The Origins of Firm Heterogeneity: A Production Network Approach

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We explore firm size heterogeneity in production networks. In comprehensive data for Belgium, firms with more customers have higher total sales but lower sales and lower market shares per customer.

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Downstream factors, especially the number of customers, explain the vast majority of firm size dispersion. We rationalize these facts with a model of network formation and two-dimensional firm heterogeneity. Higher productivity generates more matches and larger market shares among customers. Higher relationship capability generates more customers and higher sales. Model estimates suggest a strong negative correlation between productivity and relationship capability and potentially large welfare gains from improving relationship capability.

I. Introduction

Even within narrowly defined industries, there is massive dispersion in firm outcomes such as sales, employment, and productivity. In Belgium, a firm at the 90th percentile of the size distribution has turnover 32 times greater than a firm at the 10th percentile within the same industry.[1] Understanding the origins of firm size heterogeneity is a fundamental question in economics and has important micro- and macroeconomic implications. At the micro level, bigger firms perform systematically better along many dimensions, including survival, innovation, and participation in international trade (e.g., Bernard et al. 2012). At the macro level, the skewness and granularity of the firm size distribution affect aggregate productivity, the welfare gains from trade, and the impact of idiosyncratic and systemic shocks (e.g., Pavcnik 2002; Gabaix 2011; di Giovanni, Levchenko, and Mejean 2014; Melitz and Redding 2015; Gaubert and Itskhoki 2021).

This paper examines the firm size distribution in a production network with firm heterogeneity and buyer-supplier connections.[2] The basic premise of the analysis is intuitive: firms can be large because (1) they have inherently attractive capabilities, such as productivity or quality; (2) they have low marginal costs from matches with more or better suppliers; and/or (3) they have higher sales due to more or bigger buyers. Higher-order effects can also be important, as the customers of customers also ultimately affect firm economic outcomes.

While research has made progress in identifying underlying firm-specific supply- and demand-side factors driving firm size (e.g., Hottman, Redding, and Weinstein 2016), much less is known about the role of buyer-supplier

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[1] The p90/p10 ratio is averaged across all NACE four-digit Belgian industries in 2014. Hottman, Redding, and Weinstein (2016) report even larger sales dispersion across firms using barcode data on US retail sales, and Autor et al. (2020) find that 52.6% of sales in the average US industry is accounted for by the 20 largest firms, representing less than half a percent of the total number of firms.

[2] Throughout the paper, firm size refers to sales or turnover, and we use both terms interchangeably.

linkages in production networks. In particular, the focus on the supply side has been on heterogeneity in either firm productivity (e.g., Jovanovic 1982; Hopenhayn 1992; Melitz 2003; Luttmer 2007) or organizational capital (e.g., Prescott and Visscher 1980; Luttmer 2011), whereas work on the demand side has centered on final consumer preferences (e.g., Fitzgerald, Haller, and Yedid-Levi 2016) or firm-specific demand shocks (e.g., Foster, Haltiwanger, and Syverson 2016). To the extent that the literature has considered firm-to-firm trade, it has typically remained anchored in one-sided heterogeneity by assuming that firms source inputs from anonymous upstream suppliers or sell to anonymous downstream buyers, without accounting for the heterogeneity of all trade partners in the production network.

Our contribution is threefold. First, we document new stylized facts about production networks using data on the universe of firm-to-firm domestic transactions in Belgium, and present the first extensive analysis of how upstream, downstream and final demand heterogeneity translate into firm size heterogeneity. Second, we provide a theoretical framework for an endogenous production network with firm heterogeneity in both productivity/quality and relationship capability. Third, we estimate the parameters of the model using simulated method of moments (SMM) to explore the relative importance of the two dimensions of firm heterogeneity and their interaction across firms.

We report three stylized facts from the production network data that motivate the subsequent analysis and model. First, the distributions of total sales and buyer-supplier connections exhibit high dispersion. The enormous dispersion of sales across firms is also found in the production network in terms of the number of links to buyers and suppliers. Second, firms with more customers have higher sales but lower average sales per customer and lower market shares (shares of input purchases) among their customers. Finally, there is negative degree assortativity between buyers and suppliers: sellers with more customers match with customers who have fewer suppliers on average.

Taken together, these facts both confirm intuition and challenge existing models of firm heterogeneity. The large variation in sales across firms within an industry is intuitively related to variation in the number of customers: large firms have more customers. However, firms with more customers have lower average sales per customer, connect with less well-connected customers, and account for a smaller share of those customers' input purchases. Models that emphasize heterogeneity in productivity across firms cannot explain all three facts simultaneously. In particular, such models imply that firms with more customers should also sell more to each of their customers: they should have larger—not smaller—market shares.

A key advantage of the Belgian production network data is that sales from firm i to j can be decomposed into seller-, buyer- and match-specific components.[3] This allows us to understand how much of the value of pairwise sales is due to the seller, the buyer, or the match itself. High dispersion in seller effects means that firms vary in how much they sell to their customers, controlling for demand by those customers; that is, firms differ in their average market share across customers. Conversely, high dispersion in buyer effects means that some firms match with large customers while others do not, leading to larger sales even as the average market share remains the same. Given estimates of these fixed effects, the total sales of a firm can be decomposed into three distinct factors: (1) an upstream component that captures the firm's ability to obtain large market shares across its customers, (2) a downstream component that captures the firm's ability to attract many and/or large customers, and (3) a final demand component that captures the firm's ability to sell relatively more outside the domestic network to final consumers at home or to foreign customers.

The results are striking: 81% of the variation in firm sales within narrowly defined industries (four-digit code according to the Statistical Classification of Economic Activities in the European Community [Nomenclature Statistique des Activités Économiques dans la Communauté Européenne; NACE]) is associated with the downstream component, while the upstream component contributes only 18%. The variation in firm size is largely unrelated to the relative importance of sales to final demand (1%). These findings imply that trade in intermediate goods and the number of firm-to-firm connections are essential to understanding firm performance and, consequently, aggregate outcomes.

Motivated by these stylized facts and decomposition results, we develop a quantitative general equilibrium model of firm-to-firm trade. In the model, firms use a constant elasticity of substitution (CES) production technology that combines labor and inputs from upstream suppliers. Firms sell their output to final consumers and to domestic producers. Firms differ in two dimensions—productivity and relationship capability—defined as production efficiency and (the inverse of) the fixed cost of matching with a customer, respectively. The two dimensions are potentially correlated. Suppliers match with customers if the gross profits of the match exceed the supplier-specific fixed matching cost. Marginal costs, employment, prices, and sales are endogenous outcomes because they depend on the outcomes of all other firms in the economy. A link between two firms increases the total sales of both the seller and the buyer; for the seller, this occurs mechanically because it gains a customer, while for the buyer, this arises because a larger supplier base lowers the marginal cost of production.

[3] Our approach is similar to the intertemporal analysis of matched employer-employee data (e.g., Abowd, Kramarz, and Margolis 1999).

The model equilibrium involves three nested fixed points. A backward fixed point determines the price of a firm as a function of its marginal cost, which in turn depends on the prices of its suppliers. A forward fixed point pins down the sales of a firm as a function of demand by its customers, which in turn depends on their sales to their customers. A link function fixed point relates the likelihood of a link to the profit from the match, which is itself a function of the network structure. Jointly, these determine the endogenous structure of the network in terms of connections, the value of bilateral sales for each link, and the total sales of the firm.

We estimate the model parameters using SMM. These parameters comprise the variance of productivity, the mean and variance of relationship capability, and the correlation between productivity and relationship capability across firms. The results reveal high dispersion in relationship capability across firms and a strong negative correlation between the two firm characteristics. Firms with higher productivity have lower relationship capability. This negative relationship is crucial for matching the stylized fact that firms with more customers have lower average sales per customer and lower market share in those customers. A canonical model without this negative relationship instead produces a strongly positive relationship between the number of customers and average sales (or average market share). The model does well at matching untargeted moments, such as the variances of total sales and value added. In addition, it does well at matching moments on the upstream side of the production network, including the variances of the number of suppliers and total input purchases. Importantly, both dimensions of firm heterogeneity are necessary to match the data: shutting down one at a time results in poor model fit, including the inability to replicate the negative relationship between the number of customers and average sales per customer.

Why are the two dimensions of firm capabilities negatively related? While it is outside the scope of this paper to offer a fully specified explanation, a possible answer is imperfect reward and incentive systems. For example, Holmstrom and Milgrom (1991) offer a multitasking theory where agents will focus their effort on observable and rewarded tasks at the expense of other tasks. If the principal rewards only one dimension of performance (e.g., finding customers), this might reduce other dimensions of firm performance, such as product quality or productivity.[4]

Finally, we use the estimated model to quantify the role of firm heterogeneity in productivity and relationship capability for aggregate outcomes. Specifically, we cut relationship costs across all firms by 50%. We do so in two versions of our model: in the baseline estimated model and then in a restricted model with no correlation between productivity and relationship

[4] In a recent study, Hong et al. (2018) find that workers trade off quality for quantity under a bonus scheme that rewards quantity (but not quality), as predicted by multitasking theory.

costs. The counterfactual reveals that the real wage gains from lowering relationship costs are substantial and much larger in our baseline model compared with the model with no correlation structure. The reason is that the fall in relationship costs benefits the high-productivity firms relatively more in the baseline model, as they are more constrained by high relationship costs.

This paper contributes to several strands of literature. Most directly, the paper adds to the large literature on the extent, causes, and consequences of firm size heterogeneity. The vast dispersion in firm size has long been documented, with recent emphasis on the skewness and granularity of firms at the top end of the size distribution (e.g., Gibrat 1931; Syverson 2011). This interest is motivated by the superior growth and profit performance of bigger firms at the micro level as well as by the implications of firm heterogeneity and superstar firms for aggregate productivity, growth, international trade, and adjustment to various shocks (e.g., Gabaix 2011; Bernard et al. 2012; Freund and Pierola 2015; Oberfield 2018; Gaubert and Itskhoki 2021).

Traditionally, this literature has analyzed own firm characteristics on the supply side as the driver of firm size heterogeneity. The evidence indicates an important role for firms' production efficiency, management ability, and capacity for quality products (e.g., Jovanovic 1982; Hopenhayn 1992; Melitz 2003; Sutton 2007; Bloom et al. 2017; Bender et al. 2018). Recent work has built on this by also considering the role of either upstream suppliers or downstream demand heterogeneity but not both. Results suggest that access to inputs from domestic and foreign suppliers matters for firms' marginal costs and product quality and thereby performance (e.g., Goldberg et al. 2010; Bøler, Moxnes, and Ulltveit-Moe 2015; Manova, Wei, and Zhang 2015; Antràs, Fort, and Tintelnot 2017; Fieler, Eslava, and Xu 2018; Bernard et al. 2019; Boehm and Oberfield 2020), while final consumer preferences affect sales on the demand side (e.g., Fitzgerald, Haller, and Yedid-Levi 2016; Foster, Haltiwanger, and Syverson 2016).

By contrast, we provide a comprehensive treatment of both own firm characteristics and production network features on both the upstream and the downstream sides. The paper is related to Hottman, Redding, and Weinstein (2016), who also find that demand-side factors—such as variation in firm appeal and product scope—rather than prices (marginal costs) drive firm size dispersion. However, as these authors do not observe the production network, they cannot distinguish between the impact of serving more customers, attracting better customers, and selling large amounts to (potentially few) customers. Since they have no information on the supplier margin, they also cannot separate own from network supply factors. On the other hand, while rich in network features, our data do not provide information on prices or products and thus do not allow for a comparable decomposition into firm appeal and product scope.

The paper also adds to a growing literature on buyer-supplier production networks (for a recent survey, see Bernard and Moxnes 2018). Bernard, Moxnes, and Saito (2019) study the impact of domestic supplier connections on firms' marginal costs and performance in Japan, whereas Eaton et al. (2016), Bernard, Moxnes, and Ulltveit-Moe (2018), and Eaton, Kortum, and Kramarz (2018) explore the matching of exporters and importers using data on firm-to-firm trade transactions for United States–Colombia, Norway, and France, respectively. Using the Belgian production network data, Magerman et al. (2016) analyze the contribution of the network structure of production to aggregate fluctuations, while Tintelnot et al. (2021) examine the effect of trade on the domestic production network. In recent work, Acemoglu et al. (2012), Lim (2018), Baqaee and Farhi (2019), and Baqaee and Farhi (2020) consider how microeconomic shocks shape macroeconomic outcomes in networked environments. Our work departs from this literature by focusing on the dispersion of firm outcomes and their relationship to upstream and downstream features of the network.

