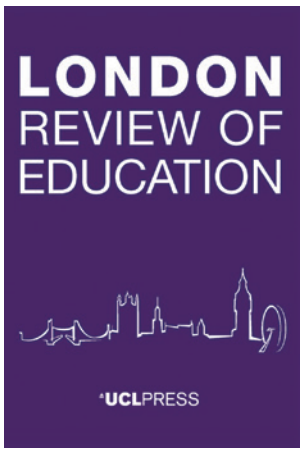

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Curriculum design through lesson study

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Abstract

Lesson study is increasingly prevalent as a collaborative activity in which teachers take part to explore their practice. There are many variations in how lesson study manifests itself, even in Japan, where it originated. However, in Japan, fundamental to lesson study is a focus on collaboration in researching teachers' professional practice. In this article, we draw on experiences of our collaborative research with colleagues in Japan to provide theoretical insights into how we might conceptualize and inform future developments of lesson study as action research that informs curriculum design and implementation. The approach taken develops ideas of the theory of didactical situations, and draws on the construct of boundary objects to understand Japanese lesson study. We identify a class of artefacts, *didactical devices*, that may provide a useful form of boundary object that supports the collaborative action research of lesson study. Although the particular focus of the work presented here is mathematics, the lessons that we draw should have applicability across the curriculum more widely.

Keywords: lesson study, action research, professional learning, didactical situations, boundary objects

Introduction

Japanese lesson study (JLS) is a translation of the Japanese words *jugyou* (instruction or lesson) and *kenkyuu* (research or study). It is the latter of these two words that indicates the focus of lesson study as an activity that is somewhat different from other forms of teacher collaboration, one in which a focus on research into classroom practice is central, be it in terms of such things as pedagogy, didactics, resources or use of manipulatives. In Japan, lesson study is framed around a form of action research that has, over time, come to establish a well-defined process, and it is well and truly embedded in professional practice. Although the context of the work reported here is mathematics education, the ideas relating to lesson study, action research and teacher learning have applicability more widely, and this is discussed in our concluding remarks.

Since the day when Stigler and Hiebert (1999) first wrote about JLS as a model for improvement of classroom practice in the United States, it has become a popular 'travelling reform' (Steiner-Khamsi and Waldow, 2012). In a landscape of increased education policy borrowing (Gorur, 2007; Grek, 2007), JLS has emerged as an increasingly popular approach to teacher professional learning in the United Kingdom and beyond, at local, regional and national scales.

In this article, we draw on our work in England and our collaborative research with colleagues in Japan to consider a theoretical analysis of lesson study that draws in particular on the theory of didactical situations and the construct of boundary objects to consider how JLS might be best used to facilitate curriculum design and

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implementation in ways that emphasize curriculum coherence. In doing so, we introduce a new construct: that of the *didactical device*, which we consider as having the potential as a boundary object to facilitate collaborative action research by communities of teachers significantly in ways that can inform curriculum design, implementation and impact. This work is particularly timely in England, where the latest Ofsted framework for inspection of schools 'is built around the connectedness of curriculum, teaching, assessment and standards' (Ofsted, 2019: 3).

In the next section, we provide a brief summary of lesson study as an activity, providing insight into the different stages of a lesson study cycle, as adopted and adapted in the context of England, and draw attention to how lesson study focuses on both student and teacher learning. We follow this with a section that emphasizes the research nature of lesson study, highlighting how each lesson study cycle is framed as a research study, and in this sense demonstrates key attributes of collaborative action research (Takahashi and McDougal, 2016), and how in the *kyozai kenkyu* ('study of material relevant to the research theme') planning phase of the study, teachers actively engage in a particular form of curriculum research. In the fourth section, we focus on teacher learning in lesson study by considering the work of Clivaz (2018), who developed Brousseau's (1997) theory of didactical situations to explore this issue. In the penultimate section, we further develop and theorize this approach by drawing on our recent work that has been considering how we might conceptualize developing curriculum coherence being facilitated by a number of didactical devices considered as boundary objects (Wake *et al*., 2020). Finally, our theoretical considerations allow us to draw some implications for lesson study as a particular form of action research in ways that can support curriculum design and implementation.

