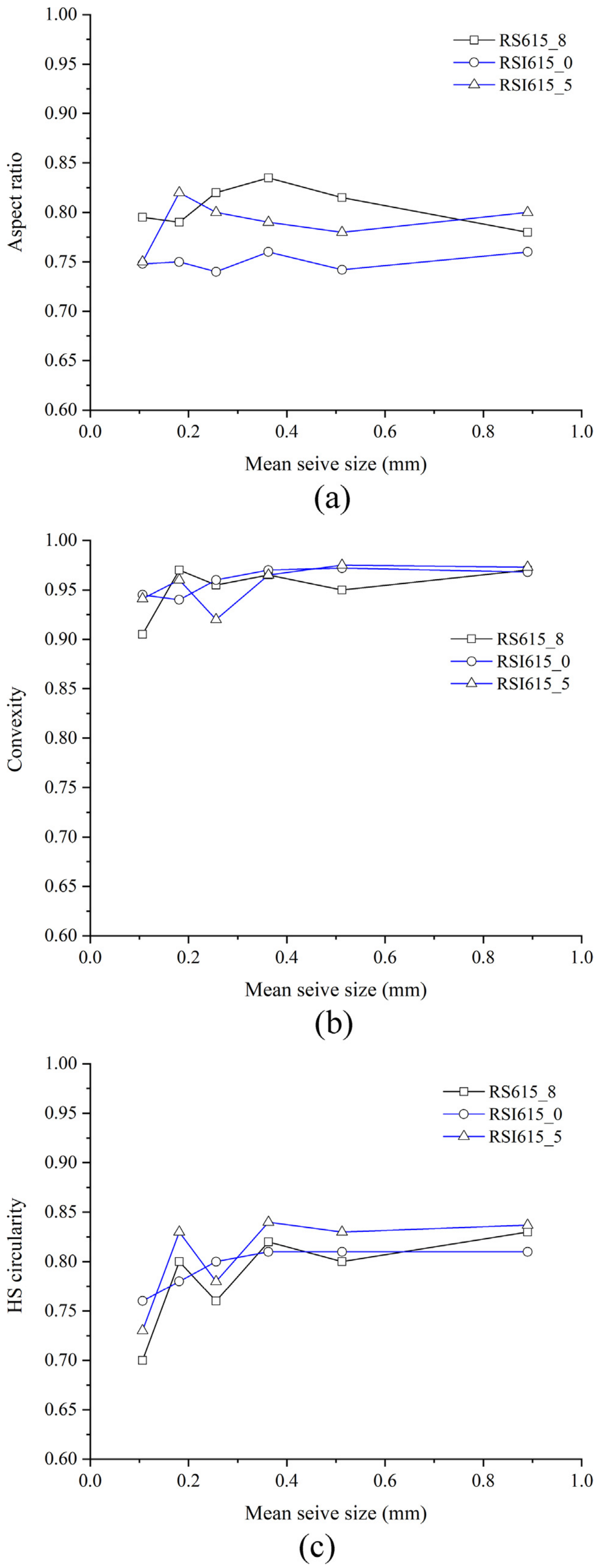

Fig. 11. Particle shape factors of crushed and intact natural sands: (a) Aspect ratio; (b) Convexity; (c) HS circularity.

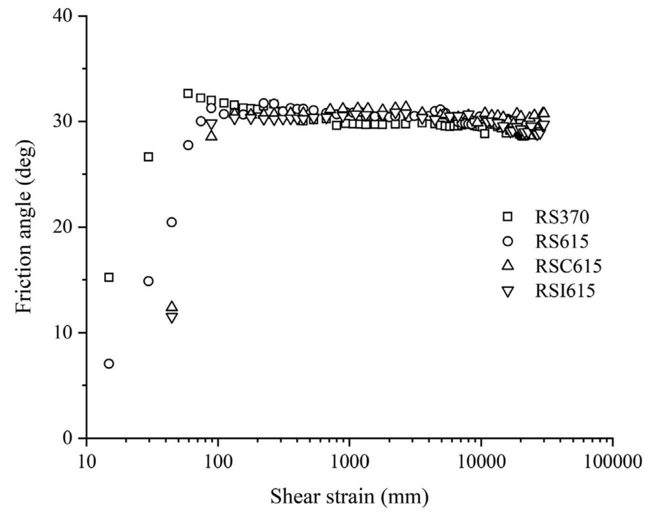

Fig. 12. Evolution of mobilised angle of shearing resistance with shear strain.

