Joint Radar-Communication Transmission: A Generalized Pareto Optimization Framework

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Abstract—Dual-functional radar-communication (DFRC) has been envisioned as a key enabler for a number of emerging applications. Nevertheless, the intrinsic difference between communication and sensing functionalities has imposed challenges in fully realizing the promise of the technology. In this paper, we propose a Pareto optimization framework for a multi-antenna DFRC system, where a single transmitter with multiple antennas communicates with multiple users and detects radar targets simultaneously. To begin with, we define an achievable performance region that is built upon the peak sidelobe level (PSLL) of the radar spatial beampattern and the signal-to-interference-plus-noise ratio (SINR) of each communication user, which is proved to be compact and normal. Then, we formulate a fairness-profile optimization problem with a lowest acceptable performance level of dual functionalities and a pre-defined direction to attain the Pareto boundary of the achievable performance region. Also, we provide a bisection search algorithm to solve the Pareto optimization problem with polynomial complexity, and prove its optimality can be achieved by single-stream transmission. As a further step, we extend the discussion to the DFRC beamforming design given hardware impairments. Finally, we evaluate the effectiveness of the proposed design and the impact brought by the impairments by numerical simulations.

Index Terms—Spectrum sharing, radar-communication, Pareto optimization, peak sidelobe level, signal to interference and noise ratio, transmitter impairments.

I. INTRODUCTION

With the rapid deployment of both modern radar and wireless communication systems, the last decade has witnessed a growing demand for wireless radio spectrum. The sub-6 GHz band is used by a variety of radar systems conceived for tasks such as weather observation and traffic control [1]. On the other hand, a considerable number of wireless communication systems including long term evolution (LTE) and WiFi are working in the same band [2]. While the millimeter wave (mmWave) band was traditionally occupied by automotive radars and high-resolution imaging radars [3], it has now been assigned to the 5G network as the new licensed band [4]. As a result, communication and radar spectrum sharing (CRSS) has attracted significant research attention for efficient usage of the wireless radio spectrum.

One research direction of CRSS is radar-communication coexistence (RCC), which aims for developing efficient in-

terference management to achieve harmonic spectral coexistence between both systems. In [5], RCC was studied from the viewpoint of cognitive radio, where the radar and the communication systems were regarded as the primary user and the secondary user, respectively. Meanwhile, the transmit power of the communication system is controlled subject to the tolerable interference temperature of the radar. Following the idea of zero-forcing precoding for multi-input and multi-output (MIMO) communication, null-space projection (NSP) was adopted by the radar given the interference channel state information in [6]. The precoded radar signal was projected onto the null-space of the channel in order to completely mitigate the interference generated to the communication system. To avoid serious performance loss incurred by the NSP approach, a relaxed NSP precoder was further designed in [7] for ensuring a controllable interference level. In [8], the covariance matrix of the communication signal was jointly optimized with the sub-sampling matrix of the MIMO radar subject to power and capacity constraints, where Lagrangian dual decomposition and alternating minimization were adopted to solve this problem. To address the more practical scenario of multi-user (MU) communication, the work of [9] further discussed the coexistence problem of the MU-MIMO communication and MIMO radar, where the detection probability of the radar was maximized subject to the transmit power budget and the users' signal to interference and noise ratio (SINR) constraints. To cope with the issue of imperfect channel state information (CSI), the authors of [10] proposed a robust beamforming design for the coexistence of the MU-MIMO communication and MIMO radar.

Another research direction of CRSS is dual-functional radar-communication (DFRC), which aims for implementing both communication and sensing functionalities not only over the same frequency band but also on the same hardware platform. By defining the channel capacity of the radar system as the maximum information that can be recovered from the echo wave, the performance bound of the DFRC system was derived by the work of [11] from the viewpoint of information theory. To avoid the mutual interference between communication and sensing functionalities, a time-division framework was proposed to allow the radar and the communication working on different time slots [12]. Motivated by the orthogonal property for resource allocation in the frequency domain, the orthogonal frequency division multiplexing (OFDM) signal was employed to realize both remote sensing and wireless communication in [13]. The authors of [14] further adopted OFDM communication signals to detect the velocity and the range parameters of vehicles.

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To improve the spatial-domain performance of both communication and sensing functionalities, multiple antennas have been employed by the DFRC system. For the radar functionality, various performance metrics have been adopted to optimize the adaptive MIMO radar waveforms, namely the Cramér-Rao bound (CRB) [15], SINR [16], mean square error (MSE) [17], [18], and mutual information (MI) [19]. For the dual functionalities, the work of [20] designed a transmit beampattern for MIMO radar to embed the information in sidelobe levels. To enable MU-MIMO transmission, the authors of [21] proposed various beamforming designs for multi-antenna DFRC system. The work of [22] studied the optimal probing waveform design that also carried communication signals for multi-antenna DFRC system, and it also provided a tradeoff design between radar and communication performance through weighted optimizations. Further, a novel transceiver architecture and frame structure of DFRC system were proposed in [23] using the hybrid analog-digital beamforming technique.

The existing works of beamforming design for multi-antenna DFRC system generally optimized the radar performance metrics under the communication constraints or vice versa. However, they cannot provide an optimal performance tradeoff between the dual functionalities due to a lack of Pareto optimization framework for multi-antenna DFRC system, which makes its beamforming design rather challenging. First of all, it is non-obvious how to measure DFRC system performance given the isolated radar and communication metrics. CRB, SINR, MSE, and MI are classical performance metrics for the design of MIMO radar, while a series of performance metrics connected to the user-experienced service quality are used to design a multi-user communication system. Second, there exists inherent conflict and tradeoffs for the DFRC system to optimize the performance of radar and communication simultaneously. It naturally incurs multi-objective optimization or tradeoff between multiple objectives. Third, the complexity of the beamforming design for DFRC system has been greatly increased, since both communication and radar constraints should be considered.

Motivated by the above observations, we propose a Pareto optimization framework for joint beamforming design of the multi-antenna DFRC system in this work. It is worth mentioning that the Pareto optimization framework has been fully studied for MU-MIMO communication system [24]–[26], where the performance of each user was coupled with each other through inter-user interference. However, it becomes more difficult for multi-antenna DFRC system due to different performance metrics and the inherent tradeoffs of dual functionalities. To the best of our knowledge, it has been rarely investigated in the existing literature.

In this work, we first define the achievable performance region based on the peak sidelobe level (PSLL) of the radar and the SINR of each communication user (CU). Given the compactness and the normalness of the achievable performance region, one can readily prove that the global optimum is on its Pareto boundary for monotonically increasing system utility. Then, a fairness-profile optimization problem is formulated. Both the radar and the CUs have a lowest acceptable performance level and attains the Pareto boundary of the achievable

performance region following a pre-defined direction. A bisection search solution with polynomial complexity is provided. And the optimality achieved by single-stream transmission is also proved, which greatly reduces the searching space for optimal solutions. Further, we study the performance of DFRC system limited by transmitter impairments. The optimization problem is formulated under a transmitter impairment model, and a corresponding solution is given. The main contributions of this work are summarized as follows.

- A Pareto optimization framework of the DFRC system is introduced, where the defined achievable radar-communication performance region is compact and normal. Thus, it can be applied to any system utility that increases monotonically in the performance of both radar and communication.
- A fairness-profile system utility function is adopted between the communication users' SINR and the radar beampattern's PLSR. It is shown that this is a general problem of max-min PSLL optimization with SINR constraints and max-min SINR optimization with PSLL constraints. Also, its optimality can be achieved by single-stream transmission.
- Based on the above Pareto framework, we further consider hardware impairments at the transmitter. We adopt existing additive distortion models for both the radar and communication functionalities. And the corresponding fairness-profile optimization problem is formulated and solved with polynomial complexity.

The remainder of the paper is organized as follows. Section II presents the system model. Section III discusses the Pareto optimization framework for DFRC system. The study is further extended to the DFRC transmitter considering impairments in Section IV. Simulation results are provided in Section V, followed by concluding remarks in Section VI.

Notation: We use boldface lowercase letter to denote column vectors and boldface uppercase letters to denote matrices. Superscripts $(\cdot)^H$ and $(\cdot)^T$ stand for Hermitian transpose and transpose, respectively. $\text{tr}(\cdot)$, $\text{diag}(\cdot)$, and $\text{rank}(\cdot)$ represent the trace operation, the vector formed by the diagonal elements and the rank operator, respectively. $\mathbb{R}^{m \times n}$ is the set of real-valued $m \times n$ matrices, $\mathbb{R}_+^{m \times n}$ is the set of positive real-valued $m \times n$ matrices, and $\mathbb{C}^{m \times n}$ is the set of complex-valued $m \times n$ matrices. $\mathbb{E}(\cdot)$ denotes the statistical expectation.

II. SYSTEM MODEL

As illustrated in Fig. 1, we consider a multi-antenna DFRC system with a DFRC base station (BS), K CUs indexed by $k \in \{1, \dots, K\}$, and a radar target. The DFRC BS is equipped with a uniform linear array (ULA) of N antennas, serving single-antenna CUs while detecting the radar target at the same time.

