

A fuzzy DEMATEL-ANP-VIKOR analytical model for maintenance strategy selection of safety critical assets

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Abstract

In most industries, such as aerospace, manufacturing, transport and energy sectors, maintenance plays a vital role in improving the performance of safety critical equipment and facilities. It also helps industries achieve the largest possible efficiency, ensure workplace and environmental safety, and reduce unnecessary breakdowns and costs. Therefore, it is crucial for industries to adopt an optimal maintenance strategy for their critical systems and infrastructure. In this study, we aim to propose a novel analytical multi-criteria decision-making (MCDM) methodology for selecting the most suitable maintenance strategy in distillation units of oil refinery plants. The alternative maintenance strategies include run-to-failure (RTF), preventive maintenance (PM), condition-based maintenance (CBM), and reliability centered maintenance (RCM), which are evaluated with respect to 12 sub-criteria in three categories of economical, safety, and sustainability issues. The MCDM methodology consists of a DEMATEL-based analytic network process (ANP) method to determine the importance weights of decision criteria and a VIKOR method to rank the maintenance strategies. Also, interval type-2 fuzzy sets are used to capture uncertainty in experts' individual judgments. Finally, a real case study is provided to show the applicability of the proposed methodology to an oil refinery plant. The results show that, thanks to advances in degradation modeling, sensor technology, and data analytics platforms, the RCM and CBM are the superior maintenance strategy for crude oil distillation systems.

Keywords

Maintenance strategy selection, multi-criteria decision-making, DEMATEL, ANP, VIKOR, interval type-2 fuzzy

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Introduction

Over the past decades, industrial maintenance has evolved from a set of tasks executed by operators to maintain equipment into a more strategic management issue.¹ A survey conducted by MIT showed that over \$200 billion is spent annually on maintenance by companies in North America.² It is indispensable to mention that maintenance costs may rise to 70% of the total operational expenses or even could exceed annual

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net profit in some cases.³ Thus, industries are under increasing pressure to reduce their expenditure while enhancing customer service.⁴ To achieve this aim, organizations and companies need to adopt an efficient and effective maintenance strategy for their critical equipment and facilities.

The development of a maintenance strategy can help businesses provide a plan of action containing specific recommendations on how to maintain their assets in a safe and serviceable condition. The implementation of this plan can result in a significant increase in the availability of assets, workplace safety, and environmental integrity. Up to now, many different types and approaches of asset maintenance management have been proposed by researchers and practitioners. Recent advances in technology and data science have also made it possible to detect faults within the system and predict potential breakdowns. New maintenance strategies such as condition-based maintenance (CBM), reliability centered maintenance (RCM), and predictive maintenance (PdM) are considered as promising technologies to monitor the condition of the equipment and determine its health and performance. However, the initial cost of implementing such technologies can be very high as they require a significant investment in sensor equipment and/or staff up-skilling.

Determining an optimal maintenance management strategy is one of the most important decision-making processes in industrial organizations.⁵ Choosing the most suitable maintenance strategy among a set of available options for a piece of equipment involves numerous evaluation criteria, such as cost, safety, time, added-value, reliability, etc. In addition, today's concerns about global warming, depletion of energy resources, and increased greenhouse gas emission levels have introduced several environmental, social and governance factors, known as "sustainability" indicators, to consider in the maintenance decision-making.⁶ Poor maintenance practice in industrial plants may result in increased energy consumption, waste, and greenhouse gas emissions, and may cause severe water, air, and soil pollutions. Every unplanned shutdown could also have negative impacts on habitats in the neighborhood area as well as the families of personnel and customers. Therefore, sustainability factors play an important role in the evaluation of maintenance strategies.

To solve the maintenance strategy selection problem, there is often a need to collect qualitative and quantitative failure data from many sources (such as interviews of stakeholders, observations from the field, historical records, etc.) and then compare different maintenance strategies with each other with respect to multiple criteria. Therefore, the maintenance strategy selection process is considered as a group multi-criteria decision making (MCDM) problem.^{7,8} MCDM is a

technique that can be used to solve decision-making problems where multiple criteria, often conflicting, must be considered.⁹ In recent years, several MCDM techniques and approaches such as simple additive weighting (SAW), analytic hierarchy process (AHP), analytical network process (ANP), technique of order preference similarity to the ideal solution (TOPSIS), etc. have been suggested to determine the optimal maintenance strategy for industrial assets. For a comprehensive review of the literature regarding the application of MCDM in maintenance decision-making, the readers can refer to Shafiee.¹⁰

The diversity of components, complexity of failure mechanisms, and existence of various dependencies among components in engineering systems have caused the process of maintenance strategy selection to be very complicated.¹¹ In addition, some of the maintenance evaluation criteria are non-financial and therefore hard to convert into a sensible measure.⁴ In such cases, it is more convenient for experts to express their opinions in linguistic terms than in numerical terms. The emerging methodology of fuzzy-set theory provides the necessary tools for dealing with such judgmental imprecision and uncertainty. The fuzzy set theory uses fuzzy numbers to capture the imprecision or vagueness in expert linguistic assessments. In the present study, interval type-2 fuzzy numbers which are a subset of type-2 generalized fuzzy numbers are used to handle uncertainty arising from judgment of multiple experts.

The objective of this paper is to propose a novel hybrid fuzzy MCDM methodology for selecting the most suitable maintenance strategy in safety critical equipment and facilities. The alternative maintenance strategies include RTF, PM, CBM, and RCM which are evaluated with respect to three distinct criteria, namely *cost*, *safety*, and *sustainability*. These criteria are further broken down into 12 sub-criteria, such as: cost of materials, cost of manpower, mean-time between failures (MTBF), mean-time to failure (MTTR), acceptance by personnel, energy consumption, and environment protection. The decision-making process consists of a DEMATEL-based ANP method to determine the importance weights of decision criteria and a VIKOR method to rank the maintenance strategies. The applicability of the proposed methodology is shown through a real case study of an oil refinery plant. The results indicate that our methodology has huge potential to enhance the performance of maintenance decision-making process, making it more user-friendly and efficient in use.

The remainder of this study is organized as follow. Section 2 provides a comprehensive review of the literature related to the selection of an optimal maintenance management strategy with the use of MCDM methodology. Section 3 gives a brief introduction of interval type-2 fuzzy sets. Section 4 presents the proposed

methodology. Section 5 discusses the results of the case study. Finally, section 6 concludes the paper and suggests further possible works.

Literature review

A brief review of the literature shows that many studies have been conducted to identify an optimal maintenance strategy for different industry sectors, ranging from oil and gas to railway transportation and pharmaceutical industry to mining. The MCDM methodology, as one of the most common maintenance decision-making approaches, has received reasonable attention from the research community over the last two decades. Table 1 summarizes some of the most relevant studies with a focus on the use of MCDM techniques to solve the maintenance strategy selection problem.

Some observations from the literature review are as follows:

- The classical MCDM methods have their own strengths and weaknesses. The integration of MCDM models can overcome the limitations of the individual methods and improve the efficiency of the decision-making process. The hybrid MCDM methods have received the most attention lately. However, to the best of authors knowledge, there is no study integrating the MCDM techniques of DEMATEL, ANP, and VIKOR to solve the maintenance strategy selection problem. This study proposes a hybrid DEMATEL-ANP-VIKOR approach to determine the importance weights of decision-making criteria and evaluate the performance of different maintenance strategies.
- Fuzzy set theory is a powerful tool to deal with vagueness of human thoughts and take the imprecision of qualitative assessments into consideration. The fuzzy MCDM approaches have received increasing attention for the analysis of maintenance strategies. This study, for the first time, proposes an interval type-2 fuzzy set (as a generalization of the interval-valued fuzzy sets in Vahdani and Hadipour⁹) to characterize the uncertainties associated with experts' judgments.
- Although cost and safety are important criteria, the sustainability factors must not be ignored. The review showed that there is very little knowledge about how sustainability factors affect maintenance decision-making. This paper involves all the economic, societal, and environmental factors related to sustainability of maintenance strategies.
- No study was found investigating the efficiency of different maintenance strategies for

distillation units in the oil refining industry. This paper provides a real case study of determining the best maintenance strategy in a crude oil distillation unit.

Interval type-2 fuzzy sets

The theory of type-2 fuzzy set (T2FS) was introduced for the first time by Zadeh³⁵ as an extension for traditional or type-1 fuzzy set (T1FS). Mendel et al.³⁶ provided numerous examples about the interval type-2 fuzzy sets (IT2-FS). T2FS is called a "fuzzy-fuzzy" set since it has a membership function whose membership grade is a T1FS in the [0,1] interval. Although T2FS is more practical for characterizing the uncertainty and imperfection in the data, IT2-FS is exploited to overcome computational complexity and difficulties of T2FS in practical settings.³⁷ In this section, some fundamental concepts of IT2-FS are briefly introduced, which will be used in the subsequent sections.

Definition 1. A type-2 fuzzy set denoted by \tilde{A} is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, and is defined by equation (1):³⁸

$$\tilde{A} = \{(x, u), \mu_{\tilde{A}}(x, u)\} | \forall x \in X, \forall u \in J_x \subseteq [0, 1] \quad (1)$$

It can also be represented by equation (2):

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{\mu_{\tilde{A}}(x, u)}{(x, u)} = \int_{x \in X} \left(\int_{u \in J_x} \frac{\mu_{\tilde{A}}(x, u)}{u} \right) / x \quad (2)$$

The symbol \int switches to \sum for discrete space.

Definition 2. The footprint of uncertainty (FOU) is a bounded region for uncertainty in \tilde{A} , and is defined as follows:³⁸

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x = \bigcup_{x \in X} \{(x, u) | u \in J_x \subseteq [0, 1]\} \quad (3)$$

Definition 3. \tilde{A} is called an IT2-FS when $\mu_{\tilde{A}}(x, u) = 1$. Therefore, an IT2-FS can be considered as a special case of a type-2 fuzzy set and is expressed by equation (4)³⁹:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{1}{(x, u)} = \int_{x \in X} \left(\int_{u \in J_x} \frac{1}{u} \right) / x \quad (4)$$

Definition 4. \tilde{A}_U and \tilde{A}_L are two trapezoidal IT2-FSs with heights between zero and one for fuzzy numbers. In general, two type-1 membership functions constitute the upper and the lower membership functions of an IT2-FS. Let $\tilde{A} = (\tilde{A}_U, \tilde{A}_L) = ((a_1^U, a_2^U, a_3^U, 4^U; H_1(\tilde{A}_U), H_2(\tilde{A}_U)), a_1^L, a_2^L, a_3^L, a_4^L;$

Table I. Summary of some important studies on the use of MCDM to solve maintenance strategy selection.

