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2 **Ship resistance when operating in floating ice floes: derivation, validation,**

3 **and application of an empirical equation**

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10

11 **Abstract:** With the effects of global warming, the Arctic is presenting a new environment where
12 numerous ice floes are floating on the open sea surface. Whilst this has improved Arctic shipping
13 navigability in an unprecedented way, the interaction of such floes with ships is yet to be understood to
14 aid the designing of ships and route planning for this region. To further explore this topic, the present
15 work develops a procedure to derive an empirical equation that can predict the effects of such floes on
16 ship resistance. Based on a validated computational approach, extensive data are extracted from
17 simulations of three different ships with varying environmental conditions. The ice-floe resistance is
18 shown to strongly correlate with ship beam, ship buttock angle, ship waterline angle, ship speed, ice
19 concentration, ice thickness and floe diameter, and the regression powers of each of the parameters on
20 resistance are ascertained. This leads to a generic empirical equation that can swiftly predict ice-floe
21 resistance for a given ship in a given condition. Subsequently, demonstrations are given on the
22 incorporation of the derived equation into a set of real-time Arctic ship performance model and voyage
23 planning tool, which can predict a ship's fuel consumption in ice-infested seas and dynamically suggest
24 a route with the least safety concern and fuel consumption. Moreover, the equation is validated by
25 providing ice resistance prediction for experimental and full-scale conditions from multiple sources,
26 showing high accuracy. In conclusion, the empirical equation is shown to give valid and rapid estimates
27 for ice-floe resistance, providing valuable insights into ship designs for the region, as well as facilitating
28 practical applications for polar navigation.

29 **Keywords:** Arctic shipping, ice floe, ship resistance, empirical equation, derivation, validation.

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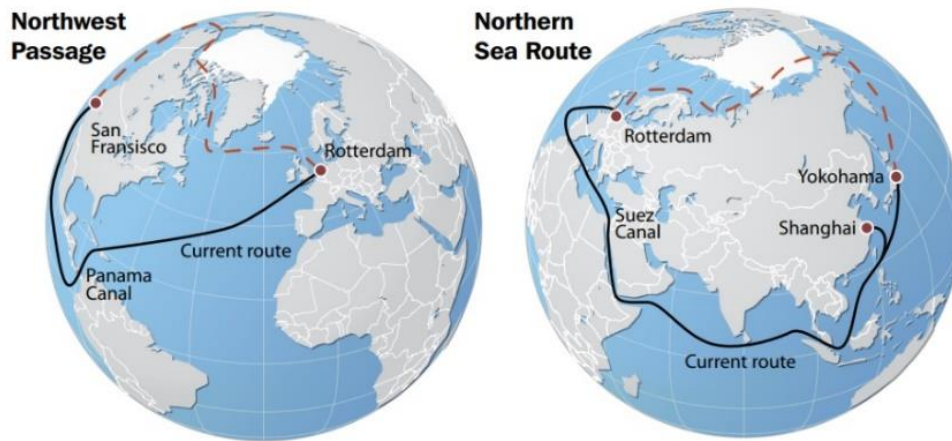
Nomenclature

α	Waterline angle
γ	Buttock angle
ρ_{ice}	Ice density
A	Ice resistance coefficient
B	Ship beam at waterline
C	Ice concentration
D	Equivalent ice diameter of upper surface
Fr	Froude number
g	Gravitational acceleration
h	Ice thickness
L_{pp}	Ship length between perpendiculars
R_{ice}	Ship resistance induced by ice
R_{water}	Ship resistance induced by water
U	Ship speed
AIV	Arctic In-service Vessel
ASPM	Arctic Ship Performance Model
CFD	Computational Fluid Dynamics
DEM	Discrete Element Method
FSICR	Finnish Swedish Ice Class Rules
ITTC	International Towing Tank Conference
JBC	Japan Bulk Carrier
KCS	KRISO Container Ship
NSR	Northern Sea Route
NWP	Northwest Passage
RPM	Revolutions Per Minute
VPT	Voyage Planning Tool

33 **1. Introduction**

34 Climate change has caused Arctic sea ice to melt dramatically, in turn causing an extensive transition
35 from level-ice coverage to broken ice-floe fields and open water [1]. The changing conditions make the
36 Arctic more accessible to ships, with new waterways allowing improved access for oil and gas
37 extraction, mining, fishing and tourism. Two major cargo-shipping routes are becoming viable: the
38 Northwest Passage (NWP) and the Northern Sea Route (NSR), alternatives to the Panama and Suez
39 canals to connect Europe, Asia and America. Compared with current routes, both new routes can reduce
40 travel distance by up to 40%, leading to substantial fuel, cost, time and emission savings [2], as
41 illustrated in Figure 1.

42



43

44 Figure 1: Comparison between the Arctic shipping routes (red dashed line) and the traditional
45 shipping routes (solid black line) [3]

46

47 The practicality for employing the NSR is currently greater than for the NWP. As introduced by Ryan
48 et al. [4], the NWP is made up of straits through the Canadian Arctic Archipelago that are both narrow
49 and shallow. These straits are easily clogged by free floating ice, and are still insufficiently surveyed,
50 presenting the very real risks of grounding or becoming stuck in ice; By contrast, the NSR presents a
51 less complex situation, yet has several choke points where ships must pass through shallow straits
52 between islands and the Russian mainland. Apart from the geographical factor, politics has also been
53 providing increasing impetuses for adopting the NSR; for example, China has indicated its plans to
54 establish a Polar Silk Road as part of the Belt and Road Initiative [5], which aims to build infrastructure
55 and perform voyages through the NSR. There are four typical sub-passages in the NSR, as outlined in
56 Figure 2, i.e. coastal route, middle route, high-latitude route and transpolar route. During the summer
57 there is normally no ice appearing along the coastal or the middle line. Even in winter, the sea ice has
58 significantly retreated in Kara and Barents seas [6–8].



60

61 Figure 2: Illustration of four typical sub-passages in the NSR: coastal route, middle route, high-
 62 latitude route and transpolar route [9]

63

64 The Arctic sea ice extent and thickness are seeing a continuous decline [10], with increasing navigable
 65 days for non-icebreaking vessels [11]. There have been a significant number of complete transits via
 66 the emerging Arctic sea routes. In 2019 alone, 24 and 35 ships transited through the NWP and NSR
 67 respectively, and there were over 2000 cargo voyages completed via segments of the NSR [12, 13].
 68 Considering climate, economic and political factors, multiple predictive models have indicated an
 69 ongoing increase in the scale of vessels and voyages through these routes [14], and \$1 trillion investment
 70 is planned for constructing infrastructure in the next 15 years to fulfil the needs of potential Arctic
 71 maritime operations [15]. These trends are attracting significant research interest in Arctic shipping, one
 72 aspect of which is to identify potential ice conditions and conduct corresponding ship design, power
 73 estimates and route planning.

74 Traditional polar ship design has focused on the level-ice condition, as the Arctic region tended to be
 75 covered by consolidated ice all year round and was only accessible to icebreakers. A large number of
 76 models have been developed to predict the level-ice resistance of ships [16–18]; in particular, significant
 77 recent progress has been made through high-fidelity computational modelling of the ice-breaking
 78 process, see Ni et al. [19], Li et al. [20, 21], Lilja et al. [22], and the review of Xue et al. [23]. Similar
 79 efforts have also been made to other traditional polar shipping scenarios, e.g. brash ice [24, 25] and ice
 80 ridges [26, 27]. The above models have been applied in practice and evolved into empirical equations
 81 and international guidelines, such as the Finnish-Swedish Ice Class Rules (FSICR) [28].

82 On the other hand, completed Arctic voyages in recent years have reported very different conditions

83 from the traditional level, brash or ridge ice. The emerging shipping routes are observed to be infested
 84 by broken ice floes, especially during the summer season [1]. These floating floes range in sizes and
 85 only partly cover the sea surface. In addition, those floes tend to be circular due to the effect of wave
 86 wash and floe–floe collisions [14, 29, 30], as shown in Figure 3. Because of the flat, round appearance,
 87 this ice feature is commonly known as pancake ice. Figure 4 demonstrates the ice-floe condition (green
 88 and blue) along Arctic shipping routes, reported by field observations in the autumn of 2015, while level
 89 ice (red) occupies a small portion. With global warming, it can be anticipated that the proportion of ice-
 90 floe conditions will continue increasing, and the average size of floes will decrease. However, in the
 91 first half of the twenty-first century the shipping season will remain variable and unreliable [31]. Ship
 92 operations through level ice and ice ridges will remain important, as such challenging ice conditions
 93 will provide safety concerns for polar ships. Accordingly, almost all contemporary design standards and
 94 safety studies for marine structures are still based on the level ice condition [32, 33].

