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Progressive pedagogies made visible: Implications for equitable mathematics teaching

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Abstract

This paper makes a significant contribution to contemporary debates over the direction curriculum reforms should take. It challenges claims that progressive pedagogies can exclude disadvantaged learners from gaining access to powerful knowledge and argues that greater attention needs to be given to learner agency and subject didactics. It reports on the findings from the Visible Maths Pedagogy research project, which aimed to develop and evaluate strategies for making progressive pedagogies more visible to mathematics learners. Evidence collected from student surveys and interviews suggests that these novel strategies were successful in heightening students' appreciation of the teacher's pedagogic rationale for employing progressive teaching approaches. They appeared to have a positive impact on students' mathematical engagement and awareness of how to achieve success in the secondary school mathematics classroom, particularly for those from disadvantaged backgrounds. The findings highlight the potential of progressive pedagogies made visible for establishing an alternative didactic situation based on socio-mathematical norms associated with 'sense making', rather than 'answer getting', which can help develop students' individual and collective agency. We argue that such a didactic situation offers a pathway towards a more equitable mathematics curriculum that enables wider access to powerful knowledge, and which forms an integral part of a school curriculum designed to address the environmental, economic and social challenges currently faced by our global society.

KEYWORDS
pedagogic rationale, powerful knowledge, progressive pedagogy, visible pedagogy

INTRODUCTION

This paper reports on a participatory action research project carried out collaboratively by the authors (an academic researcher and two teacher researchers) in Stoke Newington School, a non-selective state secondary (age 11–18) school in London. This study explored the impact of adopting strategies for making progressive pedagogy more visible on students' learning. In this paper, we consider the implications of the project's findings for contemporary debates around curriculum reform, particularly those relating to powerful knowledge and learner agency. We explore the potential contribution that progressive pedagogies can make to developing an equitable and empowering mathematics curriculum capable of addressing contemporary challenges facing society. Given its apparently contentious nature, we have been advised on several occasions to avoid using the term 'progressive' in describing the focus of our study and in disseminating its findings. However, we use the term unapologetically in this paper for reasons we hope will become clearer in the next section. When we talk of 'progressive pedagogy', we are referring to collaborative, discursive, open-ended, problem-solving teaching approaches, which recognise multiple solutions to problems and embrace challenges, errors and misconceptions as learning opportunities (Boaler, 2008; Swan, 2006).

THEORETICAL FRAMEWORK

Curriculum making and ideology

Curriculum making is a complex process that involves multiple agents, representing different interests, agreeing on how to help make sense of human experience for future generations. It has been constrained in the past by disagreements over what should be taught (curriculum content) and how it should be taught, that is binary oppositions between traditional/progressive and child-centred/teacher-led pedagogies (Philippou & Priestley, 2019). Disagreements between policymakers with differing educational ideologies have been particularly acute in the field of mathematics education (Ernest, 1991). In the 1970s and 1980s, a 'cognitive revolution' in mathematics education in the United States (Schoenfeld, 2004), with a move towards more progressive and utilitarian (or 'reformist') teaching approaches, was followed by a conservative backlash from traditionalists, who advocated more teacher-led learning of discrete items of mathematical knowledge within traditional subject boundaries, resulting in a bitter conflict that became known as the Math Wars (Schoenfeld, 2004). Similar, although less overt, tensions accompanied mathematics curriculum reforms in England. The Cockcroft (1982) Report endorsed a move towards more progressive teaching approaches in schools, which was reversed by the imposition of the National Curriculum in 1989 by the Conservative-led UK government (Wright, 2012). A subsequent shift back towards progressive ideas was evident in the revised National Curriculum introduced in 2007 by a Labour-led government, which emphasised problem-solving skills, communication and critical understanding, and encouraged collaborative working, engaging with open tasks and out-of-school contexts (Wright, 2017). This policy was again reversed by a Conservative-led government which introduced the 2013 National Curriculum, with far greater emphasis on subject content and disregard for pedagogical approaches and learning contexts (Manyukhina & Wyse, 2019).

Increasing involvement of politicians in educational policymaking since the 1980s has resulted in a global shift towards new curricula foregrounding national competitiveness which focus on developing generic skills or competencies that enable learners to contribute to the economy. More recently, increasing concerns about global warming, populist politics, rising right-wing extremism and racism have highlighted the need for more humanistic approaches to the curriculum (Priestley & Philippou, 2019). A growing appreciation of how misleading adverts, statistics and media reports can influence the beliefs, behaviour and voting habits of millions of people (Alderson, 2020) has resulted in a renewed focus on the school curriculum as a way of cultivating the collective knowledge and critical understanding needed to promote human rights, cultural diversity, equality and social justice (UNESCO, 2015) and to address environmental, economic and social challenges faced by global society (OECD, 2018). However, what a curriculum that promotes equity and social justice might look like remains a source of contention particularly in mathematics education (Xenofontos et al., 2021).

