The Timing and Method of Payment in Mergers when Acquirers Are Financially Constrained*

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Abstract

Although acquisitions are a popular form of investment, the link between firms' financial constraints and acquisition policies is not well understood. We develop a model in which financially constrained bidders approach targets, decide how much to bid and whether to bid in cash or in stock. In equilibrium, financial constraints do not affect the identity of the winning bidder, but they lower bidders' incentives to approach the target. Auctions are initiated by bidders with low constraints or high synergies. The use of cash is positively related to synergies and the acquirer's gains from the deal and negatively to financial constraints. (D44, G32, G34)

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Ample research shows that firms' financial constraints are a key determinant of their investment policies.[1] Because acquiring other firms or divisions of firms is one of the most common forms of corporate investment, it is important to understand how financial constraints of potential acquirers affect the market for corporate control. Evidence, both anecdotal and empirical, suggests that bidders pay considerable attention to their existing cash reserves and ability to raise cash when deciding on their acquisition policies.[2] The goal of our paper is to understand the theoretical connection between bidders' financial constraints and their acquisition decisions: maximum willingness to pay for the target, whether to make the payment in cash or stock, and the decision to approach the target in the first place. We argue that the effects of financial constraints are not obvious. For example, one might expect a more financially constrained bidder to be less aggressive at bidding, conditional on having the same valuation of the target. Similarly, one might expect that financial constraints of rival bidders encourage a potential acquirer to initiate a bid, because of the expectation of weaker competition. Among other results, we show that these conjectures are incorrect and the interplay between financial constraints and acquisition policies is more subtle.

To study this connection, we propose a tractable dynamic model. In our model, there are three agents: a target and two potential acquirers. Their stand-alone values fluctuate with the state of the market. Each bidder has a private signal about potential synergies it can realize by acquiring the target. At any time, each bidder can approach the target by expressing an interest in acquiring it, thereby initiating the auction. Participation in the auction is costly, implying that a bidder does not initiate before the state reaches a high enough threshold. Upon being approached, the target invites the other bidder to participate, opens its books, and both bidders learn their synergies. The bidders then compete for the target in an ascending-price auction, in which the winning bidder wins at the price and the losing bidder quits. The winner then decides on the combination of stock and cash to pay to the target to deliver the promised payment. On the one hand, paying in cash is costly, because there is a wedge between the value of a dollar to shareholders of the target and to the bidder, which measures how financially constrained the bidder is. On the other hand, paying in stock can be costly, because a rational target understands that the acquirer has a lower incentive to pay in stock when synergies are higher. Thus, paying in cash becomes a costly signal of the acquirer's synergies.

[1] See Fazzari, Hubbard, and Petersen (1988), Lamont (1997), Almeida and Campello (2007), and Whited and Wu (2006), among others.

[2] For anecdotal evidence, see article Rappaport and Sirower (1999). For empirical evidence, see Harford (1999) and Faccio and Masulis (2005).

The equilibrium in this model has the following structure. First, consider the auction stage. Perhaps surprisingly, financial constraints do not affect a bidder's maximum willingness to pay for the target and, hence, how "strong" the bidder is. Regardless of financial constraint, the bidder continues bidding up to the point at which the price equals the value of the target under the bidder's ownership. This result comes from the bidder's ability to pay in stock. When a bidder wins the auction at the price equal to its maximum willingness to pay, only the lowest-synergy type could possibly win at this price. A rational target then perceives that only this type of bidder will submit an all-stock offer: any lower type would have dropped out of the auction before, whereas any higher type would have included some cash in the bid to signal its type. The bidder then submits an all-stock offer when the net payoff from the deal is zero, so financial constraint does not affect the bidder's maximum willingness to pay, that is, the aggressiveness of the bidder. However, the financial constraint does affect the bidder's expected payoff from the auction through the payment method it uses. If a bidder wins at a price less than its valuation of the target, the bidder submits a mixed cash-stock offer, where the cash portion of the bid is just enough to signal the bidder's synergy. A higher constraint increases the cost of this signaling, reducing the payoff of the bidder from the auction. It also reduces the cash portion of the total payment, because the same cash portion is a stronger signal of the bidder's valuation when the bidder is more constrained.

Second, consider the initiation stage. In equilibrium, the bidder initiates the auction when the state of the market reaches an upper threshold. This equilibrium threshold is affected by the bidder's signal about synergy as well as by financial constraints of both the bidder and the rival bidder. First, it is decreasing in the signal of the bidder: all else equal, a more optimistic bidder is more likely to be the initiator and, hence, the winner. Second, the equilibrium threshold is increasing in the bidder's own financial constraint because it reduces the bidder's expected payoff from initiating the auction. Thus, even though a financial constraint does not affect the bidder's strength (i.e., maximum willingness to pay) when it is able to pay in stock, it does affect the bidder's decision to initiate a bid. Interestingly, the equilibrium initiation threshold is also increasing in the financial constraint of the rival bidder. This effect is subtle, since the rival bidder's maximum willingness is unaffected by how constrained it is. Intuitively, when the rival bidder is more constrained, it is more reluctant to initiate the auction, given the same information. Hence, observing that the rival bidder has not approached the target yet, the bidder does not downgrade its belief about the rival bidder's signal as much as if it were less constrained. Hence, the bidder

perceives that the rival is a stronger competitor at each date. As a result, it expects to obtain a lower payoff from the auction, which reduces the incentives to initiate it in the first place.

The model delivers three groups of implications, many of which are consistent with the empirical evidence and some have not been tested yet. The first group of implications concerns the impact of bidders' financial constraints on their acquisition activity. As discussed above, financial constraints do not make bidders weaker, because they have the ability to bid in stock, in contrast to models of auctions with financially constrained bidders in which bids must be made in cash (e.g., Che and Gale 1998). Despite this, they reduce each bidder's incentives to initiate a bid. Thus, financial constraints do not affect *who* acquires the target, rather they impact efficiency by changing *when* the acquisition occurs. Importantly, both a bidder's own and the rival bidder's constraint reduce a bidder's incentive to approach the target. Thus, an unexpected tightening of financial constraints in the economy reduces a bidder's propensity to acquire targets even if its own ability to pay in cash is unaffected, in line with Harford (2005), who finds that the occurrence of mergers is strongly related to aggregate liquidity in the market.

These implications of financial constraints are driven by two assumptions. First, synergies that a bidder realizes are independent of its financial constraint. Second, a bidder can circumvent the costs of financial constraints by making a payment in stock. These assumptions are consistent with models in which financial constraints are driven by transaction costs but may be inconsistent with models in which financial constraints are driven by agency problems between a bidder's management and investors. Thus, our paper suggests that the nature of financial constraints is important for their relation to merger and acquisitio (M&A) outcomes.

The second group of implications concerns the method of payment used in acquisitions. In equilibrium, the cash portion of the total payment is determined by the acquirer's financial constraint and by the difference between its valuation of the target and the valuation of the target by the rival bidder. The former implies that acquisitions rely more on cash when bidders are less financially constrained. The latter implies that the payoff to the winning bidder and synergies in a deal are positively related to the cash portion of the payment, consistent with the empirical evidence on acquisition announcement returns (Travlos 1987; Eckbo, Giammarino, and Heinkel 1990).

Finally, because the model endogenizes both initiation and bidding decisions, it delivers implications about the identities of initiating and winning bidders, which have not been empirically examined yet. All else equal, the auction is more likely to be initiated by a less constrained bidder

and by a bidder with more positive information about its synergies. In contrast, the identity of the winner is determined by synergies, but not by financial constraints. As a consequence, the initiating bidder is more likely to have high maximum willingness to pay and to win the auction than the noninitiating bidder in the same auction. Our model implies that this effect is stronger if the initiating bidder is more constrained than the noninitiating bidder. Whether the initiating bidder wins the auction is also related to the payment method. If bidders have similar constraints, then the cash portion of the payment is higher when the identities of the initiating and winning bidders coincide than when they are different. Intuitively, because the initiating bidder is more likely to have high synergies, events in which the initiating bidder wins the auction are more likely to correspond to events in which the gap between bidders' synergies is substantial, implying that the initiating bidder needs to signal its valuation by including substantial cash in the payment.

Relatively little work studies the effects of bidders' financial constraints on bidders' acquisition activity. Burkart et al. (2014) examine the role of legal investor protection and bidders' financial constraints in a tender offer setting of Grossman and Hart (1980). Our paper also examines the role of bidders' financial constraints, but it focuses on different aspects of the problem and employs a very different model. Specifically, we study takeovers in the form of mergers (as opposed to tender offers) and focus on the role of bidders' private information and dynamics. Another literature, started by Che and Gale (1998), studies standard auctions and auction design when bidders face budget constraints. It does not consider bids in stock or other securities, which is our focus. We show that stock bids relax financial constraints, but come at a cost of the adverse selection discount. Li, Taylor, and Wang (2017) estimate a static auction model with both financial constraints and stock bids. The model has several similarities to our auction stage but has a different information structure and form of financial constraints.

In addition to these papers, our paper is related to two other strands of the literature. First, it is related to the literature that studies mergers and acquisitions as stopping time problems (see, e.g., Lambrecht 2004; Hackbarth and Morellec 2008).[3] Existing papers assume that acquirers and targets have the same information, a fact that makes bids in cash and in securities equivalent and financial constraints irrelevant, because the parties can always transact in securities. Our contribution is to introduce bidders' private information into their dynamic decision-making process. Private information is central for the implications of our paper: it makes bids in cash and stock

[3]Other papers that study mergers and acquisitions as real-options problems include Morellec and Zhdanov (2005, 2008), Alvarez and Stenbacka (2006), Lambrecht and Myers (2007), Margsiri, Mello, and Ruckes (2008), and Hackbarth and Miao (2012).

different, leading to the importance of financial constraints for initiation decisions and auction outcomes.

Second, the paper is related to information-based models of means of payment in mergers and acquisitions and, more generally, to the literature on auctions in which bids can be made in securities, recently summarized by Skrzypacz (2013).[4] Information-based models of means of payment are provided by Hansen (1987), Fishman (1989), Eckbo, Giammarino, and Heinkel (1990), Berkovitch and Narayanan (1990), and Rhodes-Kropf and Viswanathan (2004). These papers consider static models with various information structures and assumptions about the bidding process. In contrast, our model focuses on bidders' financial constraints and their dynamic decision-making: each bidder not only decides on bids but also on when to initiate the contest. Hence, the model delivers implications about the role of financial constraints and the identities of initiating and winning bidders, relating them to payment method and the timing of the deal. Finally, our paper is related to two recent papers that study initiation of auctions and bargaining in models with privately informed players, but with different ingredients and results. Cong (2017) studies the interplay between post-auction moral hazard and the seller's strategic timing of auctioning the asset in a security-bid auction framework. Chen and Wang (2015) study initiation of mergers in a bargaining problem with two-sided private information.

1 Model Setup

Consider a setting in which the risk-neutral target attracts two potential risk-neutral acquirers, or bidders. The roles of the target and the bidders are exogenous. As a stand-alone entity, the target yields a cash flow of $\Pi_T X_t$ per unit time, and each bidder yields a cash flow of $\Pi_B X_t$ per unit time. Common state X_t evolves as a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \tag{1}$$

where μ and $\sigma > 0$ are constant growth rate and volatility, dB_t is the increment of a standard Brownian motion, and X_0 is low enough. The discount rate r is constant. To guarantee finite values, we assume $r > \mu$.

[4] Security-bid auctions are studied by Hansen (1985), Rhodes-Kropf and Viswanathan (2000), DeMarzo, Kremer, and Skrzypacz (2005), Gorbenko and Malenko (2011), Liu (2016), Sogo, Bernhardt, and Liu (2016), and Cong (2017). Vladimirov (2015) examines the link between making bids in securities and the financing of cash bids via issuing securities to investors.

If bidder i acquires the target at time τ, the combined firm produces a cash flow of $(\Pi_T + \Pi_B + v_i) X_t$ per unit time at any time $t > \tau$. Here, $v_i \in \{v_l, v_h\}$, $v_h > v_l > 0$ is the synergy that captures an improvement from combining operations of the target and bidder i.[5] At the start of the game, bidder i privately learns signal s_i about its synergy v_i, where $\Pr[v_i = v_h | s_i] = s_i$. Each bidder's signal is a draw from a distribution with p.d.f. $f(s) > 0$ on $[\underline{s}, \bar{s}]$, where $0 < \underline{s} < \bar{s} < 1$. The signals and synergies the two bidders are independent. Both are soft signals of each bidder that cannot be credibly conveyed other than via the initiation and bidding decisions described below.

The model consists of two stages: the initiation stage and the auction stage, illustrated in Figure 1. We describe each stage below.

1. **Stage 1: Initiation.** Prior to the auction, each bidder i knows s_i, but not v_i. In practice, acquisitions by strategic buyers are usually initiated by a bidder (Fidrmuc et al. 2012). To reflect this practice, we assume that each bidder has a real option to approach the target at any time. When the bidder exercises this option, the target invites the other bidder to participate in the auction and opens its books. Participating in the auction costs $I > 0$ to each bidder, which is a one-time nonmonetary cost prior to the start of the auction. Cost $I > 0$ is small enough so that both bidders choose to participate in the auction in equilibrium (the parameter restrictions are provided in Section 3). After the auction is initiated, each bidder learns its synergy v_i.

2. **Stage 2: Auction.** At the auction stage, the bidders compete in an ascending-bid (English) auction, in which offers can be made in a combination of cash and stock. We formalize the auction in the following way. The auctioneer (the target) continuously raises price p from zero. As p rises, each bidder confirms its participation until deciding to quit. Once one bidder withdraws, the remaining bidder is declared the winner. The winning bidder chooses a combination of $b \geq 0$ in cash and a fraction $\alpha \geq 0$ of the stock of the combined company, subject to the "no default" condition that the value of the bundle, evaluated according to the beliefs of the target, is at least p, the price at which the rival bidder quit. This formalization extends "clock" models of an English auction in all-cash bids (Milgrom and Weber 1982) and all-stock bids (Hansen 1985) for bids in combinations of cash and stock.[6]

[5] For example, a bidder and the target can reduce the cost of making a product by a certain percentage. As the size of the market grows, the value of this synergy also grows one-to-one with the size of the market.

[6] This bidding protocol is robust to allowing the method of payment to be part of the auction process in the following sense. Consider a modified bidding protocol, in which at each price p, a bidder confirms participation or drops out. A bidder who confirms participation also picks any combination of cash and stock, the value of which

Finally, each bidder is financially constrained. Specifically, the cost to bidder i of a payment of b in cash is $\lambda_i b$, where $\lambda_i > 1$.[7] This is a lump-sum cost borne by the bidder at the time of the deal. Difference $\lambda_i - 1$ reflects the wedge between the value of a dollar in cash to the shareholders of the target and the cost to the acquirer.[8] It is related to the concept of external finance premium in financial accelerator models in macroeconomics (Bernanke and Gertler 1989). The values of λ_1 and λ_2 are common knowledge.

It remains to define the equilibrium concept. At the auction stage, when a bidder chooses a combination of stock and cash to offer to the target, we have a signaling game where the target guesses the synergy of the bidder from the offer (b, α) it makes. We assume that the belief of the seller following off-equilibrium offers satisfies the Cho and Kreps (1987) Intuitive Criterion (CKIC), which is defined as follows.

Assumption 1 (CKIC). Suppose that bidder i wins the auction at price p and makes an off-equilibrium offer (b, α) to the target. If type v_l is worse off acquiring the target for (b, α) than playing its equilibrium strategy, while type v_h is better off acquiring the target for (b, α) than playing its equilibrium strategy, then the target believes that bidder i's synergy is v_h.

Assumption 1 is a standard restriction in signaling games. Intuitively, it is unreasonable for a low-synergy bidder to submit an offer, which makes it worse off even if the offer is accepted. As we show in the next section, the auction has a unique equilibrium satisfying Assumption 1.

At the initiation stage, we look for separating equilibria in continuous and monotone threshold strategies. These are equilibria where bidder i with signal s follows a strategy of initiating the auction at threshold $\bar{X}_i(s)$, provided that the rival bidder has not initiated the contest before, where $\bar{X}_i(s)$ is continuous and monotone. In what follows, we refer to them simply as equilibria. As we show below, $\bar{X}_i(s)$ is strictly decreasing in s, implying that all else equal the bidder approaches the target earlier if it is more optimistic about potential synergies.

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the target estimates to be p if the bidder acquires the target for this combination. After the rival bidder drops out, the remaining bidder picks the final combination of cash and stock. An equilibrium in our bidding game is also an equilibrium in this modified bidding game, because only the final combination of cash and stock is relevant for the payment that the winner makes.

[7] We also generalized the base model to convex costs of paying in cash: specifically, if the cost to bidder i of a payment of b in cash is $l_i\left(\frac{b_i}{X_t}\right)b$, where $l_i(\cdot)$ satisfies $l_i(\cdot) > 1$ and $l_i'(\cdot) \geq 0$. With two types of synergies, this model essentially leads to the same equilibrium as our formulation.

[8] The following toy model can capture this cost. Suppose the firm has no internal storage technology, so that it pays out all cash flows as dividends. To pay b in cash, the firm must raise it from existing owners. However, for each dollar, $\lambda_i - 1$ dollars is "wasted" in transaction costs. Hence, it costs owners $\lambda_i b$ to pay b in cash.