Lesson study as an activity

In Japan, lesson study is often, particularly in the elementary school phase, well-established in the professional culture of the school. It is carried out in regular cycles throughout the year, within boundaries defined by an overarching research theme. All teachers in a school will take part in three or four cycles of lesson study per year, with each of these cycles having a particular research question. Important in each cycle is a first phase of the process, in which teachers work in collaborative groups to carry out *kyozai kenkyu* ('study of material relevant to the research theme'). This study leads to the production of a collaboratively written plan for a research lesson. This detailed plan is written over several meetings. As Fujii (2016) pointed out, this plan might better be thought of as a 'research proposal' that considers carefully what we already know from a range of research, textbooks and supporting teacher guides, as well as from the collective experience of the group. The final documents provide much detail for the lesson, including anticipated student responses, how the teacher will respond to these, and expectations of how the narrative of the lesson will lead to the desired learning. As such, the 'lesson plan' embodies a shared understanding of teaching and learning *intentions* and provides an *image* of the intended lesson (Roth and Radford, 2011). Our experience has proved that it is difficult to develop this first phase of the process in English schools in ways that maintain both a broad focus on curriculum and a tight focus on the lesson itself. This latter focus is perhaps implied by the term *lesson* study, and we note that it has been rebadged in some countries, such as Sweden, as learning study in recognition of Marton's work and an agreed focus on variation theory (see, for example, Marton *et al*., 2019), and in the United States by Takahashi and McDougal (2016) as *collaborative lesson research*.

The research lesson is taught by a nominated teacher, who is a member of the collaborative planning group. Other members of the group act as silent observers, collecting evidence of student learning in relation to the intentions of the agreed plan, allowing for later analysis of the enactment of the lesson. As an immediate follow-up to the lesson, the collaborative group meet in a post-lesson discussion to formally discuss the evidence gathered, following a set of conversation protocols. An important part of this phase of the cycle is input from an outside expert (*koshi*, in Japan) who will sum up the lesson and provide carefully considered advice that aims to move the thinking of the group on in ways that might enhance their understanding and knowledge. The learning of the group in relation to the research theme is identified and recorded by the discussion chair. It is intended that this learning informs subsequent cycles of research and the overall research theme of the school.

Subsequent cycles of lesson study are planned and taught, now drawing on the findings from previous post-lesson discussions. These are new lessons and not revisions or reteachings of previous research lessons; rather, all lessons work towards collaborative professional learning around a particular theme.

The above description of the JLS cycle gives a somewhat idealized view of the process, one that we have seen parts of in many Japanese schools. In its transformation into other countries and schools, it is found that there is much variation in practice. Seleznyov's (2018) systematic literature review explores how faithful international models of lesson study are to these critical components. Of 97 studies published between 2005 and 2015:

- 33 per cent did not include the identification of a research theme
- 63 per cent did not include *kyozai kenkyu*
- 60 per cent involved the revising and reteaching of a lesson or focused on polishing a 'perfect' lesson
- 55 per cent did not engage an outside expert 'other' or *koshi*
- 61 per cent did not mention mobilizing knowledge between lesson study groups.

In summary, studies of the translation of JLS into contexts beyond Japan show that there is not an internationally shared understanding of JLS. It is interesting to note that several of the often-missing components are those that distinguish JLS as a research process, instead of merely a collaborative approach to professional development. Important to JLS are: having a research theme, exploring the literature base, learning from others' process and research, and generalizing beyond the lesson itself. Indeed, several studies of the translation of JLS beyond Japan have observed that teachers who are unfamiliar with the process of research engagement can lack the skills to engage effectively in JLS. Both Murata (2011) and Fernandez (2002) found that US teachers engaged in JLS struggled to develop a research hypothesis, design an appropriate classroom experiment, gather and use appropriate evidence, and generalize findings. In the next section, we consider lesson study as a particular form of action research, and in the remaining sections we consider one way in which lesson study can be (re) focused on curriculum design.

