

Empirical Essays on Non-Linear Income Taxation and Labour Supply

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Ph.D Thesis

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Abstract of Thesis

It is well documented that elasticities from labour supply studies vary from one study to another (with or without taxes) and naturally from one country to another. A standard explanation has been that researchers have used different statistical methods and different data sets and it seems to be the case that there are no generally accepted robust way to estimate labour supply functions (with taxation). In chapter 2 we use two different data sets for the same individuals. One is the survey data and the other is register based data provided by tax authorities. We use method developed by Burtless and Hausman (1978) and we are able to overcome some of the main criticism expressed concerning the piece-wise linear type of modelling. Special attention is given to constructing budget constraints for all individuals and we are able to estimate model taking account factors usually neglected in similar kind of analysis, like individual tax deductions and local tax rates. We estimate models using register based income information and using survey based information. Both fixed preference model and random preference model are estimated. Results indicate that using survey information compensated wage elasticities are significantly higher than using register based information. In their seminal paper MaCurdy et al. (1990) suggested a new approach which utilises a differentiable budget constraint approach to approximate piece-wise linear marginal tax rate function. We estimate labour supply function using both methods in chapter 3. Our results support the view that if one is able to mimic actual budget set closely and the degree of progression is high then these two methods are likely to give similar results. On the other hand, if the above mentioned factors are not present then the differentiable budget constraint approach is likely to be better choice. Non-linearities in budget constraints may arise for variety of reasons - the structure of the tax/benefit scheme, overtime rates etc. Non-linearities also cause problems when it comes to interpreting the policy implications of the estimates. In chapter 4 we use results got from earlier chapters and construct microsimulation model to analyse different income tax regimes and systems to labour supply behaviour. Our simulation results show that none of the reforms conducted are self-financing. Revenue neutral proportional tax system do not have major effects on labour supply. Biggest behavioural responses are achieved if we reduce the marginal tax rates from the lower end of the state income tax schedule.

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Chapter 1

Labour Supply and Nonlinear Income Taxation: An Overview

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1.1 Introduction

The motivation for the scientific research on income taxation's effect on labour supply largely stems from various sources. It has long time interested economic theorists, econometricians and policy makers. Theorists are interested in it for various reasons. From the micro perspective labour supply is an interesting application of the theory of consumer demand and taxation brings into the framework the questions of economic welfare and justice. For macroeconomists' labour supply has important implications for understanding the nature and origins of business cycles. For econometricians labour supply has been a long time a subject which offers demanding empirical applications. It is probably fair to state that labour supply research has been in a frontline of applied microeconomic research in recent decades. Policy makers are interested of it because in almost all countries income taxation contributes a significantly to state and municipal tax revenues.

Economic research has increased our understanding concerning the determinants behind the individual's labour supply behaviour and how these determinants are linked to institutional arrangements. For example, we know that various child care systems and social security systems affect differently for the female labour supply than for the male labour supply, as an example. We also know that changes in income taxation schemes can potentially have significant effects on labour force

participation or hours supplied by the workers. Thus, important welfare reform issues are almost without an exception related to the labour supply behaviour and it is useful to have some kind of tools to analyse these behavioural responses.

This thesis tries to offer some evidence concerning the labour supply behaviour among Finnish females. It contains empirically orientated research on labour supply which mainly concentrates on how to model progressive income taxation when estimating labour supply function. The reasons for this relatively narrow choice are the following ones. First, it is not clear how non-linear taxation should be taken into account in empirical work and does it really cause big effects on labour supply behaviour. This is still an open question in the literature. Secondly, we wish to offer some practical tools of how to analyse different tax reforms in the case of Finnish economy, because this is still a very much uncovered topic in Finland. Thirdly, we hope that this thesis may open some new questions for the future research in this field.

1.2 Labour supply in Finland: some historical trends

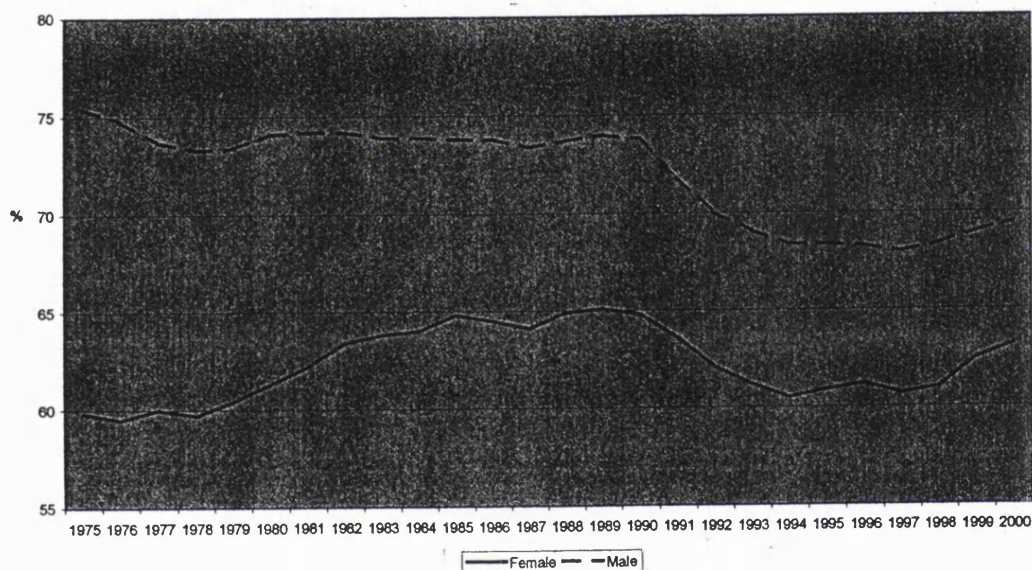
Throughout this thesis information on labour supply behaviour is based on the labour force surveys conducted by the Statistics Finland. In these surveys labour force is defined as the sum of employed and unemployed persons. The labour force participation rate is defined as the percentage ratio of labour force to population.¹

From 1970 to 2000 the population of working age, 15–64 years, increased by some 415 000 individuals. The increase in labour force(employed + unemployed) was about 380 000. The number of employed persons increased by some 210 000 and thus the number of unemployed has increased sharply during the last decades.

¹In recent years there has been some changes in definitions due to the attempt to harmonise statistics in the European Union, but these changes do not change the picture presented here or affect our analysis conducted in later chapters.

The behaviour of men and women with respect to the labour market has differed markedly. The labour force participation rate for men decreased considerably in the 1960s and it continued to decrease more moderately still in 1970s. The rate for females increased steadily in the 1960s and early 1970s. From late seventies it started to increase rapidly (due to the tax and social security reforms) and the difference between the rates for males and females narrowed during the 1980s, as can be seen from the figure 1.1.

Figure 1.1: Labour force participation rates in 1975–2000



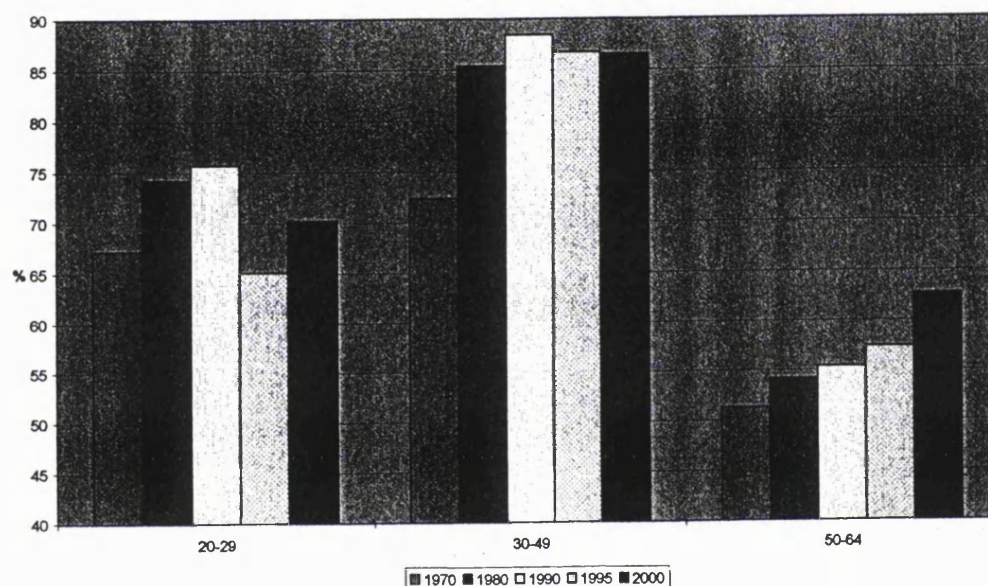
The rapid decrease in these figures in early 1990s is due to the unseeingly severe recession. Unemployment rate rose from 4 per cent (1989) to 19 per cent (1993) and afterwards it has come down quite slowly during the last years even when the economy has boomed again.

