Mapping components of verbal and visuospatial working memory to mathematical topics in seven- to fifteen-year-olds

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Background. Developmental research provides considerable evidence of a strong relationship between verbal and visuospatial working memory (WM) and mathematics ability across age groups. However, little is known about how components of WM (i.e., short-term storage, processing speed, the central executive) might relate to mathematics sub-categories and how these change as children develop.

Aims. This study aimed to identify developmental changes in relationships between components of verbal and visuospatial WM and specific mathematics abilities.

Sample. Children ($n = 117$) were recruited from four UK schools across three age groups (7–8 years; 9–10 years; and 14–15 years).

Methods. Children's verbal and visuospatial short-term storage, processing speed, and central executive abilities were assessed. Age-based changes in the contributions from these abilities to performance on mathematics sub-categories were examined.

Results. When WM was examined both as an amalgamation of its component parts, and individually, relationships with mathematics were more evident in younger children compared to the middle and older age groups. However, when unique variance was examined for each WM predictor (controlling for the other components), many of those relationships disappeared. Relationships with processing speed and the central executive were found to be more evident in the older age groups.

Conclusions. The WM-mathematics relationship changes dependent on age and mathematical sub-component. Overlap in individual WM abilities in younger children, compared to reliance on the central executive and processing speed in older children, suggests a set of fluid resources important in mathematics learning in younger children but separating out as children grow older.

Working memory (WM) is commonly defined as a limited capacity system which holds information in mind for a known purpose, while concurrently processing other information. Importantly, it is relied on when a situation is novel (Shallice & Burgess, 1996), for example when learning new information (Cowan, 2014). Developmental

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research provides evidence for a link between WM and mathematical learning (Allen, Giofrè, Higgins, & Adams, 2020; Allen, Higgins, & Adams, 2019; Cragg, Keeble, Richardson, Roome, & Gilmore, 2017; Friso-van den Bos, Van der Ven, Kroesbergen, & Van Luit, 2013; Geary, 2011; Lee & Bull, 2016; Toll, Van der Ven, Kroesbergen, & Van Luit, 2011; Van der Ven, van der Maas, Straatemeier, & Jansen, 2013).

Although WM is a broad construct with many theoretical explanations (see Barrouillet, Bernardin, & Camos, 2004; Barrouillet & Camos, 2007; Cowan, 1999; Cowan et al., 2005 for attention-based models), Baddeley and Hitch (1974; Baddeley, 1996, 2000) posited the enduring and influential multicomponent WM model. This consists of two domain-specific short-term stores for verbal and visuospatial information and a domain-general central executive, responsible for allocating limited attentional resources to processing and storage. The multi-component nature of this model has led to investigation of its sub-components and evidence has been found for the differing roles of verbal WM (Allen et al., 2019, 2020; De Smedt et al., 2009; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Toll et al., 2011), visuospatial WM (Allen et al., 2020; De Smedt et al., 2009; Fanari, Meloni, & Massidda, 2019; Geary, 2011; Imbo & LeFevre, 2010; Meyer et al., 2010; Toll et al., 2011; Van der Ven et al., 2013), and the central executive (De Smedt et al., 2009; Henry & MacLean, 2003; Holmes & Adams, 2006; Meyer et al., 2010; Swanson, 2006; Toll et al., 2011) in mathematics ability.

These studies commonly measure WM using complex span tasks, designed to simulate the requirement to process and temporarily store information concurrently. For example, Counting Span (Case, Kurland, & Goldberg, 1982) requires participants to count an array of shapes (processing) and remember the number of shapes presented (storage). The maximum number of items consistently correctly recalled in serial order denotes the participant's span score, which reliably indexes WM capacity (Conway et al., 2005). Such tasks suggest the possible role of processing speed in WM, as Case et al. (1982) found children's processing time predicted their storage capacity. They interpreted this as demonstrating that increases in processing speed release cognitive resources for information storage and thus explain developmental increases in WM capacity (i.e., the resource-sharing hypothesis). This was challenged by Towse and Hitch (1995; Towse, Hitch, & Hutton, 1998) who manipulated both complexity and time in the processing task and found that increased processing time, but not complexity, resulted in lower span scores. They proposed a task-switching account positing that time-based forgetting (i.e., time spent processing and not maintaining information) determines WM capacity. These studies are important as they argue for a direct role of processing speed in WM, and further research has demonstrated how faster processing times might explain relationships with mathematics (Formoso et al., 2018; Geary, 2011; Gordon, Smith-Spark, Newton, & Henry, 2020; Li & Geary, 2013).