A. Communication Model

For the communication function, the received signal of the CU k can be expressed as

Figure 1. System model of multi-antenna DFRC network.

$$y_k = \mathbf{h}_k^H \mathbf{x} + z_k = \mathbf{h}_k^H \sum_{k=1}^K \mathbf{s}_k + z_k, \quad (1)$$

where $\mathbf{x} \in \mathbb{C}^N$ is the transmit signal of the DFRC BS containing all CUs' data signal, $\mathbf{s}_k \in \mathbb{C}^N$ is the transmit signal intended for the CU k , $\mathbf{h}_k \in \mathbb{C}^N$ is the flat-fading channel between the DFRC BS and the CU k , and $z_k \in \mathbb{CN}(0, \sigma_z^2)$ is the additive white Gaussian noise (AWGN) at the CU k . Then, the communication performance of the CU k can be evaluated by its receive SINR, i.e.,

$$\gamma_k(\mathbf{S}_1, \dots, \mathbf{S}_K) = \frac{\mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}{\sum_{i \neq k} \mathbf{h}_k^H \mathbf{S}_i \mathbf{h}_k + \sigma_z^2} \quad (2)$$

where $\mathbf{S}_k$ is the signal covariance matrix of the CU k and the transmit signals of different CUs are assumed to be independent. The transmit power constraint of the DFRC BS can be given by

$$\text{tr} \left(\sum_{k=1}^K \mathbf{S}_k \right) \leq P_0, \quad (3)$$

where P_0 is the power constraint of the DFRC BS.

B. Radar Model

For the radar function, the power of the probing signal given the angle θ of target can be calculated as

$$P_{pro}(\theta) = \mathbf{a}^H(\theta) \mathbf{E}(\mathbf{x} \mathbf{x}^H) \mathbf{a}(\theta) = \mathbf{a}^H(\theta) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta), \quad (4)$$

where $\mathbf{a}(\theta) = [1, e^{j2\pi d \sin(\theta)}, \dots, e^{j2\pi(N-1)d \sin(\theta)}] \in \mathbb{C}^N$ is the steering vector of the transmit antenna array, d is the interval between adjacent antennas being normalized by the wavelength. Then, the radar performance of the probing beampattern can be evaluated by the PSLL η_m of the sidelobe angular θ_m , i.e.,

$$\eta_m = \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) - \mathbf{a}^H(\theta_m) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_m), \quad (5)$$

where the main beam is centered at θ_0 , $\theta_m \in \{\theta_1, \dots, \theta_M\}$ is the prescribed sidelobe region, and M is the cardinality of the sidelobe region set. And the 3dB main beam width constraint of the DFRC BS can be given by

$$\begin{aligned} 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) &= \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_a), \\ 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) &= \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_b), \end{aligned} \quad (6)$$

where $\theta_a - \theta_b$ determines the 3 dB main beam region.

It can be seen that the selection of the transmit signal covariance matrices $\{\mathbf{S}_1, \dots, \mathbf{S}_K\}$ determines the SINR of the CUs in (2) and the PSLL of the radar in (5). Also, its selection is constrained by the transmit power constraint in (3) and the main beam width constraint in (6).

III. JOINT RADAR AND COMMUNICATION OPTIMIZATION

In this section, we define and discuss the achievable performance region of the communication function and the radar function. A fairness-profile optimization framework is proposed, which represents the generalization of radar performance optimization with communication performance constraint and communication performance optimization with radar performance constraint.

A. Achievable Performance Region

The performance of the multi-antenna DFRC system includes the performance of each CU and the performance of the radar probing signals. Since the performances of the communication and the radar are coupled via the transmit signal covariance matrices $\{\mathbf{S}_1, \dots, \mathbf{S}_K\}$, it is instructive to consider the set of all feasible operating points, i.e., the achievable performance region of the multi-antenna DFRC system.

Definition 1. (Achievable performance region) The achievable performance region $\mathcal{R} \subseteq \mathbb{R}^{K+M}$ is defined as

$$\mathcal{R} = \{\gamma_1, \dots, \gamma_K, \eta_1, \dots, \eta_M\} : \forall \{\mathbf{S}_1, \dots, \mathbf{S}_K\} \in \mathcal{S} \quad (7)$$

where γ_k is the SINR of the k -th CU in (2), η_m is the PSLL of the m -th sidelobe angular θ_m in (5), and $\mathcal{S}$ is the set of feasible transmit strategies considering transmit power constraint and main beam width constraint, i.e.,

$$\mathcal{S} = \{\mathbf{S}_1, \dots, \mathbf{S}_K\} : (3) \text{ \& } (6). \quad (8)$$

The dimension of the achievable performance region is $K + M$, which describes the performance simultaneously achieved by all CUs' SINR and the radar's PSLL. Its shape depends on the channel gains of the CUs, the properties of the ULA, the power constraint, and the main-beam width constraint. Due to the non-convexity of the SINR expression, $\mathcal{R}$ is generally a non-convex set, but it can be proved to be compact and normal.

Definition 2. (Compact and normal set) A set $\Upsilon \subset \mathbb{R}_+^L$ is called compact if it is closed and bounded. And it is called normal, if for any point $\mathbf{x} \in \Upsilon$, all $\mathbf{x}' \in \mathbb{R}_+^L$ satisfying $\mathbf{x}' \leq \mathbf{x}$ with an element-wise inequality also satisfy $\mathbf{x}' \in \Upsilon$.

Lemma 1. ($\mathcal{R}$ is compact and normal) If the dimension of the sidelobe region is larger than the number of the CUs, i.e., $M \geq K^1$, the achievable performance region $\mathcal{R}$ of the multi-antenna DFRC system is compact and normal.

Proof. The proof is given in Appendix A $\square$

Lemma 1 indicates that for any point $\mathbf{x} \in \mathcal{R}$, all points that provide worse performance than $\mathbf{x}$ are also in $\mathcal{R}$. Thus, the points in $\mathcal{R}$ which provide good performance all lie on the Pareto boundary of $\mathcal{R}$, which is defined as follows.

Definition 3. (Pareto boundary) A point $\mathbf{x}$ is called Pareto optimal point of a compact and normal set Υ , if $\mathbf{x} \in \Upsilon$ while $\mathbf{x}' \in \mathbb{R}_+^L$ and $\{\mathbf{x}' > \mathbf{x}\} \cap \Upsilon = \emptyset$. The set of all Pareto optimal points is called the Pareto boundary of the set Υ , which is denoted as $\partial^+ \Upsilon$.

B. Fairness-Profile Optimization

In order to compare the merits of different Pareto optimal points, a system utility function should be defined. The system utility function $f : \mathcal{R} \rightarrow \mathbb{R}$ that take any point in $\mathcal{R}$ as input and produces a scalar value describing how preferable this point is.

In this work, we adopt the minimum weighted compromise function as the system utility function, i.e.,

$$f(\gamma_1, \dots, \gamma_K, \eta_1, \dots, \eta_M) = \min_{k,m} \left\{ \frac{\gamma_k - a_k}{\alpha_k}, \frac{\eta_m - b_m}{\beta_m} \right\}, \quad (9)$$

where $a_k \in \mathbb{R}$, $\alpha_k \in \mathbb{R}_+$ are the performance fairness parameters of the communication, and $b_m \in \mathbb{R}$, $\beta_m \in \mathbb{R}_+$ are the performance fairness parameters of the radar.

Remark 1. (Significance of performance fairness) For the communication, the CUs have different SINR requirements. For example, the CUs with high throughput or low latency require a large SINR. Thus, the SINR of the CU k has a lowest acceptable level $\gamma_k \geq a_k$, and α_k defines the direction from a_k to the final Pareto optimal point. For the radar, the sidelobe angulars have different PSLL requirements. For example, the

sidelobe angulars near the main beam require a large PSLL. Thus, the PSLL of the angular θ_m has a lowest acceptable level b_m , and β_m defines the direction from b_m to the final Pareto optimal point.

In light of the above discussion, the corresponding system utility optimization problem can be formulated as

$$\begin{aligned} \text{P1 : } \quad & \max_{\{\mathbf{S}_k\}} \min_{k,m} \left\{ \frac{\gamma_k - a_k}{\alpha_k}, \frac{\eta_m - b_m}{\beta_m} \right\} \\ \text{s.t.} \quad & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k \right) \leq P_0, \\ & 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ & = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_a), \\ & 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ & = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_b), \end{aligned} \quad (10)$$

where γ_k and η_m are given by (2) and (5), respectively. It is a fairness-profile optimization, where each CU and sidelobe angular has a lowest acceptable performance level, and the aggregate performance above this level with a predefined rate.

Given different fairness parameters, the proposed problem (P1) can be regarded as a generalization of the PSLL optimization problem with SINR constraint and the SINR optimization problem with PSLL constraint.

Remark 2. (PSLL optimization with SINR constraint) Set $a_k = \Gamma_k$, $\alpha_k = \delta$, $b_m = 0$, and $\beta_m = 1$, where δ is a positive constant, and Γ_k is the SINR constraint of the CU k . When $\delta \rightarrow 0$, the problem (P1) converges to a max-min PSLL optimization problem with each CU's SINR constraint, i.e.,

$$\begin{aligned} \text{P1.1 : } \quad & \max_{\{\mathbf{S}_k\}} \min_m \eta_m \\ \text{s.t.} \quad & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k \right) \leq P_0, \\ & 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ & = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_a), \\ & 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ & = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_b), \\ & \gamma_k \geq \Gamma_k, \forall k = 1, \dots, K, \end{aligned} \quad (11)$$

where the problem (P1.1) maximizes the minimum PSLL of the prescribed sidelobe region with the transmit power constraint, the main beam width constraint, and each CU's SINR constraint.