The breakdown of employed persons between wage-earners and self-employed has changed considerably in favour of wage-earners. The main reason for this has been the rapid decrease in persons working in agriculture and as unpaid family

workers. Also the composition of persons not in the labour force has changed quite a lot during the last 20 years or so. The number of persons performing domestic work has decreased by two thirds.

The changes in female participation rates by age groups from 1970 to 2000 are shown in figure 1.2.

Figure 1.2: Labour force participation rates for women by age groups.



From the figure we see a big rise in the participation rate of females in the most active age range till 1990s. It can also be seen that recession hit most severely to the younger cohorts. But, it is interesting to realise that recession did not hit to the mature workers and their participation rates have steadily increased throughout the years. In the early 1960s the labour force participation rate for married women was considerably lower than for unmarried women. Since those days, the rise in the rate for married women has been so substantial that there are no longer any difference between these two groups. One can state that the major part of the increase in the supply of labour during the last 30 years is due to

changes in the labour force participation rate for married women. This fact makes it important also for macroeconomic reasons to try to find the determinants of labour supply for this group.

To obtain a better understanding of the choices made by working age females between employment and the main non-working categories, let us look for the flows of working age population.² The main (two-way) flows are between employed and student, employed and homemaker and employed and unemployed categories. However, it is generally the case that persons employed in period $t-1$ have a very high probability of being employed in period t . This fact is a clear indication of the life-cycle nature of labour supply. But, there is still a room for voluntary choices between working and not working, especially between employment and homemaking as well as between employment and studying. There might also be a voluntary component in the flow from employed to unemployed.

Changes in the labour supply are not only due to changes in participation rates but also to changes in hours worked among participants. The average number of hours worked among employed wage and salary earners has declined considerably during the last 40 years. The main reason for this has been the legislated increases in the length of yearly holidays. Also the number of normal weekly hours has diminished. It is clear that Finnish labour market is quite rigid with respect to working time. The proportion of part-time workers (i.e. those working 1–29 hours per week) is low by international comparisons. It is also noteworthy that the proportion of part-time employed did not increase during the 1980s even though the service industry grew rapidly. This proportion stayed at approximately 7.5 per cent throughout the decade. After the severe recession this share has started to increase. According to labour force surveys a greater share of females would be willing to do part-time work had such jobs been available. Having a second job or working overtime seems to be the easiest ways to adjust working hours. For example, in year 1989 approximately 9 per cent of the employed population worked overtime and some 7 per cent had a second job. Generally about two

²These flows have been surprisingly similar during the last decades.

thirds of employed women normally work between 35–40 hours per week. For further and more detailed picture from Finnish labour markets can be found from the various Labour Force Survey publications published by the Statistics Finland.

There are many different factors which are important for labour supply behaviour and all of these cannot be analysed in a single study. A short list of relevant factors would include among others:

- General demand for labour
- Socioeconomic factors
- Family size and compositions
- Flexibility of working time
- Taxation and transfer payment systems
- Pension system
- Mobility in the labour market
- Role of labour unions

In a cross-section study like this we have to concentrate on the factors that are most relevant to short run decision making, and therefore we focus here on factors such as hourly wage, unearned income, taxation and socioeconomic variables. In addition, data limitations also dictates the possible research subjects.

1.3 Some remarks on labour supply theory

In most of the cases labour supply behaviour is analysed within the neoclassical framework.³ In this framework labour supply is the mirror image of leisure. Individuals utility depends on the amount of (market) goods c and hours of leisure time l . Individuals face two constraints, namely the budget constraint and the time constraint. The budget constraint implies that spending on the market goods $p * c$ (p is the price vector of market goods) must be equal to (total) income from work, $w * h$ (w is the gross wage rate and h is the hours worked) and the nonlabour income y . Time constraint implies that the total amount of time for the individual per period (e.g. day, week, year) is fixed and we denote it by T . T can be allocated to working hours h and leisure hours l . Now, it is essential to realise the dual role of the given market-determined wage rate. First, the wage rate appears in the role of a price for leisure. Secondly, it appears as part of the budget in valuing the time endowment and thus change in w has effects on behaviour over and above those of a change in the price of a good and this alters the analysis of income and substitution effects.

So, the consumers face the following optimisation problem (note that geometrically on the horizontal axis we have a leisure time and on the vertical axis we have a composite commodity (consumption)).

$$MAX u = u(l, c) \tag{1.1}$$

subject to

$$wh + y = pc$$

and

$$h + l = T.$$

³Usually the term labour supply is used to refer to the measurable number of hours of work or labour force participation and other aspects like work effort are ignored.

We can rewrite the above constraints directly in terms of c and l to get the following form

$$pc + wl = y + wT. \quad (1.2)$$

Now we see that w , the price of leisure, appears not only in the normal role of a price on the left-hand side, but also as a part of the budget in valuing the time endowment T . In the above equation the quantity $y + wT$ is called full income, i.e. the total purchasing power available to the consumer to be spent on leisure and goods. From now on we denote it by X . This small and simple thing is good to keep in mind because it makes the analysis a bit more complicated than in a traditional demand analysis where the increase of single price of good has not immediate effect on individuals resources.

The comparative statistics of this model can be done compactly and straightforwardly using the dual theory and the Slutsky equation has the the following form

$$\frac{\partial l}{\partial w} = \frac{\partial l}{\partial w} \Big|_u + \frac{\partial l}{\partial X} * h, \quad (1.3)$$

where X is the total purchasing power available (i.e. $y + wT = X$) to be spent on leisure and goods. Above Slutsky equation decomposes the total effect on the leisure of a *gross wage* change into a substitution and an income effect (note, that because, w , the price of leisure appears also as part of the budget valuing the time endowment T , there is an extra effect called 'Revaluation-of-time-endowment effect'). From this equation we can derive the following empirically testable hypothesis. First, substitution effect for the leisure time is negative (because its price has increased) and thus for the labour supply it is positive. Secondly, if leisure is normal good then the total income effect is always positive for the leisure time (total income effect composes to the conventional income effect which is negative if leisure time is a normal good, and to a revaluation-of-time endowment effect which is always positive) and correspondingly it is always negative for the labour supply.

The next natural step is see how we can incorporate income taxation into this

basic neoclassical model. The simplest option to start is the effect of the proportional(linear) tax. If we denote the linear tax as t , we can write the budget constraint as

$$pc + (1 - t)wl = \underbrace{(1 - t)(wT + y)}_{full\ income} = X \quad (1.4)$$

Price for the leisure time p_l can be derived from the above constraint by derivating it with respect to leisure. Now, using the above Slutsky equation we can easily derive the total effect of the linear income tax on the demand of leisure(or labour supply) as

$$\begin{aligned} \frac{\partial l}{\partial t} &= -w \left[\frac{\partial l}{\partial p_l} \Big|_u - l * \frac{\partial l}{\partial X} \right] + \frac{\partial l}{\partial X} (-wT - y) \\ &= -ws - [w(T - l) + y] \frac{\partial l}{\partial X} \end{aligned} \quad (1.5)$$

where s stands for the substitution effect. As from the above equation can be easily seen, we cannot, *A priori*, determine the linear income taxation's effect on leisure demand or labour supply. We can also derive how the change in hourly wage rate affects on leisure time or labour supply in presence of the linear income tax. Using the above presented notation and equations we can derive this to be as

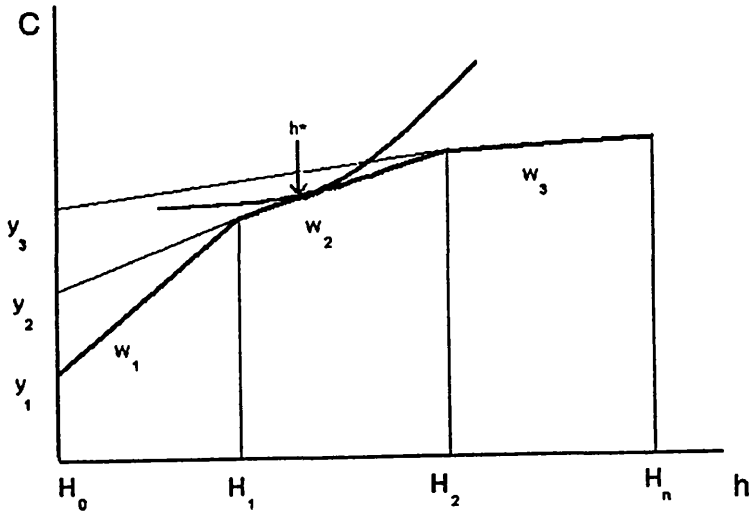
$$\frac{\partial l}{\partial w} = (1 - t) \left[s + (T - l) \frac{\partial l}{\partial X} \right] \quad (1.6)$$

and we see that the role of the linear tax is only a scalar one and it dampens the substitution and the income effects. This is logical because only a change in the after tax wage rate affects consumer behaviour.