2 Equilibrium at the Auction Stage

Consider the auction that occurs at time t. We present a heuristic derivation of the equilibrium in this section and formalize it in the proof of Proposition 1 in the appendix. It is useful to introduce the post-auction value of the combined firm with synergy v, $V(v) = \frac{\Pi_T+\Pi_B+v}{r-\mu}X_t$, and the post-auction value of the losing bidder, $V_o = \frac{\Pi_B}{r-\mu}X_t$, where we supersede the time subscript for brevity.

Suppose that bidder i's equilibrium strategy is to bid up to price $p_i^*(v)$ for $v \in \{v_l, v_h\}$. If bidder i wins the auction when the rival drops out at price p, which is very close to $p_i^*(v_h)$, the target infers that bidder i has high synergies, as otherwise it would have dropped out earlier. Because paying in cash is costly and the target believes that the synergies are high, it is optimal for bidder i to make an all-stock offer in this case. Because the bidder does not make a cash payment and its valuation of the stock coincides with the target's valuation of stock, the cost of the payment to bidder i is p. It follows that a high-synergy bidder finds it weakly dominant to bid up to its valuation of the target:

$$p_i^*(v_h) = p^*(v_h) = V(v_h) - V_o = \frac{\Pi_T + v_h}{r-\mu}X_t. \tag{2}$$

Similarly, consider the case, in which bidder i wins the auction when the rival drops out at price $p \leq p_i^*(v_l)$. In this case, the target is uncertain about the synergy of bidder i. It can signal its synergy via the mixed cash-stock offer. Because raising cash is costly and the bidder values stock of the combined company more if it has high synergies, the high-synergy bidder can separate from the low-synergy bidder by including a sufficient amount of cash in its offer. Because the most pessimistic belief that the target can have is v_l, it is optimal for the low-synergy bidder to make an all-stock offer. It follows that a low-synergy bidder also finds it weakly optimal to bid up to its valuation of the target:

$$p_i^*(v_l) = p^*(v_l) = V(v_l) - V_o = \frac{\Pi_T + v_l}{r-\mu}X_t.$$

It remains to solve for the amount of cash that the high-synergy bidder i offers if winning against the low-synergy rival, that is, at price $p^*(v_l)$. The combination of cash and stock (b_i^*, α_i^*)

must satisfy:

$$\begin{aligned}(1-\alpha_i^*)\,V(v_l)-\lambda_i b_i^* &\leq V_o,\\ \alpha_i^* V(v_h)+b_i^* &\geq p^*(v_l)\end{aligned}$$

The first condition ensures that bidder i with low synergies does not want to deviate from its all-stock bid and payoff of V_o to mimic bidder i with high synergies. Note that this condition depends on bidder i's own financial constraint and not on the financial constraint of the rival. Because the seller knows the identity of each bidder, the high-synergy type of bidder i needs to separate from the low-synergy version of itself, not from the rival bidder. The second condition is the requirement that the value of the offer to the target is no lower than price $p^*(v_l)$, at which the bidder wins the auction. The optimal offer for the high-synergy bidder that satisfies the above conditions is such that both inequalities bind. That is, it uses just enough cash so that the low-synergy bidder does not mimic it, and the value of the mixed offer is exactly $p^*(v_l)$.

The above argument does not imply that the equilibrium is unique. In standard signaling models with two types, the Intuitive Criterion (Assumption 1) selects the least-cost separating equilibrium as the unique one. In the proof of Proposition 1, we show that this is also the case in our model.

Proposition 1 (equilibrium bidding). *There exists a unique equilibrium at the auction stage, in which bidding is in weakly dominant strategies and the beliefs satisfy CKIC (Assumption 1). A bidder with synergy $v \in \{v_l, v_h\}$ drops out at price*

$$p^*(v) = V(v) - V_o = \frac{\Pi_T + v}{r-\mu} X_t. \tag{3}$$

If bidders' synergies are equal at $v \in \{v_l, v_h\}$, both bidders drop out at price $p^*(v)$, the winner is determined at random, and it makes an all-stock payment of fraction $\frac{p^*(v)}{V(v)}$ of the stock of the combined company. If bidders' synergies differ, the high-synergy bidder wins at price $p^*(v_l)$ and makes a payment of $(1-\gamma_i)\,p^*(v_l)$ in cash and fraction $\gamma_i \frac{p^*(v_l)}{V(v_h)}$ of the stock of the combined company, where $\gamma_i = \left(1+\frac{1}{\lambda_i - 1}\left(1-\frac{V_o+p^*(v_l)}{V(v_h)}\right)\right)^{-1} = \left(1+\frac{1}{\lambda_i-1}\frac{v_h-v_l}{\Pi_T+\Pi_B+v_h}\right)^{-1}$ represents the proportion of stock in the total offer value.

The economics of Proposition 1 are as follows. In the presence of financial constraints, the

acquirer's internal valuation of cash exceeds that of the shareholders of the target. Hence, it pays in cash only if its all-stock offer is undervalued by the target. This happens if the acquirer's valuation of the target exceeds the payment made to the target, determined by the rival bidder's dropout price. In contrast, if the acquirer already commits to pay a high takeover premium, which is the case when the rival's bidder synergy is high, there is no benefit in paying in cash, because the high takeover premium by itself signals the high synergy of the acquirer. Proposition 1 implies that the synergies of both bidders are revealed in the course of the auction: The price at which the losing bidder drops out reveals its synergy, while the method of payment reveals the synergy of the winning bidder.

2.1 Expected payoffs from the auction

Given the equilibrium in the auction derived in Proposition 1, we can calculate the expected surplus from the auction for each bidder, defined as its post-auction value less the stand-alone value of V_o. If a bidder loses the auction or wins against the rival with the same synergy, its surplus from the auction is zero. If the two bidders differ in synergies, the high-synergy bidder wins and obtains the surplus equal to its maximum willingness to pay, $V(v_h) - V_o$, less the cost of a bid, which is the sum of the value of the bid to the target, $p^*(v_l)$, and the cost of signaling, $(\lambda_i - 1)(1 - \gamma_i) p^*(v_l)$. Simplifying, this is equal to $\psi_i(v_h, v_l) \frac{X_t}{r-\mu}$, where

$$\psi_i(v_h, v_l) \equiv v_h - v_l - (\lambda_i - 1)(1 - \gamma_i)(\Pi_T + v_l). \tag{4}$$

To get bidder i's expected surplus from the auction, we need to multiply the conditional surplus $\psi_i(v_h, v_l) \frac{X_t}{r-\mu}$ by the probability that its synergy is high and the rival's synergy is low. The post-auction payoff of this bidder is then the sum of this surplus and its stand-alone value V_o.

Using this logic, consider the expected surplus of the initiating bidder, denoted bidder 1, given its initiation of the auction at time t. Because the payoff from the auction is increasing in the bidder's signal s, the equilibrium initiation threshold of each bidder is decreasing in signal s.[9] Because a bidder with a higher signal approaches the seller at an earlier threshold, at time t bidder 1 believes that the signal of the rival bidder s_2 is below some cutoff, denoted $\hat{s}$, to be determined in equilibrium. In addition, bidder 1 knows its signal s_1. Therefore, the expected

[9] See Lemma 1 in the appendix. This result follows from an application of Topkis's theorem (Topkis 1978).

surplus of the initiating bidder from the auction is

$$s_1 \left(1 - \mathbb{E}\left[s_2 | s_2 \leq \hat{s}\right]\right) \psi_1 \left(v_h, v_l\right) \frac{X_t}{r - \mu}. \tag{5}$$

Prior to the auction, the expected payoff of the initiating bidder is the sum of its expected surplus from the auction (5) and its stand-alone value $V_o = \frac{\Pi_B}{r-\mu} X_t$, less the cost of participating in the auction I.

Similarly, consider the expected surplus of the noninitiating bidder, denoted bidder 2, from the auction initiated by bidder 1 at threshold X_t. If bidder 2 observes initiation at X_t, it believes that the signal of bidder 1 is $\tilde{s}_1 = \bar{X}_1^{-1}\left(X_t\right)$. In addition, bidder 2 knows its signal s_2. Therefore, the expected surplus from the auction of the noninitiating bidder is

$$s_2 \left(1 - \tilde{s}_1\right) \psi_2 \left(v_h, v_l\right) \frac{X_t}{r - \mu}. \tag{6}$$

Prior to the auction, the expected payoff of the noninitiating bidder is the sum of (6) and its stand-alone value $V_o = \frac{\Pi_B}{r-\mu} X_t$, less the cost of participating in the auction I.

3 Equilibrium at the Initiation Stage

Having derived the equilibrium in the auction, we solve for the equilibrium at the initiation stage.

To obtain the present values of bidders' payoffs, we use the following result (e.g., Dixit and Pindyck 1994). If τ is the first passage time by X_t of an upper threshold $\hat{X}$, then the time-0 present value of a security that pays \$1 at time τ equals $\mathbb{E}\left[e^{-r\tau}\right] = \left(\frac{X_0}{\hat{X}}\right)^{\beta}$, where β is the positive root of the quadratic equation $\frac{1}{2}\sigma^2 \beta \left(\beta - 1\right) + \mu\beta - r = 0$:

$$\beta = \frac{1}{\sigma^2}\left[-\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 1. \tag{7}$$

We start with the case in which bidders have the same financial constraints: $\lambda_i \equiv \lambda$. We then consider the case in which financial constraints are different.

3.1 Same financial constraints

The same financial constraints lead to the same bidding strategies, as shown in Proposition 1. In particular, when bidders differ in synergies, the proportion of the stock in the total offer of the

winner, $\gamma_i = \gamma = \left(1 + \frac{1}{\lambda - 1} \frac{v_h - v_l}{\Pi_T + \Pi_B + v_h}\right)^{-1}$ is the same for both bidders. The surpluses from the auction of the initiating and the noninitiating bidders are given by (5) and (6), respectively, with $\psi_i(v_h, v_l) = \psi(v_h, v_l)$, $i \in \{1, 2\}$. Note that because $\psi(v_h, v_l)$ does not depend on X_t, expected payoffs of bidders depend on X_t linearly.

If bidders face the same financial constraints, there is a symmetric equilibrium, in which a bidder with signal s initiates the auction at threshold $\bar{X}(s)$, provided that the rival bidder has not initiated yet. Consider bidder i with signal s, who expects the rival bidder j with signal z to initiate at threshold $\bar{X}(z)$, where $\bar{X}(z)$ is a strictly decreasing function. As time goes by and bidder i observes that bidder j has not initiated the auction yet, bidder i truncates its belief about bidder j's signal by tracking the highest realization of X_t: at time t, the highest possible signal of bidder j is $\bar{X}^{-1}\left(\max_{u \in [0,t]} X_u\right)$. Therefore, if bidder i initiates at threshold $\hat{X}$, its expected value at any time t prior to reaching threshold $\hat{X}$ is

$$\Pi_B \frac{X_t}{r - \mu} + \left(\frac{X_t}{\hat{X}}\right)^{\beta} \int_{\underline{s}}^{\bar{X}^{-1}(\hat{X})} \left(s(1 - z)\psi(v_h, v_l)\frac{\hat{X}}{r - \mu} - I\right) \frac{dF(z)}{F\left(\bar{X}^{-1}(\max_{u \in [0,t]} X_u)\right)}$$
$$+ \int_{\bar{X}^{-1}(\hat{X})}^{\bar{X}^{-1}\left(\max_{u \in [0,t]} X_u\right)} \left(\frac{X_t}{\bar{X}(z)}\right)^{\beta} \left(s(1 - z)\psi(v_h, v_l)\frac{\bar{X}(z)}{r - \mu} - I\right) \frac{dF(z)}{F\left(\bar{X}^{-1}(\max_{u \in [0,t]} X_u)\right)}, \qquad (8)$$

where we set $\bar{X}^{-1}(x) = \bar{s}$ for any $x < \bar{X}(\bar{s})$. The intuition behind (8) is as follows. Prior to the auction, bidder i's value is $V_o = \Pi_B \frac{X_t}{r - \mu}$, which is the first summand in (8). The second and third summands in (8) represent the adjustments to bidder i's value following the auction. If bidder j's signal z is low, so that $\bar{X}(z) > \hat{X}$, then the auction is initiated by bidder i at threshold $\hat{X}$. In this case, the rival's signal is inferred to be below $\bar{X}^{-1}\left(\hat{X}\right)$. Using expression (5), the expected surplus of bidder i from the auction in this case is $s\left(1 - \mathbb{E}\left[z | z \leq \bar{X}^{-1}\left(\hat{X}\right)\right]\right)\psi(v_h, v_l)\frac{\hat{X}}{r - \mu}$. In addition, the bidder incurs the participation cost I. Combining the two yields the second summand in (8). If bidder j's signal is high, so that $\bar{X}(z) < \hat{X}$, then the auction is initiated by bidder j at threshold $\bar{X}(z)$. In this case, bidder i infers that the signal of bidder j is z from its initiation threshold. Using expression (6), the expected surplus to bidder i from the auction in this case is $s(1 - z)\psi(v_h, v_l)\frac{\bar{X}(z)}{r - \mu}$. Integrating over the possible realizations of bidder j's signal yields the third summand in (8).

To solve for the equilibrium $\bar{X}(s)$, we maximize (8) with respect to $\hat{X}$ and apply the equilibrium condition that the maximum must be reached at $\bar{X}(s)$. To ensure existence and uniqueness of the equilibrium, we impose the following assumptions:

Assumption 2 (monotone payoff of the initiating bidder). $sm(s)$ is strictly increasing in s, where $m(s) \equiv 1 - \mathbb{E}[z|z \leq s]$ is the probability that the synergy of the rival bidder is v_l, conditional on its signal being below s.

Assumption 3 (no entry deterrence). The parameters of the model satisfy conditions (20) and (24) in the appendix.

Assumption 2 means that the expected payoff from the auction of the initiating bidder is strictly increasing in its signal. Intuitively, if the signal of the initiating bidder is higher, there are two effects. The first-order effect is that the initiating bidder is more likely to have a high synergy. This effect increases the payoff of the initiating bidder. The second-order effect is that the rival bidder facing a stronger initiating bidder is also more likely to have a high synergy, because its distribution is truncated less at the initiation stage. This effect decreases the payoff of the initiating bidder. Assumption 2 restricts the distribution of types to be such that the latter effect does not dominate the former, meaning that a higher signal is always "good news" for the payoff of the initiating bidder. As an example, it is satisfied for uniform distribution over $[\underline{s}, \overline{s}]$, used in our numerical examples, if $\frac{s}{2} + \overline{s} < 1$.

Assumption 3 ensures that the equilibrium features participation of both bidders in the auction. Specifically, condition (20) means that a bidder always prefers to join the auction initiated by the rival bidder at threshold $\bar{X}(s)$, $s \in [\underline{s}, \bar{s}]$. Condition (24) means that the bidder with the highest signal is better off initiating the auction at threshold $\bar{X}(\bar{s})$ and facing competition from the rival bidder rather than speeding up to initiate the auction earlier and deterring the entry of some types of the rival bidder.

Proposition 2 shows that under Assumptions 2 and 3, there exists a unique symmetric equilibrium in the initiation game:

Proposition 2 (equilibrium initiation when financial constraints are the same). *There exists a unique symmetric equilibrium in the initiation game. A bidder with signal s initiates the auction when X_t reaches upper threshold*

$$\bar{X}(s) = \frac{\beta}{\beta - 1} \frac{(r - \mu)I}{sm(s)\psi(v_h, v_l)}, \tag{9}$$

provided that the target has not been approached before. If the auction is initiated by a rival bidder

at $\hat{X} \in \left[\bar{X}\left(\bar{s}\right), \bar{X}\left(\underline{s}\right)\right]$*, the bidder always participates in it.*

It is useful to consider two special cases: when the bidders have no financial constraints ($\lambda \to 1$) and when the financial constraints are extreme ($\lambda \to \infty$). In this case, (9) yields

$$\lim_{\lambda \to 1} \bar{X}(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu)I}{sm(s)(v_h - v_l)}, \tag{10}$$

$$\lim_{\lambda \to \infty} \bar{X}(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu)I}{sm(s)(v_h - v_l)\left(1 - \frac{p^*(v_l)}{V(v_h)}\right)}. \tag{11}$$

In particular, (11) is always higher than (10). In general, $\bar{X}(s)$ is increasing in λ: higher financial constraints make it costlier for a high-synergy winner to signal its type when acquiring the target at a low price. This results in a lower expected profit at the initiation stage and further initiation delay. We provide formal proofs in Section 4.

