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Dynamic Process Modeling and Hybrid Intelligent Control of Ethylene Copolymerization in Gas-phase Catalytic Fluidized-bed Reactors

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ABSTRACT

BACKGROUND: Polyethylene (PE) is the most extensively consumed plastic in the world and gas-phase based processes are widely used for its production due to their flexibility. The sole type of reactor that can produce PE in gas-phase is fluidized bed reactor (FBR) and effective modeling and control of FBRs are of great importance for design, scale-up and simulation studies. This paper discusses these issues and suggests a novel advance control structure for these systems.

RESULTS: A unified process modeling and control approach is introduced for ethylene copolymerization in FBRs. The results show that our previously developed two-phase model is well confirmed using real industrial data and is exact enough to further develop different control strategies. It is also shown that due to high system nonlinearities, conventional controllers are not suitable for this system and advance controllers are needed for this system. Melt flow index (MFI) and reactor temperature are chosen as vital variables and intelligent controllers were able to sufficiently control them. Performance indicators show that advanced controllers have a superior performance in comparison with conventional controllers.

CONCLUSION: Based on control performance indicators, the adaptive neuro-fuzzy inference system (ANFIS) controller for MFI control and the hybrid ANFIS-PID controller for temperature control perform better regarding the disturbance rejection and set point tracking in comparison with the conventional controllers.

Keywords: Polyethylene; Process control; Fluidized-Bed Reactor; Dynamic Modeling; Solid Elutriation; Artificial Intelligence

Short Title: Dynamic Modeling and Advanced Control of Ethylene Copolymerization in FBRs

Declarations of interest: none

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NOMENCLATURE			
A	cross sectional area of the reactor (m^2)	M_n	number average molecular weight of polymer (kg/kmol)
ALDMC	adaptive linear DMC	MPC	model predictive controller
AI	artificial intelligence	M_w	weight average molecular weight of polymer (kg/kmol)
$AlEt_3$	triethyl aluminum co-catalyst	MWD	molecular weight distribution
ANFIS	adaptive neuro-fuzzy inference system	mw_i	molecular weight of monomer i (g/mol)
APMBC	adaptive Predictive Model-Based Control	$N(0, j)$	uninitiated site of type j produced by formation reaction
Ar	Archimedes number	NARMAX	Nonlinear Auto Regressive Moving Average with eXogenous input
B_i	moles of reacted monomer bound in the polymer in the reactor	$N^*(j)$	potential active site of type j
CFD	computational fluid dynamics	$N_d(j)$	spontaneously deactivated site of type j
$C_{p,pol}$	specific heat capacity of solid product (J/kg·K)	$N_{dth}(0, j)$	impurity killed sites of type j
C_{pg}	specific heat capacity of gaseous stream (J/kg·K)	N_H	uninitiated site of type j produced by transfer to hydrogen reaction
C_{pi}	specific heat capacity of component i (J/kg·K)	$N_j(r, j)$	living polymer molecule of length r , growing at an active site of type j , with terminal monomer M
CSTR	continuous stirred tank reactor	P	pressure (Pa)
d_b	bubble diameter (m)	NLMPC	nonlinear model predictive controllers
d_{b0}	initiate bubble diameter (m)	PBE	population balance equation
D_g	gas diffusion coefficient (m^2/s)	PID	proportional-integral-differential
DMC	dynamic matrix control	PDI	polydispersity index
d_p	particle diameter (m)	$Q(r, j)$	dead polymer molecule of length r produced at a site of type j
D_t	reactor diameter (m)	r	number of units in polymer chain
ETC	error trajectory controller	Re_{mf}	Reynolds number of particles at minimum fluidization condition
FBR	fluidized-bed reactor	R_i	instantaneous consumption rate of monomer (kmol/s)
F_{cat}	catalyst feed rate (kg/s)	R_p	production rate (kg/s)
FDM	fuzzy decision making	R_v	volumetric polymer outflow from the reactor (m^3/s)
f_i	fraction of total monomer in the reactant gas which is monomer M_i	SISO	Single-input-single-output
FIS	fuzzy inference system	T	temperature (K)
FH	fuzzy Hammerstein	t	time (s)
FLC	fuzzy logic controller	T_{in}	temperature of the inlet gaseous stream (K)
g	gravitational acceleration (m/s^2)	T_S	Takagi-Sugeno fuzzy inference system
GPC	gel permeation chromatography	T_{ref}	reference temperature (K)
H	height of the reactor (m)	U_t^*	dimensionless terminal falling velocity coefficient
H_2	hydrogen	U_0	superficial gas velocity (m/s)
H_{bc}	bubble to cloud heat transfer coefficient ($W/m^3 \cdot K$)	U_b	bubble velocity (m/s)
H_{bc}	bubble to emulsion heat transfer coefficient ($W/m^3 \cdot K$)	U_{br}	bubble rise velocity (m/s)
H_{ce}	cloud to emulsion heat transfer coefficient ($W/m^3 \cdot K$)	U_{mf}	minimum fluidization velocity (m/s)
HDPE	high-density polyethylene	U_t	terminal velocity of falling particles (m/s)
ICA	induced condensing agent	V	reactor volume (m^3)
i	monomer type	V_p	volume of polymer phase in the reactor (m^3)
I_m	impurity such as carbon monoxide ($kmol/m^3$)	W_b	weight of solids in the bubble phase (kg)
J	active site type	W_e	weight of solids in the emulsion phase (kg)
K_b	elutriation constant in bubble phase ($kg \cdot m^2/s$)	$X(n, j)$	n th moment of chain length distribution for dead polymer produced at a site of type j
K_{bc}	bubble to cloud mass transfer coefficient (1/s)	$Y(n, j)$	n th moment of chain length distribution for living polymer produced at a site of type j
K_{bc}	bubble to emulsion mass transfer coefficient (1/s)	Z-N	Ziegler-Natta catalyst
K_{ce}	cloud to emulsion mass transfer coefficient (1/s)	Greek letters	

$k_{dt}(j)$	deactivation by impurities rate constant for a site of type j	ΔH_R	heat of reaction (J/kg)
$k_{ds}(j)$	spontaneous deactivation rate constant for a site of type j	δ	volume fraction of bubbles in the bed
K_e	elutriation constant in emulsion phase (kg·m ² /s)	ϵ_b	void fraction of bubble for Geldart B particles
$k_f(j)$	formation rate constant for a site of type j	ϵ_e	void fraction of emulsion for Geldart B particles
$k_{ft}(j)$	transfer rate constant for a site of type j with terminal monomer M_i reacting with hydrogen	ϵ_{mf}	void fraction of the bed at minimum fluidization
$k_{fm}(j)$	transfer rate constant for a site of type j with terminal monomer M_i reacting with monomer M_k	μ	gas viscosity (Pa·s)
$k_{ft}(j)$	transfer rate constant for a site of type j with terminal monomer M_i reacting with $AlEt_3$	ρ_g	gas density (kg/m ³)
$k_{ft}(j)$	spontaneous transfer rate constant for a site of type j with terminal monomer M_i	ρ_{pol}	polymer density (kg/m ³)
k_g	gas thermal conductivity (W/m·K)	ϕ_s	sphericity for sphere particles
$k_{ri}(j)$	rate constant for reinitiating of a site of type j by monomer M_i	Subscripts and superscripts	
$k_{ri}(j)$	rate constant for reinitiating of a site of type j by cocatalyst	1	ethylene
$k_i(j)$	rate constant for initiation of a site of type j by monomer M_i	2	1-butene
$k_{pik}(j)$	propagation rate constant for a site of type j with terminal monomer M_i reacting with monomer M_k	b	bubble phase
k_{pTi}	propagation rate constant (m ³ /kmol·s)	e	emulsion phase
LDMC	linear DMC	i	component type number
LDPE	low density polyethylene	j	active site type number
LLDPE	linear low-density polyethylene	mf	minimum fluidization
MFI	melt flow index (g/10min)	pol	polymer
$[M_i]$	concentration of component i in the reactor (kmol/m ³)	ref	reference condition
$[M_i]_{in}$	concentration of component i in the inlet gaseous stream	T, TT	Pseudo kinetic rate constants
MIMO	Multi-input-multi-output		

1 INTRODUCTION

2 Union Carbide's process technology, known as "UNIPOL" to produce polyethylene (Figure 1)
 3 is characterized by moderate operating conditions (e.g. low pressure and temperature) and is
 4 less energy intensive than the traditional high-pressure process (2000-3000 atm). High purity
 5 ethylene and catalyst are continuously fed into the fluidized-bed reactor, where ethylene and
 6 co-monomers such as 1-butene or 1-hexene are polymerized together at a pressure of 20-30
 7 atm with a narrow temperature range.¹

8 FIGURE 1

9 A crucial aspect is the tight temperature control in gas phase polymerization
 10 reactors. The aim is to make sure that the reaction zone temperature stays higher than the

1 reactants' dew point, but under the polymer's melting point; this is to avoid melting and
2 subsequent particle agglomeration.² Consequently, most of the commercial gas phase
3 polyolefin fluidized bed reactors work in such a small temperature range of 248 to 400 K.³
4 Even so, temperature excursions have to be prevented since they can cause some remarkable
5 deviations in product properties and lead to low catalyst productivity.⁴ Lacking temperature
6 control, industrial polyolefin reactors have the tendency to be subjected to undesired situations
7 such as unsteady states, or temperature trips toward undesirable high reaction zone temperature
8 which is above the polymer softening point.⁵⁻⁹ Also noteworthy is that the maximum
9 polyethylene production rates take place when they work in the unstable steady states which
10 surely need a proper controller to make stable steady states.

11 A vital polymer property that should be carefully controlled is the polymer MFI.
12 Even though MFI is a quantity to measure polymer processability and its rheological behavior,
13 it has been shown that MFI correlate with the final product properties and is one of the main
14 variables to decide on the properties of a specific polymer grade. Two main standards are used
15 to decide the polymer MFI by putting the polymer under a certain weight and temperature and
16 measure the weight of melted polymer that passes an orifice with a certain size. ASTM D1238
17 and ISO 1133 are the standards used throughout the industry which differ on the testing
18 specifications. For example, MFI of different linear low-density polyethylene (LLDPE) grades
19 may range from 0.1 to 5 g/10min or even higher for special uses. LLDPEs with high MFIs may
20 be used in producing geomembrane, high strength film, food and industrial packaging, and
21 shopping bags while applications such as molded containers, chemical and water tanks and
22 traffic cone need polymers with lower MFI. Although MFI was impossible to be measured on-
23 line previously, recent modern technology with invention of new methods and instruments gave
24 us the opportunity to control this vital polymer property online.¹⁰

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3 1 Despite the non-linear attribute of the chemical processes, most of them are still
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5 2 regulated by the linear controllers aided by the plant models that are linearized.^{11,12} The
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7 3 Arrhenius behavior of the rate constants is one of the main reasons behind the non-linearity in
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9 4 chemical reactors in terms of energetic transport phenomena, however, reaction kinetics and
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11 5 complex flow patterns also have effects on the nonlinearity of this process. Similarly,
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13 6 constrained manipulated variables possibly will give saturation non-linearity. During recent
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15 7 decades, non-linear controller designs were suggested to control specific highly non-linear
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17 8 systems where precise control was the important criterion. It is easy to design the input-output
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19 9 linearizing controllers, but they are restricted to minimum-phase systems since the zero
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21 10 dynamics of the process must be stable for the non-linear inversion.

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26 11 Several researchers have studied controlling the main variables in olefin
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28 12 polymerization process using FBRs. McAuley and Macgregor¹³ studied the non-linear model
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30 13 inverse-based control of polymer melt flow index (MFI) and density during grade changeover
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32 14 by manipulating hydrogen and butene feed concentration. They concluded that the nonlinear
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34 15 controller shows superior performance compared to a linear one and how essential it is to take
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36 16 nonlinearities into account for good product property control. Dadebo et al.² controlled the
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38 17 temperature and showed that the nonlinear error trajectory controller (ETC) shows better
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40 18 performance when compared to an optimally-tuned PID controller and showed that nonlinear
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42 19 controllers have an advantage as they are more capable in rejecting disturbances and ensure
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44 20 stability. Ali et al.¹⁴ have tried to control monomer concentration and reactor temperature using
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46 21 improved single-input-single-output (SISO) and multi-input-multi-output (MIMO) PI-based
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48 22 controllers by manipulating the catalyst feed rate, U_0/U_{mf} ratio and feed temperature. Sato et
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50 23 al.¹⁵ designed a model inverse optimal MIMO servo controller with integral actions plus an
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52 24 attached nonlinear compensator to control the MFI and polymer density by altering the fresh
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54 25 hydrogen and butene feed concentrations.
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1 A number of researchers tackled this topic by employing nonlinear model
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4 1 predictive controllers (NLMPC). Hedengren et al.¹⁶ used moving horizon estimation (MHE) to
5 2 estimate new model parameters based on new plant data and have used the model for an
6 3 NLMPC to control 26 control variables by altering 10 manipulated variables. In another work,
7 4 Ali and Ali¹⁷, used NLMPC to control polymer molecular weight distribution (MWD) by
8 5 manipulating the ratio of monomer to hydrogen.
9

10 7 In relation to using intelligent controllers for gas-phase olefin polymerization in
11 8 FBRs, one two works were found to utilize artificial intelligence (AI) or expert system-based
12 9 controllers to maintain the system at optimal conditions and reject disturbances. Ghasem¹⁶
13 10 discussed that an effective way of handling nonlinearities present in this process is to use fuzzy
14 11 logic. He recommended that because of the high accuracy in using PI controller at steady state
15 12 and high performance of fuzzy logic in transient state, a grouping between the two can give an
16 13 optimal solution. The author applied this hybrid controller in regulation of the reaction
17 14 temperature by manipulating the cooling water inlet temperature to the heat exchanger. The
18 15 results showed that the hybrid controller can significantly improve the severe oscillations that
19 16 a sole fuzzy logic controller (FLC) produces. Ibrehem et al.¹⁸ developed a model to take into
20 17 account the presence of particles participating in the reaction with emulsion and catalyst phases
21 18 and implemented a neural-network based predictive controller for controlling the temperature
22 19 of the polyethylene system. He compared it with conventional controllers and showed that their
23 20 neural network based adaptive controller gives better results in comparison with those PI
24 21 controllers. Although polymer MFI is a very important polymer property, controlling it was
25 22 not discussed in these two studies. Furthermore, using fuzzy logic alone needs an expert
26 23 knowledge to define membership functions while neural networks provides a great tool to do
27 24 this task for nonexpert designers. Benefiting from both these tools was the aim of designing
28 25 the controller that is presented in this study.
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3 1 In this study, a newly published dynamic two-phase ethylene copolymerization
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5 2 model¹⁹ is used to simulate the process in order to study different control structures. This is a
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7 two-phase model which takes particle entrainment into account and the related equations are
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10 4 integrated with kinetic, hydrodynamic and transport phenomena equations. In the process
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12 5 control study, application of intelligent controllers on this highly nonlinear system is examined.
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14 6 For this objective, a SISO hybrid adaptive neuro-fuzzy inference system (ANFIS)-PID
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16 7 controller is used to both stabilize temperature at setpoints below the polyethylene's melting
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18 8 point and to reject temperature excursions resulting from disturbances. Furthermore, the
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20 9 ANFIS controller is used to control the polymer MFI with the aim of minimizing settling time
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22 10 and overshoot which are necessary for grade transition practices.
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28 11 **METHODOLOGY**