95 The operating limit of a ship in ice can refer to the Ice Class. During ship design, an Ice Class may be
 96 assigned according to the hull structural and machinery installation, which suggests a safe magnitude
 97 of equivalent ice thickness (ice thickness times ice concentration) that the ship may operate in. The
 98 relationship between Ice Class and intended equivalent ice thickness for operation is given in Table 1.
 99 In other words, with suitable ice-strengthening, a commercial ship can navigate in the emerging ice-
 100 floe without ice-breaking capabilities. This is making the ice-floe condition become a principal scenario
 101 of future Arctic shipping.

102

103 Table 1: Rules and Regulations of the Lloyd’s Register - Ice Class and the Intended Equivalent Ice
 104 Thickness for Operation [34]

Ice Class	Equivalent ice thickness (m)
1AS	1.0
1A	0.8
1B	0.6
1C	0.4

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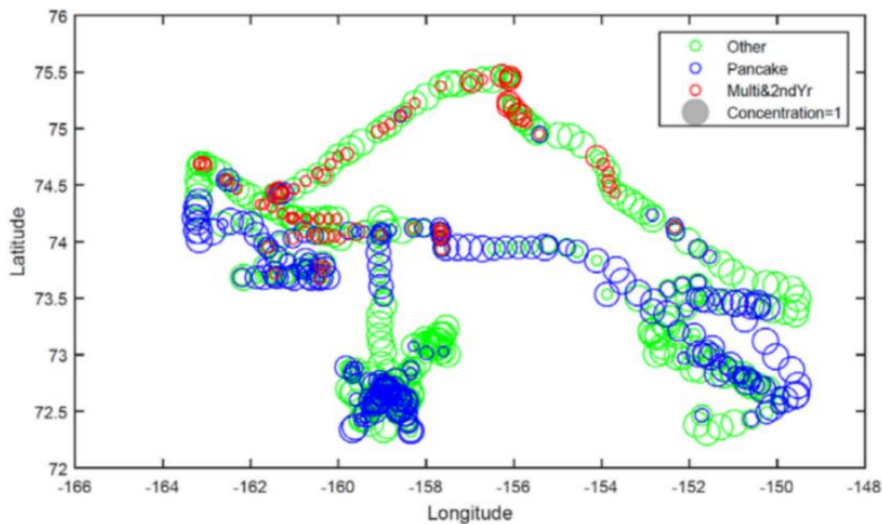
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Figure 3: A ship advancing in floating ice floes (photo credit: Alessandro Toffoli)



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Figure 4: Ice floes (pancake and other) observed as the primary environment of shipping routes in West Arctic [1]

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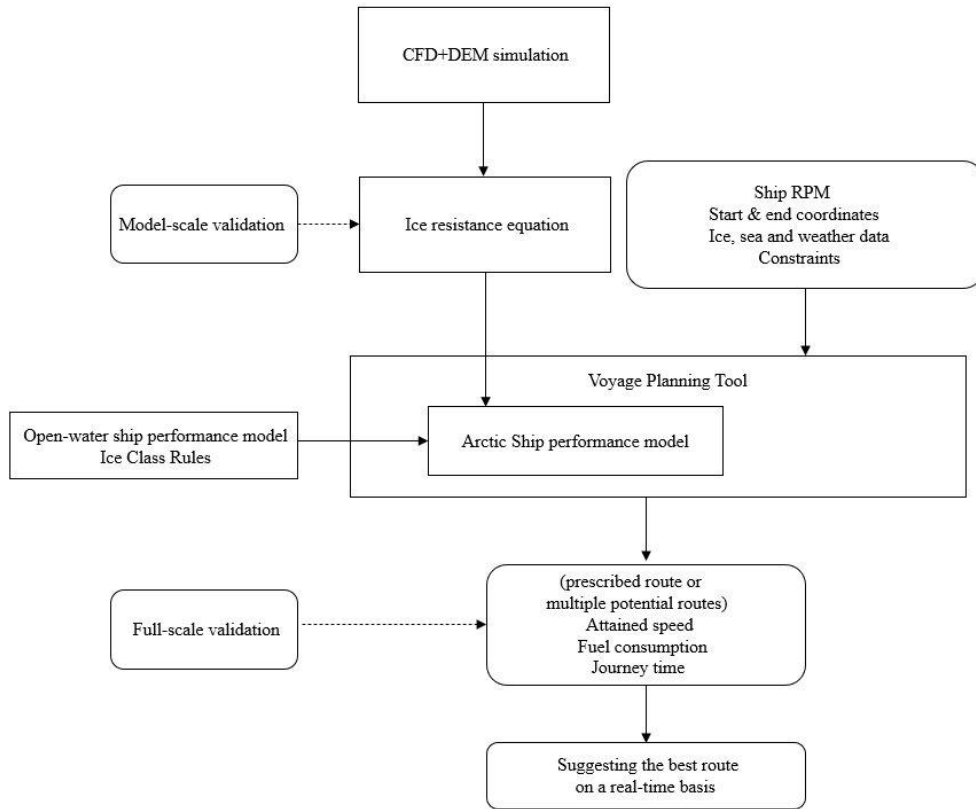
The process of a ship advancing in floating ice floes can be summarised as the following ship-wave-ice interaction: ship advancement generates waves; waves interact with ice floes; ice floes make contact with each other and with the ship [35–39]. In this scenario, ships tend to push the floes aside rather than break them [40], which means that the mechanism of ice resistance on the ship is very different from in level ice. In other words, existing resistance predictions for the level-ice condition may not be applicable for the emerging Arctic shipping conditions. For ship resistance in broken ice floes, Guo et al. [41] and Kim et al [42] conducted model tests to acquire data. Woolgar and Colbourne [43] presented regression analyses based on experimental data to derive the relationship of ice-floe load on a moored vessel with ice drift speed, ice friction, floe size and ice concentration. However, as their tests were conducted to

122 study ice loads on a moored structure, the examined ice speed conditions are very small compared with
123 normal shipping speeds, thus the relationships cannot be directly applied to ship resistance in ice floes.
124 On the whole, relevant research on structures in floe ice is still very scarce [44], partially because the
125 topic has only emerged recently, and also due to the prohibitive costs and complexity of ice experiments
126 involving parameter matrices.

127 To provide a cost-effective solution to understand ship-wave-ice interaction and provide reliable
128 resistance prediction, Huang et al. [45] developed a high-fidelity computational model using a
129 combined CFD+DEM (Computational Fluid Dynamics and Discrete Element Method) approach that
130 can simulate the operation of a ship in floating ice floes. This approach benefits from CFD that can
131 obtain fully nonlinear fluid solutions, as well as DEM that can solve solid contacts to account for ship-
132 ice collisions and ice-ice collisions. Two recent review articles of the field [23, 46] suggest that Huang
133 et al. [45] is the first work that has coupled ship hydrodynamics with the ship-ice interaction process to
134 achieve ship-wave-ice coupling. Experiments have confirmed the accuracy of this approach in
135 predicting the ice-floe resistance. Moreover, ship and ice parameters such as hull form, ship speed and
136 ice dimensions can be easily changed, thus allowing the consideration of extensive input combinations.

137 Despite the relative affordability of a computational approach, it may still be impractical to run a
138 simulation each time a resistance estimate is needed. Therefore, there is the need to develop empirical
139 equations for quick estimation of ice resistance for a given condition, so as to meet real-time purposes
140 e.g. control system, decision support and training simulator [46–48]. One particular example is the
141 Arctic Voyage Planning Tool (VPT). The VPT has the purpose of improving the safety and efficiency
142 of cargo vessels operating in the Arctic, by optimising shipping routes in ice-infested waters [49]. Since
143 the voyage planning process links with dynamic weather systems and works in real-time, it needs a
144 rapid estimate of ice resistance for each potential route to provide decision-making support for the crew.

