Factors that Influence the Terminal Grading of Sands

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Nomenclature

B_r relative breakage (Hardin, 1985)

d fractal dimension

DBS Dogs Bay sand

DG decomposed granite

H_0 initial sample height

δ_s shear displacement

γ shear strain

φ' angle of shearing resistance

σ'_v vertical stress

Abstract

Coop et al. (2004) found that for a carbonate sand shearing to extremely large strains allowed the soil to reach a stable grading; findings also reported by others for a limited range of sands, typically sedimentary and often weak grained. This paper describes similar tests on a soil of very different geological origin, a weathered soil, that confirms similar patterns of behaviour. Tests investigating factors affecting the final grading for both soils indicated small effects of shearing velocity but significant effects of the initial grading, highlighting that the final fractal dimension is not a constant.

Keywords: Terminal Grading, Shearing, Granular soil. (Fractals, Sands, Shear strenght)

Introduction

The existence of a final grading has become a central assumption in the mechanics of coarse grained soils. In compression, McDowell & Bolton (1998) found that breakage caused the gradings to tend towards a fractal, plotting as a straight line on a log:log particle size distribution with a fractal dimension d of around 2.5. Coop et al. (2004) then investigated final gradings in a ring shear apparatus, finding a final fractal distribution with a slightly higher d of 2.57 that also corresponded to a cessation of volumetric compression. Einav (2007a, b) and Muir Wood (2008) moved to quantifying breakage, based on a final fractal grading, within their constitutive models.

Ultimate fractal distributions seemed to be supported by natural gradings of some less common soils that have undergone intense shearing in-situ, notably sub-glacial tills, debris flow materials and fault gouges (Sammis et al., 1987; Hooke and Iverson, 1995) and Altuhafi et al. (2011) found that a fractal glacial till did not undergo further breakage during either compression or shear. However, in each case the fractal dimensions were not constant. Further doubts about the immutability of the final grading arise from the observation that, in Coop et al. (2004), the final stable grading was different at each stress level, so it is not known what would have been achieved had the capacity of the apparatus permitted tests at larger stresses. They also only tested a uniform grading of one particularly weak grained carbonate sand, although in a different type of ring shear Sadrekarimi and Olson (2014) had reached terminal gradings for three sands with stronger grains. Their final gradings also depended on stress level, while the locus of the voids ratios at these terminal states plotted well below the critical state line from triaxial tests.

Other doubts about the fundamental nature of the ultimate grading arose from work on gap graded soils where in both compression and shear the knee in the grading prevented a simple fractal

developing (Zhang and Baudet, 2013; Miao and Airey, 2013; Zhang et al., 2017). Miao and Airey also identified that even for simpler gradings, original features were retained in the final grading.

This paper draws together unpublished data from tests using the same techniques as Coop et al. (2004) but which illustrate other factors that may influence the final particle breakage.

Apparatus and Experimental Procedures

The tests were carried out in a Bishop ring shear (Bishop et al. 1971), with 203mm OD, 152mm ID and 23mm sample height, modified for fast shearing so that new tests at 190mm/min could be compared with previous ones at 1.9mm/min. The principal advantages of this apparatus are that the shearing is at the mid-height and the loss in vertical stress due to friction on the side walls may be measured, by lifting the upper ring and measuring the downwards force. This is done in stages as the gap between the rings causes soil losses, limiting the strains that may be reached. As the sample height reduces so do the friction losses, so that even if the vertical stress is known accurately, which it is not in some ring shear designs, the changing friction means it is quite variable. For each test, a range of net vertical stresses is therefore given in Table 1 and tests must be compared in which the vertical stresses are not exactly the same, which is not ideal.

Tests were carried out on two sands; the Dog's Bay biogenic carbonate sand, DBS, used by Coop et al. (2004) and a Korean decomposed granite, DG (Lee and Coop, 1995). The DG is of Grade V (Geological Society, 1990) with a mineralogy dominated by quartz and feldspar, with smaller quantities of mica, smectite and kaolin. The DBS was tested from a variety of initial gradings while one grading of the decomposed granite was used, which is given in Fig.1. This grading is close to a fractal dimension of 2.65 applied over the size limits 0.3-2mm; Miao and Airey (2013) had tested a carbonate sand with an initial fractal grading limited between bounds

defined by the grading (the effects of applying bounds to fractal distributions are discussed below). The tests on the DBS were carried out at the same nominal stresses used by Coop et al., 100, 400 and 1000kPa, while for the DG, only 400kPa was used.

The more uniformly graded samples were created by pluviation through water and wet compaction was used for the better graded samples, typically in five layers, with gentle tamping to create the desired density, although in most cases the samples were made in loose states. The advantage of using a ring shear over a triaxial apparatus is that the very much larger strains permit the particle breakage to stabilise, while the strains imposed in a triaxial test are far from sufficient to reach such a state. However, the disadvantage is the much less uniform strains within the sample. Loose samples minimise the effects of strain localisation although Luzzani and Coop (2002) showed that even so there is localisation of strains around a central zone of about 5mm thickness. After each test, the soil from this central zone was collected and wet sieved; sedimentation was used in some cases to establish the fines distribution.

The Evolution of Grading in Decomposed Granite

The evolution of the grading curves for the DG is shown in Fig.1. Shear strain γ was calculated as:

[1] $$\gamma = \frac{\delta_s}{H_0}$$

where δ_s is the shear displacement and H_0 the initial height; because of localisation, this is only a nominal strain.

