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Nonintrusive investigation of large Al-kaolin fractal aggregates with slow settling velocities



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12 *Nonintrusive Investigation of Large Al-kaolin Fractal Aggregates*
13 *with Slow Settling Velocities*

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42 **Abstract**

43 Although a combination of aggregate characteristics dictate particle settling, it is commonly
44 assumed that large particles have higher terminal velocities. This simplifying assumption
45 often leads to overprediction of large aggregate settling velocities which in turn negatively
46 impacts on estimates of sedimentation clarification efficiency. Despite its importance, little
47 attention has been given to large aggregates with slow-settling velocities. This paper
48 addresses this gap by investigating slow-settling velocities of large, heterodisperse and multi-
49 shape Al-kaolin aggregates using non-intrusive methods. A particle image velocimetry
50 technique (PIV) was applied to track aggregate velocity and a non-intrusive image technique
51 was used to determine aggregate characteristics, including size (d_f), three-dimensional fractal
52 dimension (D_f), density (ρ_f), aggregate velocity (V_{exp}) and Reynolds number (Re). Results
53 showed no strict dependence of settling velocity on large aggregate size, shape and density, as
54 Al-kaolin aggregates with the same size exhibited different settling velocities. A comparison
55 of the results with the well-known Stokes' law for velocity modified by a shape factor showed
56 that the settling velocities measured here can vary from 2 to 14 fold lower than the predicted
57 values for perfect sphere-shape aggregates with the same density and size. Furthermore,
58 results have also shown large Al-kaolin aggregate's drag coefficient (C_d) to be around $56/Re$,
59 for average fractal aggregate sphericity of around 0.58.

60 **Keywords:** flocculation, fractal dimension, settling velocity, aggregate density, sedimentation

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65 **1- Introduction**

66 Sedimentation is widely used as a technique for separating suspended material in water
67 treatment before filtration. This stage is commonly preceded by coagulation and flocculation
68 processes, which destabilize colloids and promote their subsequent agglomeration, favouring
69 the formation of large aggregates, which are commonly assumed as more likely to settle in
70 sedimentation tanks. This comes from the belief that size and density are dependent features
71 and that aggregates can be considered as spheres, as described by Stokes' law. However,
72 some studies (e.g. Chakraborti et al., 2000; Johnson, Li and Logan, 1996; Vahedi and
73 Gorczyca, 2014) have shown that the complex mechanisms involving aggregation, breakage,
74 restructuring of multi-shape primary particles and different relations between size, shape and
75 density can result in different terminal settling velocities, even for fractal aggregates with the
76 same size. As such, practitioners have reported that large aggregates may still remain in the
77 supernatant water due to their very slow-settling velocity, which can be attributed to features
78 other than their size (Vahedi and Gorczyca, 2012).

79 The aggregates formed from the flocculation of colloidal material are known as fractal objects
80 (Jiang and Logan, 1991; Gregory, 1997), i.e. they have non-spherical shape and porous
81 structures, and hence, cannot be fully geometrically represented by a sphere. Despite this, to
82 reduce the complexities of aggregate settling hydrodynamics, it is still common to evaluate
83 the sedimentation of particles by assuming impervious and perfect-shape sphere aggregates
84 (Bushell et al., 2002). It is known that this oversimplification may lead to inaccurate
85 predictions of settling velocity with significant errors, with actual velocity estimates varying
86 from 4 to 8 times higher, as shown by Johnson et al. (1996) to 5 times lower, as shown by
87 Vahedi and Gorczyca (2012). This makes it difficult to fully understand the phenomena and
88 their relevance to engineering (Johnson et al., 1996, Gregory, 1997, Li et al., 2006 and Vahedi

89 and Gorczyca, 2012). Therefore, for practical reasons, it is necessary to gain a better
90 understanding of the complex relation between the characteristics of large fractal aggregates
91 produced by coagulation and flocculation, and their settling velocities.

92 Fractal dimension can be related to aggregates' porosity, density, strength of flocs,
93 sedimentation velocity, collision models and flocculation kinetics. Settling velocity is mainly
94 dependant on the density and size of aggregates, which in turn may be strongly affected by the
95 interrelated parameters of porosity and permeability. Low fractal dimensions change the
96 density-size relationship, thus affecting aggregate mass, whereas higher porosity may also
97 affect flocs permeability, favouring water flow through flocs, and decreasing flow resistance.

98 Some studies incorporated the understanding of fractal geometry into the dynamics of
99 aggregate removal. Gregory (1997) and Jarvis et al., (2005) followed a theoretical and
100 conceptual approach, while Vahedi and Gorczyca (2014) focused on modelling and
101 simulation. So far, few papers have combined experimental with fundamental approaches
102 (e.g. Johnson et al., 1996; Tang et al., 2002; Vahedi and Gorczyca, 2012, Chakraborti and
103 Kaur, 2014).

104 In terms of floc size and settling velocity, Vahedi and Gorczyca (2012) have studied lime
105 softening flocs (denser than Alum flocs), where an average of $2.37 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$ was measured
106 for an average size of $124 \mu\text{m}$ of equivalent diameter. However, this settling velocity was
107 slower than those presented by Johnson et al. (1996), who have studied fractal aggregates (2 -
108 $40 \mu\text{m}$) formed from latex microspheres, and coagulated with NaCl solution at a shear rate of
109 5 s^{-1} . Johnson et al. (1996) found settling velocities to be 4 to 8 times higher than Stokes' law
110 prediction, i.e. 1.0×10^{-4} to $1.0 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$. These results differ from the expected ones, and it is

111 difficult to explain why such high velocities were measured, even considering high
112 permeability and the flow-through floc effect, as pointed by Bushell et al. (2002).
113 Although primary particles in real flocs are heterodisperse and multi-shaped, latex spheres of
114 known sizes were still used in many of these works to form aggregates. Nevertheless,
115 Chakraborti and Kaur (2014) have also shown that even for large latex spheres (size larger
116 than 100 μm) settling velocities may also be lower than those predicted by Stokes' law
117 equation. They concluded that drag coefficients (C_d) assumed in the Stokes' law velocity
118 expression may be not applicable for larger particles, even with $Re < 1$. Furthermore, primary
119 particles affect aggregates' shape and there is evidence that large aggregates have fractal-like
120 shapes, with lower dimensions, resulting in lower density compared to smaller aggregates
121 (Johnson et al., 2016; Vahedi and Gorczyca, 2012 and Moruzzi et al., 2017). According to
122 Bushell et al. (2002), the impact of aggregation of primary particles on hydrodynamics may
123 result either in the increase of drag forces compared to spherical primary particles, or in the
124 formation of aggregates that can become permeable, decreasing the drag force. This may
125 impact the settling velocity and more investigation is needed to confirm which effect prevails
126 on large aggregates.

127 This study aims to systematically investigate and analyse the features and dynamics of slow
128 settling Al-kaolin fractal large aggregates. The focus here is specifically, large, heterodisperse
129 and multi-shape flocculated kaolin particles settled by gravity. To avoid the assumption that
130 particle size and drag force are intrinsically related, a non-intrusive imaging method has been
131 used as a means to characterise aggregates' behaviour, rather than settling velocity. Particle
132 image velocimetry (PIV) is used to measure aggregates' settling velocity as well as their size
133 and shape. Furthermore, a novel procedure is applied to convert 2D fractal dimensions into

134 3D from image analysis. Size and density were analysed and correlated from two approaches,
135 individually and from the entire population of aggregates. The shape, Reynolds number and
136 drag coefficients of large Al-kaolin fractal aggregate velocities are determined, and
137 experimental velocities are compared with estimates using Stokes' equation modified by a
138 shape factor, which has been widely used in predicting settling velocities.