145 Therefore, following the development of the CFD+DEM model approach [45], the present work aims
146 to develop a quick empirical equation to express ship resistance in ice floes and integrate it into VPT
147 applications. This paper is organised as follows: Section 2 presents a systematic series of computational
148 simulations for three different ships and for varying input conditions to identify the influential
149 parameters for ship resistance in ice floes. Based on these simulation results, Section 3 proposes a non-
150 dimensional derivation process, which is used to develop an empirical equation for ice-floe resistance.
151 The derived equation is validated by multiple existing experiments. Section 4 introduces further
152 applications of the rapid empirical equation, demonstrating how the equation is incorporated within a
153 set of Arctic ship performance model and VPT that can predict ship fuel consumption in ice-infested
154 waters and suggest shipping routes. In Section 5, the VPT incorporated with the derived equation is
155 used to simulate a historical voyage and the predicted ship performance is validated against
156 corresponding full-scale measurement data. Finally, Section 6 summarises this work with its
157 implications. The design flow chart of this work is portrayed in Figure 5.



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Figure 5: Design flow chart of this work

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161 2. Candidate ships and computational modelling

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Three ships are studied in this work: the KRISO Container Ship (KCS), the Japan Bulk Carrier (JBC), and a real Arctic In-service Vessel (AIV). These are three typical ship types that are expected to operate on future Arctic shipping routes, and they have significantly different hull forms that represent the diversity of current shipping fleets. Their dimensions and hull geometries are shown in Table 2 and Figure 6.

167

168

Table 2: Main Particulars of the Candidate Ships

	KCS	JBC	AIV
Ship type	Container ship	Bulk carrier	General cargo carrier
Length between perpendiculars [m]	230.0	280.0	186.4
Waterline beam [m]	32.2	45.0	28.5
Draught midships [m]	10.8	16.5	11.0
Block coefficient [-]	0.651	0.858	0.79
Wetted surface area [m ²]	9424.0	19556.1	8153.0

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(a) Profile view

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(b) Plan view

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179

(c) Front view

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Figure 6: Geometries of the KCS, JBC and AIV ships, in the order of KCS, JBC and AIV (image

181

sizes correspond to the actual ship sizes)

182

183 These three ships were separately incorporated into the CFD+DEM model, as shown in Figure 7. The
184 numerical theories and settings were detailly introduced in Huang et al. [45]. In brief, the CFD+DEM
185 model consists of (a) a standard CFD model for ship advancement in open water that obtains fluid
186 solutions, including the ship-generated wave; (b) a DEM used to model ice floes (assumed to be rigid
187 bodies) and account for their collisions with the ship and nearby floes; these floes obtain fluid force
188 from the CFD solution so that ship-wave-ice coupling is achieved; (c) original floe-distribution
189 algorithms to import natural ice-floe fields into the CFD+DEM model, in which the floes are randomly
190 distributed and have a range of sizes according to field measurements.

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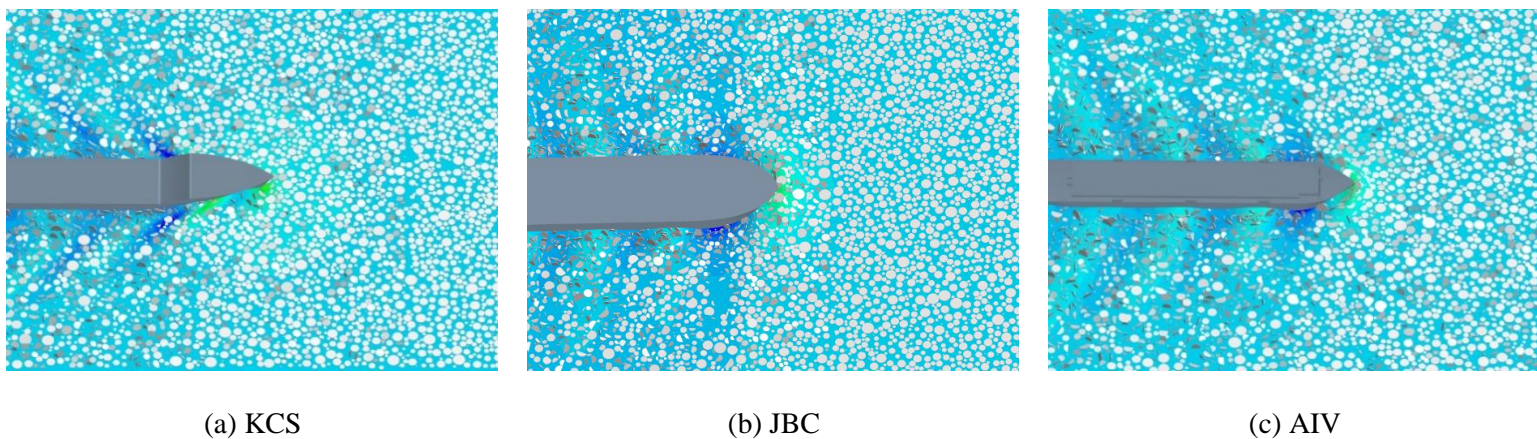


Figure 7: Simulations of different ships advancing in ice floe fields

192

193 For each ship, systematic simulations were conducted to study relevant environmental variables, which
194 is done by varying one parameter whilst holding others constant. The inserted hull geometries are in a
195 model scale of 1:52.667, which was chosen to allow comparison of the simulations against model tests
196 [41]. This process identified a series of influential parameters on ice floe resistance (R_{ice}): ship beam,
197 ship speed, ice concentration, ice thickness and floe size, as introduced in Table 3.

198 The relationship of R_{ice} with these parameters is shown in Figures 8-12, with KCS taken as the example.
199 In Figure 8-9 the ship speed and ice concentration were examined in sufficient ranges to obtain their
200 regression powers, which are approximately 1.2 and 1.5. Similarly, the relationships of R_{ice} with ship
201 beam, ice thickness and floe size were found to be linear, as plotted in Figures 10-12. These regression
202 powers obtained for KCS do not have a notable difference from those obtained for JBC and AIV.

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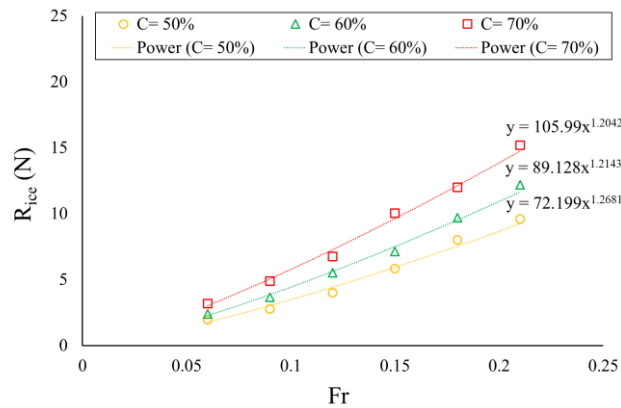
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Table 3: Essential Variables for R_{ice}

Parameter	Definition	Symbol	[unit]
Ship beam	Maximal width on the ship's design waterline	B	[m]
Ship speed	Straight-line speed (can be converted to Froude number $Fr = U/\sqrt{g \times L_{pp}}$, where g is gravitational acceleration)	U	[m*s ⁻¹]
Ice concentration	The proportion of a certain sea surface covered by ice	C	[-]
Ice thickness	The average thickness of all floes	h	[m]
Ice diameter	The equivalent diameter of the ice upper surface	D	[m]

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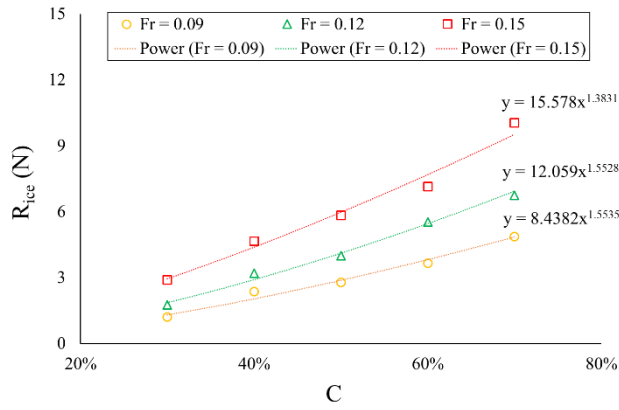
211 Figure 8: Ice-floe resistance for varying ship speed, obtained when $h = 0.02$ m; results of 1:52.667