size group (0.15 ~ 0.212 mm, 0.212 ~ 0.3 mm, 0.3 ~ 0.425 mm, 0.425 ~ 0.6 mm and 0.6 ~ 1.18 mm), thus a total of 600 particles per specimen. A representative value was taken at 50% from the cumulative distribution of each shape factor. Each 100-grain sample was scanned twice, with good repeatability as shown in Fig. 6 for the natural, uncrushed sand. The uniformly graded sand, of size 0.6–1.18 mm, had an average aspect ratio of 0.76, HS circularity of 0.82, and average convexity of 0.97 before the test. During shearing, the convexity and HS circularity of the same size range and of the grains created by breakage from bigger size particles, hardly changed, as shown in Fig. 7. The small particles created during shearing are initially more elongated, with lower aspect ratios, tending to higher aspect ratios at larger strains. This is more marked for the tests performed under 615 kPa than those under 370 kPa, suggesting that the main mode of breakage in the sand was splitting, which Miao and Airey (2013) attributed to the requirement of breakage to minimise contact stresses. At very large strains, the shape factors tend to similar values, in parallel to the stabilisation of the grading (Zhang et al., 2018).

3.3. *Evolution of particle size distribution of well-graded specimens*

The sand specimens RSC, which were created by remoulding the pre-sheared sample RS615_8, did not show any more sign of significant breakage with further shearing to 50,000% (Fig. 8). Therefore, although the particles with the lost soil were re-arranged, with different contacts created and the fines distributed more homogeneously within the sand, the stable grading reached in the previous shearing was near "ultimate" for those particles. A slightly different fractal dimension of 2.57 was calculated. A small change in particle shape factors was found, as shown in Fig. 9, with the aspect ratio and HS circularity increasing slightly with increasing shear strain, while the change in aspect ratio was negligible compared to the variation seen previously between RS615_1 and RS615_8 in the initially uniformly graded specimen. This suggests that the remoulding allowed additional abrasion to occur at the new contacts, but changes in the main grain form, quantified by the aspect ratio, were limited by the self-similar nature of the distribution.

The well-graded specimens prepared from the intact grains with a fractal distribution of sizes ($D = 2.52$), on the other hand, experienced breakage as shown in Fig. 10. A stable grading was achieved at shear strains larger than 70,00%, with a fractal dimension of 2.68. Fig. 11 shows the difference in shape between the well-graded samples made of pre-sheared grains (RS615_8) and the samples made of the intact natural grains (RSI615_0). As expected from the previous section, the main difference lies in the aspect ratio, with the intact particles more elongated than the pre-crushed ones. Shearing of the fractal sample made of intact particles caused a similar increase in aspect ratio, while the convexity and circularity were again not significantly affected (Fig. 11). Both for the uniformly graded specimens and this well-graded specimen, which were created with the intact sand grains, particle breakage occurred during shearing, mostly as splitting and thus altering the main form.

3.4. Strength mobilisation during shearing

Fig. 12 summarises the mobilised angles of shearing resistance found for all the tested samples, plotted on a logarithmic strain scale to enable detail of the initial part of the tests. The angles were calculated as $\tan^{-1} \tau/\sigma$, where τ is the applied shear stress and σ the normal effective stress. All samples reached an approximately constant value of friction angle as the shear strain reached about 100%, with no tendency for any degradation of peak angles despite the particle breakage and the changes of particle contact and particle shape. This observation is consistent with previous findings that the critical state angle of shearing resistance for carbonate sand is constant for all whatever the stress level and does not reduce with particle breakage (Coop, 1990), and more generally, is also consistent with previous works without breakage showing that residual strength does not depend on initial grading (Muir Wood and Maeda, 2008; Voivret et al., 2009; Li et al., 2013; Azéma et al., 2017; Cantor et al., 2018; Linero et al., 2019). It is interesting to see that the initial shape of the particles did not affect the friction angle either, but it seems that the final distribution of particle shape factors, which is roughly the same at the end of all tests, is a determinant factor.

4. Conclusions

It was shown, using data from ring shear tests on carbonate sands to very large strains, that a stable, fractal grading, with stable particle shapes, can be achieved for a given soil sample sheared under a given normal stress. Remoulding the crushed sample at its stable, fractal grading and shearing it did not cause any further significant change in particle sizes or shapes, with only limited abrasion observed. A sample of uncrushed particles at the same fractal grading however experienced quantifiable changes in the particles' main form as well as in the particle size distribution, and the fractal dimension increased by about 6%. These data highlight that not only do the particle size distribution and particle arrangement play a role in particle breakage, but the shape of the particles is also a major contributor. The angle of shearing resistance seems to be only affected by the ultimate particle size and shape distributions.

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