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31 12 A lot of research has been done on either mathematical modeling²⁰⁻²² or computer fluid
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33 13 dynamics (CFD) modeling²³⁻²⁶ of this system. For testing the control structures, our newly
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35 14 modified dynamic model for ethylene copolymerization¹⁹ is adopted. It is a two-phase and two-
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37 15 site copolymerization kinetic and hydrodynamic modeling structure for 1-butene and ethylene
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39 16 to help to get a further pragmatic view on heterogeneous ethylene copolymerization over Z-N
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41 17 catalyst in an FBR. Particle elutriation is embedded in the model to consider particle loss from
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43 18 the reactor bed which brings about a more improved model. This model helps to design and
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45 19 test new intelligent controllers that is discussed in this work.
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51 20 **Polymerization model**

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54 21 As established in the present work, the kinetic model suggested Abbasi et al.¹⁹ produced an
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56 22 inclusive mechanism which details the 1-butene and ethylene copolymerization over Z-N
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58 23 catalyst. Parallel and series reactions in catalytic ethylene copolymerization with alpha-olefin
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1 copolymers contribute to the complication of the process. The detailed multi-site kinetic model
 2 recommended weighs upon heterogeneous olefin copolymerization over Z-N catalysts.
 3 Important polymerization reactions as mentioned in their work can be referred to in Table 1.
 4 The most often used technique for polymerization modeling is the method of moments. The
 5 reason for this is that, by using this method, polymer properties and branching frequency,
 6 besides the means to estimate operational variables, can be foretold. These moment formulas
 7 are provided in Table 2 in which index *i* and *j* refer to the monomer and active site types
 8 respectively. Table 3 gives each reaction's rate constant for both site types and their literature
 9 sources.

10 **TABLE 1**

11 **TABLE 2**

12 **TABLE 3**

13 With the assumption that monomers are largely used during the propagation reactions, each
 14 component's rate of consumption can be obtained after solving the moment equations:²²

$$R_k = \sum_i^m \sum_j^{ns} [M_k] Y(0, j) k_{pik} \quad (1)$$

16 Where *m* is the number of monomer and *ns* is the number of active site types. Subsequently,
 17 the overall polymer production rate can be attained:

$$R_p = \sum_{k=1}^m m w_k R_k \quad (2)$$

19 The polymerization is presumed to occur in both bubble and emulsion phases in this model.
 20 The equations required to compute the mass and heat transfer coefficients, velocities in the
 21 emulsion and bubble phases, and other valuable details in the copolymerization model are set
 22 down in Table 4. The list of assumptions made in the dynamic two-phase copolymerization
 23 model is given below:

- 1 • Polymerization reactions are presumed to happen in both bubble and emulsion phases.
- 2 • The emulsion phase is entirely mixed, and it does not stay at the minimum fluidization
- 3 condition.
- 4 • The bubbles are constant size spheres and move with a velocity that stays unchanged
- 5 over the bed at the plug flow condition.
- 6 • Temperature and concentration radial gradients in the reactor resulting from the rigidity
- 7 of mixing resulting from the up-flowing gas are ignored.
- 8 • Entrainment of solids is considered at the uppermost segment of the bed.
- 9 • Heat and mass transfer resistance among solid particles and gas in the bubble and
- 10 emulsion phases are not accounted for.
- 11 • Consistent particle size over the bed is taken into account.

12 Mass and energy balances that are formed considering these assumptions are given in Table 5.

13 One does not fail to include the solid entrainment from the reactor topmost section since there
14 are occasions in which the elutriation rate cannot be dismissed. In normal circumstances, a lot
15 of the coarse particles stay in the reactor bed while the fluidizing gas makes the smaller ones
16 to leave from the reactor. Nonetheless, when gas velocity becomes several times higher than
17 the terminal velocity, particles with bigger size may be entrained from the bed as well.²⁷ This
18 case is labeled particle entrainment or carry over and is highly crucial in both designing and
19 operating of FBRs. Meanwhile, particle elutriation which is sometimes used interchangeably
20 with particle entrainment which occurs in fact in the FBR cyclone that is placed outside the
21 reactor in which the solids are moved apart from the fluidizing gas before returning to the
22 reactor and after some processing. The implication is that in situations where the particle
23 entrainment arises, their effect on the process must be deliberated. In effect, in the model used
24 in this study, solid entrainment is considered to perform mass and energy balances to give more

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- 4 1 realistic results when compared to the real data. The correlations for calculating the rate of
- 5
- 6 2 solids entrainment are highlighted in
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- 1 Table 6. Furthermore, MWD which is a significant polymer property is also included in the
- 2 model to examine the model validity when it is compared to the industrial data.

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3 **TABLE 4**

4 **TABLE 5**

5 **TABLE 6**

6 The following initial conditions are used to find the solution of these equations:

$$[M_i]_{b,t=0} = [M_i]_{in}$$

$$T_{b,t=0} = T_{in}$$

$$[M_i]_{e,t=0} = [M_i]_{in}$$

$$T_{e,t=0} = T_{in}$$

7 Molecular weight and Melt Flow Index (MFI) are polymer properties which are
8 crucial for estimating the polymer quality, have been computed using the modified model used
9 in this work.

10 The number averaged molecular weight is described as the mean of all the
11 molecular weights of polymer sample's chains, defined by:

$$\bar{M}_n = \frac{\sum N_i M_i}{\sum N_i} \quad (3)$$

12 where N_i is the number of chains and M_i is the chain's molecular weight. \bar{M}_n is quantified using
13 methods that give the sum of molecules present in a weight's sample and can be estimated
14 through various polymerization mechanisms. If \bar{M}_n is stated for a specific MWD, this writes
15 down that equal numbers of molecules exist on both sides of the M_n in the distribution.

16 Conversely, the weight average molecular weight is defined by:

$$\bar{M}_w = \frac{\sum N_i M_i^2}{\sum N_i M_i} \quad (4)$$

17 Compared to \bar{M}_n , \bar{M}_w takes a chain's molecular weight into account to figure out how it
18 commits to the average molecular weight. If the chain gets bigger, the influence of chain on
19 M_w would be greater. Weight average molecular weight, in lieu of the sum of molecules, is
20 given by methods that quantify the molecular size e.g. light scattering techniques. If \bar{M}_w is

1 stated for a specific MWD, this says that equal weight of molecules exists on both sides of M_w
2 in the distribution.

3 The relationship between the molecular weight and MFI of polyethylene leans on
4 the equation suggested by McAuley et al.²², in which its constants are modified to be
5 compatible with the real industrial data and is defined by:

$$MFI = 3.346 \times 10^{17} \bar{M}_w^{-3.472} \quad (5)$$

7 **Advanced control strategies**

8 For the control studies of MFI and temperature, the conventional PID controllers were first
9 tested to see how they perform with this nonlinear system. The PID controllers were tuned
10 using Ziegler-Nichols' closed loop method. Then, to overcome the limitations of the
11 conventional methods, a neuro-fuzzy subclass of intelligent controllers, namely ANFIS
12 controller, was used.

13 ANFIS is a type of artificial neural network that is constructed on Takagi–Sugeno
14 fuzzy inference system²⁸. The fuzzy logic approach is based on predefined rules (if-then) that
15 does not have the ability to learn and adjust them to new conditions. Thus, to overcome this
16 drawback, Jang²⁹ hybridized a fuzzy inference system (FIS) with an artificial neural network
17 (ANN) to form ANFIS. The ANFIS methodology can be considered as an adaptive system in
18 the form like ANN in which by training the system the parameters of the fuzzy membership
19 functions (antecedent parameters) and the parameters of the fuzzy system output function
20 (consequent parameters) are adapted³⁰. ANFIS possesses the advantage of both FIS and ANN
21 and it has solved the drawbacks of both systems, where the complicated procedures of neural
22 networks are bypassed by applying linguistic variables of FIS system, and the weakness of FIS
23 is solved by applying the neural inference system which create the ability to learn and adapt

1 itself to new condition. Further details about ANFIS is given in the attached Appendix:
 2 Adaptive Neuro-Fuzzy Inference System (ANFIS)

3 The Hybrid ANFIS-PID controller combines two controller types in one structure
 4 to receive help from individual capabilities of them to achieve one goal. The hybrid ANFIS-
 5 PID controller is based on a selecting or switching hybrid structure. In this structure, the
 6 controller action is divided into two regions. The output of PID controller is the main control
 7 signal and the output of ANFIS controller is the recovery control signal which is switched
 8 based on the value of three variables to determine the selecting hybrid controller which are the
 9 specified error threshold (ΔE_{error}), PID control signal (U_{PID}) and ANFIS control signal (U_{ANFIS}).
 10 The error threshold is defined based on the requirements of the studied system. The block
 11 diagram and decision-making flowchart of hybrid PID-ANFIS controller is shown in Figure 2.

$$U_{hybrid} = \begin{cases} U_{PID}, & \text{for } Error < \Delta E_{error} \\ U_{ANFIS}, & \text{for } Error \geq \Delta E_{error} \end{cases} \quad (6)$$

FIGURE 2

14 Figure 3 and Figure 4 show the type and values of membership functions for inputs of the
 15 designed MFI and temperature ANFIS controllers respectively.

FIGURE 3

FIGURE 4

18 Table 7 shows the specifications of the designed ANFIS controllers for controlling both MFI
 19 and temperature.

TABLE 7

21 Modeling and simulation of the proposed model and control scheme was done using
 22 MATLAB[®] and Simulink[®] suit. The model of the process was written in MATLAB[®] and was
 23 linked to Simulink[®] as a process block with several inputs and outputs. The dynamic equations
 24 are solved using a variable step numerical solver given the inputs to produce the outputs. This

1 makes it possible to design and test controller to control desired outputs by manipulating the
2 block inputs. Each simulation takes almost 2 to 10 seconds using a normal desktop computer
3 utilizing an Intel® Core™ i7-6700HQ processor.

4 **RESULTS AND DISCUSSION**

5 The validation of the model proves to be crucial. This was carried out using industrial data¹⁹
6 that include the production rate, MWD and transient reactor temperatures. The results rest on
7 the BP LL0209 grade polyethylene operating conditions which is a licensed linear low-density
8 polyethylene.

9 Polymer production rate of the FBR versus residence time is illustrated in Figure
10 5. This figure demonstrates the production rate evolution from when the Ziegler-Natta catalyst
11 goes into the reactor and the reaction begins up until the bed fully fluidizes and the production
12 becomes steady. This figure as well highlights the production rate evolution of the polymer in
13 both the emulsion and bubble phases over time in the reactor. The computed total value ascends
14 from nearly 7 tons to almost 10 tons within the first and second hours of operation and turn
15 steady after nearly 5 hours of production at 13.44 tons. To demonstrate that the model is right
16 while confirming the results, the polymer production rate was drawn versus the actual industrial
17 data. The horizontal line that shows the actual steady state data is the production rate value in
18 the industrial FBR. As can be seen, the steady state value of production rate was predicted
19 accurately enough by the model with only 0.4 ton deviation. To illustrate the effect of taking
20 particle elutriation into account, the results of another work from literature³¹ that do not
21 consider this fact is given in this figure. When particles leave the column, less catalyst exist in
22 the reactor resulting in lower reactions and less production rate. The figure also shows that
23 almost 60 percent of the polyethylene is made in the emulsion phase and nearly 40 percent of
24 the overall production occur inside the bubble phase.

FIGURE 5

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1 Additionally, the computed stable state MWD was compared with industrial MWD data points
2
3 in Figure 6. The actual data was produced through gel permeation chromatography. In this
4
5 figure, the calculated cumulative weight fraction derivative is plotted versus the logarithm of
6
7 weight average molecular weight, where it is a characteristic gel permeation chromatography
8
9 (GPC) result. Notably, other than the very slight differences, these sets of data have shown a
10
11 great consistency.

FIGURE 6

9 To add, the industrial data was plotted versus the estimated reactor temperatures in Figure 7.
10
11 The model was found to have the potential to accurately estimate the temperature of the reactor
12
13 for this polymer grade. The mean 0.6 **percent** difference from the industrial data is acceptable
14
15 and small in the engineering context. That said, by giving some weight on the heat transfer
16
17 resistance among solid particles and gas in both phases, particle size distributions and radial
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19 temperature gradients in the reactor, the model can be improved further, leading to better
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21 polymer properties and reactor parameters predictions. Nevertheless, the model intricacy and
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23 computational labors will be increased.

FIGURE 7

MFI control

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The modified two-phase model¹⁹ as has been discussed earlier, is employed for the control
studies since it has been validated with the actual process. To show the non-linear performance
of the ethylene copolymerization FBR, the model underwent a simulation for step changes in
the inlet hydrogen concentration, as the process key parameter to affect the polymer MFI. The
open-loop simulation results can be referred to in Figure 8.

1
2
3 1 In Figure 8, the inlet hydrogen concentration was altered after the reactor reached
4
5 2 the steady state. The inlet hydrogen concentration has a considerable impact on the polymer
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7 3 MFI. In this case, inlet hydrogen concentration experience incremental step changes from 2
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9 4 percent to 18 percent of hydrogen with 2 percent increase in each of the open-loop runs. This
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11 5 figure indicates that the hydrogen concentration in the feed has considerable effects on the MFI
12
13 6 and non-symmetric responses are yielded. This means that the polymer MFI variates with the
14
15 7 changes in inlet hydrogen concentration nonlinearly. When talking about the nonlinear
16
17 8 behavior, implementation of conventional controllers results in deficient control of the process
18
19 9 variables. This clarifies using a better suited advanced control system to adjust the impact of
20
21 10 hydrogen concentration on the desired variables within the process. This justifies the
22
23 11 implementation of a more effective advanced control system to appropriately control the
24
25 12 hydrogen concentration effect on the process variable. Moreover, Figure 8 also shows that MFI
26
27 13 has inherently very slow dynamics and it takes 5-10 hours to reach steady state after a step
28
29 14 change in the open loop configuration.