Reference	Summary
Bevilacqua and Braglia ¹²	This study adopted an AHP methodology to determine the most appropriate maintenance strategy for an Italian oil refinery plant. The maintenance strategies included: CM, PM, OM, CBM, and PdM, which were compared with respect to four criteria of damage, applicability, added value, and cost. The results showed that PM and PdM were the most suitable strategies for safety critical machines, whereas CM was found to be a proper strategy for non-critical components in the plant.
Mechefske and Wang ⁴	This study proposed a fuzzy MCDM methodology to evaluate and select the best maintenance strategy among BM, SM, and CBM (based on three techniques of thermal monitoring, oil analysis, and vibration analysis). The approach was then applied to different types of machines, including centrifugal pumps, centrifugal compressors, and reciprocating compressors.
Al-Najjar and Alsyouf ¹³	This study proposed a fuzzy MCDM methodology to select the most efficient maintenance strategy among FBM, TBPM, VBM, TPM, RCM, and TQMain. The approach was tested on two case studies: a paper mill bearing and a pump station.
Alsyouf ⁷	This study developed MCDM models using the SAW method to select the most cost-effective maintenance strategy in power stations, paper mills, and hydraulic machines within different Swedish industries. It was shown that the use of VBM for maintenance planning saved repair costs to some extent.
Sharma et al. ¹⁴	This study proposed a fuzzy linguistic modeling approach to select the best maintenance strategy among BM, TBPM, CBM, TPM, and RCM for a gear system. Inputs of the fuzzy model included historical data related to failure of the gear system such as its operation mode, load, speed, and lubricant quality. The results demonstrated that CBM and TPM were the most advantageous maintenance strategies.
Bertolini and Bevilacqua ¹⁵	This study proposed an integrated AHP-GP (goal programming) approach to determine the most suitable maintenance strategy for centrifugal pumps in an oil refinery. The maintenance strategies included CM, TBPM, and PdM, which were compared with respect to how much they contributed to the reduction in occurrence, severity, and detection of failures.
Wang et al. ¹⁶	This study proposed a fuzzy AHP methodology with a new prioritization technique to solve the maintenance strategy selection problem in a power plant. Safety, cost, added value, and feasibility were considered as evaluation criteria for the decision-making process. The results of the study showed that PdM was the most suitable maintenance strategy for boilers.
Gaonkar et al. ¹⁷	This study presented an approach based on Saaty's priority theory and fuzzy arithmetic (α -cuts) to select the best maintenance strategy among CM, TBPM, and PdM. The evaluation criteria included: cost, reliability, safety, product quality, inventory, return on investment, acceptance by labor, and competitiveness. The results showed that the most suitable strategy was PdM.
Ierace and Cavalieri ¹	This study proposed a comparative analysis between the AHP technique and a fuzzy linguistic approach for selecting the most suitable maintenance strategy in an Italian manufacturing firm. The comparison was made in terms of three performance metrics, namely, reliability of the model, easiness to construct, and easiness to use the model. The results showed that the AHP was better in terms of the first two metrics, however the fuzzy approach was user friendlier.
Ahmadi et al. ¹⁸	This study presented an MCDM approach, which was a combination of AHP, VIKOR, TOPSIS, and benefit-cost analysis methods, to rank various maintenance strategies based on different preferences. The model was tested on an aircraft system and the results revealed that "prognostics and health management (PHM)" was the most favorable maintenance strategy, followed by "functional check" and "restoration."
Arunraj and Maiti ¹⁹	This study proposed an integrated AHP-GP approach for evaluating the maintenance strategies in a benzene extraction unit of a chemical plant. The results showed that, considering risk as a criterion, CBM was a preferred strategy over TBPM.
Vahdani and Hadipour ⁹	This study proposed a fuzzy ELECTRE method to solve the maintenance strategy selection problem, where the weights of criteria were expressed by interval-valued fuzzy numbers. The maintenance alternatives included: FBM, TBPM, and CBM, which were compared with each other with respect to six criteria of cost, acceptance by labors, reliability, competitiveness, product quality, and inventories.
Bashiri et al. ²⁰	This study proposed an interactive fuzzy linear assignment method for selecting the most suitable maintenance strategy between CM, TBPM, CBM, and PdM based on six criteria. The experts' opinions are incorporated by linguistic variables using trapezoidal fuzzy numbers.
Tan et al. ²¹	This paper adopted an AHP and ANP methodology to determine the most suitable maintenance strategy for Fujian Oil Refinery ISOMAX unit in China. The decision-making criteria were same as those considered in Wang et al. ¹⁶ The results showed that RCM was the best strategy for unsatisfactory areas, PM for critical areas, and CM for tolerable areas.
Chan and Prakash ²²	This study developed a fuzzy MCDM approach to choose the most appropriate maintenance strategy in manufacturing firms. The evaluation criteria included: capital cost, running cost, maintenance downtime, reliability, capability, repair load, operator skills, flexibility, efficiency, facility utilization, and resource availability.

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Reference	Summary
Nezami and Yildirim ⁶	This study proposed a fuzzy VIKOR methodology with sustainability considerations to solve the maintenance strategy selection problem in a manufacturing company. The concept of factor analysis was applied to determine the leading factors of the sustainability pillars.
Nakhjavani et al. ²³	This study proposed a combined model of ANP and DEMATEL to select the most suitable maintenance strategy for transformer equipment in the pharmaceutical industry. Seven criteria, including output quality, availability, overall equipment effectiveness (OEE), productivity, maintenance quality, and maintainability were considered. The results ranked the RCM strategy as the most suitable maintenance strategy.
Baidya and Ghosh ²⁴	This study adopted the AHP methodology to evaluate the effectiveness of four CBM strategies, including: vibration analysis, acoustic emission, oil analysis, and shock pulse. The results showed that acoustic emission is the most effective technology for gear and bearing systems.
Vishnu and Regikumar ⁵	This study adopted the AHP methodology to find the most appropriate maintenance strategy for a titanium dioxide process plant based on the data obtained from an RCM program. Experts from both maintenance and production departments were involved in the decision-making.
Tajadod et al. ²⁵	This study proposed a comparative analysis between AHP, ANP, fuzzy AHP, and fuzzy ANP techniques for selecting the most suitable maintenance strategy in a dairy manufacturing factory. The experts' opinions were incorporated by triangular fuzzy numbers and combined using an AIP (aggregation of individual priorities) approach. The priority vectors of decision elements were calculated by the Mikhailov's fuzzy preference programming.
Seiti et al. ¹¹	This study proposed a risk based AHP method to choose the most appropriate maintenance strategy in a steel rolling company. The maintenance alternatives included: BM, TBPM, TPM, and CBM, which were compared with each other with respect to four criteria of added value, efficiency, damage, and reliability.
Srivastava et al. ²⁶	This study provided a comparative analysis between fuzzy AHP, fuzzy TOPSIS, and graphical methodologies to evaluate and prioritize five maintenance strategies, including: CM, TBPM, CBM, PdM, and OM according to four criteria of cost, safety, added value, and the ease of execution. All methods concluded that PdM was the most appropriate maintenance strategy for a steam generating unit in a power plant.
Baidya et al. ²⁷	This study proposed a method which was the combination of quality function deployment (QFD), AHP, and benefit of doubt (BoD) to prioritize the CBM techniques studied in Baidya and Ghosh. ²⁴ The method was tested on a bearing system, a gear system, and a lubricating system in an Indian manufacturing organization.
Borjalilu and Ghambari ²⁸	This study proposed a fuzzy ANP method to select the best maintenance strategy for a 5-MW powerhouse unit. The maintenance strategies included CM, TBPM, CBM, RCM, and PdM which were evaluated with respect to five criteria: organization, safety, administration, staff, and technical requirements. The results showed that PdM was the best maintenance strategy.
Hemmati et al. ²⁹	This study adopted a fuzzy ANP model to select the best maintenance strategy for different types of equipment in an acid manufacturing plant (e.g. boiler, molten sulfur ponds, cooling towers, absorption tower, converter, sulfur fuel furnace, and heat exchanger). The maintenance strategies included CM, BM, TBPM, and CBM, which were evaluated with respect to three criteria of risk, added value, and cost.
Asuquo et al. ²	This study adopted the TOPSIS methodology to rank the maintenance strategies of marine and offshore machinery with respect to their costs and benefits. The method allowed to incorporate and aggregate the subjective opinions of multiple decision makers.
Ighravwe and Oke ³⁰	This study proposed a model by combining four methods of stepwise weight assessment ratio analysis (SWARA), weighted additive sum product assessment (WASPAS), fuzzy axiomatic design (FAD), and additive ratio assessment (ARAS) to select the most appropriate maintenance strategy for public buildings. PdM was ranked as the best strategy, followed by PM and CBM.
Shafiee et al. ³¹	This study proposed a model which was a combination of ANP and cost-risk criticality analysis to choose the best maintenance strategy for multi-component systems. The aim of the model was to find the best maintenance strategy that meets the goals of minimum cost and maximum reliability for the system. The approach was tested on a new wind turbine technology consisting of several mechanical, electrical, and auxiliary components at the design stage.
Wang and Piao ³²	This study adopted the AHP methodology to select the best maintenance strategy for a single piece of equipment and then proposed a fuzzy MCDM approach to determine the maintenance priorities of equipment components in the initial stage of operation and maintenance (O&M).
Ighravwe et al. ³³	This study adopted a TOPSIS methodology to evaluate and rank four maintenance strategies for an off-grid PV-powered street lightning system. The methodology simultaneously contemplated all the social, technical, economic, environmental and policy factors and presented a SWOT (strength, weakness, opportunity, and threats) matrix to analyze the maintenance strategies. The most appropriate maintenance strategy was chosen to be CBM.

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Reference	Summary
Panwar et al. ³⁴	This study proposed an integrated AHP-VIKOR methodology to rank the maintenance strategies for a pulping system in the paper industry. The ranking was made based on five criteria: failure likelihood, detectability, downtime duration, standby parts, and safety risks.

BM: breakdown maintenance; CBM: condition-based maintenance; CM: corrective maintenance; FBM: failure-based maintenance; OM: opportunistic maintenance; PdM: predictive maintenance; RCM: reliability-centered maintenance; SM: scheduled maintenance; TBPM: time-based preventive maintenance; TPM: total productive maintenance; TQMain: total quality maintenance; VBM: vibration-based monitoring.

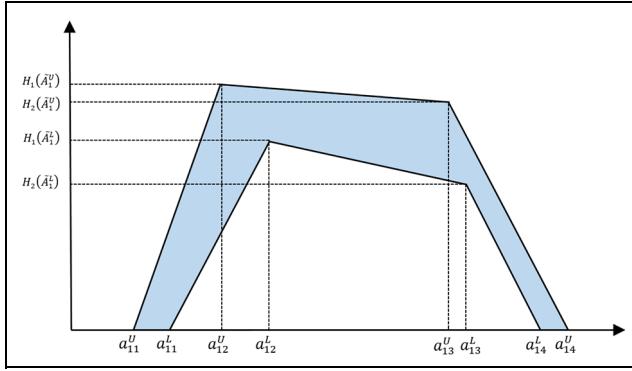


Figure 1. The geometrical representation of a trapezoidal IT2-FS number.

$H_1(\tilde{A}_1)$, $H_2(\tilde{A}_1)$) be a trapezoidal IT2-FS, where $a_1^U, a_2^U, a_3^U, a_4^U, a_1^L, a_2^L, a_3^L$ and a_4^L are real numbers which hold for $a_1^U \leq a_2^U \leq a_3^U \leq a_4^U \leq a_1^L \leq a_2^L \leq a_3^L \leq a_4^L$. $H_j(\tilde{A}_U) \in [0, 1]$ for $1 \leq j \leq 2$ is the membership value of the element $a_{(j+1)}^U$ in upper trapezoidal membership function \tilde{A}_U , and $H_j(\tilde{A}_L) \in [0, 1]$ for $1 \leq j \leq 2$ is the membership value of the element $a_{(j+1)}^L$ in lower trapezoidal membership function \tilde{A}_L .⁴⁰ The geometrical representation of a trapezoidal IT2-FS number is shown in Figure 1.