Remark 3. (Beamforming optimization with beam pattern constraint) Set $a_k = 0$, $\alpha_k = 1$, $b_m = \Lambda_m$, and $\beta_m = \delta$, where δ is a positive constant, and Λ_m is the PSLL level constraint

¹This assumption can be easily realized in the practical situation. The sidelobe area is continuous, and the number of the sidelobe angular $\{\theta_m\}$ can be increased by increasing the number of samples in the sidelobe area.

Figure 2. Illustration of bisection search on the Pareto boundary of achievable performance region.

of the sidelobe angular θ_m . When $\delta \rightarrow 0$, the problem (P1) converges to a max-min SINR optimization problem with each sidelobe angular's PSLL constraint, i.e.,

$$\begin{aligned}
 \text{P1.2: } & \max_{\{\mathbf{S}_k\}} \min_k \gamma_k \\
 \text{s.t. } & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k \right) \leq P_0, \\
 & 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\
 & = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_a), \\
 & 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\
 & = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_b), \\
 & \eta_m \geq \Lambda_m, \forall m = 1, \dots, M,
 \end{aligned} \tag{12}$$

where the problem (P1.2) maximizes the minimum SINR of all CUs with the transmit power constraint, the main beam width constraint, and the PSLL constraints of the prescribed sidelobe region.

Although the problem (P1) is a generalization of the optimization problems (P1.1) and (P1.2), it is nonconvex and generally NP-hard. Fortunately, the achievable performance region $\mathcal{R}$ is compact and normal according to Lemma 1, we only need to search the Pareto boundary $\partial^+ \mathcal{R}$ to solve this problem.

Lemma 2. (Global optimum on Pareto boundary) If the system utility function $f: \mathcal{R} \rightarrow \mathbb{R}$ is strictly increasing, and the achievable performance region $\mathcal{R}$ is a compact and normal set, the global optimum (if it exists) to $\max_{\mathbf{x} \in \mathcal{R}} f(\mathbf{x})$ is attained on the upper boundary of the achievable performance region, i.e., $\partial^+ \mathcal{R}$.

Proof. Assume that $\mathbf{x}' \notin \partial^+ \mathcal{R}$ is a global optimum point to $\max_{\mathbf{x} \in \mathcal{R}} f(\mathbf{x})$. According to the definition of Pareto boundary, there exists a point $\mathbf{x}'' \in \partial^+ \mathcal{R}$ satisfying $\mathbf{x}'' \geq \mathbf{x}'$ with an element-wise inequality. For the strictly increasing system

utility function f , one has $f(\mathbf{x}'') \geq f(\mathbf{x}')$, which is a contradiction of the global optimum point $\mathbf{x}'$. $\square$

Thus, we can start from a point in the achievable performance region, and follow a ray in a pre-defined direction until the global optimal point on the Pareto boundary is found. First, we consider a feasible transmit strategy problem with fixed SINR and PSLL constraints, i.e.,

$$\begin{aligned}
 \text{P2: } & \text{find } \{\mathbf{S}_1, \dots, \mathbf{S}_K\} \\
 \text{s.t. } & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k \right) \leq P_0, \\
 \text{Radar } & \begin{cases} 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_a), \\ \eta_m \geq \Lambda_m, \forall m = 1, \dots, M, \\ 0.5 \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k \right) \mathbf{a}(\theta_b), \end{cases} \\
 \text{Comm. } & \gamma_k \geq \Gamma_k, \forall k = 1, \dots, K,
 \end{aligned} \tag{13}$$

where Λ_m are fixed PSLL of the sidelobe angular m , and Γ_k are fixed SINR of the CU k . Observe that the Problem (P2) only has linear constraints, which leads to a convex optimization problem.

Then, we define a continuous vector-valued function governed by a single scalar to provide fixed SINR and PSLL constraints, i.e.,

$$\mathbf{r}(\tau) = [\Gamma_1(\tau), \dots, \Gamma_K(\tau), \Lambda_1(\tau), \dots, \Lambda_M(\tau)], \tag{14}$$

where

$$\Gamma_k(\tau) = \alpha_k \tau + a_k \tag{15}$$

and

$$\Lambda_m(\tau) = \beta_m \tau + b_m. \tag{16}$$

Observe that $\mathbf{r}(\tau)$ is strictly linearly increasing function of the scalar $\tau \in [0, \tau_{\max}]$, which describes a one-dimensional curve between $\mathbf{r}(0)$ and $\mathbf{r}(\tau_{\max})$. Thus, we have the following proposition.

Proposition 1. (Bisection search solution) Assuming that $\mathbf{r}(0) \in \mathcal{R}$ and $\mathbf{r}(\tau_{\max}) \notin \mathcal{R}^2$, the curve $\mathbf{r}(\tau)$ intersects the Pareto boundary of the achievable performance region $\mathcal{R}$ exactly once. Thus, the Problem (P1) can be solved by bisection search over $[0, \tau_{\max}]$. Specifically, given τ' , if the problem (P2) is feasible, all $\tau < \tau'$ where $\tau \in [0, \tau_{\max}]$ is removed. Otherwise, if the problem (P2) is infeasible, all $\tau > \tau'$ where $\tau \in [0, \tau_{\max}]$ is removed.

Proof. According to Lemma 1, the achievable performance region $\mathcal{R}$ is compact and normal. Thus, according to Lemma 2,

$2\tau_{\max}$ can be obtained by checking the feasible problem (P2). Given any τ , one has the corresponding Γ_k and Λ_m . If the problem (P2) is feasible, we have to increase τ until the problem (P2) is infeasible. Then, $\tau_{\max} = \tau$.

Algorithm 1 Bisection search to optimize system utility.

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Initialize  $\{\alpha_k\}, \{\alpha_k\}, \{b_m\}, \{\beta_m\}$ .
Initialize  $\tau_{\min} = 0$  and  $\tau_{\max}$ , where  $\mathbf{r}(\tau_{\max}) \notin \mathcal{R}$ .
Initialize the convergence threshold  $\Delta$ , and  $t = 0$ .
Initialize  $\tau^*(t) = (\tau_{\max} + \tau_{\min})/2$ .
repeat
  Set  $t = t + 1$ .
  Fix the performance constraints according to  $\mathbf{r}(\tau^*)$  in
  (14). If {The problem (P2) is feasible}
     $\tau^*(t) = [\tau^*(t-1) + \tau_{\max}]/2$ 
     $\tau_{\min} = \tau^*(t)$ 
  Else
     $\tau^*(t) = [\tau^*(t-1) + \tau_{\min}]/2$ .
     $\tau_{\max} = \tau^*(t)$ .
until  $|\tau^*(t) - \tau^*(t-1)| < \Delta$ 
Fix the performance constraints according to  $\mathbf{r}(\tau^*)$ .
Solve the feasible problem (P2) and obtain the optimal
transmit signal covariance matrices  $\{\mathbf{S}_1, \dots, \mathbf{S}_K\}$ 

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a ray from a point within the region meets its upper boundary $\partial^+ \mathcal{R}$ in a unique point. Also, because the utility function $f[\mathbf{r}(\tau)]$ in (9) is strictly increasing, the global optimum to $f[\mathbf{r}(\tau)]$ is attained on the upper boundary of the achievable performance region $\partial^+ \mathcal{R}$, which completes the proof. $\square$

Based on Proposition 1, the specific algorithm to solve the problem (P1) can be given as Algorithm 1. And the bisection search of the optimal solution on the Pareto boundary of achievable performance region is illustrated in Fig. 2.

Remark 4. (Algorithm complexity) The number of iterations in the bisection search scales only logarithmically with the convergence threshold Δ in Algorithm 1, i.e., $N_{\text{iter}} = \lceil \log_2(\tau_{\max}/\Delta) \rceil$. The feasible problem (P2) is semidefinite programming (SDP). It has K matrix variables of size $N \times N$, and $M + K + 3$ linear constraints. Assuming the solution accuracy of the termination is ε , interior point methods will take $O(K^3 N^6 + (M + K)KN^2)$ arithmetic operations at most [27]. Thus, the feasible problem P2 can be solved with a computational complexity that only scales polynomially with the number of antennas N , the number of CUs K , and the dimension of the achievable performance region $K + M$. Thus, the worst-case computational complexity of the problem (P1) is also polynomial in the number of antennas N , the number of CUs K , the dimension of the achievable performance region $K + M$, and the number of iterations $\lceil \log_2(\tau_{\max}/\Delta) \rceil$.

C. Optimality achieved by Single-Stream Transmission

In this section, we will prove that the optimality of the problem (P1) can be achieved by single-stream transmission, which can greatly reduce the searching space for optimal solutions. The rank of $\{\mathbf{S}_k\}$ determines the number of streams transmitted to each CU by the DFRC BS. The rank-one optimality has been well known for the beamforming design for multi-antenna communication system. However, it has never been discussed for the DFRC system. Due to the complex

performance region, the proof of the rank-one optimality becomes more difficult.