Some researchers have further examined the sub-components of WM using complex span tasks to better understand the WM-mathematics relationship. Bayliss, Jarrold, Gunn, and Baddeley (2003) found that domain-general processing speed and domain-specific storage related to mathematical ability in 7- and 8-year-olds. Similarly, Gordon et al. (2020) examined time and accuracy in processing and storage in 7- to 8-year-olds using three complex span tasks measuring visuospatial, verbal, and numerical WM. They found that only processing speed and storage predicted general mathematics ability as defined by the UK curriculum. Moreover, only processing speed in the numerical WM task explained variance in mathematics when all performance indices from all WM tasks were considered.

However, mathematics is not a single ability but one that encompasses various topics including arithmetic, fractions, algebra, measurement, geometry, and handling data (Department for Education, 2014). Therefore, it is problematic to conflate all topics into a single measure and then claim that WM, and its components, are important. Considering the aforementioned evidence that basic arithmetic can involve both verbal and visuospatial WM (Allen et al., 2019), the central executive (De Smedt et al., 2009), and processing speed (Gordon et al., 2020), it is important to further examine how these different aspects of WM map onto the distinct mathematics topics.

The WM-mathematics relationship grows in complexity when considering the influence of age on how these constructs might inter-relate. Studies have found a stronger reliance on visuospatial WM during mathematics learning in younger children, with a move to verbal WM in older children (De Smedt et al., 2009; Van der Ven et al., 2013; Van de Weijer-Bergsma et al., 2015). One explanation is that, as children develop, they create memories of mathematics facts which can be verbally recalled to contribute to completing mathematical tasks (De Smedt et al., 2009). However, Van der Ven et al. (2013) found that the relationship between visuospatial WM and elementary mathematics tasks such as addition and subtraction decreased with age but not the relationship with the more complex operations of multiplication and division. Importantly, relationships between visuospatial WM and different mathematics domains were stronger at the age when the new material was introduced into the curriculum.

Furthermore, there is evidence that the importance of WM, regardless of domain, reduces when mathematical procedures become more familiar (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Imbo & Vandierendonck, 2007). Generally, when a task is not novel, there is less reliance on effortful attentional abilities and more reliance on automatic behaviours (Norman & Shallice, 1986; Shallice & Burgess, 1996). Therefore, a reduction in the novelty of mathematical procedures as children grow older, rather than age per se, may reduce reliance on WM (Tronsky, 2005). As the central executive is viewed as responsible for allocating limited attentional resources in WM, such findings imply a shift in the role of this ability in mathematics. This is at odds with research that suggests increases in mathematical ability as children grow older are related to increases in executive abilities (Bull & Scerif, 2001), but is supported by recent evidence that inhibition is less important in mathematics as children grow older and other factors, such as strategy use, come into play (Avgerinou & Tolmie, 2020).

To understand which cognitive abilities are relied upon in mathematical learning a comprehensive developmental investigation of the different aspects of WM and how they relate to each mathematical topic is required. The aim of the current study was to assess verbal short-term storage (STS), visuospatial STS, processing speed, and the central executive and their separate contributions to performance on different mathematics topics. How these relationships change as children are exposed to novel and increasingly complex mathematical concepts and procedures through primary and secondary school was then examined. Specifically, three age groups were used to identify where developmental shifts occur. 7- to 8-year-olds were selected as this represents a developmental stage where more sophisticated WM abilities, which correlate with educational outcomes, begin to emerge (Gathercole, Pickering, Ambridge, & Wearing, 2004). By the age of 9–10 years, WM is more firmly identified as a cognitive construct separate from general cognitive abilities and learning processes (Brydges, Fox, Reid, & Anderson, 2014; Thompson et al., 2019). At 14–15 years of age, the relationship between WM and mathematics is firmly established (Gathercole et al., 2004). Therefore, these age

groups were used to identify these particular developmental shifts in relation to components of WM and subsequent links with mathematics ability.

The aim of the present study was to test the hypothesis that different components of WM explain performance in individual mathematical topics and that these relationships change as children move through primary school to secondary school. Specifically, the predictions were as follows:

1. Visuospatial STS predicts mathematical ability in younger children (7–8 years), but verbal STS predicts this ability in older children (14–15 years).
2. The central executive predicts ability in younger children but this reliance reduces in older children as procedures become automated.
3. Processing speed predicts mathematics ability in older children when there is a requirement to hold information in STS and faster processing prevents decay. No specific hypotheses were identified for particular mathematics topics given the lack of clear evidence from past research.