Powerful knowledge and social justice

Muller and Young (2019) adopt a social realist perspective in claiming that certain types of knowledge are inherently powerful. They draw on Bernstein's (2000) argument that 'powerful knowledge' is abstract, specialised, formal and coherent and, by enabling learners to extend their horizons, allows individuals and societies to think 'the unthinkable' and the 'not-yet-thought'. Social realists' concerns for social justice focus on challenging the distributive rules that restrict access to powerful knowledge for some groups in society (Wheelahan, 2007). Muller and Young (2019) propose a framework that categorises curricula into three types. 'Future 1' is considered 'under-socialised' as it foregrounds the transmission of discrete items of knowledge and ignores the context in which these are generated. It is associated with the 'traditionalist' or 'conservative' pedagogies described above, which reinforce outmoded forms of knowledge within rigid subject boundaries and preserve the interests of the powerful (Morgan et al., 2019). 'Future 2' is considered 'over-socialised' as it views knowledge as primarily socially constructed and focuses on learners' experiences, social identity and self-realisation rather than disciplinary knowledge (Shalem & Allais, 2019). It is seen as arising directly from a critique of Future 1 by those with reformist or progressive views.

Social realists contend that the competence-based learning associated with Future 2 ignores disciplinary boundaries, is designed primarily to meet the needs of employers and denies some learners access to the powerful knowledge they need to engage meaningfully in society (Wheelahan, 2007). They argue that the relatively unstructured nature of progressive pedagogies, in which success criteria are less clear to students, renders them ‘invisible’ (Morgan et al., 2019), concurring with findings that suggest students from less-privileged backgrounds may be further disadvantaged by their adoption (Lubienski, 2004). They propose ‘Future 3’ as an alternative to the Future 1/Future 2 binary, built on the premise that powerful knowledge, located within traditional disciplinary boundaries, is an educational benefit that all learners should have equal access to (Hoadley et al., 2019). How to achieve this, however, remains a contentious question.

Resurgence of teacher-led pedagogies

Unfortunately, some academics, who share an opposition to Future 2 with social realists, have misinterpreted Future 3 as an invitation to return to teacher-led pedagogies, based on rote-learning of disconnected curriculum content and emphasising testing, which are more characteristic of Future 1 (Hoadley et al., 2019). Hudson (2018) describes one such approach, ‘Core Knowledge’ advocated by the ‘Academy of Ideas’, as a mutated version of mathematics of ‘low epistemic quality’ and with a tendency towards rule-following. This view of curriculum as a list of topics devoid of coherence, and ignoring issues of pedagogy, had a significant influence on the 2013 National Curriculum in England (Muller & Young, 2019).

Teacher-centred approaches, including Direct/Explicit Instruction in the United States and Mathematics Mastery in England, have enjoyed a recent resurgence in popularity. They rest on the premise that learning is most effective when concepts are presented by teachers in a highly structured and unambiguous way. Examples are carefully chosen and sequenced, with minimal conceptual steps, to enable learners to draw correct inferences and to avoid cognitive overload. Emphasis is placed on maintaining a fast pace, consolidating learning through regular guided practice and routine use of testing to correct students' errors and to ensure work is not too easy/difficult (Doabler & Fien, 2013; Rosenshine, 2012). Teacher-led pedagogies, in which teachers typically model the application of a procedure to solve a closed problem and students practise a series of almost identical problems before being tested on their understanding, have caused widespread damage to students' engagement with mathematics, resulting in increasing levels of anxiety and alienation of students over the course of their schooling (Foster, 2013; Grootenboer, 2013; Hudson, 2018; Williams & Choudry, 2016). While Direct Instruction and similar approaches claim to offer more than just transmission of knowledge, they are associated with similar problems relating to the disengagement and disempowerment of learners (Ewing, 2011). Given that Direct Instruction is used primarily with students exhibiting difficulties in learning mathematics and those from disadvantaged backgrounds (Doabler & Fien, 2013), this has worrying implications for equitable curriculum access.

A focus on disciplinary meaning

The curricula described above are a long way from the Future 3 envisioned by Muller and Young (2019). While they may help students make connections between different concepts, in foregrounding content they ignore the “inner dynamic property” of a discipline (p. 206), which determines how knowledge is generated. Powerful knowledge is more than a collection of isolated propositions and must include an appreciation of ‘disciplinary meaning’, that is how those propositions become accepted within the discipline. Limiting the educational experiences of low-attaining and disadvantaged students to content-focused curricula denies them access to the generative principles for creating new knowledge (Wheelahan, 2007). A Future 3 curriculum based on principles of social justice must engage learners with historical debates around truth criteria within a discipline and make explicit the relationship between formal abstract knowledge and real-world problems (Shalem & Allais, 2019).