Lesson study as collaborative action research

Lesson study is a cyclical process that is designed to develop teacher knowledge using a systematic process over a sustained period of time. Importantly, the learning of the group is socially negotiated and constructed, with strategic input

from 'outsiders' who often have a long-standing relationship with the group and, as such, may be considered as part of the team. As we have highlighted, fundamental to JLS, as practised in Japan, is its focus on collaborative practitioner research. This situates lesson study firmly in the paradigm of action research, which Elliott (1991: 69) defines as 'the study of a social situation with a view to improving the quality of action within it'. As Posch (2019) suggested, action research focuses on both developmental interests (what practitioners would like to develop or change) and cognitive or research interests (what practitioners would like to find out or understand). In this sense, action research focuses on both practical skills and knowledge.

As described earlier, even in its many different manifestations, in terms of scope and scale, a fundamental aspect of JLS is a clear sense of teacher inquiry into – often very detailed – aspects of teachers' professional practice.

In their introduction to a special issue of the journal *Educational Action Research*, Hanfstingl *et al*. (2019) summarize that action research, lesson study and learning study approaches share many commonalities. Their summary highlights the cyclical nature of each as an activity, but suggests that lesson study is focused 'on a team-oriented instructional design and shared responsibility for the instructional outcome' (ibid.: 455), not explicitly recognizing the research nature of lesson study. We contest this concerning lesson study as practised in Japan. It is our experience that in Japan, the research focus is fundamental to JLS. As Hanfstingl *et al.* (ibid.) point out, there are researchers that do claim strong connections between JLS and action research, recognizing its focus on practitioner inquiry into practice in a systematic cyclical way, and these researchers suggest that JLS is a highly evolved form of action research that has a distinct set of processes and protocols (see, for example, Dudley, 2014; Austin, 2017). This is consistent with the notion of Stenhouse, whose work in action research has been seminal, and considers that action research can be used effectively to test curriculum proposals (Stenhouse, 1983) in alignment with lesson study as a form of professional collaborative research.

It is our view that JLS is very clearly a form of action research, indeed a form of collaborative action research that has as its objective teacher learning in relation to their knowledge of curriculum implementation, both in the immediacy of particular lessons and more widely in terms of the subject discipline in its totality. That is, lesson study provides teachers with opportunities for developing insight into the interconnectedness of mathematics topics across the curriculum and over time. Important to our stance is recognizing our own position as having dual roles, as both researchers and educators with a deep concern about our own particular curriculum area. In our experience, it is through research that focuses on teaching in the reality of our classrooms, with the diverse experiences, needs and aspirations of learners, that we can best understand the curriculum in action.

Theoretical considerations 1: Didactical situations and devices

Fundamental to the whole endeavour of lesson study is the pursuit of teacher learning and the collaborative inquiry/research that relates to it. In this section we provide, from a theoretical position, a view of what this entails and how it is supported in Japan. In doing so, we provide insight into why it is likely that adoption/adaptation of JLS as a practice may be difficult in another country and educational culture, and, importantly, how the process of adoption might be usefully supported.

We draw on two complementary perspectives, developing on thinking that relates (a) to the theory of didactical situations (Brousseau, 1997) and the recent thinking of Clivaz (2018), who has examined lesson study from this perspective, and (b) to that of boundary objects and boundary crossing, as developed in cultural historical activity theory (Akkerman and Bakker, 2011) and elsewhere. In this section we consider the former of these; we consider the latter perspective in the next section.

The theory of didactical situations in mathematics education provides insight into the role of classroom teaching by defining a particular type of learning situation: an adidactical situation, in which teachers provide a carefully designed milieu (Brousseau, 1997) in relation to which students engage in learning of the subject. Fundamental to this is a carefully designed task that facilitates students in developing new mathematical understanding by scaffolding from what they already know and understand (Vygotsky, 1987). In Japan, such tasks are designed in the most minute detail in ways that often prompt misconceptions to be surfaced and reconciled through dialogic interactions between students themselves and, at a later stage, between students and teacher. Japanese mathematics problem-solving lessons have a well-defined structure that first facilitates dialogic learning between students and, then, dialogic learning between students and teacher. In both episodes of dialogic learning, different thinking and approaches to solving the problems that have emerged in the class are discussed, and conclusions are drawn about the new mathematics at issue and about problem-solving methods. It is in the phase of the lesson known as seat work (*kikan-shido*) that students are effectively engaged with the milieu, during which, in Brousseau's terms, they engage in action and formulation (where they develop their ideas). Validation (where students convince themselves and each other of the validity of their ideas) and institutionalization (in which the teacher returns to the situation and formalizes the learning of the class) take place at a later stage of the mathematics problem-solving lesson. Validation and institutionalization occur in the *neriage* phase of the lesson, in which the teacher draws on individual student thinking and, through carefully orchestrated discussion, enables students in the class to arrive at an agreed understanding of the mathematics. In the *matome* phase, the learning of the class is summarized and formalized.