There is an evolution towards a unique final grading for strains of 17400% and above, with only minor changes after the test at 6910%. In the log:log plot, the fractal terminal grading d=2.57, determined by Coop et al. (2004) for the DBS at 1MPa nominal is indicated. However, the shape

of the final grading is more complex and cannot be perfectly fitted by a single fractal as Miao and

Airey (2013) highlighted and as Coop et al. found for the tests on DBS at stress levels of 100 and

400kPa. Miao and Airey also determined lower fractal dimensions for the fines which were not

measured here.

For consistency with the previous work, the particle breakage has been quantified with B_r,

defined by Hardin (1985) as the ratio of the area between the initial and final gradings curves to

the area above the initial curve, however the cut-off used here is of 63µm, rather than the 74µm

originally proposed. This is clearly not an ideal means of quantifying breakage as it ignores any

contribution to breakage below the cut-off. However, it allows the breakage to be quantified by a

single number and avoids any assumption about a terminal grading, the uniqueness of which is

challenged by the data presented in this paper.

The B_r data for the DG sheared at the slow speed is shown on Fig.2, along with the data of

Coop et al. (2004) for DBS. The asymptotes at zero strain for the 1MPa tests on DBS and the new

tests on DG are provided by oedometer tests. The DG reaches a final value of B_r, indicating the

cessation of breakage, at a strain of about 5000-6000%, similar to the DBS tests at similar stress

levels but different B_r values partly because of the different initial grading. Most previous work on

terminal gradings has been on weak bioclastic carbonate sands (e.g. Coop et al., 2004; Miao and

Airey, 2013), clastic sedimentary soils and basaltic glacial sediments (Sadrekarimi and Olson,

2014; Zhang et al., 2017) and these data confirm that similar behaviour is seen in a soil with a

weathered rock origin, despite the rather different breakage processes that these soils undergo. Lee

& Coop (1995) had shown that the DG is composed of aggregated particles of different minerals,

the clay minerals being attached to the non-clay minerals and Madhusudhan & Baudet (2014)

found that the clay minerals would only become detached and act as separate particles under

vigorous washing. Zhao et al. (2015) investigated in detail the breakage mechanisms of a similar DG, finding that the mode of breakage was rather dissimilar to other sands, being by means of separation of particles along mineral boundaries, splitting along the cleavage of minerals that had one, and fractures along pre-existing flaws such as micro-fissures. It is interesting that even with such different forms of breakage, the patterns of grading evolution in the DG are still similar to the DBS.

Mobilised Strength of the Decomposed Granite

Coop et al. (2004) found that for the DBS the mobilised angle of shearing resistance, φ', remained constant at all stress levels and for all shear strains. Similar data are shown for the DG in Fig.3, for the gap-open stages, when the stresses are accurately measured. The mobilised φ' could only be defined at small strains for the slow tests, where the denser sample DG7 has a peak strength. At larger strains, the data confirm that the mobilised φ' is insensitive to the intense breakage. The mobilised strength could be affected by both the grading and the distribution of particle shapes, although the shapes were not measured here. While Altuhafi & Coop (2011) found a small decrease of sphericity in their ring shear tests on a quartz sand, Miao & Airey (2013) found an increase of aspect ratio for their tests on a carbonate sand. Differences in evolution of shape for different sands might explain why no change of mobilised φ' was seen either in these DG tests or those on DBS by Coop et al. (2004) while Sadrekarimi & Olson (2010) saw a small increase of φ' that they believed corresponded to an increase of angularity in three predominantly quartz sands.

The Influence of Speed of Shearing

Most of the tests on the DG were carried out at the faster velocity of 190mm/min, but three were at 1.9mm/min. On Fig. 1, the gradings for slow shearing plot slightly higher than those of the fast, and this is confirmed in Fig.4(a) in terms of slightly higher B_r. The mobilised φ' on Fig.3 are insensitive to the shear rate. It was not possible to shear fast at a nominal stress of 1MPa, and so tests were carried out on the DBS at the two lower stresses. Figure 4(b) again only shows a small effect of velocity, and at both stress levels there is a lower breakage at low strains for fast shearing but a slightly higher final breakage. Miao and Airey (2013) concluded that speed did not make a difference to the breakage, but their fast and slow tests were carried out in two very different apparatus, while here, in otherwise identical tests it is clear that there are effects, although small and not consistent between different soil types.

The Influence of Initial Grading Curve

The final fractal of 2.57 given in Coop et al. (2004) was based on the final state of poorly graded DBS. These data are replotted in Fig.5(a) for test RS3 along with the fractal dimension of 2.57. The comminution limit is around 1µm for many rock minerals and this was confirmed for a soil by Vilhar et al. (2013). Miao and Airey (2013) followed Einav (2007a) in adapting the fractal distribution to accommodate the comminution limit, imposing a minimum particle size D_{min} within the grading, using Equation 2:

$$[2] \qquad \%\ \text{passing} = \frac{D^{(3-d)} - D_{min}^{(3-d)}}{D_{max}^{(3-d)} - D_{min}^{(3-d)}}$$

where D is the particle size. If this is used, then by trial and error the best fit fractal dimension d for RS3 becomes 2.68. The imposition of a comminution limit means that the fractal particle size

distribution must break down as that limit is approached, since on a log : log particle size distribution the curve cannot be straight, self-similarity must cease, and the curve must dive towards minus infinity. At larger particle sizes however, there is only a small influence of the limit so the curves using d=2.68 in Equation 2 or 2.57 without the limit are close. For clarity the distribution given by Equation 2 is referred to within this paper as a "limited fractal". In fact, even if a comminution limit should be expected, none of the distributions on Fig.5(a) could be represented by Equation 2 and a limiting size at 1μm, as used by Miao and Airey (2013) because the "limited fractal" grading plunges asymptotically towards that limit below about 0.02mm. This discrepancy may be the result either of deficiencies of the sedimentation method or a resolution issue of the Morphologi system of Miao and Airey. However, even at larger particle sizes above 0.02mm, it is the straight true fractal with d=2.57 that fits RS3 better than the slight curvature still caused at that size by Equation 2.