In school mathematics, therefore, learners should be given opportunities to experience processes that mathematicians go through in generating and accepting new knowledge, involving argumentation and refutation amongst peers, which highlight the fallible nature of the discipline (Ernest, 1991). These processes include working collaboratively (most new mathematical knowledge is generated by teams), making conjectures, posing questions, following new lines of inquiry, explaining and justifying findings to others, considering alternative solutions/interpretations and constructing mathematical arguments (Mason et al., 1985). Students should be encouraged to employ strategies that mathematicians use in solving real-life problems, such as considering assumptions needed to model a problem, selecting the most appropriate mathematical tools for solving it, interpreting the solution and considering its limitations given the initial assumptions. Mathematicians often find themselves in challenging situations where the methods needed for solving a problem are not immediately obvious and where initial paths prove unproductive. Progressive pedagogies, by embracing collaborative, discursive, problem-solving teaching approaches in which challenges, errors and misconceptions are welcomed as learning opportunities (Boaler, 2008; Swan, 2006), can provide those experiences (described above) that students need to develop an appreciation of disciplinary meaning.

Hudson (2018, p. 394) outlines how powerful knowledge can serve as a useful starting point for enabling ‘progressive arguments for teaching to be reconnected with the emancipatory ambitions of education’. He argues that ‘subject didactics’, which investigates relationships within/between disciplines and school subjects, while popular in Scandinavia, Germany and many French-speaking countries, is overlooked by Young in his conception of powerful knowledge. Brousseau's (1997) ‘didactic contract’ involves a system of implicit rules that regulate how mathematics students and teachers behave towards each other in the generation of knowledge. Schoenfeld (2012) describes how the didactic contract is shaped by students' experiences, with ‘socio-mathematical norms’ determined by often-tacit classroom culture. He argues for the replacement of the orthodox didactic contract, which focuses on ‘answer getting’, with an alternative didactic contract that encourages ‘sense making’, through engaging with rich mathematical tasks and encouraging students to explain their mathematical reasoning. This resonates with Brousseau's (1997) ideal of a ‘didactic situation’ in which students develop as autonomous learners through cultivating habits of mind that enable them to select and use appropriate mathematical tools to solve problems (an important element of powerful knowledge). Hudson (2018) reports on an action research project in which primary school teachers succeeded in cultivating such habits of mind in their students, through exploring topic-based approaches to mathematics learning. He describes how the progressive teaching approaches adopted, while increasing levels of engagement, motivation and fulfilment, also proved to be of ‘high epistemic quality’, that is they enabled students to engage in critical thinking, learn from mistakes, generate multiple solutions and appreciate the fallible nature of mathematics.

Learner agency within the curriculum

Manyukhina and Wyse (2019) claim that the social realist vision of Future 3 neglects learner agency, which has two dimensions: ‘sense of agency’ (a feeling of control over one's own learning) and ‘agentic behaviour’ (exercising control by making decisions and taking actions). From their critical realist perspective, abstract knowledge alone cannot be considered powerful, as its power depends on the agency of the ‘knower’, hence it is an ‘epistemic fallacy’ to locate the power necessary for advancing social justice primarily within knowledge (Alderson, 2020). This would imply that school is a level playing field, ignoring how everyday routines convey hidden messages about inclusion and respect, and the systemic advantage that schools afford children from more privileged backgrounds by assigning greater value and recognition to their social and cultural resources.

Critical realists argue that the structure in which learning takes place and the agency of learners have mutual causality. Allowing students to become actively engaged in their learning and to influence its direction has a positive impact on their academic achievement and their view of themselves and their place in the world. Conversely, adopting more progressive and context-sensitive teaching approaches helps develop learners' sense of agency and their capacity to exercise that agency (Philippou & Priestley, 2019). Greater attention needs to be devoted to learner agency, both individual and collective, in considering how the school curriculum should respond to the environmental, economic and social challenges of the 21st century outlined earlier. Focusing too narrowly on the pursuit of powerful knowledge that misrepresents power as abstract, apolitical and inhuman diverts attention away from tackling global problems facing society (Alderson, 2020). If learners are to be empowered to make effective use of mathematics in arguing for change later in life, then attention needs to be given to establishing ‘didactic situations’ that enable them to develop as autonomous learners (Brousseau, 1997).

We have argued above that progressive teaching approaches can promote the development of powerful knowledge, in particular an appreciation of disciplinary meaning, within the mathematics classroom. We have also argued that developing agency is essential to enable mathematics learners to harness and employ such powerful knowledge. Fears from those with genuine concerns for social justice, that competence-led curricula fail to take adequate account of disciplinary knowledge and meaning, can be allayed by locating progressive pedagogies firmly within disciplinary boundaries. However, the concern that the less-structured nature of progressive pedagogies can render them invisible to learners (Lubienski, 2004), with the consequence that disadvantaged students are less likely to access powerful knowledge, is one that must be addressed. In the following sections we report on a research project which aimed to make the teachers' rationale for adopting progressive pedagogies more visible to learners, and we consider the implications for developing an emancipatory and empowering mathematics curriculum capable of addressing the 21st century challenges facing society.