Method

Participants
One hundred and seventeen children (70 girls) were recruited from three primary schools and one secondary school as follows: Group A: 32 (19 girls) 7- to 8-year-olds; Group B: 56 (28 girls) 9-to 10-year-olds; and Group C: 29 (23 girls) 14- to 15-year-olds. No children had any known documented neurodevelopmental disorders or learning difficulties.

Procedure
Participants were tested over a three-week period in school during regular hours. A total of seven tasks were completed. All tasks were administered individually, except the mathematics assessment, which was administered in a group session. These sessions were run for each age group in each school.

Measures
Reading and Non-verbal Reasoning were assessed to confirm that the groups were representative of a typical developmental trajectory.

Reading ability
Reading was assessed using the Word Reading test from the British Ability Scales 3rd edition (BAS-III, Elliot & Smith, 2011). The BAS III Word Reading test is suitable for children aged 3–17 years of age. It has good internal consistency (α = .79–.91) and test–retest reliability (r = .64). Participants were asked to read each word aloud from a list of 90 words grouped in blocks of 10, with increasing difficulty across blocks. One point was awarded for each word pronounced correctly. Testing was stopped after eight incorrect responses in a block.

Non-verbal reasoning

Non-verbal reasoning (NVR) was measured using the matrices task from the BAS-III, for the five to 17 years age range. There were five practice and 33 test trials. In each trial, a matrix of nine cells containing black and white geometric designs was presented to each child. For each matrix, one of the nine cells was empty, denoting an incomplete pattern. The child was asked to select, from six options, the correct component which would complete the pattern. Level of complexity increased across trials, with it becoming increasingly difficult to identify which piece would complete the pattern. One mark was given for each correct answer, which denoted the NVR score.

Processing speed

Processing speed was assessed using the Speed of Processing Information sub-test from the BAS III. Group A (7- to 8-year-olds): Children were shown a series of circles in rows. Each of the circles was filled with a number of smaller squares. The number of squares in each circle never exceeded four. For each row, they were asked to mark the circle with the greatest number of squares. Group B (9- to 10-year-olds): Children were shown a series of two-digit numbers in rows. They were asked to mark the highest numerical digit in each row. Group C (14- to 15-year-olds): Children were shown a series of three-digit numbers in rows and were asked to mark the highest numerical digit per row. For all groups, there were two practice trials and six assessment trials. Participants were awarded one mark for each correct answer (raw score). The possible scores for all tasks ranged from 0 to 36. The task was timed from the moment the child marked the first circle in the first row until the moment the child marked a circle in the last row. A score between zero to six was awarded based on six time intervals. If a child made more than three errors on a single trial, they were scored zero for that trial, regardless of their time. Total score was calculated by adding the scores for all six trials together. A higher score indicated a faster processing speed.

Verbal STS

Verbal STS was measured using the Working Memory Test Battery for Children (WMTB-C) (Gathercole & Pickering, 2001) digit span task. A sequence of random, non-repetitive digits was read at a rate of one digit per second. The participant was then asked to recall the list in serial order. There were seven blocks of trials with six trials per block. Trials initially included two numbers and increased by one number in each block until the participant was unable to recall four correct trials in a block. No score was awarded for a block after three errors. One point was awarded for each correct trial when four or more trials were correctly recalled. The sum of scores denoted the total score.

Visuospatial STS

Spatial STS was measured using the WMTB-C block recall task. For the block recall task, the participant was shown a plastic tray consisting of an array of nine fixed, three-dimensional cubes. The researcher pointed to a random and non-repetitive sequence of cubes (locations) at a rate of one per second. The participant was required to repeat the sequence in correct serial order. There were seven blocks of trials with six trials per block. Trials initially included two locations and increased by one location in each block until the participant was unable to recall four correct trials in a block. No score was awarded for a

block after three errors. One point was awarded for each correct trial when four or more trials were correctly recalled. The sum of scores denoted the total score.

Central executive

The backward digit span task from the WMTB-C was used to measure the Central Executive. This simple task was used to reduce possible cognitive 'noise' that can be a limitation of executive function tasks (see Denckla, 1994; Rabbitt, 1997). Administration was similar to that for digit span except the child was asked to recall the numbers in reverse serial order. Failure points and scoring protocols were as per digit span.