The rest of the paper is organized as follows. Section II introduces the data and presents stylized facts about the Belgian production network. Section III decomposes firm sales into upstream, downstream, and final demand components. Section IV develops a theoretical framework with heterogeneous firms and endogenous matching in a production network. Section V estimates the parameters of the model and quantitatively assesses the two dimensions of firm heterogeneity. Section VI concludes.

II. Data and Stylized Facts

A. *Data Sources and Preparation*

The empirical analysis draws from four micro-level data sets on Belgian firms and their sales relationships administered at the National Bank of Belgium (NBB). These include (1) the universe of domestic firm-to-firm relationships within Belgium from the NBB B2B Transactions Dataset, (2) standard firm characteristics from the annual accounts, (3) additional information on sales and inputs from the value-added tax (VAT) declarations, and (4) the sector of main economic activity and postal code of the firm from the Crossroads Bank of Enterprises. These data sets cover firms and their transactions in all economic activities over the years 2002–14. Firms are identified by a unique enterprise number, which allows unambiguous merging across all data sets. A detailed description of the construction of the data sets is also provided in online appendix A.

Central to this paper is the NBB B2B Transactions Dataset (Dhyne, Magerman, and Rubinova 2015), which is used to construct the production network of Belgian firms. It is based on the VAT listings that all VAT liable firms have to submit to the VAT authorities at the end of each calendar year.[5] An observation in this data set reports the yearly sales value in euro of firm i selling to firm j within Belgium (excluding VAT). Sales values are the sum of invoices from i to j, which implies that we observe the value but not the content of the flows. All yearly sales values of at least 250 euro have to be reported, and pecuniary sanctions by the tax authorities on late or erroneous reporting ensure a very high quality of the data.

The other data sets contain information on firm characteristics. From firms' annual accounts, we retain information on firm-level sales, input expenditures, employment, and labor costs. Flow variables are annualized pro rata from fiscal years to calendar years to match the reporting in calendar years in the NBB B2B data set. All firm have to report employment and labor costs. Depending on size thresholds, small firms can submit abbreviated annual accounts, which omits information on turnover and inputs expenditures.[6] We fill in these values for small firms using the VAT declarations, which contain information on sales and inputs for all VAT liable firms.[7] The main economic activity of the firm is extracted at the NACE four-digit level (harmonized over time to the NACE revision 2 [2008] version) from the Crossroads Bank of Enterprises. To control for geographical heterogeneity, we also extract the postal code of the firm from this data set.

We include firms with at least one full-time equivalent to avoid potential issues with shell or management companies. We keep only the set of firms that are active in the production network. The main analysis is within NACE four-digit industries on the basis of the seller's industry. To avoid potential incidental parameter problems (e.g., estimates of means, fixed effects), we drop sectors with fewer than five observations. Results are robust to changing this cutoff. We calculate final demand as a firm's turnover minus its B2B sales to other enterprises in the domestic network. Consistent with national accounting standards, final demand thus contains both sales to final consumers at home and exports.

We use the 2014 cross section for the main analysis and provide additional results in online appendixes A–E. The main sample for 2014 ultimately contains 94,147 firms and all their production network connections.

[5] VAT listings templates are available at https://www.dropbox.com/s/kxvm49bjrqyi7cl/VAT%20listings%20(nl).pdf?dl=0 (Dutch) and https://www.dropbox.com/s/s00x7shubts6kgj/VAT%20listings%20(fr).pdf?dl=0 (French).

[6] Size criteria are available at https://www.nbb.be/en/central-balance-sheet-office/drawing/size-criteria/size-criteria-companies.

[7] VAT declaration templates are available at https://www.dropbox.com/s/kshw7ajdf67s7y2/VAT%20declaration%20(nl).pdf?dl=0 (Dutch) and https://www.dropbox.com/s/x2t9fropx90827n/VAT%20declaration%20(fr).pdf?dl=0 (French).

B. *Stylized Facts*

We start by documenting three empirical facts about firm size, firm-to-firm relationships, and their correlations in the Belgian production network. These facts point to inconsistencies with standard models of one-dimensional firm heterogeneity and will motivate the foundations of our model in section IV.

FACT 1. The distributions of firm sales and supplier-buyer connections are highly dispersed.

Even within narrowly defined industries, firms show significant heterogeneity along several dimensions.[8] Figure 1 documents the distributions of firm size and the number of customers and suppliers for Belgian firms. All variables are demeaned at the four-digit NACE industry.[9] Figure 1*A* shows the firm size distribution expressed in total sales value. As is well known, the distribution spans several orders of magnitude: relative to the average firm in its industry, some firms are up to four orders of magnitude larger, and they coexist with very small firms several orders of magnitude smaller than the average. Figure 1*B* reports the distribution of the number of customers of these firms. Here as well, firms can have over 1,000 times as many customers as their industry average, again coexisting with firms that have few customers. Similarly, in figure 1*C*, the number of suppliers is shown. While less excessive, again this distribution spans several orders of magnitude. Online appendix B reports additional moments on both demeaned and raw variables.

FACT 2. Firms with more customers have higher sales but lower sales per customer.

A sharp pattern in the data is that firms with more customers have higher sales but lower sales per customer. Figure 2*A* displays the binned scatterplot of firm sales to other producers in the network (y-axis) against the number of customers (x-axis) on a log-log scale. Both variables are demeaned by their four-digit industry average, and observations are binned into 20 quantiles. The elasticity of sales with respect to the number of buyers is 0.77. Therefore, sales increase in the number of customers but less than proportionally. This directly implies that sales per customer decrease with the number of customers, as illustrated in figure 2*B*, with an elasticity of -0.23.[10] This pattern is not driven by

[8] These patterns mirror findings for firm-to-firm linkages in the domestic production network in Belgium (Dhyne, Magerman, and Rubinova 2015) and Japan (Bernard, Moxnes, and Saito 2019) and for firm-to-firm export transactions in Norway (Bernard, Moxnes, and Ulltveit-Moe 2018).

[9] We demean variables by regressing log variables on four-digit sector fixed effects and retain the residuals as demeaned log variables.

[10] We construct total domestic network sales as $S_i^{\text{net}} = \Sigma_j m_{ij}$. We thus obtain an identity between S_i^{net}, the number of customers n_i^c, and the average sales per customer as $S_i^{\text{net}}/n_i^c = (1/n_i^c)\Sigma_j m_{ij}$. Note that this identity implies that the elasticities in fig. 2 amount to

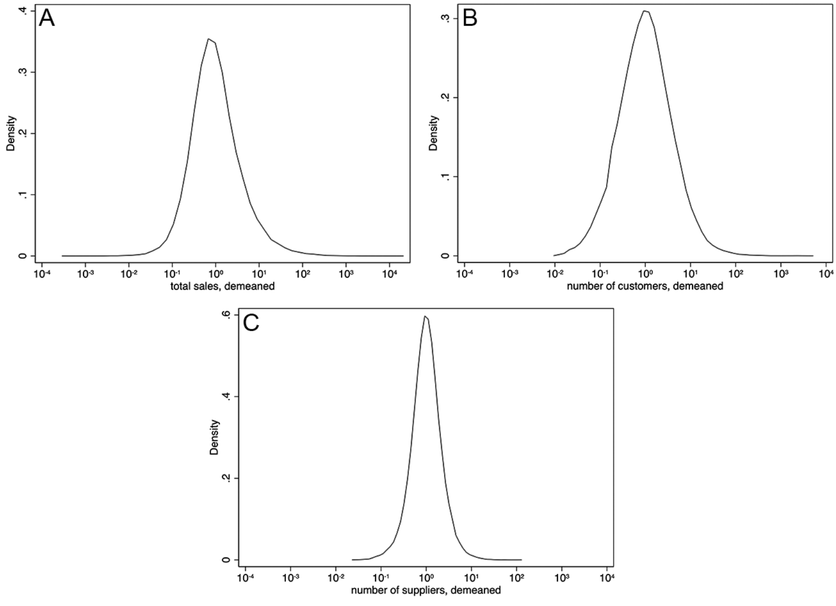

FIG. 1.—Distribution of firm sales, number of customers, and number of suppliers. A color version of this figure is available online.

composition effects among customers. Figure 3*A* demonstrates that sales per customer fall with the number of customers for both big and small customers. For each firm, we calculate the 10th, 50th, and 90th percentiles of sales across its buyers and plot these percentiles against the firm's number of buyers. The slope coefficients are negative and range between -0.13 and -0.22. This implies that firms do not systematically tend to sell relatively more or less to their top customers at the expense of their bottom customers when they add more buyers.

One may also wonder whether the decline in average sales per customer is driven by selection. If sellers match with smaller buyers when they grow their customer base, they would record lower average and median bilateral sales. To address this concern, we leverage the network data and calculate a firm's weighted average market share among its customers: the geometric mean of m_{ij}/M_j^{net}, where m_{ij} is sales from i to j and M_j^{net} is total network purchases by firm j, using sales shares m_{ij}/S_i^{net} as weights.[11] If selection were the main mechanism, this weighted average market share would be increasing

$0.77 - (-0.23) = 1$. As we observe total sales S_i but not the number of customers in export destinations, this identity would no longer hold if using S_i instead of S_i^{net}. However, all results are very similar and qualitatively the same when using total sales instead of network sales.

[11] That is, the weighted average market share is $\bar{\delta}_i = \prod_j (m_{ij}/M_j^{\text{net}})^{m_{ij}/S_i^{\text{net}}}$. The weighted average puts less emphasis on fringe customers. Using the unweighted average, however, produces similar results.

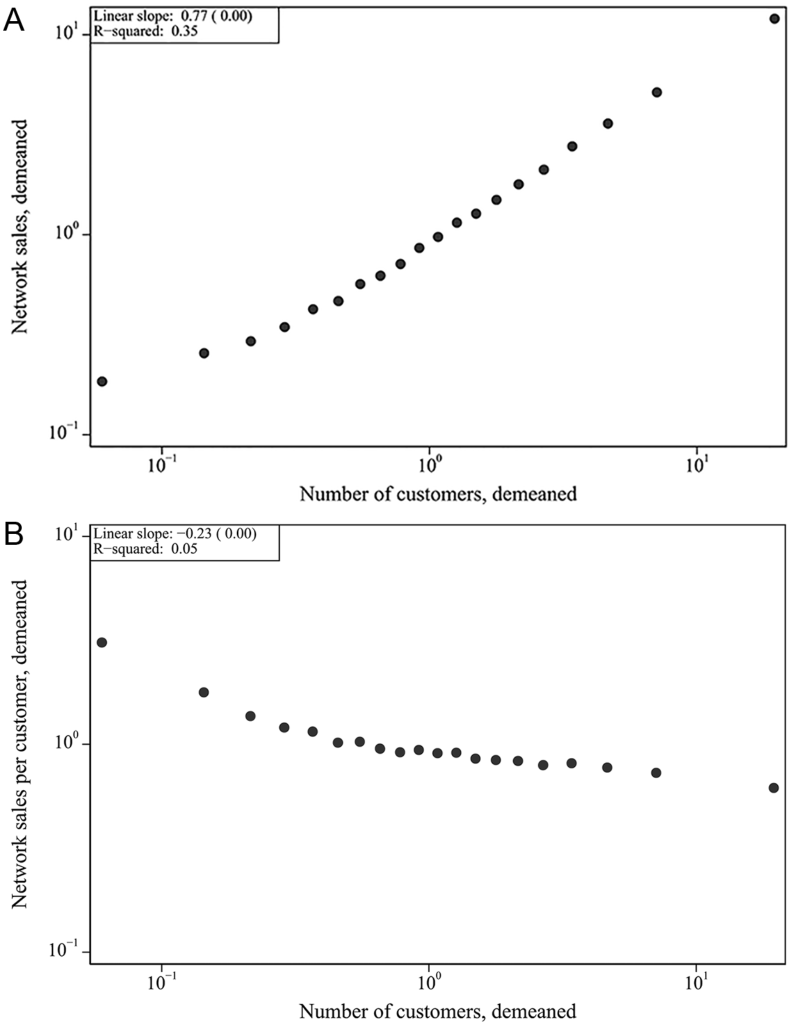

Fig. 2.—Total network sales, average sales, and number of customers. The binned scatterplots group firms into 20 equally sized bins by log number of customers and compute the mean of the variables on the *x*- and *y*-axes in each bin. Network sales refer to a firm's total sales to customers in the domestic production network. All variables are demeaned by NACE four-digit industry averages. Implied elasticities and R^2 from OLS regressions with NACE four-digit industry fixed effects are reported.

in, or unrelated to, the number of customers. Figure 3*B* shows that this is not the case: firms' weighted average market share also declines with their number of customers, with an elasticity of -0.08.