Let us now proceed to the more realistic cases where we have to deal with a progressive taxation and/or with a different kind of transfer programs organised by government and/or municipalities. Unfortunately, the algebra becomes quite complicated very quickly and we cannot derive neat results like those above using the differential calculus very easily. Let us start with a simple case where we have a progressive income tax on labour income so that the marginal tax rate

is nondecreasing.

Figure 1.3: Piece-wise linear budget constraint



The income tax system in figure 1.3 consists three tax brackets and thus three marginal tax rates; t_1 , t_2 , and t_3 . Inside the tax brackets the marginal tax rate is constant but it increases with the income. As we can see from the figure the budget constrained faced by individuals is piece-wise linear. As above, c is the composite commodity and h is labour supply. Note that we have now changed the leisure time for the hours in our geometrical presentation.

Marginal tax rate t_1 leads to the after-tax net wage $w_1 = (1 - t_1)w$ and this corresponds to the first segment in the above figure. Correspondingly the net wage rate in the second segment is $w_2 = (1 - t_2)w$ etc. H_0 , H_1 and H_2 are kink points where the marginal tax rate changes and H_n stands for the upper limit of labour supply. y_1 is the exogenous income component and thus does not depend on hours supplied.⁴ Note that this component is directly observed from the data. y_2 and y_3 are called 'virtual income' terms and must be calculated recursively.

⁴This assumption is questionable, at least in the long run, and we will discuss more about it in chapter 2.

So, the problem of consumer is to maximise a strictly quasiconcave utility function $u(c, l)$ increasing in c and locally increasing in l , subject to a nonlinear constraint. Before proceeding we need to define taxable income, tax function and virtual incomes associated with each tax brackets. Let B_t and B_{nt} be taxable and nontaxable income respectively and E is the exemption level. Taxable income can then be written as $x = wh + B_t - E$. Let the income tax be of the form

$$Tax(x) = \int_0^x t(z)dz, \quad (1.7)$$

where t is an increasing step function such that $t(z) = t_i$ for $A_{i-1} < z \leq A_i$, $i = 1, \dots$ and A is the tax bracket. As in the above figure the slope of the i :th segment is $w_i = w(1 - t_i)$ and the upper limit of the corresponding interval on the h -axis is $H_i = (A_i + E - B_t)/w$. The virtual incomes (i.e the intercepts on the x -axis) can be calculated recursively as $y_i = y_{i-1} + (w_i - w_{i-1})H_i = y_{i-1} + (t_i - t_{i-1})(A_{i-1} + E - B_t)$, $i = 2, \dots$ Now, if $B_t > E$ but $B_t - E < A_1$, then $y_1 = B_{nt} + E + (1 - t_1)(B_t - E) - T$

Corresponding to each budget segment (H_{i-1}, H_i) we can define the indirect utility function

$$v_i(y_i, w_i, p) = MAX u(c, l); pc + w_i h \leq y_i + w_i T. \quad (1.8)$$

Properties of this indirect utility function v_i are

- 1) $v_i(y_i, w_i, p)$ is continuous at all $y_i > 0, w_i > 0, p > 0$. This follows directly from the theorem of maximum, see eg. Varian[58].
- 2) $v_i(y_i, w_i, p)$ is homogenous of degree 0 in (y_i, w_i, p) . This means that if y_i, w_i, p are all multiplied by a positive number, the budget set does not change at all. Thus, $v_i(ty_i, tw_i, tp) = v_i(y_i, w_i, p)$ for $t > 0$.
- 3) $v_i(y_i, w_i, p)$ is nondecreasing in y_i and w_i and nonincreasing in p . That is, if $w'_i \geq w_i$, $v_i(y_i, w'_i, p) \geq v_i(y_i, w_i, p)$ and similarly for y_i and p . For the proofs again see Varian[58].⁵

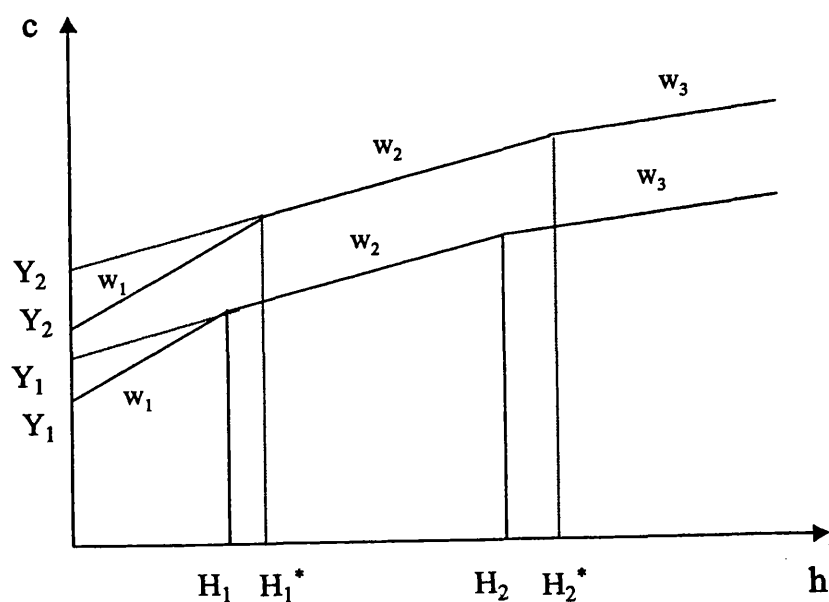
Using these properties and assuming that leisure time is a normal good, we are

⁵In the case of w_i the proof is a bit more complicated than in other cases.

able to consider the effects on labour supply of variations in exemption level, tax brackets, marginal tax rate in a certain tax bracket and gross wage rate.

As an example let us work through how change in exemption level affects labour supply.

Figure 1.4: Change in exemption level



First of all, it

- 1) increases all virtual income components,
- 2) does not change net wages,
- 3) increases all kink points,

as can be seen from the above figure.

So, individuals whose prior optimums are in the intervals $(0, H_1)$ and $(H_i^*, H_{i+1}), i = 1, \dots$ will decrease their labour supply. This is because there is only an income effect.

But, individuals with prior optimums in intervals $(H_i, H_i^*), i = 1, \dots$ may increase or decrease their labour supply. However, it can increase at most up to the upper

limit H_i^* . The proof goes in a following way. Let the original optimum be on a segment $i + 1$ in the interval (H_i, H_i^*) . Suppose (for the purpose of contradiction) that the new optimum is to the right of H_i^* on segment $i + 1, i + 2, \dots$. Since leisure is a normal good the prior optimum must have been to the right of H_i^* on the prior budget constraint, which is a contradiction. Hence the new optimum must be less than or equal to H_i^* .

From the above discussion we realise that the comparative statics in the case of a piece-wise linear budget constraint yield a number of predictions about how changes in tax parameters affect labour supply. Changes (most likely small ones) in wages and/or incomes may keep the individuals on their original segment and in this case the predictions from the standard theory are valid. But, it may be the case that changes in above mentioned factors lead to changes in segments. It is also possible that some individuals may stay at the kink point where the wage or income effect is zero. At the kink point utility maximisation is compatible with many different marginal rates of substitution. Here we have just analysed the convex cases but the analysis can be extended to handle also nonconvexities. All this is clearly in contrast to the case of proportional taxation where we cannot say much. Above discussion also motivates the empirical work which concentrates on the nonlinearities in budget constraints.

Some generalisations to the static model

Despite that the focus of this thesis is on nonlinear taxes and labour supply in a static framework it is useful to mention very shortly other dimensions of labour supply research. An extensive survey of different approaches to labour supply research can be found, for example, from Blundell and MaCurdy[7].