3.2 Heterogeneous financial constraints

Consider the case in which bidders differ in their financial constraints, $\lambda_1 \neq \lambda_2$. In this case, the expected payoffs from the auction differ for the two bidders. As a consequence, the equilibrium in the initiation game is asymmetric. Consider bidder i with signal s, who expects the rival bidder j with signal z to initiate at threshold $\bar{X}_j(z)$, where $\bar{X}_j(z)$ is a strictly decreasing function. As time goes by and bidder i observes that bidder j has not initiated the auction yet by time t, bidder i updates the belief about the signal of the rival bidder to $\left[\underline{s}, \bar{X}_j^{-1}\left(\max_{u\in[0,t]} X_u\right)\right]$.Therefore, if bidder i follows the strategy of initiating the auction at threshold $\hat{X}$, provided that the rival has not initiated the auction yet, its expected payoff at any time t prior to reaching threshold $\hat{X}$ is

$$\begin{aligned}
&\Pi_B \frac{X_t}{r-\mu} + \left(\frac{X_t}{\hat{X}}\right)^{\beta} \int_{\underline{s}}^{\bar{X}_j^{-1}(\hat{X})} \left(s(1-z)\psi_i(v_h, v_l)\frac{\hat{X}}{r-\mu} - I\right) \frac{dF(z)}{F\left(\bar{X}_j^{-1}(\max_{u\in[0,t]} X_u)\right)} \\
&+ \int_{\bar{X}_j^{-1}(\hat{X})}^{\bar{X}_j^{-1}\left(\max_{u\in[0,t]} X_u\right)} \left(\frac{X_t}{\bar{X}_j(z)}\right)^{\beta} \left(s(1-z)\psi_i(v_h, v_l)\frac{\bar{X}_j(z)}{r-\mu} - I\right) \frac{dF(z)}{F\left(\bar{X}_j^{-1}(\max_{u\in[0,t]} X_u)\right)},
\end{aligned} \tag{12}$$

where we set $\bar{X}_j^{-1}(x) = \bar{s}$ for any $x < \bar{X}_j(\bar{s})$ and $\bar{X}_j^{-1}(x) = \underline{s}$ for any $x > \bar{X}_j(\underline{s})$.[10] Condition $\bar{X}_j^{-1}(x) = \underline{s}$ for any $x > \bar{X}_j(\underline{s})$ is a natural restriction on off-equilibrium beliefs of bidder i.[11] The logic behind (12) is similar to that in the model with same financial constraints. The notable difference concerns the impact of bidder i's own and its rival's constraints. Bidder i's constraint, λ_i, affects its expected payoff through $\psi_i(v_h, v_l)$, because it determines how costly it is to signal its type when the acquisition price is low. The rival's financial constraint, λ_j, affects the payoff indirectly through the rival's initiation strategy $\bar{X}_j(\cdot)$, which bidder i uses to infer the rival's signal.

To solve for the equilibrium initiation thresholds, we maximize (12) for each bidder with respect to $\hat{X}$, and apply the equilibrium condition that the maximums must be reached at $\bar{X}_i(s)$. We impose restrictions, analogous to Assumptions 2 and 3, which correspond to the monotone payoff of the initiating bidder in its signal s and entry deterrence not occurring. The equilibrium is summarized in Proposition 3, which is similar to Proposition 2:

Proposition 3 (equilibrium initiation when financial constraints are different). *Let a pair $\bar{X}_i(s)$, $i \in \{1,2\}$ be a solution to the system of equations:*

$$\bar{X}_i(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu)I}{s\psi_i(v_h, v_l) m\left(\bar{X}_j^{-1}(\bar{X}_i(s))\right)}, \tag{13}$$

where $j \neq i$ and $\psi_i(v_h, v_l)$ is defined in (4). Suppose that $sm\left(\bar{X}_1^{-1}(\bar{X}_2(s))\right)$ and $sm\left(\bar{X}_2^{-1}(\bar{X}_1(s))\right)$ are strictly increasing in s, and conditions (19) and (23) in the appendix hold. Then, thresholds $\bar{X}_i(s)$, $i \in \{1,2\}$ give an equilibrium. Bidder i with signal s initiates the auction when X_t reaches upper threshold $\bar{X}_i(s)$, provided that the target has not been approached before. If the auction is initiated by a rival bidder at $\hat{X} \in \left[\bar{X}_j(\bar{s}), \bar{X}_j(\underline{s})\right]$, bidder i always participates in it. Furthermore, there is no separating equilibrium in continuous threshold strategies that does not solve (13).

Note that if $\lambda_i \equiv \lambda$ then $\bar{X}_j^{-1}(\bar{X}_i(s)) = s$ and $\psi_i(v_h, v_l) = \psi(v_h, v_l)$, so Proposition 3 embeds Proposition 2. Unlike in Proposition 2, we could not prove existence and uniqueness of the

[10] As we show in Proposition 5, for any signal s, the less financially constrained bidder initiates the auction earlier: $\bar{X}_2(s) < \bar{X}_1(s)$, if $\lambda_2 < \lambda_1$. Thus, learning is different for two bidders: for example, in the region of thresholds between $\bar{X}_2(\bar{s})$ and $\bar{X}_1(\bar{s})$, bidder 2 does not update its belief about the signal of bidder 1, while bidder 1 updates its belief about the signal of bidder 2.

[11] In particular, any other belief would imply that bidder i believes that bidder j is stronger than $\underline{s}$ if it does not initiate at threshold $\bar{X}_j(\underline{s})$, which is opposite of the fact that the lack of initiation is a signal that the rival bidder is (weakly) weaker at any other threshold.

equilibrium in the model with asymmetric financial constraints. However, the unique equilibrium exists in all our numerical examples.

4 Model Implications

In this section, we discuss four implications of the model: (1) the relation of the financing constraints of a bidder and its rival to the bidder's propensity to initiate an acquisition; (2) endogenous selection of deals into stock-only deals and deals that involve cash payments; (3) links between identities of initiating bidders and winning bidders; and (4) the relation between the equilibrium timing of the deal and the optimal one.

4.1 Effects of financial constraints

The following proposition shows the comparative statics of the equilibrium initiation threshold $\bar{X}_i(s)$ in the parameters of the model:

Proposition 4 (comparative statics). *Suppose that the equilibrium is continuous and differentiable in the parameter of interest $\theta \in \{\sigma, r, \mu, I, \Pi_T, \Pi_B, v_h, \lambda_i, \lambda_j\}$ and consider a marginal change in θ. For any s, $\bar{X}_i(s)$ is: (1) strictly increasing in σ, r, I, Π_T, and λ_i; (2) strictly increasing in λ_j for s such that $\bar{X}_i(s) \in \left[\bar{X}_j(\bar{s}), \min_{i\in\{1,2\}} \bar{X}_i(\underline{s})\right]$ and weakly for remaining s in $[\underline{s}, \bar{s}]$; (3) strictly decreasing in μ, Π_B, and v_h.*

Our central comparative statics are with respect to the bidder own's (λ_i) and its rival's (λ_j) financial constraints. The effect of the bidder's own constraint is intuitive. If it has a high synergy and faces a low-synergy rival, the bidder needs to include cash in the bid to signal its synergy and thereby avoid having its stock undervalued by the target. The cost of such signaling is higher if the bidder is more constrained. Therefore, a higher financial constraint reduces the expected payoff of the bidder from initiating the auction and leads to more delay.

The effect of the rival bidder's financial constraint is subtler. A naive conjecture could be that an increase in the rival bidder's constraint makes it a weaker rival, which increases the payoff of the other bidder giving it more incentive to initiate the auction. This conjecture is incorrect. As we saw in Section 2, the expected payoff of a bidder from the auction depends only on its own constraint, but not on the constraint of its rival, which would be different if bids were restricted

to be in cash (Che and Gale, 1998). Furthermore, each bidder learns about the strength of its opponent by observing that the rival has not initiated the auction yet. If the rival bidder is more constrained, it initiates the auction later for any signal s. Thus, conditional on the rival not initiating the auction, the bidder believes that the rival is more likely to have high synergies than if the rival were less constrained. Therefore, a bidder is also less likely to approach the target if the rival bidder's financial constraint goes up. Figure 2 illustrates the result of Proposition 4 that both a bidder's own and a rival bidder's financial constraint delay the initiation of the auction. Figure 3 illustrates the comparative statics of all parameters in Proposition 4.

Proposition 4 yields an empirical prediction: an unexpected tightening in the aggregate financial constraint reduces acquisition activity. In fact, a bidder's acquisition activity declines even if it is unaffected by the shock, as long as other potential acquirers are affected. Empirically, Harford (2005) finds that the timing of mergers is related to aggregate liquidity in the market, which is in line with Proposition 4.

The results that bidders' financial constraints do not affect the identity of the acquirer and that a bidder delays approaching the target when the rival bidder is more constrained are surprising, so we would like to stress three assumptions driving them. The first one is that synergies that a bidder realizes are independent of its financial constraint. If synergies and financial constraints are connected, for example, via an agency problem between a bidder and its investors, the model could imply "intuitive" results that a higher financial constraint of a bidder reduces its maximum willingness to pay and makes the rival bidder more willing to initiate a contest, because it expects a weaker rival. The second important assumption is that a payment in stock is possible and does not result in a loss to a bidder-target pair. This assumption is consistent with models in which financial constraints come from transaction costs of issuing securities (e.g., fees to underwriters) that are avoided if the acquirer pays in stock. However, other models may be inconsistent with this assumption.[12] Finally, to some extent, it is also important that there is no entry deterrence. As we show in Section 5.1, relaxing this assumption adds nuances to these results.

A related question is about the cross-sectional relation between potential acquirers' financial constraints and their propensity to initiate acquisitions. Figure 2 illustrates that for any signal s, the more financially constrained bidder initiates the auction later than the less constrained one. Proposition 5 formalizes this result:

[12] For example, suppose that financial constraints occur because of the moral hazard of the manager: issuing new stock lowers the "skin in the game" of the manager and reduces firm value by lowering "effort." In such a model, a bidder would be unable to circumvent financial constraints by paying in stock.

Proposition 5. *If* $\lambda_i > \lambda_j$, *then* $\bar{X}_i(s) > \bar{X}_j(s)$ *for any* $s \in [\underline{s}, \bar{s}]$.

Proposition 5 implies that bidders learn about the rival's signal in an asymmetric way. When the signal of the less financially constrained bidder j exceeds $\bar{X}_j^{-1}\left(\bar{X}_i(\bar{s})\right)$ ($\hat{s}_2$ on Figure 2), bidder j does not learn anything about the signal of the rival bidder prior to initiating the auction. Therefore, the initiation threshold of type $\bar{s}$ of the less financially constrained bidder does not depend on the financial constraint of the more financially constrained rival, which is illustrated in Figure 2, where the two solid lines coincide at $\bar{s}$. In contrast, type $\bar{s}$ of the more financially constrained bidder i learns about the signal of the rival bidder as state X_t approaches bidder i's initiation threshold $\bar{X}_i(\bar{s})$. Hence, unlike $\bar{X}_j(\bar{s})$, $\bar{X}_i(\bar{s})$ depends on the financial constraint of the rival bidder. This is illustrated in Figure 2, where the two dashed lines do not coincide at $\bar{s}$.

4.2 Method of payment

In the equilibrium at the auction stage, the use of cash in the payment increases in the difference in the valuations of the winning bidder and its rival and decreases in the financial constraint of the winning bidder. Two implications follow. First, the payoff to the winning bidder is higher in deals involving a cash payment than in all-stock deals, in line with empirical evidence that acquirers' announcement stock returns are lower in stock acquisitions than in cash and mixed deals (Travlos 1987; Eckbo, Giammarino, and Heinkel 1990). In fact, if the market has the same information about bidders' valuations as the target, the announcement of an all-stock deal leads to a decline of the acquirer's stock price, also consistent with the empirical evidence. The price reaction is negative neither because the acquisition in stock is necessarily a signal of low valuation (indeed, a high-synergy acquirer pays in stock if the rival bidder's synergy is also high) nor because the acquirer overpays, but rather because the acquirer pays in stock if the required payment is close to its valuation. Participation costs then outweigh the acquirer's gains from the deal. Second, cash deals have higher synergies than stock deals on average. Intuitively, a cash deal requires the winning bidder to have high synergies, because otherwise it would have been better off paying in stock. This implication also appears to be consistent with the data (e.g., Andrade, Mitchell, and Stafford 2001; Bhagat et al. 2005).

Because the timing of the deal is endogenous, the model links payment method to the size of the deal and expected synergies. To evaluate this, we perform the following numerical analysis.

For a set of parameters in Table 1, we draw bidders' signals from uniform distribution over $[\mathbb{E}[s]-\Delta, \mathbb{E}[s]+\Delta]$ and simulate their synergies. We then relate the average deal size (measured by the threshold at which the deal takes place) and the average fraction of cash in the total payment. For illustrative purposes, we define "cash" ("stock") deals as deals that contain more (no more) than 50% cash of the total payment. Table 2 gives definitions of all studied quantities.

The bottom panels of Figure 4 illustrate that selection into cash and stock deals based on synergies leads to a smaller expected size of cash versus stock deals. This result arises because for baseline parameters the bidder is more likely to have low synergies than high synergies, implying a positively skewed distribution of synergies.[13] Thus, a typical stock deal occurs when both bidders have low synergies, who tend to have low signals and hence approach the target late. In contrast, a cash deal occurs when one of the bidders has high synergies. This bidder typically also has a high signal and hence approaches the target early.

The top panels of Figure 4 illustrate the effect of parameters on the average proportion of cash in the total payment. It is driven by two factors: first, by the proportion of cash in the payment, $1-\gamma$, when the high-synergy bidder wins against the low-synergy rival, and, second, by the probability of this event. An increase in Π_B, Π_T, or λ leads to a decrease in the proportion of cash $1-\gamma$. On the other hand, the probability of unequal synergies is given by $2\mathbb{E}[s](1-\mathbb{E}[s])$, which is independent of Π_B, Π_T, or financial constraints, and increases in $\mathbb{E}[s]$ when $\mathbb{E}[s] < \frac{1}{2}$, that is, the distribution of synergies has a positive skew. Thus, the model implies that more cash is used on average when bidders are less financially constrained and more optimistic about synergies, consistent with evidence in Martin (1996), Faccio and Masulis (2005), and Alshwer, Sibilkov, and Zaiats (2011).

Summarizing the above discussion, the model implies that more cash in a deal is associated with higher synergies, higher payoffs to the acquirers, and lower financial constraints of the acquirer. In addition, if the distribution of synergies has positive skew, mergers of smaller companies are more likely to rely on cash.

4.3 Initiating and winning bidders

Because each potential acquirer in the model makes both initiation and bidding decisions, the model derives links between the identities of initiating and winning bidders. Information about deal initiation is available in deal backgrounds, so these implications can be empirically examined.

[13] Positive skew is arguably more plausible than negative skew for the distribution of synergies.

As shown in Section 2, the identity of the winning bidder is determined by the bidders' synergies: The bidder with the highest synergy wins the auction. However, as shown in Section 3, the identity of the initiating bidder is also affected by relative financial constraints of the two bidders: given the same signal s, the less financially constrained bidder is the initiator. An implication follows:

Implication 1. *The probability that a bidder initiates the contest is decreasing in its financial constraint, holding the financial constraint of the rival bidder constant.*

The left panel of Figure 5 illustrates this implication. Combined with the fact that the identity of the winning bidder is determined by synergies and not by financial constraints, it leads to the following implication:

Implication 2. *The probability that the initiating bidder is also the acquirer exceeds 50%, if* $\lambda_i = \lambda_j$. *If* $\lambda_i \neq \lambda_j$, *the conditional probability that the initiating bidder wins the auction is higher for the more constrained bidder.*

The first part follows from the fact that the initiating bidder has a higher signal than the noninitiating bidder, when the two bidders have the same financial constraints. The second part follows from Proposition 5. For any signal, the more constrained bidder is less likely to be the initiating bidder. Hence, conditional on a bidder initiating the auction, it is more likely to have high synergies and win the auction if it is more constrained. The central panel of Figure 5 illustrates this implication.

Finally, the right panel of Figure 5 examines how the payment method relates to the identities of the initiating and winning bidders. When $\lambda_i = \lambda_j$, the average proportion of cash in the deal is higher when the identities of the initiating and the winning bidder coincide than when they are different. Recall that cash is used when the synergies of the two bidders differ but not when they are equal. Because the initiating bidder is more likely to have high synergies, the events in which the initiating bidder wins are more likely to correspond to events in which the two bidders' synergies are unequal. These are exactly the events in which the high-synergy bidder uses cash in the payment. On the other hand, the events in which the noninitiating bidder wins, for a positively-skewed distribution of synergies, are more likely to correspond to events in which the two bidders' synergies are low, resulting in stock payments. If one bidder is more constrained

than the other, the more constrained bidder includes less cash in the payment, so the average proportion of cash in deals done by the more constrained bidder declines.