Clivaz (2018) considers teacher learning in the lesson study cycle from this perspective. In doing so, he identifies the work of teachers in studying the curriculum and planning the lesson – the *kyozai kenkyu* phase – as being adidactical for them. That is, in this phase, teachers engage in a (adidactic) milieu, with a focus on a professional problem in relation to the curriculum. Clivaz (ibid.) suggests that the milieu is provided by the development of the research lesson, for it is this on which the teachers act, and the writing of the lesson plan or lesson proposal provides a formality to the formulation stage. Later in the lesson study cycle, in the post-lesson discussion, the actual lesson, both as enacted and planned, provides a milieu for further potential teacher learning in what can be considered to correspond to the validation and institutionalization stages in Brousseau's (1997) terms.

Here, we focus particularly on the *kyozai kenkyu* phase of the lesson study cycle, as it is here that we believe we detect an important issue relating to lesson study as practised in Japan and elsewhere. The issue is one of the lack of careful depth of curriculum knowledge and understanding in *kyozai kenkyu* as practised outside Japan. It is our contention that lesson study as practised in Japan provides implicit and important features that facilitate potential learning in *kyozai kenkyu* and post-lesson discussion stages. Further to this, we detect a potentially important feature of lesson study work in Japan that ensures that the milieu of the *kyozai kenkyu* and post-lesson discussion phases facilitate teacher learning in relation to the mathematics

curriculum: we note that discussion in relation to curriculum as implemented in the research lesson, and as developed more widely over time, is supported by a class of devices that we will explore in further detail below.

Particularly outside Japan, teacher collaboration with colleagues in *kyozai kenkyu* contrasts unfavourably as a learning situation when compared to situations for student learning in lessons, where the design of tasks is much more carefully and deliberately engineered (Margolinas and Drijvers, 2015). Important in the design of tasks for mathematics lessons is careful understanding and consideration of the mathematical structure that underpins the topic of mathematics to be learned. In our research, in collaboration with colleagues in Japan, we have identified a number of what we have termed *didactical devices* that are important in facilitating professional learning in relation to curriculum connections and coherence. We use the term to distinguish how such devices provide for careful structuring of the curriculum and its teaching and learning in ways that support discussion about coherence, both within and across topics and over time. Such devices become part of the cultural setting in which mathematics teaching and learning is situated in Japan, and they provide an additional resource that is important in the milieu of *kyozai kenku*. This effectively means that the milieu of *kyozai kenkyu* in Japan is not quite so open as at first sight might be experienced in other countries.

Let us exemplify this by reference to the number line as a didactical device. In interviews with lead authors of a popular series of elementary and secondary school textbooks in Japan, we have discovered that there is an 18-page document that sets out ideas relating to the use of number lines and associated thinking, which is referred to by authors and used in the professional development of teachers. This document illustrates how the notion of the number line will be developed from the very early stages of working with mathematics, through to its more formal introduction and use to consider adding and subtracting numbers. For example, Figure 1 illustrates how

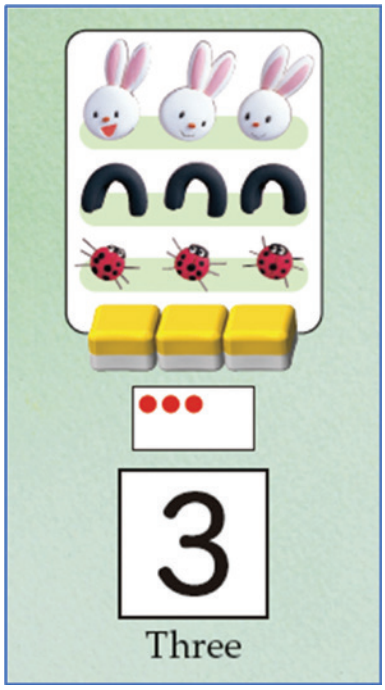

Figure 1: Early counting arranged horizontally in a Japanese textbook, to encourage number line development
Source: Hironaka and Sugiyama (2006a)

even in early counting situations, the textbooks illustrate that teachers (and students) should consider ideas of one-to-one correspondence working horizontally, thus leading naturally at a later stage to the introduction of the number line as a horizontal tool to support conceptual understanding of a range of important mathematical ideas.