THE VISIBLE MATHS PEDAGOGY RESEARCH PROJECT

The Visible Mathematics Pedagogy research project was a collaboration between the paper's three authors: an academic researcher (Pete) and two teacher researchers (Alba and Tiago), referred to collectively as the ‘research team’. It was conducted between 2017 and 2019 in Stoke Newington School, a non-selective state secondary (age 11–18) school in London with a higher-than-average proportion of students from disadvantaged backgrounds. The mathematics department was already committed to using progressive teaching approaches and developing its scheme of work to incorporate open-ended questions, rich tasks and ‘low threshold/high ceiling’ activities. It had recently decided to transition from setting students by prior attainment to teaching in mixed-attainment groups. The aim of the project was to explore the impact of strategies for making progressive pedagogies more visible on students' mathematical engagement and their appreciation of success criteria, particularly for those identified as disadvantaged. In this paper we report primarily on the findings from the second year of the project.

Over the 2 years, we held 15 research team meetings that focused on reviewing existing practice in relation to research literature, devising and evaluating the strategies, agreeing and revising the project's research design and planning how to collect evidence to support the analysis. In each of four action research cycles completed, Alba and Tiago taught one research lesson to their own classes (which they planned together) in which the strategies were tried out. Data were collected through administering anonymous surveys to all students who consented to take part and audio recording semi-structured interviews with three students from each class and the discussions that took place during research team meetings. Students participating in the interviews were selected from those identified as ‘pupil premium’ (a measure of disadvantage), ensuring a range of prior attainment and engagement with mathematics. Informed consent was obtained from students and parents/carers. All research lessons were video recorded, but these recordings were used only to stimulate discussion in research team meetings and did not form part of the data.

During the first year of the project (cycles 1 and 2), we worked with two Year 7 (age 11–12) classes. Students were surveyed in cycle 1 and six students were interviewed soon after the research lesson in cycle 2. During the second year (cycles 3 and 4), we worked with two Year 8 (age 12–13) classes. All students were surveyed at the beginning of cycle 3 (2–3 days before the research lesson) and at the end of cycle 4 (8–9 days after the research lesson) and six students were interviewed soon after the research lessons in both cycles. All four classes were broadly mixed-attainment and some, but not all, students were involved in both years including two students who were interviewed (classes were restructured at the beginning of Year 8). The surveys were designed to assess students' appreciation of the success criteria for mathematics learning. They included the same four questions in both cycles:

1. How successful [on a scale of 1 to 5] do you think you are in maths in general?
2. How do you know?
3. What do you think you can do to be more successful in maths?
4. What does your teacher do to help you to be successful in maths?

The interviews were designed to assess students' engagement with progressive pedagogies employed by the teacher and their appreciation of the rationale behind them. They included questions such as:

- Did you enjoy yesterday's lesson? Why?
- Did you notice anything different about yesterday's lesson?
- How well did you do in yesterday's lesson? How do you know?
- Why do you think I asked you to …?

STRATEGIES FOR MAKING PROGRESSIVE PEDAGOGY MORE VISIBLE

The three strategies described below aimed to make progressive pedagogies more visible to learners and were tried out during the second year of the project. The first two strategies, ‘model solution’ (used in cycle 3) and ‘boxing up’ (used in cycle 4), involved the teacher researchers reflecting carefully on their reasons for employing a specific teaching approach (the ‘pedagogic rationale’) and then facilitating a discussion with students in which these reasons are made explicit. The third strategy, ‘card sort’ (used in cycle 3 and revised for use in cycle 4), was seen as a generic strategy for use with a variety of progressive teaching approaches.

The ‘model solution’ strategy

Students have a go at a problem on factorising/substitution on their own and then present their solutions to others. The teacher then facilitates a whole -class discussion to agree a model solution and to draw out the characteristics that make this a 'good' solution, for example clear explanation of reasoning and unambiguous use of language. Students then copy this agreed model solution into their books and use it for reference in solving a series of similar problems and presenting good solutions. Finally, the teacher prompts another whole-class discussion in which the pedagogic rationale for agreeing and using a model solution is made explicit. The rationale here is for students to learn from each other through sharing ideas, appreciate multiple ways of solving a problem (comparing differences, similarities and merits of different methods) and to develop skills in communicating mathematical reasoning and solving problems independently.

The 'boxing up' strategy (see 'Acknowledgements')

Students are provided with a series of generic prompt questions, printed in boxes on a laminated card (see Figure 1), to use when solving a series of challenging problems (in this case on probability). The teacher facilitates an initial discussion with students on the reasons for using the 'boxing up' questions (in this case limited to: 'What is the question asking me? What information do I already have?') by asking: 'Why is this useful? What does this question allow you to do?' Students then tackle the problems. Finally, the teacher prompts another whole-class discussion in which the pedagogic rationale for using the boxing up questions is made explicit. The rationale here is to support students in developing metacognitive skills, in this case recognising the value of making a plan before jumping in to solve a problem and identifying key information needed to decode a question.