We explore the potential impact of additional dimensions of customer heterogeneity in online appendix B. In particular, we control for

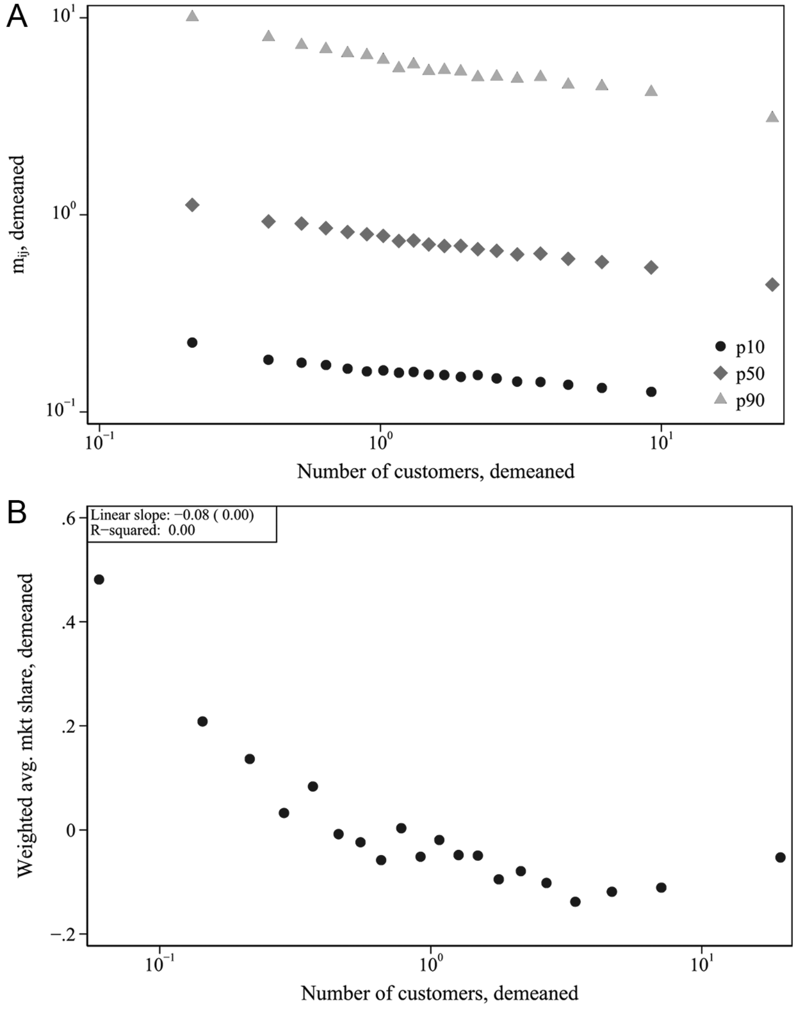

FIG. 3.—Sales distribution, market share, and number of customers. The binned scatterplots group firms into 20 equally sized bins by log number of customers and compute the mean of the variables on the *x*- and *y*-axes in each bin. All variables are demeaned by NACE four-digit industry averages. p10/p50/p90 refers to the 10th/50th/90th percentile of firm-to-firm sales m_{ij} across buyers j within firm i. Average market share refers to the weighted geometric mean of the market shares m_{ij}/M_j^{net} of firm i in the total network purchases of its buyers j, using sales shares m_{ij}/S_j^{net} as weights. A color version of this figure is available online.

heterogeneity in input requirements across customers within the seller's industry. We also consider the role of fringe buyers, that is, relatively unimportant customers in terms of m_{ij}. In all cases, our empirical findings retain the same message. Taken together, these empirical regularities

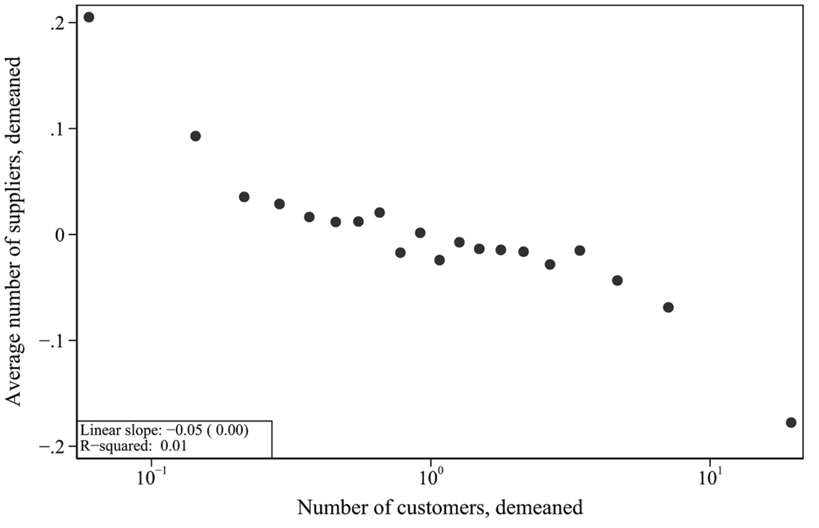

FIG. 4.—Degree assortativity. The binned scatterplot groups firms into 20 equally sized bins by log number of customers and computes the mean of the variables on the x- and y-axes in each bin. Average number of suppliers refers to the geometric mean of the number of suppliers serving the customers of firm i. All variables are demeaned by NACE four-digit industry averages. Implied elasticities and R^2 from OLS regressions with NACE four-digit industry fixed effects are reported.

present a puzzle: big firms match with many buyers, but they are unable to gain a large market share among those buyers. By contrast, in canonical one-dimensional models of firm heterogeneity or models with two or more (but independent) dimensions (e.g., Arkolakis 2010; Bernard, Moxnes, and Ulltveit-Moe 2018; Eaton, Kortum, and Kramarz 2018; Lim 2018), highly productive firms would both attract many customers and have a high market share among those customers. The empirical evidence therefore calls for a model with an additional element of firm heterogeneity, where firm size is determined not only by productivity but also by a second firm attribute that enables firms to match with more buyers.

FACT 3. Sellers with more customers match with customers who have fewer suppliers on average.

An important property of networks is the extent to which a well-connected node is linked to other well-connected nodes, so-called degree assortativity. The production network is characterized by negative degree assortativity. In other words, better-connected firms match to less well-connected firms on average.[12] Figure 4 shows a binned scatterplot of the

[12] Negative degree assortativity has been documented in earlier research on production networks (e.g., Bernard, Moxnes, and Ulltveit-Moe 2018; Lim 2018; Bernard, Moxnes, and Saito 2019).

average number of suppliers to firm i's customers on the y-axis against the number of i's customers on a log-log scale. The fitted regression line has slope −0.05, such that doubling the number of customers is associated with a 5% decline in the average customer's number of suppliers. We also find a robust and more negative relationship between a firm's number of suppliers and the average supplier's number of customers (see table 5).

Negative degree assortativity motivates our choice of a parsimonious matching model, in which firm connections form whenever the gross profits of a match exceed the fixed cost of forming a relationship. In this class of models, the marginal (and average) customer of more capable firms is less capable, generating a pattern of negative degree assortativity.

III. An Exact Decomposition

In this section, we develop an exact variance decomposition of firm sales into upstream, downstream, and final demand margins. The downstream component reflects characteristics of a firm's customers (i.e., their number and size), while the upstream component captures firm characteristics that remain constant across customers (i.e., average sales to customers, controlling for their size). Final demand includes factors unrelated to the domestic production network (i.e., sales to final consumers or foreign customers). This method exploits the granularity of the firm-to-firm transaction data in a way that would not be feasible with standard firm-level data sets, and its results provide the rationale for the structural framework in section IV.

A. Methodology

We start by estimating buyer, seller, and buyer-seller match effects using data on sales between firms in the production network. We then use these estimates to decompose the variance of firm sales. The specification is a two-way fixed effects regression for firm-to-firm sales:

$$\ln m_{ij} = \ln G + \ln \psi_i + \ln \theta_j + \ln \omega_{ij}, \tag{1}$$

where $\ln m_{ij}$ is log sales from i to j and $\ln G$ is the mean of $\ln m_{ij}$ across all ij pairs. The seller effect $\ln \psi_i$ reflects the amount of sales by i to its average customer j, controlling for total purchases by j via θ_j. The seller effect is therefore related to the average market share of i among her customers.[13] Analogously, the buyer effect $\ln \theta_j$ captures the value of input purchases by j from its average supplier i, controlling for total sales by i via ψ_i. Intuitively, attractive buyers (high θ_j) purchase a disproportionate

[13] This is shown formally in online app. C.2.

share of suppliers' sales. Finally, $\ln \omega_{ij}$ is the residual from the regression. A positive $\ln \omega_{ij}$ reflects match-specific characteristics that induce a given firm pair to trade more with each other, even if they are not fundamentally attractive trade partners.

To illustrate the advantage of the bilateral sales data, consider first an extreme case in which the variation in $\ln m_{ij}$ is only due to ψ_i. Seller i is then larger than seller i' because i sells more to every customer, while there is no variation in how much each of these customers buys from i. In this case, firm size heterogeneity is only driven by seller characteristics ψ_i; who you are as a seller explains firm size. Consider next the opposite case in which the variance in $\ln m_{ij}$ is only due to θ_j. Seller i now dominates seller i' because i matches with bigger customers than i', while sales to common customers j are identical. In this case, firm heterogeneity is only driven by differences in matching ability across sellers; who you meet as a seller explains firm size. In standard firm-level data sets, we cannot differentiate between these two scenarios because they are observationally equivalent. Estimating equation (1) using ordinary least squares (OLS) poses some threats to identification. First, in order for us to obtain unbiased estimates, the assignment of suppliers to customers must be exogenous with respect to ω_{ij}, so-called conditional exogenous mobility (Abowd, Kramarz, and Margolis 1999). This assumption as well as tests for exogenous mobility and functional form relevance are discussed at length in the appendix. Overall, we find strong support for the log-linear model and the conditional exogenous mobility assumption.

Second, to identify the fixed effects, firms must have multiple connections. Specifically, identifying a seller fixed effect requires a firm to have at least two customers, and identifying a buyer fixed effect requires a firm to have at least two suppliers. Therefore, single-customer and single-supplier links are dropped in the estimation procedure. Also, dropping customer A might result in supplier B having only one customer left. Supplier B is then also removed from the sample. This iterative process continues until a connected network component remains (i.e., a within-projection matrix of full rank), in which each seller has at least two customers and each buyer has at least two suppliers. This component is known as a mobility group in the labor literature on firm-employee matches.

Identification is obtained from cross-sectional variation. Compared with related work on firm-employee matches in the labor literature (e.g., Abowd, Kramarz, and Margolis 1999), this works to our advantage.[14] First, it attenuates an incidental parameter problem, as the number of suppliers per customer and the number of customers per supplier is relatively large: the median number of customers and suppliers is 26

[14] In Abowd, Kramarz, and Margolis (1999), identification comes from workers who move across firms over time.

and 53, respectively (see online app. B). Second, we do not require the otherwise standard assumption that the fixed effects be constant over time, as identification comes from a single cross section.