4.4 Deviations from the optimal timing of acquisitions

Finally, we explore deviations of the equilibrium timing of the acquisition from the optimal one. Consider the planner's problem of choosing the timing of the auction, given signals s_1 and s_2. The probability that at least one of the bidders has high synergies is $s_1 + s_2 - s_1 s_2$. Hence, the optimal timing of the auction is given by threshold

$$\begin{aligned} X^*(s_1, s_2) &= \arg\max_{\hat{X}} \left(\frac{X_t}{\hat{X}}\right)^{\beta} \left(\frac{\hat{X}}{r-\mu}\left(v_l + (v_h - v_l)(s_1 + s_2 - s_1 s_2)\right) - 2I\right) \\ &= \frac{\beta}{\beta - 1} \frac{2I(r-\mu)}{v_l + (v_h - v_l)(s_1 + s_2 - s_1 s_2)}. \end{aligned} \tag{14}$$

We focus on the case of equal financial constraints. In this case, the auction is initiated at threshold (9) with signal $s_m \equiv \max\{s_1, s_2\}$. Its comparison with (14) can go either way because of two opposite effects. On the one hand, the initiating bidder pays a portion of the total value to the target but bears the full cost I. This effect delays the equilibrium timing of the auction compared to the optimal one. On the other hand, the initiating bidder imposes an externality on the rival bidder by forcing it to participate in the auction and pay the cost earlier than it would want to. This effect speeds up the equilibrium timing of the auction. The former effect dominates, that is, the auction occurs inefficiently late, if the initiating bidder's payoff from the auction is small, which happens when the support of signals $[\underline{s}, \bar{s}]$ is narrow (i.e., bidders are similar in their expected synergies).[14] In this case, a marginal relaxation of financial constraints will lead to a more efficient timing of deals.

In contrast, if the support of signals $[\underline{s}, \bar{s}]$ is wide (i.e., bidders are sufficiently different in their expected synergies), the latter effect dominates and the deal occurs inefficiently early. In this case, a marginal tightening of financial constraints will lead to a more efficient timing of deals.

[14] Intuitively, if $\underline{s}$ and $\bar{s}$ are close, the bidder with the lower signal is quite strong, which implies that the payoff of the initiating bidder is small.

5 Extensions and Discussion

5.1 Entry deterrence

In the base model, we assume that the parameters are such that the noninitiating bidder always participates in the auction in equilibrium. In practice, there are many single-bidder contests, so it is worth examining how our analysis is affected if the "no entry deterrence" constraint is violated. We assume that if a single-bidder contest occurs, the bidder makes a take-it-or-leave it offer (b, α) that the target accepts if and only if it perceives its value to be at least the valuation of the low-synergy bidder, $V(v_l) - V_o$.[15] For simplicity, we focus on the case in which both bidders have the same financial constraints λ. Like in the base model, we look for an equilibrium in which a bidder with signal s initiates an auction at threshold $\tilde{X}(s)$, where $\tilde{X}(s)$ is strictly decreasing.

If a bidder with signal s observes that the rival initiates the auction when X_t reaches threshold $\tilde{X}(\tilde{s})$, it participates if and only if its expected payoff from the auction, $s(1-\tilde{s})\psi(v_h, v_l)\frac{\tilde{X}(\tilde{s})}{r-\mu}$, exceeds I. Because the former is decreasing in $\tilde{s}$, a bidder that prefers not to participate in an auction initiated at some threshold also prefers not to participate in an auction initiated at a lower threshold. It is therefore natural to look for an equilibrium with the following structure: Types $[\underline{s}, \hat{s}_1)$ initiate the auction late enough, so that all types of the noninitiating bidder participate (the "no deterrence" region); when types $(\hat{s}_1, \hat{s}_2)$ initiate the auction, low enough types of the rival bidder prefer not to participate, while high types of the rival bidder participate (the "partial deterrence" region); finally, types $(\hat{s}_2, \bar{s}]$ initiate the auction early enough, so that no type of the rival bidder participates (the "full deterrence" region). To see the argument better, we construct the equilibrium assuming that all regions are nonempty, though depending on the parameters some of them may be empty. We only give the key steps of the construction here and leave the details for the Online Appendix.

First, consider a bidder with signal s in the "no deterrence" region, $s \in [\underline{s}, \hat{s}_1)$. Because both bidders always participate in the auction initiated at any threshold in the neighborhood of $\tilde{X}(s)$, the bidder's marginal incentives are the same as those used in the basic model. It follows that the equilibrium initiation strategy is the same in this range: $\tilde{X}(s) = \bar{X}(s)$, where $\bar{X}(s)$ is given by (9).

Second, consider a bidder with signal s in the "partial deterrence" region, $s \in (\hat{s}_1, \hat{s}_2)$. Because the set of types of the rival bidder that do not enter the auction depends on its belief about the

[15] In other words, a single-bidder contest is equivalent to a two-bidder auction in which one bidder has low synergies with probability one.

initiating bidder's signal, the bidder's initiation problem is a signaling game where the timing of initiation is a costly signaling device.[16] Intuitively, the initiating bidder wants to make the rival bidder believe that its signal is higher to keep more rival types away from the auction. In equilibrium, the rival correctly infers the signal of the initiating bidder from the timing of initiation, but signaling incentives speed up initiation. In the online appendix, we show that the equilibrium threshold in this range, $\tilde{X}(s) = \tilde{X}_{pd}(s)$, satisfies differential equation (A16).

Finally, consider a bidder with signal s in the "full deterrence" region, $s \in (\hat{s}_2, \bar{s})$. In this region, the initiating bidder has no signaling incentives, because the rival never enters the auction. The decision when to initiate the auction in this range is determined by the following trade-off. On the one hand, by initiating the auction earlier, the bidder foregoes the option to acquire the target in the future. On the other hand, delaying is costly for two reasons: the time value of money and the risk that the bidder is preempted if the rival bidder initiates the auction first. The latter cost is absent in the base model, where the identity of the initiating bidder does not affect payoffs from the auction. In the online appendix, we show that in this range, this trade-off results in the equilibrium threshold $\tilde{X}(s) = \tilde{X}_{fd}(s)$, which satisfies differential equation (A18). There, we also derive the initial value conditions $\tilde{X}_{pd}(\hat{s}_1)$ and $\tilde{X}_{fd}(\hat{s}_2)$, and the cutoff types $\hat{s}_1$ and $\hat{s}_2$. Intuitively, the expected value from the auction must be continuous at $\hat{s}_1$ and $\hat{s}_2$, because otherwise types just above or just below the cutoff would be better off deviating from their equilibrium initiation strategies. Also, thresholds $\tilde{X}_{pd}(\hat{s}_1)$ and $\tilde{X}_{fd}(\hat{s}_2)$ must be optimal for the cutoff types.

Figure 6 illustrates the equilibrium. A bidder with a sufficiently low synergy does not participate in the auction initiated early enough, that is, if the initiating bidder signals that the initiating bidder is strong enough. Compared to Fishman (1988), entry deterrence happens due to signaling via initiation timing, as opposed to the size of the bid. The potential to deter entry erodes the value of the option to delay approaching the target and speeds up the acquisition.[17]

The next proposition explores the role of financial constraint λ and shows that, somewhat surprisingly, it has no effect on equilibrium entry deterrence:

Proposition 6. *Consider the equilibrium described above. Cutoff types $\hat{s}_1$ and $\hat{s}_2$ do not depend on the level of financial constraint λ.*

[16] A class of option exercise games with signaling incentives is analyzed by Grenadier and Malenko (2011).

[17] This erosion becomes extreme in the limit case $\underline{s} \to \bar{s}$ and $\bar{s} \to 1$ when information about synergies becomes almost complete. In this case, a bidder initiates the auction at its "zero-NPV" threshold, at which the gains from the auction just cover cost I, and never joins the auction initiated by the rival.

Proposition 6 implies that, like in the basic model, financial constraints affect outcomes by changing the timing of the sale rather than the allocation within the auction. Figure 6 illustrates this result: A higher level of λ increases the initiation threshold but has no effect on equilibrium entry of the rival bidder.

The intuition for Proposition 6 is that there are two effects of higher λ. On the one hand, it reduces a bidder's expected payoff from the auction by a fraction in each of the three entry deterrence regimes. On the other hand, bidders delay initiation, implying that the target is bigger and, hence, the bidder's expected payoff from the auction is higher when the auction happens. With multiplicative synergies and financial constraints, the two effects exactly cancel out, implying that equilibrium entry is unaffected by financial constraints. While strong, this result relies on the multiplicative nature of the problem and on the assumption that both bidders have the same financial constraints.[18] However, it illustrates a broader point that financial constraints do not necessarily make bidders less aggressive.

5.2 Continuous distribution of synergies

We have assumed a binary distribution of synergies, which simplifies the solution. The downside is that the base model does not deliver rich implications for the takeover premium: The equilibrium takeover premium takes only two values. In addition, it implies that the average takeover premium is lower in cash deals than in stock deals, because a cash deal only occurs when the rival bidder's synergy is low. In this section, we analyze the model with a continuous distribution of synergies.

Suppose that synergies are distributed with conditional c.d.f. $G(v_i|s_i)$ with full support on $[v_l, v_h]$. In addition, suppose that a higher signal corresponds to a more optimistic distribution of synergies in the sense of first-order stochastic dominance: $\forall s < s'$ and $v \in (v_l, v_h)$, $G(v|s) > G(v|s')$. We restrict the analysis to the case of equal financial constraints ($\lambda_1 = \lambda_2 = \lambda$).

Consider the auction at time t. The analogous equilibrium to the base model is as follows. Each bidder drops out from the auction when the price reaches its valuation of the target, $V(v) - V_o$. The winning bidder makes an all-stock offer if its valuation of the target equals the price at which

[18] If the bidders differ in financial constraints, the cutoff types from Proposition 6 will be different for different bidders and will depend on both λ_1 and λ_2. In this case, the effect of a marginal increase in the rival's financial constraint (e.g., λ_2) on the bidder's initiation threshold (e.g., $\bar{X}_1(s)$) may also depend on s. For very low (in the "no deterrence" region) and very high s, a marginal increase in λ_2 will increase $\bar{X}_1(s)$; in constrast, in the intermediate region, it may decrease $\bar{X}_1(s)$, because bidder 1 will have greater incentives to speed up initiation to preempt bidder 2.

it wins the auction (i.e., if its synergy equals the synergy of the rival bidder). If the winning bidder's valuation of the target exceeds the price at which it wins the auction, its payment is a combination of cash and stock with the proportion of cash strictly increasing in the gap between the bidder's valuation and the winning price:[19]

Proposition 7. *At the auction stage, there exists a unique equilibrium in weakly dominant bidding strategies, in which different types of the winning bidder separate via cash-stock mixes, whose equilibrium values equal the price at which the losing bidder drops. A bidder with synergy v drops out once the price reaches*

$$p^*(v) = V(v) - V_o = \frac{\Pi_T + v}{r - \mu} X_t. \tag{15}$$

The higher-synergy bidder wins. If type v wins after the rival drops out at p, it offers

$$(b^*(p,v), \alpha^*(p,v)) = \left(p\left(1 - \gamma(p,v)\right), \frac{p}{V(v)} \gamma(p,v) \right), \tag{16}$$

where $\gamma(p,v) = \left(\frac{V_o + p}{V(v)}\right)^{\frac{1}{\lambda - 1}}$ is the fraction of stock in total offer value. For any v, $\gamma(p^*(v), v) = 1$.

Bidding strategies in Proposition 7 are similar to the ones in the model with binary synergies. Thus, the implications are also similar. First, a bidder's financial constraint affects the offer and its payoff from the auction but not its maximum willingness to pay. Second, the proportion of cash in the offer is driven primarily by two factors: the financial constraint of the acquirer and by how much the acquirer's valuation exceeds the price at which it wins.

The equilibrium at the auction stage pins down the expected payoffs from the auction of the initiating and the noninitiating bidder. Similarly to the base model, we can proceed with solving for the separating equilibrium in the continuous threshold strategies. The resultant equilibrium initiation threshold is similar to that in Proposition 2:

Proposition 8. *Let $m(v|s) \equiv E[G(v|z)|z \leq s]$ be the probability that the rival's synergy is be-*

[19] The difference from the binary model is that the separating equilibrium is no longer the unique equilibrium satisfying the CKIC criterion (Assumption 1). A stronger refinement, such as D1 (see Cho and Kreps 1987 for the definition), needs to be imposed to rule out nonseparating equilibria. Ramey (1996) shows that D1 selects the separating equilibrium as the unique one in a large class of signaling games with a continuum of types. We conjecture that his result extends to our model.

low v, conditional on its signal being below s. Let $m'(v|s) \equiv E[g(v|z)\,|z \leq s]$ be the corresponding density, and assume that $g(v|s)\,m'(w|s)$ is strictly increasing in s for any v and w. Let

$$\bar{X}(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu)I}{\int_{v_l}^{v_h} \int_{v_l}^{v} g(v|s) M'(w|s) \psi(v,w) dw dv}, \tag{17}$$

and suppose that conditions (32)–(33) in the appendix are satisfied. Then there exists a unique symmetric equilibrium in the initiation game. A bidder with signal s initiates the auction when X_t reaches upper threshold $\bar{X}(s)$, provided that the target has not been approached before. If the auction is initiated by the rival bidder at $\hat{X} \in \left[\bar{X}(\bar{s}), \bar{X}(\underline{s})\right]$, the bidder always participates in it.

The assumptions behind Proposition 8 are similar to Assumptions 2 and 3. The assumption that $g(v|s)\,m'(w|s)$ is strictly increasing in s ensures that the payoff of the initiating bidder is increasing in its signal, and conditions (32)–(33) ensure that there is no entry deterrence. The novel aspect of this model is that it generates richer implications for the acquisition premium. Figure 7 illustrates that the ranking between the average, across synergies, takeover premium in cash deals and in stock deals depends on model parameters. This is in contrast to the takeover premium conditional on the acquirer's synergy, which is always higher in stock deals. When the distribution of synergies has a positive skew and financial constraints are low, stock deals typically occur when both bidders have low synergies, leading to lower average premiums in stock deals. As Figure 7 shows, the average takeover premium in cash deals is higher than in stock deals when Π_B is high, the distribution of synergies is less skewed, or the financial constraints are high. For example, when bidders have high financial constraints, even deals with moderate differences between synergies are mostly completed in stock. As a result, average synergies of both the winning and the losing bidders in stock (cash) deals increase (decrease), leading to an increase (decrease) in average premiums in stock (cash) deals.

5.3 Active target

We have assumed that the auction is always initiated by a bidder, rather than the target. In this section, we show that this assumption is consistent with an equilibrium even if we allow for the target to be active. Consider the model in which the agreement of the target is necessary for the transaction to occur. In addition, suppose that initiation is publicly observed: If a bidder approaches the target to initiate the auction and the target declines, the rival bidder observes it.

First, we argue that the target has no incentive to delay the auction further when it is approached by a bidder.[20] Suppose that the target is approached by bidder i at some threshold $\bar{X}_i(s_i)$. At this point, the target infers bidder i's signal s_i, as well as the posterior distribution of bidder j's signal $s_j \sim [\underline{s}, \hat{s}(s_i)]$, where $\hat{s}(s_i) \equiv \bar{X}_j^{-1}\left(\bar{X}_i(s_i)\right)$. The payoff of the target from selling itself immediately when it is approached is thus $\frac{\bar{X}_i(s_i)}{r-\mu}\phi(s_i)$, where $\phi(s_i) \equiv \left(\Pi_T + \mathbb{E}\left[\min_{i\in\{1,2\}} v_i | s_i, s_j \leq \hat{s}(s_i)\right]\right)$. Suppose that if the target deviates to delaying the auction when the bidder approaches it, the other bidder does not approach it before the target initiates the auction voluntarily. Then, the expected payoff of the target from delaying the auction until threshold $\hat{X} > \bar{X}_i(s_i)$ is $\left(\frac{\bar{X}_i(s_i)}{\hat{X}}\right)^{\beta}\frac{\hat{X}}{r-\mu}\phi(s_i)$. This payoff is strictly decreasing in $\hat{X}$. Thus, the target does not benefit from delaying the auction when it is approached by the bidder.

Second, consider incentives of the target to accelerate the auction prior to being approached by either bidder. Suppose that the target invites both bidders to participate in the auction via a take-it-or-leave-it offer at at threshold $\bar{X}_T$ prior to either bidder approaching it.[21] We show that the target's commitment to such an offer is dynamically inconsistent, that is, not credible. At $\bar{X}_T$, the target infers posterior distributions of bidders' signals $s_i \sim [\underline{s}, \hat{s}_i]$, $i \in \{1,2\}$, where $\hat{s}_i \equiv \bar{X}_i^{-1}(\bar{X}_T)$. Suppose, prior to the target's offer, that each bidder believes that no type of the rival bidder accepts the target's offer to sell itself. By backward induction, if the target's offer is rejected by bidders and later, bidder i attempts to initiate the auction at some threshold X_t, there can be two outcomes. First, the target follows its promise, rejects conducting the auction, and obtains the payoff of $\frac{X_t}{r-\mu}\Pi_T$. Second, the target reneges on its promise, accepts the bidder's offer, and obtains $\frac{X_t}{r-\mu}\phi(s_i, s_j)$, where $\phi(s_i, s_j) \equiv \left(\Pi_T + \mathbb{E}\left[\min_{i\in\{1,2\}} v_i | s_i \leq \hat{s}_i, s_j \leq \hat{s}_j\right]\right)$. The second case yields a strictly higher payoff, implying that the target will find it optimal to renege on its promise. Thus, if each bidder believes the rival will not accept the target's offer to participate in the auction, the bidder also ignores the offer due to the target's inability to commit not to sell itself in the future. Hence, an equilibrium of the base model remains in this extension.[22]

[20] This argument applies to the basic model, in which the "no entry deterrence" assumption holds. If there is entry deterrence in equilibrium, like in Section 5.1, the target may prefer to delay the auction if approached by the bidder early. Intuitively, by delaying the auction, it makes it more competitive, because the noninitiating bidder will be more likely to enter. Allowing the target to be active in the model of Section 5.1, will introduce a participation constraint of the target and potentially make entry deterrence less likely.