In most cases we are only interested in individual's behaviour, but it is evident that in some circumstances this approach is too narrow. In family labour supply models decision making is extended to deal with two labour supply decisions. The so-called basic family labour supply model deals with the behaviour of a

household consisting two working age individuals. Children and other dependents are included in the vector of other observable characteristics. It is assumed that families maximise joint utility over consumption and leisure time of both adults. This framework is quite easy to extend for intertemporal labour supply analysis or it can be used to analyse taxation and family labour supply. Finally it is also convenient to introduce nonparticipation into the analysis. For empirical applications see Ransom[52] and van Soest[56].

The optimal labour supply choices in the basic model satisfy all the standard consumer demand restrictions of symmetry, negative semidefiniteness of Slutsky substitution matrix etc. All these can be tested in empirical analysis. One of the main disadvantage is the assumption that one can combine all sources of non-labour income into a single unearned (exogenous) income measure. This implies that the source of non-labour income is irrelevant in within family labour supply decisions.

Researchers have tried to relax the assumptions of symmetry and income pooling by using the tools from efficient bargaining theory. Intuition behind this approach is to assume that individual utilities are weakly separable over consumption and leisure. Family utility consists the sub-utilities of husband and wife. When family utility has this weakly separable form individuals follow two-stage budgeting. First total household income is allocated among household members and then they act as if they are making labour supply and consumption decisions conditional on this initial stage. The main advantage is that it relaxes the income allocation rule among household members. Now allocation may depend on variables that reflects the bargaining position of members within the family. Even when household members are altruistic and allocations are Pareto efficient, the allocation rule can be different from the optimal rule in the traditional model. More thorough discussion concerning the economic theory behind this approach can be found from the series of papers by Chiappiori[11][12][13]. For empirical work readers may consult Kooreman and Kapteyn[34] and Fortin and Lacroix[20].

Second generalisation deals with the time dimension of labour supply. It is apparent that individuals go through different life situations and then also labour supply behaviour varies. Individuals might want to get more education for the possible higher earnings in the future or changes in family composition might affect labour supply behaviour for a certain time period. Note that one can deal family labour supply also in an intertemporal framework, but the analysis becomes quite complicated very soon, especially if nonlinear taxation is also included.

In most of the cases one has to assume that utility is separable in time to get empirically tractable specifications. So, individuals maximise time separable utility function with respect to standard intertemporal budget constraint. From the first order conditions we can derive the result that within period marginal rate of substitution condition still continues to characterise the relative amount of leisure and consumption. The addition is to find a some sort of statistic that captures the other periods impact on this periods decision. In most cases this is done by utilising the so called two-stage budgeting. Because the within-period marginal rate of substitution conditions still characterise behaviour, we need an allocation of full income to each period to allow each maximisation problem to be solved exactly as it was in static problem. Thus, we proceed in two stages. First, we determine an allocation of wealth across periods. Secondly, within each period we solve the standard static problem. In practise we first maximise each periods utility given some amount of full income. Result is the indirect utility function for each period. Then we substitute these indirect utility functions in to the direct utility functions and choose the amount of full income to maximise this function given current wealth and future wages. This approach has been utilised by Macurdy[41][42] in his seminal papers.

In principle it is possible to introduce nonlinear taxation into the above presented framework. Problem is the changing nature of tax legislation. If marginal tax rates varies from year to year, then the likelihood function becomes very complicated and estimation problem will be an untractable one.

1.4 The econometrics of labour supply with progressive taxes

1.4.1 Some general remarks

One can state that the most important reason to conduct empirical research is to estimate the elasticities needed in policy analysis. This is neat and informative way to describe individuals labour supply behaviour, but once again these elasticities are not without problems. It is hard to believe that one representative elasticity value represents the whole population and thus partly due to this reason a common practise has been to study different groups separately. Especially, one very distinguishing feature in empirical labour supply analysis has been the fact that gender is the dividing factor. Empirical research typically deals with either female or male labour supply, and especially a significant amount of effort has been put on female labour force participation and hours of work choice. This is natural because there is a specific social phenomena behind all this research interest, namely that the number of married females entered to the labour market have increased significantly during the last few decades or so.

Empirical findings also support the claim that married females usually do more household work than males or unmarried females and this gives an additional motivation to study this group. Basically, time allocation between household work, leisure time and market work is the one that matters. Now, it is natural to think that market work and household work are substitutes and indeed, Ehrenbers and Smith[53] note that a larger substitution effect is plausible for married women because household and market work can be seen as close substitutes and when the incentives to pursue one activity change then a large response in time spent doing the other can be expected. Unfortunately, in most of the times we have to group leisure time and household work time because the available data sets do not include enough information to separate them.

Above discussion leads us to another factor which encourages researchers to study female labour supply. The substitutability between market work and household work can be seen as an explanation for the higher flexibility of female labour supply compared to male labour supply. In other words, we can see almost from all relevant data sets that there is more variations between female working hours than in male working hours. Because hours are the endogenous variable this also leaves us more sensible statistical problem to study. Additional factor is that in the case of females we usually have a significant number of nonparticipants and thus the statistical problem comes more interesting.

Next we will shortly describe how researchers have tackled the problem of nonlinear income taxation in empirical labour supply research. In the following we will not discuss about how to tackle participation problem in econometric analysis. Discussion of Tobit and Heckit models would need an own chapter and because this is not main topic of the thesis we'll skip it. A good presentations can be found from Davidson and Mackinnon[16] and from Greene[21], among others.

1.4.2 First generation models

One of the earliest work on the effects of taxation on labour supply was the study by Kesters[35] who assumed a proportional income tax. Hall[22] was among the first who took the progressive nature of income taxation explicitly into account. He replaced the true nonlinear constraint with a straight line tangent to the true constraint at the point of actual hours of work. Instead of the gross wage and non-labour income, he included the net wage and virtual income (defined as the intercept of the linearised budget set at zero hours) in his regressions. This method was used in many studies in the 1970's, e.g. Boskin[9], and Wales and Woodland[59].

In stochastic environment above mentioned procedure is not appropriate anymore. The reasons are basically following ones. The observed net wage rate is it-

self endogenous, since it depends on the number of hours worked, and is therefore correlated with the disturbance term. This leads to the inconsistent parameter estimates got from the OLS procedure. Despite an individual may be observed to be on a given segment of a piece-wise linear constraint, this observed position is the sum of two components – the utility maximising position plus a random disturbance. Hence individual's true utility maximising position may be on a segment of the piece-wise linear constraint other than the observed one. In this case the net wage rate that should be used in the labour supply equation is the one corresponding to the utility maximising position, not the observed one. For a more detailed discussion can be found from Pudney[51].

A natural way to try to solve the endogeneity problem is to estimate a simultaneous equation model relating hours worked to the net wage rate. Taking also participation equation into the consideration we are faced with the following simultaneous equations model.

$$w_i = \gamma X_i + \alpha_1 y_i + \varepsilon_{wi} \quad (1.9)$$

$$y_i = \lambda G_i + \alpha_2 w_i + \varepsilon_{yi} \quad (1.10)$$

$$h_i = \beta Z_i + \alpha_3 w_i + \alpha_4 y_i + \varepsilon_{hi}, \quad (1.11)$$

if $h > 0$.

In above Z is a vector of observable taste factors that influence the marginal rate of substitution between leisure and consumption(reservation wage). X and G are vectors of personal characteristics and lower case letters are as before. These type of models to estimate labour supply have been used by e.g. Leuthold[40] and Merz[46] In Leuthold's study selectivity issues are ignored and in Merz's study endogeneity of virtual income is ignored but selectivity issues are considered.

We can also think the labour supply problem in a following way. Let us assume a budget constraint with only two segments. In this case individual may occupy one of the four states of the world: zero hours of work, the first budget segment, kink point or the second segment. As a consequence of convexity of preferences and the constraint set, a local comparison of the marginal rate of substitution

function and the net wage at the kink points determines the location of an individual on the budget set. The individual chooses not to work if

$$m(y_1, 0, \varepsilon) \geq (1 - t_1)w.$$

The consumer works in the interval $(0, H_1)$ if

$$m(y_1, 0, \varepsilon) < (1 - t_1)w \quad \text{and} \quad m(y_2, H_1, \varepsilon) > (1 - t_1)w.$$

and etc. Using this strategy we can easily construct an ordered probit model(see e.g. Maddala[44]). In this framework hours of work data is not utilised at all. Categorical data on the individual's location on the budget constraint is only required. This model can be easily extended to utilise hours of work information and then the problem has similar features than the basic Heckman model and estimation is usually done by two-step procedure. But, in two-step procedure we assume that hours worked are measured without error. For this reason this approach is feasible only with a small number of budget segments. Zabalza[60] utilised this approach in a model with two budget segments.