Once we have estimated parameters $\Psi = \{\psi_i, \theta_j, \omega_{ij}\}$, firm sales can be exactly decomposed into upstream, downstream, and final demand factors. Total sales of firm i are by construction $S_i = \Sigma_{j\in\mathcal{C}_i} m_{ij} + \mathcal{F}_i$, where $\mathcal{C}_i$ is the set of firm i's customers and $\mathcal{F}_i$ is sales to final demand (i.e., sales outside of the domestic network). Therefore, total sales can be expressed as $\ln S_i = \ln S_i^{\text{net}} + \ln \beta_i$, where $S_i^{\text{net}} \equiv S_i - \mathcal{F}_i$ is network sales and β_i is total sales relative to network sales, $\beta_i \equiv S_i/S_i^{\text{net}} \geq 1$, that is, an inverse measure of (the share of) network sales.

As shown in online appendix C.1, total sales can be decomposed as

$$\ln S_i = \ln G + \ln \psi_i + \ln n_i^c + \ln \bar{\theta}_i + \ln \Omega_i^c + \ln \beta_i, \tag{2}$$

where n_i^c is the number of customers, $\bar{\theta}_i \equiv \left(\prod_{j\in\mathcal{C}_i}\theta_j\right)^{1/n_i^c}$ is the average buyer fixed effect among customers, and $\Omega_i^c = (1/n_i^c)\Sigma_{j\in\mathcal{C}_i}\omega_{ij}\theta_j/\bar{\theta}_i$ is an interaction term between the buyer fixed effect and match quality.[15]

Each of these components has an intuitive economic interpretation. The ψ_i component represents *upstream* fundamentals that shape firm size: if sales dispersion is only due to variance in ψ_i, then large firms have larger market shares among their customers than small firms, while the number of customers is the same. The n_i^c, $\bar{\theta}_i$, and Ω_i^c components represent *downstream* fundamentals that shape firm size: firms face high network demand if (1) they are linked to many customers (high n_i^c), (2) their average customer has high input purchases (high $\bar{\theta}_i$), and/or (3) the interaction term Ω_i^c is large, that is, large customers (high θ_j) also happen to be good matches (high ω_{ij}). If sales dispersion is only due to variance in these downstream factors, large firms transact with more, bigger, and/or better-matched customers than small firms, while market shares are the same across customers. Finally, β_i represents the share of sales that goes to final demand, capturing all downstream variation outside the production network.

Note that all elements in equation (2) are known: S_i, β_i, n_i^c, and G come directly from the data, while ψ_i, $\bar{\theta}_i$, and Ω_i^c are estimated from equation (1). In order to assess the role of each margin, we follow the literature (Eaton, Kortum, and Kramarz 2004; Hottman, Redding, and Weinstein 2016) and perform a simple variance decomposition on equation (2). All observed and constructed variables are first demeaned by their NACE four-digit industry average to difference out systematic variation across industries. We then regress each component ($\ln \beta_i$, $\ln \psi_i$, $\ln n_i^c$, $\ln \bar{\theta}_i$, and $\ln \Omega_i^c$) separately on log sales. By the properties of OLS and from the exact nature of

[15] By the properties of OLS, the average term $(1/n_i^c)\Sigma_{j\in\mathcal{C}_i} \ln \omega_{ij} = 0$ and is therefore omitted from the expression above.

TABLE 1
BUYER AND SELLER EFFECTS

	N	$\frac{\text{var}(\ln \psi_i)}{\text{var}(\ln \psi_i + \ln \theta_j)}$	$\frac{\text{var}(\ln \theta_j)}{\text{var}(\ln \psi_i + \ln \theta_j)}$	$\frac{2\text{cov}(\ln \psi_i, \ln \theta_j)}{\text{var}(\ln \psi_i + \ln \theta_j)}$	R^2	Adjusted R^2
$\ln m_{ij}$	17,054,274	.66	.32	.02	.43	.39

NOTE.—The table reports the (co)variances of the estimated seller and buyer fixed effects from equation (1). The estimation is based on the high-dimensional fixed effects estimator from Correia (2016).

the decomposition, the coefficients from these regressions sum to unity and represent the share of the overall variation in firm size explained by each margin.

B. *Results*

The results from estimating equation (1) are reported in table 1.[16] Three patterns stand out. First, the adjusted R^2 from the regression is 0.39, indicating that the buyer and seller fixed effects explain a sizable share of the variation in firm-to-firm sales. Second, the variation in the seller effect $\ln \psi_i$ is larger than that in the buyer effect $\ln \theta_j$. Third, the correlation between the fixed effects is close to zero.

The results in table 1 shed light on the variation in transaction values, m_{ij}, but not on the variation in firm sales, S_i. Table 2 reports the results for the exact firm sales decomposition in equation (2). Relative differences in final demand across firms, as captured by the ratio of total to network sales, $\ln \beta_i$, account for an economically negligible 1% of the overall variation in firm size. Thus, large firms are not systematically selling relatively more (or less) to final demand than small firms. The upstream factor $\ln \psi_i$ represents, roughly speaking, the average market share of i among its customers. Being an important supplier to one's customers is weakly related to overall firm success, contributing 18% of the variation in firm sales. The three final rows report the magnitude of the downstream margins $\ln n_i^c$, $\ln \bar{\theta}_i$, and $\ln \Omega_i^c$. In total, the downstream side accounts for 81% of the size dispersion across firms. Most of the variation in the downstream component across firms can be attributed to the extensive margin, that is, the number of (domestic) buyers, $\ln n_i^c$. The average sourcing capability across a firm's customers, $\ln \bar{\theta}_i$, and the customer interaction term, $\ln \Omega_i^c$, explain a more modest 5% and 25%, respectively.

Therefore, the single most important advantage of large firms is that they successfully match with many buyers, whereas the characteristics of these buyers play a smaller role. These findings suggest that a key to understanding the vast firm size heterogeneity observed in modern economies is

[16] After removing firms with unidentified fixed effects, 99% of the links and 95% of the value of all transactions remain in the estimation sample.

TABLE 2
FIRM SIZE DECOMPOSITION ($\ln S_i$)

	Component	Coefficient	Standard Error
Relative final demand	$\ln \beta_i$	.01	.00
Upstream	$\ln \psi_i$	.18	.00
Number of customers	$\ln n_i^c$	.51	.00
Average customer capability	$\ln \bar{\theta}_i$	.05	.00
Customer interaction	$\ln \Omega_i^c$	.25	.00

NOTE.—The table reports coefficient estimates from separate OLS regressions of a firm size margin (as indicated in the row heading) on $\ln S_i$. All variables are first demeaned by their four-digit NACE industry average. The number of firms in the core sample is 94,147.

how firms manage their sales activities and, specifically, how they match and transact with buyers in the production network. Online appendix D provides several robustness checks and reports results for business groups, individual industries, and different years, reinforcing our main conclusions.

C. *Correlations*

We conclude this section by documenting the correlations among various firm characteristics in the data. Column 1 in table 3 shows that firm sales are strongly positively correlated with both the upstream ($\ln \psi_i$) and the various downstream components ($\ln n_i^c$, $\ln \bar{\theta}_i$, $\ln \Omega_i^c$). The number of customers ($\ln n_i^c$) and the upstream component ($\ln \psi_i$) are negatively correlated. This mirrors the findings in section II and implies that firms with many customers tend to have smaller average market shares among those customers. Our interpretation of this pattern is that firms are unlikely to succeed along both the extensive and the intensive margins: some firms become large by accumulating a broad customer base, while other firms become large by being important suppliers to their clients, and few firms manage to do both.

These results, coupled with the stylized facts and sales decomposition, are difficult to reconcile with canonical heterogeneous firm models. They suggest that both upstream and downstream dimensions of firm activity

TABLE 3
CORRELATION MATRIX

Firm Size Component	$\ln S_i$	$\ln \psi_i$	$\ln n_i^c$	$\ln \bar{\theta}_i$	$\ln \Omega_i^c$	$\ln \beta_i$
$\ln S_i$	1					
$\ln \psi_i$	.23	1				
$\ln n_i^c$	.49	−.33	1			
$\ln \bar{\theta}_i$	.20	.22	−.18	1		
$\ln \Omega_i^c$	.45	.16	.09	.23	1	
$\ln \beta_i$	.02	−.36	−.33	−.16	−.42	1

NOTE.—All correlations are significant at the 5% level. All variables are demeaned at the NACE four-digit level.

underpin sales dispersion when firms interact in production networks. One interpretation of our findings is that firm attributes that matter for matching with customers and suppliers are orthogonal or even negatively related to firm attributes that determine sales conditional on a match.

IV. Theoretical Framework

Motivated by the stylized facts, this section develops a theoretical framework of a buyer-supplier production network with two-sided firm heterogeneity and endogenous match formation. This framework admits a two-step analysis: we first present the model conditional on a set of firm-to-firm links and subsequently introduce a parsimonious firm-to-firm matching model.

Our starting point is a framework in which firms are heterogeneous in two dimensions. First, firms within an industry have different productivities, which implies that they have different marginal costs and prices.[17] Second, firms have different relationship capabilities. These capabilities determine their ability to match with customers conditional on their (quality-adjusted) prices. We model relationship capability as a fixed cost that the firm must incur for each customer it chooses to serve. A firm with lower relationship fixed costs will endogenously be able to match with more customers, all else equal. In contrast to much of the earlier literature, productivity and relationship capability are potentially correlated.

Firms operate in a production network, sourcing their inputs from other firms and selling their output to both other firms and final demand. In addition to productivity and relationship capability, a firm's size thus also depends on its input prices. Input prices are low and sales high if the firm has many low-price (or high-quality) suppliers.

A. *Technology and Demand*

The economy consists of a unit continuum of firms, each with the following production function:

$$y(i) = \kappa z(i) l(i)^{\alpha} v(i)^{1-\alpha},$$

where $y(i)$ is output (in quantities) of firm i, $z(i)$ is productivity, $l(i)$ is the amount of labor used by firm i, α is the labor share, and $\kappa > 0$ is a normalization constant.[18] $v(i)$ is a CES input bundle:

[17] As is standard in this class of models, under the assumption of CES preferences and monopolistic competition, productivity and product quality enter equilibrium firm revenue in exactly the same way.

[18] $\kappa \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. This normalization maps the production function to the cost function and simplifies the expression for the cost function, without any bearing on our results.

$$v(i) = \left(\int_{\mathcal{S}(i)} \nu(k,\,i)^{(\sigma-1)/\sigma}\,dk\right)^{\sigma/(\sigma-1)},$$

where $\nu(k,\,i)$ is the quantity purchased from firm k, $\mathcal{S}(i)$ is the set of suppliers to firm i, and $\sigma > 1$ is the elasticity of substitution across suppliers. The corresponding input price index is $P(i) = (\int_{\mathcal{S}(i)} p(k)^{1-\sigma}dk)^{1/(1-\sigma)}$, where $p(k)$ is the price charged by supplier k. Setting the wage w as the numeraire, we find that the marginal cost of the firm is

$$c(i) = \frac{P(i)^{1-\alpha}}{z(i)}. \tag{3}$$

Final demand.—Final consumers have a CES utility function with the same elasticity of substitution σ across output varieties. The representative consumer is the shareholder of all firms, so that aggregate profits Π become part of consumer income. Aggregate income X is therefore the sum of aggregate labor income and aggregate corporate profits, $X = wL + \Pi$, where L is inelastically supplied labor.

B. *Firm-to-Firm Sales*

Each firm faces demand from other firms as well as from final consumers. Given the assumptions about technology, sales from firm i to firm j are

$$m(i,j) = p(i)^{1-\sigma}P(j)^{\sigma-1}M(j), \tag{4}$$

where $M(j)$ are total intermediate purchases by firm j, $M(j) = \int_{\mathcal{S}(j)} m(i,j)\,di$.

The market structure is monopolistic competition, such that firms charge a constant markup over marginal costs, $p(i) = \mu c(i)$, where $\mu \equiv \sigma/(\sigma - 1)$. After rearranging, we can express sales from i to j as

$$m(i,j) = \left[\frac{z(i)}{\mu P(i)^{1-\alpha}}P(j)\right]^{\sigma-1}M(j). \tag{5}$$

The model thus delivers a simple log linear expression for firm-to-firm sales, just as in the reduced-form equation (1).