[21] The case of $\bar{X}_T$ being the upper threshold, which can possibly be crossed more than once, is similar.

[22] There may be other equilibria, in which alternative beliefs about the rival's actions result in target-initiated auctions. Their characterization is beyond the scope of this paper.

6 Conclusion

We study how bidders' financial constraints affect the M&A market, that is, their incentives to approach targets, size of bids, and the payment method. We propose a tractable model based on three building blocks: dynamic decision-making, private information of bidders, and financial constraints. Four main results are derived. First, because of ability to pay in stock, financial constraints do not affect bidders' maximum willingness to pay, in contrast to models in which bids are restricted to be made in cash. Second, financial constraints affect bidders' decisions to approach the target in the first place. Specifically, both a bidder's own and a rival bidder's constraint discourage the bidder from initiating the auction. Third, auctions are initiated by bidders with low financial constraints or high synergies. Fourth, cash as a method of payment is positively associated with synergies and the acquirer's gains from the deal and negatively with financial constraints.

Two extensions of our model could be fruitful. First, it can be interesting to incorporate aggregate shocks to financial constraints, for example, by modeling them as a two-state Markov chain, like in Bolton, Chen, and Wang (2013). Second, the structure of private information could be made richer by allowing for two-sided private information, when the target is privately informed about its stand-alone value, in addition to bidders being informed about synergies, or for one-sided two-dimensional private information, when a bidder is privately informed about its synergies and the value of its assets in place. In the latter case, financial constraints would likely affect the identity of the winning bidder, since the size of the stock bid at a bidder's indifferent price point would not be fully revealing when the bidder's private information is two-dimensional. Li, Taylor, and Wang (2017) estimate a static model of this kind and find misallocation to be quantitatively small.

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Appendix: Proofs

Proof of Proposition 1. Consider bidder i. Let $p_i^*(v)$ be the dropout price for bidder i with synergy $v \in \{v_l, v_h\}$. Let $(b_i^*(p,v), \alpha_i^*(p,v))$ be the equilibrium offer of bidder i with synergy $v \in \{v_l, v_h\}$ if it wins at price $p \leq p_i^*(v)$. We prove the proposition in a sequence of seven steps. Steps 1–4 consider what happens if bidder i wins at or below price $p_i^*(v_l)$, that is, the price at which the low-synergy type of bidder i quits the auction. Step 1 shows if the offers of the low- and high-synergy types of bidder differ, then type v_l makes an all-stock offer. Step 2 shows that if the offers of the low- and high-synergy acquirers differ, then type v_h of bidder i includes just enough cash to dissuade type v_l of bidder i from mimicking. Step 3 shows that, under Assumption 1, offers of the low- and high-synergy types indeed differ. Finally, Step 4 solves for $p_i^*(v_l)$ and shows that it does not depend on λ_i: $p_i^*(v_l) = p^*(v_l)$. Steps 5 and 6 consider what happens if the high-synergy type bidder i wins at or close to price $p_i^*(v_h)$, that is, the price at which the high-synergy type of bidder i quits the auction. Steps 5 shows that it makes an all-stock offer in this case. Intuitively, that bidder i stayed in the auction until such a high price by itself signals that its synergy is high, so there is no need it include cash in the offer. Step 6 solves for $p_i^*(v_h)$ in a way similar to Step 4 and shows that it does not depend on λ_i. Finally, Step 7 calculates the amount of cash that the high-synergy type of bidder i needs to include in the bid to signal its type when it wins at price $p^*(v_l)$. Online Appendix provides a detailed proof of each step.

Proof of Proposition 2. Please refer to the proof of Proposition 3. The case $\lambda_1 = \lambda_2$ is discussed at the end of that proof.

Proof of Proposition 3. First, we solve the problem under the assumption that the noninitiating bidder participates in all initiated deals. We then characterize parameter restrictions, under which the derived equilibrium exists when the noninitiating bidder chooses whether to participate in the auction strategically.

Taking the derivative of (12) with respect to $\hat{X}$ and canceling the terms, we obtain

$$\begin{aligned} & -\beta X_t^{\beta} \hat{X}^{-\beta-1} \int_{\underline{s}}^{\bar{X}_j^{-1}(\hat{X})} \left(s(1-z)\psi_i(v_h, v_l) \frac{\hat{X}}{r-\mu} - I \right) \frac{dF(z)}{F\left(\bar{X}_j^{-1}(\max_{u\in[0,t]} X_u)\right)} \\ + \quad & X_t^{\beta} \hat{X}^{-\beta} \int_{\underline{s}}^{\bar{X}_j^{-1}(\hat{X})} s(1-z)\psi_i(v_h, v_l) \frac{1}{r-\mu} \frac{dF(z)}{F\left(\bar{X}_j^{-1}(\max_{u\in[0,t]} X_u)\right)}. \end{aligned}$$

The first-order condition equates this derivative to zero, which yields

$$s\psi_i(v_h, v_l) \frac{\hat{X}}{r-\mu} \left(1 - \mathbb{E}\left[z | z \leq \bar{X}_j^{-1}(\hat{X})\right]\right) = \frac{\beta}{\beta - 1} I. \tag{18}$$

Since $\bar{X}_j(s)$ is decreasing in s, $\bar{X}_j^{-1}(\hat{X})$ is decreasing in $\hat{X}$. Therefore, the left-hand side of (18) is strictly increasing in $\hat{X}$, taking values from zero to infinity. Hence, given $\bar{X}_j(\cdot)$, there exists a unique threshold $\hat{X}$ that solves (18). This threshold is a local maximum. Since there is only one local maximum and there are no local minima, it is also a global maximum.

In equilibrium, threshold $\hat{X}$ that solves (18) must be given by $\bar{X}_i(s)$. We obtain a system of two

equations:

$$\bar{X}_i(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu)I}{s\psi_i(v_h, v_l) m\left(\bar{X}_j^{-1}(\bar{X}_i(s))\right)}.$$

If the system of two equations has solutions $\bar{X}_i(s)$ and $\bar{X}_j(s)$, then this is an equilibrium, since given $\bar{X}_j(\cdot)$, threshold $\bar{X}_i(s)$ maximizes the payoff of bidder i, as shown in the previous paragraph. Otherwise, no separating equilibrium in continuous threshold strategies exists. For our numerical examples, we establish the existence and uniqueness of the equilibrium numerically.

For the case $\lambda_1 = \lambda_2$, $\bar{X}_j^{-1}(\bar{X}_i(s)) = s$, so instead of having a system of two equations, we have a single equation for $\bar{X}_i(s) = \bar{X}(s)$:

$$\bar{X}(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu)I}{sm(s)\psi(v_h, v_l)}.$$

Hence, the symmetric equilibrium exists and is unique.

In the above derivations, we made two assumptions:

1. Any noninitiating bidder i always joins the auction initiated at threshold $\bar{X}_j(s)$ for any $s \in [\underline{s}, \bar{s}]$, $j \neq i$.

2. No bidder i with signal s is better off deviating to a low enough threshold $\hat{X}$ that deters entry of some of the types of the rival bidder.

We follow by deriving restrictions on the parameters of the model, such that these two statements hold as results. The restrictions depend on the expected payoff of the bidder in the auction without the rival bidder. To obtain conditions that guarantee the equilibrium for any possible such payoff, assume that it is the highest possible: if only the initiating bidder joins the auction, it acquires the target for the value of the low-synergy bidder $p^*(v_l)$. In this case, the payoff of a bidder with signal s from initiating the auction at time t is $s\psi_i(v_h, v_l)\frac{X_t}{r-\mu}$.

Consider the first condition. Take a noninitiating bidder i with signal s and an auction initiated by bidder j at threshold $\hat{X} \in \left[\bar{X}_j(\bar{s}), \bar{X}_j(\underline{s})\right]$. Using (6), bidder i is better off participating in the auction if and only if

$$s\left(1 - \bar{X}_j^{-1}\left(\hat{X}\right)\right)\psi_i(v_h, v_l)\frac{\hat{X}}{r-\mu} \geq I.$$

The left-hand side is strictly increasing in s and $\hat{X}$. Therefore, it is optimal for a noninitiating bidder i to participate in the auction for any signal $s \in [\underline{s}, \bar{s}]$ and threshold $\hat{X} \in \left[\bar{X}_j(\bar{s}), \bar{X}_j(\underline{s})\right]$ if and only if

$$\underline{s}(1 - \bar{s})\psi_i(v_h, v_l)\frac{\bar{X}_j(\bar{s})}{r-\mu} \geq I.$$

Plugging in $\bar{X}_j(s)$ from (13),

$$\underline{s} \geq \frac{\beta - 1}{\beta}\frac{\psi_j(v_h, v_l)}{\psi_i(v_h, v_l)}\frac{\bar{s}}{1-\bar{s}} m\left(\bar{X}_i^{-1}(\bar{X}_j(\bar{s}))\right).$$

Since this condition must hold for each bidder,

$$\underline{s} \geq \frac{\beta - 1}{\beta}\frac{\bar{s}}{1-\bar{s}} \max\left\{\frac{\psi_1(v_h, v_l)}{\psi_2(v_h, v_l)} m\left(\bar{X}_2^{-1}(\bar{X}_1(\bar{s}))\right), \frac{\psi_2(v_h, v_l)}{\psi_1(v_h, v_l)} m\left(\bar{X}_1^{-1}(\bar{X}_2(\bar{s}))\right)\right\}. \tag{19}$$

When the two bidders have the same financial constraints, this inequality simplifies to

$$\underline{s} \geq \frac{\beta - 1}{\beta} \frac{\bar{s}}{1 - \bar{s}} m(\bar{s}). \tag{20}$$

As an illustration, consider our baseline parameters: $\beta = 1.285$, $\bar{s} = 0.25$, and uniform distribution of signals. Then (20) implies $\underline{s} \geq 0.062$, which is satisfied by our baseline value $\underline{s} = 0.1$. To illustrate condition (19), consider the set of parameters in Table 1 and the uniform distribution of signals over $[\mathbb{E}[s] - \Delta, \mathbb{E}[s] + \Delta]$. Fix $\mathbb{E}[s] = 0.175$, which is the level we use in our figures, and vary Δ from 0. In this case, (19) holds if and only if Δ is below 0.099. If we fix $\mathbb{E}[s] = 0.5$, then (19) holds if and only if Δ is below 0.206.

Consider the second condition. Suppose that bidder i with signal s deviates to initiating the auction at threshold $\hat{X} < \bar{X}_i(\bar{s})$. Upon deviation, bidder j perceives the signal of bidder i to be $\bar{s}$. Using (6), bidder j is better off entering the auction if and only if

$$s_j \geq \hat{s}_j\left(\hat{X}\right) \equiv \frac{(r - \mu) I}{\hat{X}(1 - \bar{s}) \psi_j(v_h, v_l)}.$$

Thus, bidder j with signal $s_j < \hat{s}_j\left(\hat{X}\right)$ does not participate in the auction, that is, its entry is deterred. Let $U_i\left(s, \hat{X}\right)$ denote the expected payoff at the auction time to bidder i with signal s, when bidder i deviates to initiating the auction at threshold $\hat{X} < \bar{X}_i(\bar{s})$:

$$U_i\left(s, \hat{X}\right) = \Pi_B \frac{\hat{X}}{r - \mu} + s\psi_i(v_h, v_l) \frac{\hat{X}}{r - \mu} \left(1 - \int_{\left[\hat{s}_j\left(\hat{X}\right)\right]_{\underline{s}}^{\bar{s}}} z dF(z)\right) - I, \tag{21}$$

where $\left[\hat{s}_j\left(\hat{X}\right)\right]_{\underline{s}}^{\bar{s}}$ denotes $\min\left\{\max\left\{\underline{s}, \hat{s}_j\left(\hat{X}\right)\right\}, \bar{s}\right\}$, that is, $\hat{s}_j\left(\hat{X}\right)$ truncated at $\underline{s}$ from below and at $\bar{s}$ from above. In contrast, bidder i's expected payoff as of this time from following the strategy of initiating the auction at threshold $\bar{X}_i(s)$, if it has not been initiated before, is

$$\begin{aligned} V_i\left(s, \hat{X}\right) &= \Pi_B \frac{\hat{X}}{r - \mu} + \int_{\underline{s}}^{\bar{s}} \left(\frac{\hat{X}}{\min\left\{\bar{X}_i(s), \bar{X}_j(z)\right\}}\right)^{\beta} \left(s(1 - z)\psi_i(v_h, v_l) \frac{\min\left\{\bar{X}_i(s), \bar{X}_j(z)\right\}}{r - \mu} - I\right) dF(z) \\ &= \Pi_B \frac{\hat{X}}{r - \mu} + \max_{\tilde{X}} \int_{\underline{s}}^{\bar{s}} \left(\frac{\hat{X}}{\min\left\{\tilde{X}, \bar{X}_j(z)\right\}}\right)^{\beta} \left(s(1 - z)\psi_i(v_h, v_l) \frac{\min\left\{\tilde{X}, \bar{X}_j(z)\right\}}{r - \mu} - I\right) dF(z) \end{aligned} \tag{22}$$

Let us show that it is sufficient to verify that the bidder with signal $\bar{s}$ does not benefit from deviating to initiating the auction at $\hat{X} < \bar{X}_i(\bar{s})$. Suppose that there exists a pair $\left(s, \hat{X}\right)$ at which $U_i\left(s, \hat{X}\right) >$

$V_i\left(s,\hat{X}\right)$. Differentiating (21) and (22) in s, where for the latter we use the envelope theorem:

$$\begin{aligned}
\frac{\partial U_i\left(s,\hat{X}\right)}{\partial s} &= \psi_i\left(v_h,v_l\right)\frac{\hat{X}}{r-\mu}\left(1-\int_{\left[\hat{s}_j\left(\hat{X}\right)\right]_{\underline{s}}^{\bar{s}}}^{\bar{s}} z dF\left(z\right)\right)\\
\frac{\partial V_i\left(s,\hat{X}\right)}{\partial s} &= \frac{\psi_i(v_h,v_l)}{r-\mu}\int_{\underline{s}}^{\bar{s}}\frac{\hat{X}^{\beta}}{\min\left\{\bar{X}_i\left(s\right),\bar{X}_j\left(z\right)\right\}^{\beta-1}}\left(1-z\right)dF\left(z\right)\\
&< \frac{\psi_i(v_h,v_l)}{r-\mu}\left(1-\int_{\underline{s}}^{\bar{s}} z dF\left(z\right)\right) < \frac{\partial U_i\left(s,\hat{X}\right)}{\partial s},
\end{aligned}$$

where the first inequality holds from $\min\left\{\bar{X}_i\left(s\right),\bar{X}_j\left(z\right)\right\} > \hat{X}$. Therefore, $U_i\left(s,\hat{X}\right) > V_i\left(s,\hat{X}\right)$ implies $U_i\left(s',\hat{X}\right) > V_i\left(s',\hat{X}\right)$ for any $s' > s$. Hence, it is sufficient to verify the condition for the bidder with the highest signal $\bar{s}$.