139

140 2- Methodology

141 2.1 Synthetic water preparation

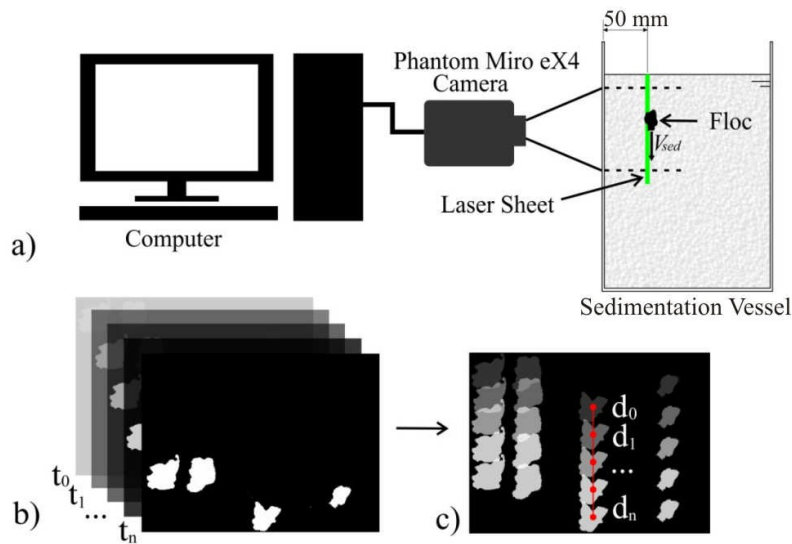
142 For the present study, commercial kaolin (Sigma-Aldrich) was used as primary particle and
143 synthetic water was prepared from a stock suspension (turbidity 5000 ± 200 NTU), as
144 recommended by Yukselen and Gregory (2004). For each assay, a volume of 10 mL from stock
145 solution was diluted in 2 L of deionized water to produce water with turbidity 25 ± 2 NTU (15.8
146 ± 1.3 mg TSS· L⁻¹), as previously investigated by Moruzzi et al. (2017). This range of low-to-
147 medium turbidity was selected to represent conditions found in many freshwater sources during
148 dry seasons, e.g. in Sao Paulo State, Brazil (CETESB, 2015) or in the USA (Swenson, 1965),
149 respectively. This type of water has been also studied by other researchers (e.g. Li et al., 2008;
150 Wei et al., 2010; Liu et al., 2019; Eman et al., 2010). A MALVERN Mastersizer 2000 particle
151 size analyser and a Scanning Electron Microscope (SEM) were used to measure kaolin size
152 distribution and to define the representative pixel size, i.e. the ratio between image resolution
153 and size that better describes flocs, for the kaolin bulk particles which form the aggregates, as
154 discussed by Moruzzi et al. (2017). Analytical grade alum ($\text{Al}_2(\text{SO}_4)_3 \cdot 14 \cdot \text{H}_2\text{O}$) from Sigma-
155 Aldrich was used as coagulant and analytical grade sodium bicarbonate (0.1 M, NaHCO_3) was
156 used as a pH buffer during coagulation tests. Here, the coagulation conditions obtained by

157 Oliveira et al. (2015) and Moruzzi et al. (2017) were applied, i.e. 2 mg Al·L⁻¹ and pH of 7.5.
158 The aggregates were obtained after flocculation with velocity gradients (G_f) ranging from 20 to
159 60 s⁻¹ for 15 minutes of flocculation time in order to consider aggregates of different sizes and
160 shapes. The sedimentation vessel had a cross-sectional area of 150 × 150 mm. The temperature
161 was kept at about 20 ± 1 °C during all experiments.

162 2.2 Non-intrusive image analysis

163 To study large aggregates with slow settling velocity, only the ones that remained in the
164 supernatant after 5 minutes of sedimentation were monitored. This meant that only aggregates
165 with settling velocities slower than 4.6 × 10⁻⁴ m·s⁻¹ (40 m·day⁻¹) would remain in the sampling
166 point, which is equivalent to the loading rate of conventional settling tanks. Furthermore, low
167 aggregates remain in the supernatant and therefore minor hydrodynamic interference can be
168 assumed (Goula et al. 2008). This was also to ensure minimum inertial fluid motion after
169 flocculation, and the predominance of aggregate vertical trajectory. In total, 118 aggregates
170 were individually monitored applying a non-intrusive image acquisition system using a *High-*
171 *Speed Miro EX-4* camera with interchangeable lenses. This approach avoids the assumption of
172 an explicit relationship between drag coefficient and aggregate size, as mentioned by Bushell
173 et al. (2002). The lighting system was set up as proposed by Moruzzi et al. (2019), and images
174 were taken at a section located 50 mm from the jar wall to avoid hydrodynamic interaction. A
175 schematic of the experimental setup is shown in Figure 1-a. For all experiments, a sampling
176 frequency of 25 Hz was used for 40s at a resolution of 800 x 600 elemental units (pixels),
177 with a field of view of 6 x 8 mm, and a shutter frequency of 800 μs. For these conditions, the
178 pixel size was 10 μm and a total of 1000 images were obtained at time intervals of 40 ms
179 (milliseconds) from t_0 to t_n , with n changing from 1 to 1000 (Figure 1-b).

180 For the Particle Image Velocimetry (PIV) evaluation, *Image-Pro Plus*® software was used to
 181 analyse the images, i.e. conversion from 2^8 to 2^1 bits (i.e. from 256 grayscale to black & white
 182 image), enhancement, measurement and tracking of aggregates from centroid distances d_0 to
 183 d_n , with n varying from 1 to 1000 frames, as shown in Figure 1-c.
 184 Only aggregates with vertically projected cross sectional areas larger than 200 pixels and sizes
 185 greater than 15 pixels were chosen for accuracy in accordance with Chakraborti et al. (2003),
 186 Moruzzi and Silva (2018) and Moruzzi et al. (2019). Data were statically analysed for 95%
 187 significance.



188
 189 Figure 1 – (a) Schematic of the experimental arrangement used in the experiments; (b) image
 190 pack screened from time intervals from t_0 to t_n ; and (c) tracking aggregates distances from d_0
 191 to d_n using Particle Image Velocimetry (PIV) tool.

192

193 2.3 Fractal aggregate features

194 The three-dimensional fractal dimension (D_f) for Al-kaolin aggregates was considered
 195 individually based on Jiang and Logan (1991) and Jarvis et al. (2005):

$$196 \quad N = b \left[\frac{d_f}{d_p} \right]^{D_f} \quad (1)$$

197 where D_f is the three-dimensional fractal dimension for individual Al-kaolin aggregates, d_p is
 198 the primary particle size (m), here determined from the median size (d_{50}) of primary kaolin
 199 particle size distribution, d_f is the floc size (m), N is the number of particles of size d_p per floc
 200 of size d_f and b is the structure factor which comprises the packing and shape factors for
 201 aggregates and primary particles, as introduced by Bushell (2002).

202 The number of pixels counted in the floc area was considered representative of N from
 203 Equation 1, as it agrees with the median size (d_{50}) of around 10 μm , discussed later on this
 204 paper. The volume of the fitted ellipsoid of revolution (E) to the aggregate was used to derive
 205 the fractal dimension. This was calculated by rotating the encased ellipsoid around the longest
 206 size of E (d_{max}) limited by the smallest dimension (d_{min}), as proposed by Chakraborti et al.
 207 (2000). From these data, the three-dimensional fractal dimension (D_{fp}) was calculated for the
 208 set of aggregates, fitting the areas (A) and volumes (V) with aggregate longest length (d_{max}) in
 209 *log-log* plots, using Equations 2 and 3, respectively:

$$210 \quad A \sim d_{max}^{D_{fp}'} \quad (2)$$

$$211 \quad V \sim d_{max}^{D_{fp}} \quad (3)$$

212 where A is the projected floc area on the image plane, d_{max} is the longest dimension of the floc
 213 (m), D_{fp}' is the two-dimensional fractal dimension for the set of aggregates, V is the volume of
 214 the ellipsoid containing the floc (m^3) and D_{fp} is the three-dimensional fractal dimension for
 215 the set of aggregates.

216 Finally, Equation 4 recently proposed by Moruzzi et al. (2020) was applied to determine the
 217 three-dimensional fractal dimension per aggregate (D_f), based on the ratio D_{fp}/D_{fp}' from the
 218 entire aggregate population and on 2D fractal dimension calculated individually:

$$D_f = \frac{D_{fp}}{D_{fp'}} \left(\frac{\text{Log}N}{\text{Log}(d_{max}/d_p)} \right) \quad (4)$$

The key assumption here is that the D_{fp}/D_{fp}' ratio, determined from the entire aggregate population, could be applied to convert 2D to 3D fractal dimensions for individual aggregates using images. This has never been described before in the literature; however it is expected that the entire population of aggregates can provide good approximations for the shape of individual flocs.

The density of the aggregates was determined individually from the mass balance between floc, particle and voids occupied by the liquid (Jiang and Logan, 1991 and Johnson et al., 1996):

$$\rho_f = \rho_l + \left(b \left(\frac{d_f}{d_p} \right)^{D_f-3} (\rho_p - \rho_l) \right) \quad (5)$$

where ρ_f is the density of aggregate ($\text{kg}\cdot\text{m}^{-3}$), ρ_l is the density of water ($\text{kg}\cdot\text{m}^{-3}$) and ρ_p is the density of the primary particle ($\text{kg}\cdot\text{m}^{-3}$).

It is important to note that although Equation 5 describes density in terms of three-dimensional fractal dimensions, it is still assumed that primary particles are perfect spheres, as pointed by Vahedi and Gorczyca (2014). Furthermore, it was also assumed that the entire aggregate size could be represented as a homogeneous aggregation of mono-sized primary particles.