Further to this, the number line is introduced and used to consider the size and position of whole numbers, their addition and subtraction, likewise with fractions and decimals, and so on. A double number line is introduced when working with situations involving proportionality (see Figure 2), and this is eventually developed further to consider Cartesian graphs of such relationships and other functions by considering one of the number lines being rotated vertically.

More generally in our research we have identified a range of devices upon which teachers draw to facilitate classroom learning of mathematics. We differentiate these into three classes: general pedagogies, mathematics-specific pedagogies and what we have termed *didactical devices*. To clarify, we consider there to be a class of general pedagogies that are non-subject specific, with applicability across many lessons in many subjects (for example, dialogic teaching (Alexander, 2008) as a pedagogy involves teachers in supporting students in meaningful peer-to-peer and teacher–student dialogue about key ideas and understanding); a class of mathematics-specific pedagogies that draw attention to mathematical structure (such as asking students to give an example that demonstrates understanding of a particular concept and then asking for 'another', 'and another', 'and another' (Watson and Mason, 2006)); and, finally, didactical devices, which are more closely connected with specific aspects of mathematical knowledge and its structuring (with the number line being used to exemplify this type of device).

It is our contention that didactical devices, although recognized explicitly in the case of number lines in Japan, are not always recognized so explicitly, and that they are important to the development of coherence in the mathematics curriculum, in the sense of both the specified and the implemented curriculum. It is noteworthy that a comparison of curriculum specifications in the UK and Japan highlights that in Japan much greater attention is paid to specifying how subject-specific content should be considered in terms of teaching. In Japan, the 'Course of study' document (Takahashi *et al.*, 2008) is regarded as a legal document. Alongside the legal documents that specify the mathematics curriculum, the Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT) has published guides that set out more detailed guidance regarding curriculum design and implementation – for example, guides that support the development of the curriculum, justifying why aspects of the subject as an academic school discipline are included and how it might be taught (MEXT, 2008). In Japan, these guides are not legally binding, but they do inform the

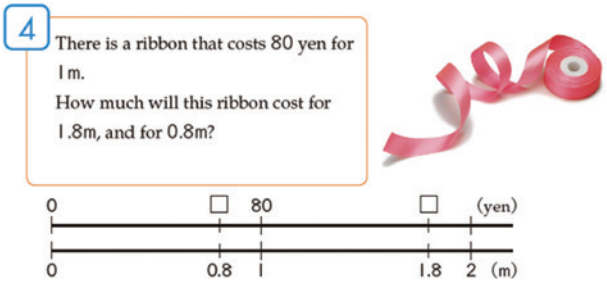

Figure 2: Double number line used in Japanese textbook
Source: Hironaka and Sugiyama (2006b: 36)

writing of textbooks, and the authorization of textbooks by MEXT. There is even greater detailed support for teacher development in further guides that support classroom research and, for example, in publisher-led professional development that draws on a document that sets out how the number line is developed as a didactical device (in our terms) in and across their textbooks.

It is important to emphasize that, as is the case with all the devices identified above, didactical devices require positive action by teachers as individuals in their classrooms for effective activation. In this sense, we consider the devices as artefacts that can be drawn upon to mediate student learning. In the classroom in general, access to artefacts is controlled by the teacher. For example, tasks that are selected by the teacher to stimulate learning, texts, manipulatives, technology, mathematical techniques, representations and so on are all artefacts that the teacher can potentially bring into use. How these artefacts are made *instrumental* in action (Drijvers and Trouche, 2008), and how the teacher therefore facilitates students' development of their conceptual understanding, is central to teaching and learning. In the development of the theory of instrumental genesis, Drijvers and Trouche (ibid.) make the distinction between an artefact, which has only the *potential* to support actions, and its instrumental use, which requires the user to have a mental scheme that supports both technical and conceptual abilities to realize this potential in a specific situation. For example, the number line as an artefact can be considered as a material object with the potential to be used to consider the position and magnitude of numbers such as fractions. For the number line to be used instrumentally in this way, the user must understand both the potential and the appropriateness of doing so and have the technical expertise and conceptual understanding to do this. In our analysis, the lesson study team's work in planning implicitly focuses on this issue: how can teachers bring artefacts to instrumental use to facilitate students' conceptual understanding?

