

Reasoning about patterns

Helen Thouless, Sue Gifford, Karen Moses and Ruth James share their work with young children exploring patterns.

Two boys in the reception class had succeeded in making a continuous pattern with pom-poms around a rectangular mirror. This involved a repeating unit of three white pom-poms, two yellow and one blue, resulting in a wavy border around the rectangle. When Sue asked them if they could make it fit better, one of the boys said, “I might change the pattern” and then removed one of the yellow pompoms from each unit of repeat. This seemed an impressive example of patterning by a five-year-old. Instead of simply removing one of the repeating units they had altered the structure of the unit (see Figure 1). Asked how they would tell someone else to make the pattern, the boy said, “White, white, white- three whites” and the other finished “... blue, yellow and start over and over.”



Figure 1: Pom-pom pattern.

These boys were in Karen Moses' class, who had been doing a lot of work on patterning. When we visited, they had progressed from making linear patterns to border patterns around different shapes.

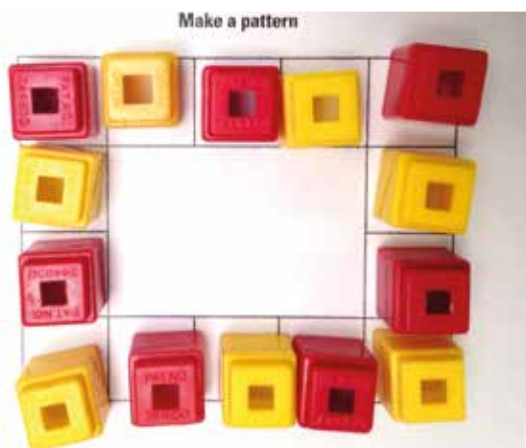


Figure 2: Continuous repeating pattern.

Following a whole class introductory activity, the boys were responding to Karen's challenge to choose a shape and objects to make a border pattern that 'worked'.

Patterns

Karen and colleagues at Christ Church C of E Primary School Surbiton, were part of the *Pattern Project*, a group of early years teachers led by Sue and Helen, who were working on developing pattern awareness, inspired by the work of Papic, Mulligan and Mitchelmore (2011) in Australia. We had developed a progression of activities for linear repeating patterns, including spotting mistakes and identifying the unit of repeat, progressing from AB to more complex pattern structures such as ABB, or ABBC (see NCETM early years resources <https://www.ncetm.org.uk/resources/52504> and NRIC <https://nrich.maths.org/13250>).

Patterns can be defined as “any predictable regularity, usually involving numerical, spatial or logical relationships” (Mulligan and Mitchelmore, 2009, p. 34). Research tells us that developing preschool children's pattern awareness can improve their mathematical performance (Papic, Mulligan, and Mitchelmore, 2011) and is a predictor of later mathematical achievement (Rittle-Johnson et al., 2017). It is not clear why developing pattern awareness helps children learn mathematics but it may be partly because “students who recognise the structure



Figure 3: Fine motor difficulties.

of mathematical processes and representations acquire deep conceptual understanding.” (Mulligan and Mitchelmore, 2009, p. 33).

Continuous Repeating Patterns

In this particular class, the children had explored linear repeating patterns and wanted more challenge, so their teacher had introduced continuous repeating patterns, as found in Mulligan, Mitchelmore and Stephanou (2015) (see Figure 2). Their version of continuous repeating patterns involves rectangular borders with a finite number of spaces, which posed multiple problems for young children. Some children found the fine motor challenge of placing one block on each square difficult (see Figure 3). Other children were able to complete an AB pattern when the pattern was linear but found turning the corner difficult (see Figure 4). Other children got frustrated when their more interesting pattern did not fit into the fixed number of squares (see Figure 5).