C. *Equilibrium Conditional on Network*

We characterize the equilibrium in two separable steps. This section first describes properties of the partial equilibrium conditional on a fixed network structure. Section IV.D then develops the firm-to-firm matching model and specifies the general equilibrium with endogenous match formation.

To proceed, we introduce additional notation. A firm i is characterized by the tuple $\lambda = (z, F)$, where z is productivity and F is a relationship fixed cost in units of labor. z and F are potentially correlated, and $dG(\lambda)$ denotes the (multivariate) density of λ. We define the link function $l(\lambda, \lambda')$ as the share of seller-buyer pairs (λ, λ') that match.[19]

1. Backward Fixed Point

For a given network structure, the equilibrium can be found by solving for two fixed points sequentially. Using the pricing rule $p(\lambda) = \mu c(\lambda)$ and the equation for marginal costs (3), we can solve the input price index by iterating on a backward fixed point problem:

$$P(\lambda)^{1-\sigma} = \mu^{1-\sigma} \int P(\lambda')^{(1-\sigma)(1-\alpha)} z(\lambda')^{\sigma-1} l(\lambda', \lambda) dG(\lambda'). \tag{6}$$

The input cost index of firm λ, $P(\lambda)$, depends on the input cost index and productivity of all its suppliers λ', $P(\lambda')$ and $z(\lambda')$.

2. Forward Fixed Point

Sales of a type λ firm are the sum of sales to final and intermediate demand: $S(\lambda) = \mathcal{F}(\lambda) + \int m(\lambda, \lambda') l(\lambda, \lambda') dG(\lambda')$, where $m(\lambda, \lambda')$ now denotes sales by supplier λ to buyer λ'. Final demand is $\mathcal{F}(\lambda) = p(\lambda)^{1-\sigma} \mathcal{P}^{\sigma-1} X$, with the consumer price index equal to $\mathcal{P}^{\sigma-1} = \int p(\lambda)^{1-\sigma} dG(\lambda) = \mu^{1-\sigma} \int P(\lambda)^{(1-\sigma)(1-\alpha)} z(\lambda)^{\sigma-1} dG(\lambda)$. Also note that total input purchases are $M(\lambda) = S(\lambda)(1-\alpha)/\mu$. Using this together with equation (3) yields

$$S(\lambda) = \mu^{1-\sigma} z(\lambda)^{\sigma-1} P(\lambda)^{(1-\sigma)(1-\alpha)} \left(\frac{X}{\mathcal{P}^{1-\sigma}} + \frac{1-\alpha}{\mu} \int \frac{S(\lambda')}{P(\lambda')^{1-\sigma}} l(\lambda, \lambda') dG(\lambda') \right). \tag{7}$$

Sales of a type λ firm depend on final demand, X, the productivity and input price index of the firm itself, $z(\lambda)$ and $P(\lambda)$, and the sales and input prices of its customers, $S(\lambda')$ and $P(\lambda')$. Online appendix C.3 proves the existence and uniqueness of the equilibrium.

D. *Firm-to-Firm Matching*

We now consider the general equilibrium when the production network is endogenous and sellers match with buyers if and only if the profits

[19] Because of idiosyncratic pairwise fixed cost shocks, the link function will take values between 0 and 1 (see sec. IV.D).

from doing so are positive. The seller incurs a relationship fixed cost $F\epsilon$ for every buyer it chooses to sell to, where F varies across sellers and ϵ is an idiosyncratic component that varies across firm pairs. This matching model is similar to Bernard, Moxnes, and Ulltveit-Moe (2018) and Lim (2018), but in contrast to these papers, F is a firm-specific attribute that can be correlated with the productivity of the firm, z.[20]

The share of seller-buyer pairs (λ, λ') that match and trade with each other is then

$$l(\lambda, \lambda') = \int I[\ln \epsilon < \ln \pi(\lambda, \lambda') - \ln F] dH(\epsilon), \tag{8}$$

where $I[]$ is the indicator function, $dH(\epsilon)$ denotes the density of ϵ, and the gross profits from the potential match are

$$\pi(\lambda, \lambda') = \frac{m(\lambda, \lambda')}{\sigma}.$$

The introduction of idiosyncratic match costs ϵ is not needed to solve the model or to rationalize the stylized facts presented earlier in the paper. However, ϵ will play a role in the structural estimation in section V. Formally, dispersion in ϵ ensures that the link function is continuous in the parameters of the model, such that standard gradient-based numerical methods can be used to minimize the objective function. Intuitively, ϵ can be justified with seller-buyer specific costs that affect the profitability of the relationship, such as the fixed cost of adapting the seller's output to the buyer's production needs.

This link function is also a fixed point problem. The gross profits from a potential match, $\pi(\cdot)$, determine link probabilities according to equation (8), and the link probabilities determine gross profits via the backward and forward fixed points in equations (6) and (7).

The general equilibrium of the model can be solved by a simple nested fixed point algorithm. (1) Start with a guess for the link function $l()$. (2) Solve for $P(\lambda)$ and $S(\lambda)$ using the backward and forward fixed points in equations (6) and (7) sequentially. (3) Calculate gross profits for all potential matches using equation (5). (4) Calculate the share of seller-buyer pairs (λ, λ') that match according to equation (8). (5) Go back to step 2 until the link function converges. We do not have a formal proof of existence and uniqueness. In practice, however, the nested fixed point problem is numerically well behaved and always converges to the same solution irrespective of the chosen starting values.

[20] On the other hand, sales to final demand incur no fixed costs and vary across firms only because of differences in output prices.

E. Discussion

We conclude the exposition of the model by discussing some key implications and features. We start by considering the role of each dimension of firm heterogeneity in determining equilibrium outcomes on its own. Conditioning on relationship capability, firms with higher productivity have lower marginal costs, lower prices, and higher profits from a match with any given buyer (see eq. [3]). As a result, higher-productivity firms match with more buyers (see eq. [8]) and have greater sales (market share) conditional on a match (see eq. [5]). Larger total sales and input purchases make higher productivity firms more attractive partners for upstream firms (see eq. [5]). The increased number of upstream suppliers contributes to an additional reduction in marginal cost through the firm's input price index (see eq. [6]).

Conditioning on productivity, we find that firms with better relationship capability (lower F) are able to match with more buyers (see eq. [8]) and, as a result, have greater sales and greater input purchases. As with higher productivity, the greater input demand makes these firms relatively attractive to upstream suppliers, and the greater number of suppliers lowers their marginal cost of production through the input price index. The lower marginal cost results in greater sales (market share) to any given buyer.

Thus, considered by itself, either higher productivity or better relationship capability leads to higher sales through both the extensive margin of more downstream buyers and the intensive margin of greater sales per buyer.

Several features of the model grant it analytical and quantitative tractability as well as transparency in illustrating the main mechanisms. First, we consider a unit continuum of firms in the economy. This implies that individual sellers take other sellers' prices and all buyers' input price indexes as given when deciding whether to match with a particular buyer and how much to sell to that buyer.

Separately, we focus on the costs that sellers incur to match with buyers and assume that buyers do not face corresponding costs of matching with suppliers. Even with this assumption, in equilibrium the number of both suppliers and buyers varies across firms. This choice lends tractability because firms make separable sales decisions with respect to different buyers and do not internalize the effect of their match decisions on buyers' input demand. It also avoids the well-known problem of interdependence of sourcing decisions in frameworks where buyers choose suppliers (see Antràs, Fort, and Tintelnot 2017).

Finally, the model focuses on the domestic production network and does not directly consider the role of exports and imports, both of which are important in the Belgian context. Exports are implicitly included in final demand, even though this almost surely understates the importance of firm-to-firm sales, as almost all export sales are to firms rather than consumers. Similarly, while we do not model imports, they can be added to production without changing the implications for firm outcomes.[21]

V. Estimation and Results

This section provides a model-based assessment of the origins of firm heterogeneity. Specifically, we exploit the Belgian production network data to parameterize the model above, allowing for heterogeneity in both productivity and relationship capability across firms. We then estimate the model under alternative scenarios to evaluate the quantitative importance of each firm attribute.

A. *Simulated Method of Moments*

The general equilibrium model is estimated by SMM. We assume that firm productivity z and relationship capability F are distributed joint log-normal with expectations $\mu_{\ln z} = 0$ and $\mu_{\ln F}$, standard deviations $\sigma_{\ln z}$ and $\sigma_{\ln F}$, and correlation coefficient ρ.[22] In sum, there are four unknown parameters to be estimated, $\Upsilon = \{\sigma_{\ln z}, \mu_{\ln F}, \sigma_{\ln F}, \rho\}$.

In addition to the unknown parameters, information is needed on α (labor cost share), μ (markup), and X (aggregate income). α is constructed by dividing labor costs by total costs for each firm and then taking the simple average across firms. μ is computed by dividing sales by total costs for each firm and then taking the simple average across firms. X is inferred from sales going out of the network, that is, $X = \Sigma_i S_i - \Sigma_i \Sigma_{j \in C_i} m_{ij}$. The idiosyncratic matching cost ϵ is assumed log-normal, with mean $\mu_{\ln \epsilon} = 0$ and standard deviation $\sigma_{\ln \epsilon}$. The standard deviation is chosen so that the objective function is smooth in the parameters of the model.[23] Table 4 summarizes the parameters of the model, their definitions, and the values assigned to them.

We choose seven moments in the data to estimate Υ. While all moments jointly pin down all unknown parameters in general equilibrium, there is

[21] See Bernard et al. (2019) for a static model of a domestic production network with imports in the production function and idiosyncratic match-specific shocks.

[22] The mean of ln z is not identified, and it is therefore normalized to zero. This normalization is appealing on conceptual grounds. Consider a shift in the productivity distribution, such that productivity increases for all firms. While this would lower prices and increase welfare, it would not change firms' market shares or the network structure of the economy that are of interest to us.

[23] If the dispersion in ϵ is small relative to the dispersion in z and F, the share of links for some (λ, λ') pairs will be close to zero. This complicates the SMM estimation using standard gradient-based methods, as the objective function is no longer smooth in the parameters of the model. In practice, we set $\sigma_{\ln \epsilon} = 4$, which makes the problem sufficiently smooth, similar to the scale factor in the logit-smoothed accept-reject simulator (McFadden 1989). Other choices of $\sigma_{\ln \epsilon}$ do not significantly improve the fit of the model.

TABLE 4
MODEL PARAMETERS

Parameter	Definition	Value	Source
α	Labor cost share	.24	Mean of $w_i L_i/(w_i L_i + M_i)$
μ	Markup	1.24	Mean of $S_i/(w_i L_i + M_i)$
X	Aggregate final demand	€470 billion	$\Sigma_i S_i - \Sigma_i \Sigma_{j\in C_i} m_{ij}$
$\sigma_{\ln \epsilon}$	Pair matching cost dispersion	4	

an intuitive mapping between them. First, the mean log number of customers across firms, mean $(\ln n_i^c)$, helps identify the mean of the relationship costs. Second, the variance of log number of customers, var $(\ln n_i^c)$, and the variance of network sales, var $(\ln S_i^{\text{net}})$, together identify the variances of productivity and relationship costs. Third, the slope coefficient from the regression of average market share on the number of buyers, $\ln \bar{\delta}_i = \alpha + \beta \ln n_i^c + \varepsilon_i$, helps identify the correlation coefficient ρ (see fig. 3): implicitly, a smaller (or more negative) slope coefficient suggests that firms with low relationship costs and therefore high $\ln n_i^c$ are relatively less productive and thus have lower $\ln \bar{\delta}_i$. Finally, we include the contribution of key decomposition margins in table 2: the number of customers $(\ln n_i^c)$, average customer capability $(\ln \bar{\theta}_i)$, and the customer interaction term $(\ln \Omega_i^c)$. Intuitively, the contribution of these margins to firm size dispersion inform the role of relationship costs versus productivity in determining firm size.[24] When we collect the targeted empirical moments in vector x and the corresponding simulated moments in vector $x^s(\Upsilon)$, the SMM estimates for Υ solve

$$\arg\min_{\Upsilon}(x - x^s(\Upsilon))'(x - x^s(\Upsilon)).$$

We obtain standard errors by bootstrapping these estimates (for details, see online app. E).[25]

B. *Results*

The estimated parameters are summarized in column 2 of panel A in table 5. A striking result is the large positive correlation between productivity and relationship costs (ρ). In other words, firms that are more efficient at converting inputs into outputs on the production side have lower relationship capability in matching with buyers on the sales side.