Definition 5. Assume \tilde{A}_1 and \tilde{A}_2 are two IT2-FSs given by:

$$\begin{aligned}\tilde{A}_1 &= (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U), a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L))), \\ \tilde{A}_2 &= (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U), a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L))),\end{aligned}$$

Therefore, the main arithmetic operations for IT2-FSs are defined as follows:⁴¹

$$\tilde{A}_1 + \tilde{A}_2 = (\tilde{A}_1^U, \tilde{A}_1^L) + (\tilde{A}_2^U, \tilde{A}_2^L) = \left(\left(\begin{array}{c} a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \\ \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)) \end{array} \right), \right), \quad (5)$$

$$\tilde{A}_1 - \tilde{A}_2 = (\tilde{A}_1^U, \tilde{A}_1^L) - (\tilde{A}_2^U, \tilde{A}_2^L) = \left(\left(\begin{array}{c} a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U; \\ \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)) \end{array} \right), \right), \quad (6)$$

$$\tilde{A}_1 \times \tilde{A}_2 = (\tilde{A}_1^U, \tilde{A}_1^L) \times (\tilde{A}_2^U, \tilde{A}_2^L) = \left(\left(\begin{array}{c} a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \\ \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)) \end{array} \right), \right), \quad (7)$$

$$k\tilde{A}_1 = \left(\left(\begin{array}{c} (k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \\ (k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \end{array} \right), \right), \quad (8)$$

$$\frac{\tilde{A}_1}{k} = \left(\left(\begin{array}{c} (\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \\ (\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \end{array} \right), \right). \quad (9)$$

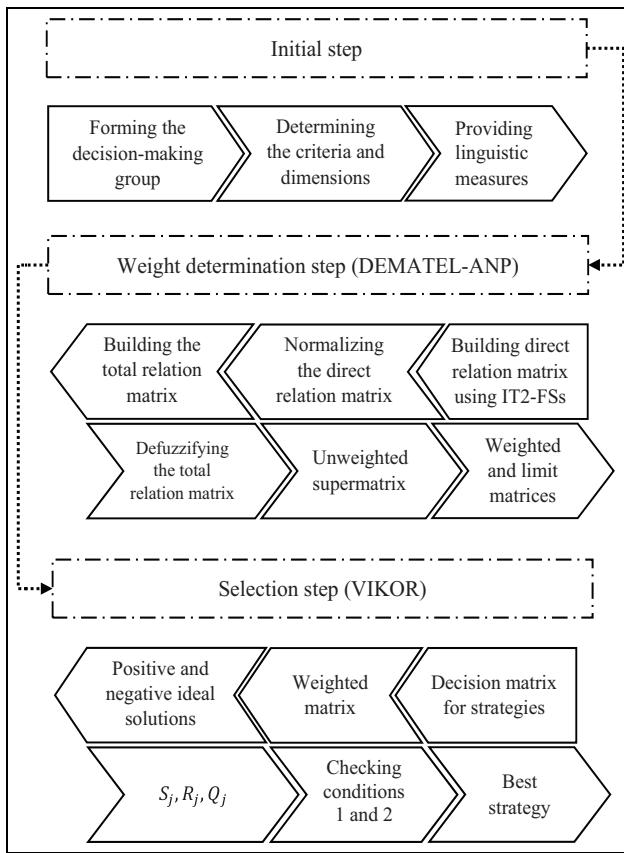


Figure 2. The proposed fuzzy group MCDM methodology for maintenance strategy selection.

Definition 6. The deviation degree of two trapezoidal IT2-FS numbers is defined by equation (10):⁴¹

$$D(\tilde{A}_1, \tilde{A}_2) = \sqrt{D^U + D^L}, \quad (10)$$

where:

$$\begin{aligned} D^U &= (a_{11}^U - a_{21}^U)^2 + (a_{12}^U H_1(\tilde{A}_1^U) - a_{22}^U H_1(\tilde{A}_2^U))^2 \\ &\quad + (a_{13}^U H_2(\tilde{A}_1^U) - a_{23}^U H_2(\tilde{A}_2^U))^2 + (a_{14}^U - a_{24}^U)^2, \\ D^L &= (a_{11}^L - a_{21}^L)^2 + (a_{12}^L H_1(\tilde{A}_1^L) - a_{22}^L H_1(\tilde{A}_2^L))^2 \\ &\quad + (a_{13}^L H_2(\tilde{A}_1^L) - a_{23}^L H_2(\tilde{A}_2^L))^2 + (a_{14}^L - a_{24}^L)^2. \end{aligned}$$

Definition 7. The crisp value of a trapezoidal IT2-FS number is defined by equation (11):⁴⁰

$$CR(\tilde{A}) = \frac{[a_1^U + a_2^U H_1(\tilde{A}_U) + a_3^U H_2(\tilde{A}_U) + a_4^U + a_1^L] + [a_2^L (1 + H_1(\tilde{t}_{ij}^L)) + a_3^L (1 + H_2(\tilde{t}_{ij}^L)) + a_4^L]}{8} \quad (11)$$

For further reading on the interval type-2 fuzzy sets the readers can refer to Li et al.⁴²

The proposed methodology

In real-world situations, the decision-makers might encounter uncertainty when assigning numerical values to their preferences. The maintenance decision-making process involves subjective judgments by domain experts, which may be inconsistent or sometimes contradictory. In this study, we propose a fuzzy group decision-making methodology to overcome difficulties of subjective assessments.

Figure 2 represents the flowchart of the proposed maintenance strategy selection methodology in the IT2-FS environment. As can be seen, a three-step process is employed to solve the decision-making problem. These steps include: (1) initial step, where the decision makers define linguistic measures for the evaluation of criteria, dimensions, and maintenance strategies using IT2-FS models; (2) weight determination step, where the DEMATEL-ANP (DANP) method is applied to identify the importance weights of criteria and sub-criteria; and (3) maintenance selection step, where the VIKOR method is used to prioritize the maintenance strategies.

In what follows, the details of implementing the interval type-2 fuzzy DEMATEL-ANP-VIKOR technique are presented.

Interval type-2 fuzzy DANP

The DANP is a novel MCDM method that combines the individual DEMATEL and ANP techniques. The DANP method is used to identify the relationships among criteria, and then obtain the importance weights of criteria. This hybrid MCDM method is evolved from Dinçer et al.⁴³ and contains the following steps:

Step 1: The experts' opinions are collected in the form of fuzzy preference relations. The experts are asked to fill out the relation matrix using some linguistic variables as defined in Table 2.

Step 2: The linguistic variables are converted to IT2-FS numbers, and the aggregated initial relation matrix $\tilde{\mathbf{K}}$ is computed by equation (12):

$$\tilde{\mathbf{K}} = [\tilde{\kappa}^{(1)} + \tilde{\kappa}^{(2)} + \dots + \tilde{\kappa}^{(P)}]/P \quad (12)$$

where P is the number of decision makers. Therefore, the aggregated initial direct relation matrix is given by:

$$CR(\tilde{A}) = \frac{[a_1^U + a_2^U H_1(\tilde{A}_U) + a_3^U H_2(\tilde{A}_U) + a_4^U + a_1^L] + [a_2^L (1 + H_1(\tilde{t}_{ij}^L)) + a_3^L (1 + H_2(\tilde{t}_{ij}^L)) + a_4^L]}{8} \quad (11)$$

Table 2. Linguistic variables used for comparing criteria.

Linguistic variables	IT2-FS numbers
Very low (VL)	((0,0,0,0;1,1,0,0,0,0;0.9,0.9))
Low (L)	((0,0,0,0.1;1,1,0,0,0,0.05;0.9,0.9))
Low medium (LM)	((0,0,1,0.1,0.3;1,1,0.05,0.1,0.1,0.2;0.9,0.9))
Medium (M)	((1,0,0.3,0.3,0.5;1,1,0.2,0.3,0.3,0.4;0.9,0.9))
High medium (HM)	((0.3,0.5,0.5,0.7;1,1,0.4,0.5,0.5,0.6;0.9,0.9))
High (H)	((0.5,0.7,0.7,0.9;1,1,0.6,0.7,0.7,0.8;0.9,0.9))
Very high (VH)	((0.7,0.9,0.9;1,1,0.8,0.9,0.9,0.95;0.9,0.9))

$$\tilde{K} = \begin{bmatrix} 0 & \tilde{k}_{12} & \dots & \tilde{k}_{1m} \\ \tilde{k}_{21} & 0 & \dots & \tilde{k}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{k}_{m1} & \tilde{k}_{m2} & \dots & 0 \end{bmatrix} \quad (13)$$

where $\tilde{k}_{ij} = (\alpha_{ij}^U, \beta_{ij}^U, \gamma_{ij}^U, \delta_{ij}^U; H_1(\tilde{k}_{ij}^U), H_2(\tilde{k}_{ij}^U), \alpha_{ij}^L, \beta_{ij}^L, \gamma_{ij}^L, \delta_{ij}^L; H_1(\tilde{k}_{ij}^L), H_2(\tilde{k}_{ij}^L))$.

Step 3: The direct relation matrix is normalized. To do so, the initial direct relation matrix \tilde{K} is converted to eight $m \times m$ matrices, namely $\tilde{K}_{\alpha^U}, \tilde{K}_{\beta^U}, \tilde{K}_{\gamma^U}, \tilde{K}_{\delta^U}, \tilde{K}_{\alpha^L}, \tilde{K}_{\beta^L}, \tilde{K}_{\gamma^L}$, and \tilde{K}_{δ^L} , as given below:

$$\begin{aligned} \tilde{K}_{\alpha^U} &= \begin{bmatrix} 0 & \alpha_{12}^U & \dots & \alpha_{1m}^U \\ \alpha_{21}^U & 0 & \dots & \alpha_{2m}^U \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1}^U & \alpha_{m2}^U & \dots & 0 \end{bmatrix}, \\ \tilde{K}_{\beta^U} &= \begin{bmatrix} 0 & \beta_{12}^U & \dots & \beta_{1m}^U \\ \beta_{21}^U & 0 & \dots & \beta_{2m}^U \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1}^U & \beta_{m2}^U & \dots & 0 \end{bmatrix}, \dots, \\ \tilde{K}_{\delta^L} &= \begin{bmatrix} 0 & \delta_{12}^L & \dots & \delta_{1m}^L \\ \delta_{21}^L & 0 & \dots & \delta_{2m}^L \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{m1}^L & \delta_{m2}^L & \dots & 0 \end{bmatrix} \end{aligned} \quad (14)$$

The largest element in \tilde{K}_{δ^U} matrix is found and used to obtain the normalization coefficient (Y) as defined in equation (15):

$$Y = \max \left(\max_{1 \leq i \leq m} \sum_{j=1}^m \tilde{K}_{\delta_j^U}, \max_{1 \leq j \leq m} \sum_{i=1}^m \tilde{K}_{\delta_i^U} \right) \quad (15)$$

The normalized direct relation matrix, $\tilde{\Omega}$ is given by equation (16):

$$\tilde{\Omega} = \begin{bmatrix} 0 & \tilde{\omega}_{12} & \dots & \tilde{\omega}_{1m} \\ \tilde{\omega}_{21} & 0 & \dots & \tilde{\omega}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\omega}_{m1} & \tilde{\omega}_{m2} & \dots & 0 \end{bmatrix} \quad (16)$$

where the matrix elements are calculated by equation (17):

$$\begin{aligned} \tilde{\omega}_{ij} &= \frac{\tilde{k}_{ij}}{Y} \\ &= \left(\begin{array}{l} \frac{\tilde{K}_{\alpha_j^U}}{Y}, \frac{\tilde{K}_{\beta_j^U}}{Y}, \frac{\tilde{K}_{\gamma_j^U}}{Y}, \frac{\tilde{K}_{\delta_j^U}}{Y}; H_1(\tilde{k}_{ij}^U), H_2(\tilde{k}_{ij}^U), \\ \frac{\tilde{K}_{\alpha_j^L}}{Y}, \frac{\tilde{K}_{\beta_j^L}}{Y}, \frac{\tilde{K}_{\gamma_j^L}}{Y}, \frac{\tilde{K}_{\delta_j^L}}{Y}; H_1(\tilde{k}_{ij}^L), H_2(\tilde{k}_{ij}^L) \end{array} \right) \end{aligned} \quad (17)$$