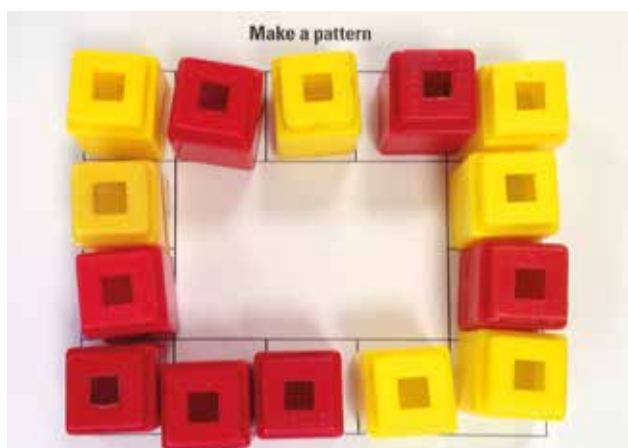


Figure 4: Turning the corner.



Figure 5: Unit of repeat.

Having observed these frustrations, another teacher in our group suggested using a paper plate as a structure for a border because this removed both

the corners and the fixed number of spaces (see Figure 6). Karen worked with this idea in her class and, having spoken with her early years colleagues at work, introduced another innovation: using large glitter foam shapes as the structure for a border pattern (see Figure 7). These shapes reintroduced corners but did not require the children to fit a fixed number of objects into the border.



Figure 6: Paper plate patterns.



Figure 7: Glittery foam shapes.

Whole Class Session

Earlier in the year we had observed Karen beginning a lesson with the class sitting around the edges of the carpet looking at a pattern bordering a sparkly foam triangle with orange and green bottle tops (from baby yoghurt pots, all contributed by parents). However, the AB pattern had a mistake of two oranges together, so when Karen asked, “Have I got it right?” there was a chorus of, “No.” When asked, “What’s wrong?” a girl explained, “One green, one orange, one green, two oranges”. Karen invited her to “fix it”, and she came out and proceeded to change the pattern to an AABB structure. However, she got stuck going round a corner, said, “I have to think” and then, “I need some help”. Karen asked her if she wanted to, “Phone a friend” and she chose another girl to come out and help her. The two girls collaborated to finish the pattern, with the friend turning the corners.

Karen then asked, “What’s different about their pattern and my pattern?” One child answered, “It’s two oranges and two greens and yours was one orange and one green”. And another said, “It has two the same colour”. However, the pattern did not ‘work’, as there were four oranges meeting around a corner. Karen asked, “Does it work? Hands up who thinks ‘yes’”. Lots of children voted ‘yes’. She then said, “Hands up who thinks ‘no’”, and, “Tell the person next to you why you think yes or no”. When she asked, “What’s wrong?” one child said, “Orange next to orange”, another said, “Four of them” and another, “There’s four oranges together”.

Two boys then came out to try to fix the pattern, but one put in extra oranges while the other removed oranges and put extra greens, resulting in moving the problem around the triangle, so their solution had one single green in place of one pair. A child pointed out, “There’s only one green”. So Karen asked, “What can we do?” One child suggested, “Take one orange and one green out” and then came and did this, so the pattern structure returned to AB. Karen said, “Everyone, thumbs up if it works” and then, “Shall we check it and see?” The class joined in while Karen checked, saying, “AB, AB, AB” all the way round.

Karen concluded the session by summarising their findings on a whiteboard: “So on this shape (How do you know it is a triangle? Class: It has three sides) does an AB pattern work?” When they agreed she wrote **AB** ✓. She then asked, “Does an AABB work?” and recorded their response as **AABB** x. Finally, she asked, “What else might we try?” Following suggestions, she wrote **AAB** and **ABB** on board, so it showed:

AB ✓
AABB x
AAB
ABB

Karen’s challenge to the class was, “Can you make one of these work? (pointing to AAB and ABB) but not just with a triangle.” (There were a range of other shapes, including rectangles, to choose from.) “We don’t have enough bottle tops: what are you going to use?” To which a child responded, “Everything!” Finally, Karen asked, “Do you think you can do this?” to which the class shouted: “Yes!” This rally was repeated several times, and then the children dispersed in twos and threes to make their patterns.

We found this lesson with four and five year olds very

impressive on several counts, and it provided some clues to the kind of thinking the class were doing by the end of the year. We also found it notable that they displayed almost all the Characteristics of Effective Learning (Standards & Testing Agency, 2019, p22), including:

- Playing with what they know.
- Being willing to ‘have a go’.
- Being involved and concentrating.
- Finding ways to solve problems.
- Making links and noticing patterns.
- Making predictions.
- Testing their ideas.
- Checking how well their activities are going.
- Changing strategy as needed.