15 **FIGURE 8**

16 **MFI control- setpoint tracking**

17 Figure 9 shows the MFI set point changes between 1 and 1.4 g/10min tracked by PID and
18 ANFIS controllers with 0 to 20 percent saturation values respectively. This figure illustrates
19 that both controllers are capable of set-point tracking. Even though the PID controller reaches
20 the set-point in every step change after a while, the ANFIS controller performance outperforms
21 the PID controller as it has very small overshoots and has much less response times. For
22 example, during the second step change from 1.4 to 1.3 g/10min at 2×10^5 seconds, it takes the
23 PID controller around 3 hours to settle but it takes only 13 minutes for the ANFIS controller to
24 settle with a negligible post-transition undershoot of 0.015 g/10min in the MFI value.

FIGURE 9

The controller moves of ANFIS and PID controllers in MFI set point tracking are further shown in Figure 10. The controllers are constrained to a maximum 20 percent of hydrogen inlet concentration to make polymerization workable and realistic. To illustrate, it is found that the starting point of hydrogen concentration must be 9.6 percent to set the MFI at 1 g/10min. However, after a step change to 1.4 g/10min, the controllers should set the inlet concentration to 11.5 percent. The controller moves of the ANFIS controller is fast and realistic. It fluctuates two times from minimum to the maximum of the manipulated variable and then some oscillations before reaching a steady state and this takes around 25 minutes for the ANFIS controller. On the other hand, PID controller jumps to the maximum and suddenly to the minimum of the manipulated variable before starting to rise slowly to reach the right value after a prolonged period of almost 8 hours. This shows that the ANFIS controller was much faster and robust in following the MFI setpoint, while the PID moves were more extreme and drastic in its controller moves.

FIGURE 10**MFI control-disturbance rejection**

To verify that a controller can be employed in the industrial trials, it must be prepared to handle regulatory complications successfully as well. In the case of MFI, as Figure 11 shows, the concentration of ethylene has nonlinear and non-proportional effects on the MFI value such that increasing ethylene concentration in the feed leads to polymers with higher density and lower MFI which may get above or below allowable specifications. As a result, variation in ethylene concentration was used as disturbance to test predictability and robustness of these controllers.

FIGURE 11

1
2
3 1 Figure 12 displays the MFI response when it is controlled by ANFIS and PID controllers with
4
5 2 a disturbance of 50 percent deduction in inlet ethylene concentration after 1.5×10^5 seconds.
6
7
8 3 This figure shows clearly that the ANFIS controller is capable of disturbance rejection in a
9
10 4 faster and more effective manner as compared to the PID controller.

5 **FIGURE 12**

14 6 As shown, since MFI has slow dynamics, ANFIS controller can completely omit the effect of
15
16 7 the disturbance by rapidly changing the manipulated variable within just 8 minutes (Figure 13).
17
18 8 However, although the disturbance makes little deviation from the set point in case of the PID
19
20 9 controller, it takes almost 19 hours for it to retract the system to the stable set-point. This is due
21
22 10 to the less sensitiveness of the PID controller in this case with very slow decrease that it makes
23
24 11 in the manipulated variable for compensating the disturbance effect.

12 **FIGURE 13**

30 13 The measures of controlled system performance in terms of the integral of time absolute error
31
32 14 (ITAE), the integral of absolute error (IAE), integral of squared error (ISE), and absolute
33
34 15 percentage error (APE) for each MFI controller in both disturbance rejection and set-point
35
36 16 tracking cases was also computed and is listed in Table 8. It is shown by the error values that
37
38 17 the ANFIS controller showed improved performance compared to PID since all the
39
40 18 performance index values for the ANFIS controller is lower than the PID controller in both the
41
42 19 disturbance rejection and set-point tracking studies.

20 **TABLE 8**

21 **Temperature control**

52 22 Characteristically, the ethylene copolymerization reaction is very much exothermic. To keep
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54 23 the polyethylene production rates at the preferable values, it is important to make sure that the
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56 24 reactor's temperature is higher than reactants' dew point to prevent gas condensation within
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1 the reactor. Retaining the temperature below the polymer melting point is also necessary so
2 that the particle melting, and agglomeration can be prevented, or the reactor will shut down.
3 Consequently, an effective temperature control system must be implemented.

4 In this study, cooling water flow rate in terms of its valve opening percentage to
5 the recycle stream heat exchanger is used as the manipulated variable of the controller. Figure
6 14 shows how the reactor temperature changes with incremental changes in valve opening for
7 the cooling water flow rate. Exothermic reaction takes place in this reactor, therefore, reducing
8 the cool water flow rate of the recycle stream heat exchanger leads to higher temperatures in
9 the reactor.

10 **FIGURE 14**

11 Figure 15 illustrates how fully closing and opening the cool water valve impacts the reactor
12 temperature. As seen, unlike the case of MFI, here we have faster and responsive dynamics.
13 While it took nearly 10 hours for the MFI to reach steady state after changing the manipulated
14 variable from 10 percent to 20 percent, it takes the temperature only 2.5 hours to get to the
15 steady value after altering the manipulated variable from 0 percent to 100 percent and vice
16 versa. This shows the response dynamics are different in this case and we may need an
17 advanced controller design here.

18 **FIGURE 15**

19 **Temperature control – setpoint tracking**

20 Temperature control design also needs to be tested with step changes in set point and see how
21 the controller follows it. Firstly, a conventional PID controller with 0 to 100 saturation values
22 was tuned using the Ziegler-Nichols' closed loop method and tested in the servo scenario which
23 is shown in Figure 16. As illustrated, rising, or falling time is short (200-400 seconds) and the
24 controller eventually reaches and follows the setpoint. However, there is overshoots in both

1 kinds of step changes where the duration and peak of it depends on the increase or decrease
 2 percentage in the setpoint. For increasing cases, it takes around 2.5 hours with a peak of almost
 3 2 K overshoot in temperature while decreasing the setpoint leads to some oscillation with
 4 around ± 1 K overshoot and undershoots and with settling times of nearly half an hour.

5 The difference in temperature response to raising or lowering of the set points has
 6 a reason. In the case of a temperature increase the only thing that controller can do is to fully
 7 close the cool water valve and let temperature rise in the system because of the exothermic
 8 reaction. On the other hand, to decrease the temperature, controller fully opens the valve. Since
 9 rising time for temperature increase is higher than falling time for temperature decrease, system
 10 gets cool faster than it gets hot in this system and with this specific heat exchanger capacity.

11 In this nonlinear system case, the only problem with the PID controller is the
 12 overshoots and undershoots. To alleviate this issue, we need to think of other advanced
 13 controllers which can control the process efficiently.

14 **FIGURE 16**

15 An ANFIS controller was designed and trained for controlling the temperature by manipulating
 16 the cool water valve and the result is presented in Figure 17. The ANFIS controller improves
 17 the performance of the PID controller disadvantages but shows some oscillations around the
 18 setpoint. Depending on the amount of step change, these oscillations are within ± 0.2 K of the
 19 setpoint with periods of around 5 minutes. This effect is because of the fast dynamics of the
 20 system and fuzzy rule changes of the ANFIS controller.

21 **FIGURE 17**

22 To mitigate the problems with both controllers, both controllers were used to control the
 23 temperature in a hybrid design and the result is illustrated in Figure 18. The main controller of
 24 this hybrid controller structure is the ANFIS controller and when the PID controller has passed
 25 its overshoot period, the controller switches to the PID to stick to the setpoint with no

1 oscillations. This is made possible by setting a switching threshold of 0.2 degrees of error
2 meaning the controller switches to the PID controller only after its error is less than this value.

3 **FIGURE 18**

4 Figure 19 shows the control moves for all three types of the temperature controllers. As can be
5 seen, hybrid controller was again superior in performance since it greatly reduced the
6 oscillations. The cold-water valve opening and closing gradually takes place almost every 3
7 minutes within the oscillatory range which is acceptable for a control valve.

8 **FIGURE 19**

9 **Temperature control – disturbance rejection**

10 Several different step changes in the velocity of superficial gas was applied on the model to
11 examine possible disturbances on the reactor temperature, as a key process parameter. The
12 open-loop simulation result is shown in Figure 20. In this figure, after the reactor continues to
13 work in steady state for a while, superficial gas velocity was altered with both additive and
14 deductive increments which led to non-symmetrical responses. This figure suggests that
15 superficial gas velocity has a significant effect on the reactor temperature. Furthermore,
16 positive steps in the superficial gas velocity have less effects on the reactor temperature in
17 comparison with the matching negative steps. To put simply, the reactor temperature changes
18 in a non-linear manner with the superficial gas velocity. With the demonstration of the
19 nonlinear behavior, the conventional controllers lead to the process variables being poorly
20 controlled. This justifies clearly the need to implement a more effective control system to
21 control the superficial gas velocity effect on the process variable efficiently.

22 **FIGURE 20**

23 To examine the system response to superficial gas velocity disturbance, a high reduction of 50
24 percent in the process variable is introduced at 5×10^4 seconds. The controller responses are

1 given in Figure 21. Once again, PID was unable to mitigate the disturbance and responded with
2 an overshoot and a settling time of almost 3 hours. ANFIS controller starts to gradually diverge
3 the oscillations from the set point with increase in the periods of oscillations. On the other hand,
4 hybrid controller has successfully absorbed the effect of the disturbance and performed like a
5 step change case where the ANFIS controller takes over right after the change and switches to
6 PID afterwards.

7 **FIGURE 21**

8 Controller moves of temperature controllers are given in Figure 22. The same trend of control
9 variable is seen here again. PID takes a lot of time to find the correct value, ANFIS starts to
10 destabilize after the disturbance point and hybrid controller behave as before.

11 **FIGURE 22**

12 Table 9 lists the calculated measures of the controlled system performance for different
13 temperature controller designs in both disturbance rejection and set-point tracking scenarios.
14 The info shows the superiority of the hybrid controller in terms of performance compared to
15 both PID and ANFIS alone since the ITAE, IAE, ISE and APE values for the hybrid controller
16 prove to be the smallest in both disturbance rejection and set-point tracking studies.

17 **TABLE 9**

18 **CONCLUSIONS**

19 A dynamic two-phase ethylene copolymerization model was taken up to describe the dynamics
20 in FBRs. The particle entrainment from the FBR is considered by the model. The
21 hydrodynamics is integrated with a kinetics of copolymerization (ethylene and 1-butene) to
22 give an improved knowledge of the reactor performance. The modified dynamic two-phase
23 ethylene copolymerization model performance resembled the actual industrial conditions to a
24 great deal, where the temperature foretold by the model was close to the real plant data. The

1 model was then adopted to perform further control examinations. A right choice of controllers
2 that can be used by industries were implemented to address both the servo and regulatory
3 control of the polymer MFI as a fundamental polymer property and reactor temperature as a
4 very crucial process parameter. The simulation results showed that reactor temperature can be
5 controlled by altering the cold-water flow rate and MFI by changing the hydrogen
6 concentration in the feed. Conventional controllers showed that they perform poorly compared
7 to the intelligent controllers due to their linear nature. For instance, PID controller takes around
8 2.5 hours to settle with 2K overshoot when the temperature setpoint changes from 343K to
9 350K while it takes around 30 minutes for the hybrid ANFIS-PID temperature controller to
10 settle with only 0.2K overshoot. ANFIS-based controllers showed to be promising as they have
11 faster settling times and less overshoots in both servo and regulatory control situations. This
12 work can be extended in future by adding ANFIS-based MIMO controllers which makes it
13 possible to simultaneously control several variables using one main controller. Other control
14 variables that can be considered are polymer density, MWD, polydispersity index and
15 production rate. Another interesting work would be to dynamically optimize the controller and
16 model parameters online on a pilot plant using its transitory measured data.

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APPENDIX: ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)

ANFIS was developed as an adaptive system with a set of fuzzy rules (if-then) and tunable MF (membership function) parameters in a training phase. During the training phase of ANFIS, two different parameters should be perfected to give the learning procedures:

- antecedent parameters (the MF parameters)
- consequent parameters (the fuzzy system output function)

As the consequent parameters are linear, to optimize these parameters the linear least-squares method is applied and to optimize the antecedent parameters the back-propagation algorithm in conjunction with an optimization method such as the gradient descent is applied.

Five different layers construct the ANFIS structure while each layer consists of node functions and the inputs of the nodes in the present layer are obtained from earlier layers. The consecutive layers of ANFIS structure are as follows: layer 1 is fuzzification (if-part), layer 2 is production part, layer 3 is normalization part, layer 4 is defuzzification (then-part), and eventually layer 5 is total output generation part. Figure A1 shows the structure of ANFIS with two independent variables (x and y) as input and one dependent variable f_{out} as an output.

FIGURE A1

For fuzzy inference systems, difference in the consequence of the set of fuzzy rules (if-then) and defuzzification procedures lead to two distinct types of fuzzy inference systems known as Mamdani type FIS and Sugeno type FIS.

The main difference between Sugeno type FIS and Mamdani type FIS is the way the fuzzy inputs are converted to the crisp output. In Mamdani type FIS for computing the crisp output the defuzzification technique of a fuzzy output is used while in Sugeno type FIS the weighted average method is used. As the consequents of the rules are not fuzzy in the Sugeno

1 method, the interpretability and expressive power of Mamdani output, are eliminated in this
 2 method. In comparison to Mamdani type FIS, Sugeno has faster processing time since instead
 3 of the time consuming defuzzification process, the weighted average method is applied.

4 Moreover, another difference between Sugeno and Mamdani type FIS is that
 5 Sugeno has no output membership functions while Mamdani FIS has output membership so,
 6 Sugeno method gives an output that is either linear (weighted) mathematical expression or a
 7 constant. Instead, Mamdani method gives an output that is a fuzzy set. In comparison to
 8 Mamdani type FIS, Sugeno has more flexibility in system design as latter can be integrated
 9 with ANFIS tool to model the systems more precisely. Considering ANFIS with Sugeno type
 10 FIS, so the rule base of ANFIS holds fuzzy IF-THEN rules of a first order Sugeno type FIS are
 11 told as:

12 • Rule 1: If x is A_1 and y is B_1 then z is $f_1(x, y; p_1, q_1, r_1) = xp_1 + yq_1 + r_1$

13 • Rule 2: If x is A_2 and y is B_2 then z is $f_2(x, y; p_2, q_2, r_2) = xp_2 + yq_2 + r_2$

14 where $f_i(x, y; p_i, q_i, r_i)$ is a first order polynomial function which stands for the outputs of the
 15 Sugeno type FIS, A_i and B_i are the fuzzy sets, and x and y are two different input and z is an
 16 output of ANFIS model.