The 'card sort' strategy

Groups of students are provided with a card sort comprising a series of statements, some representing what the teacher considers to be primary reasons for adopting a particular teaching approach, others potentially valid reasons not identified as primary, and the remainder considered to be invalid reasons. The teacher facilitates a group discussion in which students agree how to arrange the statements in order, with those at the top being

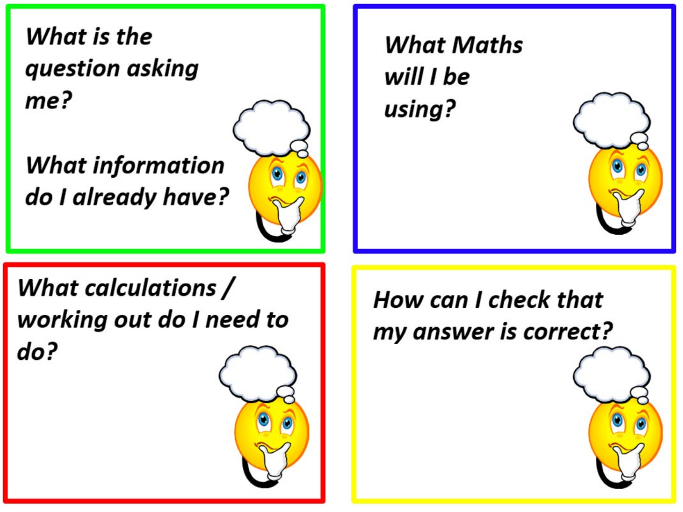

FIGURE 1 The 'boxing up' questions

those they believe best describe the teacher's reasons for adopting the approach. The aim of the 'card sort' strategy is to make the teacher's pedagogic rationale more explicit to learners.

The card sort strategy was used in cycle 3 to facilitate a discussion around the 'model solution' strategy. Students were first invited to rank 12 statements according to those that best described the reasons for 'getting the class to come up with a model solution to the first problem'. They were then invited to rank the same statements according to those that best described the reasons for 'getting students to copy down the model solution and use it to solve the other problems'. The card sort was used again in cycle 4 to facilitate a discussion around the boxing up strategy. However, this time, the research team simplified the task by reducing the number of statements. Students were provided with the following statements and invited to rank them according to those that best described the reasons for 'getting students to think about the questions in the first box' (B and F being primary reasons, A and D being invalid reasons):

A So I can work through all the problems more quickly.
B So I can make a plan to help me to solve a problem.
C So I can share my ideas with other students.
D So I can focus on my work without being distracted by others.
E So I can recognise similarities and differences between problems.
F So I can identify the key information in the question.

DATA ANALYSIS

This paper focuses on the analysis of data from the student surveys and interviews conducted during the second year of the research project (cycles 3 and 4). As there were no noticeable differences in the patterns of responses between the two classes, the results have been combined in the following analysis. There were 42 survey responses altogether in cycle 3 and 45 responses in cycle 4. The survey responses were categorised according to codes derived inductively by reading and re-reading the data with each response being assigned one or more codes. Where it was considered appropriate, codes were grouped into sets of related codes. For example, in response to the question 'How do you know [how successful you are in maths]?' the codes were grouped into four sets relating to 'students' dispositions towards learning', 'judgements about their own work output', 'perceptions about how others saw them' and 'judgements about their level of understanding'. A thematic analysis was carried out by comparing the number of responses assigned each code and comparing/contrasting responses assigned similar codes (see Online Appendix 1 for survey coding scheme).

The audio recordings of interviews were transcribed, with names of students replaced in the transcripts by pseudonyms. A combination of deductive and inductive coding was used to carry out a thematic analysis (Fereday & Muir-Cochrane, 2006). An initial coding scheme, derived from the research literature, was modified through a process of familiarisation, by reading and re-reading the transcripts. Further changes were made during the coding process as it became apparent that additional codes were needed to cater for unanticipated responses and some original codes were either duplicated or redundant. The final coding scheme (see Online Appendix 2) comprised six broad groups of codes:

1. References by students to progressive teaching approaches as defined in Section 'Introduction' (derived from the research literature, e.g., Boaler, 2008; Swan, 2006);
2. References by students to the teachers' strategies for making pedagogy more visible described in Section 'Strategies for Making Progressive Pedagogy More Visible' (devised by the research team);

3. References by students to their experiences of learning mathematics, for example challenge, frustration, anxiety, familiarity, confusion (derived from the research literature, e.g., Black et al., 2009; Boaler, 2008);
4. References by students to their dispositions towards learning mathematics, for example enjoyment, perseverance, motivation, independence and empathy (as above);
5. References by students to the teacher's pedagogical rationale as described in Section ‘Strategies for Making Progressive Pedagogy More Visible’ (devised by the research team);
6. References by students to success criteria for learning mathematics (derived from the research literature, e.g., Lubienski, 2004; Schoenfeld, 2012).

NVivo software was used to code interview transcripts and to compare text assigned similar or related codes. A process of reading and re-reading extracts of text, allowing the original meanings to be taken into account, was used to explore ‘commonalities’, ‘differences’ and ‘relationships’ between codes and to enable themes to be identified from the data (Gibson & Brown, 2009).