Mathematics

Mathematics performance was assessed using the Access Mathematics Test (AMT) (McCarty, 2008). This standardized test is based on the UK national curriculum for mathematics. Test 1 (Form A) was used to assess Groups A and B (aged 7–10 years) and Test 2 (Form B) was used to assess Group C (aged 14–15 years). Both tests accessed seven different mathematics domains (Using and Applying Mathematics, Counting and Understanding Number, Knowing and Using Number Facts/Algebra, Calculating, Understanding Shape, Measuring, and Handling data). Scores were calculated for each sub-topic and total mathematics.

Results

Some measures showed skewness and kurtosis. Therefore, the values were converted to z-scores to identify any which were ± 2.5 standard deviations from the mean. Values that were ± 2.5 standard deviations from the mean were winsorized and the corresponding true values were substituted with the closest upper or lower value (within 2.5 standard deviation). This resulted in the alteration of one case each for Understanding Shape, Verbal STS, Central Executive and NVR, two cases each for Understanding and Applying Mathematics and Measuring, three cases for visuospatial STS, and four cases for Reading. In total, these cases represented between 0.8% and 3.4% of the data set for any single variable. For similar methodology, see Bayliss et al. (2003), Bayliss, Jarrold, Baddeley, Gunn, and Leigh (2005).

Non-verbal reasoning and reading ability were used to identify whether the groups represented age-typical abilities. These are reported in Table 1. All children performed within the ability range for their age. To examine whether the three age groups represented a developmentally typical trajectory, analysis of variance (ANOVA) was conducted for reading and NVR. Significant increases in ability between age groups were found for Reading, $F(2, 114) = 47.36$, $p < .0001$. Post-hoc comparisons showed a significant increase in mean scores from Group A to Group B ($M = 18.35$, $SE = 3.26$,

Table 1. Mean (standard deviation) for age, non-verbal reasoning, and reading ability for each age group

	Age in years	Non-verbal reasoning	Reading
Group A	8.11 (0.43)	11.62 (6.00)	49.43 (20.44)
Group B	10.03 (0.52)	14.20 (5.00)	67.79 (14.18)
Group C	14.61 (0.48)	22.79 (3.87)	86.14 (4.92)

$p < .0001$), Group A to Group C ($M = 36.70$, $SE = 3.77$, $p < .0001$), and Group B to Group C ($M = 18.35$, $SE = 3.37$, $p < .001$). Significant increases in ability were also found for NVR, $F (2, 114) 41.47$, $p < .0001$. Post-hoc comparisons showed significantly higher scores from Group A to Group C ($M = 11.17$, $SE = 1.30$, $p < .0001$) and Group B to Group C ($M = 8.60$, $SE = 1.16$, $p < .0001$). There was no significant increase in NVR ability from Group A to Group B ($p = .07$).

The mean and standard deviation for each WM measure is shown in Table 2. To assess increases in WM ability from Group A to Group B to Group C, ANOVA was conducted for each of the WM measures. Significant increases in ability between age groups were found for visuospatial STS, $F (2, 114) = 10.02$, $p < .0001$; the central executive, $F (2, 114) = 19.71$, $p < .0001$; and processing speed, $F (2, 114) = 14.67$, $p < .0001$. There were no significant differences in mean scores in verbal STS between the three Groups, $F (2, 114) = .879$, $p = .418$.

Post-hoc analysis showed significant increases in visuospatial STS from Group A to Group B ($M = 3.17$, $SE = 1.09$, $p < .05$) and from Group A to Group C ($M = 5.62$, $SE = 1.34$, $p < .0001$). There was no significant difference in the mean scores between Group B and Group C ($p = .097$). For the central executive measure, there was a significant increase in ability from Group A to Group B ($M = 3.44$, $SE = .90$, $p < .01$), from Group A to Group C ($M = 6.51$, $SE = 1.04$, $p < .0001$), and from Group B to Group C ($M = 3.07$, $SE = .93$, $p < .01$). There was a significant increase in processing speed from Group A to Group C ($M = 6.92$, $SE = 1.89$, $p < .01$) and from Group B to Group C ($M = 9.09$, $SE = 1.69$, $p < .0001$). There was no significant difference in the mean scores between Group A and Group B ($p = .56$).

The means and standard deviations for the mathematics scores are reported in Table 3. To identify any significant increases in mathematics ability across age groups, an ANOVA was conducted for the overall mathematics score. There were significant increases in ability from Group A to Group B ($M = 14.81$, $SE = 2.02$, $p < .0001$) and Group A to Group C ($M = 10.35$, $SE = 2.34$, $p < .0001$) and a modest but non-significant decline from Group B to Group C ($p = .10$). However, these scores are based on different age-appropriate measures and, as such, an increase from Group B to Group C was not necessarily expected. This also further supports the subsequent analysis of abilities within, as opposed to across, the age groups.