Sphericity (Ψ) and aspect ratio (i.e. the larger and shorter aggregate length ratio) were also determined, with the value of 1 representing the shape of perfect sphere for both cases (Jarvis et al., 2005). The size of each aggregate and their associated measured velocities obtained from the experiments were also used to determine the dimensionless Reynolds number (Re) for Al-kaolin aggregates:

241 $Re = \frac{\rho_l d_f V_{exp}}{\mu}$ (6)

242 where Re is the dimensionless Reynolds number, V_{exp} is the measured Al-kaolin aggregate

243 terminal velocity by PIV ($m \cdot s^{-1}$) and μ is the absolute viscosity ($N \cdot m^{-2} \cdot s$).

244 Experimental velocities of fractal aggregates were compared with the modelling approach in a

245 wide range of fractal dimensions, through the well-known Stokes' law for settling velocity

246 modified by a dimensionless shape factor:

247 $V_{calc} = \frac{\Delta\rho g d_f^2}{\theta_{18}\mu}$, valid for $Re < 1$ and $d_f < 1mm$, so that $Cd=24/Re$ (7)

248 where $\Delta\rho$ is the differential density of aggregate and water, also named as aggregate buoyant

249 density, ($kg \cdot m^{-3}$) and θ is the shape factor (dimensionless) that comprises all limitations

250 resulting from simplifying assumptions such as: primary particles being compact and

251 perfectly sphere-shaped with homogeneous size, aggregates being perfect shapes and

252 impervious spheres, porosity being homogeneous in aggregates, aggregates presenting mono

253 structures, drag coefficient (Cd) being constant and represented by $24/Re$ for $Re < 1$.

254 The drag coefficients for Al-kaolin fractal aggregates were also calculated, based on fractal

255 homogeneous aggregate porosity (\mathcal{E}) as initially proposed by Jiang and Logan (1991), here

256 adapted for the encased ellipsoid (E).

257 $Cd = \frac{4 \Delta\rho d_f g (1-\mathcal{E})}{3 \rho_f V_{exp}^2}$ (8)

258 $\mathcal{E} = 1 - b \cdot \left(\frac{d_f}{d_p}\right)^{D_f-3}$ (9)

259 where \mathcal{E} is the Al-kaolin aggregate porosity.

260

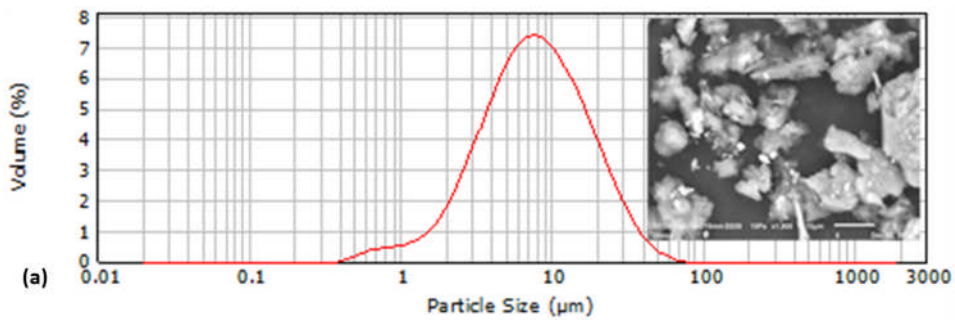
261 **3- Results and Discussion**

262 3.1 Physical attributes

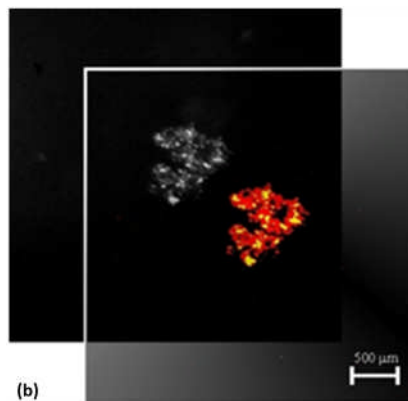
263 Figure 2-a shows the heterodisperse nature of dry kaolin powder with size distribution
264 ranging from 1 to 100 μm , and median size (d_{50}) of around 10 μm , which is in agreement with
265 the findings of other researchers (e.g. Aparício et al., 2004; Zbik and Smart, 1998). This result
266 reinforces the hypothesis that the assumption of homogeneous and perfectly sphere-shaped
267 primary particles is an oversimplification of a more complex shape and size distribution.
268 Although the pixel size of 10 μm , used here, could be assumed to represent kaolin median
269 size for volume distribution, using only one fractal dimension for flocs would result in an
270 unrealistic high-density aggregate. Aggregates cannot be presumed as a sum of side-by-side
271 primary particles, but a complex structure with multi-scale voids occupied by water and Al-
272 kaolin precipitates (Vahedi and Gorczyca, 2014). As described by Gorczyca and Ganczarzyk
273 (1999), fractal aggregates are the result of primary particles attaching onto pre-formed flocs
274 with different levels of aggregation, leading to different sizes and pore populations within
275 aggregates, i.e. *floculi*, microflocs and flocs aggregates. Furthermore, Yu et al. (2015) found
276 that when the coagulant is added to the suspension, flocs grew rapidly as primary particles
277 enmesh within the hydroxide precipitate during the flocculation. Consequently, floc
278 aggregates have different primary particle concentrations within fractal aggregates, and
279 therefore, the density calculated using only one fractal dimension for flocs does not reflect the
280 complex multilevel floc structure.

281 In order to overcome this issue, the cross sectional area of aggregates was analysed to
282 determine kaolin within its structure, by performing image analysis based on different level of
283 brightness. For this purpose, aggregates formed by Alum only were compared to those formed
284 by Al-kaolin aggregates, making it possible to define the multi threshold level for brightness.

285 Figure 2-b shows an example of the aggregate structure formed by kaolin (marked in yellow)
286 and alum gel (marked in red). Similar analysis was performed in several images and results
287 have shown that the kaolin effective cross-sectional area is about 20 % of the total cross-
288 sectional area average. This made it possible to calculate the floc effective bulk density (ρ_f) of
289 $1,300 \text{ kg}\cdot\text{m}^{-3}$, based on relative quantities of Alum and kaolin within flocs. This finding is in
290 agreement with the results reported by Tambo and Watanabe (1979).



291

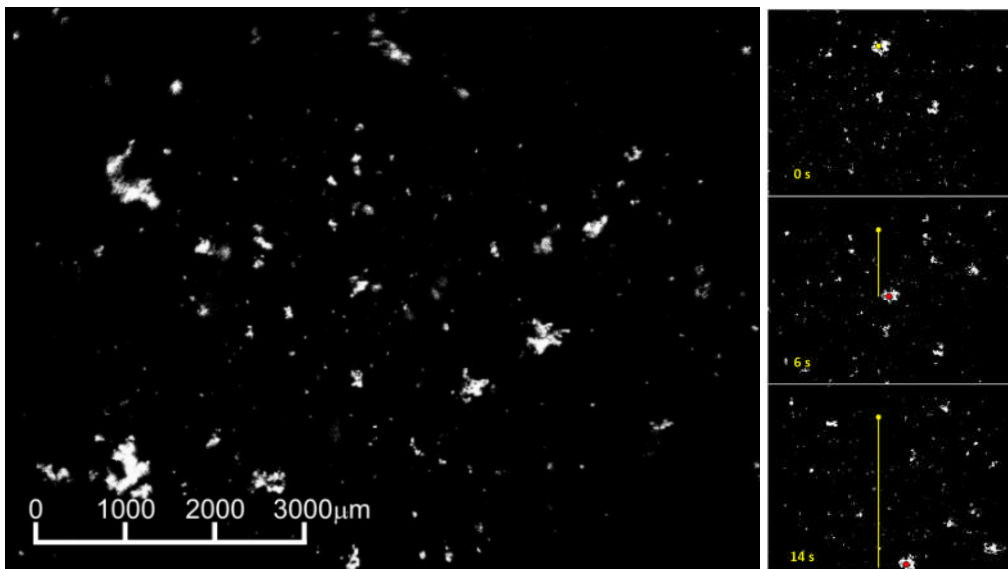


292

293 Figure 2 – (a) Volume frequency distribution for dry kaolin powder used as primary particles
294 in the tests. The image in the right hand side of (a) refers to (SEM) taken as sample from
295 kaolin, as dry material. (b) Example of aggregate formed by kaolin and alum. Background
296 shows original image and front image shows kaolin highlighted in yellow within aggregate
297 structure of Alum in red.