FINDINGS

In this section, we present findings from the second year of the project, which are organised into six themes (the first two emerging from the analysis of the student surveys and the remainder from the interviews). The six students participating in the interviews (all identified as disadvantaged) were assigned the following pseudonyms: Anna, Keira and Simon (from Alba's class); Nasri, Neal and Tom (from Tiago's class).

Students' perceptions of success in mathematics

Students generally felt comfortable with their level of success in mathematics, with over 90% in both cycles placing themselves at 3 or above on the 1–5 scale (first survey question). Most students maintained the same traditional notion of success in mathematics that they exhibited in the first year of the project. The most common codes assigned to responses to the second survey question (‘How do you know [that you are successful]?’) were ‘getting correct answers’ (43% of students in cycle 3 and 16% of students in cycle 4), ‘scoring highly in tests’ (21% and 36%), ‘finding the work easy’ (17% and 36%) and ‘completing a large amount of work’ (17% and 7%). The growing importance attributed to test scores may reflect the greater emphasis students place on them as they progress through their schooling. The following survey responses exemplify these findings: ‘Because I understand most of the tasks we do pretty easily and sometimes finish quickly’ (cycle 3); ‘I know because of the grades I get in my test … if I'm successful I know it because I get the questions correct’ (cycle 3); ‘Because in class I find the work helpful and sometimes challenging but I can answer the questions … my grades are good as well and my teacher tells me that I am doing well’ (cycle 4).

Students' self-efficacy in realising success in mathematics

Responses to survey question 3 (‘What do you think you can do to be more successful in maths?’) further reflected students' recognition of the importance of tests and assessments, for example: “Practice and revise what you learnt in the lesson, so you don't forget, go through the things you don't understand" (cycle 4). The most common strategies identified were: 'do more homework' (11 and 12 responses in cycles 3 and 4 respectively), 'revise' (10 and 9 responses), and 'practice in areas of weakness' (6 and 4 responses).

Other survey responses suggested an increase between cycles 3 and 4 in students taking responsibility for, and control over their own actions in realising success, for example: "To be more successful in maths I think that I could try a bit harder rather than just giving up all the time" (cycle 4). Strategies suggested by students included: 'do more work/try harder' (4 and 9 responses in cycles 3 and 4 respectively), 'concentrate more' (4 and 10 responses), 'focus more' (1 and 3 responses), 'persevere' (0 and 3 responses), 'listen more carefully' (6 and 7 responses).

Responses to survey question 4 ('What does your teacher do to help you to be successful in maths?') suggested that most students articulated a heavy reliance on their teachers, for example: 'He always explains the question if I don't understand and shows us how to work it out' (cycle 4). The most common codes assigned to responses were for teachers to 'explain the work fully' (10 and 9 responses in cycles 3 and 4 respectively) and 'helping students when they are stuck' (11 and 9 responses).

However, there was an increasing appreciation among some students of how teachers used progressive pedagogies to help them achieve success by 'making the lessons more interactive' (2 and 5 responses in cycles 3 and 4 respectively), 'encouraging more talk between students' (0 and 4 responses), for example: 'He makes me talk to my partner' (cycle 4), and 'engaging with more than one method' (0 and 2 responses), for example: 'When there is a method, and she always does both, even if people understand one' (cycle 4).

Students' explanations of reasons for their success in mathematics

All six students interviewed felt they were successful in the research lessons, reflecting the views of most students in the surveys about their general success in mathematics. However, there was a small shift towards attributing success in research lessons to their own effort/engagement (2 and 3 students in cycles 3 and 4 respectively), for example, Tom (interview, cycle 4) demonstrated a willingness to persevere with harder questions:

> Um … I answered some questions. And then I tried to work out the harder ones, but then, if I couldn't work them out, then I just left them, until … I did the other questions, then I had to come back to them all.

There was also a growing recognition of the benefits of working effectively with others, for example: 'And, like, getting help from my partner, as well, helped me with the work a bit' (Neal, interview, cycle 4) and 'Yeah, and me and my partner agreed very easily' (Simon, interview, cycle 4). There was less emphasis placed by students on getting correct answers and test outcomes compared to the first year of the project. Only three students in cycles 3 and 4 referred to these, for example: 'Um, because when you were marking the questions, I was … getting it, like, correct … when you would ask us questions, and I'd put my hand up and answer them, you would, like, tell me "well done"' (Keira, interview, cycle 4).

Students' awareness of the use of progressive pedagogies

Students appeared to demonstrate growing awareness of the teacher's use of progressive pedagogies during the project. Four students interviewed in cycle 4, when asked what they noticed that was different about the research lesson, described (without prompting) features of the teaching approaches associated with the boxing up strategy, for example using the boxing up questions to help in solving problems: ‘Because it wasn't, like, one of our usual lessons … we did the work that we usually do, but then we had to, like, show what's the most important thing to use … like planning's very important’ (Neal, interview, cycle 4); ‘Because we got to use something we never used before, which is the block … and you can write on it to help you with your answers’ (Nasri, interview, cycle 4).

This compared to cycle 3 in which Keira (interview, cycle 3) was the only student who noticed the collaborative teaching approaches associated with the ‘model solution’ strategy (comparable to findings from cycle 2): ‘And we had to work in partners, like, why we did the model solution, and like what it would help us with, why we had to do it’.