Relationships between the WM measures and mathematics achievement were conducted for each group. Significant values are shown in Table 4. Non-significant relationships are replaced with a hyphen for ease of interpretation. Several relationships between verbal STS and mathematics abilities were evident in Group A and Group B, but these disappeared for the oldest age group. Visuospatial STS was linked to all mathematics scores in Group A, but there were no relationships with visuospatial STS in the two older age groups. Correlations between the central executive and mathematics abilities were

Table 2. Mean (standard deviation) for verbal short-term store, visuospatial short-term store; central executive and processing speed for each age group

	Verbal STS	Visuospatial STS	Central Executive	Processing Speed
Group A	32.03 (6.05)	24.34 (5.97)	11.59 (3.95)	19.94 (8.06)
Group B	33.77 (6.59)	27.52 (4.61)	15.04 (4.17)	17.77 (7.38)
Group C	34.00 (7.14)	29.97 (4.26)	18.10 (3.94)	26.86 (6.57)

STS = short-term storage.

Table 3. Mean (standard deviation) for all mathematics measures for each age group

	Total Mathematics	UA	CN	NF	CA	SH	ME	HD
Group A	14.03 (9.33)	2.50 (1.52)	3.06 (1.97)	2.56 (1.72)	1.59 (1.32)	1.41 (1.24)	1.03 (1.36)	1.78 (1.74)
Group B	28.84 (9.38)	4.04 (1.73)	6.82 (2.61)	4.82 (1.72)	3.86 (1.71)	2.36 (1.30)	3.25 (1.64)	3.57 (1.62)
Group C	24.38 (8.31)	2.38 (1.18)	5.34 (1.93)	4.55 (1.66)	4.21 (1.80)	3.00 (1.65)	1.24 (1.09)	3.66 (1.37)

UA = Using and Applying Mathematics; CN = Counting and Understanding Number; NF = Knowing and Using Number Facts/Algebra; CA = Calculating; SH = Understanding Shape; ME = Measuring; HD = Handling Data

Table 4. Correlation between all WM measures and mathematical total and component scores

	Total Mathematics	UA	CN	NF	CA	SH	ME	HD
Group A								
Verbal STS	.503**	.401*	.504**	—	—	.402*	.385*	.590**
Visuospatial STS	.538***	.445*	.389*	.383*	.454**	.495**	.529**	.465**
Central Executive	.439*	—	.423*	.400*	—	—	—	.527**
Processing speed	.441*	.512**	.414*	.410*	—	—	—	—
Group B								
Verbal STS	.312*	.267*	.324*	.399**	—	.354**	—	—
Visuospatial STS	—	—	—	—	—	—	—	—
Central Executive	.415**	.336*	.358**	.471**	.404**	.363**	.275*	—
Processing speed	.343**	.394**	.400**	.334*	.372**	—	.429**	—
Group C								
Verbal STS	—	—	—	—	—	—	—	—
Visuospatial STS	—	.415*	—	—	—	—	—	—
Central Executive	.395*	—	—	—	—	—	—	.589**
Processing speed	.465*	—	.389*	.423*	—	.435*	.467*	—

CA = calculating; CN = counting and understanding number; HD = handling data; ME = measuring; NF = knowing and using number facts/algebra; SH = understanding shape; STS = short-term storage; UA = using and applying mathematics.
*$p < .05$; **$p < .01$.

more evident in Group B compared to Groups A and C. Some relationships between processing speed and mathematics scores were evident for all age groups but these differed in terms of mathematics abilities across the three groups.

Multiple regression was used to understand contributions from WM to Total Mathematics and each sub-component. First, all WM measures (i.e. verbal and visuospatial STS, processing speed, central executive) were entered together at Step 1. Results for each age group are shown in Table 5. Only significant models are shown. WM predicted all but one mathematics ability measure in Group A and Group B., and only Total Mathematics and Handling Data in Group C.