298

299 Figure 3-a shows an example of one of the images used to characterize the aggregate, 5
300 minutes after flocculation had finished, i.e. during sedimentation. It is clear that the
301 morphology of the aggregate formed after flocculation cannot be explained by Euclidean
302 geometry and by the assumption of impermeable spheres. Furthermore, the asymmetrical
303 shape of aggregates can also be observed. The shape and the existence of voids inside the floc
304 may alter the effective density of the aggregate, influencing the terminal velocity of the floc.
305 Figure 3-b shows an example of one of the 118 tracked Al-kaolin aggregates monitored
306 during sedimentation.
307 Figure 4 shows the value of two and three-dimensional fractal dimensions for the entire
308 population of aggregates in the experimental data, according to Equations 2 and 3. The slopes
309 of the fitted trend lines, i.e. 2.35 and 1.50, represent the three and two-dimensional fractal
310 dimension for the set of aggregates respectively, which is compatible with the findings of
311 Chackraborti et al. (2003) and Johnson and Logan (1996). A structure factor of 0.74 was
312 determined from the best fit line intercept of Figure 4-b.



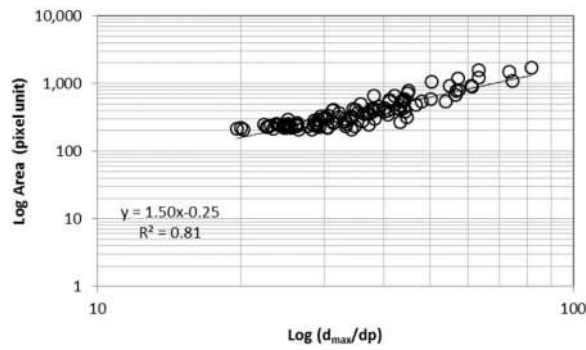
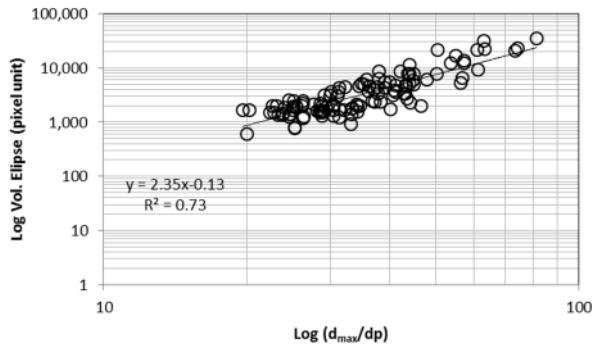
313

314

a)

b)

315 Figure 3 – (a) An example of Al-kaolin fractal aggregates image obtained after the
 316 flocculation at 15 x magnification; and (b) example of fractal aggregate tracking during
 317 sedimentation for frames extracted at 0, 6 and 14 s, 4 x magnification.



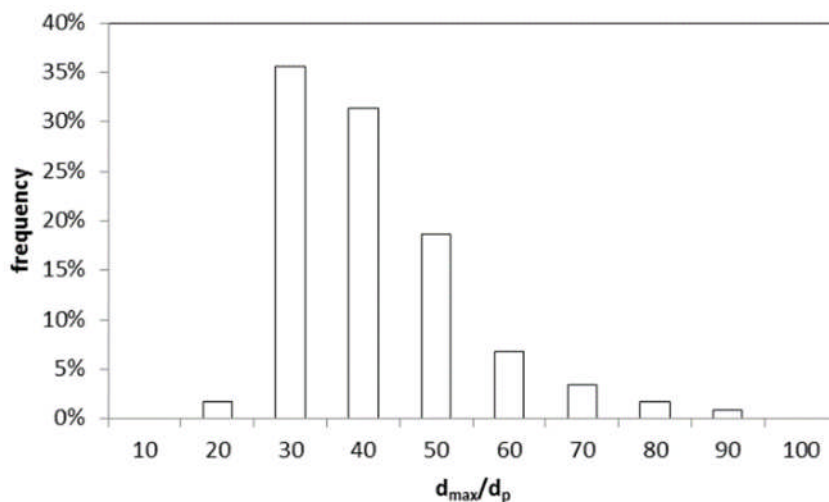
320 **a)**

b)

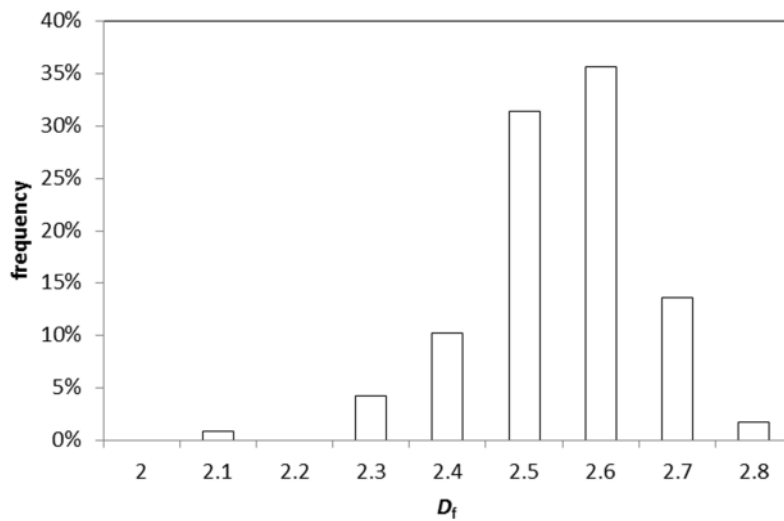
321 Figure 4 - Fractal dimension obtained from image analysis based on the set of aggregates, (a)
 322 three-dimensional fractal dimension (D_{fp}) and (b) two-dimensional fractal dimension (D_{fp}'),
 323 calculated for the entire population of aggregates by the slope of *Log-Log* plot. Moruzzi et al.
 324 (2020).

325 The frequency distribution of the dimensionless aggregate sizes, i.e. the d_{max}/d_p ratio, can be
 326 observed in Figure 5. It can be seen that 86% of the measured aggregates were within the
 327 range of $30.0 \leq d_{max}/d_p \leq 50.0 \pm 2.2$, that is, their longest lengths were between 30 and 50
 328 times larger than the mean size of primary particles of kaolin (d_{50} of 10 μm). The distribution

329 of the fractal dimension, D_f , as determined individually for the aggregates, can be seen in
 330 Figure 6. Almost 70% of the measured D_f were within in the interval of $2.60 - 2.70 \pm 0.02$,
 331 which deviates from the 2.35 fractal dimension calculated from the entire population of
 332 aggregates (Figure 4-a). However, there is no consensus on what approach yields the most
 333 accurate estimates of fractal dimensions (Chakraborti et al., 2000). Later discussion in this
 334 paper will show how these two approaches relate to each other. Sphericity (Ψ) of ordinary Al-
 335 kaolin aggregates was determined individually, and found to be around 0.58 ± 0.02 for all 118
 336 measured average aggregates, based on the encased ellipsoid (E), with average aspect ratios of
 337 about 2.7. Such results indicate irregular geometries, which cannot be adequately explained
 338 by regular plane geometry, and these findings agree with those presented by Vahedi and
 339 Gorczyca (2012).



340
 341 Figure 5 - Discrete distribution of dimensionless aggregate sizes, expressed by the ratio
 342 d_{max}/d_p .

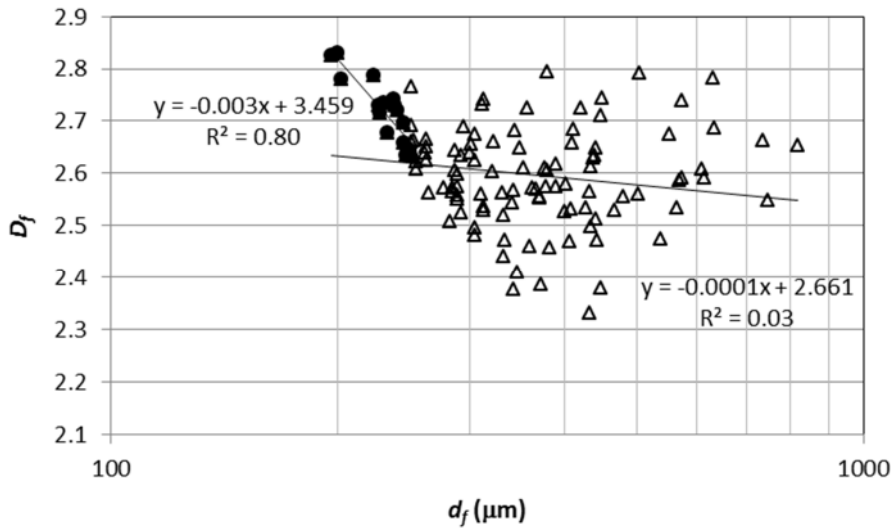


343

344 Figure 6 - Discrete distribution of the fractal dimension (D_f) determined by individual
 345 aggregates.

346 In Figure 7, the three-dimensional fractal dimension is plotted against the longest length of
 347 fractal aggregate size. It is clear that shape is independent of aggregate size for the entire
 348 population of large aggregates (triangles in Figure 7), i.e. there is no dominant fractal
 349 dimension for large Al-kaolin aggregates. This is in agreement with the findings of Jiang and
 350 Logan (1991) who identified an overlap in fractal dimension for aggregates formed from
 351 Brownian motion and differential sedimentation, despite the different aggregates sizes in
 352 which those mechanisms are likely to be dominant. The results from Figure 7 are in
 353 agreement with the findings presented by Vahedi and Gorczyca (2012) who considered a non-
 354 linear behaviour for D_f and floc size, and reinforced that the variety of aggregation
 355 mechanisms, kinetics (aggregation and breakage) and the sort of primary particles are some of
 356 the possible reasons why flocs with the same size may exhibit many different structures and
 357 different fractal dimensions. However, a closer look at particular and smaller range of
 358 aggregates, ranging from 180 to 300 μm (black circles in Figure 7), allows the identification