17 In the ANFIS structure, different layers consist of different node function. As
 18 shown in Figure A1, adaptive nodes which stand for the adjustable parameter sets are denoted
 19 by squares while fixed nodes which stand for the fixed parameter sets in the system are denoted
 20 by circles.

21 **Layer 1:** Every node in this layer is an adaptive node with a node function as follows:

$$22 \quad Q_{1,i} = \mu_{A_i}(x) \quad i = 1, 2 \quad (A.1)$$

$$23 \quad Q_{1,i} = \mu_{B_i}(y) \quad i = 3, 4 \quad (A.2)$$

1 Where x and y are the inputs to node i , A_i and B_i are linguistic labels, μ_{A_i} and μ_{B_i} are the
 2 membership functions for A_i and B_i fuzzy sets, respectively and $Q_{1,i}$ is the membership grade
 3 of a fuzzy set and considered as the output of node i in the first layer which specifies the degree
 4 to the given input (x or y) satisfies the quantifies.

5 Typically, in ANFIS, the MF (membership function) for a fuzzy set can be any
 6 parameterized membership function, such as generalized bell-shaped function, Gaussian,
 7 trapezoidal or triangular.

8 A generalized Bell-shaped MF (bell MF) is specified as follows:

$$9 \quad \mu_A(x; a, b, c) = \frac{1}{1 + \left[\frac{x-c}{a} \right]^{2b}} \quad (A.3)$$

10 A Gaussian MF is specified as follows:

$$11 \quad \mu_A(x; c, \sigma) = e^{-0.5 \left(\frac{x-c}{\sigma} \right)^2} \quad (A.4)$$

12 while σ and c decide the width and center of Gaussian MF, respectively.

13 A trapezoidal MF is specified as follows:

$$14 \quad \mu_A(x; a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) \quad (A.5)$$

15 The parameters with $a < b \leq c < d$ specify the x coordinates of the four corners for the
 16 underlying trapezoidal MF.

17 A triangular MF is specified as follows:

$$18 \quad \mu_A(x; a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right) \quad (A.6)$$

19 The parameters with $a < b < c$ specify the x coordinates of the three corners for the
 20 underlying triangular MF.

21 In this layer, the parameters a, b, c, d and σ are the antecedent parameters.

Layer 2: Every node in this layer is a fixed node whose output is the product of all the incoming signals. In this layer through multiplication of input signals the firing strength of each rule is figured out.

$$Q_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y) \quad i = 1,2 \quad (\text{A.7})$$

where w_i is output signal, which stands for the firing strength of a rule.

Layer 3: Every node in this layer is a fixed node. In this layer, the firing strength provided in earlier layer is normalized by computing the ratio of the i^{th} rule's firing strength to the sum of all rules' firing strengths.

$$Q_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2} \quad i = 1,2 \quad (\text{A.8})$$

where \bar{w} is output signal, which is the normalized firing strength of a rule.

Layer 4: In this layer every node i is adaptive with a node function.

$$Q_{4,i} = \bar{w}_i f_i \quad i = 1,2 \quad (\text{A.9})$$

where f_1 and f_2 are the fuzzy IF-THEN rules as follows:

- Rule 1: If x is A_1 and y is B_1 then z is $f_1(x, y; p_1, q_1, r_1)$
- Rule 2: If x is A_2 and y is B_2 then z is $f_2(x, y; p_2, q_2, r_2)$

Where r_i, q_i , and p_i are the parameter set, referred to as the linear consequent parameters.

Layer 5: This layer has only one fixed node that computes the overall output of ANFIS by summation of all incoming signals as:

$$Q_{5,i} = f_{out} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} = \text{overall output} \quad i = 1,2 \quad (\text{A.10})$$

The overall output is linear combination of the consequent parameters. Thus, the final output of ANFIS is expressed as

$$\begin{aligned} f_{out} &= \bar{w}_1 f_1 + \bar{w}_2 f_2 \\ &= \frac{w_1}{w_1 + w_2} f_1 + \frac{w_2}{w_1 + w_2} f_2 \\ &= (\bar{w}_1 x) p_1 + (\bar{w}_2 x) p_2 + (\bar{w}_1 y) q_1 + (\bar{w}_2 y) q_2 + (\bar{w}_1) r_1 + (\bar{w}_2) r_2 \end{aligned} \quad (\text{A.11})$$

- 2 Eventually, ANFIS applies a hybrid learning algorithm for parameter tuning. It uses the back-
3 propagation algorithm and the least squared method for updating the input MF parameters
4 (antecedent parameters) in layer 1, and training the consequent parameters, respectively.

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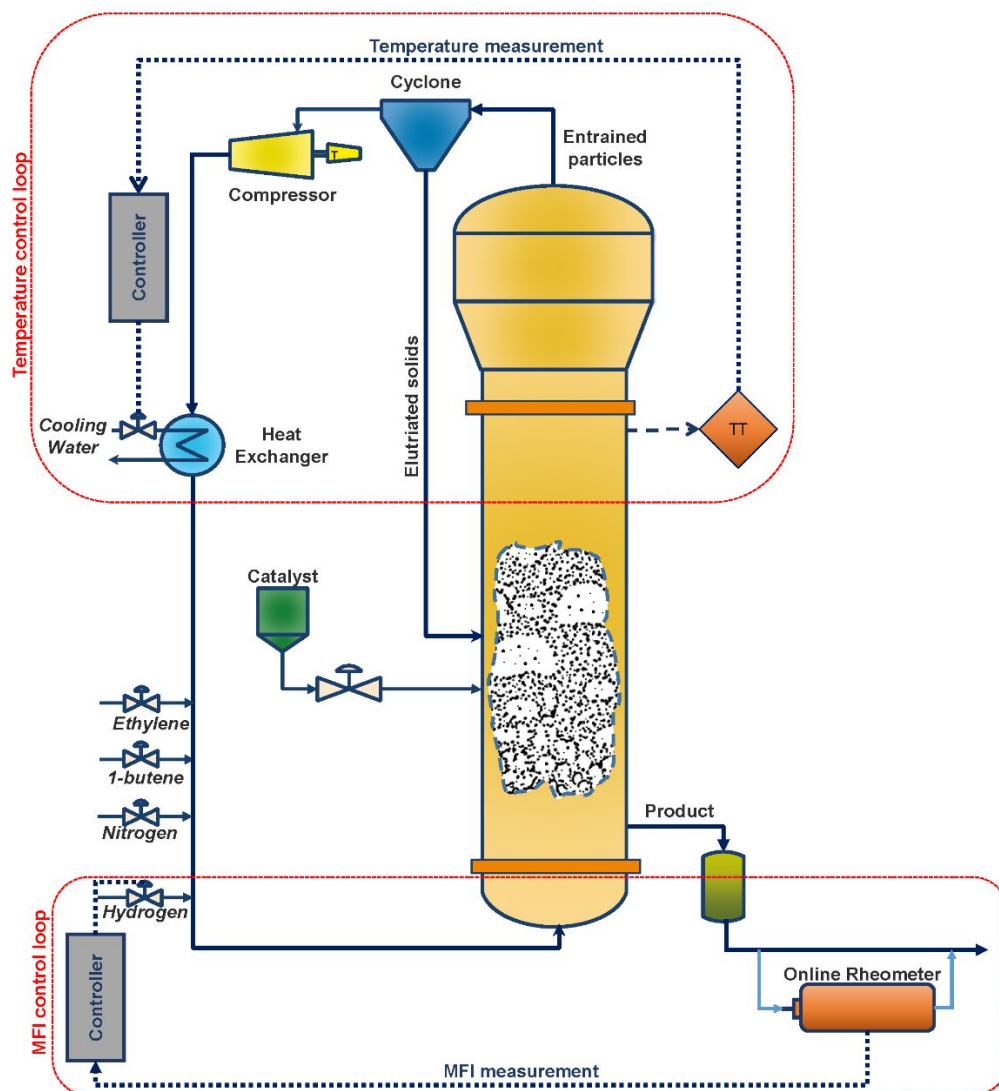


Figure 1: A typical fluidized bed polymerization reactor with temperature and MFI control loop structures.

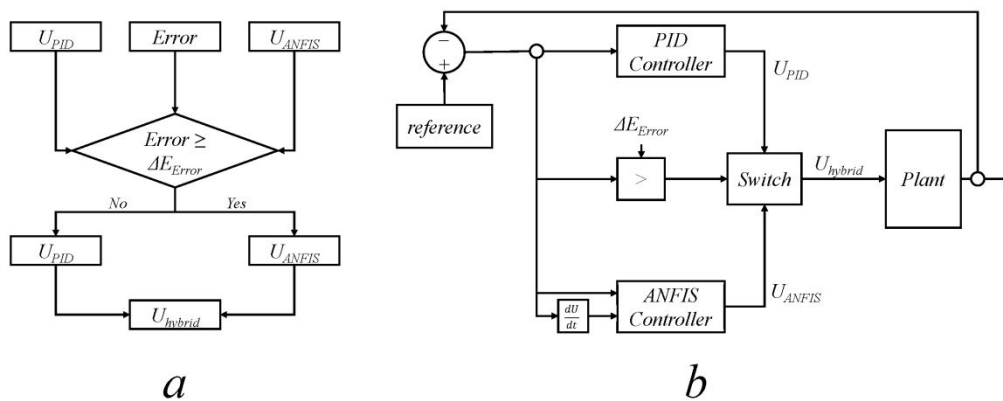


Figure 2: a) flowchart and b) block diagram of selecting hybrid PID-ANFIS controller

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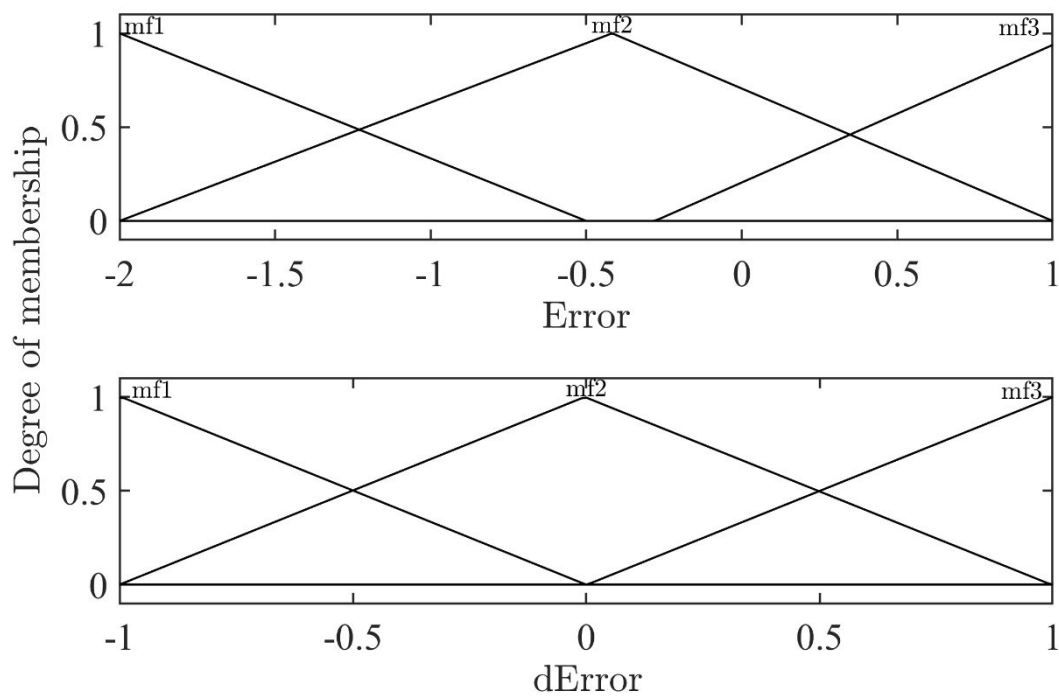