[24] There are six margins in total, which by construction sum to 1. The final demand margin is omitted as a targeted moment because final demand is measured as the difference between total sales and network sales, and it may therefore include sales to firms outside the observed network (e.g., foreign firms).

[25] We use the equally weighted minimum distance estimator. We have also estimated the model weighting the moments by the inverse of the variance-covariance matrix of the moments, which yielded similar results compared with the baseline.

TABLE 5
SMM Model Fit

	Data (1)	Estimated Models: Baseline (2)	No F (3)	No Z (4)	No Rho (5)
		A. Estimated Parameters			
$\mu_{\ln F}$		18.11	19.18	19.65	18.21
		(.02)	(.02)	(.02)	(.02)
$\sigma_{\ln z}$		.24	.13		.07
		(.00)	(.00)		(.00)
$\sigma_{\ln F}$		2.23		1.48	1.29
		(.01)		(.01)	(.01)
ρ		.86			0
		(.00)			
		B. Targeted Moments			
Mean ($\ln n_i^c$)	−8.12	−8.12	−8.27	−8.14	−8.12
var ($\ln n_i^c$)	1.87	1.86	.81	2.20	1.92
var ($\ln S_i^{\text{net}}$)	3.12	3.12	3.37	2.78	3.08
β from $\ln \bar{\delta}_i = \alpha + \beta \ln n_i^c + \epsilon_i$	−.11	−.10	1.10	.15	.28
Decomposition:					
Number of customers	.51	.52	.49	.89	.76
Average customer capability	.05	.01	.01	.03	.02
Customer interaction	.25	−.04	−.04	−.05	−.06
		C. Nontargeted Moments			
Downstream:					
var ($\ln S_i$)	1.73	2.08	1.61	.62	.90
var (ln value added/worker$_i$)	.62	.71	.15	.54	.42
β from $\ln m_i^k = \alpha + \beta \ln n_i^c + \epsilon_i$:					
10th percentile	−.25	−.14	1.07	.15	.27
50th percentile	−.28	−.15	1.06	.13	.25
90th percentile	−.29	−.17	1.02	.11	.19
Degree assortativity	−.05	−.04	−.07	−.02	−.03
Upstream:					
var ($\ln M_i^{\text{net}}$)	2.12	2.08	1.61	.62	.90
var ($\ln n_i^s$)	.60	.41	.38	.12	.18
Degree assortativity	−.18	−.18	−.07	−.15	−.15

Note.—The number of customers in col. 1, n_i^c, is normalized relative to the number of firms in the final sample. $\bar{\delta}_i$ is the geometric mean of the market share $\delta_{ij} = m_{ij}/M_j$ for seller i across its buyers j. Downstream degree assortativity refers to β from the regression for i's customers, $\ln \text{mean}_i n_j^s = \alpha + \beta \ln n_i^c + \epsilon_i$. Upstream degree assortativity refers to β from the regression for j's suppliers, $\ln \text{mean}_j n_i^c = \alpha + \beta \ln n_j^s + \epsilon_j$. $\ln m_i^k$ is the kth (10th/50th/90th) percentile of log bilateral sales, $\ln m_{ij}$, for seller i across its customers j. The three decomposition moments refer to the contribution of different margins to firm size dispersion from sec. III. All variables in col. 1 except mean ($\ln n_i^c$) are demeaned by NACE four-digit industry averages. Bootstrapped standard errors are in parentheses.

In addition, the standard deviation of log relationship costs is an order of magnitude larger than the standard deviation of log productivity. To put matching frictions into perspective, we calculate the ratio of relationship costs (F_ϵ) to firm-to-firm sales (m) for successful matches in the economy. The mean of this ratio is 0.07; that is, average relationship costs account for 7% of relationship sales. The mean of this ratio across all potential matches is orders of magnitude higher (222,760), consistent with the observation that the production network is sparse because many possible seller-buyer matches are not profitable.

Columns 1 and 2 of table 5 report targeted and untargeted moments in the data and in the simulated model with the estimated parameters. Panel B demonstrates that the model hits targeted moments well, as expected, with the exception of the contribution of the customer interaction term to the firm size decomposition. In particular, it replicates the negative relationship between the number of customers (n_i^c) and the average market share per customer ($\bar{\delta}_i$) across sellers (see fig. 3). The model hits not only the moments of the distribution of network sales but also the full distribution relatively well. Figure 5 shows the (within-industry) distribution of network sales according to the estimated model and the data.

Panel C of table 5 presents nontargeted moments. On the downstream side, panel C shows that the model slightly overpredicts the variance of total sales. It also matches relatively well the negative relationship between the 10th/50th/90th sales percentiles and out-degree documented in figure 3 as well as the pattern of negative degree assortativity downstream in figure 4.

The last three rows of panel C of table 5 reports untargeted upstream moments. These moments are interesting because the model emphasizes

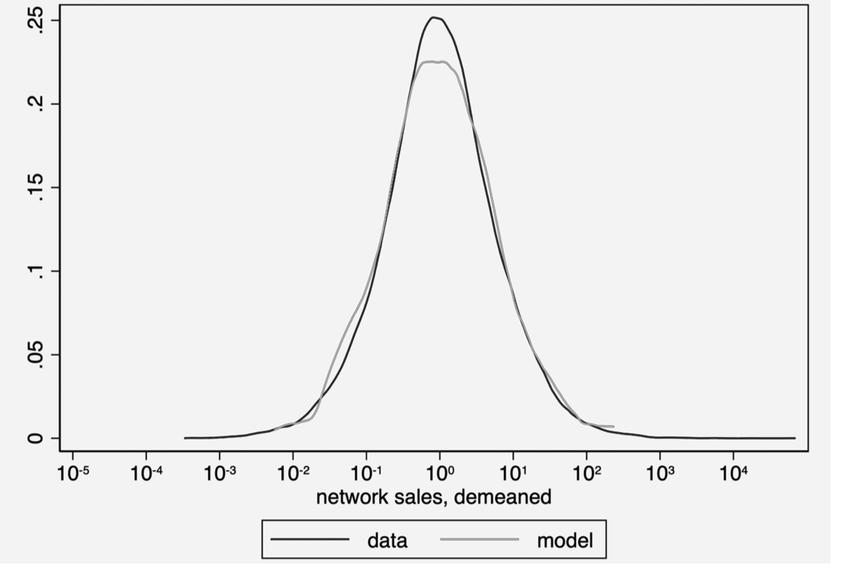

FIG. 5.—Density of network sales. The figure shows the density of network sales across firms in the data and in the model. The variable is demeaned by NACE four-digit industry averages. A color version of this figure is available online.

sellers' choice of buyers downstream but remains silent about firms' choice of partners upstream. The model does a good job matching the variance of input purchases from the network as well as the variance of the number of suppliers. There is also close correspondence between the negative upstream assortativity in the model and in the data: buyers with more suppliers have suppliers who on average have fewer customers in the network.

We also evaluate to what extent the model fits the observed dispersion in labor productivity. In this class of models, value added per production worker, $(S - M)/l$, is constant across all firms within an industry (Hsieh and Klenow 2009). In our model, however, a firm's employment is the sum of production workers and "marketing" workers, that is, workers allocated to relationship building, such that total employment is $L = l + n^c F$. Value added per worker therefore varies across firms and is increasing in value added per marketing worker.[26] In equilibrium, firms with high productivity and/or high relationship capability have higher value added per marketing worker and therefore also greater labor productivity. Table 5 confirms that the estimated model produces significant variance in log labor productivity, although dispersion in the model is slightly higher (0.71 vs. 0.62).

C. *Restricted Models*

We next illustrate the need for two firm attributes in order to rationalize observed empirical patterns, by estimating a model with heterogeneity in either (1) productivity (no F) or (2) relationship capability (no Z) but not both. Under assumption 1, there are two parameters to estimate, $\Upsilon = \{\sigma_{\ln z}, \mu_{\ln F}\}$, and we use the same moments to identify Υ. Under assumption 2, the parameters to estimate are $\Upsilon = \{\mu_{\ln F}, \sigma_{\ln F}\}$.

The estimated parameters and fit of these two restricted models are summarized in columns 3 and 4 of table 5. Both restricted models are unable to generate the negative correlation between the average market share $(\bar{\delta}_i)$ and the number of customers across sellers (the β coefficient from the regression $\ln \bar{\delta}_i = \alpha + \beta \ln n_i^c + \epsilon_i$). Figure 6 plots this relationship according to the estimated restricted models. In both cases, the model generates the opposite pattern to the empirical regularity in figure 3.

[26] Value added per worker is

$$\frac{S - M}{L} = \left(\frac{l + n^c F}{S - M}\right)^{-1} = \left(\frac{l}{S - M} + \frac{n^c F}{S - M}\right)^{-1},$$

where the first term is constant across firms and the second term is the inverse of value added per marketing worker.

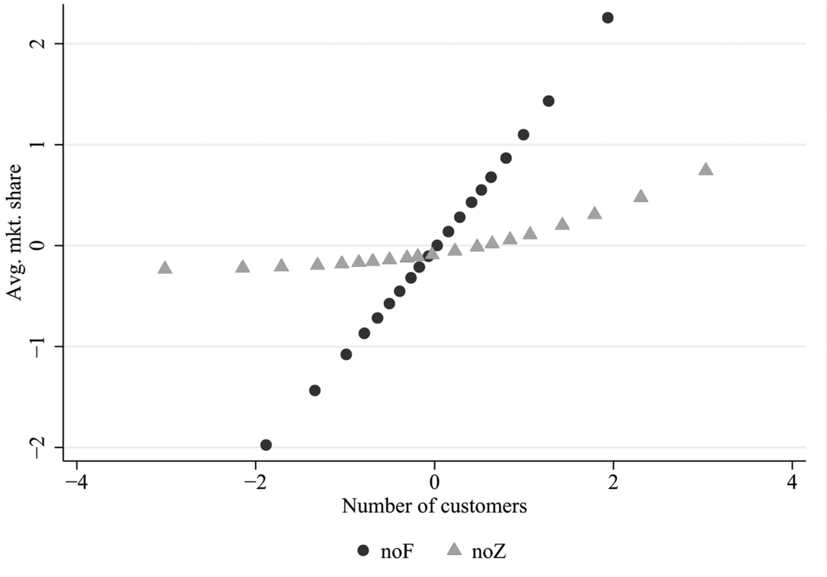

FIG. 6.—Restricted models. The figure shows the binned scatterplot of the number of customers and the average market share in buyers' input purchases across sellers in the restricted models, where $\mathrm{var}(F) = 0$ (no F) or $\mathrm{var}(z) = 0$ (no Z). A color version of this figure is available online.

In addition, the single-factor models do a relatively poor job in other dimensions. In the no F case, we more or less match dispersion in network sales, but this comes at the expense of not matching the variance in the number of customers or in value added per worker. In the no Z case, we match dispersion in the number of customers, but dispersion in total sales is too small, and the contribution of the number of customers in the firm size decomposition is too high. Both restricted models underestimate the variances of input purchases and of the number of suppliers and counterfactually imply that bilateral sales at different customer percentiles increase rather than decrease with the number of customers.

Finally, we estimate a model with heterogeneity in both productivity and relationship capability but where the correlation between them set to zero, $\rho = 0$ (no rho). Column 5 of table 5 shows that the restricted model produces a positive correlation between average market share $(\bar{\delta}_i)$ and the number of customers across sellers (the slope coefficient is 0.28), far from the slightly negative coefficient in the data. Furthermore, it does poorly for many nontargeted moments: the restricted model generates significantly less heterogeneity in both log sales, log input purchases, and log number of suppliers. This result highlights that a data-generating process with unrelated Z and F is inconsistent with our data.