Interestingly, RS5 which was similar to RS3, at a nominal stress of 1MPa, but which through the vagaries of the friction correction had a slightly higher net stress also had a slightly higher grading. More significantly, a better graded sample, RS8, gave a noticeably higher grading curve, again a true fractal, but with a dimension of 2.67 assuming no comminution limit. Two more tests were carried out to examine the effects of initial grading. DBS19 started with the same initial grading as the mean final grading of RS3 and RS5. In contrast to the findings of Altuhafi et al. (2010), additional breakage did occur, although the final grading was similar to that of RS8. DBS18 was created at a grading higher than the final ones of RS3 and RS5, and breakage still occurred, resulting in a final grading of d=2.79, again fitted using a true fractal and assuming no limit (although the fractal is curved here because of the use of a linear % passing scale on Fig.5(b)).

Figure 6 shows an investigation of the effect of the absolute size of the initial grading. As Coop et al. (2004) had indicated, test RS7, with a finer grading, shows an almost parallel shift in the final grading compared to RS3 and RS5, although the fractal dimension (not shown on the figure) is slightly higher at 2.62. Most of the tests were carried out on loose samples, but DBS1-5 were on slightly denser samples. There seems to be an effect of initial density on the final grading, for example comparing DBS2 with RS7, which were sheared to similar strains. However, the comparison is slightly compromised by the higher friction in the dense samples, meaning that the vertical stress was lower and tests in a better controlled ring shear would be needed to confirm this. It is clear though that as the initial grading becomes coarser, the final grading does change shape from concave to convex in the top of the curve, when a linear scale is used.

Conclusions

Ring shear tests on a weathered soil have confirmed similar patterns of grading evolution to those seen by Coop et al. (2004) and others on carbonate sands and other sedimentary sands. The final grading could be approximated as a fractal with dimension 2.57 as for the DBS, even if the tests were at a lower stress level, but details of the grading departed from this. Changing the rate of shearing over two orders of magnitude only had a small effect on the breakage and with inconsistent effects between the two soils. There was no significant effect of shearing speed on the mobilised angle of shearing resistance, which also seemed insensitive to the continuing breakage. Tests on the DBS at different gradings and densities again confirmed that there was nothing fixed about a final fractal grading of dimension 2.57, and that the final grading was sensitive to the initial grading and possibly the sample density, with well graded samples reaching final gradings of significantly higher fractal dimensions. The lack of a final stable grading that applies even for one

soil, calls into question the definition of breakage used by Einav (2007a, b) and Muir Wood (2008) and may mean that, with all of its defects, a simpler definition like that of Hardin (1985) may be more appropriate.

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Table 1 Summary of tests conducted (Tests LC1-4 are DBS tests from Luzzani & Coop, 2002; Tests RS1-15 are DBS tests from Coop et al., 2004).

Test	Notes	Initial grading (μm)	v prior to shearing	Nominal / Net σ'_v (kPa)	Speed (mm/min)	Final γ (%)
LC1		300-425	2.58	1000 / ≈ 805	1.9	207
LC2		300-425	2.61	1000 / ≈ 805	1.9	730
LC3		300-425	2.57	1000 / ≈ 805	1.9	104
LC4		300-425	2.56	1000 / ≈ 805	1.9	251
RS1		300-425	2.51	1000 / ≈ 670	1.9	171
RS2		300-425	2.46	1000 / ≈ 670	1.9	2860
RS3		300-425	2.55	1000 / 650-660	1.9	11100
RS4		300-425	2.52	1000 / ≈ 670	1.9	1430
RS5		300-425	2.46	1000 / 740-860	1.9	11030
RS6		300-425	2.43	1000 / 750-820	1.9	2780
RS7	Finer grading	212-300	2.41	1000 / 750-850	1.9	2910
RS8	Well graded	63-425	1.96	1000 / 725-825	1.9	11710
RS9		300-425	2.50	400 / 250-280	1.9	10920
RS10		300-425	2.47	400 / 248-346	1.9	3350
RS11		300-425	2.46	400 / 283-375	1.9	13280
RS12		300-425	2.45	400 / 296-368	1.9	1180
RS13		300-425	2.59	400 / 288-386	1.9	26650
RS14		300-425	2.60	400 / 290-343	1.9	285
RS15		300-425	2.50	100 / 60-77	1.9	147000
RS16		300-425	2.79	100 / 62-70	1.9	9040
RS17		300-425	2.72	100 / 66-80	1.9	31700
RS18		300-425	2.68	100 / 78-94	1.9	23900
RS19		300-425	2.69	100 / 68-97	1.9	37500
DBS1	Denser, finer grading	150-212	1.91	1000 / 550-700	2.1	3688
DBS2	Denser, finer grading	212-300	2.64	1000 / 550-750	2.1	3826
DBS3	Denser	300-425	2.34	1000 / 625-680	2.1	3616
DBS4	Denser, coarser grading	425-600	2.40	1000 / 530-560	2.1	3111
DBS5	Denser, coarser grading	600-1180	2.18	1000 / 650-730	2.1	3103
DBS6		300-425	2.51	400 / 330-378	190	53146
DBS7		300-425	2.79	400 / 365-380	190	2695
DBS8		300-425	2.32	400 / 330-380	190	30030
DBS9		300-425	N/A	400 / 325-374	190	472
DBS10		300-425	2.55	100 / 62-80	190	50856
DBS11		300-425	2.10	100 / 60-90	190	15016
DBS12		300-425	2.42	100 / 60-90	190	12770
DBS13		300-425	2.13	100 / 60-90	190	27011
DBS14		300-425	2.39	100 / 65-90	190	53702