Figure 3: Input membership functions of the MFI ANFIS controller

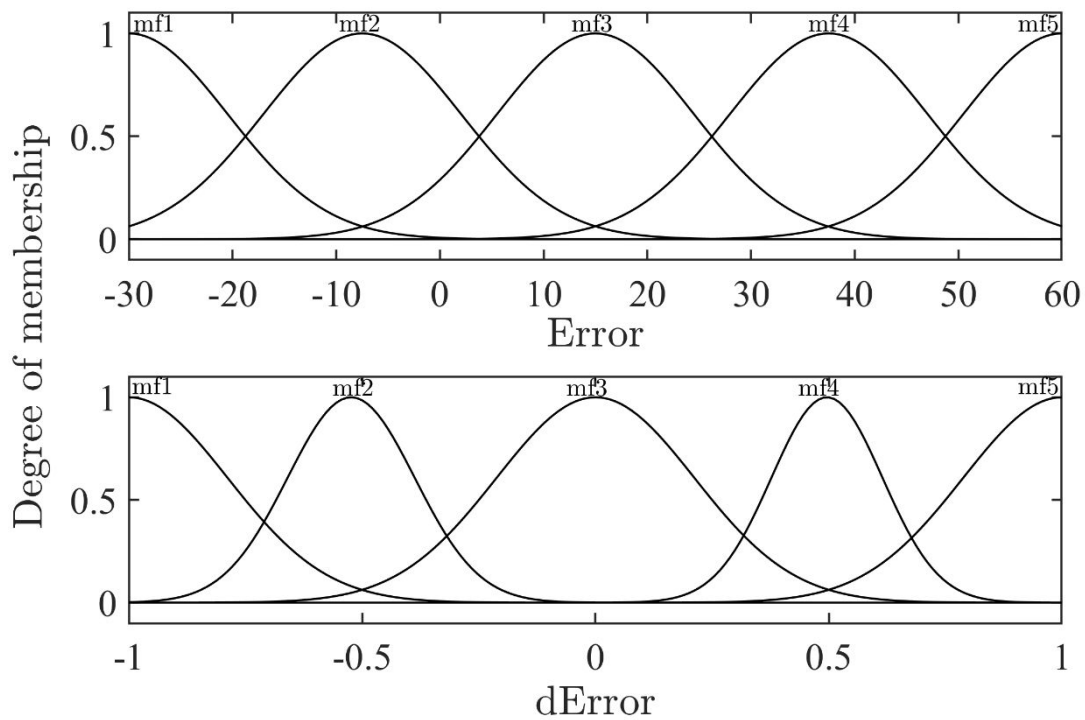


Figure 4: Input membership functions of the temperature ANFIS controller

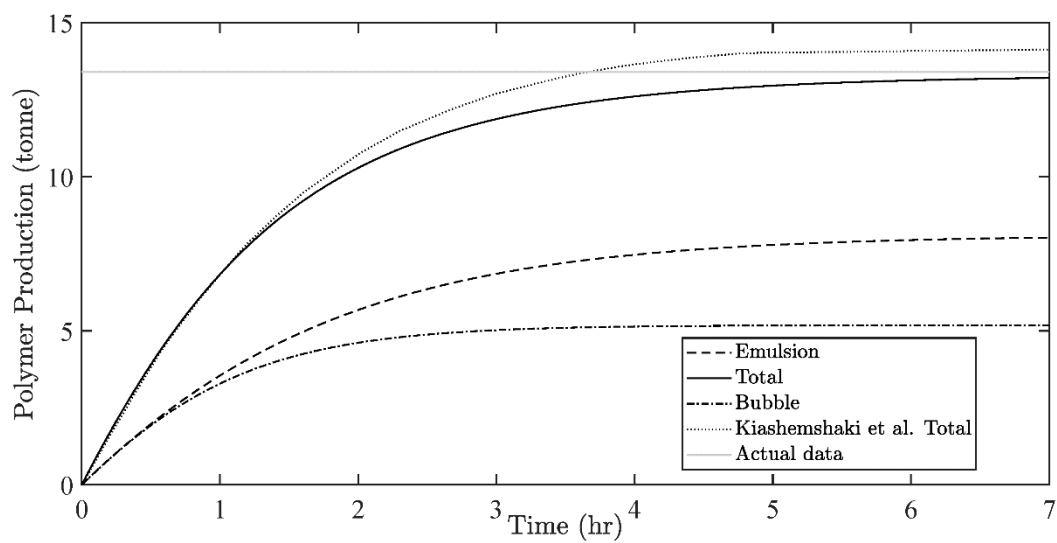


Figure 5: Polymer production rate evolution over time in the FBR

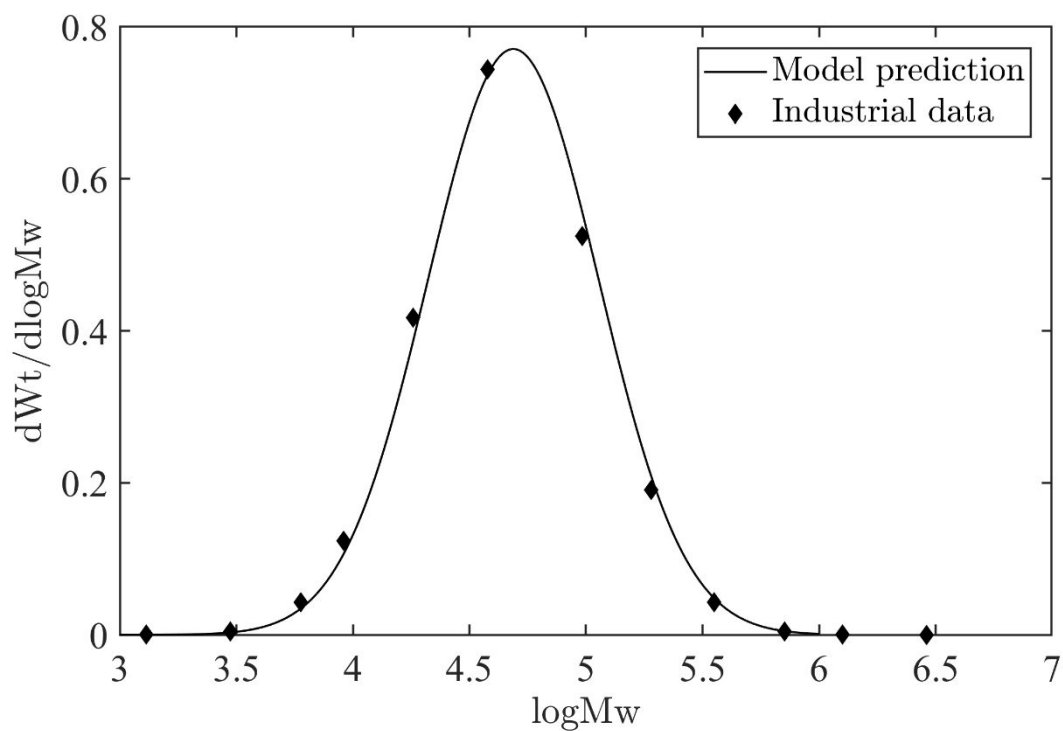


Figure 6: Model predicted molecular weight distribution comparison with industrial data

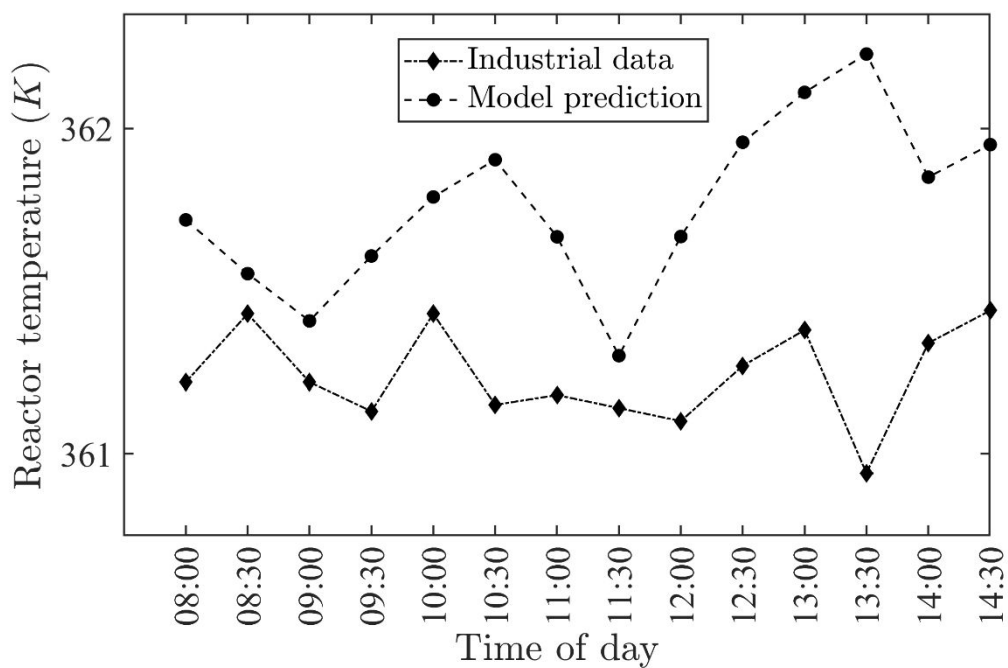


Figure 7: Reactor temperature and industrial data comparison during an operating shift

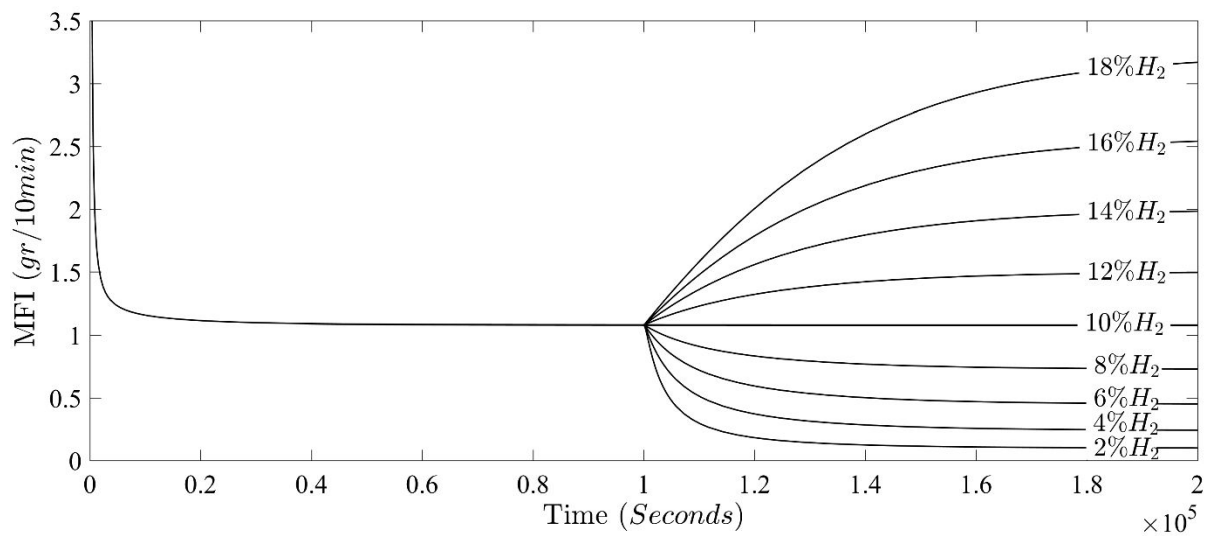


Figure 8: Open loop dynamic response of MFI for different hydrogen inlet concentrations over time

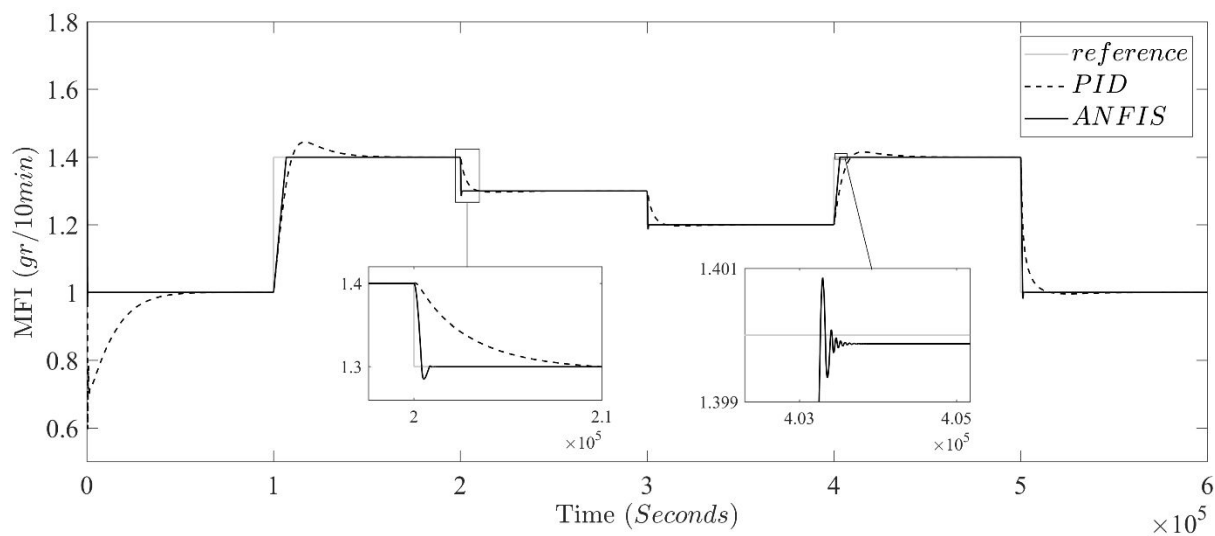


Figure 9: Comparison of MFI reference tracking for PID ($P=0.6$, $I=0.0006$, $D=-50$) and ANFIS controllers

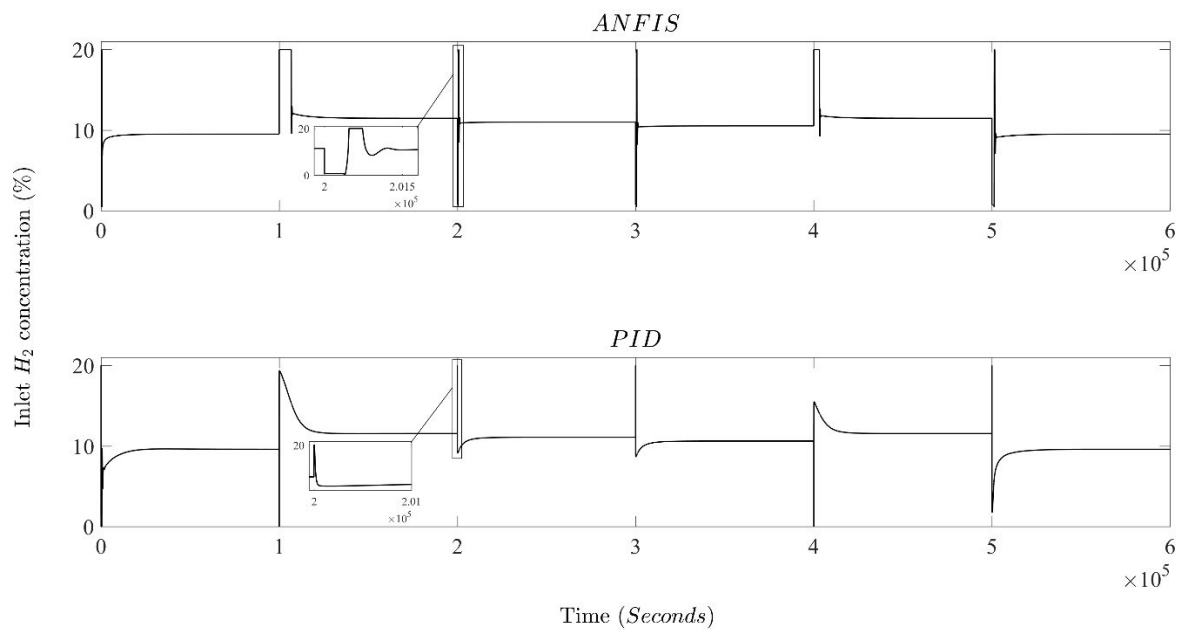


Figure 10: Controller moves in set point tracking of polymer MFI

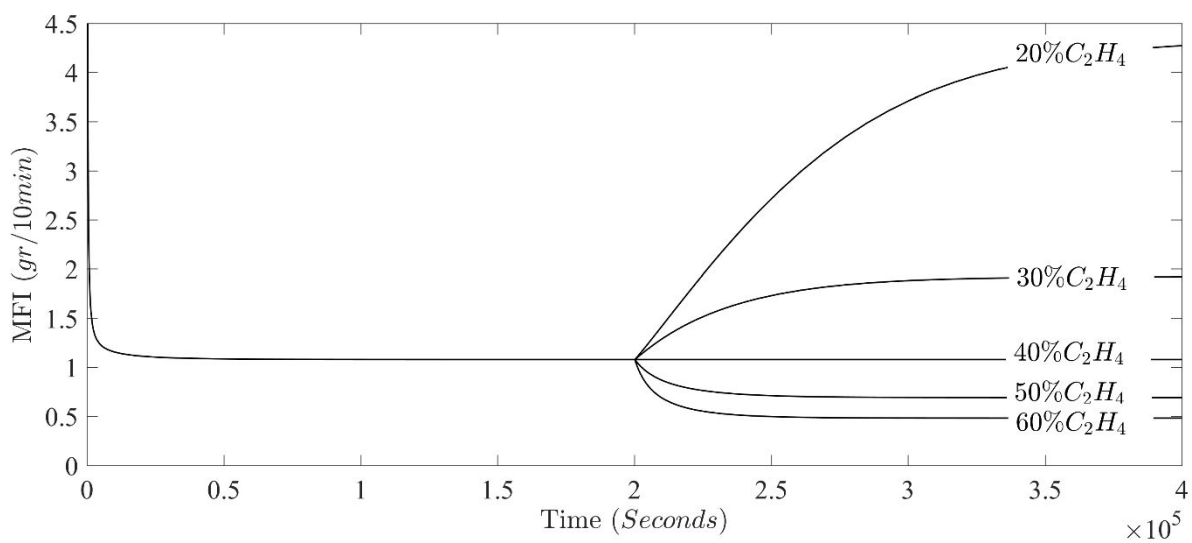


Figure 11: Effect of different inlet ethylene concentration step changes on the polymer MFI at 2×10^5 s