D. *Sensitivity*

Next, we evaluate the sensitivity of the estimates to the vector of estimation moments. We use the methodology from Andrews, Gentzkow, and Shapiro (2017). Specifically, we ask how sensitive the parameter estimate ρ (the correlation between productivity and relationship costs) is to perturbations of the various moments of the data. We consider perturbations that are additive shifts of the moment functions due to either misspecification of $x^s(\Upsilon)$ or measurement error in the empirical moments x. Andrews, Gentzkow, and Shapiro (2017) show that sensitivity can be summarized by the matrix $\Lambda = (S'WS)^{-1} S'W$, where S is the matrix of partial derivatives of $x^s(\Upsilon)$ evaluated at the true value Υ_0 (see their proposition 2). W is the method of moments weighting matrix, which in our case is the identity matrix.

Figure 7 plots the column of the estimated Λ corresponding to the parameter estimate ρ. The plot shows one-hundredth of the value of sensitivity of the parameter ρ with respect to the vector of estimation moments. The values are scaled by the standard deviation of the moments, so the values can be interpreted as the effect of a 1 standard deviation change in the moment on the parameter ρ. The plot largely confirms our intuition about identification. First, the slope coefficient β (from the regression of average

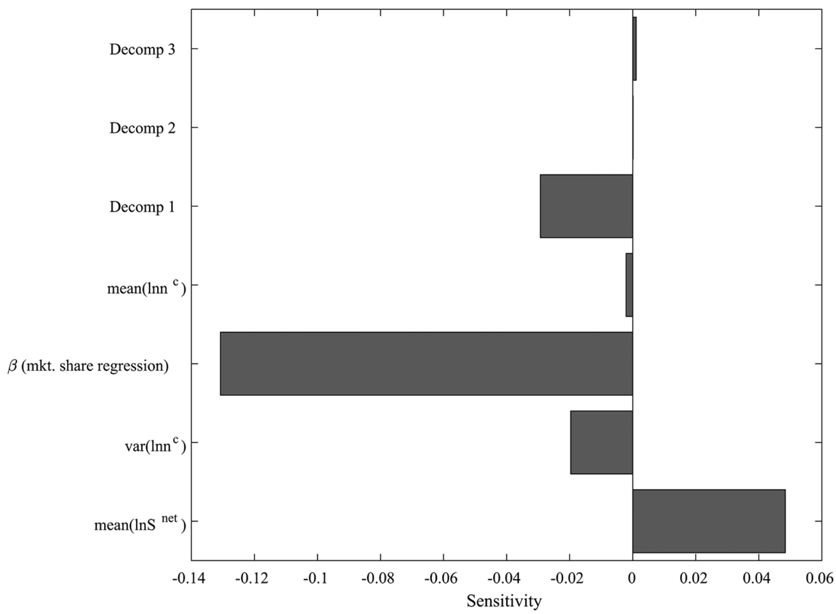

FIG. 7.—Sensitivity of parameter estimate ρ. The plot shows one-hundredth of the value of sensitivity of the parameter ρ with respect to the vector of estimation moments. The values are scaled by the standard deviation of the moments, so the values can be interpreted as the effect of a 1 standard deviation change in the moment on the parameter ρ.

market share on the number of customers) is the moment that matters the most, while the other moments are less relevant for this particular parameter. Furthermore, the direction of sensitivity is also in line with our expectations: a steeper slope coefficient has a negative impact on ρ; that is, we get a lower positive correlation between Z and F as the slope becomes steeper.

E. A Counterfactual

We end this section by quantifying the role of firm heterogeneity in productivity and relationship costs for aggregate outcomes. We do so by performing a simple counterfactual experiment: a common 50% reduction in relationship costs across all firms in the economy (i.e., a reduction in $\mu_{\ln F}$). We do so both in the baseline estimated model and in a restricted model with no correlation between productivity and relationship costs in order to illustrate the importance of the latter.

The simulation shows that real wages increase by 17% in the baseline and 12% in the model with no correlation, which implies that the welfare gains are 42% higher with correlation than without. Figure 8 shows a binned scatterplot of the counterfactual change in the log number of customers on the vertical axis against log productivity ($\ln Z$) on the horizontal

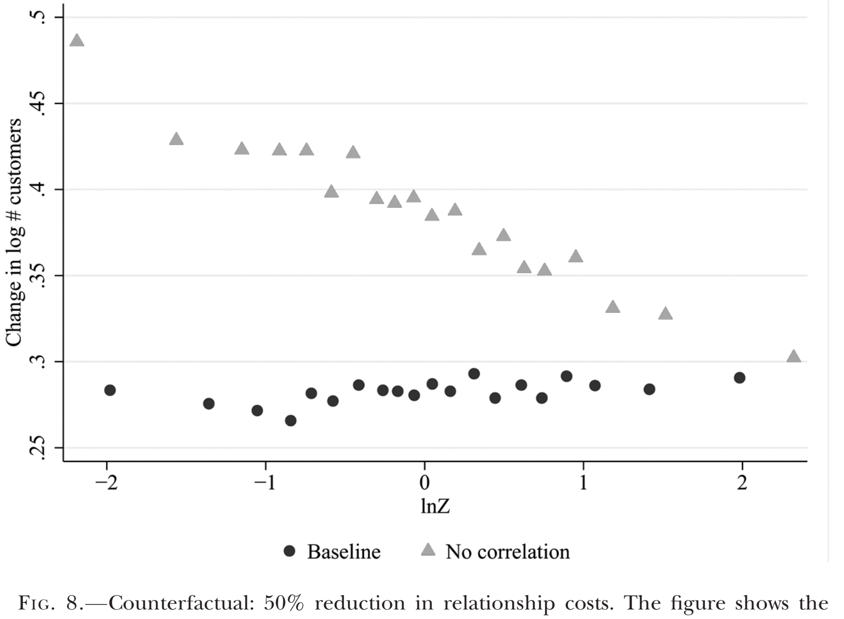

FIG. 8.—Counterfactual: 50% reduction in relationship costs. The figure shows the binned scatterplots of the change in the log number of customers in the baseline counterfactual (circles) and in the counterfactual with no correlation between Z and F (triangles) against $\ln Z$. A color version of this figure is available online.

axis. The circles refer to the baseline counterfactual, while the triangles denote the no-correlation counterfactual. In both versions of the model, lower relationship costs generate many new customers per firm. In the baseline model, the increase is relatively similar across firms with different productivity levels. In the no-correlation model, however, low-productivity firms gain many more customers relative to high-productivity firms (50% increase versus 30% in the tails of the distribution).[27] Recall that in the baseline, low-productivity firms are also firms with low relationship costs, such that the drop in $\mu_{\ln F}$ will not have a large impact on their connections. In the no-correlation model, by contrast, low-productivity firms are constrained because they face similar relationship costs as other firms. This explains the difference in slopes between the two models.

VI. Conclusion

This paper quantifies the origins of firm size heterogeneity when firms are interconnected in a production network. We report three stylized facts from the production network data that motivate the subsequent analysis and model. First, the enormous dispersion in sales across firms is also observed in the production network in terms of the number of firm-to-firm connections and the value of pairwise sales. Second, firms with higher sales have more customers but lower average sales per customer and lower market shares (of input purchases) among their customers. Finally, there is negative degree assortativity between buyers and suppliers; that is, sellers with more customers match with customers who have fewer suppliers on average.

Taken together, these facts present challenges to many existing models of firm heterogeneity. The large variation in sales across firms within an industry is intuitively related to variation in the number of customers: larger firms have more customers. However, larger firms also sell less to their customers. Models that emphasize heterogeneity in productivity across firms cannot explain these facts simultaneously. In particular, such models imply that firms with more customers should also sell more to each of their customers and have higher rather than lower market shares.

We confirm the importance of the production network in a decomposition of the variance of firm sales within narrowly defined industries. We find that 81% of the variation in firm sales is associated with the downstream component, and most of that is due to variation in the number of customers. The upstream component contributes 18%, and variation

[27] The increase in the total number of firm-to-firm connections in the network is relatively similar in the two models: 26% in the baseline and 28% in the no-correlation model, respectively.

in the share of sales outside the domestic production network plays a minor role at 1%. These findings imply that trade in intermediate goods and the number of firm-to-firm connections are essential to understanding firm performance and, consequently, aggregate outcomes.

Motivated by the stylized facts and decomposition results, we develop a quantitative general equilibrium model of firm-to-firm trade. In the model, firms differ along two dimensions—productivity and relationship capability—defined as production efficiency and (the inverse of) the fixed cost of matching with a customer, respectively. Suppliers match with customers if the gross profits from the match exceed the supplier-specific fixed matching cost. Marginal costs, employment, prices, and sales are endogenous outcomes because they depend on the outcomes of all other firms in the economy. A link between two firms increases the total sales of both the seller and the buyer; for the seller, this occurs mechanically because it gains a customer, while for the buyer, this arises because a larger supplier base lowers the marginal cost of production.

We estimate parameters of the model using SMM. The results reveal a strong negative correlation between the two firm characteristics: firms with higher productivity have lower relationship capability. Importantly, both dimensions of firm heterogeneity are necessary to match the data. Shutting down one at a time results in poor model fit, including the inability to replicate the negative relationship between the number of customers and average sales per customer.

Our results challenge current understanding of the sources of firm size heterogeneity and point to important areas for future research on the negative relationship between firm productivity and relationship capability. While we make progress in matching the relative importance of upstream and downstream factors in firm success, there is room for new models to better fit these features of the production network. In addition, research is needed to examine the factors that lead to a negative relationship between productivity and relationship capability across firms. One promising avenue for further work is examining span of control issues inside the firm and the allocation of resources to improving productivity versus acquiring more customers.

Appendix

Exogenous Mobility

We first discuss the empirical relevance of buyer and seller effects in equation (1). We then examine the necessary assumptions on the assignment process of buyers and sellers for OLS to identify the underlying parameters of interest and develop a simple evaluation of conditional exogenous mobility in the context of a production network.

A1. Buyer and Seller Effects

The log-linear relationship in equation (1) predicts the following: (1) expected sales from seller i to customer j are increasing in the average sales of i to other customers k and (2) expected purchases by buyer j from seller i are increasing in the average purchases by j from other suppliers k.

Both properties can be tested nonparametrically as follows. For each seller i and buyer j, calculate the leave-out mean of log sales ($\bar{s}_i^{-l}$) and purchases ($\bar{m}_j^{-l}$) across its buyers and suppliers, excluding customer/supplier l, respectively:[28]

$$\bar{s}_i^{-l} = \frac{\sum_{j\in C_i\setminus l} \ln m_{ij}}{n_i^c - 1},$$
$$\bar{m}_j^{-l} = \frac{\sum_{k\in S_j\setminus l} \ln m_{kj}}{n_j^s - 1}.$$

Then sort firms into decile groups based on $\bar{s}_i^{-l}$ and $\bar{m}_j^{-l}$, denoting the decile group the firm belongs to as $q_{\bar{s}} = 1, \ldots, 10$ and $q_{\bar{m}} = 1, \ldots, 10$, respectively. Finally, calculate the mean of $\ln m_{ij}$ for every decile group pair, $\overline{\ln m_{q_{\bar{s}},q_{\bar{m}}}}$, for example, the average $\ln m_{ij}$ for the seller-buyer pairs in $(q_{\bar{s}}, q_{\bar{m}}) = (1, 1)$, and so on.

Figure A1 illustrates the results using a heat map. The decile groups $q_{\bar{s}}$ and $q_{\bar{m}}$ are plotted on the horizontal and vertical axes, respectively. $\ln m_{ij}$ is increasing in the average sales from i to other customers k (moving from left to right), and $\ln m_{ij}$ is increasing in the average purchases of j from other suppliers k (moving from bottom to top).