DBS15		300-425	2.77	100 / 53-61	1.9	153321
DBS16		300-425	2.48	100 / 61-92	190	113715
DBS17		300-425	N/A	100 / 80-88	190	75151
DBS18	Well graded	<63-425	2.26	1000 / 500-800	2.1	3246
DBS19	Well graded	<63-425	2.12	1000 / 500-800	2.1	3539
DG1		300-2000	2.86	400 / 300-340	190	34700
DG2		300-2000	2.93	400 / 320-330	190	17400
DG3		300-2000	2.92	400 / 320-345	190	6910
DG4		300-2000	2.94	400 / 330-345	190	3450
DG5		300-2000	2.89	400 / 320-325	190	37000
DG6	Denser	300-2000	2.75	400 / 320-335	190	46500
DG7	Denser	300-2000	2.72	400 / 300-430	1.9	44500
DG8		300-2000	2.90	400 / 300-310	190	430
DG9		300-2000	2.92	400 / 290-335	1.9	440
DG10		300-2000	2.89	400 / 290-335	1.9	6940

(a)
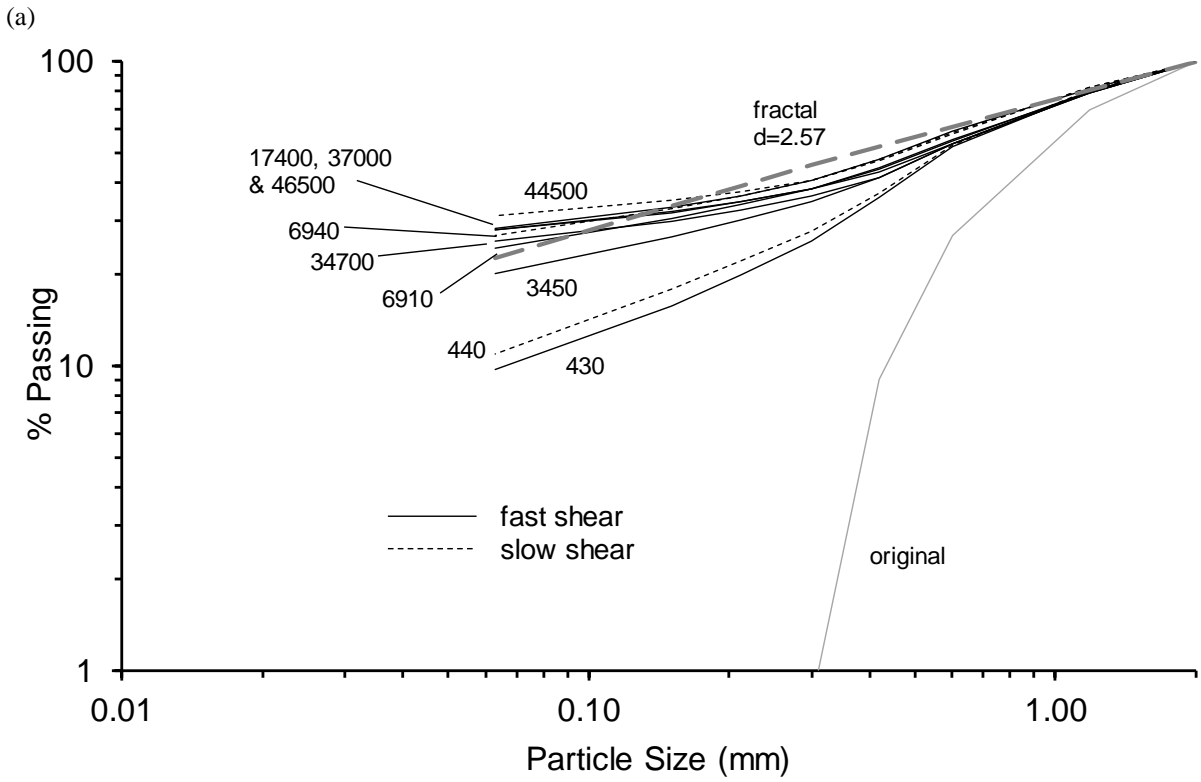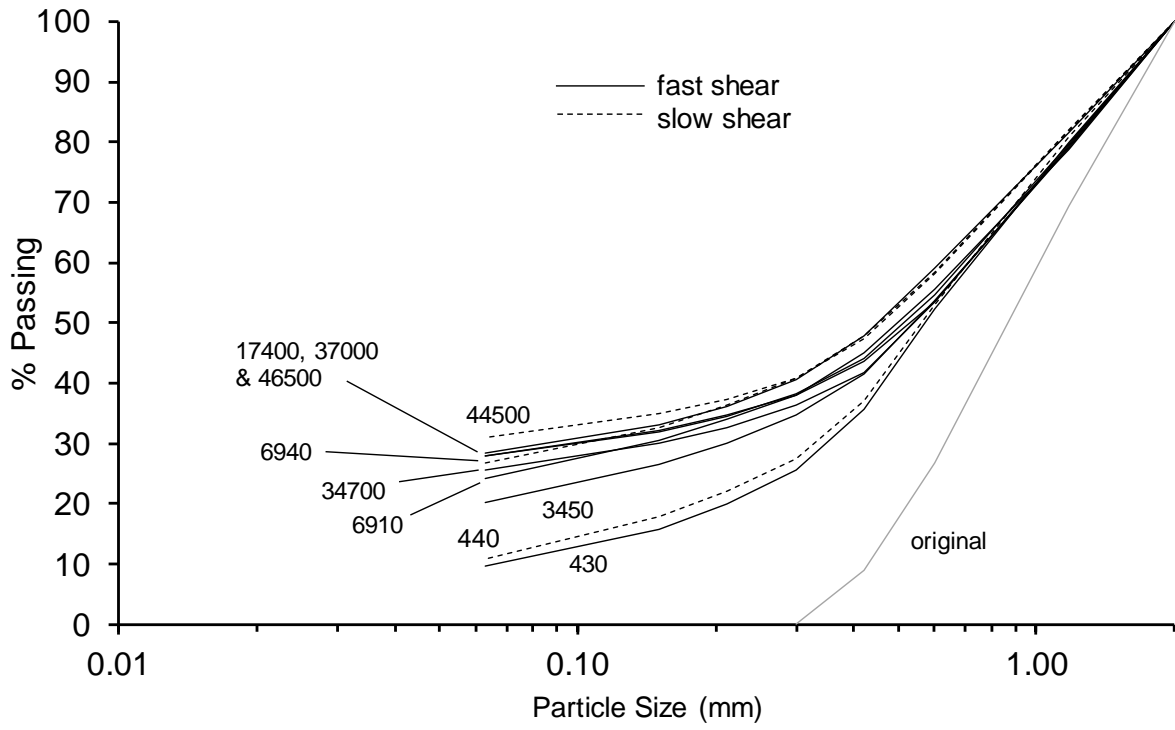
(b)