Step 4: The total relation matrix is calculated in a similar way as the initial direct relation matrix in step 3 was calculated. Thus, we consider eight $m \times m$ matrices, namely $\tilde{\Omega}_{\alpha^U}, \tilde{\Omega}_{\beta^U}, \tilde{\Omega}_{\gamma^U}, \tilde{\Omega}_{\delta^U}, \tilde{\Omega}_{\alpha^L}, \tilde{\Omega}_{\beta^L}, \tilde{\Omega}_{\gamma^L}$, and $\tilde{\Omega}_{\delta^L}$, that are given by equation (18):

$$\begin{aligned} \tilde{\Omega}_{\alpha^U} &= \begin{bmatrix} 0 & \tilde{\omega}_{\alpha_{12}^U} & \dots & \tilde{\omega}_{\alpha_{1m}^U} \\ \tilde{\omega}_{\alpha_{21}^U} & 0 & \dots & \tilde{\omega}_{\alpha_{2m}^U} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\omega}_{\alpha_{m1}^U} & \tilde{\omega}_{\alpha_{m2}^U} & \dots & 0 \end{bmatrix}, \\ \tilde{\Omega}_{\beta^U} &= \begin{bmatrix} 0 & \tilde{\omega}_{\beta_{12}^U} & \dots & \tilde{\omega}_{\beta_{1m}^U} \\ \tilde{\omega}_{\beta_{21}^U} & 0 & \dots & \tilde{\omega}_{\beta_{2m}^U} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\omega}_{\beta_{m1}^U} & \tilde{\omega}_{\beta_{m2}^U} & \dots & 0 \end{bmatrix}, \dots, \\ \tilde{\Omega}_{\delta^L} &= \begin{bmatrix} 0 & \tilde{\omega}_{\delta_{12}^L} & \dots & \tilde{\omega}_{\delta_{1m}^L} \\ \tilde{\omega}_{\delta_{21}^L} & 0 & \dots & \tilde{\omega}_{\delta_{2m}^L} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\omega}_{\delta_{m1}^L} & \tilde{\omega}_{\delta_{m2}^L} & \dots & 0 \end{bmatrix} \end{aligned} \quad (18)$$

The total relation matrix is denoted by \tilde{T} as in equation (19), and is calculated by equation (20):

$$\tilde{T} = \begin{bmatrix} 0 & \tilde{t}_{12} & \dots & \tilde{t}_{1m} \\ \tilde{t}_{21} & 0 & \dots & \tilde{t}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{t}_{m1} & \tilde{t}_{m2} & \dots & 0 \end{bmatrix} \quad (19)$$

$$\begin{aligned} \left[\tilde{t}_{\alpha_{ij}^U} \right] &= \tilde{\Omega}_{\alpha^U} \times \left(I - \tilde{\Omega}_{\alpha^U} \right)^{-1}, \\ \left[\tilde{t}_{\beta_{ij}^U} \right] &= \tilde{\Omega}_{\beta^U} \times \left(I - \tilde{\Omega}_{\beta^U} \right)^{-1}, \dots, \\ \left[\tilde{t}_{\delta_{ij}^L} \right] &= \tilde{\Omega}_{\delta^L} \times \left(I - \tilde{\Omega}_{\delta^L} \right)^{-1} \end{aligned} \quad (20)$$

Step 5: Using equation (11), the total relation matrix (\tilde{T}) is converted to a defuzzified total relation matrix (T) as given by equation (21):

$$T = \begin{bmatrix} 0 & t_{12} & \dots & t_{1m} \\ t_{21} & 0 & \dots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \dots & 0 \end{bmatrix} = [t_{ij}]_{m \times m} \quad (21)$$

Step 6: In this step, the summations of each row as well as each column of the total relation matrix T are calculated and denoted by two vector matrices of r and s , that are given by equations (22) and (23), respectively:

$$r = \left[\sum_{j=1}^m \tilde{t}_{ij} \right]_{m \times 1} = [r_i]_{m \times 1} = (r_1, \dots, r_i, \dots, r_m) \quad (22)$$

$$s = \left[\sum_{i=1}^m \tilde{t}_{ij} \right]_{1 \times m}' = [s_j]_{1 \times m}' = (s_1, \dots, s_i, \dots, s_m)' \quad (23)$$

Then, the influential network relation map is formed with respect to values of r and s .

Step 7: In this step, the unweighted supermatrix is constructed. For this purpose, a normalized matrix T_c^A is obtained from the total relation matrix T , that is given by equations (24)–(26):

$$T_c^A = \begin{bmatrix} D_{1hh} & \dots & D_j & \dots & D_n \\ c_{11} \dots c_{1m_1} & \dots & c_{j1} \dots c_{jm_j} & \dots & c_{n1} \dots c_{nm_n} \\ \vdots & & \vdots & & \vdots \\ D_1 & \begin{array}{c} T_c^{A_{11}} \\ \vdots \\ T_c^{A_{1l}} \\ \vdots \\ T_c^{A_{1n}} \end{array} & \dots & \begin{array}{c} T_c^{A_{1j}} \\ \vdots \\ T_c^{A_{lj}} \\ \vdots \\ T_c^{A_{jn}} \end{array} & \dots & \begin{array}{c} T_c^{A_{1n}} \\ \vdots \\ T_c^{A_{ln}} \\ \vdots \\ T_c^{A_{nn}} \end{array} \\ \vdots & & & & & \vdots \\ D_i & \begin{array}{c} T_c^{A_{i1}} \\ \vdots \\ T_c^{A_{il}} \\ \vdots \\ T_c^{A_{in}} \end{array} & \dots & \begin{array}{c} T_c^{A_{ij}} \\ \vdots \\ T_c^{A_{lj}} \\ \vdots \\ T_c^{A_{in}} \end{array} & \dots & \begin{array}{c} T_c^{A_{im}} \\ \vdots \\ T_c^{A_{in}} \\ \vdots \\ T_c^{A_{nn}} \end{array} \\ \vdots & & & & & \vdots \\ D_n & \begin{array}{c} T_c^{A_{n1}} \\ \vdots \\ T_c^{A_{nm}} \end{array} & \dots & \begin{array}{c} T_c^{A_{mj}} \\ \vdots \\ T_c^{A_{nn}} \end{array} & \dots & \begin{array}{c} T_c^{A_{nn}} \\ \vdots \\ T_c^{A_{nn}} \\ \vdots \\ T_c^{A_{nn}} \end{array} \end{bmatrix} \quad (24)$$

where

$$T_c^{A_{11}} = \begin{bmatrix} \frac{t_{c_{11}}^{11}}{t_1^{11}} & \dots & \frac{t_{c_{1j}}^{11}}{t_1^{11}} & \dots & \frac{t_{c_{1m_1}}^{11}}{t_1^{11}} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \frac{t_{c_{1l}}^{11}}{t_l^{11}} & \dots & \frac{t_{c_{lj}}^{11}}{t_l^{11}} & \dots & \frac{t_{c_{lm_1}}^{11}}{t_l^{11}} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \frac{t_{c_{m_1}}^{11}}{t_{m_1}^{11}} & \dots & \frac{t_{c_{mj}}^{11}}{t_{m_1}^{11}} & \dots & \frac{t_{c_{mm_1}}^{11}}{t_{m_1}^{11}} \end{bmatrix} \quad (25)$$

$$= \begin{bmatrix} t_{c_{11}}^{A_{11}} & \dots & t_{c_{1j}}^{A_{11}} & \dots & t_{c_{1m_1}}^{A_{11}} \\ \vdots & \dots & \vdots & \dots & \vdots \\ t_{c_{1l}}^{A_{11}} & \dots & t_{c_{lj}}^{A_{11}} & \dots & t_{c_{lm_1}}^{A_{11}} \\ \vdots & \dots & \vdots & \dots & \vdots \\ t_{c_{m_1}}^{A_{11}} & \dots & t_{c_{mj}}^{A_{11}} & \dots & t_{c_{mm_1}}^{A_{11}} \end{bmatrix}$$

$$t_i^{11} = \sum_{j=1}^{m_1} t_{c_{ij}}^{11}; i = 1, 2, \dots, m_1 \quad (26)$$

Step 8: The transpose of normalized matrix T_c^A is calculated by equation (27):

$$W = (T_c^A)' = \begin{bmatrix} D_{1hh} & \dots & D_i & \dots & D_n \\ c_{11} \dots c_{1m_1} & \dots & c_{i1} \dots c_{im_i} & \dots & c_{m1} \dots c_{nm_n} \\ \vdots & & \vdots & & \vdots \\ D_1 & \begin{array}{c} W^{11} \\ \vdots \\ W^{il} \\ \vdots \\ W^{n1} \end{array} & \dots & \begin{array}{c} W^{i1} \\ \vdots \\ W^{ij} \\ \vdots \\ W^{in} \end{array} & \dots & \begin{array}{c} W^{11} \\ \vdots \\ W^{in} \\ \vdots \\ W^{nn} \end{array} \\ \vdots & & & & & \vdots \\ D_i & \begin{array}{c} W^{1j} \\ \vdots \\ W^{ij} \\ \vdots \\ W^{nj} \end{array} & \dots & \begin{array}{c} W^{ij} \\ \vdots \\ W^{ij} \\ \vdots \\ W^{nj} \end{array} & \dots & \begin{array}{c} W^{ij} \\ \vdots \\ W^{ij} \\ \vdots \\ W^{nj} \end{array} \\ \vdots & & & & & \vdots \\ D_n & \begin{array}{c} W^{1n} \\ \vdots \\ W^{in} \\ \vdots \\ W^{nn} \end{array} & \dots & \begin{array}{c} W^{in} \\ \vdots \\ W^{in} \\ \vdots \\ W^{nn} \end{array} & \dots & \begin{array}{c} W^{in} \\ \vdots \\ W^{nn} \\ \vdots \\ W^{nn} \end{array} \end{bmatrix} \quad (27)$$

Equation (28) gives the matrix T_D , and equation (29) gives the matrix T_D^A which is normalized form of total influence matrix by equation (30):

$$T_D = \begin{bmatrix} t_{D_{11}}^{11} & \dots & t_{D_{1j}}^{1j} & \dots & t_{D_{1m}}^{1m} \\ \vdots & \dots & \vdots & \dots & \vdots \\ t_{D_{il}}^{11} & \dots & t_{D_{ij}}^{ij} & \dots & t_{D_{im}}^{im} \\ \vdots & \dots & \vdots & \dots & \vdots \\ t_{D_{m1}}^{m1} & \dots & t_{D_{mj}}^{mj} & \dots & t_{D_{mm}}^{mm} \end{bmatrix} \quad (28)$$

where

$$T_D^{A_{11}} = \begin{bmatrix} t_{D_{11}}^{11}/\varsigma_1 & \dots & t_{D_{1j}}^{1j}/\varsigma_1 & \dots & t_{D_{1m}}^{1m}/\varsigma_1 \\ \vdots & & \vdots & & \vdots \\ t_{D_{11}}^{i1}/\varsigma_i & \dots & t_{D_{ij}}^{ij}/\varsigma_i & \dots & t_{D_{im}}^{im}/\varsigma_i \\ \vdots & & \vdots & & \vdots \\ t_{D_{m1}}^{m1}/\varsigma_m & \dots & t_{D_{mj}}^{mj}/\varsigma_m & \dots & t_{D_{mm}}^{mm}/\varsigma_m \end{bmatrix} \quad (29)$$

$$= \begin{bmatrix} t_{11}^{A_{11}} & \dots & t_{1j}^{A_{1j}} & \dots & t_{1m}^{A_{1m}} \\ \vdots & & \vdots & & \vdots \\ t_{il}^{A_{11}} & \dots & t_{ij}^{A_{11}} & \dots & t_{im}^{A_{1m}} \\ \vdots & & \vdots & & \vdots \\ t_{ml}^{A_{11}} & \dots & t_{mj}^{A_{11}} & \dots & t_{mm}^{A_{1m}} \end{bmatrix}$$

$$\varsigma_i = \sum_{j=1}^m t_{D_{ij}}^{ij}, i = 1, 2, \dots, m \quad (30)$$

Step 9: The weighted supermatrix (W^A) is calculated by the product of two matrices, namely T_D^A and W , and is given by equation (31):

$$W^A = T_D^A \times W$$

$$= \begin{bmatrix} t_{11}^{A_{11}} \times W^{11} & \dots & t_{il}^{A_{11}} \times W^{1l} & \dots & t_{ml}^{A_{11}} \times W^{n1} \\ \vdots & & \vdots & & \vdots \\ t_{1j}^{A_{1j}} \times W^{1j} & \dots & t_{ij}^{A_{1j}} \times W^{ij} & \dots & t_{mj}^{A_{1j}} \times W^{nj} \\ \vdots & & \vdots & & \vdots \\ t_{1m}^{A_{1m}} \times W^{1m} & \dots & t_{im}^{A_{1m}} \times W^{im} & \dots & t_{mm}^{A_{1m}} \times W^{nm} \end{bmatrix} \quad (31)$$

where T_D is a $m \times m$ matrix given by equation (28) and the matrix T_D^A is calculated using equation (30).