359 of a linear relationship between D_f and size, which is in accordance with the data presented by
 360 Vahedi and Gorczyca (2014). Furthermore, Vahedi and Gorczyca (2012) have also shown
 361 greater dispersion of D_f for large flocs, with D_f varying from 2.3 to 2.9 for floc size of 200
 362 μm , for example.



363
 364 Figure 7 – Variation of the three-dimensional fractal dimension calculated individually with
 365 the longest length of fractal aggregate size. Triangles (Δ) refer to the entire population of large
 366 aggregates and black circles (\bullet) refer to subset of longest length between 180 and 300 μm .

367
 368 Figure 8-a shows the values of density (ρ) determined from the fractal dimensions per
 369 aggregate (D_f), using Equation 5. Individual aggregate densities vary from 1,020 to 1,140
 370 $\text{kg}\cdot\text{m}^{-3}$ for D_f within the range of $2.3 \leq D_f \leq 2.7$ and size range in the interval $20 \leq d_f/d_p \leq 80$.
 371 The density of the aggregates, with an average of $1,068 \pm 4 \text{ kg}\cdot\text{m}^{-3}$, seems to slightly vary
 372 with the d_f/d_p ratio and also depends, to a lesser extent, on the fractal dimension (D_f). The
 373 aggregate densities scatter in a wide range of D_f and they seem to be slightly dependent on
 374 size for large aggregates. Hence, several densities can be found for the same size, as also

375 shown when the D_f values are calculated individually, which is in agreement with the results
 376 presented by Vahedi and Gorczyca (2012). The reason for this probably lies in the fact that
 377 the shape and compactness of the flocs also have an influence on density. Thus, for Al-kaolin
 378 aggregates in the range of 180.0 to $816.0 \pm 2.2 \mu\text{m}$, there are a variety of shapes and densities,
 379 which are independent in their size only, when aggregates are considered individually.
 380 However, a linear behaviour can be observed, in the range of 180 to $300 \mu\text{m}$, which is in
 381 accordance with previous analysis and suggests that a large variation is expected for bigger
 382 aggregates.

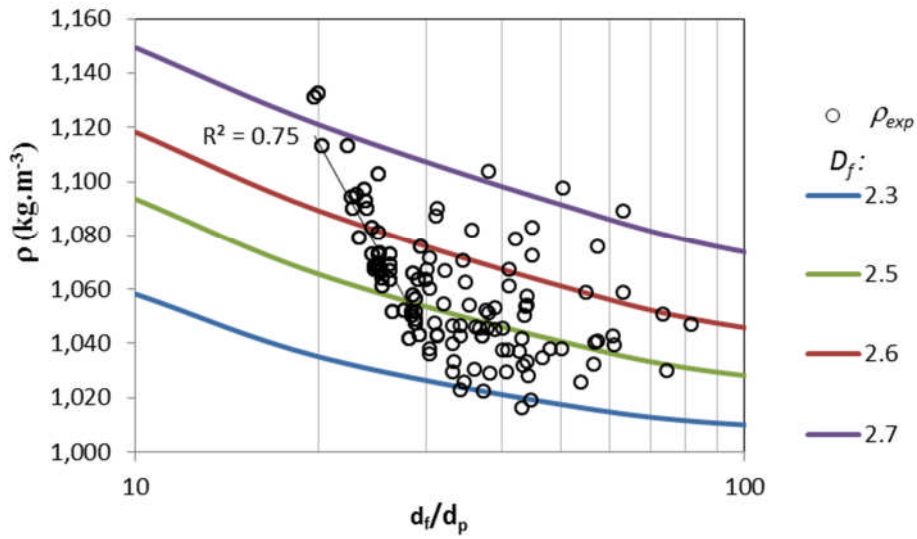
383 Another analysis for the density and size relationship can be performed by rearranging
 384 Equation 5, so that results can be expressed in the form of Equation 10, presented by Gregory
 385 (1997) for the entire population of flocs. In this case, Equation 5 transforms to Equation 11,
 386 written in *Log-Log* format.

$$387 \quad \rho_f = B d_f^{-y}, \text{ for } y = 3 - D_f \quad (10)$$

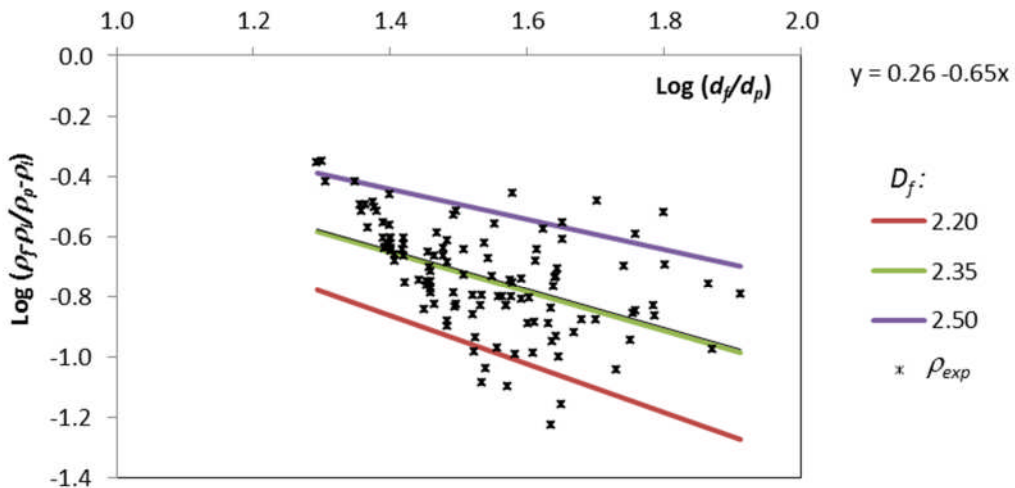
$$388 \quad \text{Log} \left(\frac{\rho_f - \rho_l}{\rho_p - \rho_l} \right) = \text{Log} b - y \text{Log} \left(\frac{d_f}{d_p} \right) \quad (11)$$

389

390 Figure 8-b was plotted using Equation 11 applied to the same data used in Figure 8-a. It is
 391 clear that the entire population of aggregates behave as expected by Gregory (1997), and from
 392 the best fit line (in green) it is possible to determine that y of Equation 11 equals to 0.65 . This
 393 results in D_f of 2.35 , which is indeed the three-dimensional fractal dimension for the entire
 394 population of flocs, and agrees with the fractal dimension previously calculated using
 395 Equation 2 and shown in Figure 4-a. Therefore, 3D fractal dimension for the whole
 396 population of aggregates can be derived from the density and size relationship determined
 397 from fractal dimension calculated individually.



398 (a)



399 (b)

400 Figure 8 – (a) Changes of calculated density (ρ), determined from fractal dimension of
 401 individual aggregates, with the d_f/d_p ratio, using Equation 5. The coloured curves represent the
 402 calculated density for three-dimensional fractal dimensions (D_f) of 2.3, 2.5, 2.6 and 2.7. (b)
 403 *Log-log* plot of density measurements against the d_f/d_p ratio using Equation 11.