1.4.3 The second generation models

The maximum likelihood method proposed by Burtless and Hausman[10] is where the whole budget constraint is taken into account in the estimation procedure. The general principle in this approach is that the consumer chooses her most preferred labour supply point on each budget segment, determine the corresponding utility of that choice and then chooses the one that yields *maximum maximorum* of utility across all segments. In our application above it means that the likelihood function takes into account the choice of hours over the entire exogenous tax schedule(it can be argued that in the cross-section studies we can treat tax schedule as exogenous) removing the endogeneity problem mentioned earlier.

Chapter 2 is devoted to this topic and a more detailed discussion can be found from there. But, to get an intuition about concerning the method let us look once again to the figure 1.3. In a presence of a convex budget set a quasi-concave utility function implies the existence of unique optimum given the maximisation. Now, we start by deriving desired labour supply on the first segment by substituting the corresponding net wage w_1 and unearned income y_1 into the labour supply function. If desired hours h^* are less than equal to zero, then the individual do not participate. If desired hours lies between zero and H_1 we have the unique optimal desired hours for the individual. If desired hours are above H_1 we move to the second segment where the net wage is w_2 and the virtual income is y_2 . Now, desired hours are determined using these figures and if the desired hours are less than H_1 , then H_1 is the unique optimum. This can be seen applying the revealed preference argument. If $H_1 < h^* < H_2$, then unique optimum can be found from the second segment(which is the case in figure 1.3) etc. So, it is evident that this method is computationally quite burdensome because we also have to use an search(tax) algorithm to find the optimal hours.

A clear advantage of this approach is the possibility to include institutional features of the tax and welfare system into the analysis. This is especially important if one believes that other methods do not provide a detailed enough information from the tax system. This method also admits randomness in hours of work arising from both measurement error and variation in individual preferences(see chapter 2) and it takes explicitly into account the endogeneity of the marginal tax rate in estimation. As Blundell and Macurdy[7] mention, the piece-wise linear method allows fixed costs of holding a job (see Cogan[14]) to be taken into account and also studying multiple program participation is possible.

Probably the most serious shortcoming is the assumption that econometrician and each individual have the perfect knowledge of the entire budget constraint in question. This is clearly not the case and this has to be kept in mind when interpreting the results. For other shortcomings look Blundell and MaCurdy[7]. Relating to the above discussion it is clear that the maximum likelihood method

requires a lot from the data. Heckman[27] argues that in most of the cases the budget constraints cannot be measured accurately enough which is the requirement for using maximum likelihood method. MaCurdy et al.[43] also argued that a proper specification of the log-likelihood over the region of a convex interior kink(s) is only possible if the estimated coefficients fulfill the condition that the compensated substitution effect is positive. In other words, if the compensated substitution effect is not positive for some observations then they will have a negative probability of locating at the kink(s).

MaCurdy et al.[43] proposed an estimation procedure which do not have (all) the drawbacks mentioned earlier and it is also simpler to estimate. We will discuss about this method more in chapter 3 and in the following we just give an short overview concerning this method. The very first step in this approach is to approximate the marginal tax function. This is done by constructing a differentiable step function. What this step function does is that it creates a smooth budget constraint which approximates the actual piecewise linear one. This means that we do not have to construct the tax algorithm to find the desired hours.

Integration of this marginal tax function yields a differentiable relation approximating the amount of total taxes paid as a function of taxable income. Now, the derivation of the likelihood is easier than in the piecewise linear case since a purely continuous distribution describes the hours worked for all employed individuals. One of the advantages of this differentiable approach is that one can quite easily try alternative stochastic assumptions.

Blomquist and Newey[6] suggested an estimation method of the labour supply function when it is generated by piecewise linear budget constraint. The basic idea is to think of the choice, hours of labour supply, as being a function of the entire budget set. In other words, to estimate nonparametric regression where the variables in the regression is the budget set. In the special case of a linear budget constraint this estimator would be the same as nonparametric regression

on wage and exogenous income. Assuming that the budget set is piece-wise linear then it will be characterised by two or more variables but this will lead to a 'dimensionality' problem. We would need a huge amount of observations if we would like to mimic the actual tax schedules accurately. They proposed a two step estimation procedure. In the first step each actual budget constraint is approximated (using either the least squares method or the interpolation method) by a budget constraint that can be represented, for example 5-6 variables. In the second step nonparametric estimation using series approximation (Blomquist and Newey argue that it is relatively easier to impose additivity using this approach rather than using the other methods) is applied using the approximate budget constraint as data. For further technical details see Blomquist and Newey[6].

1.5 Empirical evidence from the second generation studies

We have listed to the table 1 and 2 (see appendix 1) some of the most influential studies. These works can be classified as a "new generation" labour supply studies. For the summary of earlier studies see e.g. Killingsworth[33], Heckman and Killingsworth[32] and Pencavel[50].

Two studies listed in the table 1 and 2 need more closer look than the others. These are Hausman[23] and MaCurdy et. al[43]. Paper by Jerry Hausman is a seminal one, because of two reasons. First, his results suggested that taxation income taxation creates significant work disincentives for both gender. Secondly, the econometric approach he used was new.⁶ MaCurdy et al.[43] tried to replicate Hausman's study as closely as possible using the same data set and economic approach and they find that taxation seems to have no effect at all. Secondly, they criticised Hausman's econometrical approach.

⁶Actually, this approach was first suggested by Burtless and Hausman[10].

Basically all the other studies are more or less replications of these studies using different kind of data sets and statistical assumptions and in what follows we try to highlight these differences and find some explanations why results differ across the studies.

Functional form

Stern[54] gives a criteria list for choosing a functional form for the labour supply studies. These criteria are the following ones: 1) consistency with the utility theory, 2) convenience in estimation, 3) facility for incorporation in theoretical studies, 4) ease of use in applied studies and 5) flexibility in the type of response it permits. Clear majority of studies with progressive taxation have used linear labour supply function. Usually the authors have not given any justification for this choice but presumably the reasons have been the following ones: it is relatively easy to derive direct and indirect utility functions from the linear labour supply function(or vice versa). Linear labour supply function is also a very convenient when estimating models with piecewise linear budget constraints and simply because all the other studies have also use it.

Probably the most severe drawback of linear labour supply function is its inflexibility. It is quite unlikely that the labour supply curve is linear. Intuitively one should expect the wage effect to be positive for small number of hours and then after the certain point it may become negative, thus allowing the backward bending supply curve, for example. However, it is quite unlikely that the linear labour supply curve in itself generates the different results. For example, Hausman[23] reports that the compensated wage elasticity is 0.13 for males using 1975 wave of the US Panel Study of Income Dynamics. MaCurdy et al.[43] report 0.01 for the compensated wage elasticity using virtually the same data set as Hausman.⁷

⁷The sample size in Hausman's study is 1085 observations. MaCurdy et al. used the same sample selection procedure as Hausman and they ended for 1017 observations. Eklof and Sacklen[18] argues that the wage variable is constructed differently between the studies and that used by MaCurdy et. al leads to a downward biased wage effect.

Estimation method

Do different estimation methods lead to different kind of results? It is quite hard to give an exact answer to this question because there are quite a few studies which try to tackle this problem. Blomquist[4] applied different estimation methods to the same data sets and the estimated wage elasticities range from -0.19 to 0.26. According to his Monte Carlo results the OLS-method works quite badly in all sample sizes, which is expected. He also concludes that the IV-method is very sensitive for the chosen instruments. Small sample biases can be serious if one uses poor instruments. For the similar findings see also Nelson and Startz[49] and Maddala and Jeong[45]. According to Blomquist the ML-method seems to perform well in all sample sizes. Most of the studies have utilised the ML-method, so it is unlikely that this would be the main contributing factor to the different results.

Different stochastic assumptions

In the traditional linear regression context we usually are not very specific about the sources of the stochastic disturbance. These stochastic disturbances can be considered to arise from a different sources. A common view is that the relationship could be seen to be exact but the measurement error in endogenous variable leads to stochastic disturbance ϵ . We can also argue that the relationship may not be exact because the individuals whose behaviour we try to model using the regression method may be unable to achieve her desired outcome exactly. As Hausman[24] mentions, we usually see large proportion of unexplained variance in cross-section models and thus nonconstant parameter(s) seem(s) a distinct possibility.