Step 10: In this step, the limit supermatrix is constructed by raising the weighted supermatrix (W^A) to the power $2k + 1$ until it reaches convergence and the weight vector (ω) becomes accessible.

Interval type-2 fuzzy VIKOR

VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) is an MCDM technique that determines the compromise ranking of alternatives. The basic idea of VIKOR method was proposed in a PhD dissertation by Opricovic⁴⁴ and later some extensions were made to the method by Opricovic and Tzeng.^{45,46} In the recent decades, numerous studies have used the IT2-FSSs to overcome uncertainty.^{37,47-49}

Table 3. Linguistics variables for the evaluation of maintenance strategies.

Linguistic variables	IT2-FS numbers
Very poor (VP)	((0,0,0,0;1,1,0,0,0,0;0.9,0.9))
Poor (P)	((0,0,0,0,1;1,1,0,0,0,0.05;0.9,0.9))
Medium poor (MP)	((0,0,1,0,1,0.3;1,1,0.05,0,1,0,1,0.2;0.9,0.9))
Fair (F)	((0,1,0.3,0.3,0.5;1,1,0.2,0.3,0.3,0.4;0.9,0.9))
Good (G)	((0,3,0.5,0.5,0.7;1,1,0.4,0.5,0.5,0.6;0.9,0.9))
Very good (VG)	((0,5,0.7,0.7,0.9;1,1,0.6,0.7,0.7,0.8;0.9,0.9))
Best (B)	((0,7,0.9,0.9;1,1,1,0.8,0.9,0.9,0.95;0.9,0.9))

In this study, we use the IT2-FS VIKOR functions for prioritization of maintenance strategies in a distillation unit. It is worth mentioning that the weight vector for IT2-FS VIKOR is calculated by the IT2-FS DANTP method. The method is explained below in a step-by-step manner:

Step 1: The decision matrix is constructed using experts' opinion based on the preference scale given in Table 3.

Step 2: The decision matrix for ρ th decision maker, $X^\rho = [X_{ij}^\rho]_{n \times m}$ is presented by equation (32):

$$X^\rho = \begin{bmatrix} X_{11}^\rho & \dots & X_{1j}^\rho & \dots & X_{1j}^\rho \\ \vdots & & \vdots & & \vdots \\ X_{i1}^\rho & \dots & X_{ij}^\rho & \dots & X_{im}^\rho \\ \vdots & & \vdots & & \vdots \\ X_{nj}^\rho & \dots & X_{nj}^\rho & \dots & X_{nm}^\rho \end{bmatrix} \quad (32)$$

where X_{ij}^ρ is a trapezoidal IT2-FS representing the evaluation made by ρ th decision maker for i th maintenance strategy on j th criterion ($1 \leq i \leq n, 1 \leq j \leq m, 1 \leq \rho \leq P$).

Step 3: The average decision matrix (\tilde{X}) is constructed using equation (33), where \tilde{X}_{ij} represents the average evaluation of i th maintenance strategy on j th criterion.

$$\tilde{X}_{ij} = \left[X_{ij}^{(1)} + X_{ij}^{(2)} + \dots + X_{ij}^{(P)} \right] / P \quad (33)$$

Step 4: Ideal (X^*) and null (X) values for beneficial and non-beneficial criteria are computed using equations (34) and (35), respectively:

$$\tilde{X}_j^* = \max_i \tilde{X}_{ij}; \tilde{X}_j^* = \min_i \tilde{X}_{ij} \quad (34)$$

$$\tilde{X}_j^* = \min_i \tilde{X}_{ij}; \tilde{X}_j^* = \max_i \tilde{X}_{ij} \quad (35)$$

where

$$\tilde{X}_j^* = \left(\left(\alpha_j^{*U}, \beta_j^{*U}, \gamma_j^{*U}, \delta_j^{*U}; H_1(X_j^{*U}), H_2(X_j^{*U}) \right), \alpha_j^{*L}, \beta_j^{*L}, \gamma_j^{*L}, \delta_j^{*L}; H_1(X_j^{*L}), H_2(X_j^{*L}) \right),$$

$$\tilde{X}_j^\circ = \left(\left(\alpha_j^{\circ U}, \beta_j^{\circ U}, \gamma_j^{\circ U}, \delta_j^{\circ U}; H_1(X_j^{\circ U}), H_2(X_j^{\circ U}) \right), \alpha_j^{\circ L}, \beta_j^{\circ L}, \gamma_j^{\circ L}, \delta_j^{\circ L}; H_1(X_j^{\circ L}), H_2(X_j^{\circ L}) \right).$$

It is noted that defuzzified values are calculated by equation (11) to determine the minimum and maximum values.

Step 5: The normalized fuzzy differences are calculated by equation (36) for beneficial criteria and equation (37) for non-beneficial criteria:

$$Dif_{ij} = (\tilde{X}_j^* - \tilde{X}_{ij}) / D(\tilde{X}_j^*, \tilde{X}_j^\circ) \quad (36)$$

$$Dif_{ij} = (\tilde{X}_{ij} - \tilde{X}_j^*) / D(\tilde{X}_j^\circ, \tilde{X}_j^*) \quad (37)$$

Step 6: \tilde{S} and \tilde{R} values are obtained by equations (38) and (39), respectively.

$$\tilde{S}_i = (\omega_1 \times Dif_{i1}) + (\omega_2 \times Dif_{i2}) + \dots + (\omega_m \times Dif_{im}) \quad (38)$$

$$\tilde{R}_i = \min_j (\omega_j \times Dif_{ij}) \quad (39)$$

The weight vector (ω) includes real numbers between 0 and 1 which for each criterion are obtained by the IT2-FS DANP method. The best values of \tilde{S} and \tilde{R} , that is, \tilde{S}_* and \tilde{R}_* are computed by equations (40) and (41):

$$\tilde{S}_* = \min_i (\tilde{S}_i) \quad (40)$$

$$\tilde{R}_* = \min_i (\tilde{R}_i) \quad (41)$$

Step 7: \tilde{Q} values are obtained by equation (42):

$$\begin{aligned} \tilde{Q}_i &= v \times \left((\tilde{S}_i - \tilde{S}_*) - (s_i^{\circ U} - s_{i1}^{*U}) \right) \\ &\quad + (1 - v) \times \left((\tilde{R}_i - \tilde{R}_*) - (r_i^{\circ U} - r_{i1}^{*U}) \right) \end{aligned} \quad (42)$$

where v the is maximum group utility, and $s_i^{\circ U} = \max_i(s_{i4}^{*U})$ and $r_i^{\circ U} = \max_i(r_{i4}^{*U})$.

Step 8: According to the defuzzified values $CR\tilde{Q}$, maintenance strategies are ranked while the following conditions must be met:

$$\text{Condition 1: } CR\tilde{Q}^{(x^{(2)})} - CR\tilde{Q}^{(x^{(1)})} \geq \frac{1}{m-1}$$

where $CR\tilde{Q}^{(x^{(2)})}$ represents the second-smallest defuzzified value and m is the number of maintenance strategies.

Condition 2: The alternatives $CR\tilde{Q}^{(x^{(1)})}$ should be the first ranked with respect to \tilde{S} and \tilde{R} .

In case the condition 1 is not met, the inequality $CR\tilde{Q}^{(x^{(N)})} - CR\tilde{Q}^{(x^{(1)})} < \frac{1}{m-1}$ holds for $CR\tilde{Q}^{(x^{(N)})}$, then the ranking of maintenance strategies will be: $CR\tilde{Q}^{(x^{(1)})}, CR\tilde{Q}^{(x^{(2)})}, \dots$ and $CR\tilde{Q}^{(x^{(N)})}$. On the other hand, if condition 2 is not met, then both the maintenance strategies $CR\tilde{Q}^{(x^{(1)})}$ and $CR\tilde{Q}^{(x^{(2)})}$ will be ranked the best.

Case study

In this section, the proposed MCDM methodology is applied to determine an optimal maintenance strategy for a distillation unit in an oil refinery plant. Firstly, a brief explanation of the distillation unit under study is given. Secondly, applicable maintenance strategies are identified. Thirdly, performance evaluation criteria and sub-criteria are presented. Fourthly, the IT2-FS DANP method to determine the criteria's weights is presented. Lastly, the IT2-FS VIKOR method to rank the maintenance strategies is explained.

Distillation unit

The distillation unit is the heart of any oil refinery plant as it is the first process unit to receive crude oil. The primary function of the distillation unit is to distill the crude oil into numerous fractions of varied boiling ranges, each of which are then processed further in the other refinery processing units. A typical crude oil distillation unit consists of an atmospheric distillation column for separation of lighter components and a vacuum distillation column for further separation of hydrocarbons under reduced pressure. A schematic illustration of the distillation unit under study is shown in Figure 3.

The first stage in a distillation unit is pre-heating, which is performed in exchanger bay. In this step, hot products and cold crude oil are entered in shell and tube heat exchangers to interchange the heat. This process will cause an increase in crude oil temperature and decrease in the temperature of products. Now, the pre-heated crude is pumped to the primary atmospheric tower and subsequently, the primary flash distillate (PFD) and light gases will be extracted from top of the tower. The rest of the oil will pass to heaters, where its

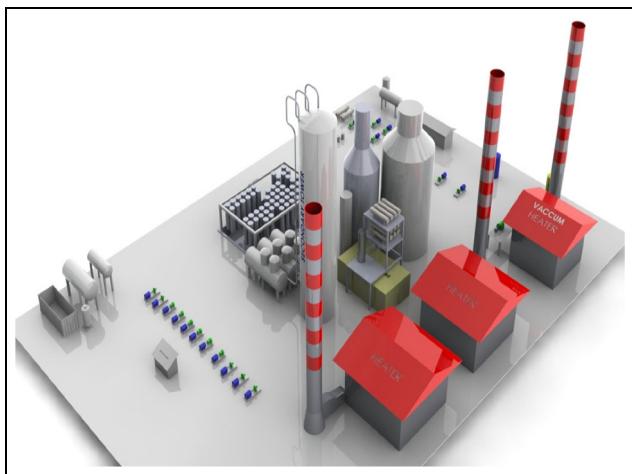


Figure 3. A schematic illustration of a crude oil desalination unit.

temperature is raised to approximately 350°C. Afterwards, it enters from the bottom into secondary atmospheric tower. The light oil products will be obtained from the distillation of vapors on different elevation trays in a way that lighter ones are on upper trays. At the bottom of the tower, a dense product called residue will be transferred to the vacuum stage. Like atmospheric site, the residue will be abstracted to a hot environment in a vacuum heater. Then, the hot residue will be distilled in the vacuum heater. The products that are obtained in this step include: vacuum gasoil, wax, slops, and vacuum bottom. The processes and the transmission of products in atmospheric and vacuum crude distillation units are shown in Figure 4.

The most critical components in distillation units include fans of heaters, crude oil pumps, heaters, and columns. The performance and operations of distillation unit intensively depend on these components and any failure of them will result in a wide range of serious issues, from feed reduction to total plant shutdown. Additionally, owing to production chain in a refinery, every failure in critical components may affect other units. Downstream units may encounter limitation for feeding upstream units, and therefore it will cause disruption. As a result, the oil refinery industry must prevent any possible failure in their critical equipment. To achieve this, selection of an efficient maintenance strategy for critical components of the distillation unit is crucial to save cost and effort, reduce energy consumption, and protect the environment.

Maintenance strategies

Four maintenance strategies for the crude oil distillation unit are taken into consideration. These strategies are explained below:

Run-to-failure maintenance. This maintenance strategy – also known as reactive or corrective maintenance – is usually recommended for non-critical and low-cost assets.² Under this strategy, no action will be taken until a failure occurs.