Next, separate simple regression models were run to understand the variance explained in mathematical ability by each predictor for each age group. This was done to identify which predictors should be included in subsequent hierarchical regression models for each mathematics score and each age group. All significant predictors are shown in Figure 1. Overall, more WM measures individually predicted mathematics

Table 5. Regression models (adjusted R^2) ANOVA including all predictors of mathematics ability for each age group

Group	A	B	C
Total Mathematics	(.38) F (4, 27) 5.79**	(.17) F(4, 51) 3.82**	(.24) F (4, 24) 3.24**
UA	(.30) F (4, 27) 4.27**	(.15) F (4, 51) 3.40*	–
CN	(.29) F (4, 27) 4.12*	(.18) F (4, 51) 4.03**	–
NF	(.20) F (4, 27) 2.88*	(.24) F (4, 51) 5.45**	–
CA	–	(.16) F (4, 51) 3.70*	–
SH	(.21) F (4, 27) 3.11*	(.11) F (4, 51) 2.76*	–
ME	(.24) F (4, 27) 3.48*	(.14) F (4, 51) 3.32*	–
HD	(.42) F (4, 27) 6.57**	–	(.43) F (4, 24) 6.29**

CA = calculating; CN = counting and understanding number; HD = handling data; ME = measuring; NF = knowing and using number facts/algebra; SH = understanding shape; UA = using and applying mathematics.
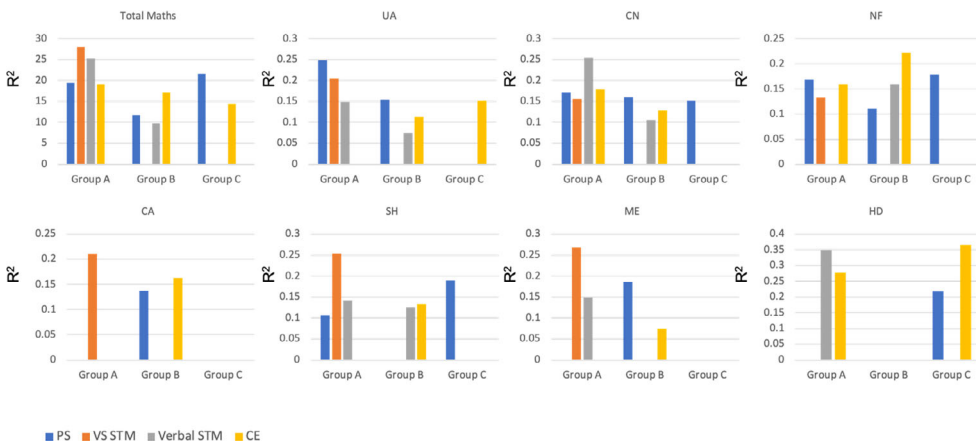*$p \leq .05$, **$p < .01$.

Figure 1. Variance explained in total mathematics and topics by WM components for each age group. WM = working memory.

ability in Group A, with the exception of Calculation, Measuring, and Handling data. For Group B, mostly verbal STS, processing speed and the central executive predicted mathematics ability. For Group C, significant models were only evident for processing speed and the central executive.

Hierarchical regression was used to identify *unique* variance in each mathematics score explained by the WM measures. The variable of interest was entered at Step 2, while all other variables were controlled for at Step 1. For example, to identify unique variance explained by processing speed, the other three predictors (verbal STS, visuospatial STS, the central executive) would be entered at step one and processing speed was entered at Step 2. This was conducted for all significant predictors from the simple regression analyses shown in Figure 1 and was repeated for each age group. The findings are shown in Table 6. Variables omitted due to not reaching significance in the aforementioned simple regression are 'greyed out'. Only values significant at $p < .05$ or less are shown for ease of interpretation.

Whereas Group A showed consistent relationships with the WM measures in the multiple and simple regressions, these relationships largely disappeared when controlling for the other significant predictors. Visuospatial STS predicted Calculation, Understanding Shape, Measuring, but none of the WM measures predicted Total Mathematics. For Group B, processing speed predicted Using and Applying Mathematics, Counting and Understanding Number, Measuring but, again, none of the WM measures predicted Total Mathematics. Group C was the only age group for which the WM measures predicted Total Mathematics (processing speed, central executive), and processing speed also predicted Counting and Understanding Number, Knowing and Using Number Facts and Understanding Shapes. In addition, the central executive predicted Using and Applying Mathematics and Handling Data.

Discussion

The samples that represented each age group were typically developing in terms of NVR and reading. Also, reading and NVR, verbal STS, visuospatial STS, the central executive, and processing speed across the three age groups were representative of the developmental trajectory expected from 7 to 15 years of age. Reading ability increased steadily across the three age groups, and NVR increased more gradually in the younger two age groups compared to the older age group. This is consistent with research that has shown reading ability has a consistent steep trajectory from 7 to 15 years of age (Berman, 2004), whereas NVR has a steeper trajectory in early adolescence compared to childhood (Cotton et al., 2005).