A2. Assumptions on the Assignment Process

Equation (1) is a two-way fixed effects model similar to the models that are used in the employer-employee literature (Abowd, Kramarz, and Margolis 1999; Card, Heining, and Kline 2013).[29] OLS estimates of $\ln \psi_i$ and $\ln \theta_j$ will identify the effect of seller and buyer characteristics if the following moment conditions are satisfied:

$$\begin{cases} E[s_i'r] = 0 \ \forall\ i, \\ E[b_j'r] = 0 \ \forall\ j. \end{cases} \tag{A1}$$

Here $S = [s_1, \ldots, s_N]$ is the $N^* \times N_s$ seller fixed effects design matrix, $B = [b_1, \ldots, b_N]$ is the $N^* \times N_b$ buyer fixed effects design matrix, r is the $N^* \times 1$ vector of residual match effects, and N^*, N_s, and N_b are the number of matches, sellers, and buyers, respectively. The first condition states that for each seller i, the average $\ln \omega_{ij}$ across buyers j is zero, while the second condition states that for each buyer j, the average $\ln \omega_{ij}$ across sellers i is zero. Intuitively, a high $\ln \omega_{ij}$ that is common

[28] Using the overall mean generates a mechanical relationship between, e.g., seller size and sales between i and j. We calculate $\bar{s}_i^{-l}$ and $\bar{m}_j^{-l}$ for all (i, l) and (j, l) pairs, respectively. Firms with only one customer or supplier are by construction omitted from the sample.

[29] The linear fixed effects approach imposes no restrictions on the seller and buyer effects, unlike random or mixed effects models. With random effects, one also needs to model the network formation game to assess the plausibility of the required distributional assumptions for unobserved heterogeneity (see Bonhomme 2020).

across customers j of i will be automatically loaded onto i's seller effect (and similarly for suppliers i of j). In other words, these moment conditions require that the assignment of suppliers to customers is exogenous with respect to ω_{ij}, so-called conditional exogenous mobility in the labor literature.

Exogenous mobility holds more generally than perhaps considered prima facie. It is instructive to review two important cases when these moment conditions hold. First, they hold if firms match on the basis of their seller and buyer effects; for example, highly productive firms match with more and/or different customers/suppliers than less productive ones. Second, the assumption holds if firms match on the basis of idiosyncratic pairwise shocks that are unrelated to $\ln\,\omega_{ij}$. One example of this is idiosyncratic fixed costs, such as costs related to search and matching, which affect profits for a potential match but not the value of bilateral sales.[30]

Now consider the case of endogenous mobility. To fix ideas, assume that matching is based on the idiosyncratic match component of sales, ω_{ij}, together with the seller effect ψ_i. In that case, only high ψ_i sellers would want to match with low ω_{ij} buyers. OLS would then give a downward bias in the estimated ψ_i, because OLS imposes that the average $\ln\,\omega_{ij}$ across customers is zero.

A3. Exogenous Mobility Evaluation

To explore the possibility that matching shocks are correlated with sales shocks, we evaluate conditional exogenous mobility as follows. Consider firm i selling to customers 1 and 2. The expected difference in bilateral sales is

$$\begin{aligned}\Delta\ln\,m_i &\equiv E[\ln\,m_{i2} - \ln\,m_{i1}\mid(i,1),(i,2)]\\ &= \ln\theta_2 - \ln\theta_1 + E[\ln\omega_{i2} - \ln\omega_{i1}\mid(i,1),(i,2)].\end{aligned}$$

Consider the case $\theta_2 > \theta_1$. Under exogenous mobility, the last expectation term is zero, and $\Delta\ln\,m_i$ is unrelated to firm i characteristics. Under endogenous mobility, the last expectation term is nonzero, and $\Delta\ln\,m_i$ is potentially a function of firm i characteristics. Now seller i will want to match with customer 1 only if ω_{i1} is sufficiently large. The expectation $E[\ln\omega_{i2} - \ln\omega_{i1}|(i,1),(i,2)]$ is then negative. Moreover, for small sellers (low ψ_i), the size of ω_{i1} is important for whether a match occurs, while for large sellers (high ψ_i), the size of ω_{i1} is less important (since matching is determined by both ψ_i and ω_{ij}). Under endogenous mobility, the expectation is therefore less negative for high-ψ_i than for low-ψ_i firms, so that $\Delta\ln\,m_i$ is greater for high-ψ_i than for low-ψ_i firms. Under exogenous mobility, by contrast, $\Delta\ln\,m_j$ should be unrelated to ψ_i.[31]

Going back to the seller and buyer decile groups constructed above, we can test these predictions by looking at $\overline{\ln\,m}_{q_s,q_m}$ when moving from a small to a big customer for different groups of sellers. Figure A2 shows the results. Each line represents the mean of log sales for a given seller decile group $(1,\dots,10)$. Within a seller group, we calculate $\overline{\ln\,m}_{q_s,q_m}$ to small customers (buyer decile group 1) and

[30] For example, see the matching framework presented in sec. IV.

[31] Card, Heining, and Kline (2013) test for endogenous mobility for employer-employee matches using a related but different test.

TABLE A1
Exogenous Mobility Test

	Buyer Decile								
Seller Decile	2-1	3-2	4-3	5-4	6-5	7-6	8-7	9-8	10-9
2-1	.01*	.02*	.02*	.03*	.02*	.01	.01*	.01	−.04*
3-2	.04*	.01*	.01	.00	.00	.01*	.00	.01	.02*
4-3	.01	.01*	.02*	.00	.01	.00	.01	.00	.03*
5-4	.00	−.01	.00	.01*	.00	.00	.00	.00	.08*
6-5	.02*	.02*	.01	.00	.01	.01	.02*	.00	.02*
7-6	.04*	.02*	.02*	.01*	.01*	.01	.01	.03*	.00
8-7	.00	.01*	.00	.00	.03*	−.01	.00	−.01	.01
9-8	−.01	.02*	.02*	.01	−.01	.02*	.01	.05*	.08*
10-9	.03*	.04*	.01	.02*	.02*	.02*	.02*	.04*	.19*

Note.—The table shows the double difference from eq. (A2). The t-values are based on Welsh's t-test.

* Significant at the 5% level.

to big customers (buyer decile group 10). Under exogenous mobility, those lines should be parallel; that is, for buyer bins $q_{\bar{m}}$ and $q'_{\bar{m}}$, $\overline{\ln m}_{q_{\bar{s}},q_{\bar{m}}} - \overline{\ln m}_{q_{\bar{s}},q'_{\bar{m}}}$ does not depend on the seller decile group.

The lines are, to a large degree, parallel, in particular for the seller decile groups 2–9. Parallel lines are a sufficient but not necessary condition for exogenous mobility: if the data-generating process is not linear in logs, then one could find nonparallel lines even under exogenous mobility.

One can test for this nonparametrically as follows. If we use the buyer and seller bins defined above, exogenous mobility implies that

$$\overline{\ln m}_{q'_{\bar{s}},q'_{\bar{m}}} - \overline{\ln m}_{q'_{\bar{s}},q_{\bar{m}}} - \left(\overline{\ln m}_{q_{\bar{s}},q'_{\bar{m}}} - \overline{\ln m}_{q_{\bar{s}},q_{\bar{m}}}\right) = 0 \tag{A2}$$

for any bins $q_{\bar{s}}$, $q'_{\bar{s}}$, $q_{\bar{m}}$ and $q'_{\bar{m}}$. We form these averages for $q'_{\bar{s}} = q_{\bar{s}} + 1$ and $q'_{\bar{m}} = q_{\bar{m}} + 1$ and test the null hypothesis that the double difference equals zero. This yields 81 separate hypothesis tests across all buyer-seller pair bins.[32] Overall, the results mirror those in figure A2: the double differences are not significantly different from zero in the middle of the distribution, whereas we find significant deviations in the tails. Significant deviations are typically relatively small: for example, moving from a 6th to 7th decile buyer yields 12% more sales for seller decile 9 and 14% more sales for seller decile 10, a difference of 2%.

We report 81 separate hypothesis tests across all buyer-seller pair bins in table A1. Each column refers to the change from buyer decile t to $t + 1$, and each row refers to the change from seller decile t to $t + 1$. For example, the cell (3-2,2-1) reports the difference $\overline{\ln m}_{3,2} - \overline{\ln m}_{3,1} - (\overline{\ln m}_{2,2} - \overline{\ln m}_{2,1})$.

[32] t-values are calculated using Welch's t-test.

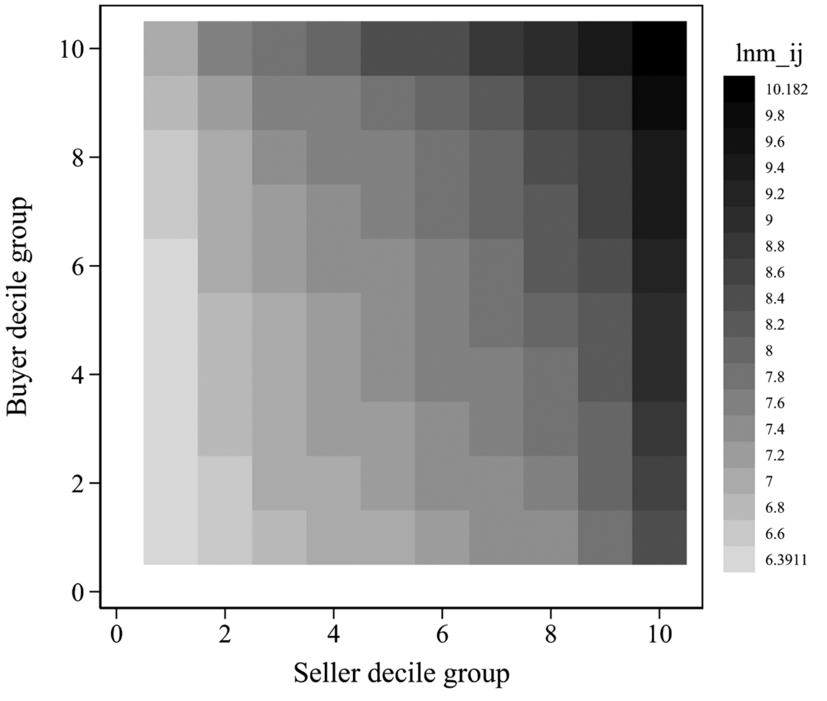

FIG. A1.—Average log sales across seller and buyer decile groups. The figure shows the average of ln m_{ij} in all decile group pairs $(q_{\bar{s}}, q_{\bar{m}})$.

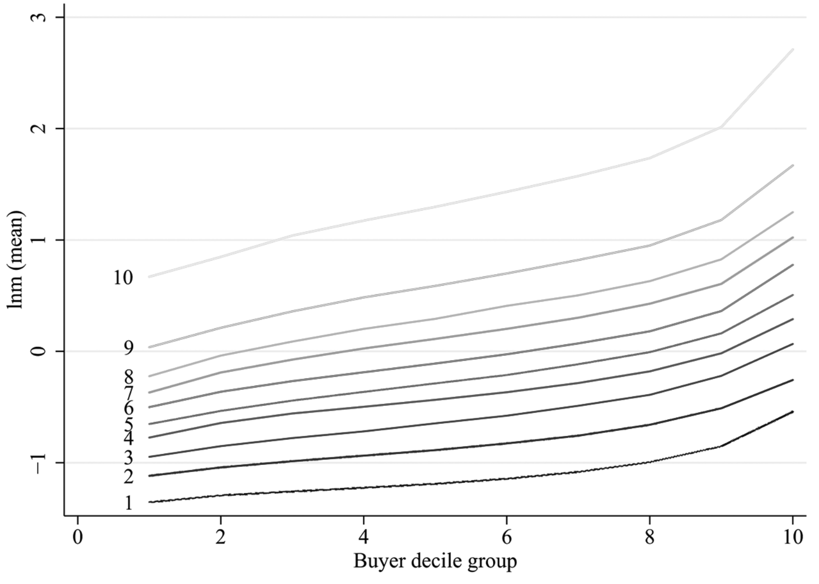

FIG. A2.—Average log sales across seller and buyer decile groups. The figure shows $\ln \bar{m}_{ij}$ across buyer decile groups $q_{\bar{m}} = 1, \ldots, 10$. Each line represents a seller decile group, $q_{\bar{s}} = 1, \ldots, 10$. A color version of this figure is available online.

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