Because the optimal solution of the problem (P1) can be obtained by bisection search based on the feasible problem (P2). In order to prove the optimality achieved by single-stream transmission for the problem (P1), it is equivalent to prove that the optimality of the problem (P2) can be achieved by single-stream transmission. The problem (P2) can be rewritten as

$$\begin{aligned}
 \text{P2.1: } \quad & \text{find} \quad \{\mathbf{S}_1, \dots, \mathbf{S}_K\} \\
 & \text{s.t.} \quad \sum_{k=1}^K \text{tr}(\mathbf{S}_k) \leq P_0, \\
 & \text{Radar} \quad \begin{cases} \sum_{k=1}^K \text{tr}(\mathbf{C}_a \mathbf{S}_k) = 0, \\ \sum_{k=1}^K \text{tr}(\mathbf{C}_b \mathbf{S}_k) = 0, \\ \sum_{k=1}^K \text{tr}(\mathbf{C}_m \mathbf{S}_k) \geq \Lambda_m, \forall m, \end{cases} \\
 & \text{Comm.} \quad \frac{\text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_k)}{\Gamma_k} - \sum_{i \neq k} \text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_i) \geq \sigma_z^2, \forall k,
 \end{aligned} \tag{17}$$

where

$$\begin{aligned}
 \mathbf{C}_a &= 0.5 \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) - \mathbf{a}(\theta_a) \mathbf{a}^H(\theta_a), \\
 \mathbf{C}_b &= 0.5 \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) - \mathbf{a}(\theta_b) \mathbf{a}^H(\theta_b), \\
 \mathbf{C}_m &= \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) - \mathbf{a}(\theta_m) \mathbf{a}^H(\theta_m), \forall m.
 \end{aligned} \tag{18}$$

As illustrated in Fig. 2, we consider 3 cases according to different sections of the Pareto boundary, i.e., radar-constrained case, communication-constrained case and both-constrained case.

First, we consider the radar-constrained case, where the communication constraints are inactive. Thus, the problem (P2.1) can be simplified as

$$\begin{aligned}
 \text{P2.2: } \quad & \text{find} \quad \{\mathbf{S}_0\} \\
 & \text{s.t.} \quad \text{tr}(\mathbf{S}_0) \leq P_0, \\
 & \text{Radar} \quad \begin{cases} \text{tr}(\mathbf{C}_a \mathbf{S}_0) = 0, \\ \text{tr}(\mathbf{C}_b \mathbf{S}_0) = 0, \\ \text{tr}(\mathbf{C}_m \mathbf{S}_0) \geq \Lambda_m, \forall m, \end{cases}
 \end{aligned} \tag{19}$$

where $\mathbf{S}_0 = \sum_{k=1}^K \mathbf{S}_k$. We can have the following lemma.

Lemma 3. (Rank-one optimality for radar-constrained case) Consider the optimization problem (P2.2). The problem is linear in $\mathbf{S}_0$. If it is feasible, it always has the optimal solution $\mathbf{S}_0^*$ with $\text{rank}(\mathbf{S}_0^*) = 1$.

Proof. The proof is given in Appendix B. $\square$

Then, we consider the communication-constrained case, where the radar constraints are inactive. Thus, the Problem (P2.1) can be simplified as

$$\begin{aligned}
\text{P2.3: find } & \{\mathbf{S}_1, \dots, \mathbf{S}_K\} \\
\text{s.t. } & \sum_{k=1}^K \text{tr}(\mathbf{S}_k) \leq P_0, \\
\text{SINR } & \frac{\text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_k)}{\Gamma_k} - \sum_{i \neq k} \text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_i) \geq \sigma_z^2, \forall k,
\end{aligned} \tag{20}$$

We can have the following lemma.

Lemma 4. (Rank-one optimality for communication-constrained case) Consider the optimization problem (P2.3). The problem is SDP. If it is feasible, it always has the optimal solution $\{\mathbf{S}_k^*\}$ with $\text{rank}(\mathbf{S}_k^*) = 1, \forall k$.

Proof. The proof is given in Appendix C. $\square$

Finally, we consider the both-constrained case. And we can have the following lemma.

Lemma 5. (Rank-one optimality for both-constrained case) Consider the optimization problem (P2.5). If it is feasible, it always has the optimal solution $\{\mathbf{S}_k^*\}$ with $\text{rank}(\mathbf{S}_k^*) = 1, \forall k$.

Proof. The proof is given in Appendix D. $\square$

Proposition 2. (Optimality achieved by single-stream transmission) It holds for the problem (P1) that if the problem is feasible, there exists single stream optimal solution of signal covariance matrices $\{\mathbf{S}_k^*\}$, i.e., $\text{rank}(\mathbf{S}_k^*) = 1, \forall k$. In other words, it is sufficient to consider signal covariance matrices $\{\mathbf{S}_k\}$ with rank one.

Proof. Based on Lemma 3, Lemma 4 and Lemma 5, it completes the proof. $\square$

IV. HARDWARE-IMPARED OPTIMIZATION

The transmitter of DFRC BS suffers from a variety of impairments that distort the transmitted signal. In the above discussion, we assume that the joint beamforming design is realized over an ideal hardware platform, where the transmitter impairments are omitted. In this section, we provide a detailed analysis of transmitter impairments for the multi-antenna DFRC system. Accordingly, a novel joint beamforming design that is robust to these impairments is formulated and solved.

A. Impairment Model

For downlink communications, the received signal of the CU k with the transmitter impairment can be rewritten as

$$y_k = \mathbf{h}_k^H \left(\sum_{k=1}^K \mathbf{s}_k + \mathbf{z}^{(t)} \right) + z_k, \tag{21}$$

where $\mathbf{z}^{(t)} \in \mathbb{CN}(\mathbf{0}, \mathbf{C}^{(t)})$ is the additive distortions due to the residual impairments at the transmitter.

The combined residual of the impairments are Gaussian or behave as Gaussian when summed up [28]–[30]. Thus, the additive distortions can be well-modeled as circular-symmetric complex Gaussian, i.e., $\mathbf{z}^{(t)} \in \mathbb{CN}(\mathbf{0}, \mathbf{C}^{(t)})$, where $\mathbf{C}^{(t)} = \text{diag}\{c_1^2, \dots, c_N^2\}$. The variance of the additive distortion

is determined by the transmit magnitude of each transmit antenna. Specifically, we have

$$c_n = \psi \left(\sqrt{\text{tr} \left(\mathbf{\Lambda}_n \sum_{k=1}^K \mathbf{S}_k \right)} \right), \tag{22}$$

where $\mathbf{\Lambda}_n = \text{diag}\{0, \dots, 1, \dots, 0\} \in R^{N \times N}$ with only the n -th diagonal element being 1 and the other diagonal elements being 0, $\text{tr} \left(\mathbf{\Lambda}_n \sum_{k=1}^K \mathbf{S}_k \right)$ is the signal power allocated to the n -th transmit antenna, and $\psi(\cdot)$ is the distortion function of impairment.

Remark 5. (Distortion function of impairment) The distortion function of impairment is a monotonically increasing continuous function, which maps the transmit magnitude to the distortion magnitude. It can be modeled as the linear function as $\psi(x) = \kappa x$, where κ is the ratio between the average distortion magnitude and the average transmit magnitude [31]. It can be also modeled as a convex function, e.g., polynomial functions with positive coefficients, due to non-linearities of the power amplifiers, where the distortion magnitude increases faster than the transmit magnitude [32], [33].

The SINR of the CU k in (2) can be rewritten as

$$\gamma'_k = \frac{\mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}{\sum_{i \neq k} \mathbf{h}_k^H \mathbf{S}_i \mathbf{h}_k + \mathbf{h}_k^H \mathbf{C}^{(t)} \mathbf{h}_k + \sigma_z^2}, \tag{23}$$

and the transmit power constraint of the DFRC BS can be given by

$$\text{tr} \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \leq P_0. \tag{24}$$

For the radar function, the power of the probing signal given the angle θ of the target encountering the transmitter impairment can be rewritten as

$$P_{pro}(\theta) = \mathbf{a}^H(\theta) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta). \tag{25}$$

Thus, the PSL of the sidelobe angular θ_m in (5) can be rewritten as

$$\begin{aligned}
\eta'_m &= \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
&\quad - \mathbf{a}^H(\theta_m) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_m).
\end{aligned} \tag{26}$$

And the 3dB main beam width constraint of the DFRC BS in (6) can be rewritten as

$$\begin{aligned}
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_a), \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_b).
\end{aligned} \tag{27}$$

B. Optimization with Impairments

According to the transmitter impairment model, the corresponding system utility problem (P1) can be reformulated as

$$\begin{aligned}
\text{P3: } \max_{\{\mathbf{S}_k\}} \min_{k,m} & \left\{ \frac{\gamma'_k - a_k}{\alpha_k}, \frac{\eta'_m - b_m}{\beta_m} \right\} \\
\text{s.t. } & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \leq P_0, \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_a), \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_b).
\end{aligned} \tag{28}$$

Thus, we can also adopt the bisection search in Proposition 1 to find the optimal solution of the system utility optimization problem by defining a continuous vector-valued function governed by a single scalar $r(\tau)$ in (14), and the corresponding feasible problem is

$$\begin{aligned}
\text{P3.1: } \text{find } & \{\mathbf{S}_1, \dots, \mathbf{S}_K\} \\
\text{s.t. } & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \leq P_0, \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_a), \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_b), \\
& \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_0) - \\
& \mathbf{a}^H(\theta_m) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{C}^{(t)} \right) \mathbf{a}(\theta_m) \geq \Lambda_m, \forall m, \\
& \frac{\mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}{\sum_{i \neq k} \mathbf{h}_k^H \mathbf{S}_i \mathbf{h}_k + \mathbf{h}_k^H \mathbf{C}^{(t)} \mathbf{h}_k + \sigma_z^2} \geq \Gamma_k, \forall k,
\end{aligned} \tag{29}$$