Given this, we obtain conditions for $U_i\left(\bar{s},\hat{X}\right) \leq V_i\left(\bar{s},\hat{X}\right)$ for any $\hat{X} < \bar{X}_i\left(\bar{s}\right)$. First, consider $\hat{X} \geq \frac{(r-\mu)I}{\underline{s}(1-\bar{s})\psi_j(v_h,v_l)}$. If bidder i deviates to initiating the auction at such threshold, all types of bidder j participate in the auction. Therefore,

$$\begin{aligned}
U_i\left(\bar{s},\hat{X}\right) &= \Pi_B\frac{\hat{X}}{r-\mu} + \bar{s}\psi_i\left(v_h,v_l\right)\frac{\hat{X}}{r-\mu}\left(1-\int_{\underline{s}}^{\bar{s}} z dF\left(z\right)\right) - I\\
&\leq \Pi_B\frac{\hat{X}}{r-\mu} + \int_{\underline{s}}^{\bar{s}}\left(\frac{\hat{X}}{\min\left\{\bar{X}_i\left(\bar{s}\right),\bar{X}_j\left(z\right)\right\}}\right)^{\beta}\left(\bar{s}\left(1-z\right)\psi_i(v_h,v_l)\frac{\min\left\{\bar{X}_i\left(\bar{s}\right),\bar{X}_j\left(z\right)\right\}}{r-\mu} - I\right)dF\left(z\right),
\end{aligned}$$

since $\bar{X}_i\left(\bar{s}\right)$ maximizes $\int_{\underline{s}}^{\bar{s}}\left(\frac{\hat{X}}{\min\left\{\tilde{X},\bar{X}_j(z)\right\}}\right)^{\beta}\left(s\left(1-z\right)\psi_i(v_h,v_l)\frac{\min\left\{\tilde{X},\bar{X}_j(z)\right\}}{r-\mu} - I\right)dF\left(z\right)$ over $\tilde{X}$. Hence, deviation to any $\hat{X} \in \left[\frac{(r-\mu)I}{\underline{s}(1-\bar{s})\psi_j(v_h,v_l)}, \bar{X}_i\left(\bar{s}\right)\right)$ cannot be profitable. Second, consider $\hat{X} < \frac{(r-\mu)I}{\underline{s}(1-\bar{s})\psi_j(v_h,v_l)}$. If bidder i deviates to initiating the auction at such threshold, types $s \in \left[\underline{s}, \hat{s}_j\left(\hat{X}\right)\right)$ choose not to participate in the auction. Then, condition $U_i\left(\bar{s},\hat{X}\right) \leq V_i\left(\bar{s},\hat{X}\right)$ can be rewritten as

$$\begin{aligned}
&\bar{s}\psi_i\left(v_h,v_l\right)\frac{\hat{X}}{r-\mu}\left(1-\int_{\min\left\{\hat{s}_j\left(\hat{X}\right),\bar{s}\right\}}^{\bar{s}} z dF\left(z\right)\right) - I\\
\leq\ &\int_{\underline{s}}^{\bar{s}}\left(\frac{\hat{X}}{\min\left\{\bar{X}_i\left(\bar{s}\right),\bar{X}_j\left(z\right)\right\}}\right)^{\beta}\left(\bar{s}\left(1-z\right)\psi_i(v_h,v_l)\frac{\min\left\{\bar{X}_i\left(\bar{s}\right),\bar{X}_j\left(z\right)\right\}}{r-\mu} - I\right)dF\left(z\right).
\end{aligned}$$

Since this condition must hold for all $\hat{X} < \frac{(r-\mu)I}{\underline{s}(1-\bar{s})\psi_j(v_h,v_l)}$, it is equivalent to

$$\begin{aligned}
&\sup_{\hat{X}<\frac{(r-\mu)I}{\underline{s}(1-\bar{s})\psi_j\left(v_h,v_l\right)}}\left\{\hat{X}^{-\beta}\left(\bar{s}\psi_i\left(v_h,v_l\right)\frac{\hat{X}}{r-\mu}\left(1-\int_{\min\left\{\hat{s}_j\left(\hat{X}\right),\bar{s}\right\}}^{\bar{s}} z dF\left(z\right)\right) - I\right)\right\} \qquad (23)\\
\leq\ &\int_{\underline{s}}^{\bar{s}}\min\left\{\bar{X}_i\left(\bar{s}\right),\bar{X}_j\left(z\right)\right\}^{-\beta}\left(\bar{s}\left(1-z\right)\psi_i(v_h,v_l)\frac{\min\left\{\bar{X}_i\left(\bar{s}\right),\bar{X}_j\left(z\right)\right\}}{r-\mu} - I\right)dF\left(z\right)
\end{aligned}$$

When the two bidders have the same financial constraints, this inequality simplifies to

$$\sup_{\hat{X}<\frac{(r-\mu)I}{\underline{s}(1-\bar{s})\psi(v_h,v_l)}}\left\{\hat{X}^{-\beta}\left(\bar{s}\psi\left(v_h,v_l\right)\frac{\hat{X}}{r-\mu}\left(1-\int_{\min\left\{\hat{s}(\hat{X}),\bar{s}\right\}}^{\bar{s}}zdF\left(z\right)\right)-I\right)\right\}\leq\left(\frac{\beta-1}{\beta}\frac{\bar{s}m(\bar{s})\psi(v_h,v_l)}{(r-\mu)I}\right)^{\beta}\frac{I}{\beta-1}dF\left(z\right). \tag{24}$$

Lemma 1. *The only equilibrium in monotone initiation thresholds that can exist is the one in which* $\bar{X}_i\left(s\right)$, $i\in\{1,2\}$ *is decreasing in* s.

Proof. See Online Appendix.

Proof of Proposition 4. Let $\zeta_i\left(x\right)=\bar{X}_i^{-1}\left(x\right)$, that is, the type of bidder i that initiates the auction at threshold x. Note that $\zeta_i\left(x\right)$ is only well defined for $x\in\left[\bar{X}_i\left(\bar{s}\right),\bar{X}_i\left(\underline{s}\right)\right]$. To define $\zeta_i\left(x\right)$ for all $x>0$, let $\zeta_i\left(x\right)=\bar{s}$, if $x<\bar{X}_i\left(\bar{s}\right)$, and $\zeta_i\left(x\right)=\underline{s}$, if $x>\bar{X}_i\left(\underline{s}\right)$. Then the system of equations (13) can be written as

$$\delta_i(x,\zeta_i(x),\zeta_{-i}\left(x\right))=0,$$

for $i\in\{1,2\}$, where

$$\delta_i\left(x,\zeta_i,\zeta_{-i}\right)\equiv x\zeta_i m(\zeta_{-i})-\frac{\beta}{\beta-1}\frac{(r-\mu)I}{\psi_i\left(v_h,v_l\right)}. \tag{25}$$

The following auxiliary result will be useful to prove the proposition:

Lemma 2. $\frac{\partial\delta_1(x,\zeta_1,\zeta_2)}{\partial\zeta_1}\frac{\partial\delta_2(x,\zeta_2,\zeta_1)}{\partial\zeta_2}-\frac{\partial\delta_1(x,\zeta_1,\zeta_2)}{\partial\zeta_2}\frac{\partial\delta_2(x,\zeta_2,\zeta_1)}{\partial\zeta_1}>0$ for any $\forall\, x\in\left[\min_{i\in\{1,2\}}\bar{X}_i\left(\bar{s}\right),\max_{i\in\{1,2\}}\bar{X}_i\left(\underline{s}\right)\right]$.

Proof of Lemma 2. See Online Appendix.

With Lemma 2, we are equipped to prove comparative statics. We use notations $\zeta_i\left(x|\theta\right)$ and $\delta_i\left(x,\zeta_i,\zeta_j|\theta\right)$, where θ is the parameter of interest. The derivative of $\delta_i\left(x,\zeta_i\left(x|\theta\right),\zeta_j\left(x|\theta\right)|\theta\right)$ in θ:

$$\frac{\partial\delta_i}{\partial\theta}+\frac{\partial\delta_i}{\partial\zeta_i}\frac{\partial\zeta_i}{\partial\theta}+\frac{\partial\delta_i}{\partial\zeta_j}\frac{\partial\zeta_j}{\partial\theta}=0.$$

Multiply the equation for δ_i by $\frac{\partial\delta_j}{\partial\zeta_j}$, the equation for δ_j by $\frac{\partial\delta_i}{\partial\zeta_j}$, and then subtract the latter from the former:

$$\begin{aligned}&\left(\frac{\partial\delta_i\left(x,\zeta_i,\zeta_j|\theta\right)}{\partial\zeta_i}\frac{\partial\delta_j\left(x,\zeta_j,\zeta_i|\theta\right)}{\partial\zeta_j}-\frac{\partial\delta_j\left(x,\zeta_j,\zeta_i|\theta\right)}{\partial\zeta_i}\frac{\partial\delta_i\left(x,\zeta_i,\zeta_j|\theta\right)}{\partial\zeta_j}\right)\frac{\partial\zeta_i\left(x|\theta\right)}{\partial\theta}\\ =\;&\frac{\partial\delta_j\left(x,\zeta_j\left(x|\theta\right),\zeta_i\left(x|\theta\right)|\theta\right)}{\partial\theta}\frac{\partial\delta_i\left(x,\zeta_i,\zeta_j|\theta\right)}{\partial\zeta_j}-\frac{\partial\delta_i\left(x,\zeta_i\left(x|\theta\right),\zeta_j\left(x|\theta\right)|\theta\right)}{\partial\theta}\frac{\partial\delta_j\left(x,\zeta_j,\zeta_i|\theta\right)}{\partial\zeta_j}.\end{aligned} \tag{26}$$

The term in the brackets on the left-hand side is positive by Lemma 2, so the sign of $\frac{\partial\zeta_i(x|\theta)}{\partial\theta}$ coincides with the sign of the right-hand side. Furthermore, as shown in Lemma 2, $\frac{\partial\delta_i\left(x,\zeta_i,\zeta_j\right)}{\partial\zeta_j}\leq 0$ and $\frac{\partial\delta_j\left(x,\zeta_j,\zeta_i\right)}{\partial\zeta_j}>0$. Therefore, if $\frac{\partial\delta_i\left(x,\zeta_j,\zeta_j\right)}{\partial\theta}$ and $\frac{\partial\delta_j\left(x,\zeta_j,\zeta_j\right)}{\partial\theta}$ have the same sign for all x, ζ_j, and ζ_j, then the sign of $\frac{\partial\zeta_i(x|\theta)}{\partial\theta}$ is the opposite of the sign of $\frac{\partial\delta_i\left(x,\zeta_j,\zeta_j\right)}{\partial\theta}$.

Effects of σ and I. Differentiate δ_i with respect to β:

$$\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j|\beta\right)}{\partial \beta}=\frac{1}{(\beta-1)^2}\frac{(r-\mu)I}{\psi_i(v_h,v_l)}>0$$

Therefore, $\frac{\partial \zeta_i(x|\beta)}{\partial \beta}<0$. Because β is decreasing in σ and σ affects the solution only through β, $\zeta_i(x)$ is increasing in σ. Therefore, $\bar{X}_i(s)$ is increasing in σ. By the same argument $\bar{X}_i(s)$ is increasing in I.

Effect of r. Take the derivative of $\delta_i(\cdot)$ in r:

$$\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j|r\right)}{\partial r}=-\frac{I}{\psi_i(v_h,v_l)}\frac{d}{dr}\left[\frac{\beta(r-\mu)}{\beta-1}\right]=-\frac{I}{\psi_i(v_h,v_l)}\frac{d}{dr}\left[r+\frac{\sigma^2\beta}{2}\right],$$

where the transformation in the brackets comes from the quadratic equation defining β, $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-r=0$. By definition of β, β is strictly increasing in r. Therefore, $\frac{\partial \delta_i}{\partial r}<0$. Hence, $\bar{X}_i(s)$ is increasing in r.

Effect of μ. In a manner similar to that above, take the derivative of $\delta_i(\cdot)$ in μ,

$$\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j|\mu\right)}{\partial \mu}=-\frac{I}{\psi_i(v_h,v_l)}\frac{\sigma^2}{2}\frac{d\beta}{d\mu},$$

where

$$\frac{d\beta}{d\mu}=\frac{1}{\sigma^2}\left[-1+\frac{\left(\mu-\frac{\sigma^2}{2}\right)}{\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2r\sigma^2}}\right]<0.$$

Therefore, $\frac{\partial \delta_i}{\partial \mu}>0$. Hence, $\bar{X}_i(s)$ is decreasing in μ.

Effects of v_h (holding v_l fixed), Π_B, and Π_T. For $\theta\in\{v_h,\Pi_B,\Pi_T\}$, we have

$$\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j|\theta\right)}{\partial \theta}=\frac{\beta}{\beta-1}\frac{(r-\mu)I}{\psi_i(v_h,v_l)^2}\frac{d\psi_i(v_h,v_l)}{d\theta}.$$

For $\theta=v_h$,

$$\begin{aligned}\frac{d\psi_i(v_h,v_l)}{dv_h} &= 1-\frac{(\Pi_T+\Pi_B+v_l)(\Pi_T+v_l)}{\left(\Pi_T+\Pi_B+v_l+\frac{\lambda_i}{\lambda_i-1}(v_h-v_l)\right)^2}\\ &> 1-\left(\frac{1}{1+\frac{\lambda_i}{\lambda_i-1}\frac{v_h-v_l}{\Pi_T+\Pi_B+v_l}}\right)^2>0,\end{aligned}$$

where the first inequality follows from $\Pi_B>0$. For $\theta=\Pi_B$,

$$\frac{d\psi_i(v_h,v_l)}{d\Pi_B}=\frac{(\lambda_i-1)^2(v_h-v_l)(\Pi_T+v_l)}{((\lambda_i-1)(\Pi_T+\Pi_B+v_h)+v_h-v_l)^2}>0.$$

For $\theta=\Pi_T$,

$$\frac{d\psi_i(v_h,v_l)}{d\Pi_T}=-\frac{((\lambda_i-1)\Pi_B+\lambda_i(v_h-v_l))(v_h-v_l)(\lambda_i-1)}{((\lambda_i-1)(\Pi_T+\Pi_B+v_h)+v_h-v_l)^2}<0.$$

Hence, $\frac{\partial \delta_i}{\partial v_h}>0$, $\frac{\partial \delta_i}{\partial \Pi_B}>0$, and $\frac{\partial \delta_i}{\partial \Pi_T}<0$. Therefore, $\bar{X}_i(s)$ is decreasing in v_h and Π_B and increasing in Π_T.

Effects of λ_i and λ_j. Differentiating in λ_i, we have

$$\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j|\lambda_i\right)}{\partial \lambda_i}=\frac{\beta}{\beta-1}\frac{(r-\mu)I}{\psi_i(v_h,v_l)^2}\frac{d\psi_i\left(v_h,v_l\right)}{d\lambda_i},$$

where

$$\frac{\partial \psi_i\left(v_h,v_l\right)}{\partial \lambda_i}=-\frac{\left(\lambda v_h-v_l\right)\left(\Pi_T+v_l\right)\left(v_h-v_l\right)}{\left(\left(\lambda_i-1\right)\left(\Pi_T+\Pi_B\right)+\lambda v_h-v_l\right)^2}<0.$$

Furthermore,

$$\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j|\lambda_j\right)}{\partial \lambda_j}=0.$$

Since $\frac{\partial \delta_j\left(x,\zeta_j,\zeta_i\right)}{\partial \zeta_j}>0$, as shown in Lemma 2, the right-hand side of (26) is positive for $\theta=\lambda_i$. Hence, $\bar{X}_i\left(s\right)$ is increasing in λ_i. As for $\theta=\lambda_j$, as shown in Lemma 2, $\frac{\partial \delta_i\left(x,\zeta_i,\zeta_j\right)}{\partial \zeta_j}\leq 0$ with strict inequality for $x\in\left[\bar{X}_j\left(\bar{s}\right),\min_{i\in\{1,2\}}\bar{X}_i\left(\underline{s}\right)\right]$ and strict equality for $x\notin\left[\bar{X}_j\left(\bar{s}\right),\min_{i\in\{1,2\}}\bar{X}_i\left(\underline{s}\right)\right]$. Hence, when $\theta=\lambda_j$, the right-hand side of (26) is positive, if $x\in\left[\bar{X}_j\left(\bar{s}\right),\min_{i\in\{1,2\}}\bar{X}_i\left(\underline{s}\right)\right]$, and equals zero, otherwise. Hence, $\bar{X}_i\left(s\right)$ is increasing in λ_j in the range of values at which $\bar{X}_i\left(s\right)\in\left[\bar{X}_j\left(\bar{s}\right),\min_{i\in\{1,2\}}\bar{X}_i\left(\underline{s}\right)\right]$, that is, which correspond to the rival bidder with some signal approaching the target at the same threshold. If $\bar{X}_i\left(s\right)\notin\left[\bar{X}_j\left(\bar{s}\right),\min_{i\in\{1,2\}}\bar{X}_i\left(\underline{s}\right)\right]$, then $\bar{X}_i\left(s\right)$ is unaffected by a marginal change in λ_j.

Proof of Proposition 5. Denote $C_i=\frac{\beta}{\beta-1}\frac{(r-\mu)I}{\psi_i(v_h,v_l)}$; $C_i>C_j$ if and only if $\lambda_i>\lambda_j$. By contradiction, suppose that there exists $s\in[\underline{s},\bar{s}]$ such that $\bar{X}_i\left(s\right)<\bar{X}_j\left(s\right)$. Because $\bar{X}_i\left(s\right)$ and $\bar{X}_j\left(\cdot\right)$ are continuous, either there exists signal s' satisfying $\bar{X}_i\left(s'\right)=\bar{X}_j'\left(s'\right)$ or $\bar{X}_i\left(s\right)<\bar{X}_j\left(s\right)$ for all $s\in[\underline{s},\bar{s}]$. The former is not possible, because it implies that

$$\bar{X}_i\left(s'\right)=\frac{C_i}{s'm_i(s')}=\frac{C_i}{s'm(s')}>\frac{C_j}{s'm(s')}=\frac{C_j}{s'm_j(s')}=\bar{X}_j\left(s'\right),$$

which contradicts $\bar{X}_i\left(s'\right)=\bar{X}_j'\left(s'\right)$. It remains that $\bar{X}_i\left(s\right)<\bar{X}_j\left(s\right)\ \forall s\in[\underline{s},\bar{s}]$, in particular, $\bar{X}_i\left(\bar{s}\right)<\bar{X}_j\left(\bar{s}\right)$. However, it is not possible, because in this case

$$\bar{X}_i\left(\bar{s}\right)=\frac{C_i}{\bar{s}m_i(\bar{s})}=\frac{C_i}{\bar{s}m(\bar{s})}>\frac{C_j}{\bar{s}m(\bar{s})}>\frac{C_j}{\bar{s}m_j(\bar{s})}=\bar{X}_j\left(\bar{s}\right),$$

which contradicts $\bar{X}_i\left(\bar{s}\right)<\bar{X}_j\left(\bar{s}\right)$. The second inequality is due to $\mathbb{E}[z|z\leq\bar{s}]>\mathbb{E}[z|z\leq\bar{X}_i^{-1}(\bar{X}_j(\bar{s}))]$ if $\bar{X}_i\left(\bar{s}\right)<\bar{X}_j\left(\bar{s}\right)$. Therefore, $\bar{X}_i\left(s\right)>\bar{X}_j\left(s\right)\ \forall s\in[\underline{s},\bar{s}]$.