When dealing with the nonlinear budget sets, the source of randomness becomes a crucial one. Whereas the measurement error ϵ tends to spread observations out evenly over the constraint, heterogeneity of the preferences tends to gen-

erate clusters of observations at the kink point(s) of a convex constraint and tends to disperse observations away from the kink point in the nonconvex case, as Moffit[47] writes. Indeed, we can see clustering in many data sets, see e.g. Blundell, Duncan and Meghir[8].

In practise⁸ we have three options to introduce the unobserved heterogeneity term. The most popular choice in later studies have been to allow the income effects to vary over the population. In case of linear labour supply function authors have usually used truncated normal distribution to force the income effect to be negative and thus to help to satisfy the Slutsky condition. Secondly, one can allow the wage effect to differ across individuals. Also in this case authors have used the truncated normal distribution to guarantee positive wage effect near the zero hours. Thirdly, one can allow the intercept term to vary across individuals. In this case one do not have to limit the distribution. Note that these models can be estimated without the measurement error term, but usually additive or multiplicative measurement term is added.

Looking the results from the table 1 and 2 we can conclude that these different stochastic assumption do not have a significant contribution to the results. Usual procedure is to estimate the model first with only the measurement error and then add the unobserved heterogeneity term. For example, Blomquist's[2] results for the income elasticity and for the uncompensated wage elasticity are -0.03 and 0.08 respectively when estimated without the unobserved heterogeneity term. After that he estimated exactly the same model allowing the income effect to vary across individuals. Estimates for the income elasticity and for the uncompensated wage elasticity were now -0.04 and 0.08 respectively. This seems to be general feature for the other studies also, except the Flood and MaCurdy[19] study where the results differ when they assume multiplicative measurement error or additive measurement error. Funnily enough, in both specifications they assume that income term can vary across the individuals but the difference comes via the

⁸In pricipale we can allow more than one parameter to vary across individuals but then the likelihood function becomes extremely cumbersome.

wage effect.

More careful reading also reveals that there might be statistical problems with these models. It seems to be either the case that the other stochastic term is not identified (usually the unobserved heterogeneity term, see also chapter 2) or then the magnitude of the term just splits into a two equal parts. Again we conclude that introducing different stochastic assumptions have not had any big influence to the results and are not the main reasons for the different outcomes.

Nonparticipants

Practically all papers which study male labour supply utilise only the data for participants. In the case of females the picture changes crucially and in some studies the proportion of nonparticipants is bigger than the proportion of participants. From the table 1 and 2 we notice that those studies where the proportion of nonparticipants is relatively large also the compensated wage effects are clearly larger than in the rest of the studies. For example Arrufat and Zabalza[1](43% nonparticipants), Colombino and Del Boca[15](60% nonparticipants), Kaiser, Essen and Spahn[31](70% nonparticipants), Van Soest, Woittiez and Kapteyn[57](59% nonparticipants) all estimated the compensated wage elasticity to be over unity. Even that the compensated elasticities are significantly higher among the females than males above mentioned relationship is evident. For further evidence, Triest[55](27% nonparticipants) and Kuusmanen(chapter 2 in this study) estimated exactly the same models for the whole sample and only for the participants and both found that the compensated wage elasticities are clearly larger when nonparticipants are included.

Above findings are consistent with the Mroz[48] paper where he found that the Tobit model seems to exaggerate both the income and wage effects. Obviously there must be some deeper reason behind this phenomena and next we will turn to investigate how studies have treated unobserved wages.

Treatment of wages in final analysis

In the Tobit specification we write the likelihood function for both the nonparticipants and participants, so we have a discontinuous and continuous parts. In the context of labour supply zero hours corresponds the discontinuous part of the likelihood and positive hours correspond the continuous part of it. This likelihood is not a fully specified because we do not observe wages for nonworkers. So, we need to specify the wage equation or use some other statistical method to produce the wage rates for the nonparticipants.

As we can see from the tables, the most common procedure is to estimate the wage equation using Heckman's[26] method to control the selection bias and then predict the wage rates for the nonparticipants(or for the all observations). Heckman and Macurdy[25] and Macurdy et al.[43] have showed that this procedure actually produces a misspecified statistical model in the context of the likelihood method. They proposed the following procedure to avoid this problem. In this method one basically formulates the likelihood function so that it recognises the absence of information on wages for nonworkers. To the best of our knowledge, nobody has used this method in practise. Even Macurdy et al. restricts themselves to study only participants and thus they avoid this problem.

A closer look to the studies reveals that authors have constructed their budget set variables in different ways. One approach is to use direct wage information if that is available. Another common approach is to construct the gross wage rate by dividing annual labour earnings by annual hours of work.⁹ This method might produce measurement error in the wage rate. Errors in the reported hours will transform to the wage rates and thus leads to the spurious negative correlation between the wage rates and hours of work.

Probably the best way to compare if different wage measures lead to different results is to analyse the Hausman's and MacCurdy et al. studies more closely

⁹Note that it quite hard to say that if the approach applied in different studies is actually the same one because of terminological differences. Some authors report that they have used annual earnings instead of annual labour earnings as a nominator. Some authors report that they have used annual labour incomes a nominator.

because they use almost identical samples. Eklof and Sacklen[18] have carried out an extensive study to compare these two papers. Hausman basically uses directly observed wage information and Macurdy et al. use annual labour income as a nominator in their final analysis. Monte Carlo results by Eklof and Sacklen suggests that wage rate measure in Macurdy et al. might cause downward biased wage effects.

Other aspects

All studies in table 1 and 2 are partial equilibrium studies and thus only supply side effects are considered. It seems natural to think that also demand side effects matters, especially if we want to stress how we should get more people to participate in the labour force. In two different studies Holm, Honkapohja and Koskela[28] and Honkapohja, Koskela and Uusitalo[29] have utilised firm level data and aggregate time series data to study how taxation affects employment in Finland. In both studies the finding is that reducing the social security costs for the employers increases the labour demand especially in the labour intensive industries. At the same time there are evidence that reducing the lowest marginal tax rates will have a strongest effects on labour supply and thus combining these two reforms would likely increase employment share in Finland.

Institutional settings in the labour markets varies significantly between countries. In some countries, for example in Scandinavia, labour unions have a strong role. It has been argued that in these countries workers basicly decide if they want to participate or not and unions then negotiate the hours they may supply. On the other hand, in a countries like U.S. it seems that workers do have more power to to decide their labour supply. If these differences are big enough then tax reforms will most likely have a different labour supply effects in different countries.

Kuismanen[37] has estimated labour supply functions for Finnish females using cross-section data sets for years 1987,1989,1991 and 1993. The first two are years when economy were really booming and the last two were years when economy

experienced an unseeingly severe recession. According to the results the model was inadequate to take account the fact that individuals were not on their labour supply curve which was the case in recession years. It would be an extremely useful to study labour supply behaviour when macroeconomic environment changes.

1.5.1 Empirical evidence from the related Finnish studies

In his work Lahdenpera[38] studied female labour supply using the labour force survey data from the year 1980. He estimates a portfolio of different models for labour force participation and worked hours. In some models taxation is taken into account by linearising the budget constraint. Results vary quite a lot between specifications, for example compensated substitution elasticity varies from 0.11 to 0.48. Ilmakunnas[30] also estimates labour supply function for Finnish females. She proceed by first estimating a linearised model and then Hausman type of model using linear labour supply functions. According the linearised method the compensated substitution elasticity is 0.26 and the income elasticity is -0.16. ML-study shows a bit bigger elasticities. Compensated substitution elasticities vary between 0.27 to 0.29 and income elasticities vary from -0.17 to -0.19.

In chapter 2 we estimate a portfolio of models. Data set is from 1989 and our models deals with married females aged 25 to 60 years. Result from the "core" model shows the compensated substitution elasticity to be 0.21. If we compare these results to the studies shown in appendix 1, the following kind of observations can be made. First, Blomquist and Hansson-Brusewitz[5] estimate the compansated substitution elasticity to vary between 0.37 to 0.75 using the Hausman type of approach in the case of Sweden. A priori one expects our results to be very much on line with Swedish ones, because of the similarities between these two countries. Compared to other international studies our results show a generally smaller labour supply effects. Compared to the study by Ilmakunnas our results are more or less in line with each others. It is also the case that our elasticities received for females are larger than elasticities received for males (see

table 2). In particular, Kuusmanen[36] has estimated labour supply function with taxation for Finnish males and he received compensated substitution elasticity to be 0.12.