Theoretical considerations 2: Boundaries

In this section we focus our attention more directly on teachers as learners through their collaborative action research into their practice. In lesson study, teachers can be considered as engaging in two main areas of activity: one aligned with their usual day-to-day practice centred on the learning of students in the research lesson classroom, and the other in collaboration with colleagues centred on their professional learning in the lesson study community. This draws our attention to notions of *boundary crossing* with members of the lesson study community active in each of the communities of classroom and lesson study group. In this way, teachers can be considered as *boundary crossers* between the two communities. In their consideration of boundaries, Akkerman and Bakker (2011: 133) identify 'a socio-cultural difference leading to a discontinuity in action or interaction' between two such communities. In their review of research into boundary crossing and boundary objects, while recognizing that the terms are used with differences by different researchers, they argue that from a sociocultural perspective, all learning involves boundaries. In lesson study, the discontinuity at the boundary between classroom and lesson study group is crucial in facilitating reflection on curriculum design, implementation and impact. It is boundary work that involves teachers as boundary crossers making sense of their daily routines of teaching in terms of researching the curriculum-in-action that has the potential to facilitate professional learning for participants. Indeed, in sociocultural terms, learning may be considered as being located in the changing relationships between an individual and the social activities in which they engage (Beach, 1999). A challenge, then, is to understand

how we might best support learning at the boundaries of classroom and lesson study group – indeed, how we might design for effective boundary crossing. It is in the understanding and use of didactical devices that we consider there is potential to affect this work in a meaningful way. It is our contention that this is part of the process in Japan, which, although hidden to outsiders and perhaps not explicit to insiders themselves, is effectively part of the process of lesson study, due to the implicit and long-established use of didactical devices.

We consider didactical devices to form a class of boundary object (Star and Greisemer, 1989) that both supports teachers' mediation of students' learning of new mathematics and facilitates their own learning in relation to the curriculum. Boundary objects are artefacts that have potential use in two or more activity systems, such as classroom and teacher lesson study group, and that have sufficient flexibility to have distinct use in each different setting while maintaining significant features across both. For example, the lesson plan itself is associated both with the lesson study research lesson, where it has meaning as a script for the lesson, and the ensuing post-lesson discussion (Wake *et al*., 2016), where it serves to set out the key aspects of research carried out in the lesson. As such, the lesson plan demonstrates the key characteristics of boundary objects in that it supports activity in the two different activity systems in which a teacher operates: that of the classroom where she works directly with students, and that of the lesson study group, where she discusses teaching issues in relation to curriculum with colleagues.

Didactical devices such as the number line we consider as other important boundary objects, which have the potential to support teachers with their work in the didactical transposition of the (specified) mathematics curriculum to classroom teaching and learning.

Boundary objects by their very nature support boundary crossing and, as we highlight above, Akkerman and Bakker (2011) in their overview of boundaries and boundary crossing point out that many studies suggest that it is at such boundaries that learning takes place. This suggests that it is in activities that explore transitions and transpositions between different communities of practice (Wenger, 1998), such as in lesson study or teacher research groups and classrooms and lessons, that new professional knowledge relating to teaching and learning can be constructed. This perspective helps inform our work in design, for example, of curriculum materials (Wake *et al*., 2013, 2014).

As we described in the previous section, it is clear, although not explicitly emphasized in the Japanese texts, that boundary objects that support didactic transposition are introduced (didactical devices). For example, the number line as a representation is used again and again across the curriculum to provide a coherence in approach to support students to develop common 'ways of seeing' concepts that they are familiar with so far, and to help to support insight into new concepts that are introduced as objects of learning. Not only does this didactical device support student learning, but we also claim that it can act as a tool that supports teacher learning in their collaborative action research.