Students' growing appreciation of the teacher's pedagogic rationale

All six students demonstrated an increasing appreciation of the teacher's pedagogical rationale for employing progressive teaching approaches. Five students articulated at least one primary purpose for the teaching approaches associated with the ‘model solution’ strategy in cycle 3. These included: encouraging students to discuss, compare and engage with each other's ideas; enabling students to communicate their mathematical reasoning; and developing strategies for solving problems independently. For example, Tom (interview, cycle 3) articulated engaging with multiple solutions: ‘So you could see each different answer and how they compare. And, like, how you did it, with the working out’. Two students described the benefits of explaining ideas to each other:

> And then someone who has the correct answer could explain, like, how they got the answer, and put it into, like, more detail. … If you get something like similar, like, you could just flip back and like check ‘Oh, how did you do that? … How do I answer the question?’.
>
> (Neal, interview, cycle 3)

> So, like, um … if we're working in partners, and it's like we're deciding on a method to use, we can say ‘this one is more efficient to use because of this’ …
>
> (Keira, interview, cycle 3)

Nasri (interview, cycle 3) highlighted the value of using model solutions for solving problems independently in future: ‘Maybe, like, if one day you decide to come up with a question like that, like, just randomly, we could, like, refer back to our books … to see how we could, like … work it out’.

All six students articulated at least one primary purpose for teaching approaches associated with the boxing up strategy in cycle 4, including developing generic strategies for solving problems and appreciating the value of making a plan, for example: ‘Planning's very important … you always have to plan before you start the work because then you, like, don't rush through it quickly, because it wouldn't really, like, clearly make sense to you’ (Neal, interview, cycle 4). Two students described how identifying the key information helps focus on the first steps needed to solve a problem: ‘So rather than, like, just focusing on what other things we can do, focus on, like, what it's asking us to do’ (Anna, interview, cycle 4); ‘And then, with that key information, you have to work from there slowly’ (Tom, interview, cycle 4). Two other students highlighted how the same boxing up questions could be used in other lessons: ‘If I'm stuck and I need help and the teacher's busy, and there's no-one to help me, I could, like, try using my own method of doing it …. like using the questions to help me’

(Nasri, interview, cycle 4); 'Cos … maybe on different lessons, you might have something similar, and then you're kind of stuck … then it helps like highlight the key information, which is how you do it' (Simon, interview, cycle 4).

The card sort appears to have been particularly effective in making the teacher's pedagogic rationale more explicit to students. Keira (interview, cycle 3) highlights how being more aware of this makes the purpose of learning clearer:

> Because it [card sort] helps us, like, understand more why we do it, because … some people would be like 'Ah, what is this going to help us with in the future?', and stuff like this. And it would just help you, like, understand more, and like why we're doing this, why we're learning this, why it's going to help us.

Students also described more generic advantages of progressive approaches, for example, five students referred to the benefits of working collaboratively, including Neal (interview, cycle 3) who highlights its potential for learning from other students:

> Imagine if your partner knows it but then you don't know it … and then, like, you'd just been like struggling on it. And you said you can talk to your partner if you want some help. And then, yeah, it helped a lot as well.

Students' engagement with progressive pedagogies

All six students claimed to have enjoyed the research lessons and, when asked why, commonly attributed this to aspects of progressive teaching approaches. Three students expressed their appreciation for having the opportunity to work collaboratively, for example: 'I like, like for me, finding the pieces, which was interesting, because you get to actually do, like, working out and stuff with our partner' (Anna, interview, cycle 3); 'Because I prefer to do things with, like, teamwork, rather than independent … and then that was, like, doing it with another person, so that's teamwork' (Simon, interview, cycle 4).

Three students attributed their enjoyment to being faced with (and managing to solve) challenging problems, for example: 'I enjoyed it because … I like getting pushed … so, when you were asking us questions, and it, like, it helped us, like, push ourselves' (Keira, interview, cycle 4); 'Um, I enjoyed answering the questions … because they were challenging but, like, if you work it out the proper way, then it was easy' (Tom, interview, cycle 3).