Laine and Uusitalo[39] evaluates changes in work incentives caused by the reform of the Finnish income transfer system. In the empirical part of the study they evaluate changes in incentives for two groups using the difference-in-difference estimator. According to their results participation elasticity with respect to wage varies from 0.12(males) to 0.22(females). For females this means something like an 1.6 per cents increase in months. These results are on line with the study by Eissa and Hoynes[17] where they study the earned income tax credit and labour force participation.

1.6 The contents of this thesis

Chapters 2 through 4 of this thesis add to the literature on labour supply and nonlinear budget sets. All works are empirically orientated. It is a first study which looks the income taxation and labour supply from different angles using Finnish microdata. The theme of this thesis is quite narrow: it only concentrates on the topic of estimating labour supply function in the presence of nonlinear income taxation scheme. This section presents summaries of the following three chapters and relates them to the literature discussed in a previous sections.

1.6.1 Summary of Chapter 2. Labour Supply and Progressive Income Taxation: An Empirical Study for Females Using Alternative Data Sets

In Chapter 2, The labour supply function for females is studied using a cross-section data set from 1989 applying the framework originally developed by Burt-

less and Hausman[10]. The motivation of the paper comes from the fact that nonlinearities and discontinuities in budget constraints may have effects on labour supply behaviour and thus it is important to try to study if this is the case. Secondly, procedure which takes the whole budget constraint into account in estimation is not free from problems. One of the most basic criticism is based on on the real root of all empirical work, namely on the data issues. This procedure requires a lot from the data set and it has been stated that it is impossible in most cases to construct budget sets accurately enough.

In section 2.3 we discuss about our data set and how we have constructed our variables used in empirical part of the study. Special attention is devoted to three crucial variables: hours of work, wage rate and unearned income. From that we move to the institutional set up of the Finnish income tax system and explain how for example tax deductions have been incorporated into the analysis. In section 2.5 we proceed using standard microeconomic tools how optimal labour supply can be defined in the presence of progressive income taxation. To go through this step is a very useful exercise to understand what lies behind the econometrics used later. What follows is the derivation of the likelihood functions. In the first state we do not make any specific assumptions about the stochastic specification. In other words, we assume that all variance in hours conditional on covariates is measurement error, which means that preferences are non-stochastic; all variation in preferences is due to the observable personal characteristics. But, there might be randomness in preferences which are not captured by the variables we include in our regression function. Due to this we also study how to derive likelihood function in the case of heterogeneity in preferences.

In empirical work we estimate labour supply function for Finnish married females aged 25-60 years. Estimates for the net wage term satisfy theoretical expectations and are precisely estimated in all cases. Exogenous income variable has negative sign as expected but it is not statistically significant even when we exclude spouse's net income from this variable. Thus, our results show a negligible income effects but a reasonable large uncompensated and compensated wage ef-

fects. Compensated wage elasticity is approximately 0.21 and it clearly smaller than the result got by Ilmakunnas[30] for married females(only participants) using 1987 data or Lahdenpera[38] using data from 1980. Lahdenpera's results are based on linearisation method. It is interesting to compare our results to the case of Sweden. Blomquist and Hansson–Brusewitz[5] obtain wage elasticities for females that vary from 0.34 to 0.75 using roughly the similar kind of statistical approach.

Next we proceed to estimate models where we have changed some crucial features of our model, namely using subjective wage measure instead of register based one, ignoring tax deductions and using constant municipality tax rate. Results are interesting. When using predictions for subjective wage measure we get higher compensated wage elasticities than earlier. Results also show that compensated wage elasticities are lower if we do not take into account individually calculated tax deductions. When we estimate the model only for participants we get lower compensated wage elasticities and this observation is on line with findings of Mroz[48].

All previous results were based on on models where preferences were assumed to non-stochastic. For participants we estimate a model where allow additive random preference term, but it does not have explanatory power at all. According to the results random variation is captured by the optimisation error term.

1.6.2 Summary of Chapter 3. Piece-wise Linear or Differentiable Budget Constraint? Estimation of Labour Supply Function in Presence of Non-Linear Income Taxation

Chapter 3 is related to the debate of how sensible it actually is to use the econometrical framework used in chapter 2. In addition of measurement problems related to the shape of budget constraint there are also other aspects which ques-

tions the use of piece-wise linear method. The fundamental assumption behind the models used in Chapter 2 is that the observed labour supply behaviour is outcome of free rational choice subject to the piece-wise linear budget constraint. In other words, we assume that econometricians and individuals who make decisions have perfect knowledge of their budget sets. It is questionable if these assumptions are actually met.

In addition MaCurdy et al.[43] have raised the doubt that ML-estimates of the labour supply function in the case of piece-wise linear approach are restricted a priori to satisfy certain properties(namely to full fill the so called Slutsky-criteria). It is still a open question if this is actually true. For example Blomquist[3][4] has written extensively from this and concludes that the above claim is not always true. To avoid these restrictions MaCurdy et al. develops a technique of differentiable budget constraint which we will utilise in Chapter 3.

The idea of this method is to approximate the tax schedule by fitting a function to the marginal tax rate and then integrating this we get a differentiable relation approximating the amount of total taxes paid as a function of taxable income. This approach is much simpler to estimate because a purely continuous distribution describes the hours of work we utilise exactly the same data set and functional form as in Chapter 2 and re-estimate the labour supply function using both approaches.

What we find out is that the results are almost identical between the two methods. This is not that surprising because the budget sets contains many linear segments and in this case the polynomial is able to mimic the actual piece-wise linear constraint quite closely. There are only a two similar studies made earlier. In Macurdy et al.[43] results differ significantly between the two approaches using U.S. data. In Flood and MaCurdy[19] they estimate labour supply model for Swedish males using these two methods. Like in our study results do not differ between differentiable and piece-wise linear approaches.

1.6.3 Summary of Chapter 4. Labour Supply and Income Tax Changes: A simulation Study for Finland

Chapter 4 is a natural next step to the earlier papers in this thesis. In this chapter we utilise our earlier results to build a small microsimulation model for Finnish economy. The additional motivations are the ongoing tax debate in Finland and to our knowledge this is the first attempt to use behavioural microsimulation approach to analyse various tax reform proposals.

In Section 4.2 we offer some motivation for microsimulation and relates it to the Finnish tax debate. Utilising two examples we will show that in the case of non-linear taxation we cannot predict the labour supply effects of tax reforms by just knowing the substitution and income elasticities. The key thing is the knowledge of each individual's location in the budget constraint. We also shortly discuss the technique of microsimulation.

Before going to the actual policy reforms we perform some preliminary simulations. Namely, we compare current progressive tax system to the proportional and to the lump-sum tax systems in a revenue neutral way. Main finding is that moving to the proportional tax system does not imply a big increases in labour supply. Next we study how changes in progressive income tax rate affect labour supply. We treat current system as a baseline scenario and then increase and decrease marginal tax rates. We find that percentage changes in mean hours and mean tax revenue relative to the baseline are bigger when decreasing the marginal tax rates. We also carry out an similar exercise in the case of proportional income tax system and find out that labour supply reactions are much smaller in this case.

We carry out three different tax reforms. In the first one we simply reduce marginal tax rate for one percentage point throughout the tax schedule. In the second reform we reduce the marginal tax rate at the lower end of the tax schedule and in the third one we lower the top marginal tax rates. The main findings are:

One percentage point reduction in the state income tax do not have a significant effect on labour supply. Biggest behavioural effects are received when we reduce the lowest marginal tax rates. It increases the willingness to enter to the labour force and also behavioural effects among the low income earners are reasonable large. Reducing the top marginal tax rates do not have any effect on labour force participation and hours changes are also quite modest. In all above cases behavioural effects are not big enough to compensate losses in tax revenues.