212

KCS hull model

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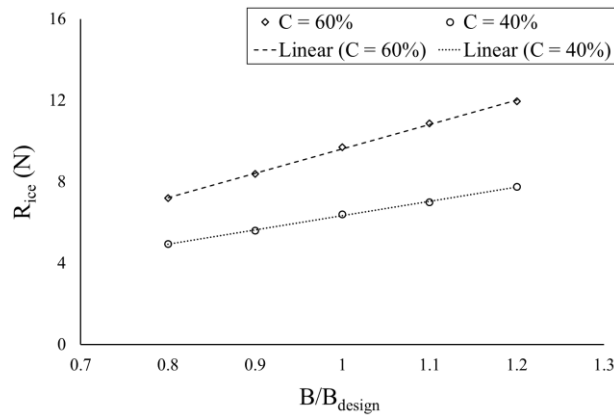
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Figure 9: Ice-floe resistance for varying ice concentration, obtained when $h = 0.02$ m; results of 1:52.667 KCS hull model

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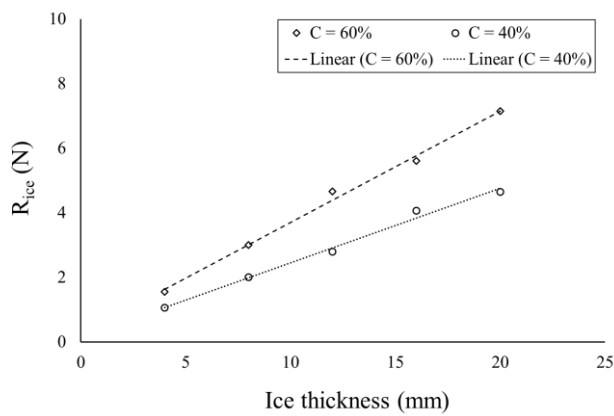
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Figure 10: Ice-floe resistance as a function of ship beam (normalised by the designed beam), obtained when $Fr = 0.18$ and $h = 0.02$ m; results of 1:52.667 KCS hull model

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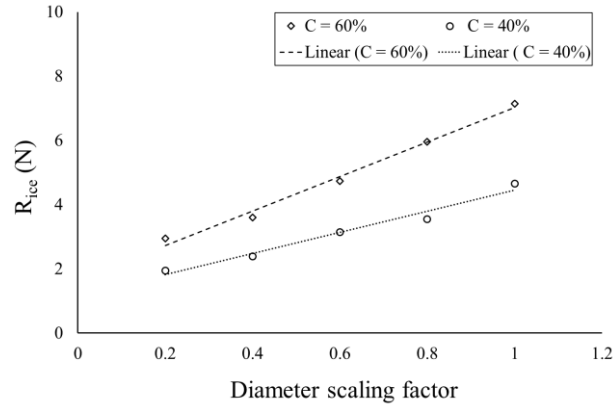


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Figure 11: Ice-floe resistance for varying ice thickness, obtained when $Fr = 0.15$; results of 1:52.667 KCS hull model

225



227

228 Figure 12: Ice-floe resistance as a function of floe size, obtained when $Fr = 0.15$ and $h = 0.02$ m;
 229 results of 1:52.667 KCS hull model. The diameter scaling factor denotes a value to be multiplied by
 230 all floe diameters applied in the experiments of [41].

231

232 3. Derivation of an empirical equation for ice-floe resistance

233 This section starts by introducing a non-dimensional procedure to derive a rational expression of R_{ice} ,
 234 based on the principal parameters and regression powers identified in the previous section. Subsequently,
 235 the ice resistance equations for the three different ships are derived, and their differences are discussed
 236 to further derive a generic equation to account for different ships. Then, the derived generic equation is
 237 validated by predicting ice resistance for previous experimental conditions. Furthermore, a discussion
 238 is given on the extrapolation of the equation from model-scale to full-scale.

239

240 3.1 Non-dimensional analysis

241 R_{ice} is first expressed using parameters shown in Table 3. This gives:

242

$$243 R_{ice} = A \times \rho_{ice}^a \times h^b \times D^c \times U^d \times B/L_{pp}^m \times C^n \quad (1)$$

244

245 where A is a coefficient dependent on the specific ship, and ρ_{ice} is the ice density used for matching up
 246 the units between the left- and right-hand sides of the equation. Subsequently, using a standard non-
 247 dimensional method to fit the units of both sides, it gives: $a = 1$, $b + c = 2$ and $d = 2$, thus,

248

$$249 R_{ice} = A \times \rho_{ice} \times h^b \times D^c \times U^2 \times B/L_{pp}^m \times C^n \quad (2)$$

250

251 Based on the regression powers shown for h , D , B and C , $b = 1$, $c = 1$, $m = 1$ and $n = 1.5$. Since the
252 power of speed was found to be 1.2, while its unit-based power is 2, a non-dimensional parameter
253 Froude number ($Fr = V/\sqrt{g \times L_{pp}}$) is introduced to fulfil both the power and unit; thereby the power
254 of Fr derives to be -0.8, therefore:

255

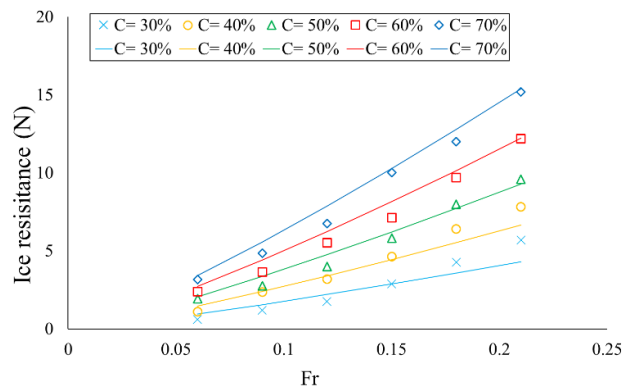
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$$R_{ice} = A \times \rho_{ice} \times h \times D \times U^2 \times B/L_{pp} \times C^{1.5} \times Fr^{-0.8} \tag{3}$$

257

258 In reality, ice floes in a given region are of different dimensions, where h has little variation while D of
259 floes can be notably different. Thus, it is recommended to input a constant h and calculate an average
260 D for the equation based on an average Aspect Ratio (AR), i.e. $D = h \times AR$. Field measurements reported
261 that a generally applicable average value for AR is 10 for ice floe fields [50]. The density of the ice,
262 ρ_{ice} , can be held constant at 900 kg/m³.

263 Then, the parameters of KCS, JBC and AIV hulls were inserted in Equation (3) to find the corresponding
264 A values. This in turn gives $A_{KCS} = 7.64$, $A_{JBC} = 11$ and $A_{AIV} = 5.5$. These coefficients can be inserted
265 in Equation (3) to provide relatively accurate predictions for ice-floe resistance, as shown in Figures
266 13-15.

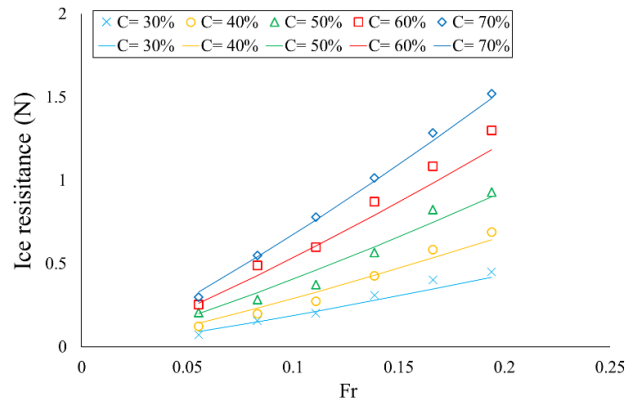
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269 Figure 13: Ice-floe resistance calculated by simulations (symbols) and Equation (3) (lines), when
270 $h = 0.02$ m; results of 1:52.667 KCS hull model