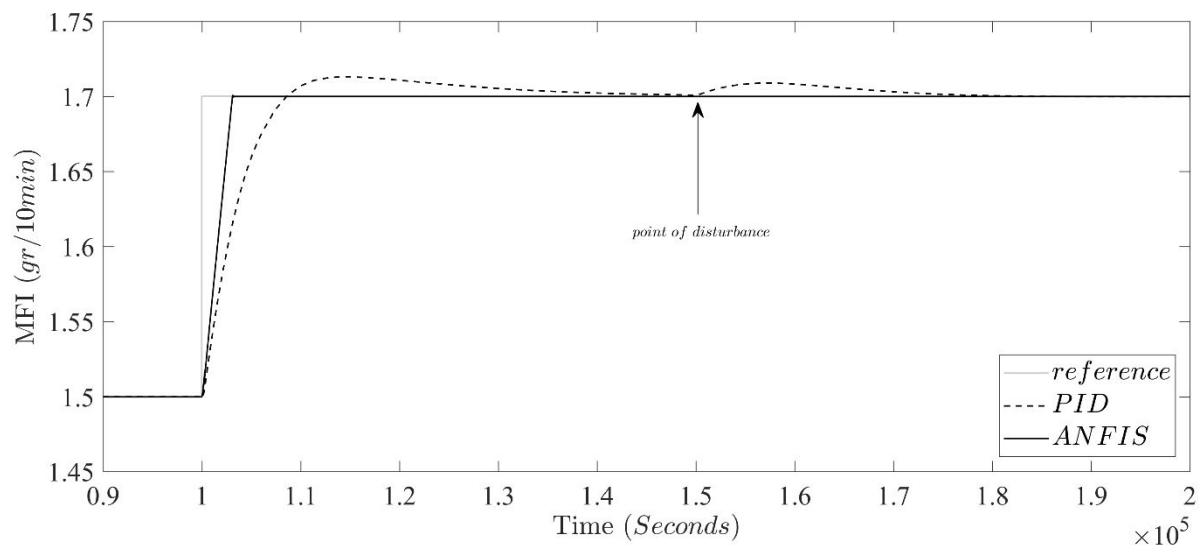
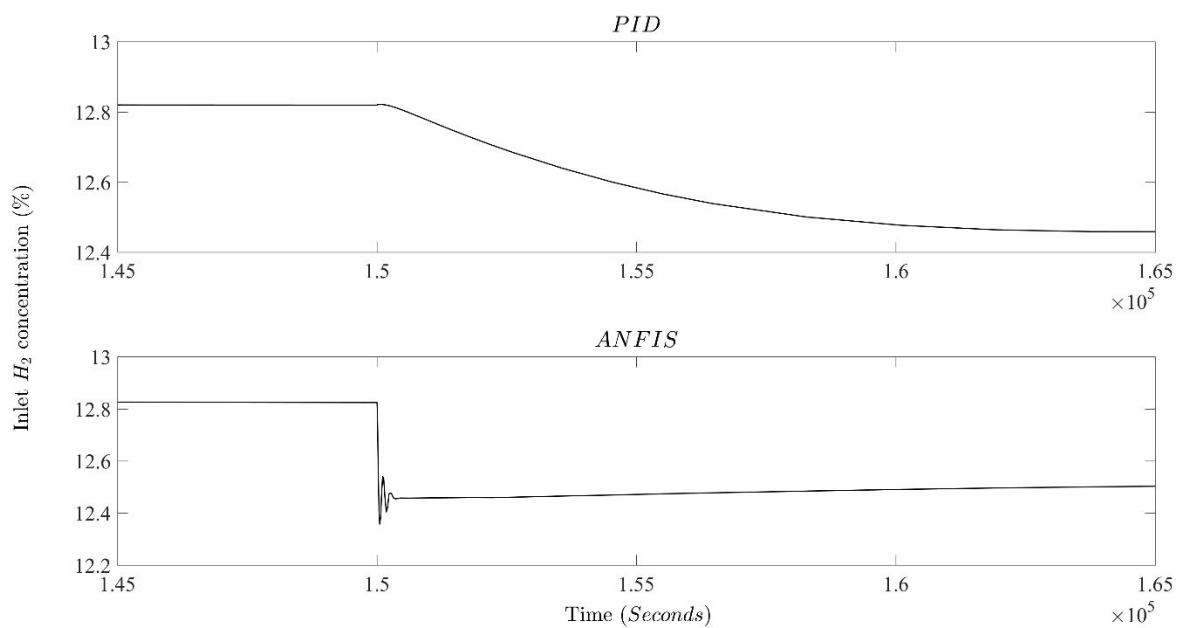


Figure 12: Performance comparison of the controllers in rejecting the effect of a 50% decrease in inlet ethylene concentration on the polymer MFI at 1.5×10^5 s



28 **Figure 13: Controller moves in disturbance rejection for a 50% decrease in inlet ethylene concentration**
29 **on the polymer MFI at 1.5×10^5 s**

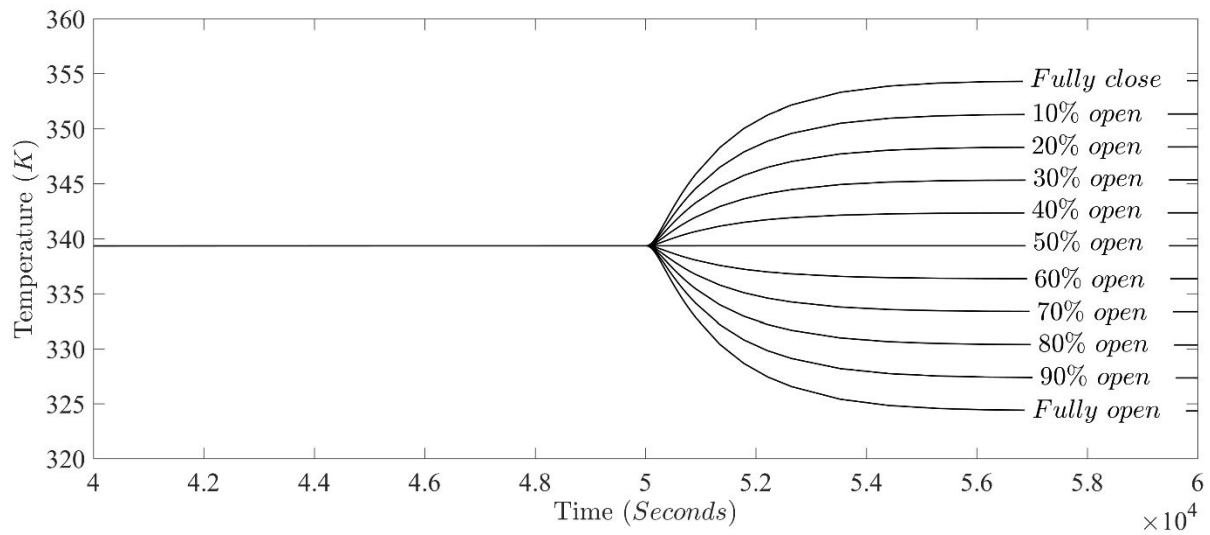


Figure 14: Open loop dynamic response of temperature to different opening percentages of the cool water valve

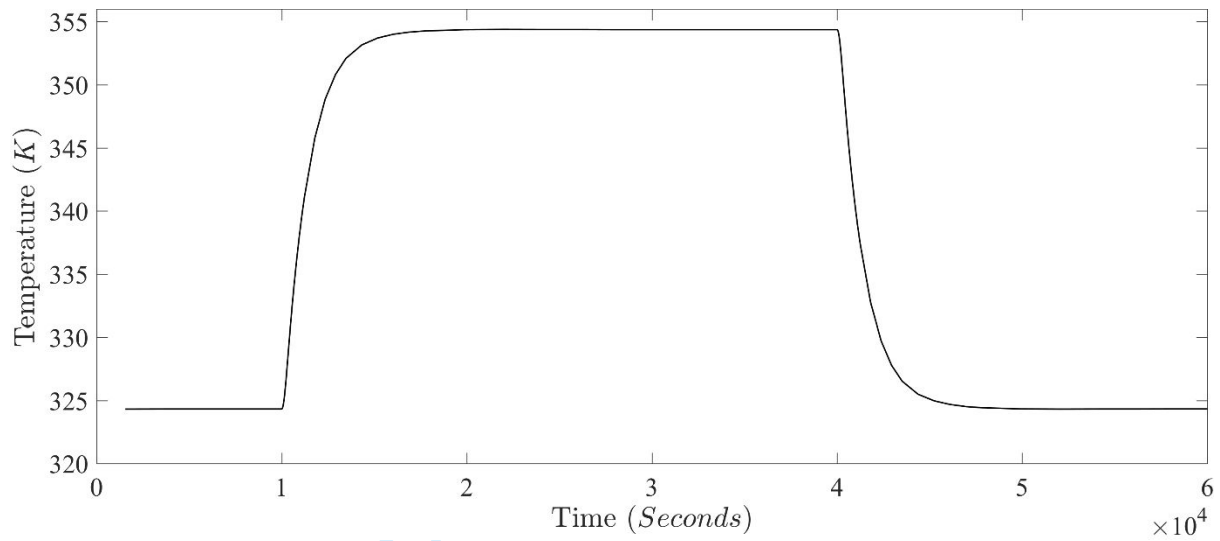


Figure 15: Open loop dynamic response of temperature to fully closing (1×10^4 s) and fully opening (4×10^4 s) of the cool water valve

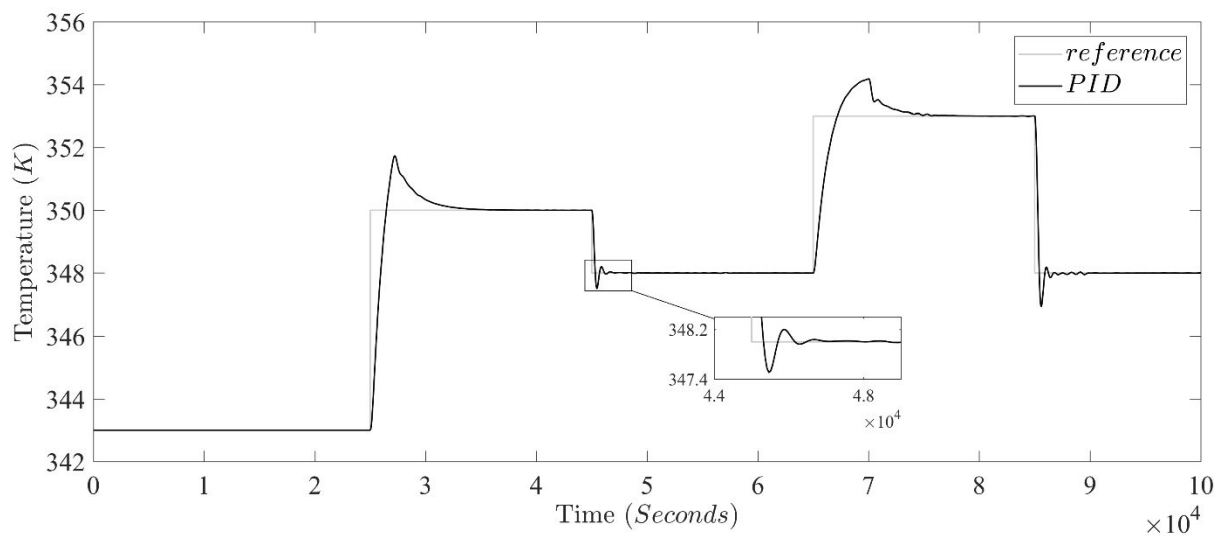


Figure 16: Temperature reference tracking using PID controller ($P=-32.61$, $I=-0.018$, $D=47.32$)