Between-group differences indicated a significant developmental trajectory for visuospatial STS, but no developmental differences in verbal STS. This is in line with research that shows verbal STS, as denoted by forward digit span, increases more gradually from 7 to 15 years, whereas as visuospatial STS measured using forward block span has a steeper trajectory (Isaacs & Vargha-Khadem, 1989). Group differences for the central executive were also consistent with the expected developmental trajectory, with significant increases shown from 7 to 15 years of age (Gathercole et al., 2004; Huizinga & Smidts, 2010). Significant, but gradual, increases in processing speed from Group A to Group C were also representative of typical development (Kail, 2000).

Analysis was undertaken to identify whether visuospatial STS predicts mathematical ability in younger children with a move to a reliance on verbal STS in older children.

Table 6. Change in adjusted R^2 (β) for unique predictors of mathematics ability. Only significant values are shown

Change in R^2 (β)	Total Mathematics	UA	CN	NF	CA	SH	ME	HD
Group A								
Verbal STS	–	–	–		–	–	–	.43* (.40)
Visuospatial STS	–	–	–		.18** (.45)	.24* (.36)	.26* (.45)	–
CE	–	.32* (.39)	–		–	–		.43* (.33)
PS	–	–	–	–	–	–	–	
Group B								
Verbal STS								
Visuospatial STS								
CE	–	.17* (.31)	.20* (.31)	.25* (.30)	.19* (.31)	–		
PS	–	–	–	–	–	–	.17**(.38)	
Group C								
Verbal STS								
Visuospatial STS								
CE	.28* (.35)	.14* (.42)				.16* (.44)		.32** (.59)
PS	.28* (.43)		.12* (.39)	.15* (.42)				–

CA = calculating; CN = counting and understanding number; HD = handling data; ME = measuring; NF = knowing and using number facts/algebra; SH = understanding shape; STS = short-term storage; UA = using and applying mathematics.
*p < .05; **p < .01.

Correlations showed links between visuospatial STS and all mathematics scores in 7- to 8-year-olds but none in the older two age groups. This is, in part, consistent with the first hypothesis. However, there were also relationships between verbal STS and mathematics abilities in 7- to 8-year-olds and 9- to 10-year-olds, but not in 14- to 15-year-olds. This does not support the second part of this hypothesis that verbal STS would replace visuospatial STS in older children. Similarly, when all WM measures were used to represent WM ability, this construct predicted all but one of the mathematics outcomes in 7- to 8-year-olds and in 9 to 10-years-olds, yet only predicted Total Mathematics and Handling Data in older children. When examined separately, more of the WM measures predicted mathematics ability in the younger two age groups compared to the older age group. This demonstrates a greater reliance on WM overall from the age of 7–10 years of age compared to the early teenage years.

When assessing unique variance in mathematics explained by the separate WM measures, links with specific WM components largely disappeared for the youngest age group, with visuospatial and verbal STS predicting few mathematics ability sub-scores. Although there is some evidence of a slightly greater reliance on visuospatial STS, there is substantially overlapping variance between the separate WM components in the application of these to mathematics ability in this age group, suggesting a relative lack of differentiation. Relationships between verbal and visuospatial STS and mathematics ability vanished in 9- to 15-year-olds. Therefore, although a reliance on visuospatial STS was demonstrated for 7- to 8-year-olds, there was no evidence for a move to verbal STS beyond that age.

The second and third hypotheses are best addressed together. These were that the central executive is important in mathematical ability in younger children but that this reliance reduces in older children as procedures become automated; and that processing speed prevents decay of information in STS in older children, who are more likely to be relying on the retrieval of mathematics facts to complete automated operations. Links between the central executive and mathematics were more evident in 9- to 10-year-olds, compared to the youngest and oldest age groups, but were not totally absent in these two groups. Similarly, relationships between processing speed and mathematics were more common in the older two age groups but still apparent to a lesser degree in the youngest group. Initially, these findings suggest that the central executive is important from the ages of 7–15 years and that processing speed becomes increasingly important across these age groups. However, the simple and hierarchical regressions demonstrated that the central executive and processing speed were the only WM measures that predicted mathematics ability in the older two age groups, whereas they only predicted a single mathematical sub-component each for the youngest age groups. These findings support the third hypothesis but only partially support the second, as it appears that reliance on both processing speed and the central executive increases with age. However, there were some definite patterns of relationships between specific types of mathematics ability and individual WM measures. For the younger age group, visuospatial STS predicted the mathematics topics that, based on previous research, ostensibly rely on visuospatial processing; namely, Understanding Shape (Holmes & Adams, 2006); Measuring (Holmes, Adams, & Hamilton, 2008); and Calculation (Zago & Tzourio-Mazoyer, 2002). Conversely, these relationships were absent in the older two age groups and visuospatial STS was replaced by the central executive and processing speed in the middle group and by processing speed in the older group. This suggests a topic-specific relationship between different aspects of WM and mathematical topics, which in part supports all three hypotheses.