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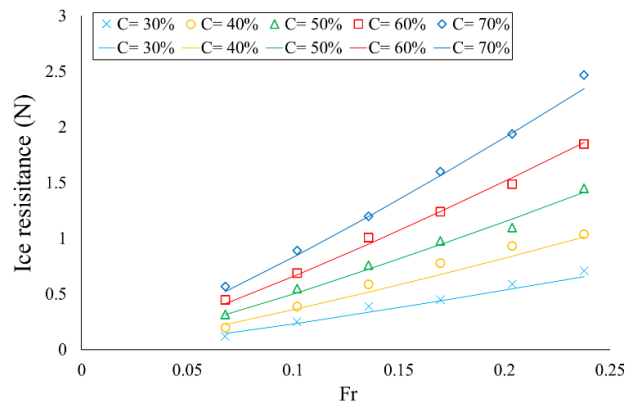
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Figure 14: Ice-floe resistance calculated by simulations (symbols) and Equation (3) (lines), when

274

$h = 0.0066$ m; results of 1:52.667 JBC hull model

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276

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Figure 15: Ice-floe resistance calculated by simulations (symbols) and Equation (3) (lines), when

278

$h = 0.0066$ m; results of 1:52.667 AIV hull model

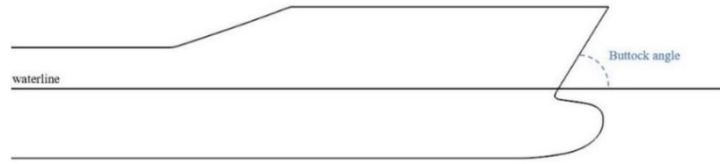
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280 3.2 Unification for multiple hull forms

281

Equation (3) has accounted for the influence of ship beam, while other parameters of the hull geometries still make the ice resistance coefficients different for each of the three ships. To investigate the underlying reasons and derive a generic equation, the ship-wave-ice interactions of these three hulls were further analysed, by which, two influential bow parameters were identified: buttock angle (γ) and waterline angle (α), which are illustrated in Figure 16.

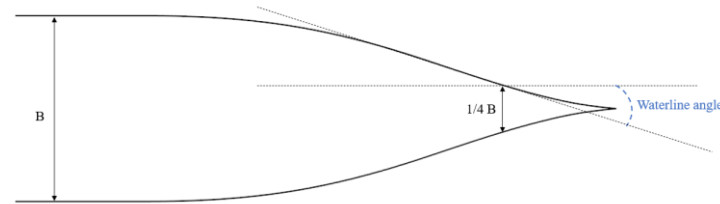
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(a) Central plane of the KCS hull



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(b) Waterline plane of the KCS hull

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Figure 16: Graphic expression of buttock angle and waterline angles, defined following the

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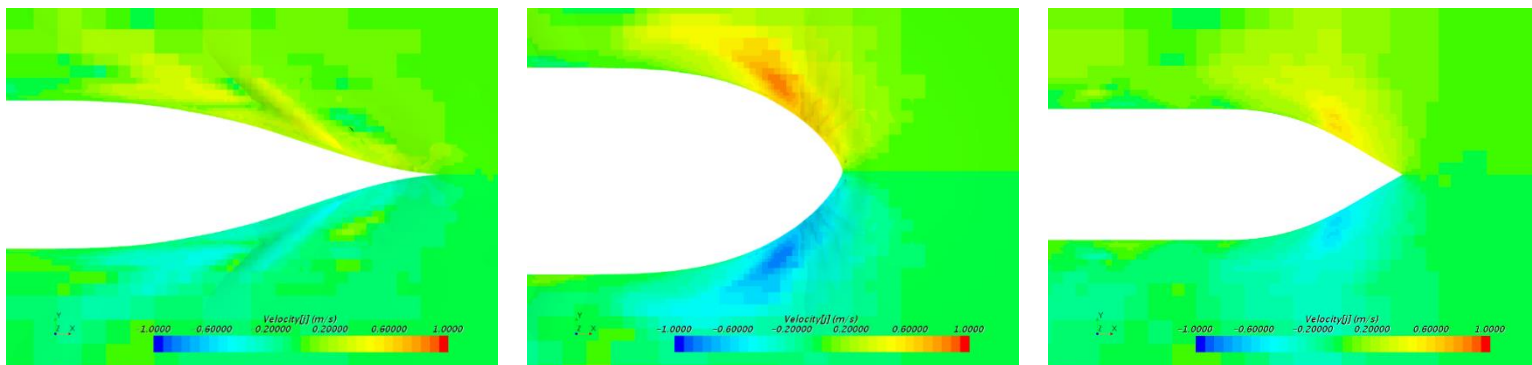
International Association of Classification Societies [51]

293

294 The buttock angle mainly influences the ship-wave-ice interaction in the vertical direction. It may be
 295 observed in Figure 6(a) that the bow of KCS or JCS has a buttock angle of approximate 60° , while the
 296 AIV has a vertical stem, i.e. buttock angle is 90° . The buttock angle dictates the contact surface between
 297 the ship and floes, which influences the ice resistance.

298 The waterline angle mainly influences the ship-wave-ice interaction in the transverse direction. The
 299 KCS has a sharp bow shape across the waterline, while JBC and AIV have relatively round shapes
 300 across the waterline. The round shape tends to generate a wave profile that pushes ice floes aside from
 301 the ship, as shown in Figure 17. This wave profile results in a reduction in the ice resistance.

302



(a) KCS

(b) JBC

(c) AIV

Figure 17: Waterline velocity fields around the bows, colour contours show the velocity magnitude in the transverse direction

303

304 Table 4: Bow Angles of the Candidate Ships and the Corresponding R_{ice} Coefficients in Equation (3)

	KCS	JBC	AIV
Buttock angle (γ) [degrees]	58	61	90
Waterline angle measured at 1/4 beam (α) [degrees]	18	50	30
Calculated ice floe resistance coefficients [-]	7.64	5.5	11

305

306 To unify the ice-floe equation for multiple hull forms, the bow angles of KCS, JBC and AIV were
307 measured and shown in Table 4. γ and α are required to be inserted in Equation (3) to account for their
308 influences. The influence of buttock angle on the ice-induced resistance force on ships was discussed
309 by Riska et al. [18], where the authors note that, for icebreaking vessels, R_{ice} tends to be proportional
310 to the tangent of γ , but their extensive experience shows that the influence of γ on R_{ice} is less
311 dominating than a tangent function for non-icebreaking vessels; thus, γ is instead taken to have a unit-
312 power correlation with R_{ice} . After normalising a unit power of γ against the ice-resistance coefficients
313 for the three ships, the reduction effect of α on R_{ice} is shown to be a cosine function. Therefore,
314 Equation (3) becomes:

315

$$316 \quad R_{ice} = 0.13665 \times \gamma \times \cos \alpha \times \rho_{ice} \times h \times D \times U^2 \times B/L_{pp} \times C^{1.5} \times Fr^{-0.8} \quad (4)$$

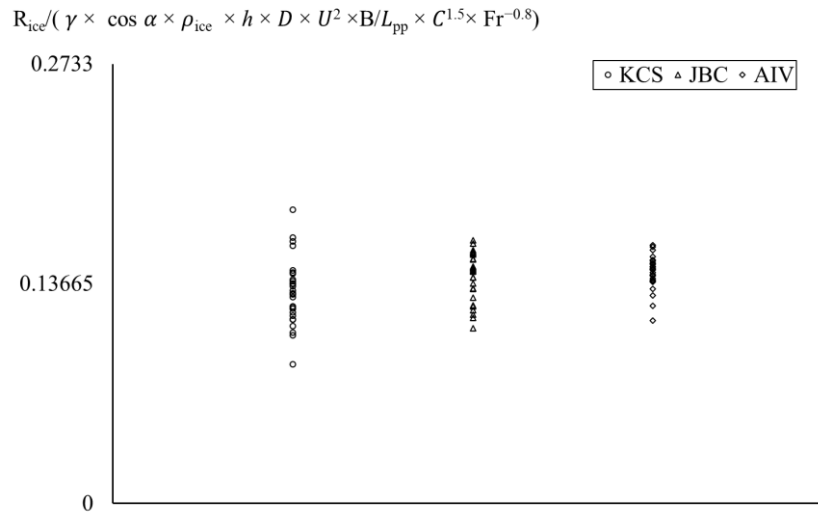
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318 Equation (3) and Equation (4) provide identical results, while Equation (4) is applicable for all the three
319 investigated ships without requiring the calculation of a specific coefficient for a specific vessel.
320 Following the same approach, further parameters and relationships could be identified when more data
321 becomes available in future work.