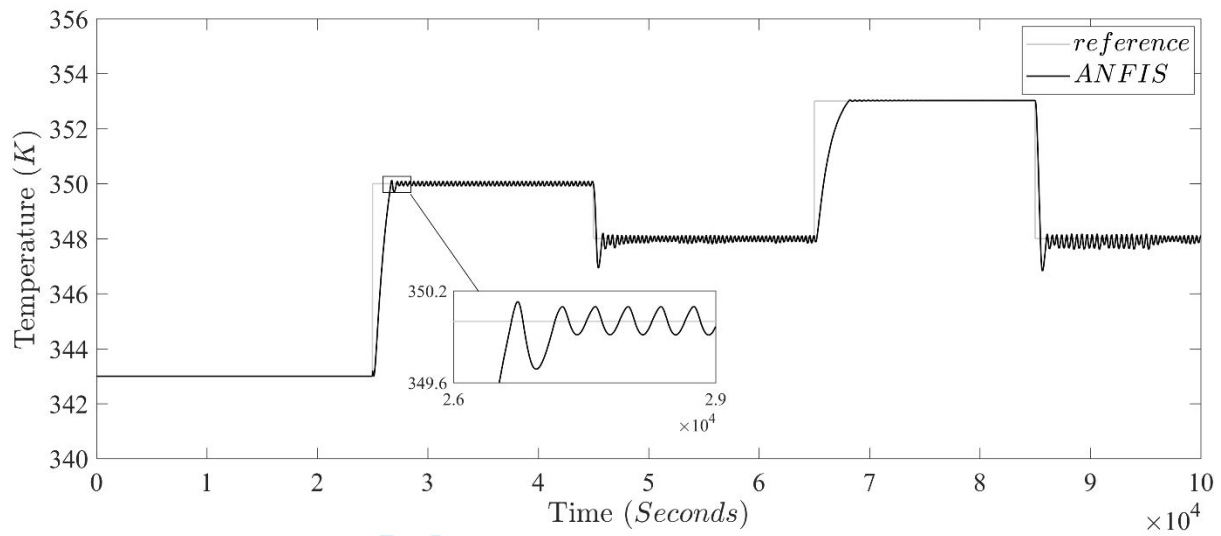


Figure 17: Temperature reference tracking using ANFIS controller

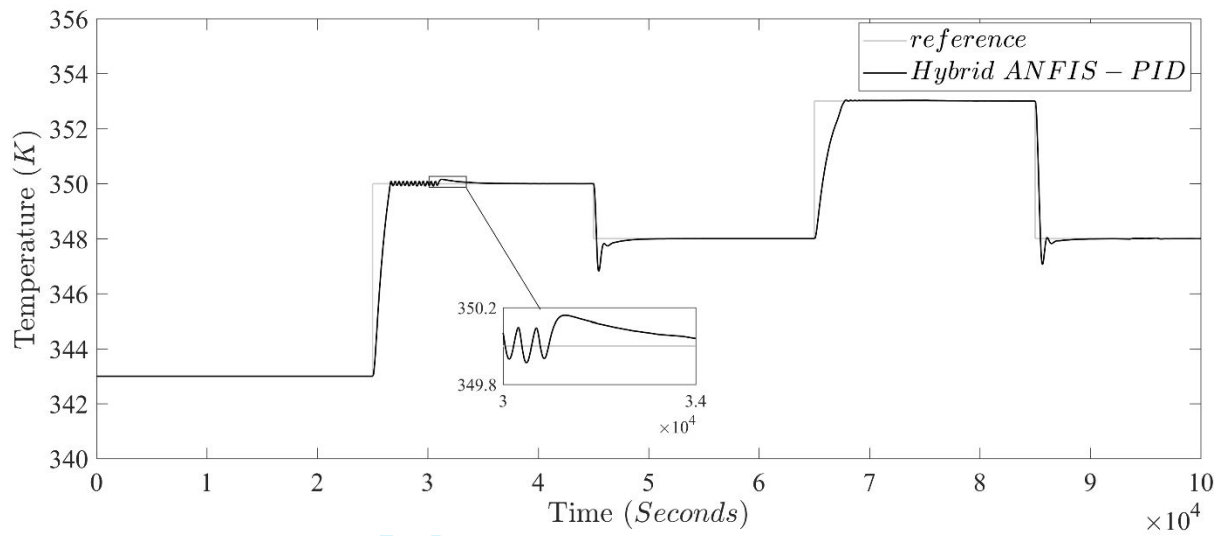


Figure 18: Temperature reference tracking using hybrid ANFIS-PID controller

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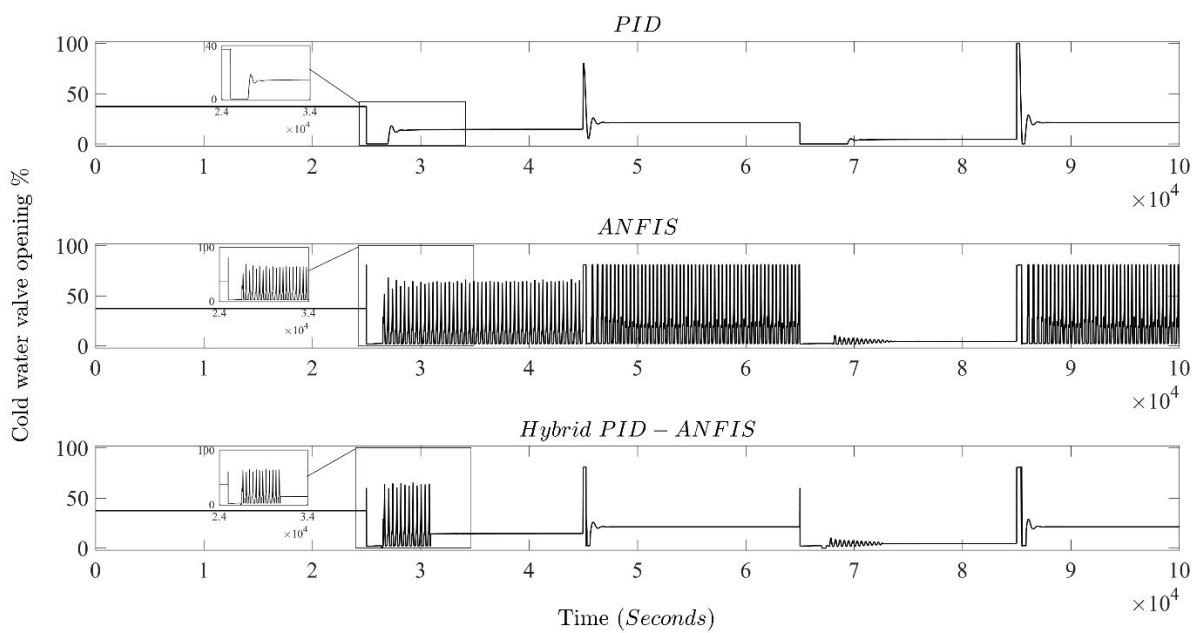


Figure 19: Controller moves in reactor temperature set point tracking

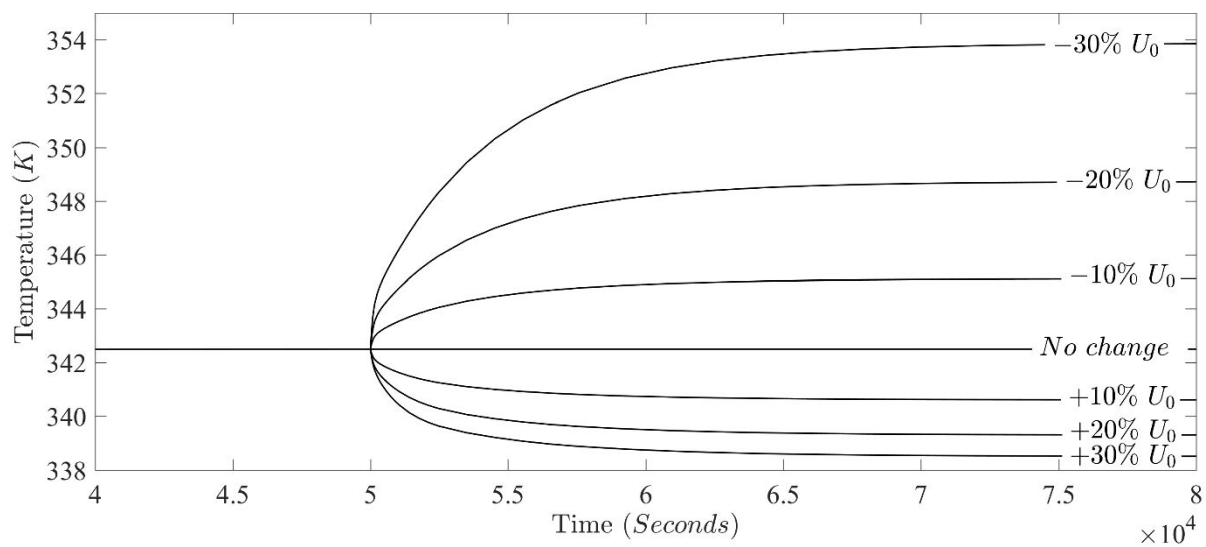


Figure 20: Effect of the superficial gas velocity step changes on the reactor temperature at 5×10^4 s

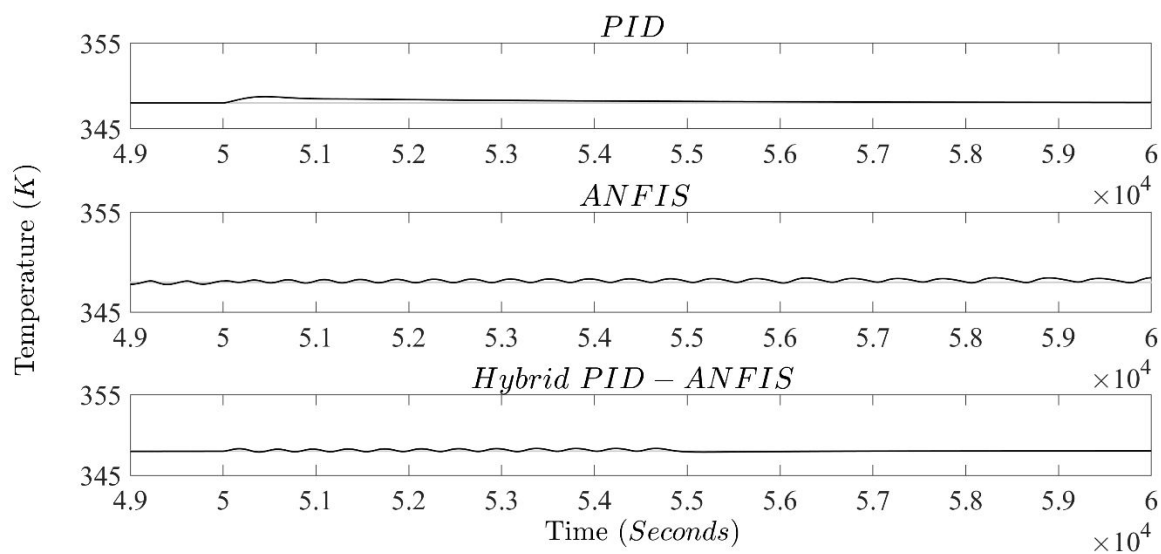


Figure 21: Performance comparison of temperature controllers in rejecting the effect of a 30% decrease in superficial gas velocity at 5×10^4 s

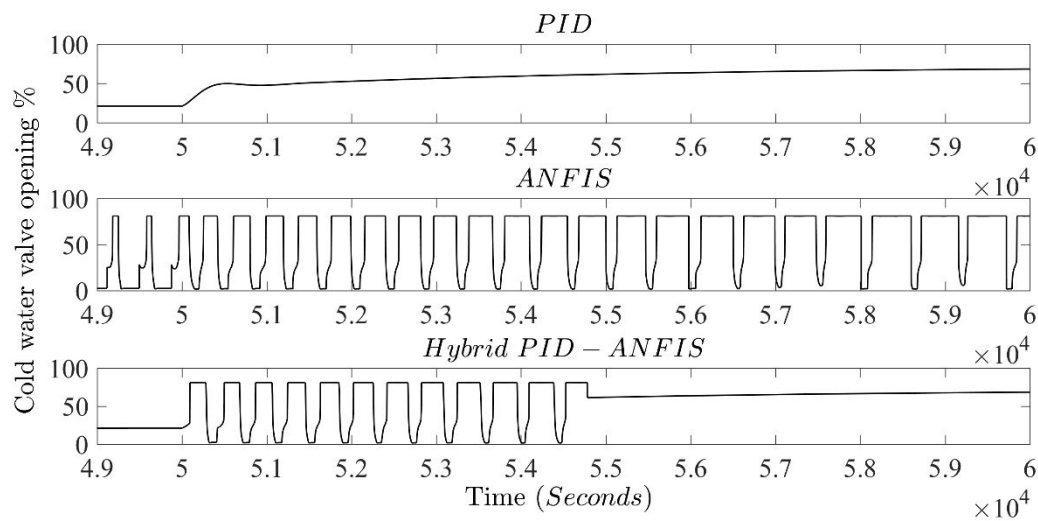


Figure 22: Controller moves in disturbance rejection during temperature control for a 30% decrease in superficial gas velocity at 5×10^4 s

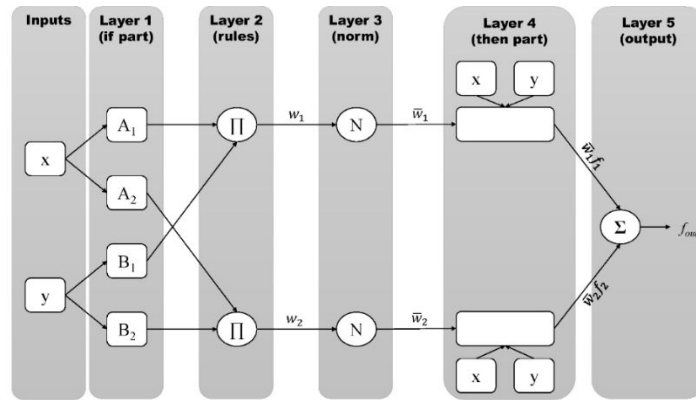


Figure A1: ANFIS structure for two inputs and one output

Table 1: Copolymerization reactions²²

Description	Reaction
Formation reaction	$N^*(j) \xrightarrow{kf(j)} N(0, j)$
Initiation reaction	$N(0, j) + M_i \xrightarrow{ki(j)} N_i(1, j) \quad i = 1, 2, \dots$
Propagation	$N_i(r, j) + M_k \xrightarrow{kp_{ik}(j)} N_k(r+1, j) \quad i = k = 1, 2, \dots$
Transfer to monomer	$N_i(r, j) + M_k \xrightarrow{kfm_{ik}(j)} N_k(1, j) + Q(r, j) \quad i = k = 1, 2, \dots$
Transfer to hydrogen	$N_i(r, j) + H_2 \xrightarrow{kfh_i(j)} N_H(0, j) + Q(r, j) \quad i = 1, 2, \dots$
	$N_H(0, j) + M_i \xrightarrow{kh_i(j)} N_i(1, j) \quad i = 1, 2, \dots$
	$N_H(0, j) + AlEt_3 \xrightarrow{kh_r(j)} N_i(1, j)$
Transfer to co-catalyst	$N_i(r, j) + AlEt_3 \xrightarrow{kfi(j)} N_1(1, j) + Q(r, j) \quad i = 1, 2, \dots$
Spontaneous transfer	$N_i(r, j) \xrightarrow{kfs_i(j)} N_H(0, j) + Q(r, j) \quad i = 1, 2, \dots$
Deactivation reactions	$N_i(r, j) \xrightarrow{kds_i(j)} N_d(0, j) + Q(r, j) \quad i = 1, 2, \dots$
	$N(0, j) \xrightarrow{kds(j)} N_d(j)$
	$N_H(0, j) \xrightarrow{kds(j)} N_d(j)$
Reactions with poisons	$N_i(r, j) + I_m \xrightarrow{kdl(j)} N_{dlH}(0, j) + Q(r, j) \quad i = 1, 2, \dots$
	$N_H(0, j) + I_m \xrightarrow{kdl(j)} N_{dlH}(0, j)$
	$N(0, j) + I_m \xrightarrow{kdl(j)} N_{dl}(0, j)$