Proof of Proposition 6. We prove a stronger result: cutoff types $\hat{s}_1$ and $\hat{s}_2$ depend only on β (i.e., on r, μ, and σ) and distribution $F\left(\cdot\right)$. To do this, we show that equilibrium threshold can be written as

$$\hat{X}\left(s\right)=\frac{\left(r-\mu\right)I}{\psi\left(v_h,v_l\right)}\tilde{Y}\left(s\right),\tag{27}$$

where $\tilde{Y}\left(s\right)$ only depends on s, β, and the distribution of signals $F\left(\cdot\right)$. This implies that $\hat{s}_1$ and $\hat{s}_2$ depend only on β and $F\left(\cdot\right)$. First, consider the "no deterrence" region. In this range, $\hat{X}\left(s\right)=\bar{X}\left(s\right)$, which from (A14) takes the form of (27) with $\tilde{Y}\left(s\right)=\beta/\left(\left(\beta-1\right)sm\left(s\right)\right)$. Second, consider the "partial deterrence" region, in which $\hat{X}\left(s\right)=\hat{X}_{pd}\left(s\right)$ is given by (A16) with initial value condition (A20). Plugging (27) into

(A20), we obtain that $\tilde{X}_{pd}(\hat{s}_1) = \frac{(r-\mu)I}{\psi(v_h,v_l)}\tilde{Y}_{pd}(\hat{s}_1)$, where $\tilde{Y}_{pd}(\hat{s}_1)$ satisfies

$$(\beta-1)\,\hat{s}_1\tilde{Y}_{pd}(\hat{s}_1)\left(1-\int_{\frac{1}{(1-\hat{s}_1)\tilde{Y}_{pd}(\hat{s}_1)}}^{\hat{s}_1} z\frac{dF(z)}{F(\hat{s}_1)}\right) = \beta - \frac{f\left(\frac{1}{(1-\hat{s}_1)\tilde{Y}_{pd}(\hat{s}_1)}\right)}{F(\hat{s}_1)}\frac{\hat{s}_1}{(1-\hat{s}_1)^2\tilde{Y}_{pd}(\hat{s}_1)}. \quad (28)$$

Because (28) includes only β, $F(\cdot)$, and $\hat{s}_1$, $\tilde{Y}_{pd}(\hat{s}_1)$ only depends on these parameters. Plugging (27) for $s \uparrow \hat{s}_1$ and $s \downarrow \hat{s}_1$ into (A19), we obtain

$$\begin{aligned}&\left(\frac{\tilde{Y}_{pd}(\hat{s}_1)(\beta-1)\hat{s}_1 m(\hat{s}_1)}{\beta}\right)^{\beta}\int_{\underline{s}}^{\hat{s}_1}\left(\frac{\beta(1-z)}{(\beta-1)m(\hat{s}_1)}-1\right)dF(z)\\ =\;&\left(\int_{\underline{s}}^{\hat{s}_1}\left(\hat{s}_1\left(1-z1\left\{z>\frac{1}{(1-\hat{s}_1)\tilde{Y}_{pd}(\hat{s}_1)}\right\}\right)\tilde{Y}_{pd}(\hat{s}_1)-1\right)dF(z)\right).\end{aligned}$$

Since this equation includes only β, $F(\cdot)$, $\hat{s}_1$, and $\tilde{Y}_{pd}(\hat{s}_1)$, and the latter only depends on β, $F(\cdot)$, and $\hat{s}_1$, we conclude that $\hat{s}_1$ only depends on β and $F(\cdot)$. Plugging (27) into (A16), we obtain $\tilde{X}_{pd}(s) = \frac{(r-\mu)I}{\psi(v_h,v_l)}\tilde{Y}_{pd}(s)$ where $\tilde{Y}_{pd}(s)$ satisfies

$$(\beta-1)\tilde{Y}_{pd}(s)\left(1-\int_{\frac{1}{(1-s)\tilde{Y}_{pd}(s)}}^{s} z\frac{dF(z)}{F(s)}\right) = \frac{\beta}{s} - \frac{f\left(\frac{1}{(1-s)\tilde{Y}_{pd}(s)}\right)}{F(s)}\frac{1-s-\frac{\tilde{Y}_{pd}(s)}{\tilde{Y}'_{pd}(s)}}{(1-s)^3\tilde{Y}_{pd}(s)}.$$

Because this equation includes only β, $F(\cdot)$, and s, and the initial value condition only depends on β and $F(\cdot)$, $\tilde{Y}_{pd}(s)$ only depends on β, $F(\cdot)$, and s. Finally, consider the "full deterrence" region, in which $\tilde{X}(s) = \tilde{X}_{fd}(s)$ is given by (A18) with initial value condition (A23). The latter takes the form of (27) with $\tilde{Y}(\hat{s}_2) = 1/(\hat{s}_2(1-\hat{s}_2))$. Plugging this and $\tilde{X}_{pd}(s) = \frac{(r-\mu)I}{\psi(v_h,v_l)}\tilde{Y}_{pd}(s)$ into (A22), we obtain

$$\int_{\underline{s}}^{\hat{s}_2}\left(\hat{s}_2\left(1-z1\left\{z>\frac{1}{(1-\hat{s}_2)\tilde{Y}_{pd}(\hat{s}_2)}\right\}\right)\tilde{Y}_{pd}(\hat{s}_2)-1\right)dF(z) = \tilde{Y}_{pd}(\hat{s}_2)^{\beta}\hat{s}_2^{\beta+1}(1-\hat{s}_2)^{\beta-1}.$$

Because this equation includes only β, $F(\cdot)$, $\hat{s}_2$, and $\tilde{Y}_{pd}(\hat{s}_2)$, and the later only depends on β, $F(\cdot)$, and $\hat{s}_2$, we conclude that $\hat{s}_2$ only depends on β and $F(\cdot)$. Plugging (27) into (A18), we obtain $\tilde{X}_{fd}(s) = \frac{(r-\mu)I}{\psi(v_h,v_l)}\tilde{Y}_{fd}(s)$ where $\tilde{Y}_{fd}(s)$ satisfies

$$(\beta-1)s\tilde{Y}_{fd}(s) = \beta + \left(s\tilde{Y}_{fd}(s)-1\right)\frac{f(s)}{F(s)}\frac{\tilde{Y}_{fd}(s)}{\tilde{Y}'_{fd}(s)}.$$

Because this equation includes only β, $F(\cdot)$, and s, and the initial value condition only depends on β and $F(\cdot)$, $\tilde{Y}_{fd}(s)$ only depends on β, $F(\cdot)$, and s. Therefore, $\tilde{X}(s)$ can be written as (27), where $\tilde{Y}(s)$ only depends on β, $F(\cdot)$, and s. In particular, cutoff types $\hat{s}_1$ and $\hat{s}_2$ do not depend on λ.

Proof of Proposition 7. Let the equilibrium strategy of bidder i with synergy v be to drop out at price $p^*(v)$ and to submit bid $(b^*(p,v), \alpha^*(p,v))$, if it wins at price $p \leq p^*(v)$. By analogy with the model

with the binary distribution of synergies,

$$p^*(v) = V(v) - V_o = \frac{\Pi_T + v}{r - \mu} X_t, \tag{29}$$

that is, the value of the target as a stand-alone entity plus the value of additional synergies. Consider the payment by the winner if the losing bidder drops out at price p. At this point, the target believes that $v \in \left[p^{*-1}(p), v_h\right]$, where, from (29):

$$p^{*-1}(p) = \frac{(r-\mu)p}{X_t} - \Pi_T.$$

In the separating equilibrium, the lowest type of the winner takes the efficient action, meaning that type $p^{*-1}(p)$ submits an all-equity bid:

$$\begin{aligned} b^*\left(p, p^{*-1}(p)\right) &= 0, \\ \alpha^*\left(p, p^{*-1}(p)\right) &= \frac{p}{V(p^{*-1}(p))}. \end{aligned}$$

No other bid could be incentive compatible for type $p^{*-1}(p)$. If this type submitted any different bid, it would strictly benefit from deviating to the all-equity bid. Not only would this deviation reduce the cost of paying with cash, but also its value could not be perceived worse, since $p^{*-1}(p)$ is the lowest possible belief that the target can hold in this subgame. Each type $v > p^{*-1}(p)$ pays a positive amount of cash $b^*(p, v)$, which is increasing in v. Consider offer (b, α) and fix the target's belief at $\tilde{v}$. Because the value of (b, α) must at least be p,

$$\alpha V(\tilde{v}) + b \geq p.$$

The equilibrium in which the target obtains the exact price at which the losing bidder dropped has $\alpha V(\tilde{v}) + b = p$. We can back out $\alpha = \frac{p-b}{V(\tilde{v})}$ and write the problem as a signaling problem with one signal b. Given b, the true type v, and the belief $\tilde{v}$, the payoff to the bidder is

$$U(v, \tilde{v}, b) = \left(1 - \frac{p-b}{V(\tilde{v})}\right) V(v) - \lambda b = V(v) - (p-b)\frac{V(v)}{V(\tilde{v})} - \lambda b.$$

If the separating equilibrium is differentiable, it can be found by solving the differential equation (e.g., Mailath 1987):

$$\frac{\partial b^*(p,v)}{\partial v} = -\frac{\frac{\partial}{\partial b} U\left(v, v, b^*(p,v)\right)}{\frac{\partial}{\partial \tilde{v}} U\left(v, v, b^*(p,v)\right)} = \left(p - b^*(p,v)\right)\frac{V'(v)}{(1-\lambda)V(v)},$$

subject to the initial value condition $b^*\left(p, p^{*-1}(p)\right) = 0$. Fixing p, this equation implies

$$-\log\left(p - b^*(p,v)\right) = \frac{1}{\lambda - 1}\log V(v) + C$$

for some integration constant C. Rewriting and imposing the initial value condition $b^*\left(p, p^{*-1}(p)\right) = 0$ yields

$$b^*(p,v) = p\left(1 - \gamma(p,v)\right), \tag{30}$$

$$\alpha^*(p,v) = \frac{p}{V(v)}\gamma(p,v), \tag{31}$$

where $\gamma(p,v) \equiv \left(\frac{V_o + p}{V(v)}\right)^{\frac{1}{\lambda - 1}}$ is the proportion of stock in the total offer value. We need to verify the

single-crossing condition to make sure that no type benefits from large deviations:

$$\begin{aligned}\frac{\frac{\partial}{\partial b}U(v,\tilde{v},b)}{\frac{\partial}{\partial \tilde{v}}U(v,\tilde{v},b)} &= \frac{\frac{V(v)}{V(\tilde{v})}-\lambda}{(p-b)\frac{V(v)}{V(\tilde{v})^2}V'(\tilde{v})} = \frac{V(\tilde{v})}{(p-b)V'(\tilde{v})} - \frac{\lambda V(\tilde{v})^2}{(p-b)V(v)V'(\tilde{v})};\\ \frac{\partial\left[\frac{\frac{\partial}{\partial b}U(v,\tilde{v},b)}{\frac{\partial}{\partial \tilde{v}}U(v,\tilde{v},b)}\right]}{\partial v} &= \frac{\lambda V(\tilde{v})^2}{(p-b)V(v)^2}\frac{V'(v)}{V'(\tilde{v})} > 0.\end{aligned}$$

Hence, the single-crossing condition is satisfied. Finally, we verify the regularity conditions in Mailath and von Thadden (2013) to ensure that there is no nondifferentiable separating equilibrium. Smoothness holds. Consider belief monotonicity:

$$\frac{\partial}{\partial \tilde{v}}U(v,\tilde{v},b) = (p-b)\frac{V(v)}{V(\tilde{v})^2}V'(\tilde{v}) > 0,$$

so belief monotonicity is verified. Consider type monotonicity:

$$\frac{\partial}{\partial v}U(v,\tilde{v},b) = V'(v) - \frac{p-b}{V(\tilde{v})}; \quad \frac{\partial^2}{\partial v \partial b}U(v,\tilde{v},b) = \frac{1}{V(\tilde{v})} > 0.$$

so type monotonicity is verified. Consider "relaxed" concavity:

$$\frac{\partial}{\partial b}U(v,v,b) = \frac{V(v)}{V(v)} - \lambda = 1-\lambda \neq 0.$$

Hence, "relaxed" concavity is satisfied. Finally, it is without loss of generality to restrict $b \leq p$, that is, to restrict signals to a compact set. Indeed, no equilibrium can feature $b > p$, since the bidder benefits from a deviation to $b = 0$. Then, according to Theorem 3 in Mailath and von Thadden (2013), the separating equilibrium $b^*(p,v)$ must be differentiable in v. Hence, (30)–(31) is indeed the unique separating equilibrium.

Proof of Proposition 8. Similarly to Proposition 3, we first solve the problem assuming that bidders participate in any initiated auction. We later give sufficient conditions for this to be the case. It is useful to calculate the payoff of bidder with synergy v if it wins against the rival with synergy $w \leq v$:

$$\begin{aligned}&\left(1-\alpha\left(p^*(w),v\right)\right)V(v) - \lambda b^*\left(p^*(w),v\right)\\ =\;& V(v) - p^*(w) - (\lambda-1)b^*(p^*(w),v).\end{aligned}$$

Hence, the incremental payoff over the stand-alone (or over the losing bidder's) value is

$$\psi(v,w)\frac{X_t}{r-\mu} \equiv V(v) - p^*(w) - (\lambda-1)b^*\left(p^*(w),v\right) - V_o.$$

Note that it is linear in X_t.

Suppose that the auction is initiated at time t; conjecture a symmetric equilibrium in strictly decreasing initiation thresholds. Let bidder 1 with signal s_1 be the initiating bidder. In any hypothetical separating equilibrium of the initiation game, bidder 1 believes s_2 is distributed over $[\underline{s}, s_1]$ with p.d.f. $\frac{f(s)}{F(s_1)}$. Consider the expected payoff of bidder 1. The synergy of bidder 1 is a draw from $G(v|s_1)$. The synergy of bidder 2 is a draw from $G(v|s_2)$. Bidder 1 perceives that s_2 is distributed over $[\underline{s}, s_1]$ with p.d.f. $\frac{f(s)}{F(s_1)}$. Hence,

given information of bidder 1, the c.d.f. of bidder 2's synergy is

$$\Pr\{v_2 \leq v\} = \int_{\underline{s}}^{s_1} G(v|s) \frac{f(s)}{F(s_1)} ds = \mathbb{E}[G(v|s)|s \leq s_1].$$

The corresponding density is $\mathbb{E}[g(v|s)|s \leq s_1]$. Hence, the expected payoff of bidder 1 from the auction is

$$\Pi_B \frac{X_t}{r-\mu} + \int_{v_l}^{v_h} g(v|s_1) \left(\int_{v_l}^{v} \mathbb{E}[g(w|s)|s \leq s_1] \psi(v,w) \frac{X_t}{r-\mu} dw \right) dv - I.$$

Consider a bidder with signal s deciding when to initiate the auction when it expects the rival bidder with signal z to initiate at threshold $\bar{X}(z)$, where $\bar{X}(z)$ is a strictly decreasing and differentiable function. If the bidder initiates at threshold $\hat{X}$, its expected payoff at any time t before initiation is

$$\Pi_B \frac{X_t}{r-\mu} + \left(\frac{X_t}{\hat{X}}\right)^{\beta} \int_{\underline{s}}^{\bar{X}^{-1}(\hat{X})} \int_{v_l}^{v_h} \int_{v_l}^{v} \left(g(v|z)g(w|z)\psi(v,w) \frac{\hat{X}}{r-\mu} dw dv - I \right) \frac{dF(z)}{F\left(\bar{X}^{-1}(\max_{\tau\in[0,t]} X_\tau)\right)}$$
$$+ \int_{\bar{X}^{-1}(\hat{X})}^{\bar{X}^{-1}(\max_{\tau\in[0,t]} X_\tau)} \left(\frac{X_t}{\bar{X}(z)}\right)^{\beta} \int_{v_l}^{v_h} \int_{v_l}^{v} \left(g(v|z)g(w|z)\psi(v,w) \frac{\bar{X}(z)}{r-\mu} dw dv - I \right) \frac{dF(z)}{F\left(\bar{X}^{-1}(\max_{\tau\in[0,t]} X_\tau)\right)},$$

where we define $\bar{X}^{-1}(X_0) = \bar{s}$. We maximize the above expression with respect to $\hat{X}$ and apply the equilibrium condition that the maximum must be reached at $\hat{X} = \bar{X}(s)$. Then, we check that we have indeed found the maximum by taking the second-order derivative of the expected payoff and evaluating it at $\hat{X} = \bar{X}(s)$. All the steps are the same as those used in the proof of Proposition 3. We obtain:

$$\bar{X}(s) = \frac{\beta}{\beta - 1} \frac{(r-\mu) I}{\int_{v_l}^{v_h} \int_{v_l}^{v} g(v|s) \mathbb{E}[g(w|z)|z \leq s] \psi(v,w) dw dv}.$$

Finally, we need to check two conditions to make sure that entry deterrence does not occur:

1. Any noninitiating bidder i always joins the auction initiated at threshold $\bar{X}(s)$ for any $s \in [\underline{s}, \bar{s}]$, $j \neq i$.