322 The uncertainty of the derivation process is analysed through nondimensionalising R_{ice} by parameters
323 of Equation (4). As shown in Figure 18, most of the nondimensionalised R_{ice} lays in a range of
324 $0.13665 \times (1 \pm 15\%)$, which indicates the inherent residual. The residual is expected due to the nature
325 of empirical derivation, and the 15% level has made an improvement than previous work (e.g. Figure
326 19 of Guo et al. [41]) by including more parameters that were not considered in previous simpler ice-
327 floe resistance derivation. However, the residual could be further reduced by applying advanced
328 machine learning algorithms [52–54]. When predicting ship fuel consumption of an ice-infested route,
329 the uncertainty level is expected to be less than 15% as highly variable ice conditions should offset the
330 residual of each other.

331

332



333

334 Figure 18: R_{ice} data from simulations nondimensionalised by the parameters of Equation (4)

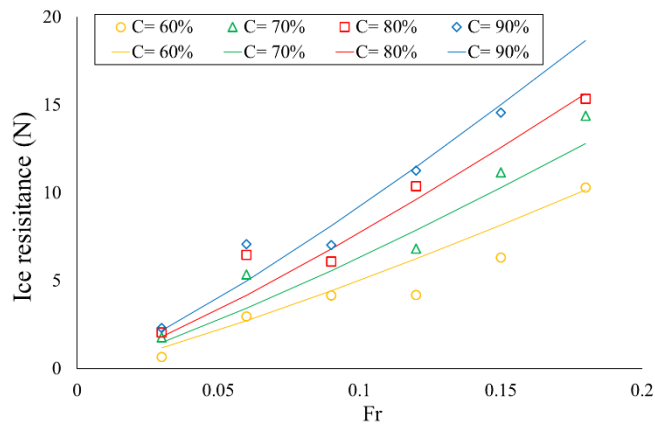
335

336 3.3 Validation

337 The equation to predict ice-floe resistance was validated against two sets of model tests with different
338 hull, ice and speed conditions:

339 Firstly, Equation (4) was used to provide prediction against experiments conducted in the towing tank
340 of Harbin Engineering University in China [41], where a model-scale KCS hull was towed through a
341 field of numerous pieces of floating paraffin wax, mimicking rigid ice floes. The floe resistance was
342 interpreted as being the total ship resistance in floes minus the corresponding calm water resistance
343 without floes. To make the comparison, Equation (4) was directly inputted with the same ship, ice,
344 operating parameters used in the experiments, where the ship speed ranges from $Fr = 0.03$ to 0.18 , ice
345 concentration ranges from $C = 60\%$ to 90% , and ice thickness $h = 0.02$ m. The comparison between
346 computational and experimental results is presented in Figure 19, where Equation (4) is shown to be
347 fairly accurate and the deviations are reasonably within the range of derivational and experimental
348 uncertainties.

349



350

351 Figure 19: Comparison between ice-floe resistance of 1:52.667 KCS hull model measured by
 352 experiments [41] (symbols) and calculated by Equation (4) (lines)

353

354 To further confirm the accuracy of Equation (4), another set of experiments were found that were
 355 recently conducted in the Korea Research Institute of Ships and Ocean Engineering [42]. Towing tests
 356 were carried out in laboratory ice floes for a model-scale Araon ship whose hull form considerably
 357 differs from the three hulls studied earlier, as shown in Figure 20. Equation (4) was directly inputted
 358 with the same ship, ice, operating parameters that used in the experiments, where the ship speeds are Fr
 359 = 0.017, 0.05 and 0.084, ice concentrations are $C = 60\%$ and 80% , and ice thickness $h = 0.057$ m. The
 360 comparison presented in Figure 21 shows good agreement; in particular, this demonstrates that the
 361 derived equation can be applicable to other hull shapes, speeds, and ice conditions that were not
 362 considered in the derivation process.

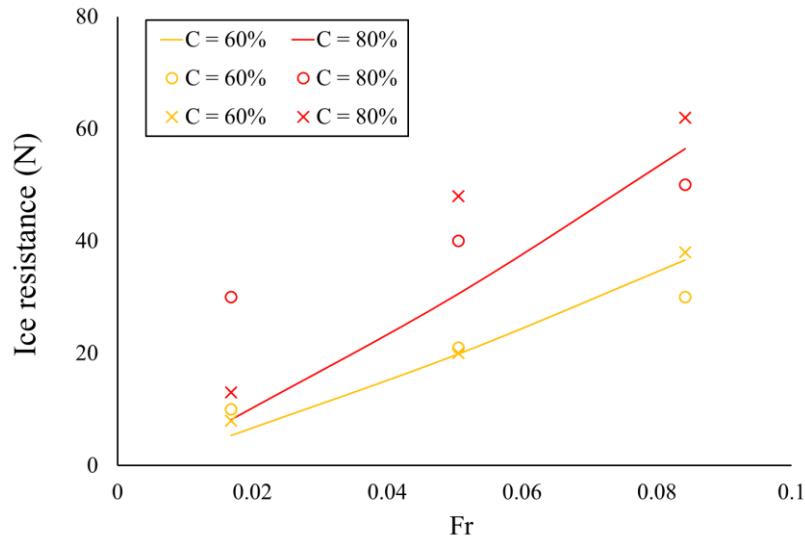
363



364

365 Figure 20: Ice model tests of 1:18.667 Araon hull model reported by Kim et al. [42]

366



367

368 Figure 21: Comparison among ice-floe resistance of 1:18.667 Araon hull model given by Equation (4)
 369 (lines), by experiments (circles) and by the finite-element method (crosses) of Kim et al. [42]

370

371 For the lowest speed and $C = 80\%$ in Figure 21, it may be seen that the experimental resistance is
 372 significantly larger than that predicted by Equation (4). Kim et al. [42] indicate that this test condition
 373 could be a special case, as the experimental value is also much higher than that predicted by their finite-
 374 element model. It is expected that R_{ice} should equal 0 when $Fr = 0$, while the experimental R_{ice} is
 375 approximate 30 N when Fr is very small at 0.017 and the slope would lead to a clearly unphysical R_{ice}
 376 (larger than 20 N) even when $Fr = 0$. The same phenomenon is discussed by Riska et al. [18], where
 377 they indicated that at very low speeds ships cannot proceed continuously in ice towing tanks, thus the
 378 measured average towing force is not directly applicable to the prediction of R_{ice} .

379 In addition, noting that the floe shape is circular in Huang et al. [45], square in Guo et al. [41], and an
 380 irregular polygon in Kim et al. [42], the coincident trends and agreement among these works indicate
 381 that floe shape may not be a critical parameter for the ice resistance of the studied conditions.

382

383 3.4 Full-scale extrapolation

384 The computational modelling was conducted at model scale to allow validation against experiments, so
 385 it is important to discuss how the derived R_{ice} equation can be applied to full-scale ships. For water
 386 resistance (R_{water}), the derivation of formulae from model tests usually needs to apply the ITTC
 387 extrapolation procedure [55], since it is impossible to ensure Froude and Reynolds numbers are both
 388 equal in full scale and model scale, in which, the former governs gravity/inertia (waves) forces and the
 389 latter governs viscous forces. R_{water} can be divided into a wave component and a frictional component;
 390 in model tests, scaling based on a consistent Froude number is practical, which scales the wave

391 component correctly yet brings about certain errors within the friction component due to changes in
392 Reynolds number. The latter may be corrected using the ITTC method [55].
393 Similarly, for the extrapolation of ice resistance, Froude and Cauchy numbers should be both equal
394 between full scale and model scale, otherwise a correction procedure would also be required [56];
395 Cauchy number relates to the elasticity of ice, whose consistency is for the elastic reaction forces of ice
396 are correctly scaled, which is essential to accurately represent the icebreaking process. However, in the
397 present ice-floe case, in principle the small floes are pushed away rather than broken by the ship, so the
398 floes can be assumed to be rigid [40], thus the Cauchy number is infinitely small in both scales and
399 relevant corrections do not need to be applied. Therefore, the present work proposes that Equation (4)
400 can be directly applied to full scale, as the non-dimensional derivation has already kept the expression
401 in line with Froude's law. However, it should be noted that this is a theoretical inference, and future
402 validation is still required to prove the applicability of scaling the ice-floe resistance equation from
403 model-scale to full-scale.