Preventive maintenance. This maintenance strategy – also known as scheduled maintenance – involves the repair or replacement of equipment components at regular time intervals. Though PM is not necessarily the most

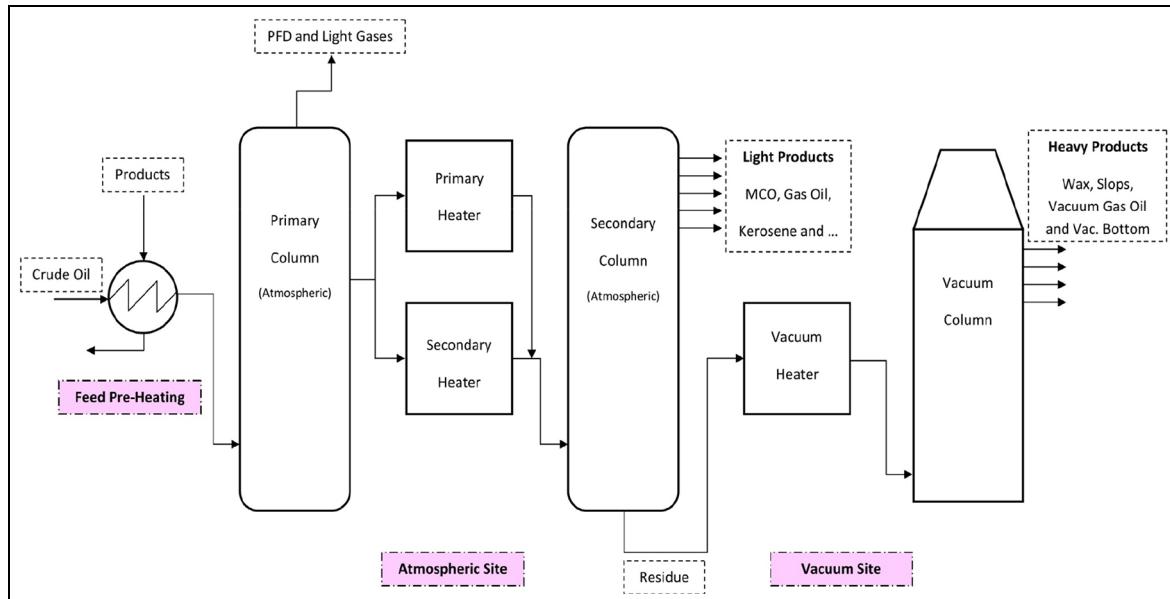


Figure 4. Atmospheric and vacuum crude distillation units.

Table 4. Criteria and sub-criteria for maintenance strategy selection.

Criteria	Sub-criteria	Abbreviation	Beneficial/non-beneficial
Cost	Cost of material and consumables	CS ₁	N
	Cost of manpower	CS ₂	N
Safety	Human health and safety	SF ₁	B
	Equipment safety	SF ₂	B
Sustainability	MTBF	ST ₁	B
	MTTR	ST ₂	B
	Product variety	ST ₃	B
	Training of personnel	ST ₄	B
	Acceptance by personnel	ST ₅	B
	Environment management system	ST ₆	B
	Energy consumption	ST ₇	B
	Environmental planning	ST ₈	B

cost-effective strategy, its effect on controlling the degradation rate and reducing the likelihood of catastrophic failures cannot be neglected. Therefore, PM improves the availability of equipment and saves O&M costs. A major limitation of this strategy is that it occasionally causes unnecessary repairs to be performed on systems that do not actually require maintenance.⁵⁰

Condition-based maintenance. This strategy involves performing maintenance based on the condition of the equipment being monitored, rather than on a fixed schedule. CBM is the best choice in situations where the asset is critical, and a reliable and economical monitoring system is accessible. The condition monitoring systems collect and analyze data describing the operating condition of the components.² These data are collected either continuously or periodically. A major limitation in implementing CBM is its relatively high costs in terms of hardware. Therefore, CBM strategy is suitable for safety critical and high value assets.⁵⁰

Reliability centered maintenance. This maintenance strategy is a systematic approach to identify the equipment function, determine failure modes associated with the function, prioritize the failure modes based on their risk, identify maintenance requirements, and select the most appropriate maintenance task. RCM aims to optimize the maintenance activities by evaluating the types of failures that affect the function of an equipment. To do this, many different tools such as the failure mode and effects analysis (FMEA) and fault tree analysis (FTA) are utilized.

Decision-making criteria and sub-criteria

After reviewing the literature and consultation with the plant stakeholders, three key criteria were considered for selecting the most suitable maintenance strategy.

These criteria include cost, safety, and sustainability, which are further broken down into 12 concrete sub-criteria. Table 4 lists the criteria and sub-criteria considered for maintenance decision-making.

The maintenance decision-making criteria and sub-criteria are described below.

Cost. Each of maintenance strategies has its own cost implications that must be taken into account. The costs associated with maintenance operations vary depending on the type of maintenance strategy adopted.⁵⁰ The cost sub-criteria considered in this study include:

- **Cost of material and consumables:** This includes expenses associated with ordering spare parts (such as bearings, mechanical seals, tubes, etc.) and consumables (such as lubricants, oils, and greases) for maintenance of the crude oil distillation unit.
- **Cost of manpower:** This includes expenses associated with training and technical assistance of maintenance team. Some strategies like CBM and RCM need a wealth of experience in maintenance and a high level of training about how to use monitoring devices or data analytics platforms.

Safety. Safety is defined as the freedom from unacceptable risk of a specific hazard that may result in loss of life, injury, or property damage. In the oil refinery industry, it is vital to protect all the machine operators and maintenance technicians against numerous hazards including fire, chemicals, and over-pressurization.² In this study, the safety criterion is broken down into two sub-criteria:

- **Human health and safety:** The failure of some assets can cause serious damage and injury to

Table 5. Linguistics values for criteria.

	Cost			Safety			Sustainability		
	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃
Cost	—	—	—	LM	LM	L	LM	LM	L
Safety	HM	VH	H	—	—	—	HM	H	HM
Sustainability	H	VH	H	M	M	LM	—	—	—

personnel. Regular maintenance can improve the health and safety of all people who work in the refinery plant.

- **Equipment safety:** The failure of some equipment can put the whole refinery plant in jeopardy. CBM can detect early signs of potential failure in the plant and initiate a detailed inspection before the failures cause further damage or risk.

Sustainability. Three sub-criteria of sustainability, namely economy, society, and environment, with their associated dimensions are described as following:⁶

Economy: The economic dimensions of sustainability concern with the added-value, efficiency, quality, and technical aspects of each maintenance strategy. These dimensions include:

- **Mean time between failures (MTBF):** This is a metric defined as the average length of the time intervals between successive failures of equipment. In general, the longer the MTBF the more reliable the equipment. Therefore, an optimal maintenance strategy should have the potential to increase the MTBF.
- **Mean time to repair (MTTR):** This is a metric defined as the average length of the time required to troubleshoot and repair a failed system. In general, the shorter the MTTF the larger the availability of the equipment. Therefore, an optimal maintenance strategy should have the potential to decrease the MTTF.
- **Product variety:** The ability to extract different products from crude oil is an important merit for an oil refinery. However, a diverse set of equipment and machines will also be required to produce diverse products in the same system, necessitating a comprehensive maintenance program for the entire plant.

Society: The society dimensions of sustainability deal with adaptability, user friendliness, training, and social acceptability of maintenance strategies. These dimensions include:

- **Training of personnel:** It is necessary that technicians and professionals learn the procedures, methods, and tools they will need to know before executing maintenance operations. These tools should be user-friendly and easy to learn.
- **Acceptance by personnel:** Successful implementation of maintenance strategies requires a high level of support and acceptance by the personnel.

Environment: The environment dimensions of sustainability concern with the toxic emissions, energy consumption, waste generation, etc. These dimensions include:

- **Environment management system:** The organizations with a system to manage the environmental hazards are better able to evaluate their risks and prepare a plan for a continuous maintenance management improvement.
- Energy consumption: The adoption of a proper maintenance strategy can reduce the energy consumption and thus also CO₂ emissions.
- Environmental planning: This is the practice of determining how maintenance strategies can be executed in a way that are not harmful for the ecosystem.

Weights of criteria

In this subsection, we obtain the weights of criteria and sub-criteria using the IT2-FS DANP method. The decision-making team includes three experts labeled as D₁, D₂, and D₃. These experts specialize in the field of oil and gas facility management and have over 15 years of experience in implementing maintenance management systems. The experts express their opinions in linguistic terms instead of crisp numbers. Tables 5 and 6 give the linguistics values by each expert to 3 criteria and 13 sub-criteria, respectively.

Using equations (12)–(31), the weights of criteria and sub-criteria were calculated. Results of the IT2-FS DANP method are presented in Tables 7 to 15. As

Table 6. Linguistics values for sub-criteria.

CS ₁			CS ₂			SF ₁			SF ₂		
D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃
CS ₁	—	—	H	VH	VH	L	LM	LM	L	VL	L
CS ₂	VL	VL	L	—	—	VL	VL	L	VL	VL	VL
SF ₁	H	H	HM	VH	H	—	—	—	M	LM	M
SF ₂	VH	VH	H	VH	VH	H	HM	HM	—	—	—
ST ₁	H	VH	H	H	VH	VH	H	HM	H	H	VH
ST ₂	M	M	LM	H	VH	HM	HM	LM	M	HM	H
ST ₃	M	H	HM	HM	VH	H	M	LM	L	HM	LM
ST ₄	H	HM	H	VH	H	H	M	L	LM	M	HM
ST ₅	M	M	LM	M	M	LM	LM	LM	L	VL	L
ST ₆	H	VH	VH	H	VH	H	M	H	LM	LM	M
ST ₇	HM	H	H	VH	H	VH	M	M	HM	L	L
ST ₈	H	H	HM	H	H	HM	M	HM	HM	LM	M
ST ₁	—	—	—	ST ₂	—	—	ST ₃	—	—	ST ₄	—
CS ₁	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	D ₁	D ₂	D ₃	D ₁	D ₂
CS ₂	L	L	VL	L	VL	L	HM	H	HM	H	HM
SF ₁	VL	VL	VL	VL	L	VL	L	M	LM	L	LM
SF ₂	LM	LM	L	M	M	LM	H	H	HM	H	HM
ST ₁	—	—	—	VH	VH	H	VH	VH	H	VH	VH
ST ₂	L	LM	L	—	—	—	H	HM	HM	H	H
ST ₃	L	VL	L	LM	VL	L	—	—	—	L	LM
ST ₄	LM	L	M	HM	HM	M	HM	H	HM	—	—
ST ₅	VL	VL	L	L	LM	L	LM	LM	M	LM	L
ST ₆	M	M	HM	M	M	HM	HM	VH	H	VH	H
ST ₇	LM	LM	M	H	HM	HM	H	H	VH	H	VH
ST ₈	M	LM	LM	HM	M	HM	H	HM	HM	H	H
ST ₅	—	—	—	ST ₆	—	—	ST ₇	—	—	ST ₈	—
CS ₁	H	H	HM	M	HM	HM	D ₁	D ₂	D ₃	D ₁	D ₂
CS ₂	HM	M	M	VL	L	L	LM	L	VL	L	VL
SF ₁	H	H	H	HM	HM	H	HM	H	HM	L	M
SF ₂	VH	VH	H	VH	H	H	VH	H	H	VH	VH
ST ₁	VH	VH	VH	VH	H	VH	VH	H	H	VH	VH
ST ₂	H	HM	H	M	M	LM	M	LM	LM	H	HM
ST ₃	M	HM	HM	L	L	LM	L	LM	VL	M	HM
ST ₄	H	HM	HM	M	HM	M	L	VL	LM	HM	M
ST ₅	—	—	—	M	LM	LM	VL	L	L	LM	LM
ST ₆	HM	H	H	—	—	—	M	M	LM	HM	H
ST ₇	H	H	VH	H	HM	H	—	—	HM	H	M
ST ₈	HM	HM	M	M	LM	M	LM	LM	L	—	—

Table 7. Aggregated initial direct relation matrix.