Proposition 3. If $\psi(\cdot)$ is monotonic increasing convex function, the problem (P3.1) can be reformulated into the following convex optimization problem:

$$\begin{aligned}
\text{P3.2: } \text{find } & \{\mathbf{S}_1, \dots, \mathbf{S}_K, d_1, \dots, d_N\} \\
\text{s.t. } & \text{tr} \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \leq P_0, \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_a) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \mathbf{a}(\theta_a), \\
& 0.5\mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \mathbf{a}(\theta_0) \\
& = \mathbf{a}^H(\theta_b) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \mathbf{a}(\theta_b), \\
& \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \mathbf{a}(\theta_0) \\
& - \mathbf{a}^H(\theta_m) \left(\sum_{k=1}^K \mathbf{S}_k + \mathbf{D} \right) \mathbf{a}(\theta_m) \geq \Lambda_m, \forall m, \\
& \frac{\mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}{\sum_{i \neq k} \mathbf{h}_k^H \mathbf{S}_i \mathbf{h}_k + \mathbf{h}_k^H \mathbf{D} \mathbf{h}_k + \sigma_z^2} \geq \Gamma_k, \forall k, \\
& \psi \left(\sqrt{\text{tr} \left(\mathbf{\Lambda}_n \sum_{k=1}^K \mathbf{S}_k \right)} \right) \leq d_n, \forall n,
\end{aligned} \tag{30}$$

where $\mathbf{D} = \text{diag}\{d_1, \dots, d_N\}$ and $\{d_n\}$ are the auxiliary optimization variables.

Proof. The auxiliary variable d_n is defined to describe the transmitter distortion magnitude at the n th antenna, i.e., $d_n = \psi(\sqrt{\text{tr}(\mathbf{\Lambda}_n \sum_{k=1}^K \mathbf{S}_k)})$. By optimizing over $\{d_n\}$, the equality can be relaxed as $\psi(\sqrt{\text{tr}(\mathbf{\Lambda}_n \sum_{k=1}^K \mathbf{S}_k)}) \leq d_n, \forall n$, which are convex constraints if the increasing function $\psi(\cdot)$ is convex. Using the auxiliary variables, the power constraint and the radar constraints are quadratic and convex. Also, the communication constraints can be reformulated as the convex second-order cone. It completes the proof. $\square$

The feasibility problem (P3.2) is a convex quadratically constrained quadratic programming (QCQP) which can be reduced to SDP. Its computational complexity is also polynomial in the number of antennas N , the number of CUs K , the dimension of the achievable performance region $K + M$. Thus, the worst-case computational complexity of the system utility optimization problem (P3) is polynomial in the number of antennas N , the number of CUs K , the dimension of the achievable performance region $K + M$, and the number of iterations $\lceil \log_2(\tau_{\max}/\Delta) \rceil$.

V. SIMULATION RESULTS

In this section, we present some simulation results to illustrate the performance of the joint radar and communication optimization for the multi-antenna DFRC system. The simulation parameters are set as follows unless specified otherwise. The transmit power constraint of DFRC BS is set as $P_0 = 20$ dBm. The interval between adjacent antennas of the DFRC

(a) The optimized beampatterns, $N = 12$, $\Gamma_k = 20$ dB, $K = 0, 1, 2, 3$.

(b) The optimized beampatterns, $N = 16$, $\Gamma_k = 20$ dB, $K = 0, 1, 2, 3$.

Figure 3. The optimized beampatterns with communication constraints.

BS is half-wavelength. We assume that $\sigma_z^2 = 0$ dBm. The channels of CUs are assumed to be Rayleigh fading³, and then the elements of the channel matrices are generated as i.i.d. complex Gaussian random variables $\mathcal{CN}(0, 1)$. The location of the main beam is 0° and the width of main beam is 10° . The average performance is obtained through 10^4 Monte-carlo simulations.

In Fig. 3, the optimized beampatterns with communication constraints are illustrated. Different numbers of antennas $N = 12$ and $N = 16$ are simulated in Fig. 3(a) and (b), respectively. The CUs' channel vectors for Fig. 3(a) are given by $\mathbf{h}_1^T = [-0.21 - 0.75i, 0.58 - 0.021i, -0.15 - 0.06i, -0.73 + 0.029i, 0.096 + 0.16i, -0.67 - 0.17i, -0.38 + 1.6i, -0.34 - 1.2i, -0.14 + 0.3i, -0.18 - 0.86i, 0.87 + 0.23i, -0.31 - 0.46i]$, $\mathbf{h}_2^T = [0.016 + 0.66i, 1.1 + 0.13i, 0.44 +$

³Rayleigh fading is viewed as a reasonable model for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals

Figure 4. The PSLR of beampattern versus different transmit antennas, $N = 10 \sim 20$, $\Gamma_k = 20$ dB, $K = 0, 1, 2, 3$.

$1.1i, 0.67 - 0.52i, 0.36 + 0.3i, -0.11 + 1.4i, 1.2 + 0.24i, -0.5 - 0.42i, -0.19 - 1.2i, -0.75 + 0.047i, -0.16 + 0.77i, -0.11 + 0.18i]$, and $\mathbf{h}_3^T = [0.036 + 0.25i, 0.33 - 1.1i, 0.13 + 0.07i, 0.22 - 0.022i, 0.18 - 0.26i, -0.1 - 1.6i, -0.62 + 0.71i, -0.83 - 0.2i, 1.1 + 0.33i, 1.1 + 0.46i, -1.1 + 0.71i, 0.2 - 0.67i]$. The CUs' channel vectors for Fig. 3(b) are given by $\mathbf{h}_1^T = [h_1^T, -0.18 - 0.93i, -0.88 - 0.039i, -0.36 + 0.25i, -2.1 + 0.17i]$, $\mathbf{h}_2^T = [h_2^T, 0.31 + 0.65i, -0.67 + 0.64i, -0.23 + 0.88i, -0.32 - 0.49i]$, and $\mathbf{h}_3^T = [h_3^T, 0.28 + i, -0.52 + 0.42i, 0 + 0.66i, 0.88 - 0.46i]$. The benchmark of the beam-pattern performance is obtained by radar-only optimization without CUs, i.e., "Radar-only". Also, the joint radar and communication Pareto optimization is provided considering different numbers of the CUs, i.e., "Joint- $K=1$ ", "Joint- $K=2$ ", and "Joint- $K=3$ ". The SINR constraint of each CU is 20 dB. It can be seen that the peak-to-sidelobe ratio (PSLR) of the beampattern decreases with the increase of the number of the CUs. And the increase of the number of antennas at the DFRC BS can improve the performance of beampattern from the viewpoint of PSLR. For example, when $N = 16$, the beampatterns of the "Radar-only" case and "Joint- $K=1$ " case are identical, which falls into the "radar-constrained" case as illustrated in Fig. 2.

The average PSLR performance of radar beampattern is provided with different numbers of DFRC BS antennas $N = 10 \sim 20$ in Fig. 4. The performance of the proposed Pareto optimization, i.e., "Pareto" is compared to that of the existing design in [21] and [22], i.e., "Similarity", which optimizes the similarity using "Radar-only" waveform as a benchmark with the communication SINR constraints. It can be seen that the proposed Pareto optimization provides a better performance. The advantage keeps constant with the number of antennas and increases with the number of the CUs. Then, the PSLR of beampattern almost logarithmically increases with the DFRC BS antennas. And the increasing rate of joint optimization is larger than that of radar-only optimization. For radar-only

Figure 5. The PSLR of beampattern versus different SINR constraints, $N = 16$, $\Gamma_k = 10 \sim 20$ dB, $K = 0, 1, 2, 3$.

optimization, multiple antennas provide waveform diversity to improve the beampattern performance. And for joint optimization, multiple antennas do not only provide transmit diversity, but also provide multiplex gain to avoid the interference among the CUs.

The average PSLR performance of radar beampattern is provided with different SINR constraints $\Gamma_k = 10 \sim 20$ dB in Fig. 5. The performance of the proposed Pareto optimization is also compared to that of the “Similarity” optimization. The proposed Pareto optimization provides a better performance, and the corresponding advantage increases with the SINR constraints. It can be also seen that the PSLR performance almost exponentially decreases with the increase of SINR constraint, and the decreasing rate increases with the number of the CUs. The benchmark PSLR of radar-only beampattern optimization is about 30.5 dB. For $K = 3$, the PSLR only decreases about 0.6 dB when the SINR constraint is 20 dB, which means that we can still push information with an ideal radar.

In Fig. 6, we evaluate the convergence performance of the proposed algorithm based on one channel realization. The SINR of the CU versus the number of iterations is provided. It can be seen that the SINR of the CU vibrates about 6–7 iterations and converges to a fixed value. The non-monotonic convergence behaviour is because we consider both PSLR of the radar beampattern and the SINR of the CU as the performance metric. Thus, the SINR of the CU will not always increase during the iteration. And the converged SINR performance decreases with the increase of the number of the CUs. That is because more CUs will increase the communication SINR constraints for the beampattern optimization.