404

405 **4. Application**

406 The achievement of Equation (4) is providing a rapid estimate of ice-floe resistance for a given ship in
407 a given ice condition, which in turn enables its incorporation into real-time applications. The derived
408 ice-floe resistance equation has already been incorporated into a new Arctic Ship Performance Model
409 (ASPM) and VPT, as presented in Li et al. [57] (in which the ASPM was named SPM-B). Hence, only
410 the most salient points are recounted in this section.

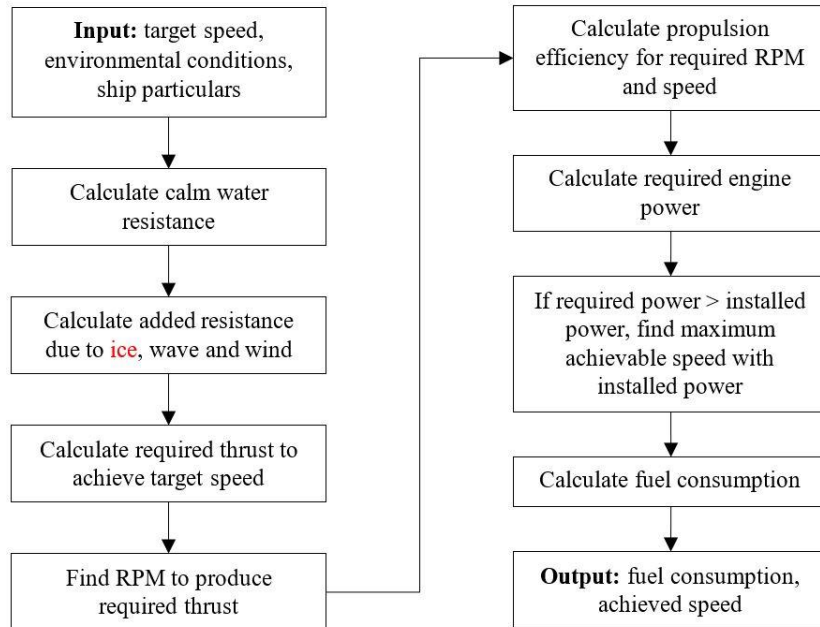
411

412 **4.1 Arctic ship performance model**

413 The calculation procedure of the ASPM is given in Figure 22. It can be seen that most of the procedure
414 can be completed using classical naval architecture methods [58–60], whilst an essential addition is the
415 calculation of added resistance due to ice. In the ASPM, ice resistance is classified into large ice floes
416 and small ice floes. These two conditions correspond to significantly different physics during the ship-
417 ice interactions. Large ice floes undergo crushing and break-up when ships are operating through them,
418 and the ultimate of this case is level ice. By contrast, small ice floes have a high degree of freedom, thus
419 their response to ships is mainly being pushed away rather than fractured. Extreme ice conditions, such
420 as ice ridges, are not considered in the ASPM, since they are designed to be detected by the crew and
421 avoided during operations. Therefore, two different methods were required to account for the ice
422 resistance in large and small ice-floe scenarios, for which the ASPM respectively incorporated the
423 empirical method provided by FSICR using the equivalent ice thickness [61] and Equation (4) derived
424 in the present work. After incorporating ice-resistance equations, the ASPM has the capability of
425 predicting the fuel consumption and attained speed for ships navigating in ice-infested routes. The

426 threshold between large and small floes in the current ASPM is $C \times h = 0.3$ m, which is based upon the
 427 classification of UK Met Office that when $C \times h > 0.3$ m first-year ice starts to grow and ship-induced
 428 fracture is expected occur, and when $C \times h \leq 0.3$ m ice types are young grey, pancake and grease floes
 429 that do not expect ship-induced fracture [62].

430



431

432

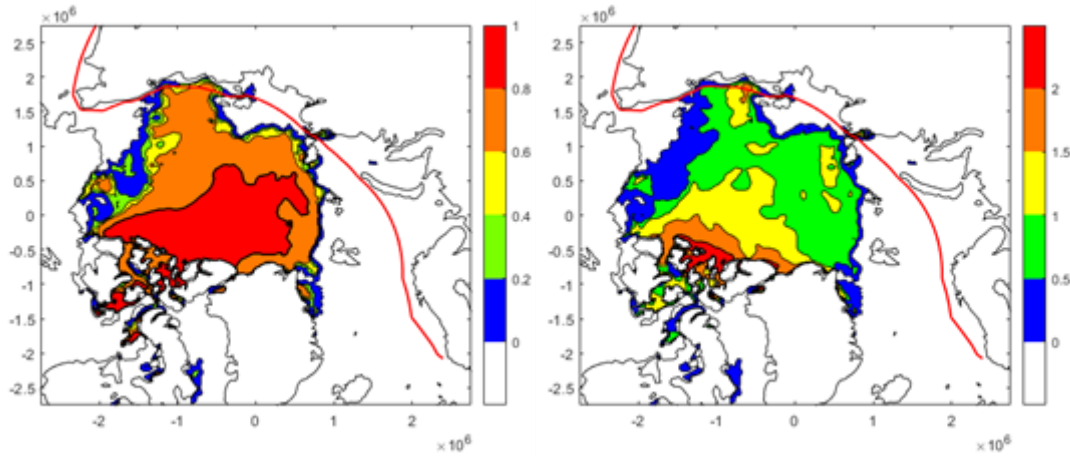
Figure 22: Calculation procedure of the Arctic Ship Performance Model

433

434 4.2 Arctic voyage planning tool

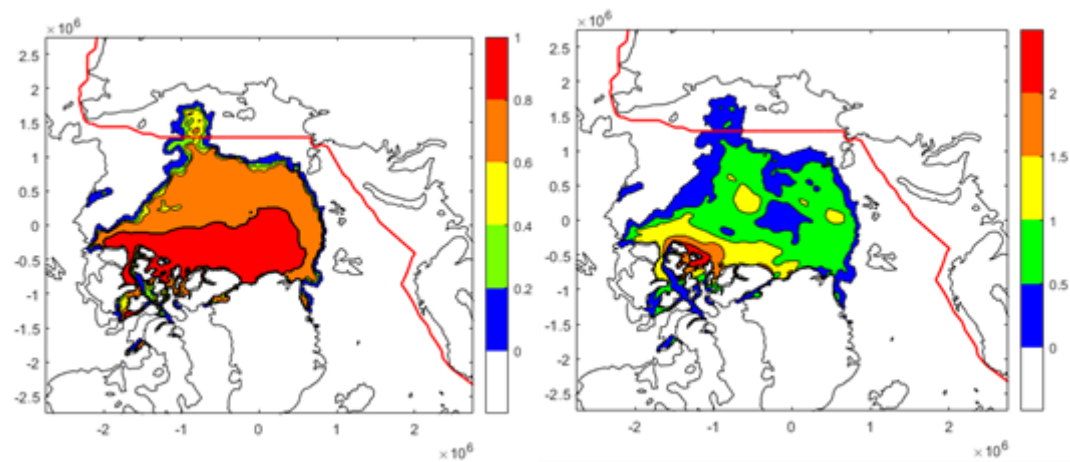
435 The ASPM was integrated into a VPT [49], which links with real-time weather and ice forecasting
 436 systems to calculate a ship's fuel consumption along all potential routes. Then the VPT performs two
 437 steps to determine the optimal route: (a) eliminate any route that contains an ice condition to violate the
 438 POLARIS standard to cause a structural risk [63] (b) suggest the route with the least fuel consumption.
 439 The coupled weather systems include the Copernicus Marine Environment Monitoring Service that
 440 provides metocean data and the UK Met Office Forecast Ocean Assimilation Model that provides sea
 441 ice data [62]. Figure 23 provides two examples of suggested routes for a ship travelling through the
 442 Northern Sea Route, obtained using the VPT based on historical metocean and ice data in 2018. In
 443 Figure 23(a), it can be seen that the majority of the Arctic was covered by sea ice at that time (early
 444 summer), and the VPT chose a route close to the Russian coastline, where the ice conditions were not
 445 severe (referred to as coastal route in Figure 2); in Figure 23(b), the Arctic sea ice reached the annual
 446 minimum (late summer), and the VPT chose a shorter route via the higher-latitude Arctic Ocean
 447 (referred to as high-latitude route in Figure 2). These route choices show the VPT can minimise the ship

448 fuel consumption based on a comprehensive consideration of shorter voyage distance and less ice
 449 impact, which signifies its practical value to Arctic shipping. Nonetheless, the shown examples are for
 450 the summer season, and intended future work is to study the VPT application in winter scenarios, in
 451 which the ice impact will be more significant as will be the benefit of route planning.