Fig.1 - Evolution of grading curves for decomposed granite with σ'$_v$ in range 290-430kPa (labels indicate final shear strain %)
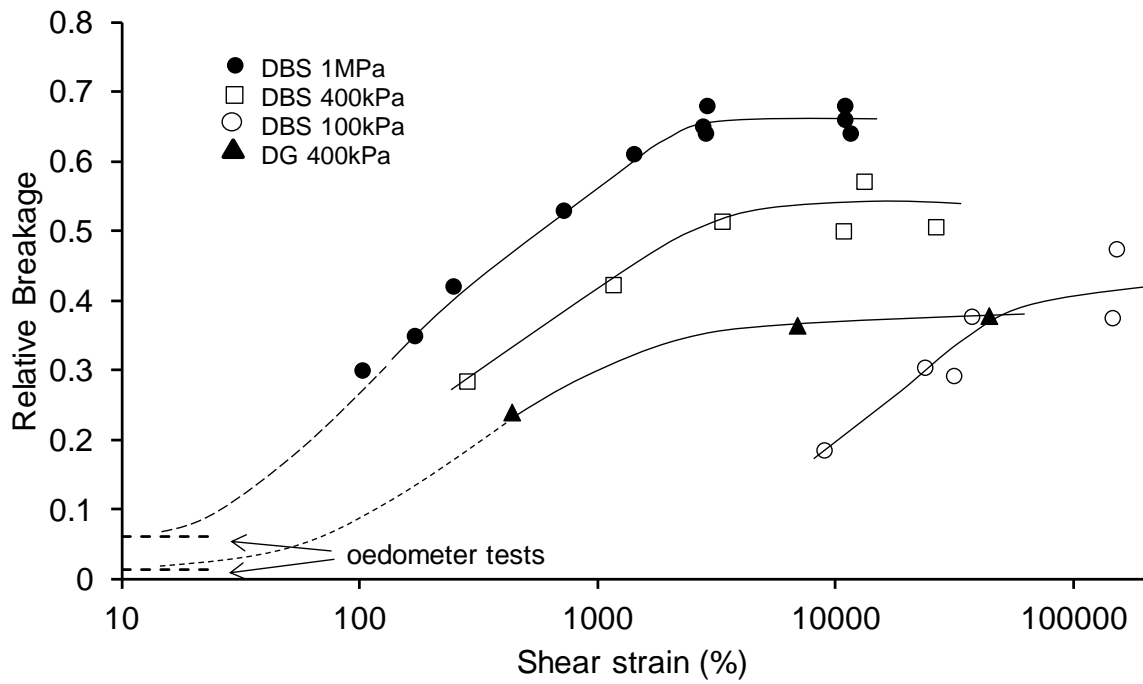