As the ongoing research of Wake (see http://m4l.org.uk/) is finding, didactical devices such as the double number line can be used by both teachers and learners to gain insight into their different conceptualization of proportionality and its effective application in a range of contextual problems. In current research, we have explored how the double number line as a didactical device can be developed as a boundary object to support teacher learning in a modified version of lesson study. However, the use of a device such as the double number line, either by a student or a teacher,

requires their insight into its potential in a range of different situations before it can be used effectively. The activation of the device or artefact – in positive classroom actions and in teachers' post-lesson discussions, in ways that support both student and teacher learning (the former of mathematics and the latter of the curriculum) – is considered as making it instrumental in use in different settings for different purposes. From this perspective, then, to inform effective curriculum implementation, it is important to ensure that we have didactical devices as artefacts with the potential to be instrumentally used in two different communities: the classroom and the post-lesson discussion. The artefact is designed so that, according to Drijvers and Trouche (2008), it becomes an instrument that both shapes the thinking of the user, in this case the teacher (the instrumentation process), and is in itself shaped by the user (the instrumentalization process). It is our contention that this theoretical perspective might inform the process of curriculum design for coherence in ways that allow for teachers to engage in co-construction of knowledge as they engage in the process of didactical transposition for effective learning.

Conclusion

We have positioned lesson study as a particular form of action research that involves teachers in collaborative research into their practice. The practice is highly evolved in Japan, where it originated, and its adoption and adaptation has become widespread internationally, particularly in the subject area of mathematics. It is important to note that in Japan lesson study is seen as having an important role to play in supporting teachers developing not only their individual professional knowledge, but also as a professional knowledge base shared more widely, initially via teacher accounts of their lesson study research, and later becoming reified in published textbooks and teacher guides. We should also remark that in Japan a social justice agenda is implicitly addressed, as the aim is always (at least in elementary and junior high schools) to teach all students in mixed attainment groups, ensuring equality of access to the academic curriculum. This agenda is often reflected in lesson study research questions and consequently discussed in post-lesson discussions.

In the transformation of JLS into other countries and cultures, we detect that the research focus of lesson study can be lost. We also detect that there are aspects of lesson study that are not necessarily considered or well understood. This is in line with claims that JLS as a process is under-theorized. Importantly in this regard, we point to a particular class of artefacts that support teachers in their curriculum work: didactical devices. These devices, we claim, go largely unrecognized as they are mainly implicit in the work of teachers, rather than being explicit and therefore more visible. Although these devices have meaning and use in individual lessons and sequences of lessons, they have the potential to provide structure to the curriculum in terms of understanding its interconnectedness internally to topics of a subject and more widely across topics, how they are sequenced and introduced temporally. Japanese textbooks embed these didactical devices in their development of the subject, and consequently permeate teachers' thinking as they develop from novice to expert.

We draw on Brousseau's (1997) theory of didactical situations and consider how this relates to phases of Japanese mathematics problem-solving lessons and note how didactical devices can be considered from this theoretical perspective to support student learning through phases of action, formulation in the adidactical milieu of the *kikan-shido* phase of the lesson, and to support the teacher in orchestrating the *neriage* and *matome* phases of the lesson as students engage in validation and

institutionalization (Brousseau's terms (ibid.)) of their mathematical understanding. Importantly, we also consider that didactical devices perform a similar role for teachers as they plan their research lessons and follow up the teaching of the lesson with discussions that focus on curriculum implementation. In this way, we consider didactical devices as important boundary objects that facilitate teacher inquiry/research into their professional practice, with the didactical devices having meaning in both classrooms and lesson study research groups.

We have argued that lesson study as a form of action research needs to be supported by careful design of process supported by boundary objects with meaning and purpose in both classroom and lesson study groups. In Japan, where lesson study is culturally long-standing and deeply embedded into the practice of teachers, such design has been developed over many years and has become part of teachers' professional practice to the extent that important details such as didactical devices have become almost invisible to practitioners. Our analysis suggests that we still have much to learn, with the theorization of the detail of lesson study still being very underdeveloped.

Notes on the contributors

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Sarah Seleznyov is Director of London South Teaching School Alliance and a PhD student at Vrije Universiteit Amsterdam, the Netherlands. Her research focuses on research approaches to teacher professional development, especially Japanese lesson study.

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