452

453 (a) 20 July 2018 - 15 August 2018, when the majority of the Arctic was covered by sea ice



454

455 (b) 01 September 2018 - 24 September 2018, when Arctic ice was the least of the year.

456 Figure 23: Two VPT-suggested routes for a ship travelling through the Northern Sea Route, obtained
 457 using the metocean and ice data of early & late summer 2018. Colour bars denote ice concentration in
 458 the left panels and ice thickness (meter) in the right panels, and the x and y axes denote the distance
 459 (meter) with respect to the North Pole origin

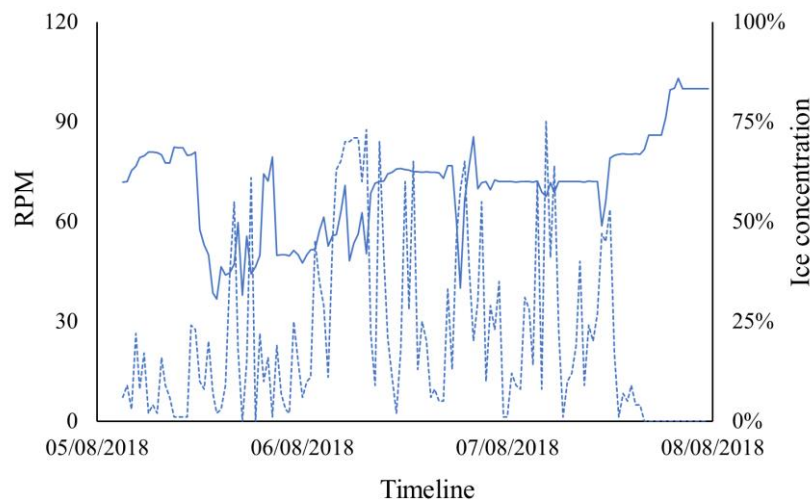
460

461 It is seen in Figure 23(a) that the ice thickness exceeds 1.5 m along a small segment of the route. In
 462 practice, icebreakers may be required to create a channel so that a commercial ship can transit through
 463 such thick ice. To account for this, the VPT now has a function to switch into “icebreaker-assistance
 464 mode” when the ice is sufficiently thick, as reported in Li et al. [64].

465 **5. Comparison with full-scale measurements**

466 Full-scale measurements were collected for the real AIV ship during a voyage through a segment of
467 NSR near the East Siberian Sea. The data were recorded from 03:00 AM 05/08/2018 to 11:30 PM
468 07/08/2018. The ship's central computer recorded the ship's engine RPM, GPS position, encountered
469 wind speed and direction, and attained speed. The encountered sea ice concentration was collected by
470 the onboard cameras that were specifically equipped for sea ice monitoring. Figure 24 plots the
471 measured RPM and ice concentration along the voyage. The figure shows that the RPM was first
472 manually reduced by the crew, before the ship entered the ice-floe field, which was to slow and protect
473 the ship. After the ship entered the ice floe field, the RPM increased due to the additional ice resistance
474 and fluctuated with the variation of ice condition. After the ship passed by the ice-floe field, the RPM
475 was manually increased to speed up the ship in open water.

476



477

478 Figure 24: Full-scale measurement of engine RPM (solid line) and sea ice concentration (dashed line).

479

480 This particular voyage was replicated using the ASPM and VPT. The inputs included (a) the onboard
481 recorded ice concentration, wind data and ship RPM; (b) ice thickness that was inputted from historical
482 satellite data of the UK Met Office, where the ice thickness did not show a notable variation along the
483 voyage, thus a constant value of 0.35 m was taken, which is the average of the encountered sea ice
484 thickness; (c) the historical metocean data from the Copernicus Marine Environment Monitoring
485 Service, which includes waves and currents. Throughout the whole voyage, the encountered sea ice was
486 observed to be small ice floes that have negligible ship-induced fracture; therefore the ice resistance
487 was calculated by Equation (4) rather than FSICR.

488 Using the ASPM and VPT, the ship was set to sail at the measured RPM following the same route as
 489 that recorded by the GPS. Based on the inputs of the environmental variables, Equation (4) and open-
 490 water ship resistance equations were used to calculate the variation of ship total resistance along the
 491 route. Then, the predicted total resistance is combined with the RPM and propulsion system (Table 5)
 492 to calculate the attained speed and fuel consumption of the ship. The measured ship speed was compared
 493 with the predicted value in Figure 25. The comparison shows good agreement, justifying the rationality
 494 of using Equation (4) to account for ice-floe resistance in such a VPT application.

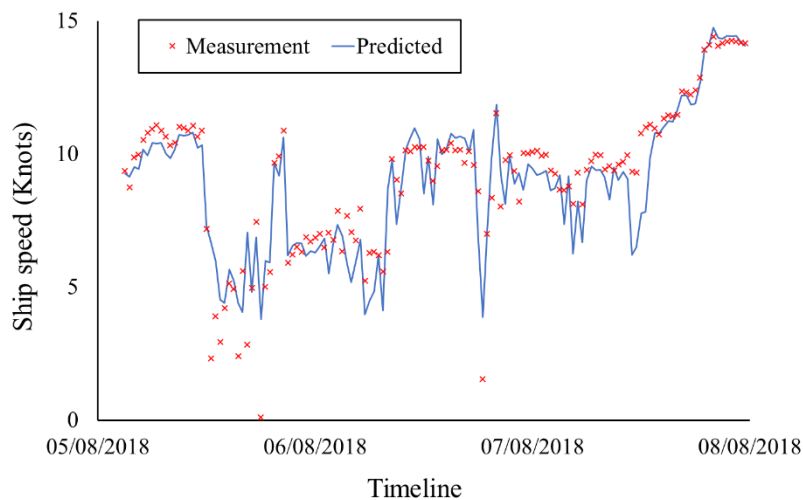
495 This example presents a workable approach that uses the derived ice-floe equation to rapidly predict
 496 ship performance in ice-infested seas, which meets relevant engineering requirements for a real-time
 497 application. Nonetheless, it should be noted that this full-scale validation has only been performed
 498 against a three-day voyage, due to the scarcity of available data containing ice-floe conditions. A
 499 complete validation procedure for a mature ice-floe equation will need more field and experimental
 500 data to help confirm its accuracy.

501

502 Table 5: Particulars of the Propulsion System of AIV.

Main engine	WinGD 6RT-flex50-D
Maximum Continuous Rating (MCR)	10,470kW * 124 r/min
Continuous Synopsis Record (CSR = 65% MCR)	6,806 kW * 107.4 r/min

503



504

505 Figure 25: Predicted ship speed against full-scale measurements.

506

507

508

509 **6. Conclusions**

510 The recent reduction in Arctic sea ice has resulted in increased navigability for commercial ships, whilst
511 also presenting a new environmental condition – floating ice floes. Addressing this emerging scenario,
512 this work provides a computationally cheap empirical equation for predicting ship resistance when
513 operating in such floes. Based upon extensive computational simulations and analyses, the equation
514 explicates how the ice resistance is related to hull, speed and ice parameters, providing valuable insights
515 for polar ship design and engine power estimation. In particular, the influences of buttock and waterline
516 angles were investigated.

517 Furthermore, the derived equation enables the quick prediction of ice-floe resistance on a given ship in
518 given ice conditions, which has facilitated the development of an Arctic ship performance model and
519 voyage planning tool. These applications reveal significant practical value through demonstrating the
520 ability to estimate fuel consumption for ice-going ships and suggest their routes on a real-time basis.

521 Validation against both model-scale and full-scale results demonstrate the rationality of the non-
522 dimensional derivation procedure and the accuracy of the proposed ice resistance equation. However,
523 since available experimental and field data on the present problem are still scarce, there could be extra
524 parameters and relationships of the equation to be identified, as more data becoming available in the
525 future.

526

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532

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