	Cost	Safety	Sustainability
Cost	((0,0,0,0;1,1),(0,0,0,0;0.9,0.9))	((0.04,0.14,0.14,0.26;1,1), (0.09,0.14,0.14,0.2,0.9,0.9))	((0.04,0.14,0.14,0.26;1,1), (0.09,0.14,0.14,0.2;0.9,0.9))
Safety	((0.42,0.52,0.52,0.58,1,1), (0.47,0.52,0.52,0.55;0.9,0.9))	((0.0,0,0;1,1),(0,0,0,0;0.9,0.9))	((0.34,0.46,0.46,0.56,1,1), (0.4,0.46,0.46,0.51;0.9,0.9))
Sustainability	((0.46,0.56,0.56,0.6,1,1), (0.51,0.56,0.56,0.58;0.9,0.9))	((0.14,0.26,0.26,0.38,1,1), (0.2,0.26,0.26,0.32;0.9,0.9))	((0.0,0,0;1,1),(0,0,0,0;0.9,0.9))

Table 15 shows, the weight vector (ω) for twelve sub-criteria is obtained as: $\omega = (0.172, 0.051, 0.179, 0.23, 0.071, 0.047, 0.032, 0.046, 0.021, 0.051, 0.046, 0.047)$.

Optimal maintenance strategy

In this subsection, we determine the optimal maintenance strategy using the IT2-FS VIKOR method. It

Table 8. Normalized initial direct relation matrix.

	Cost	Safety	Sustainability
Cost	((0,0,0,0;1,1),(0,0,0,0;0.9,0.9))	((0.03,0.11,0.11,0.22;1,1), (0.07,0.11,0.11,0.16;0.9,0.9))	((0.03,0.11,0.11,0.22;1,1), (0.07,0.11,0.11,0.16;0.9,0.9))
Safety	((0.35,0.44,0.44,0.49;1,1), (0.39,0.44,0.44,0.46;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))	((0.28,0.38,0.38,0.47;1,1), (0.33,0.38,0.38,0.43;0.9,0.9))
Sustainability	((0.38,0.47,0.47,0.5;1,1), (0.43,0.47,0.47,0.49;0.9,0.9))	((0.11,0.22,0.22,0.32;1,1), (0.16,0.22,0.22,0.27;0.9,0.9))	((0,0,0,0;1,1), (0,0,0,0;0.9,0.9))

Table 9. Total relation matrix.

	Cost	Safety	Sustainability
Cost	((0.03,0.18,0.18,0.57;1,1), (0.09,0.18,0.18,0.33;0.9,0.9))	((0.04,0.18,0.18,0.54;1,1), (0.1,0.18,0.18,0.32;0.9,0.9))	((0.04,0.21,0.21,0.6;1,1), (0.11,0.21,0.21,0.36;0.9,0.9))
Safety	((0.5,0.81,0.81,1.36;1,1), (0.63,0.81,0.81,1.02;0.9,0.9))	((0.05,0.22,0.22,0.64;1,1), (0.12,0.22,0.22,0.38;0.9,0.9))	((0.32,0.57,0.57,1.08;1,1), (0.42,0.57,0.57,0.77;0.9,0.9))
Sustainability	((0.46,0.74,0.74,1.23;1,1), (0.57,0.74,0.74,0.93;0.9,0.9))	((0.14,0.35,0.35,0.8;1,1), (0.23,0.35,0.35,0.53;0.9,0.9))	((0.05,0.22,0.22,0.65;1,1), (0.12,0.22,0.22,0.38;0.9,0.9))

Table 10. Defuzzified relation matrix.

	Cost	Safety	Sustainability
Cost	0.128	0.215	0.243
Safety	0.824	0.150	0.597
Sustainability	0.753	0.384	0.153

Table 11. Unweighted relation matrix.

	Cost	Safety	Sustainability
Cost	0.075	0.286	0.244
Safety	0.483	0.200	0.601
Sustainability	0.441	0.512	0.154

was mentioned earlier that four maintenance strategies, including RTFM, PM, CBM, and RCM were considered in this study. Also, like the previous subsection, three decision makers provided their assessments on maintenance alternatives (strategies) with respect to criteria using linguistic terms. Table 16 gives the evaluations made by the three experts.

The average decision matrix (\tilde{X}) was computed using equation (33) and it is presented in Table 17.

To calculate the ideal (X^*) and null (X^*) values of criteria, equations (34) and (35) were used and the results are presented in Table 18.

After obtaining the fuzzy differences from equation (36), we calculated \tilde{S} and \tilde{R} using equations (38) and (39). It is worth mentioning that the weight vector (ω) was identified by applying the IT2-FS-DANP method in the previous subsection. Tables 19 and 20 give the

Table 12. Defuzzified relation matrix.

	CS ₁	CS ₂	SF ₁	SF ₂	ST ₁	ST ₂	ST ₃	ST ₄	ST ₅	ST ₆	ST ₇	ST ₈
CS ₁	0.034	0.151	0.061	0.040	0.031	0.041	0.123	0.121	0.135	0.096	0.045	0.105
CS ₂	0.022	0.013	0.017	0.012	0.008	0.014	0.043	0.030	0.067	0.020	0.013	0.021
SF ₁	0.151	0.181	0.033	0.082	0.053	0.086	0.155	0.142	0.169	0.128	0.108	0.101
SF ₂	0.189	0.218	0.140	0.035	0.080	0.135	0.194	0.184	0.205	0.161	0.136	0.170
ST ₁	0.197	0.225	0.156	0.143	0.028	0.153	0.205	0.195	0.220	0.173	0.146	0.183
ST ₂	0.116	0.168	0.098	0.103	0.047	0.028	0.145	0.147	0.159	0.098	0.075	0.133
ST ₃	0.107	0.133	0.060	0.069	0.027	0.042	0.030	0.068	0.111	0.054	0.040	0.088
ST ₄	0.144	0.170	0.100	0.085	0.066	0.083	0.138	0.041	0.157	0.107	0.074	0.098
ST ₅	0.077	0.080	0.050	0.035	0.019	0.028	0.067	0.049	0.024	0.058	0.027	0.049
ST ₆	0.168	0.187	0.115	0.082	0.083	0.102	0.165	0.161	0.172	0.040	0.086	0.138
ST ₇	0.156	0.188	0.109	0.061	0.066	0.116	0.168	0.160	0.178	0.136	0.027	0.131
ST ₈	0.146	0.166	0.108	0.082	0.062	0.100	0.145	0.147	0.144	0.098	0.064	0.037

Table 13. Unweighted supermatrix.

	CS ₁	CS ₂	SF ₁	SF ₂	ST ₁	ST ₂	ST ₃	ST ₄	ST ₅	ST ₆	ST ₇	ST ₈
CS ₁	0.604	0.916	0.783	0.770	0.783	0.743	0.740	0.801	0.668	0.823	0.769	0.830
CS ₂	0.395	0.083	0.216	0.229	0.216	0.256	0.259	0.198	0.331	0.176	0.230	0.169
SF ₁	0.444	0.454	0.193	0.699	0.401	0.389	0.444	0.434	0.452	0.442	0.443	0.372
SF ₂	0.555	0.545	0.806	0.300	0.598	0.610	0.555	0.565	0.547	0.557	0.556	0.627
ST ₁	0.177	0.170	0.195	0.216	0.070	0.234	0.192	0.201	0.188	0.226	0.270	0.212
ST ₂	0.104	0.127	0.123	0.155	0.119	0.043	0.136	0.152	0.136	0.127	0.138	0.155
ST ₃	0.096	0.100	0.075	0.104	0.068	0.064	0.028	0.070	0.095	0.071	0.073	0.102
ST ₄	0.129	0.129	0.125	0.129	0.166	0.127	0.129	0.042	0.134	0.140	0.137	0.114
ST ₅	0.069	0.060	0.063	0.052	0.048	0.043	0.063	0.050	0.021	0.076	0.050	0.057
ST ₆	0.151	0.142	0.144	0.124	0.207	0.155	0.154	0.166	0.147	0.052	0.159	0.160
ST ₇	0.140	0.142	0.136	0.092	0.165	0.177	0.157	0.164	0.152	0.177	0.051	0.152
ST ₈	0.131	0.125	0.135	0.123	0.155	0.152	0.136	0.152	0.123	0.127	0.119	0.043

Table 14. Weighted supermatrix.

	CS ₁	CS ₂	SF ₁	SF ₂	ST ₁	ST ₂	ST ₃	ST ₄	ST ₅	ST ₆	ST ₇	ST ₈
CS ₁	0.045	0.069	0.224	0.220	0.191	0.182	0.181	0.196	0.163	0.201	0.188	0.203
CS ₂	0.029	0.006	0.062	0.065	0.053	0.062	0.063	0.048	0.081	0.043	0.056	0.041
SF ₁	0.214	0.219	0.038	0.140	0.241	0.234	0.267	0.261	0.271	0.266	0.266	0.223
SF ₂	0.268	0.263	0.162	0.060	0.360	0.367	0.334	0.339	0.329	0.335	0.334	0.377
ST ₁	0.078	0.075	0.100	0.111	0.010	0.036	0.029	0.031	0.029	0.034	0.041	0.032
ST ₂	0.046	0.056	0.063	0.079	0.018	0.006	0.021	0.023	0.021	0.019	0.021	0.023
ST ₃	0.042	0.044	0.038	0.053	0.010	0.009	0.004	0.010	0.014	0.011	0.011	0.015
ST ₄	0.057	0.057	0.064	0.066	0.025	0.019	0.019	0.006	0.020	0.021	0.021	0.017
ST ₅	0.030	0.026	0.032	0.027	0.007	0.006	0.009	0.007	0.003	0.011	0.007	0.008
ST ₆	0.066	0.062	0.074	0.064	0.031	0.023	0.023	0.025	0.022	0.008	0.024	0.024
ST ₇	0.062	0.063	0.070	0.047	0.025	0.027	0.024	0.025	0.023	0.027	0.007	0.023
ST ₈	0.058	0.055	0.069	0.063	0.023	0.023	0.020	0.023	0.019	0.019	0.018	0.006

Table 15. Limit supermatrix.

	CS ₁	CS ₂	SF ₁	SF ₂	ST ₁	ST ₂	ST ₃	ST ₄	ST ₅	ST ₆	ST ₇	ST ₈
CS ₁	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172
CS ₂	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051
SF ₁	0.179	0.179	0.179	0.179	0.179	0.179	0.179	0.179	0.179	0.179	0.179	0.179
SF ₂	0.230	0.230	0.230	0.230	0.230	0.230	0.230	0.230	0.230	0.230	0.230	0.230
ST ₁	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071
ST ₂	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047
ST ₃	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032	0.032
ST ₄	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
ST ₅	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
ST ₆	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051
ST ₇	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046	0.046
ST ₈	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047

values for \tilde{S} and \tilde{R} as well as their defuzzified values under $CR(\tilde{S})$ and $CR(\tilde{R})$ columns.

Based on equation (42), we obtain the values of \tilde{Q} with respect to four different values of v and they are given in Tables 21 to 24. Additionally, the defuzzified values are also calculated and presented in the Tables.

According to conditions 1 and 2, we conclude that the ranking of maintenance strategies from best to

worst performance was found to be RCM, CBM, PM, and RTFM.

Conclusion and future works

In this study, we developed a fuzzy group MCDM approach by combining three techniques of DEMATEL, ANP, and VIKOR, to determine the most

Table 16. Linguistic evaluations of maintenance strategies.

	D1				D2				D3			
	RTFM	PM	CBM	RCM	RTFM	PM	CBM	RCM	RTFM	PM	CBM	RCM
CS ₁	VP	P	B	G	P	MP	VG	G	VP	P	G	VG
CS ₂	G	B	F	MP	B	VG	G	F	VG	VG	F	MP
SF ₁	P	G	VG	B	VP	G	B	VG	VP	VG	VG	B
SF ₂	VP	F	VG	B	VP	G	B	B	VP	G	B	VG
ST ₁	P	G	VG	B	VP	G	VG	VG	MP	VG	B	B
ST ₂	VP	F	B	G	P	F	VG	G	F	VG	B	G
ST ₃	F	MP	VG	G	P	G	VG	B	F	G	B	VG
ST ₄	F	F	VG	VG	MP	F	B	VG	F	G	VG	B
ST ₅	VP	G	VG	B	VP	VG	B	VG	P	G	B	B
ST ₆	VP	G	B	B	P	G	B	VG	VP	G	VG	B
ST ₇	MP	G	B	VG	F	G	B	B	P	F	VG	VG
ST ₈	VP	G	B	VG	P	F	B	B	VP	G	VG	B

Table 17. Average decision matrix.