Fig.2 - Evolution of relative breakage for slow tests on decomposed granite (DG) at 400kPa and Dogs Bay sand at three stress levels (DBS data modified from Coop et al., 2004).
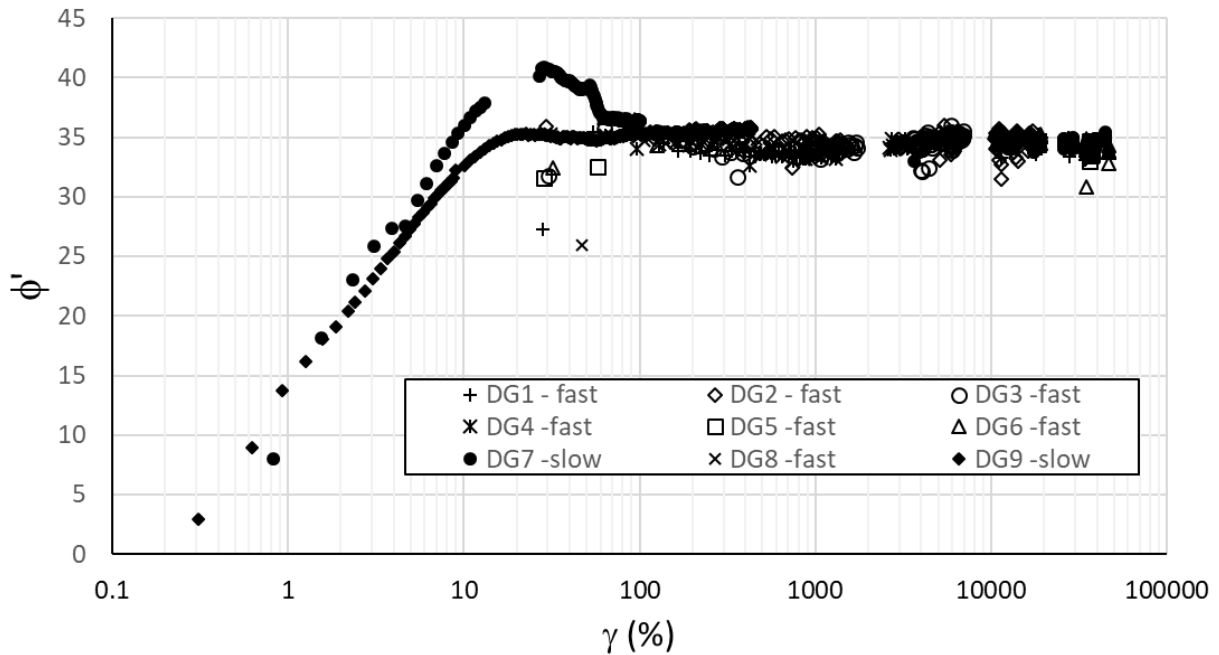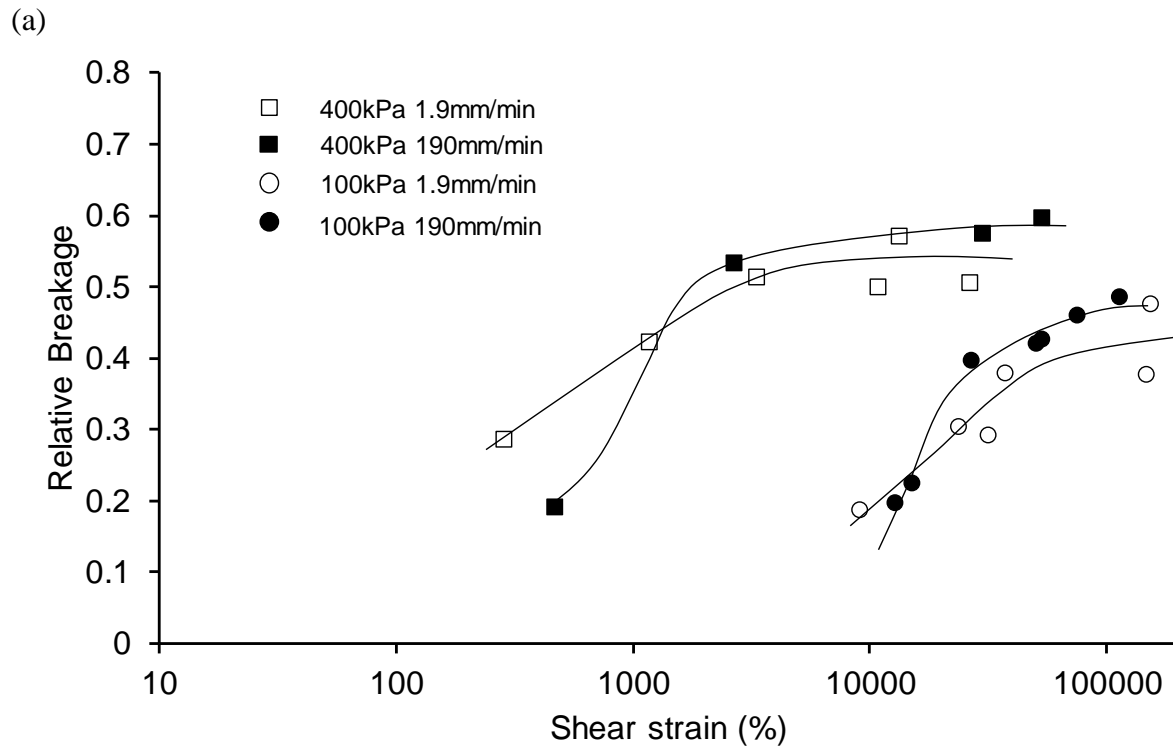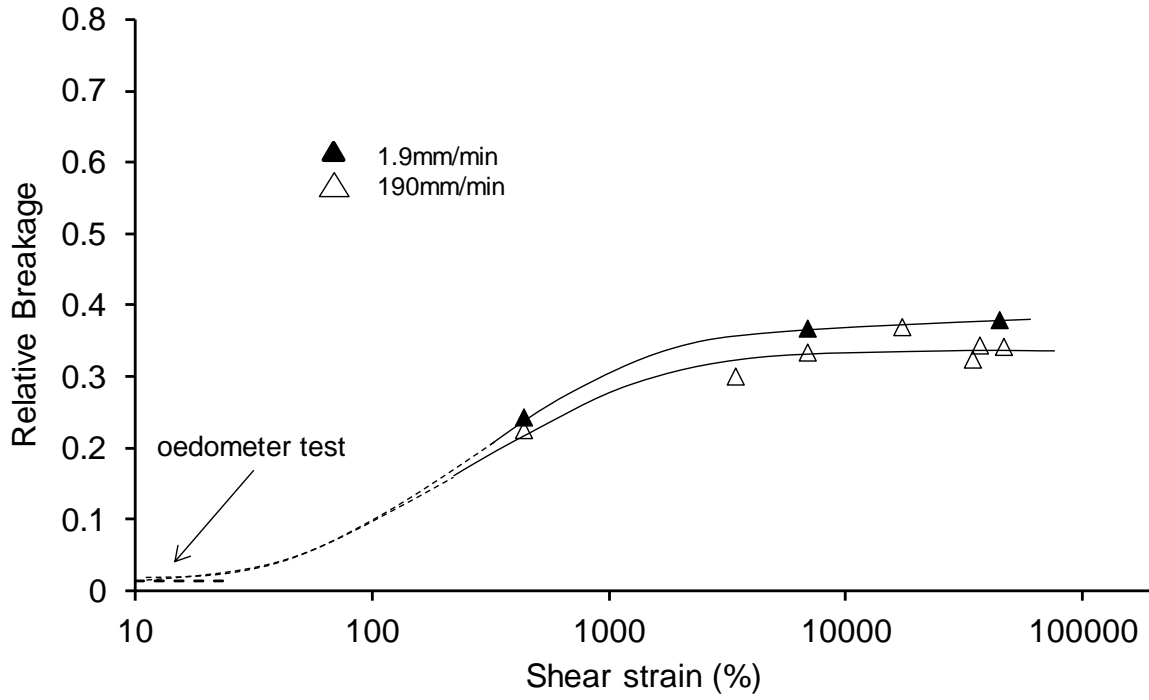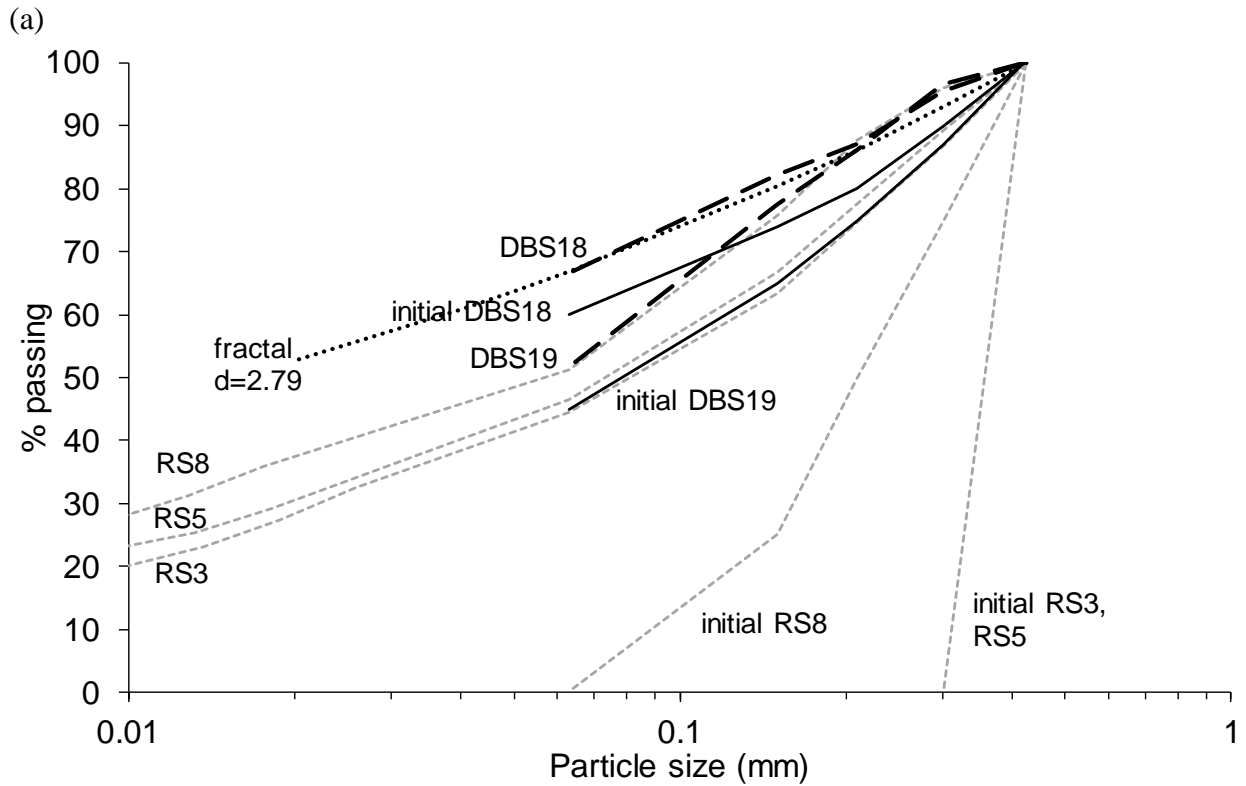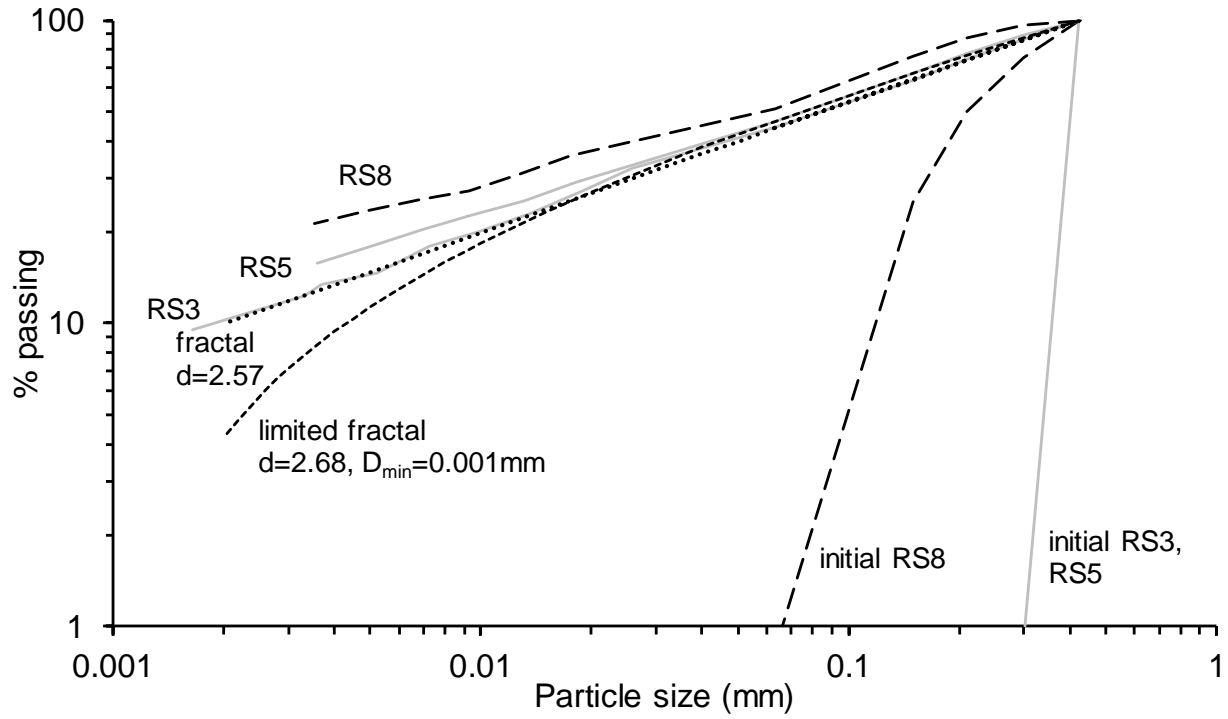