DISCUSSION

The findings from the second year of the project concurred with those from its first year in highlighting the potential of making progressive pedagogy more visible to learners for enhancing students' engagement and achievement in mathematics lessons. The strategies described in Section 'Strategies for Making Progressive Pedagogy More Visible' appeared to increase students' appreciation of the teacher's pedagogic rationale for employing progressive pedagogies, particularly those from disadvantaged backgrounds, through facilitating discussions that make this explicit (Section 'Students' Growing Appreciation of the Teacher's Pedagogic Rationale). Students appeared to become better at distinguishing features of progressive pedagogies (Section 'Students' Awareness of the Use of Progressive Pedagogies') and more clearly articulating how to respond appropriately to achieve success (Section 'Students' Explanations of Reasons for Their Success in Mathematics'). This addresses the concerns of Lubienski (2004), and social realists such as Morgan et al. (2019), that the invisible nature of progressive pedagogies might further marginalise disadvantaged students. It offers teachers and researchers a novel direction to take in developing visible forms of progressive pedagogy, rather than pursuing the alternative of teacher-led approaches, such as Direct/Explicit Instruction or Mathematics Mastery, associated with 'low epistemic quality' (Hudson, 2018), disengagement and alienation of learners (Foster, 2013; Grootenboer, 2013; Williams & Choudry, 2016). We highlighted earlier (see Section 'Theoretical Framework') how progressive pedagogies can broaden learners' access to powerful knowledge through enabling them to engage with disciplinary meaning (Muller & Young, 2019). Making these pedagogies more visible therefore offers the opportunity for mathematics learners to more successfully decipher the 'rules of the game', that is the 'recognition rules' and 'realisation rules', involved in achieving success (Bernstein, 2000).

Disadvantaged students appeared to engage positively with the progressive pedagogies employed in the research lessons, attributing their high levels of enjoyment to opportunities to work collaboratively with others in tackling challenging mathematical problems (Section 'Students' Engagement With Progressive Pedagogies'). These findings concur with those of other studies highlighting the positive impact of progressive teaching approaches on students' attitudes towards learning mathematics (Boaler, 2008; Hudson, 2018; Wright, 2016, 2017; Wright et al., 2020). Together with greater recognition of the benefits of sharing ideas and discussing mathematical reasoning with others, and developing independent problem-solving strategies (Section 'Students' Self-Efficacy in Realising Success in Mathematics'), this indicated a small shift towards establishing alternative socio-mathematical norms based on sense making rather than answer getting (Schoenfeld, 2012). It has been reported elsewhere how students sometimes misinterpret teachers' reasons for adopting progressive teaching approaches as seeking compliant behaviour (Wright et al., 2020). In contrast, the significant increase in students' appreciation of their teacher's pedagogic rationale (Section 'Students' Growing Appreciation of the Teacher's Pedagogic Rationale') suggests increasing alignment between teachers' and students' perceptions of the benefits of progressive approaches. This can avoid 'didactic tension' (Brousseau, 1984), in which teachers' attempts to make the desired mathematical behaviour explicit result in students exhibiting that behaviour without necessarily generating mathematical understanding. Developing a shared understanding of pedagogic rationale may become even more important if recent school closures resulting from the Covid-19 pandemic, necessitating greater interaction between teachers and students through online media, become more commonplace.

The small shifts in students' beliefs that they can influence their success in mathematics through their own actions (Section 'Students' Self-Efficacy in Realising Success in Mathematics') represents a modest increase in 'sense of agency' over the course of the project. Similarly, the increasing tendency of disadvantaged students to attribute their success in research lessons to effort, engagement and working effectively with others (Section Section 'Students' Explanations of Reasons for Their Success in Mathematics') rather than obtaining correct answers and high scores in tests, suggested an increase in 'agentic behaviour' (Manyukhina & Wyse, 2019). However, the inclination of students to maintain traditional notions of success and reliance on the teacher in general mathematics lessons (Sections 'Students' Perceptions of Success in Mathematics' and 'Students' Self-Efficacy in Realising Success in Mathematics') suggests 'learner agency' was still in its infancy. Students' appreciation of the benefits of peer discussion and collaborative working recurred in the findings (Sections 'Students' Self-Efficacy in Realising Success in Mathematics', 'Students' Explanations of Reasons for Their Success in Mathematics', 'Students' Growing Appreciation of the Teacher's Pedagogic Rationale' and 'Students' Engagement With Progressive Pedagogies'), highlighting the potential of making progressive mathematics pedagogies more visible for developing a shared responsibility for learning, reminiscent of Boaler's (2008) notion of 'relational equity', building solidarity and trust amongst learners and assigning greater value to communal effort (Radford, 2012). This potential contribution towards building 'collective agency', given its importance in tackling the environmental, economic and social challenges we currently face as a global society (OECD, 2018), warrants further exploration.

CONCLUSION

The findings from the Visible Maths Pedagogy research project supplement those reported elsewhere (Wright, 2020; Wright et al., 2020) in demonstrating how making progressive pedagogies more visible to learners can enhance their appreciation of the teacher's pedagogic rationale and their understanding of how to achieve success in the mathematics classroom. This paper provides a significant contribution to debates around curriculum reform by suggesting how progressive teaching approaches can be adapted to provide broader access to powerful knowledge for learners, particularly those from disadvantaged backgrounds. We argue that such a novel approach illustrates the potential of 'progressive pedagogy made visible' for developing an emancipatory and equitable mathematics curriculum that enhances learners' individual and collective agency.

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CONFLICT OF INTEREST
There is no potential conflict of interest associated with this paper.

ETHICS STATEMENT
The research reported in this paper was conducted under the approval of the Research Ethics Committee of UCL Institute of Education (ethical approval reference REC 1014) and the school's headteacher.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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SUPPORTING INFORMATION

Additional Supporting Information may be found online in the Supporting Information section.

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