Considering the impairment of the transmitter, the impairment distortion function is modeled as $\psi(x) = \kappa x$, where κ is the transmitter distortion parameter. The optimized beampattern with different transmitter distortion parameters that $\kappa = 0, 0.03, 0.06, 0.09$ is shown in Fig. 7, where the number of the CUs is set as $K = 3$. It can be seen that the

Figure 6. The SINR of each CU versus different iteration numbers $N = 16$, $\Gamma_k = 20$ dB, $K = 1, 2, 3$.

Figure 7. The optimized beampattern with different transmitter distortion parameters, $N = 16$, $\Gamma_k = 20$ dB, $K = 3$, $\kappa = 0, 0.03, 0.06, 0.09$.

performance of beampattern becomes worse with the increase of the transmitter distortion parameter.

The average SINR of each CU and the average PSLR versus different transmitter distortion parameters are shown in Fig. 8 and Fig. 9, respectively. Different numbers of the CUs are considered, with $K = 1, 2, 3$. It can be seen that both the average SINR of each CU and the average PSLR decrease with the increasing of the transmitter distortion parameter. And the decreasing rate increases with the number of the CUs. For “Radar-only” case, the PSLR is not affected by the transmitter distortion. This suggests that the transmitter impairment has more influence on the joint optimization than on the radar-only optimization.

VI. CONCLUSION

In this paper, we have proposed a Pareto optimization framework for joint beamforming design of the multi-antenna DFRC

Figure 8. The SINR of each CU versus different transmitter distortion parameters, $N = 16$, $\Lambda_m = 30$ dB, $K = 1, 2, 3$, $\kappa = 0 \sim 0.1$.

Figure 9. The PSLR versus different transmitter distortion parameters, $N = 16$, $\Gamma_k = 15$ dB, $K = 0, 1, 2, 3$, $\kappa = 0 \sim 0.1$.

system. By taking into account both radar and communication performance metrics, we have defined the achievable performance region based on the PSLR of the radar and the SINR of each CU. The achievable performance region has been proved to be compact and normal, which guarantees that the global optimum can be obtained on its Pareto boundary given any monotonically increasing system utility. Then, a fairness-profile system utility function has been adopted, which has been shown to be a generalized problem of the max-min PSLR optimization with SINR constraints and the max-min SINR optimization with PSLR constraints. Also, we have provided a bisection search solution, and the corresponding computational complexity is only polynomial in the number of antennas, the dimension of the achievable performance region, and the number of iterations. Further, the robust joint beamforming design encountered with practical impairments has been considered. The corresponding optimization problem

has been formulated and solved under a transmitter impairment model. Numerical results have verified the effectiveness of the proposed design and have illustrated the impact incurred by the transmitter impairments for the multi-antenna DFRC system.

The proposed framework can be applied to any system utility that increases monotonically in the performance of both radar and communication. Future research will focus on waveform designs with further practical system utilities. Other practical radar metrics and communication metrics will be considered. Moreover, since the optimality can be achieved by single-stream transmission, it can greatly reduce the searching space for optimal solutions. Further, some low complexity transmission techniques including closed-form transmitter and codebook design will be studied. Thus, it is easy to implement them on a hardware platform, which can further prove the effectiveness of the proposed methods in realistic scenarios.

APPENDIX A PROOF OF LEMMA 1

First, we prove that $\mathcal{R}$ is compact. According to [34, Theorem 4.14], the continuous mapping of a compact set is also a compact set. Because $\gamma_k(\mathbf{S}_1, \dots, \mathbf{S}_K)$ in (2) and $\eta_m(\mathbf{S}_1, \dots, \mathbf{S}_K)$ in (5) are all continuous functions of $\{\mathbf{S}_1, \dots, \mathbf{S}_K\}$, and the set of feasible transmit strategies

$$\mathcal{S} = \{\mathbf{S}_1, \dots, \mathbf{S}_K\} : (3) \text{ \& } (6) \quad (31)$$

is compact, $\mathcal{R}$ is a compact set.

Then, we prove that $\mathcal{R}$ is normal. Take

$$\mathbf{x} = (\gamma_1, \dots, \gamma_K, \eta_1, \dots, \eta_M) \in \mathcal{R}, \quad (32)$$

and assume that $(\mathbf{S}_1, \dots, \mathbf{S}_K)$ is the feasible strategy that attains this point. We want to show that any $\mathbf{x}' = (\gamma'_1, \dots, \gamma'_K, \eta'_1, \dots, \eta'_M)$ satisfying $\mathbf{x}' \leq \mathbf{x}$ also belongs to $\mathcal{R}$.

We consider the transmit strategy $(p_1 \mathbf{S}_1, \dots, p_K \mathbf{S}_K)$, where $(p_1, \dots, p_K) \in \mathcal{P}$ is the power allocation coefficient. Then, γ_k in (2) can be rewritten as

$$\gamma_k = \frac{p_k \mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}{\sum_{i \neq k} p_i \mathbf{h}_i^H \mathbf{S}_k \mathbf{h}_i + \sigma_z^2}. \quad (33)$$

Thus, we can derive the following equation, i.e.,

$$\mathbf{\Gamma}(\mathbf{A}\mathbf{p} + \mathbf{b}) = \mathbf{p}, \quad (34)$$

where $\mathbf{\Gamma} = \text{diag}\{\gamma_1, \dots, \gamma_K\}$, $\mathbf{p} = [p_1, \dots, p_K]^T$,

$$\mathbf{A} = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1K} \\ a_{21} & 0 & & a_{2K} \\ \vdots & & \ddots & \vdots \\ a_{K1} & & & 0 \end{bmatrix}, \quad (35)$$

with

$$a_{ki} = \frac{\sum_{i \neq k} \mathbf{h}_k^H \mathbf{S}_i \mathbf{h}_k}{\mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}, \quad (36)$$

and $\mathbf{b} = [b_1, \dots, b_K]^T$ with

$$b_k = \frac{\sigma_z^2}{\mathbf{h}_k^H \mathbf{S}_k \mathbf{h}_k}. \quad (37)$$

According to (34), we have that

$$\begin{aligned} \mathbf{p} &= (\mathbf{I} - \Gamma \mathbf{A})^{-1} \Gamma \mathbf{b} \\ &= \left(\mathbf{I} + \sum_{n=1}^{\infty} (\Gamma \mathbf{A})^n \right) \Gamma \mathbf{b}. \end{aligned} \quad (38)$$

Also, with the transmit strategy $(p_1 \mathbf{S}_1, \dots, p_K \mathbf{S}_K)$, η_m in (5) can be rewritten as

$$\begin{aligned} \eta_m &= \mathbf{a}^H(\theta_0) \left(\sum_{k=1}^K p_k \mathbf{S}_k \right) \mathbf{a}(\theta_0) \\ &\quad - \mathbf{a}^H(\theta_m) \left(\sum_{k=1}^K p_k \mathbf{S}_k \right) \mathbf{a}(\theta_m). \end{aligned} \quad (39)$$

Thus, we can derive the following equation, i.e.,

$$\mathbf{C} \mathbf{p} - \mathbf{D} \mathbf{p} = \mathbf{\Lambda}, \quad (40)$$

where $\mathbf{\Lambda} = [\eta_1, \dots, \eta_M]^T$,

$$\mathbf{C} = \begin{bmatrix} c_{1,1} & \dots & c_{K,1} \\ \vdots & & \vdots \\ c_{1,M} & \dots & c_{K,M} \end{bmatrix}, \quad (41)$$

with

$$c_{k,m} = \mathbf{a}^H(\theta_0) \mathbf{S}_k \mathbf{a}(\theta_0), \forall m, \quad (42)$$

and

$$\mathbf{D} = \begin{bmatrix} d_{1,1} & \dots & d_{K,1} \\ \vdots & & \vdots \\ d_{1,M} & \dots & d_{K,M} \end{bmatrix}, \quad (43)$$

with

$$d_{k,m} = \mathbf{a}^H(\theta_m) \mathbf{S}_k \mathbf{a}(\theta_m). \quad (44)$$

According to (40) and $M \geq K$, we have that

$$\begin{aligned} \mathbf{p} &= [\mathbf{I} - \mathbf{C}^- \mathbf{D}]^{-1} \mathbf{C}^- \mathbf{\Lambda} \\ &= \left(\mathbf{I} + \sum_{n=1}^{\infty} (\mathbf{C}^- \mathbf{D})^n \right) \mathbf{C}^- \mathbf{\Lambda}, \end{aligned} \quad (45)$$

where $\mathbf{C}^-$ is the left inverse matrix of $\mathbf{C}$, i.e., $\mathbf{C}^- = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T$.

Above all, according to (38), any increase in γ_k will increase at least one of the power allocation coefficients $(p_1, \dots, p_K)$. Also, according to (45), any increase in η_m will increase at least one of the power allocation coefficients $(p_1, \dots, p_K)$. Finally, since $\gamma_k' \leq \gamma_k, \forall k$ or $\eta_m' \leq \eta_m, \forall m$, we have $\mathbf{p}' \in \mathcal{P}$, i.e., $\mathbf{x}' \in \mathcal{R}$, which completes the proof.