Fig.3 - Evolution of mobilised angle of shearing resistance with continued shearing.

(a)

(b)

Fig.4 - The influence of speed of shearing for (a) decomposed granite at 400kPa, (b) Dogs Bay sand (slow tests from Coop et al., 2004)

(a)

(b)

Fig.5 - Influence of initial grading on final grading for tests with nominal σ′v=1MPa (slow tests only, tests RS3, RS5 and RS8 replotted from Coop et al., 2004).
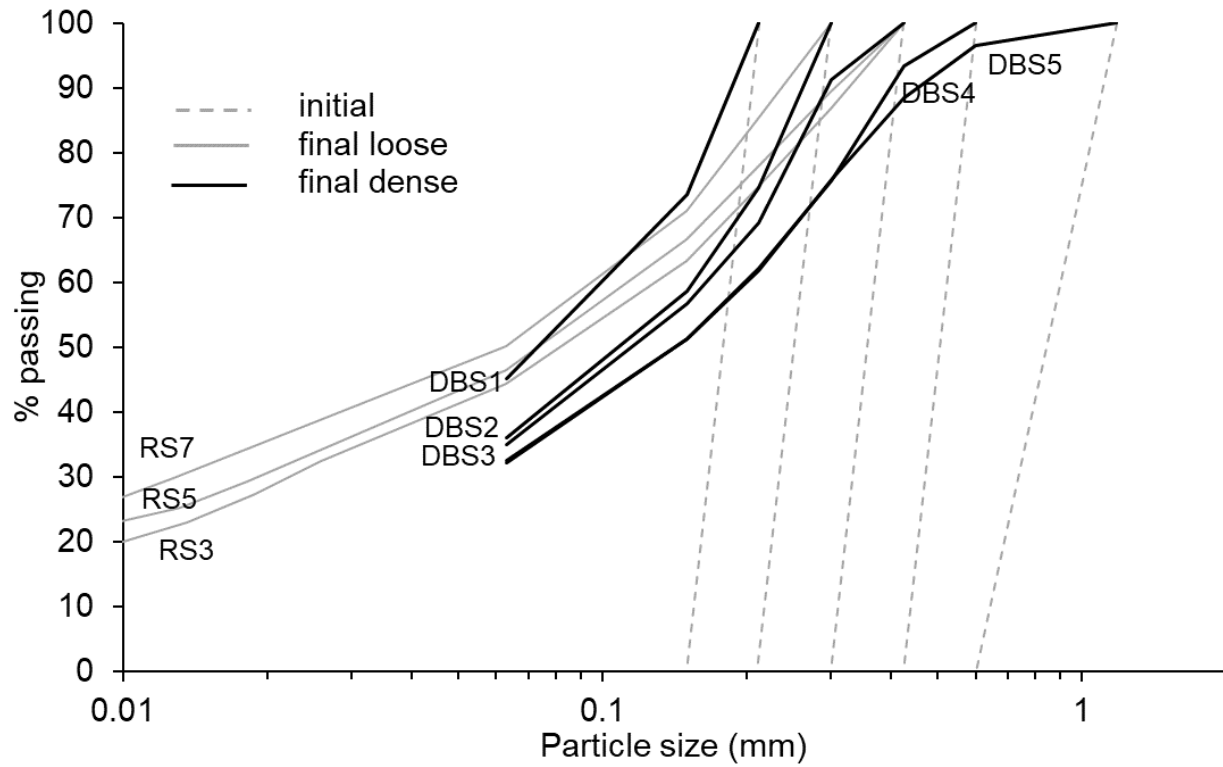
Fig.6 - Influence of absolute particle size on final grading for tests with nominal σ′v=1MPa (slow tests only, tests RS3, RS5, RS7 and RS8 replotted from Coop et al., 2004).