By assessing individual WM components and examining relationships with individual mathematics topics within separate age groups, the current study was able to demonstrate patterns of reliance on WM for mathematics learning at a universal and granular level. This is important given the prevalence of studies that use measures of WM to determine what abilities are central to mathematics achievement. Theoretical implications from such studies cannot be applied at a practical level if we do not truly understand what it is about WM that is important in a skill as broad and varied as mathematics. The challenge of applying these theoretical findings is further compounded if we do not understand how the WM-mathematics relationship might change as children develop. As this study has demonstrated, the aspects of WM that are important for mathematics learning change dependent on age and mathematical sub-component. Although WM, as an amalgamation of its component parts, was shown to be important for general mathematics for all age groups (in line with previous research findings), when mathematical topics were assessed the oldest age group relied on WM for Handling Data alone. Then, examination of WM at a granular level demonstrated a trend of diminished reliance in 9- to 10-year-olds and 14- to 15-year-olds generally, and a similar pattern for Calculation, Shape, Measurements, and Handling Data in all age groups. Additionally, it was then possible to identify a considerable overlap in the individual WM abilities in younger children with a move towards a reliance solely on the central executive and processing speed as children move through primary to secondary school.

Theoretical and practical implications

The current study demonstrated that the roles different WM components play in mathematics learning depends on mathematics topic and that this relationship changes as children develop. Of equal importance is the finding that specific components of WM uniquely predict specific mathematics abilities over and above other factors; and again, that these relationships change as children grow older. Therefore, focusing on these specific and evolving relationships can assist in tailoring age- and topic-appropriate reasonable adjustments and interventions in schools. For example, there is evidence that cognitive load reduction using techniques such as external memory aids and presenting information in smaller chunks, can improve performance in the classroom for those children with low WM capacity (Gathercole & Alloway, 2008). These approaches each aim to reduce demand on short-term memory, processing speed, and the central executive. If we understand at which point in primary and secondary school these cognitive abilities become important in mathematics learning, and for which topics, we can introduce these interventions and reasonable adjustments with more precision. Future research could also consider the potential diagnostic benefits of assessing children on these individual WM abilities to inform screening programmes aimed at early identification of mathematical learning difficulties.

Limitations

While a standardized mathematics test was used, exact calibration in terms of relative difficulty for the age groups is not available. However, it should be noted that the Access Mathematics Test provides detailed comparisons against norms and patterns in the UK National Curriculum for each age and year group. It is noted though, that the number of individual questions within the mathematics test assessing the separate mathematics topics was limited. Future studies should incorporate comprehensive, standalone

assessments for each of the individual mathematics topics to develop a more in-depth understanding of individual ability as it relates to WM components.

Summary

All WM measures together predicted mathematics ability for most specific topics in the younger and middle age group, but these relationships were almost entirely absent in the older age group. This is consistent with the idea that automatic processing is more important in mathematics learning as children grow older and there is less reliance on attentional and executive abilities. When the WM measures were examined individually, relationships with mathematics were more evident in the younger age group and gradually diminished for the middle and older age groups. However, when hierarchical regression was used to identify unique variance over and above that explained by other significant predictors, many of those relationships disappeared. This suggests a set of fluid cognitive resources that work together to facilitate mathematics learning in younger children but these separate out as children grow older. More consistent relationships with processing speed and to a lesser degree the central executive, in the older age groups suggests an increasing reliance on these abilities in mathematics learning up to 15 years of age. This supports future research that can examine the relationships between the specific WM abilities and specific mathematics topics in more detail to better understand how such relationships might inform interventions and reasonable adjustments in the classroom.

Conflicts of interest

All authors declare no conflict of interest.

Author contributions

Emily Whitelock (Investigation) Arzoo Mukarram (Data curation; Investigation; Project administration) Rebecca Gordon (Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Project administration; Resources; Supervision; Validation; Visualization; Writing – original draft; Writing – review & editing) Danila Santana De Morais (Data curation; Formal analysis; Investigation; Writing – original draft).

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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