404 3.2 Settling velocity

405 Figure 9-a, presents the discrete distribution of aggregate velocities during sedimentation. In
406 general, the measured velocities of the selected 118 slow-settling Al-kaolin aggregates ranged
407 from 1.0×10^{-4} to $2.0 \times 10^{-3} \pm 3 \times 10^{-5} \text{ m} \cdot \text{s}^{-1}$. It was verified that 75% of the aggregates had
408 settling velocities in the range of 3.0×10^{-4} to $5.0 \times 10^{-4} \pm 3 \times 10^{-5} \text{ m} \cdot \text{s}^{-1}$ and 10% of the
409 aggregates settled at a velocity greater than $1.0 \times 10^{-3} \text{ m} \cdot \text{s}^{-1}$, which is equivalent to the settling
410 rate expected for a sphere of $170 \mu\text{m}$ size and ρ_f of $1,300 \text{ kg} \cdot \text{m}^{-3}$, far lower than the average
411 floc size of $360 \mu\text{m}$. However, 15% of the aggregates settled with velocity of less than
412 $2.0 \times 10^{-4} \text{ m} \cdot \text{s}^{-1}$. On average, the settling velocity for Al-kaolin aggregates was found to be
413 $3.5 \times 10^{-4} \text{ m} \cdot \text{s}^{-1}$ (i.e. $30 \text{ m} \cdot \text{day}^{-1}$), which is slower than common values adopted for hydraulic
414 loading rate of conventional settling tanks (around $4.6 \times 10^{-4} \text{ m} \cdot \text{s}^{-1}$, i.e. $40 \text{ m} \cdot \text{day}^{-1}$), explaining
415 why those large aggregates can be dragged out of sedimentation tanks. Also, settling
416 velocities here measured for Al-kaolin aggregates were considerably lower than those
417 measured by Vahedi and Gorczyca (2012), probably due to the denser primary material
418 (lime) used to form flocs, and much lower than those measured by Johnson et al. (1996), who
419 have studied smaller flocs sizes.

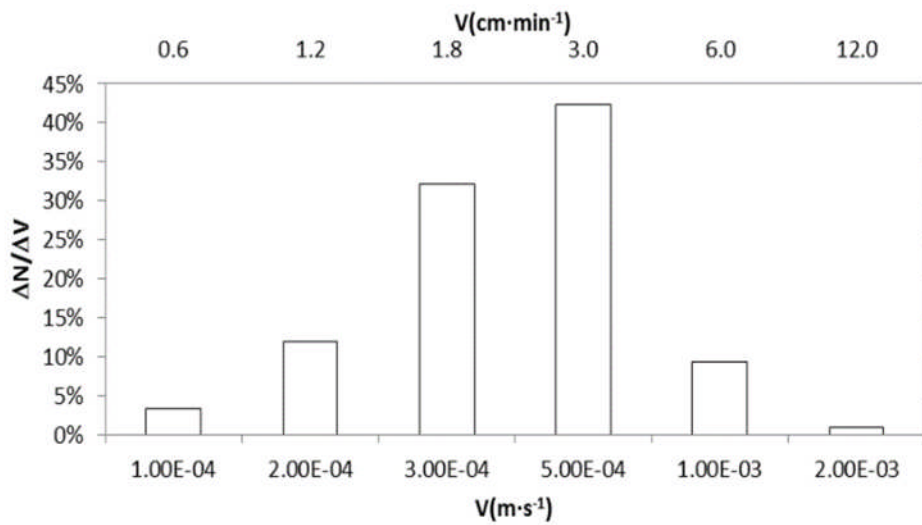
420 Floc-floc collision was not observed during these experiments, perhaps because it is less
421 likely to occur in low-concentration suspension, as also observed by Goula et al. (2008).
422 However, it must be highlighted that higher suspended particles can favour aggregate
423 collisions, potentially increasing floc size and changing settling velocities during
424 sedimentation (Zhao et al., 2018; Shi et al., 2019).

425 The intrinsically slow-settling Al-kaolin aggregates investigated here certainly influences the
426 average values presented and contributes to the observed difference, once the focus was on
427 slow settling of large aggregates. Furthermore, aggregates have far from the ideal

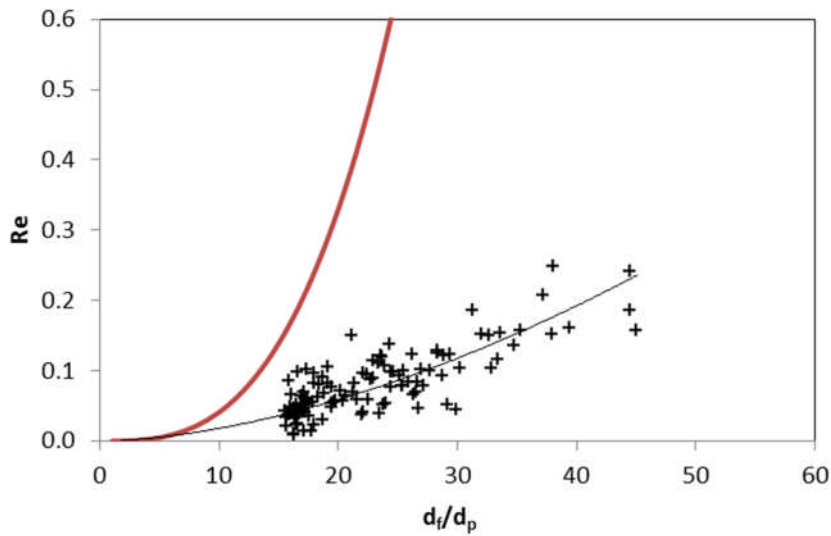
428 impermeable spheres assumed for the Stokes' law simplification and this can also explain
429 such behaviour, as previously mentioned.

430 It is clear from Figure 9-b that Re values calculated from experimental measurements are
431 lower than those back-calculated from Stokes' law for spheres using a drag coefficient (C_d)
432 equal to $24/Re$. The overestimation for Re of large latex spheres ($< 160 \mu\text{m}$) was also
433 observed by Chakraborti and Kaur (2014), who concluded that greater deviation may be
434 expected as particle size increases, probably due to an increase in drag coefficient with
435 relatively large-particle Re numbers compared to the value used in Stokes' law. Here, those
436 effects on drag coefficient are likely more significant, as fractal aggregates were calculated
437 individually. Furthermore, fractal aggregates formed from kaolin primary particles arise from
438 more complex mechanisms, such as hydrodynamic effects, than those formed from latex
439 spheres, and so, the observed deviation for Re is even higher than those reported by
440 Chakraborti and Kaur (2014).

441 Figure 9-b also shows that most Re values were less than 0.2, thus making the Stokes' law
442 modified by a shape factor applicable as presented by Wang (1988) and also used by Vahedi
443 and Gorczyca (2012).



444 (a)



445 (b)

446

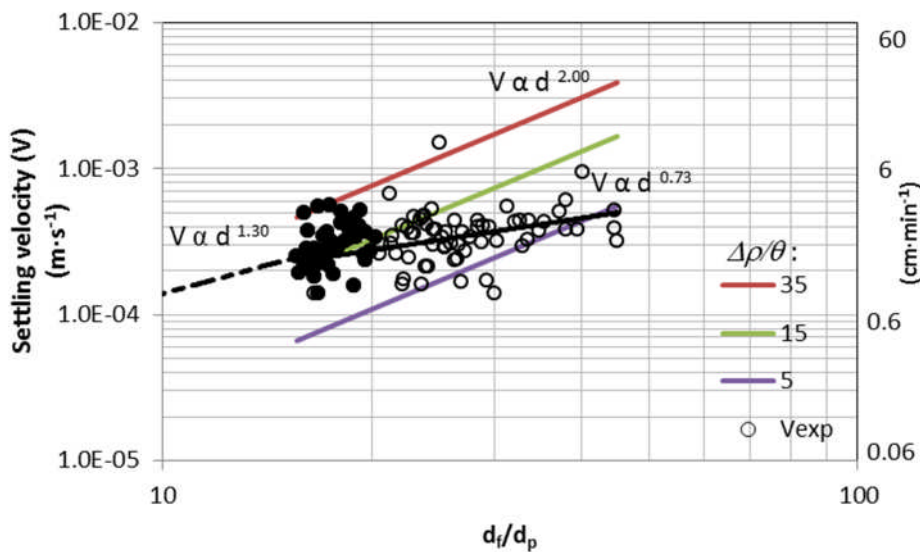
447 Figure 9 – (a) Discrete distribution for settling velocities of Al-kaolin aggregates with
 448 measured average size ranging from 150 to $450 \pm 1.3 \mu\text{m}$ and (b) calculated dimensionless
 449 Reynolds number (Re) from experiments and back-calculated Re values from Stokes' Law
 450 (continuous red line) against dimensionless aggregate size (d_f/d_p).