Table 2: Table 2: Moment equations obtained from Table 1

$$\begin{aligned}
& \frac{dY(0, j)}{dt} = [M_T] \{k_{i_r}(j)N(0, j) + k_{h_r}(j)N_H(0, j)\} + k_{h_r}(j)N_H(0, j)[AlEt_3] \\
& -Y(0, j) \left\{ k_{f_{m_T}}(j)[H_2] + k_{f_{s_T}}(j) + k_{d_s}(j) + k_{d_l}(j)[I_m] + \frac{R_v}{V_p} \right\} \\
& \frac{dY(1, j)}{dt} = [M_T] \{k_{i_r}(j)N(0, j) + k_{h_r}(j)N_H(0, j)\} \\
& + k_{h_r}(j)N_H(0, j)[AlEt_3] + [M_T]k_{p_{TT}}(j)Y(0, j) \\
& + \{Y(0, j) - Y(1, j)\} \{k_{f_{m_{TT}}}(j)[M_T] + k_{f_{r}}(j)[AlEt_3]\} \\
& -Y(1, j) \left\{ k_{f_{m_T}}(j)[H_2] + k_{f_{s_T}}(j) + k_{d_s}(j) + k_{d_l}(j)[I_m] + \frac{R_v}{V_p} \right\} \\
& \frac{dY(2, j)}{dt} = [M_T] \{k_{i_r}(j)N(0, j) + k_{h_r}(j)N_H(0, j)\} \\
& + k_{h_r}(j)N_H(0, j)[AlEt_3] + [M_T]k_{p_{TT}}(j) \{2Y(1, j) - Y(0, j)\} \\
& + \{Y(0, j) - Y(2, j)\} \{k_{f_{m_{TT}}}(j)[M_T] + k_{f_{r}}(j)[AlEt_3]\} \\
& -Y(2, j) \left\{ k_{f_{m_T}}(j)[H_2] + k_{f_{s_T}}(j) + k_{d_s}(j) + k_{d_l}(j)[I_m] + \frac{R_v}{V_p} \right\} \\
& \frac{dX(n, j)}{dt} = \{k_{f_{m_{TT}}}(j)[M_T] + k_{f_{r}}(j)[AlEt_3] + k_{f_{m_r}}(j)[H_2] + k_{f_{s_r}}(j) + k_{d_s}(j) + k_{d_l}(j)[I_m]\} \\
& \{Y(n, j) - N_T(1, j)\} - X(n, j) \frac{R_v}{V_p} \quad n = 0, 1, 2
\end{aligned}$$

Table 3: Reaction rate constants of two-site ethylene copolymerization²²

Reaction	Rate constant	Unit	Site type 1	Site type 2
Formation	$k_f(j)$	s^{-1}	1	1
Initiation	$k_{i_1}(j)$	L/kmol s	1	1
	$k_{i_2}(j)$	L/kmol s	0.14	0.14
Propagation	$k_{h_1}(j)$	L/kmol s	1	1
	$k_{h_2}(j)$	L/kmol s	0.1	0.1
	$k_{h_r}(j)$	L/kmol s	20	20
	$k_{p_{11}}(j)$	L/kmol s	85	85
	$k_{p_{12}}(j)$	L/kmol s	2	15
	$k_{p_{21}}(j)$	L/kmol s	64	64
	$k_{p_{22}}(j)$	L/kmol s	1.5	6.2
Transfer	$k_{fm_{11}}(j)$	L/kmol s	0.0021	0.0021
	$k_{fm_{12}}(j)$	L/kmol s	0.006	0.11
	$k_{fm_{21}}(j)$	L/kmol s	0.0021	0.001
	$k_{fm_{22}}(j)$	L/kmol s	0.006	0.11
	$k_{fh_1}(j)$	L/kmol s	0.088	0.37
	$k_{fh_2}(j)$	L/kmol s	0.088	0.37
	$k_{fr_1}(j)$	L/kmol s	0.024	0.12
	$k_{fr_2}(j)$	L/kmol s	0.048	0.24
	$k_{fs_1}(j)$	L/kmol s	0.0001	0.0001
	$k_{fs_2}(j)$	L/kmol s	0.0001	0.0001
Deactivation	$k_{ds}(j)$	s^{-1}	0.0001	0.0001
	$k_{dl}(j)$	L/kmol s	2000	2000
Impurity	$k_a(j)$	s^{-1}	0.0003	0.0003

Table 4: Hydrodynamic equations

Parameter	Formula	Reference
Minimum fluidization velocity	$Re_{mf} = [29.5^2 + 0.375Ar]^{0.5} - 29.5$	32
Bubble velocity	$U_b = U_0 - U_{mf} + U_{br}$	33
Bubble rise velocity	$U_{br} = 0.711(gd_b)^{0.5}$	33
Emulsion velocity	$U_e = \frac{U_0 - U_b}{(1 - \delta)}$	34
Bubble diameter	$d_b = d_{b0} [1 + 27(U_0 - U_e)]^{0.33} (1 + 6.84H)$ $d_{b0} = 0.0085(\text{for GeldartB})$	35
Mass transfer coefficient	$k_{be} = \left(\frac{1}{k_{bc}} + \frac{1}{k_{ce}} \right)^{-1}$ $K_{bc} = 4.5 \left(\frac{U_e}{d_b} \right) + 5.85 \left(\frac{D_g^{0.5} g^{0.25}}{d_b^{1.25}} \right)$ $K_{ce} = 6.77 \left(\frac{D_g \varepsilon_e U_{br}}{d_b} \right)$	33
Heat transfer coefficient	$H_{be} = \left(\frac{1}{H_{bc}} + \frac{1}{H_{ce}} \right)^{-1}$ $H_{bc} = 4.5 \left(\frac{U_e \rho_g C_{pg}}{d_b} \right) + 5.85 \frac{(k_g \rho_g C_{pg})^{0.5} g^{0.25}}{d_b^{1.25}}$ $H_{ce} = 6.77 (\rho_g C_{pg} k_g)^{0.5} \left(\frac{\varepsilon_e U_{br}}{3} \right)^{0.5}$	33
Bubble phase fraction Emulsion	$\delta = 0.534 \left[1 - \exp \left(-\frac{U_0 - U_{mf}}{0.413} \right) \right]$	36
Emulsion phase porosity	$\varepsilon_e = \varepsilon_{mf} + 0.2 - 0.059 \exp \left(-\frac{U_0 - U_{mf}}{0.429} \right)$	36
Bubble phase porosity	$\varepsilon_b = 1 - 0.146 \exp \left(-\frac{U_0 - U_{mf}}{4.439} \right)$	36
Volume of polymer phase in the emulsion phase	$V_{Pe} = AH(1 - \varepsilon_e)(1 - \delta)$	37
Volume of polymer phase in the bubble phase	$V_{Pb} = AH(1 - \varepsilon_b)\delta$	37
Volume of the emulsion phase	$V_e = A(1 - \delta)H$	37
Volume of the bubble phase	$V_b = A\delta H$	37

Table 5: Mass and energy balances attained by considering the assumptions

Type	Equation
Mass balance (emulsion)	$\frac{d}{dt}(V_e \varepsilon_e [M_i]_e) = [M_i]_{e,(in)} U_e A_e - [M_i]_e U_e A_e - R_v \varepsilon_e [M_i]_e$ $+ K_{be} ([M_i]_b - [M_i]_e) V_e \left(\frac{\delta}{1 - \delta} \right) - (1 - \varepsilon_e) R_{ie} - \frac{K_e V_e \varepsilon_e A_e [M_i]_e}{W_e}$
Mass balance (bubble)	$\frac{d}{dt}(V_b \varepsilon_b [M_i]_b) = [M_i]_{b,(in)} U_b A_b - [M_i]_b U_b A_b - R_v \varepsilon_b [M_i]_b$ $- K_{be} ([M_i]_b - [M_i]_e) V_b - (1 - \varepsilon_b) \frac{A_b}{V_{PFR}} \int R_{ib} dz - \frac{K_e V_e \varepsilon_e A_e [M_i]_e}{W_e}$
Energy balance (emulsion)	$U_e A_e (T_{e,(in)} - T_{ref}) \sum_{i=1}^m [M_i]_{e,(in)} C_{pi} - U_e A_e (T_e - T_{ref}) \sum_{i=1}^m [M_i]_e C_{pi}$ $- R_v (T_e - T_{ref}) \left(\sum_{i=1}^m \varepsilon_e C_{pi} [M_i]_e + (1 - \varepsilon_b) \rho_{pol} C_{p.pol} \right)$ $+ (1 - \varepsilon_e) R_{pe} \Delta H_R - H_{be} V_e \left(\frac{\delta}{1 - \delta} \right) (T_e - T_b)$ $- V_e \varepsilon_e (T_e - T_{ref}) \sum_{i=1}^m C_{pi} \frac{d}{dt} ([M_i]_e)$ $- \frac{K_e A_e}{W_e} (T_e - T_{ref}) \left(\sum_{i=1}^m \varepsilon_e C_{pi} [M_i]_e + (1 - \varepsilon_b) \rho_{pol} C_{p.pol} \right)$ $= V_e \left(\varepsilon_e \sum_{i=1}^m C_{pi} [M_i]_e + (1 - \varepsilon_e) \rho_{pol} C_{p.pol} \right) \frac{d}{dt} (T_e - T_{ref})$
Energy balance (bubble)	$U_b A_b (T_{b,(in)} - T_{ref}) \sum_{i=1}^m [M_i]_{b,(in)} C_{pi} - U_b A_b (T_b - T_{ref}) \sum_{i=1}^m [M_i]_b C_{pi}$ $- R_v (T_b - T_{ref}) \left(\sum_{i=1}^m \varepsilon_b C_{pi} [M_i]_b + (1 - \varepsilon_e) \rho_{pol} C_{p.pol} \right)$ $+ (1 - \varepsilon_b) \frac{A_b \Delta H_R}{V_{PFR}} \int R_{pb} dz + H_{be} V_b (T_e - T_b)$ $- V_b \varepsilon_b (T_b - T_{ref}) \sum_{i=1}^m C_{pi} \frac{d}{dt} ([M_i]_b)$ $- \frac{K_b A_b}{W_b} (T_b - T_{ref}) \left(\sum_{i=1}^m \varepsilon_b C_{pi} [M_i]_b + (1 - \varepsilon_e) \rho_{pol} C_{p.pol} \right)$ $= \left(V_b \left(\varepsilon_b \sum_{i=1}^m C_{pi} [M_i]_b + (1 - \varepsilon_e) \rho_{pol} C_{p.pol} \right) \right) \frac{d}{dt} (T_b - T_{ref})$

Table 6: Solids elutriation equations used in the model

Parameter	Formula	Referenc
Elutriation rate constant in emulsion phase	$K_e = 23.7 \rho_g U_0 \frac{A}{W_e} \exp\left(\frac{-5.4U_t}{U_0}\right)$	38
Elutriation rate constant in bubble phase	$K_b = 23.7 \rho_g U_0 \frac{A}{W_b} \exp\left(\frac{-5.4U_t}{U_0}\right)$	38
Weight of solids in the emulsion phase	$W_e = AH(1 - \varepsilon_e) \rho_{pol}$	33
Weight of solids in the bubble phase	$W_b = AH(1 - \varepsilon_b) \rho_{pol}$	33
Terminal velocity of falling particles	$U_t = \frac{U_t^*}{\left[\mu \rho_g^{-2} (\rho_{pol} - \rho_g) g\right]^{0.33}}$	39
Dimensionless terminal falling velocity coefficient	$U_t^* = \left[18(d_p^*)^{-2} + (2.335 - 1.744\varnothing_s)(d_p^*)^{-0.5}\right]^{-1}$	39
dimensionless particle size	$d_p^* = d_p \left[\mu_g^{-2} \rho_g (\rho_{pol} - \rho_g) g\right]^{0.33}$ for $0.5 < \varnothing_s \leq 1$	39

Table 7: Specifications of the developed ANFIS structures for MFI and temperature control

Parameter	MFI ANFIS	Temperature ANFIS
Fuzzy structure	Sugeno	Sugeno
Membership function type	Triangular	Gaussian
Number of inputs	2	2
Number of outputs	1	1
Optimization method	Hybrid (least square and back propagation technique)	Hybrid (least square and back propagation technique)
Number of fuzzy rules	9	25

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Table 8: Performance indexes for MFI controllers

	Setpoint tracking		Disturbance rejection	
	PID	ANFIS	PID	ANFIS
ITAE	1.44×10^9	4.38×10^8	2.13×10^9	6.52×10^8
IAE	9826	2390	1.16×10^4	2886
ISE	1748	771.6	1911	870.4
APE	8687	1879	1.01×10^4	2302

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Table 9: Performance indexes for temperature controllers

	Setpoint tracking			Disturbance rejection		
	PID	ANFIS	Hybrid	PID	ANFIS	Hybrid
ITAE	1.53×10^9	1.08×10^9	9.81×10^8	1.59×10^9	1.63×10^9	1.16×10^9
IAE	1.14×10^5	6.98×10^4	6.67×10^4	1.14×10^5	7.62×10^4	6.91×10^4
ISE	1.01×10^6	7.80×10^5	7.82×10^5	7.83×10^5	7.83×10^5	7.79×10^5
APE	3.31×10^2	2.03×10^2	1.93×10^2	3.34×10^1	2.21×10^2	2.01×10^2

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