APPENDIX B PROOF OF LEMMA 3

The proof is given following that of [24, Lemma 1.6], i.e., consider the optimization problem

$$\begin{aligned} \max \quad & \text{tr}(\mathbf{A} \mathbf{V}) \\ \text{s.t.} \quad & \text{tr}(\mathbf{B}_m \mathbf{V}) \leq b_m, m = 1, \dots, M \end{aligned} \quad (46)$$

with an arbitrary Hermitian matrix $\mathbf{A}$, Hermitian matrix $\mathbf{B}_m \succeq 0$ that satisfy $\sum_{m=1}^M \mathbf{B}_m \succ 0$, and scalars $b_m \geq 0, \forall m$. This problem is linear in $\mathbf{V}$ (and hence convex) and always has optimal solutions with $\text{rank}(\mathbf{V}) \leq 1$.

As we have assumed the Problem (P2.2) is feasible, without Hermitian matrix $\mathbf{B}_m \succeq 0$ that satisfy $\sum_{m=1}^M \mathbf{B}_m \succ 0$, and scalars $b_m \geq 0, \forall m$, strong duality still holds and the KKT conditions are necessary and sufficient for any optimal solution to the Problem (P2.2). To prove the sufficient of rank-one solutions of $\mathbf{S}_0 = \mathbf{s}_0 \mathbf{s}_0^H$, we consider the following alternative optimization problem

$$\begin{aligned} \text{P2.4:} \quad & \text{find} \quad \{\mathbf{s}_0\} \\ & \text{s.t.} \quad \mathbf{s}_0^H \mathbf{s}_0 \leq P_0, \\ & \text{Radar} \quad \begin{cases} \mathbf{s}_0^H \mathbf{C}_a \mathbf{s}_0 = 0, \\ \mathbf{s}_0^H \mathbf{C}_b \mathbf{s}_0 = 0, \\ \mathbf{s}_0^H \mathbf{C}_m \mathbf{s}_0 \geq \Lambda_m, \forall m \end{cases} \end{aligned} \quad (47)$$

We can show that the problem (P2.4) and the problem (P2.2) have the same dual problem, i.e.,

$$\begin{aligned} \min_{\mu_0, \mu_a, \mu_b, \{\mu_m\}} \quad & \mu_0 P_0 - \sum_{m=1}^M \mu_m \Lambda_m \\ \text{s.t.} \quad & \sum_{m=1}^M \mu_m \mathbf{C}_m + \mu_a \mathbf{C}_a + \mu_b \mathbf{C}_b - \mu_0 \mathbf{I} \succeq 0 \end{aligned} \quad (48)$$

where $\mu_0, \mu_a, \mu_b, \mu_m, \forall m$ are the dual variables associated with the transmit power constraint and the radar constraints, respectively. Also, every optimal solution to the Problem (P2.4) also satisfies the KKT conditions for the Problem (P2.2), which are omitted for brevity, and thus is optimal for the Problem (P2.2). Because the constraint functions of the Problem (P2.4) are convex, the KKT conditions are necessary, which completes the proof.

APPENDIX C PROOF OF LEMMA 4

The Lemma can be proved by applying the results in [35, Lemma 1], i.e., consider a separable SDP in the following form:

$$\begin{aligned} \min_{\{\mathbf{X}_l\}} \quad & \sum_{l=1}^L \text{tr}(\mathbf{B}_l \mathbf{X}_l) \\ \text{s.t.} \quad & \sum_{l=1}^L \text{tr}(\mathbf{A}_{ml} \mathbf{X}_l) \triangleright_m b_m, m = 1, \dots, M \\ & \mathbf{X}_l \succ 0, l = 1, \dots, L \end{aligned} \quad (49)$$

where $\mathbf{B}_l, \mathbf{A}_{ml}, \forall l, m$ are Hermitian matrices (not necessarily positive semidefinite), $b_m \in \mathbb{R}, \triangleright_m \in \{\leq, \geq, =\}, \forall m$, and $\mathbf{X}_l, \forall l$ are Hermitian matrices. Suppose that the above problem

is feasible. Then it always has an optimal solution $\{\mathbf{X}_l^*\}$ such that $\sum_{l=1}^L [\text{rank}(\mathbf{X}_l^*)]^2 \leq M$.

According to this result, the optimal solutions for the problem (P2.3) satisfy $\sum_{l=1}^K [\text{rank}(\mathbf{S}_k^*)]^2 \leq K + 1$. And it can be verified that due to the SINR constraint of each CU, $\mathbf{S}_k^* \neq \mathbf{0}$. Thus, one has $\text{rank}(\mathbf{S}_k^*) = 1$, which completes the proof.

APPENDIX D PROOF OF LEMMA 5

In order to prove rank-one optimality for this case, we assume the transmit signal of the DFRC BS can be rewritten as $\mathbf{x} = \sum_{k=1}^K \mathbf{s}_k + \mathbf{s}_0$, where $\mathbf{s}_0$ is the dedicated radar signal to guarantee the radar constraints and is independent of the data signals of the CUs. Thus, the transmit strategy becomes $\{\mathbf{S}_0, \mathbf{S}_1, \dots, \mathbf{S}_K\}$, where $\mathbf{S}_0 = \mathbf{E}(\mathbf{s}_0 \mathbf{s}_0^H)$. Then, the Problem (P2.1) can be rewritten as

$$\begin{aligned} \text{P2.5: find } & \{\mathbf{S}_0, \mathbf{S}_1, \dots, \mathbf{S}_K\} \\ \text{s.t. } & \sum_{k=1}^K \text{tr}(\mathbf{S}_k) + \text{tr}(\mathbf{S}_0) \leq P, \\ & \begin{cases} \sum_{k=1}^K \text{tr}(\mathbf{C}_a \mathbf{S}_k) + \text{tr}(\mathbf{C}_a \mathbf{S}_0) = 0 \\ \sum_{k=1}^K \text{tr}(\mathbf{C}_b \mathbf{S}_k) + \text{tr}(\mathbf{C}_b \mathbf{S}_0) = 0, \\ \sum_{k=1}^K \text{tr}(\mathbf{C}_m \mathbf{S}_k) + \text{tr}(\mathbf{C}_m \mathbf{S}_0) \geq \Lambda_m, \forall m \end{cases} \\ & \text{Radar} \\ & \begin{cases} \frac{\text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_k)}{\Gamma_k} - \sum_{i \neq k} \text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_i) - \\ \text{tr}(\mathbf{h}_k \mathbf{h}_k^H \mathbf{S}_0) \geq \sigma_z^2, \forall k \end{cases} \\ & \text{Comm.} \end{aligned} \quad (50)$$

It can be easily seen that the dedicated radar signal cannot improve the radar performance but incurs additional interference to the CUs. Thus, the optimal $\mathbf{S}_0^* = \mathbf{0}$.

Because the Problem (P2.5) is convex, it satisfies the Slater's condition and thus has a zero duality gap [36]. Its Lagrangian can be given by

$$\mathcal{L} = \mu_0 P_0 - \sum_{k=1}^K \lambda_k \sigma_z^2 - \sum_{m=1}^M \mu_m \Lambda_m + \sum_{k=1}^K \text{tr}(\Phi_k \mathbf{S}_k) + \text{tr}(\Phi_0 \mathbf{S}_0), \quad (51)$$

where

$$\begin{aligned} \Phi_k &= \Pi + \frac{\lambda_k \mathbf{h}_k \mathbf{h}_k^H}{\Gamma_k} - \sum_{i \neq k} \lambda_i \mathbf{h}_i \mathbf{h}_i^H - \mu_0 \mathbf{I} \\ \Phi_0 &= \Pi - \sum_{i=1}^K \lambda_i \mathbf{h}_i \mathbf{h}_i^H - \mu_0 \mathbf{I}, \\ \Pi &= \mu_a \mathbf{C}_a + \mu_b \mathbf{C}_b + \sum_{m=1}^M \mu_m \mathbf{C}_m \end{aligned} \quad (52)$$

$\mu_0, \mu_a, \mu_b, \mu_m, \forall m, \lambda_k, \forall k$ are the dual variables associated with the transmit power constraint, the radar constraints and the communication constraints, respectively. Then, the dual problem of the problem (P2.5) can be given by

$$\begin{aligned} \text{P2.6: } & \min_{\mu_0, \mu_a, \mu_b, \{\mu_m\}, \{\lambda_k\}} \mu_0 P_0 - \sum_{k=1}^K \lambda_k \sigma_z^2 \\ \text{s.t. } & \Phi_0 \prec \mathbf{0}, \Phi_k \prec \mathbf{0}, \forall k \end{aligned} \quad (53)$$

Based on the optimal solutions of the problem (P2.5), the resulting $\Phi_0, \{\Phi_k\}$ are $\Phi_0^*, \{\Phi_k^*\}$. Consider the optimal solutions of the problem (P2.5) $\mathbf{S}_0^*, \{\mathbf{S}_k^*\}$, the following complementary slackness conditions should be satisfied, i.e.,

$$\text{tr}(\Phi_0^* \mathbf{S}_0^*) = 0, \quad (54)$$

and

$$\text{tr}(\Phi_k^* \mathbf{S}_k^*) = 0, \forall k \quad (55)$$

Consider (54), if $\text{rank}(\Phi_0^*) \neq N$, we can find $\mathbf{S}_0^* \neq \mathbf{0}$ satisfying $\text{tr}(\Phi_0^* \mathbf{S}_0^*) = 0$, which induces a contradiction that $\mathbf{S}_0^* = \mathbf{0}$. Thus, $\text{rank}(\Phi_0^*) = N$. Furthermore,

$$\Phi_k^* = \Phi_0^* + \left(1 + \frac{1}{\Gamma_k}\right) \lambda_k^* \mathbf{h}_k \mathbf{h}_k^H, \quad (56)$$

one has that

$$\begin{aligned} \text{rank}(\Phi_k^*) &\geq \text{rank}(\Phi_0^*) - \text{rank}\left[\left(1 + \frac{1}{\Gamma_k}\right) \lambda_k^* \mathbf{h}_k \mathbf{h}_k^H\right] \\ &= N - 1. \end{aligned} \quad (57)$$

Consider (55) and $\mathbf{S}_k^* \neq \mathbf{0}$, one has that

$$\text{rank}(\Phi_k^*) \leq N - \text{rank}(\mathbf{S}_k^*) \leq N - 1. \quad (58)$$

Thus, one has $\text{rank}(\Phi_k^*) = N - 1$. Because $\mathbf{S}_k^* \neq \mathbf{0}$ and $\text{tr}(\Phi_0^* \mathbf{S}_0^*) = 0$, one has $\text{rank}(\mathbf{S}_k^*) = 1$, which completes the proof.