451

452 Figure 10 shows the experimental results, based on settling velocities (V) and normalised
 453 sizes (d_f/d_p). Coloured continuous lines describe the relation $V \approx (d_f/d_p)^2$, limited by the range
 454 of $5 \leq \Delta\rho/\theta \leq 35 \text{ kg}\cdot\text{m}^{-3}$, as Equation 7. The best fit lines in black refer to the minimum least
 455 square relation between aggregate velocity and size for two subsets of data: i) the entire set of
 456 data, drawn in continuous black line, where the slope is 0.73 ($V \approx (d_f/d_p)^{0.73}$); ii) the subset
 457 limited by the average size $100 \leq d \leq 200 \text{ }\mu\text{m}$, dashed line, where the slope is 1.30 ($V \approx$
 458 $(d_f/d_p)^{1.30}$).
 459 As observed above for fractal dimension and density, Al-kaolin aggregates with similar sizes
 460 can have different settling velocities. This is also in agreement with results reported by
 461 Vahedi and Gorzzyka (2012), who observed several settling velocities for one aggregate size.
 462 It is clear that experimental velocities of Al-kaolin aggregates, for average size (d) within 150
 463 to $450 \pm 1.3 \text{ }\mu\text{m}$, are predominantly (95%) encased within the limits of the lines $5 \leq \Delta\rho/\theta \leq 35$
 464 $\text{kg}\cdot\text{m}^{-3}$ (with slope of 2) using Equation 7, i.e. in the shape factor interval of $2 \leq \theta \leq 14$. A
 465 θ value equal to one would be expected for perfectly spherical-shape and impermeable
 466 aggregates, settling in accordance with Stokes' law for $\text{Re} < 1$, i.e. at drag coefficient of
 467 $24/\text{Re}$. This means that assumptions based on Stokes' law do not represent large Al-kaolin
 468 aggregates and settling velocities were over-predicted from 2 to 14 fold. Using the raw data
 469 published by Tambo and Watanabe (1979) for primary particles of clay-aluminium of $3.5 \text{ }\mu\text{m}$
 470 size, Bushell et al. (2002) found a drag and structure factor coefficient of 5.42, which derives
 471 from actual velocities lower than the Stokes' law prediction. Therefore, aggregates took on
 472 non-spherical shapes resulting in increased drag force and, consequently, had slow settling
 473 rates compared to spherical particles, as pointed out by Bushell et al. (2002).

474 According to the Stokes' law, the terminal velocity of settling aggregates is proportional to
 475 size squared, as given by Equation 7. Results of the analysis here have shown that density and
 476 fractal dimension can be related in a wide range of size, and therefore, multiple settling
 477 velocities were observed for a given floc size. For the entire population of the aggregates, the
 478 experimental velocity was found to vary with size to the power of 0.73 (black line in Figure
 479 10), which agrees with results presented by Tambo and Watanabe (1979), who found values
 480 between 0.5 and 1.0. However, for the average size range of 100 – 200 μm , the exponent for
 481 size is found to be 1.30 (dashed black line in Figure 10), in accordance to the exponent given
 482 by $D_f - 1$, presented by Vahedi and Gorczyca (2012).
 483 Therefore, it is important to note that the shape factor, used here to encase experimental
 484 values in the border limit of Stokes law's theoretical settling velocities, comprises a wide
 485 range of simplifications over porosity, permeability, size and shape. In general, results have
 486 shown that large fractal aggregates settled too slowly for Stokes' law to apply, and they may
 487 behave differently for the same size.

488



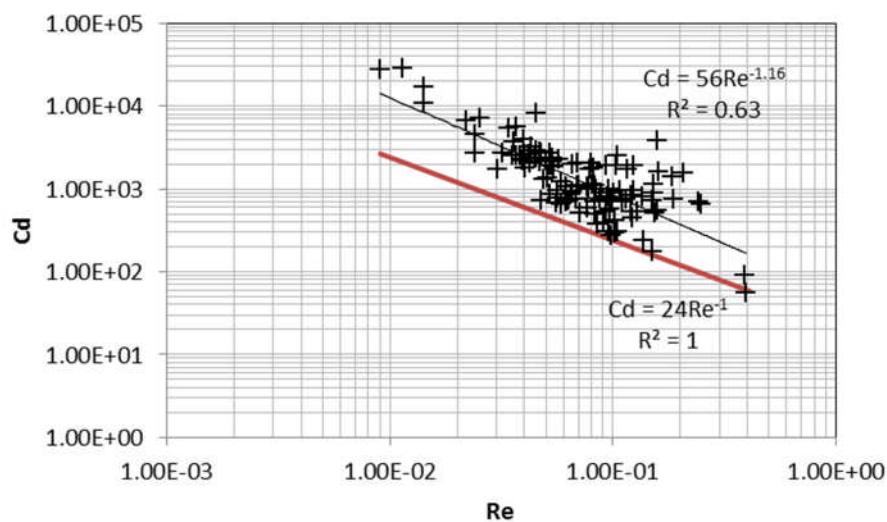
489

490 Figure 10 - Settling velocities for Al-kaolin fractal aggregates against dimensionless size
491 (d_f/d_p) . Coloured lines represent the limits of $\Delta\rho/\theta$ with $V \approx (d_f/d_p)^2$ from Equation 7. The
492 continuous black line represents the best fit for all measured data, and the dashed line is the
493 best fit for a subset of aggregate sizes.

494 Figure 11 shows the relationship between Reynolds number (Re) and drag coefficient (Cd),
495 determined from experimental results as described by Equations 6 and 8, respectively. The
496 inversely proportional relation between Cd and Re was confirmed, and a proportionality
497 constant of 56 was determined from the fitted line. Tambo and Watanabe (1979) calculated
498 the proportionality constant of 45 for Ψ of 0.80, confirming that deviation from sphericity
499 leads to a higher drag coefficient, despite the fact that both shape and orientation of
500 aggregates in flow affect results, as pointed by Bushell et al. (2002).

501 Results in Figure 11 also confirmed that for Al-kaolin aggregates, the impact of aggregation
502 of non-sphere primary particles of 10 μm median on the hydrodynamics resulted in the
503 increase of drag forces, when compared with perfect spheres of the same density settling
504 according to Stokes' law. The results, discussed in previous paragraphs, indicated that Al-
505 kaolin aggregates are asymmetrical, as the three-dimension fractal dimensions presented in
506 Figure 7 were found to be independent of size, or sphericity (Ψ) and far from spherical shape
507 ($\Psi = 0.58$) with aspect ratio of about 2.7. Therefore, these aggregates may spin or wobble as
508 they settle, as also mentioned by Bushell et al. (2002). Finally, the results have also confirmed
509 that permeability did not play an important role on high fractal dimension aggregates, as also
510 revealed by Gregory (1997) and Bushell (2002), especially as voids are potentially filled with
511 hydrolysed coagulant species (Vahedi and Gorczyca, 2014).

512 In general, the results presented here suggest that the settling velocity of Al-kaolin large
 513 fractal aggregates is influenced by shape, density, and porosity as well as size, and that these
 514 features may change the equilibrium configuration between the gravitation and drag forces
 515 acting on the particle during its sedimentation. Finally, it must be stressed that the high ionic
 516 strength water here studied (0.1 M NaHCO₃) means that electrical effects would be negligible
 517 and the diffusion of the large flocs would be insignificant. Therefore, sedimentation velocity
 518 is not remarkably affected by particle charge (Gregory, 1997), and for this reason, particle
 519 charge was not investigated in this study.



520
 521 Figure 11 – Drag coefficient determined from experimental results. The black line is the best
 522 fit to experimental data whilst the red line was determined using drag coefficient (C_d) of
 523 $24/Re$ for $Re < 1$.

524
 525 **Conclusions**

526 In this paper, large slow-settling Al-kaolin fractal aggregates' features and Re numbers were
 527 determined using a non-intrusive image technique, and were found to differ from values

528 reported in other studies on obtaining aggregate characteristics from sedimentation. This
529 approach avoids the assumption of an explicit relationship between drag coefficient and
530 aggregate size.

531 It was proposed that the 3D fractal dimension for the whole population of aggregates can be
532 derived from the density and size relationship, using fractal dimensions calculated
533 individually, by means of the rearrangement of the mass balance equation.

534 It was found that Al-kaolin large aggregates may exhibit different settling velocities for the
535 same size and the velocities based on Stokes' law do not accurately represent large
536 aggregates, where settling velocities were over-predicted from 2 to 14 fold.

537 The impact of aggregation of non-spherical kaolin primary particles of 10 μm median on Al-
538 kaolin aggregate hydrodynamics results in an increase of the drag force, when compared with
539 perfect spheres of same density, settling in accordance with Stokes' law. The inversely
540 proportional relation between C_d and Re was confirmed, and a proportionality constant of 56
541 ($C_d = 56/Re$) was determined graphically, compared to the Stokes' relation of $24/Re$.