2. No bidder i with signal s is better off deviating to a low enough threshold $\hat{X}$ that deters entry of some of the types of the rival bidder.

Consider the first condition. The noninitiating bidder i with signal s is better off participating in the auction initiated at threshold $\hat{X}$ if and only if

$$\int_{v_l}^{v_h} \int_{v_l}^{v} g(v|s) g(w|\bar{X}^{-1}\left(\hat{X}\right)) \frac{\hat{X}}{r-\mu} \psi(v,w) dw dv \geq I.$$

Since the left-hand side is strictly increasing in s and $\hat{X}$, it is sufficient to verify the condition for $s = \underline{s}$ and $\hat{X} = \bar{X}(\bar{s})$:

$$\int_{v_l}^{v_h} \int_{v_l}^{v} g(v|\underline{s}) g(w|\bar{s}) \psi(v,w) dw dv \geq \frac{I(r-\mu)}{\bar{X}(\bar{s})}. \tag{32}$$

Consider the second condition. Suppose that a bidder with signal s deviates to initiating the auction at threshold $\hat{X} < \bar{X}(\bar{s})$. By the same argument made in Proposition 3, it is sufficient to verify the condition for the bidder with the highest signal $\bar{s}$. The rival bidder with signal z is better off entering the auction if

and only if

$$\int_{v_l}^{v_h}\int_{v_l}^{v} g(v|z)g(w|\bar{s})\frac{\hat{X}}{r-\mu}\psi(v,w)dwdv \geq I.$$

Let $\hat{z}\left(\hat{X}\right)$ denote the lowest signal z at which this inequality is satisfied. Since the left-hand side is strictly increasing in z, the rival bidder enters the auction if and only if $z \geq \hat{z}\left(\hat{X}\right)$. Let $\Upsilon\left(s,\hat{X}\right)$ denote the expected payoff at the auction to the bidder with signal s, if it initiates the auction at threshold $\hat{X} < \bar{X}(\bar{s})$:

$$\Upsilon\left(\bar{s},\hat{X}\right) = \frac{\hat{X}}{r-\mu}\left(\Pi_B + F\left(\hat{z}\left(\hat{X}\right)\right)\int_{v_l}^{v_h} g(v|\bar{s})\,\psi(v,\underline{v})\,dv + \int_{v_l}^{v_h}\int_{v_l}^{v}\int_{\hat{z}\left(\hat{X}\right)}^{\bar{s}} g(v|\bar{s})\,g(w|z)\,f(z)\,\psi(v,w)dzdwdv\right) - I.$$

The second condition is satisfied if and only if

$$\Upsilon\left(\bar{s},\hat{X}\right) \leq V\left(\bar{s},\hat{X}\right)\ \forall \hat{X} < \bar{X}(\bar{s}). \tag{33}$$

Table 1: Benchmark model parameters

Variable	Description	Value
r	Discount rate	0.05
μ	Growth rate of the state	0.03
σ	Volatility of the state	0.25
Π_B	Bidder's cash flow multiple	2
Π_T	Target's cash flow multiple	1
I	Participation cost	1
λ	Financial constraint	1.02 (low) or 1.2 (high)
$\mathbb{E}[s]$	Average signal	0.175
$F(s)$	Distribution of signals	$\mathbb{E}[s]$+Uniform$[-0.075, 0.075]$
Binary distribution of synergies only (Sections 1–4)		
v_l	Low value of synergy	0
v_h	High value of synergy	1
$G(v\|s)$	Distribution of synergies given signal	$\Pr\{v_i = v_h\|s\} = s$
Continuous distribution of synergies only (Section 5.2)		
v_l	Lowest value of synergy	0
v_h	Highest value of synergy	1
$G(v\|s)$	Distribution of synergies given signal	$(v/v_h)^{2s}$

Table 2: Definitions of expected outcomes for Sections 4.2, 4.3, and 5.2

The dependence of equilibrium γ on both bidders' synergies is made explicit in the definitions. In computation of conditional expected outcomes for the model with binary synergies, we account for the fact that if $v_1 = v_2$, each bidder wins with 50% probability.

Quantity	Definition
Unconditional expected outcomes (Sections 4.2 and 5.2)	
Average share of cash	$\mathbb{E}[1 - \gamma(v_1, v_2)\|s_1, s_2, v_1, v_2]$
Initiation threshold of a cash deal	$\mathbb{E}[\min_{i\in\{1,2\}} \bar{X}_i(s_i)\|\ s_1, s_2, v_1, v_2 \text{ s.t. } \gamma(v_1, v_2) < 0.5]$
Initiation threshold of a stock deal	$\mathbb{E}[\min_{i\in\{1,2\}} \bar{X}_i(s_i)\|\ s_1, s_2, v_1, v_2 \text{ s.t. } \gamma(v_1, v_2) \geq 0.5]$
Average premium in a cash deal	$\mathbb{E}\left[\frac{\min_{i\in\{1,2\}} p^*(v_i)}{\Pi_T/(r-\mu)} - 1 \middle\| s_1, s_2, v_1, v_2 \text{ s.t. } \gamma(v_1, v_2) < 0.5\right]$
Average premium in a stock deal	$\mathbb{E}\left[\frac{\min_{i\in\{1,2\}} p^*(v_i)}{\Pi_T/(r-\mu)} - 1 \middle\| s_1, s_2, v_1, v_2 \text{ s.t. } \gamma(v_1, v_2) \geq 0.5\right]$
Conditional expected outcomes (Section 4.3)	
% deals initiated by i	$\mathbb{E}\left[\mathbb{I}\{\bar{X}_i(s_i) < \bar{X}_j(s_j)\} \middle\| s_1, s_2\right]$
% deals won by i if initiated by i	$\mathbb{E}\left[\mathbb{I}\{v_i > v_j\} \middle\| s_1, s_2, v_1, v_2 \text{ s.t. } \bar{X}_i(s_i) < \bar{X}_j(s_j)\right]$
Average share of cash if initiated (won) by i (i)	$\mathbb{E}\left[1 - \gamma(v_1, v_2) \middle\| s_1, s_2, v_1, v_2 \text{ s.t. } \bar{X}_i(s_i) < \bar{X}_j(s_j), v_i > v_j\right]$
Average share of cash if initiated (won) by i (j)	$\mathbb{E}\left[1 - \gamma(v_1, v_2) \middle\| s_1, s_2, v_1, v_2 \text{ s.t. } \bar{X}_i(s_i) < \bar{X}_j(s_j), v_i < v_j\right]$

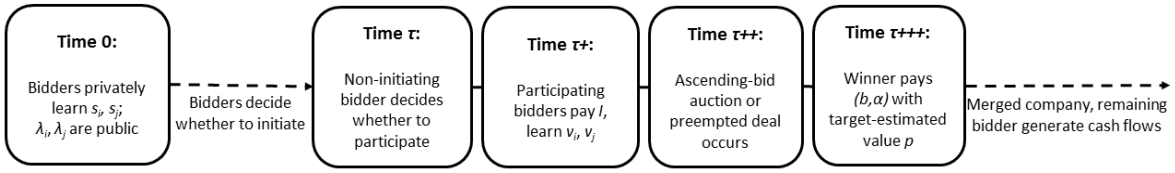

Figure 1. Time line of the model

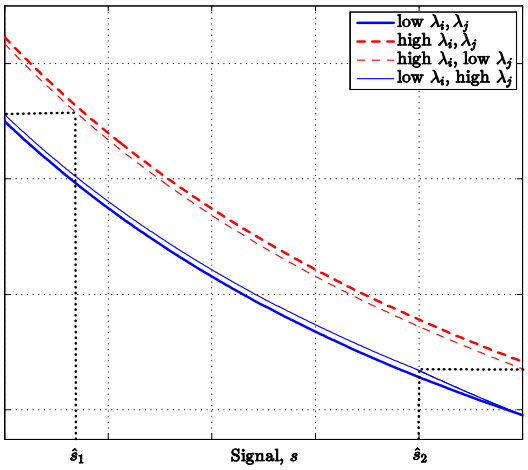

Figure 2. Equilibrium initiation thresholds
The figure plots the equilibrium initiation threshold of bidder i, $\bar{X}_i(s)$, as a function of its signal for different combinations of its own (λ_i) and the rival's (λ_j) financial constraint parameters. The thick lines show the equilibrium initiation thresholds when both bidders have the same financial constraints. The thin lines show the equilibrium initiation thresholds when bidders have different financial constraints: when bidder i's financial constraint parameter is low (high), her initiation threshold is shown by the solid (dashed) line, and the bidder j's initiation threshold is shown by the dashed (solid) line. For the case of different financial constraints, $\hat{s}_1$is the signal below which the more constrained bidder never initiates on equilibrium path; $\hat{s}_2$is the signal, above which the less constrained bidder always initiates first and does not learn about its more constrained rival's signal.

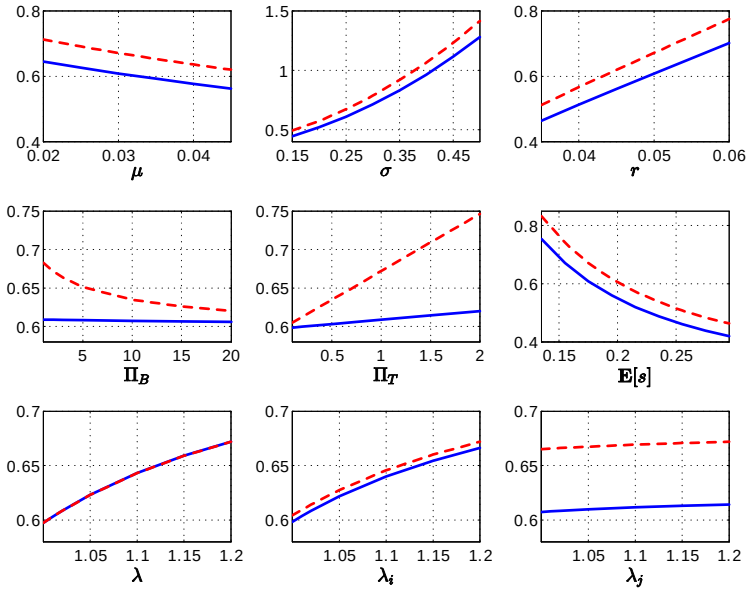

Figure 3. Equilibrium initiation threshold: Comparative statics The figure plots the equilibrium initiation threshold of a bidder with a signal fixed at the expected level, $E[s]$, as a function of various parameters of the model. The parameters are listed in Table 1. The solid (dashed) line is the initiation threshold of a bidder with low (high) financial constraints.

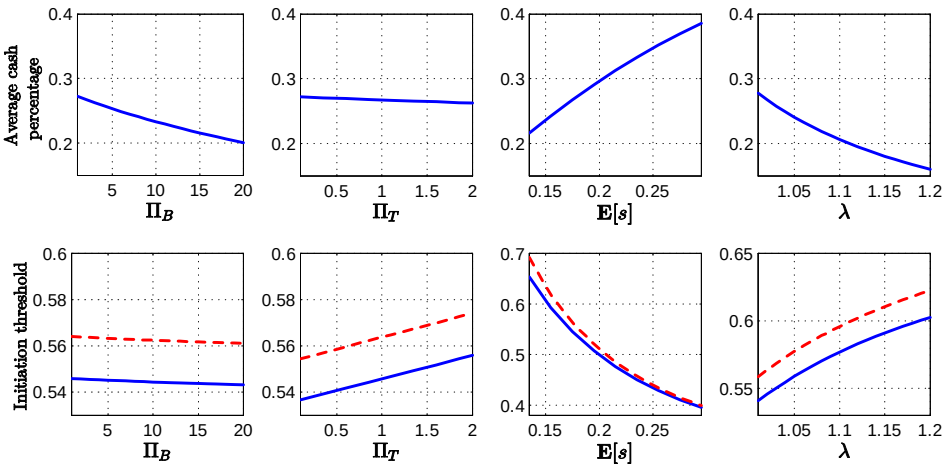

Figure 4. Average fraction of cash in the payment and average timing of cash and stock deals as functions of model parameters. The top panel of the figure plots the expected fraction of cash in the total payment of the acquirer as a function of model parameters. The bottom panel of the figure plots the expected initiation threshold, conditional on the deal being a "cash" deal (more than 50% of the payment is paid in cash; solid line) and a "stock" deal (no more than 50% of the payment is paid in cash; dashed line). We use $\lambda_1 = \lambda_2 = 1.02$. The other parameters are listed in Table 1.

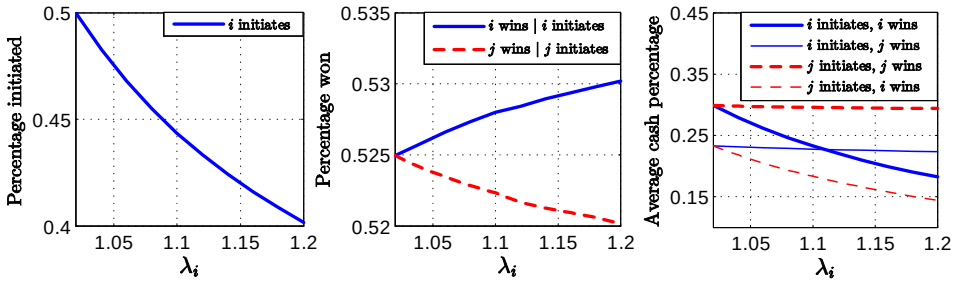

Figure 5. Initiating versus winning bidders
The figure plots the probability that the deal is initiated by bidder i (the left panel), the probability that the initiating bidder is the winner (the central panel), and the average fraction of cash in the total payment for different combinations of identities of the initiating and winning bidders (the right panel), as functions of bidder i's financial constraint parameter λ_i. We use $\lambda_j = 1.02$. Table 1 lists the other parameters.

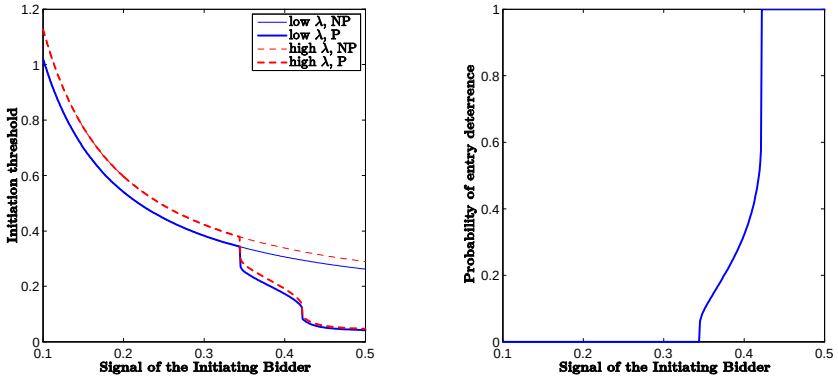

Figure 6. Equilibrium in the model with entry deterrence The left panel plots the equilibrium initiation threshold of a bidder as a function of its signal, when $\lambda = 1.02$ ("low λ, P") and when $\lambda = 1.2$ ("high λ, P"). It compares them to the equilibrium initiation threshold in the model in which each bidder is assumed to always enter the auction initiated by the rival bidder ("low λ, NP" and "high λ, NP", respectively). The right panel plots the equilibrium probability that the rival bidder is preempted (i.e., a single-bidder contest) as a function of the signal of the initiating bidder. Table 1 lists the parameters.

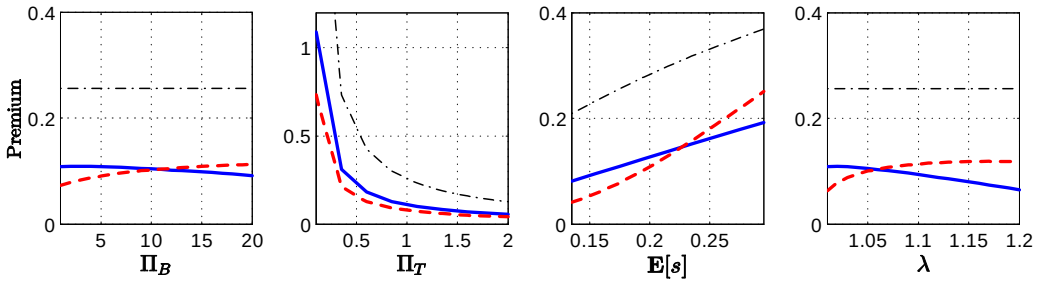

Figure 7. Acquisition premium in the model with continuous distribution of synergies The figure plots the average acquisition premium, as a function of parameters, conditional on the deal being a "cash" deal (more than 50% of the payment is paid in cash; solid line) and a "stock" deal (no more than 50% of the payment is paid in cash; dashed line). We vary one parameter and keep the others at the level listed in Table 1. We use $\lambda_1 = \lambda_2 = 1.02$. The dash-dotted line shows the average synergies, divided by the value of the target as a stand-alone.