REFERENCES

- [1] H. Griffiths, L. Cohen, S. Watts, E. Mokole, C. Baker, M. Wicks, and S. Blunt, "Radar spectrum engineering and management: Technical and regulatory issues," *Proc. IEEE*, vol. 103, no. 1, pp. 85–102, Jan 2015.
- [2] J. Zhang, M. Wang, M. Hua, T. Xia, W. Yang, and X. You, "LTE on license-exempt spectrum," *IEEE Commun. Surv. Tutor.*, vol. 20, no. 1, pp. 647–673, Firstquarter 2018.
- [3] J. Choi, V. Va, N. Gonzalez-Prelcic, R. Daniels, C. R. Bhat, and R. W. Heath, "Millimeter-wave vehicular communication to support massive automotive sensing," *IEEE Commun. Mag.*, vol. 54, no. 12, pp. 160–167, December 2016.
- [4] W. Roh, J. Seol, J. Park, B. Lee, J. Lee, Y. Kim, J. Cho, K. Cheun, and F. Aryanfar, "Millimeter-wave beamforming as an enabling technology for 5G cellular communications: theoretical feasibility and prototype results," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 106–113, February 2014.
- [5] R. Saruthirathanaworakun, J. M. Peha, and L. M. Correia, "Opportunistic sharing between rotating radar and cellular," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 10, pp. 1900–1910, November 2012.
- [6] A. Khawar, A. Abdelhadi, and C. Clancy, "Target detection performance of spectrum sharing MIMO radars," *IEEE Sens. J.*, vol. 15, no. 9, pp. 4928–4940, Sep. 2015.
- [7] A. Babaei, W. H. Tranter, and T. Bose, "A nullspace-based precoder with subspace expansion for radar/communications coexistence," in *2013 IEEE Global Communications Conference (GLOBECOM)*, Dec 2013, pp. 3487–3492.
- [8] B. Li, A. P. Petropulu, and W. Trappe, "Optimum co-design for spectrum sharing between matrix completion based MIMO radars and a MIMO communication system," *IEEE Trans. Signal Process.*, vol. 64, no. 17, pp. 4562–4575, Sep. 2016.

- [9] F. Liu, C. Masouros, A. Li, T. Ratnarajah, and J. Zhou, "MIMO radar and cellular coexistence: A power-efficient approach enabled by interference exploitation," *IEEE Trans. Signal Process.*, vol. 66, no. 14, pp. 3681–3695, July 2018.
- [10] F. Liu, C. Masouros, A. Li, and T. Ratnarajah, "Robust MIMO beamforming for cellular and radar coexistence," *IEEE Wirel. Commun. Lett.*, vol. 6, no. 3, pp. 374–377, June 2017.
- [11] A. R. Chiriyath, B. Paul, G. M. Jacyna, and D. W. Bliss, "Inner bounds on performance of radar and communications co-existence," *IEEE Trans. Signal Process.*, vol. 64, no. 2, pp. 464–474, Jan 2016.
- [12] L. Han and K. Wu, "Joint wireless communication and radar sensing systems—state of the art and future prospects," *IET Microw. Antennas Propag.*, vol. 7, no. 11, pp. 876–885, 2013.
- [13] D. Garmatyuk, J. Schuerger, and K. Kauffman, "Multifunctional software-defined radar sensor and data communication system," *IEEE Sens. J.*, vol. 11, no. 1, pp. 99–106, Jan 2011.
- [14] C. Sturm and W. Wiesbeck, "Waveform design and signal processing aspects for fusion of wireless communications and radar sensing," *Proc. IEEE*, vol. 99, no. 7, pp. 1236–1259, July 2011.
- [15] J. Li, L. Xu, P. Stoica, K. W. Forsythe, and D. W. Bliss, "Range compression and waveform optimization for MIMO radar: A Cramér–Rao bound based study," *IEEE Trans. Signal Process.*, vol. 56, no. 1, pp. 218–232, Jan 2008.
- [16] G. Cui, X. Yu, V. Carotenuto, and L. Kong, "Space-time transmit code and receive filter design for colocated MIMO radar," *IEEE Trans. Signal Process.*, vol. 65, no. 5, pp. 1116–1129, March 2017.
- [17] S. Herbert, J. R. Hopgood, and B. Mulgrew, "MMSE adaptive waveform design for active sensing with applications to MIMO radar," *IEEE Trans. Signal Process.*, vol. 66, no. 5, pp. 1361–1373, March 2018.
- [18] —, "Computationally simple MMSE (a-optimal) adaptive beam-pattern design for MIMO active sensing systems via a linear-gaussian approximation," *IEEE Trans. Signal Process.*, vol. 66, no. 18, pp. 4935–4945, Sep. 2018.
- [19] B. Tang and J. Li, "Spectrally constrained MIMO radar waveform design based on mutual information," *IEEE Trans. Signal Process.*, vol. 67, no. 3, pp. 821–834, Feb 2019.
- [20] A. Hassanien, M. G. Amin, Y. D. Zhang, and F. Ahmad, "Dual-function radar-communications: Information embedding using sidelobe control and waveform diversity," *IEEE Trans. Signal Process.*, vol. 64, no. 8, pp. 2168–2181, April 2016.
- [21] F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "MU-MIMO communications with MIMO radar: From co-existence to joint transmission," *IEEE Trans. Wirel. Commun.*, vol. 17, no. 4, pp. 2755–2770, April 2018.
- [22] F. Liu, L. Zhou, C. Masouros, A. Li, W. Luo, and A. Petropulu, "Toward dual-functional radar-communication systems: Optimal waveform design," *IEEE Trans. Signal Process.*, vol. 66, no. 16, pp. 4264–4279, Aug 2018.
- [23] F. Liu, C. Masouros, A. Petropulu, H. Griffiths, and L. Hanzo, "Joint radar and communication design: Applications, state-of-the-art, and the road ahead," *IEEE Trans. Commun.*, pp. 1–1, 2020.
- [24] E. Björnson, E. Jorswieck *et al.*, "Optimal resource allocation in coordinated multi-cell systems," *Foundations and Trends® in Communications and Information Theory*, vol. 9, no. 2–3, pp. 113–381, 2013.
- [25] E. Björnson, G. Zheng, M. Bengtsson, and B. Ottersten, "Robust monotonic optimization framework for multicell MISO systems," *IEEE Trans. Signal Process.*, vol. 60, no. 5, pp. 2508–2523, 2012.
- [26] E. Björnson, N. Jalden, M. Bengtsson, and B. Ottersten, "Optimality properties, distributed strategies, and measurement-based evaluation of coordinated multicell OFDMA transmission," *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 6086–6101, 2011.
- [27] Y. Ye, *Interior point algorithms: theory and analysis*. John Wiley & Sons, 2011, vol. 44.
- [28] X. Zhang, M. Matthaiou, M. Coldrey, and E. Björnson, "Impact of residual transmit RF impairments on training-based MIMO systems," *IEEE Trans. Commun.*, vol. 63, no. 8, pp. 2899–2911, 2015.
- [29] Q. Zhang, T. Q. S. Quek, and S. Jin, "Scaling analysis for massive MIMO systems with hardware impairments in Rician fading," *IEEE Trans. Wirel. Commun.*, vol. 17, no. 7, pp. 4536–4549, 2018.
- [30] R. Nie, L. Chen, N. Zhao, Y. Chen, F. R. Yu, and G. Wei, "Relaying systems with reciprocity mismatch: Impact analysis and calibration," *IEEE Trans. Commun.*, pp. 1–1, 2020.
- [31] H. Holma and A. Toskala, *LTE for UMTS: Evolution to LTE-advanced*. John Wiley & Sons, 2011.
- [32] Y. Wu, Y. Gu, and Z. Wang, "Efficient channel estimation for mmwave MIMO with transceiver hardware impairments," *IEEE Trans. Veh. Technol.*, vol. 68, no. 10, pp. 9883–9895, 2019.
- [33] G. Ding, X. Gao, Z. Xue, Y. Wu, and Q. Shi, "Massive MIMO for distributed detection with transceiver impairments," *IEEE Trans. Veh. Technol.*, vol. 67, no. 1, pp. 604–617, 2018.
- [34] W. Rudin *et al.*, *Principles of mathematical analysis*. McGraw-hill New York, 1964, vol. 3.
- [35] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 664–678, 2010.
- [36] S. Boyd and L. Vandenberghe, *Convex Optimization*. USA: Cambridge University Press, 2004.