542 Therefore, it was found that Al-kaolin large, heterodisperse and multi-shape aggregates can
543 settle sufficiently slowly for Stokes-type expressions to apply. The asymmetrical shape and
544 the size-density relatively independence here verified for large aggregates play an important
545 role on Al-kaolin large aggregates with slower settling velocities. Evidently, more research is
546 needed in order to better understand the complex mechanisms behind the settling rates of
547 large fractal aggregates with slow settling velocities. For example, those mechanisms
548 referring to the effects of collision and restructuring during sedimentation and flocs alignment
549 with flow direction should be investigated.

550

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556 **References**

557 Aparício, P., Pérez-Bernal, J. L., Galán, E., Bello, A., 2004. kaolin fractal dimension.

558 Comparison with other properties. Clay Minerals. v. 39, p. 75-84.

559 Bushell, G. C., Yan, Y. D., Woodfield, D., Raper, J. U. D. Y., Amal, R. O. S. E., 2002. On

560 techniques for the measurement of the mass fractal dimension of aggregates. Advances
561 in Colloid and Interface Science, 95(1), 1-50.

562 CETESB, 2015. Qualidade das águas superficiais no estado de São Paulo 2014. Available at:

563 <https://cetesb.sp.gov.br/aguas-interiores/wp->

564 [content/uploads/sites/12/2013/11/Cetesb_QualidadeAguasSuperficiais2014_ParteI_vers](https://cetesb.sp.gov.br/aguas-interiores/wp-content/uploads/sites/12/2013/11/Cetesb_QualidadeAguasSuperficiais2014_ParteI_vers%C3%A3o2015_Web.pdf)

565 [%C3%A3o2015_Web.pdf](https://cetesb.sp.gov.br/aguas-interiores/wp-content/uploads/sites/12/2013/11/Cetesb_QualidadeAguasSuperficiais2014_ParteI_vers%C3%A3o2015_Web.pdf)

566 Chakraborti R.K., Kaur J., 2014. Noninvasive Measurement of Particle-Settling Velocity and

567 Comparison with Stokes' Law. Journal of Environmental Engineering.

568 5;140(2):04013008.

569 Chakraborti, R.K., Atkinson, J.F., Van Benschoten, J.E., 2000. Characterization of alum floc

570 by image analysis, Environmental Science and Technology 34, 3969–3976.

571 Chakraborti, R.K., Gardner, K.H., Atkinson, J.F., Van Benschoten., J.E., 2003. Changes in

572 fractal dimension during aggregation. Water Res, 37, 873–883.

573 Eman N, A., Suleyman A, M., Hamzah M, S., Md Zahangir, A., & Mohd Ramlan M, S.,
574 2010. Production of natural coagulant from *Moringa oleifera* seed for application in
575 treatment of low turbidity water. *Journal of Water Resource and Protection*, 2010.

576 Goula, A. M., Kostoglou, M., Karapantsios, T. D., & Zouboulis, A. I., 2008. A CFD
577 methodology for the design of sedimentation tanks in potable water treatment: Case
578 study: The influence of a feed flow control baffle. *Chemical Engineering Journal*,
579 140(1-3), 110-121.

580 Gregory, J., 1997. The density of particle aggregates. *Water Science and Technology*, v. 36, n.
581 4, p. 1-13.

582 Jarvis, P., Jefferson, B., Gregory, J., Parsons, S.A., 2005. A review of floc strength and
583 breakage. *Water Res.*, 39, 3121–3137.

584 Jiang Q, Logan B.E., 1991. Fractal dimensions of aggregates determined from steady-state
585 size distributions. *Environmental Science & Technology*, 25(12):2031-8.

586 Johnson, C.P., Li, X., Logan, B.E., 1996. Settling velocities of fractal aggregates. *Environ.*
587 *Sci. Technol.*, 30, 1911-1918.

588 Johnson, M., Peakall, J., Fairweather, M., Biggs, S., Harbottle, D., & Hunter, T. N., 2016.
589 Characterization of multiple hindered settling regimes in aggregated mineral
590 suspensions. *Industrial & Engineering Chemistry Research*, 55(37), 9983-9993. ISSN
591 0888-5885.

592 Li, T., Zhu, Z., Wang, D., Yao, C., Tang, H., 2006. Characterization of floc size, strength and
593 structure under various coagulation mechanisms. *Power Technology*, 168, 104-110.

594 Li, X. Y., Zhai, X. D., Chu, H. P., & Zhang, J. J., 2008. Characterization of the flocculation
595 process from the evolution of particle size distributions. *Journal of Environmental*
596 *Engineering*, 134(5), 369-375.

597 Liu, Z., Wei, H., Li, A., & Yang, H., 2019. Enhanced coagulation of low-turbidity micro-
598 polluted surface water: properties and optimization. *Journal of environmental*
599 *management*, 233, 739-747.

600 Moruzzi, R. B., da Silva, P. G., Sharifi, S., Campos, L. C., & Gregory, J., 2019. Strength
601 assessment of Al-Humic and Al-kaolin aggregates by intrusive and non-intrusive
602 methods. *Separation and Purification Technology*, 217, 265-273.

603 Moruzzi, R. B., de Oliveira, A. L., da Conceição, F. T., Gregory, J., & Campos, L. C., 2017.
604 Fractal dimension of large aggregates under different flocculation conditions. *Science of*
605 *the Total Environment*, 609, 807-814.

606 Moruzzi, R.B., Bridgeman, J. and Silva, P.A.G., 2020. A combined experimental and
607 numerical approach to the assessment of floc settling velocity using fractal geometry.
608 *Water Science and Technology*.

609 Moruzzi, R.B., Silva, P. A. G., 2018 Reversibility of Al-kaolin and Al-Humic aggregates
610 monitored by stable diameter and size distribution. *Brazilian Journal of Chemical*
611 *Engineering*, v. 35, n. 3, p. 1029-1038.

612 Oliveira, A.L.de, Moreno, P., Silva, P.A.G. da, Julio, M.D., Moruzzi, R.B., 2015. Effects of
613 the fractal structure and size distribution of flocs on the removal of particulate matter.
614 *Desalination and Water Treatment*. DOI:10.1080/19443994.2015.1081833. Taylor &
615 Francis, 1-12.

616 Raşa, M., & Philipse, A. P., 2004. Evidence for a macroscopic electric field in the
617 sedimentation profiles of charged colloids. *Nature*, 429(6994), 857-860.

618 Shi, Z., Zhang, G., Zhang, Y., He, T., & Pei, G. (2019). Modeling of Flocculation and
619 Sedimentation Using Population Balance Equation. *Journal of Chemistry*, 2019.

620 Swenson, H. A., 1965. A primer on water quality. US Department of the Interior, Geological
621 Survey.

622 Tambo, N., Watanabe, Y., 1979. Physical characteristics of flocs—I. The floc density
623 function and aluminium floc. *Water Research*, v. 13, n. 5, p. 409-419.

624 Tang, P., Greenwood, J., & Raper, J. A. 2002. A model to describe the settling behavior of
625 fractal aggregates. *Journal of colloid and interface science*, 247(1), 210-219.

626 Vahedi, A., Gorczyca, B., 2012. Predicting the settling velocity of flocs formed in water
627 treatment using multiple fractal dimensions, *Water Res.* 46, 4188 – 4194.

628 Vahedi, A., Gorczyca, B., 2014. Settling velocities of multifractal flocs formed in chemical
629 coagulation process. *Water Research*, v. 53, p. 322-328.

630 Wang, R. F. T., Brooks, N. H., Koh, R. C., 1988. Laboratory analysis of settling velocities of
631 wastewater particles in seawater using holography. *Environmental Quality*.

632 Wei, J. C., Gao, B. Y., Yue, Q. Y., & Wang, Y., 2010. Strength and regrowth properties of
633 polyferric-polymer dual-coagulant flocs in surface water treatment. *Journal of*
634 *Hazardous Materials*, 175(1-3), 949-954.

635 Yu, W., Gregory, J., Campos, L. C., & Graham, N., 2015. Dependence of floc properties on
636 coagulant type, dosing mode and nature of particles. *Water Research*, 68, 119-126.

637 Yukselen, M.A., Gregory, J., 2004. The reversibility of floc breakage, *Int. J. Mineral Process.*
638 73, 251–259.

- 639 Zbik, M.; Smart, St.C. R., 1998. Nanomorphology of kaolinites: comparative SEM and AFM
640 studies. *Clays and Clay Minerals*. v. 46. n.2, p. 153-160.
- 641 Zhao, J., Wang, A., Wei, L., Ge, W., Chi, Y., & Lai, Y. (2018). Effect of kaolin on floc
642 properties for reactive orange removal in continuous coagulation process. *Water*
643 *Science and Technology*, 78(3), 571-577.