Apart from the problem solving and reasoning involved in fixing pattern errors, there was serious engagement in the problem by this very young class. Karen was clearly committed to building a mathematical learning community in which children felt confident and comfortable in trying to solve problems, getting stuck and asking for help, offering analyses and suggesting solutions. They were expected to relish challenges, collaborate, discuss and make their own choices.

Challenge time

Once the whole class session had finished the children had free time which was always referred to in class as “challenge time”. The children know this is their independent time, when they can choose what to do within the provision of the reception classroom, but the expectation is that they will keep learning and challenging themselves (encouraged by Karen’s rallying cry earlier). Most children chose to keep working on border patterns, taking up Karen’s challenge to find out different units of repeat that would fit around a variety of shapes. Having been exposed to reasoning in the whole-class segment of the class, the children continued to use reasoning during their free play. This was an inclusive activity offering a variety of challenges at different levels. For example, some children still found it challenging to turn the corner (see Figure 8) and needed support to figure out how to continue their pattern in a new direction.

The activity also offers rich options, in making the pattern ‘work’ and fit closely around the shape, using a range of objects, shapes and pattern structures.

One group of children challenged themselves to make a pattern with coloured magnetic letters around a triangle. They investigated different pattern structures saying, “We won’t do AB because that’s too easy. We can try three first.” Having finished making the pattern they said, “It’s an ABC pattern, we tried it and we wrote down that it works – we even did ticks and smiley faces!” The girls then realised that they had left spaces at the corners (see Figure 9) and added another unit of repeat to resolve this.



Figure 8: Turning the corner

Some children reasoned about different units of repeat and which would work best. Dylan worked with his friends to continue a pattern they had decided upon around a rectangle (see Figure 10). They didn’t keep all of the bottle tops close to the shape however, preferring to make the pattern ‘work’. Once completed, Dylan commented, “It’s not fair because we haven’t made the pattern fit properly – look, we’ve left lots of spaces.” He then set about fixing this



Figure 10: Dylan trying different units

problem, which then meant that his pattern would not ‘work’. Having found that an ABC pattern did not fit around the rectangle, Dylan reasoned, “We need to try a different pattern, ABC doesn’t work. We can try AB first because that’s two and it might work better.”



Figure 9: Needing to add a unit

Byron used the carpet session square and started to change the AB pattern into an AABB pattern because, “I want to see if an AABB pattern will fit around the square, AB did so two works but I don’t know about four yet.” He built the pattern around the shape, lifted the shape at the end and checked whether his pattern worked (see Figure 11), “It does work – look! AABB goes on and on all the way round.”

One group of children worked together to create a pattern using AABC and then separated the units of repeat out so everyone could see. When explaining what they had done they said, “There’s three units of repeat and there’s three of us, we can put our names on one each!” (see Figure 12). These children were able to reason with a composite unit, meaning they were able to reason about two quantities, the one that is iterated and the number of iterations which shows multiplicative thinking (Steffe, 1992).



Figure 11: Byron trying different unit



Figure 12: Composite units.



Figure 13: Pictorial representation.

Dylan continued with his investigations into pattern around shapes and decided that he did not need the physical shape because he can draw it himself. He then began to design ABB (see Figure 13) and ABC patterns which will fit around his shapes. Dylan was moving that a concrete representation to a more pictorial representation of pattern.

We thought it would be overclaiming to suggest the children's use of letters meant that they were doing algebra, as the letters did not represent variables in a strictly mathematical sense. However, some of these children were showing the early algebraic thinking practices of "generalising, representing, justifying, and reasoning with mathematical relationships" (Blanton et al., 2015, p. 521). This seems to indicate that patterning can engage young children in early algebraic thinking.

Implications

When considering how to design meaningful mathematics learning opportunities such as these in the early years it seems effective to both include some short whole class activities and give the children time and opportunities to continue exploring these ideas independently and over time. During the whole

class sessions, Karen was working to develop the classroom community, the language of patterns and the expectations that reasoning is a necessary part of exploring mathematics. Once these expectations had been set up, and revisited on a regular basis, the children were able to use, explore and expand their understanding of the mathematics of patterns.

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