	RTFM	PM	CBM	RCM
CS ₁	((0,0,02,0,02,0,1;1,1), (0,01,0,02,0,02,0,06;0,9,0,9))	((0,02,0,1,0,1,0,22;1,1), (0,06,0,1,0,1,0,16;0,9,0,9))	((0,42,0,52,0,52,0,58;1,1), (0,47,0,52,0,52,0,55;0,9,0,9))	((0,34,0,46,0,46,0,56;1,1), (0,4,0,46,0,46,0,51;0,9,0,9))
CS ₂	((0,42,0,52,0,52,0,58;1,1), (0,47,0,52,0,52,0,55;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))	((0,22,0,34,0,34,0,46;1,1), (0,28,0,34,0,34,0,4;0,9,0,9))	((0,1,0,22,0,22,0,34;1,1), (0,16,0,22,0,22,0,28;0,9,0,9))
SF ₁	((0,0,02,0,02,0,1;1,1), (0,01,0,02,0,02,0,06;0,9,0,9))	((0,34,0,46,0,46,0,56;1,1), (0,4,0,46,0,46,0,51;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))
SF ₂	((0,0,0,0,06;1,1), (0,0,0,0,03;0,9,0,9))	((0,26,0,38,0,38,0,5;1,1), (0,32,0,38,0,38,0,44;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))
ST ₁	((0,02,0,08,0,08,0,18;1,1), (0,05,0,08,0,08,0,13;0,9,0,9))	((0,34,0,46,0,46,0,56;1,1), (0,4,0,46,0,46,0,51;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))
ST ₂	((0,06,0,12,0,12,0,22;1,1), (0,09,0,12,0,12,0,17;0,9,0,9))	((0,26,0,38,0,38,0,48;1,1), (0,32,0,38,0,38,0,43;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))	((0,3,0,42,0,42,0,54;1,1), (0,36,0,42,0,42,0,48;0,9,0,9))
ST ₃	((0,12,0,22,0,22,0,34;1,1), (0,17,0,22,0,22,0,28;0,9,0,9))	((0,22,0,34,0,34,0,46;1,1), (0,28,0,34,0,34,0,4;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))	((0,42,0,52,0,52,0,58;1,1), (0,47,0,52,0,52,0,55;0,9,0,9))
ST ₄	((0,14,0,26,0,26,0,38;1,1), (0,2,0,26,0,26,0,32;0,9,0,9))	((0,22,0,34,0,34,0,46;1,1), (0,28,0,34,0,34,0,4;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))
ST ₅	((0,0,02,0,02,0,1;1,1), (0,01,0,02,0,02,0,06;0,9,0,9))	((0,34,0,46,0,46,0,56;1,1), (0,4,0,46,0,46,0,51;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))
ST ₆	((0,0,02,0,02,0,1;1,1), (0,01,0,02,0,02,0,06;0,9,0,9))	((0,3,0,42,0,42,0,54;1,1), (0,36,0,42,0,42,0,48;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))
ST ₇	((0,08,0,18,0,18,0,3;1,1), (0,13,0,18,0,18,0,24;0,9,0,9))	((0,26,0,38,0,38,0,5;1,1), (0,32,0,38,0,38,0,44;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))	((0,46,0,56,0,56,0,6;1,1), (0,51,0,56,0,56,0,58;0,9,0,9))
ST ₈	((0,0,02,0,02,0,1;1,1), (0,01,0,02,0,02,0,06;0,9,0,9))	((0,26,0,38,0,38,0,5;1,1), (0,32,0,38,0,38,0,44;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))	((0,5,0,58,0,58,0,6;1,1), (0,54,0,58,0,58,0,59;0,9,0,9))

sustainable maintenance strategy for distillation units in the oil refinery industry. To overcome the deficiencies of crisp decision-making approaches, we utilized interval type-2 fuzzy sets which are generalization of interval-valued fuzzy sets. Four main maintenance strategies were considered for the evaluations, including run-to-failure maintenance (RTFM), preventive maintenance (PM), condition-based maintenance (CBM), and reliability centered maintenance (RCM). After reviewing the literature and consultation with the plant stakeholders, three competing criteria including cost,

safety, and sustainability were identified. These criteria were further broken down into 12 concrete sub-criteria. The IT2-FS DEMATEL-ANP (DANP) method was applied to determine the importance weights of criteria and sub-criteria, whereas the IT2-FS VIKOR method was proposed to prioritize the maintenance strategies from the perspective of three experts. The results showed that, thanks to advances in degradation modeling, sensor technology, and data analytics platforms, the RCM and CBM were the superior maintenance strategies for crude oil distillation systems.

Table 18. Ideal and null values.

	\mathbf{X}^*	\mathbf{X}_*
CS ₁	((0.42,0.52,0.52,0.58;1,1),(0.47,0.52,0.52,0.55;0.9,0.9))	((0,0.02,0.02,0.1;1,1),(0.01,0.02,0.02,0.06;0.9,0.9))
CS ₂	((0.46,0.56,0.56,0.6;1,1),(0.51,0.56,0.56,0.58;0.9,0.9))	((0.1,0.22,0.22,0.34;1,1),(0.16,0.22,0.22,0.28;0.9,0.9))
SF ₁	((0,0.02,0.02,0.1;1,1),(0.01,0.02,0.02,0.06;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
SF ₂	((0,0,0.06;1,1),(0,0,0.03;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
ST ₁	((0.02,0.08,0.08,0.18;1,1),(0.05,0.08,0.08,0.13;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
ST ₂	((0.06,0.12,0.12,0.22;1,1),(0.09,0.12,0.12,0.17;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
ST ₃	((0.12,0.22,0.22,0.34;1,1),(0.17,0.22,0.22,0.28;0.9,0.9))	((0.46,0.56,0.56,0.6;1,1),(0.51,0.56,0.56,0.58;0.9,0.9))
ST ₄	((0.14,0.26,0.26,0.38;1,1),(0.2,0.26,0.26,0.32;0.9,0.9))	((0.46,0.56,0.56,0.6;1,1),(0.51,0.56,0.56,0.58;0.9,0.9))
ST ₅	((0,0.02,0.02,0.1;1,1),(0.01,0.02,0.02,0.06;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
ST ₆	((0.42,0.52,0.52,0.58;1,1),(0.47,0.52,0.52,0.55;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
ST ₇	((0.46,0.56,0.56,0.6;1,1),(0.51,0.56,0.56,0.58;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))
ST ₈	((0,0.02,0.02,0.1;1,1),(0.01,0.02,0.02,0.06;0.9,0.9))	((0.5,0.58,0.58,0.6;1,1),(0.54,0.58,0.58,0.59;0.9,0.9))

Table 19. Fuzzy and defuzzified values of \tilde{S} .

Strategy	\tilde{S}	CR (\tilde{S})
RTFM	((0.18,0.3,0.3,0.37;1,1),(0.24,0.3,0.3,0.34;0.9,0.9))	0.28
PM	((-0.01,0.13,0.13,0.24;1,1),(0.05,0.13,0.13,0.18;0.9,0.9))	0.12
CBM	((-0.02,0.07,0.07,0.16;1,1),(0.02,0.07,0.07,0.12;0.9,0.9))	0.07
RCM	((-0.04,0.06,0.06,0.15;1,1),(0.01,0.06,0.06,0.11;0.9,0.9))	0.06

Table 20. Fuzzy and defuzzified values of \tilde{R} .

Strategy	\tilde{R}	CR (\tilde{R})
RTFM	((0.06,0.08,0.08,0.08;1,1),(0.07,0.08,0.08,0.08;0.9,0.9))	0.08
PM	((0,0.02,0.02,0.05;1,1),(0.01,0.02,0.02,0.04;0.9,0.9))	0.02
CBM	((0.04,0.06,0.06,0.07;1,1),(0.05,0.06,0.06,0.07;0.9,0.9))	0.06
RCM	((0.03,0.05,0.05,0.07;1,1),(0.04,0.05,0.05,0.06;0.9,0.9))	0.05

Table 21. Fuzzy and defuzzified values of \tilde{Q} with $v = 0.3$.

Strategy	$\tilde{Q}(v = 0.3)$	CR (\tilde{Q})	Ranking
RTFM	((0.017,0.112,0.112,0.181;1,1),(0.065,0.112,0.112,0.149;0.9,0.9))	0.105	4
PM	((-0.083,0.019,0.019,0.116;1,1),(-0.034,0.019,0.019,0.07;0.9,0.9))	0.017	3
CBM	((-0.062,0.027,0.027,0.109;1,1),(-0.017,0.027,0.027,0.07;0.9,0.9))	0.025	2
RCM	((-0.074,0.018,0.018,0.106;1,1),(-0.027,0.018,0.018,0.065;0.9,0.9))	0.017	1

Table 22. Fuzzy and defuzzified values of \tilde{Q} with $v = 0.5$.

Strategy	$\tilde{Q}(v = 0.5)$	CR (\tilde{Q})	Ranking
RTFM	((0.019,0.15,0.15,0.247;1,1),(0.085,0.15,0.15,0.2;0.9,0.9))	0.140	4
PM	((-0.109,0.032,0.032,0.164;1,1),(-0.04,0.032,0.032,0.1;0.9,0.9))	0.029	3
CBM	((-0.098,0.022,0.022,0.137;1,1),(-0.037,0.022,0.022,0.081;0.9,0.9))	0.020	2
RCM	((-0.11,0.013,0.013,0.133;1,1),(-0.048,0.013,0.013,0.075;0.9,0.9))	0.012	1

The work done in this study can be extended in many directions in future. For example, the results of this study can be compared with other hybrid MCDM

methods which have been used to solve the maintenance strategy selection problem. Some other theories, such as grey theory and rough set theory would also be

Table 23. Fuzzy and defuzzified values of \tilde{Q} with $v = 0.7$.

Strategy	$\tilde{Q}(v = 0.7)$	$CR(\tilde{Q})$	Ranking
RTFM	((0.021, 0.187, 0.187, 0.313; 1, 1), (0.104, 0.187, 0.187, 0.251; 0.9, 0.9))	0.175	4
PM	((−0.135, 0.046, 0.046, 0.212; 1, 1), (−0.045, 0.046, 0.046, 0.13; 0.9, 0.9))	0.041	3
CBM	((−0.133, 0.017, 0.017, 0.164; 1, 1), (−0.058, 0.017, 0.017, 0.092; 0.9, 0.9))	0.016	2
RCM	((−0.146, 0.008, 0.008, 0.16; 1, 1), (−0.069, 0.008, 0.008, 0.085; 0.9, 0.9))	0.007	1

Table 24. Fuzzy and defuzzified values of \tilde{Q} with $v = 0.9$.

Strategy	$\tilde{Q}(v = 0.9)$	$CR(\tilde{Q})$	Ranking
RTFM	((0.023, 0.225, 0.225, 0.379; 1, 1), (0.124, 0.225, 0.225, 0.302; 0.9, 0.9))	0.210	4
PM	((−0.161, 0.059, 0.059, 0.259; 1, 1), (−0.051, 0.059, 0.059, 0.159; 0.9, 0.9))	0.053	3
CBM	((−0.169, 0.012, 0.012, 0.192; 1, 1), (−0.078, 0.012, 0.012, 0.102; 0.9, 0.9))	0.011	2
RCM	((−0.182, 0.002, 0.002, 0.187; 1, 1), (−0.09, 0.002, 0.002, 0.095; 0.9, 0.9))	0.002	1

promising methods to determine the optimal maintenance strategy under uncertain conditions. The proposed maintenance strategy selection model can also be adopted in other industries, such as renewables.

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