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Appendix 1

Labour Supply Results

Table 1. Estimates of Labour Supply Elasticities: Nonlinear Budget Sets, Males

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Method	Elasticities		
							Comp. w	inc.	uncomp. w
Blomquist (1983)	Swedish Inst. for Social Research Survey 1973 # of obs: 688 all participants.	H: Yearly Hours W: Directly observed Y: Spouse's net income + family allowances + net capital income	Calculated wage rates used in the final analysis	Linear	Convex (piecewise linear)	ML	0.11	-0.03	0.08
					Prog. taxes	ML-Random preference (income term)	0.1	-0.04	0.08
Blomquist & Hansson-Bruswitz (1990)	Swedish Level of Living Survey 1981. # of obs: 602, all participants	H: Yearly Hours W: Directly observed Y: Spouse's net income + family allowances + net capital income	Calculated wage rates used in the final analysis	Linear and quadratic (wage)	Convex and nonconvex (piecewise linear)	LINEAR: ML-Convex	0.08	0.002	0.08
						ML-Nonconvex	0.08	-0.008	0.08
						ML-Rand. pref. (income)	0.14	-0.01	0.13
					Prog. taxes	QUADRATIC ML-Convex	0.12	0.00	0.12
Bourgignon & Magnac	French Labour Force Survey 1985. # of obs: 1992, all participants	H: Normal weekly hours W: Hourly net wage (earnings/h hours) Y: Not specified	Calculated wage rates used in the final analysis	Linear	Convex (piecewise linear) Prog. taxes, benefits	ML	0.10	0.00	0.10

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Medhod	Elasticities		
							Comp. w	inc.	uncomp. w
Flood & MaCurdy (1992)	Swedish Household Market and Nonmarket Survey (HUS) 1984. # of obs: 492, all participants	H: Yearly hours W: Hourly wage (annual income/ annual hours) Y: Capital returns, unemp. ins., housing allowances etc.	Calculated wage rates used in the final analysis	Linear and semi- logarithmic	Convex (piecewise linear and differentiable) Prog. taxes	PIECEWISE LINEAR: ML-random pref. (income) DIFFERENTIA- BLE & LINEAR: ML-random. pref. (income) Add. err.	0.24	-0.10	0.16
						ML-random pref. (income) Multipl. err.	0.21	-0.09	0.14
						ML-random. pref (income) No meas. err.	0.10	-0.07	0.04
						DIFFERENTIA- BLE & SEMILOG: ML-random. pref. (income) Mult. err.	0.13	-0.08	0.07
						ML-random. pref. (income) No meas. err.	0.28	-0.09	0.21
						IV-METHOD: 7 different spesifications	0.33	-0.10	0.25
							from -0.28 to 0.30	from -0.11 to 0.04	from -0.25 to 0.21

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Method	Elasticities		
							Comp. w	inc.	uncomp. w
Friedberg (1995)	US March Current Population Survey 1983 ± 3 years % of obs: 4112 observations of which 80% do not work	H: Usual hours worked last year*weeks worked last year W: net hourly wage (annual earnings/hours) Y: Family income minus earnings and soc. security benefits.	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex and piecewise linear Prog. taxes and social security earnings test	ML	1.12	-0.76	0.36
Hausman (1981)	US Panel Study of Income Dynamics 1975. # of obs. 1085 all participants.	H: Annual hours of work W: Directly reported hourly wage rates. Y: 8% return to financial assets.	Calculated wage rates used in the final analysis	Linear	Convex and nonconvex (piecewise- linear) Prog. taxes and benefits	ML-random pref. (income)	0.13	0.00	-0.13
Kaiser, Essen & Spahn (1992)	German Socio Economic Panel 1983. # of obs: 2382 participants 939 nonparticipants	H: Yearly Hours W: Hourly wage (annual income/ annual hours) Y: Income from rents, capit. income and transf. payments.	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex (piecewise linear) Prog taxes	ML	0.28	-0.28	-0.00

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Medhod	Elasticities		
							Comp. w	inc.	uncomp. w
Macurdy, Green and Paarch (1990)	US Panel Study of Income Dynamics 1975. # of obs. 1017 all participants.	H: Annual hours of work W: Average hourly wage (incomes/annual hours) Y: Rent, interest, dividends etc.	Calculated wage rates used in the final analysis	Linear	Convex and nonconvex (piecewise- linear and diffrentiable budget constraints) Prog. taxes and benefits	ML-random pref.(income) (for the other results see the article)	0.01	-0.01	0.00
Triest (1990)	US Panel Study of Income Dynamics 1983 # of obs: 978 all participants.	H: Yearly hours in all job held in 1983. W: Average hourly earnings (incomes/annual hours) Y: Rents, dividends, interests, trust funds etc.	Calculated wage rates used in the final analysis	Linear	Convex and piecewise linear. Prog. taxes and Benefits	ML	0.00	0.00	0.00
van Soest, Woittiez & Kapteyn (1990)	Dutch Strategic Labour Market Research Survey 1985. # of obs: 801 participants 49 non- participants	H: Average weekly working hours W: Net hourly wage(incomes/ hours) Y: Not spesified	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex (piecewise linear) Prog taxes	ML	0.13	-0.01	0.12

Table 2.

Estimates of Labour Supply Elasticities: Nonlinear Budget Sets, Females

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Method	Elasticities		
							Comp. w	inc.	uncomp. w
Arrufat & Zabalza(1986)	Bristish General Household Survey 1974. # of obs: 2002 participants 1493 nonparticipants	H: Income-leisure ratio W:Gross hourly earnings. Y: Net weekly unearned fam. income+husb. earnings.	Wage rates predicted for all using the sample selection technique	Linear	Convex (Piecewise linear) Prog. taxes and benefits	ML Random pref. Add. err.	1.21	na	na
Blomquist & Hansson-Brusewitz (1990)	Swedish Level of Living Survey 1981. # of obs: 795,full sample 640,participants	H:Yearly Hours W: Directly observed Y: Spouse's net income+fam. allowances + net capital income.	Wage rates predicted for non-participants using the sample selection technique	Linear and quadratic (wage)	Convex and nonconvex (piecewise linear) Prog. taxes	LINEAR:	0.42	-0.03	0.39
						ML Nonconvex	1.03	-0.24	0.79
						Nonconvex ML Rand.Pref. (income)	0.84	-0.06	0.78
						QUADRATIC: ML Convex	0.6	-0.0	0.58
Bourgignon & Magnac (1990)	French Labour Force Survey 1985. # of obs: 1175 participants 817 non-participants	H: Normal weekly hours W: Hourly net wage (earnings/ hours) Y: Not spesified	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex (piecewise linear) Prog. taxes and benefits	Tobit	0.30	-0.20	0.10
						ML	1.0	-0.30	0.70

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Medhod	Elasticities		
							Comp. w	inc.	uncomp. w
Colombino & Del Boca (1990)	Survey of 1000 couples living in Turin 1979. # of obs: 338 participants 494 non- participants	H: Yearly hours (weeks worked* average weekly hours) W: Hourly wage (annual income/ annual hours) Y: Total net nonlabour income (not spesified)	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex (piecewise linear) Prog. taxes	ML	1.18	-0.66	0.52
Hausman (1981)	US Panel Study of Income Dynamics 1975. # of obs: 575 participants 510 non- participants.	H: Annual Hours of work W: Directly reported hourly wage rates. Y: 8% return to financial assets	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex and nonconvex (piecewise- linear) Prog. taxes and AFDC	ML-Random pref. (income)	0.68	-0.18	0.50
Kaiser, Essen & Spahn (1992)	German Socio Economic Panel 1983. # of obs: 1076 participants 2284 nonparticipants	H: Yearly Hours W: Hourly wage (annual income/ number of hours) Y: Income from rents, cap. income and transf. payments.	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex (piecewise linear) Prog taxes	ML	1.22	-0.18	1.04

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Medhod	Elasticities		
							Comp. w	inc.	uncomp. w
Kuismanen (1997)	Finnish Labour Force Survey and Finnish Tax Register Data (1989) # of obs. 1541 participants 485 non-participants	H: Yearly Hours W: Hourly Wage (Annual Income/ Number of Hours) Note: Calculated from Both Data Sets Y: Income from rents, dividends, cap. income etc.	Wage rates predicted for all using the sample selection technique	Semi-logarithmic	Convex (Piecewise Linear) Prog. taxes. Local tax rates. Tax deductions calculated for all individuals. Benefits	SURVEY DATA ML (all obs.)	0.28	-0.01	0.27
						TAX REGIST. DATA ML (all obs)	0.21	-0.01	0.20
						ML (participants)	0.10	0.00	0.10
						ML – random pref(intercept). (participants)	0.11	0.00	0.11
Triest (1990)	US Panel Study of Income Dynamics 1983 # of obs: 715 participants 263 non-participants	H: Yearly hours in all job held in 1983 W: Average hourly earnings (incomes/hours of work) Y: Rents, dividends, interests, trust funds etc.	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex and piecewise linear.	ML-working wives	0.42	-0.16	0.26
						ML-All obs.	1.31	-0.32	0.99

Author(s)	Data	Labour supply, wage and income variab.	Treatment of wages in the labour supply function	Functional Form	Budget Set	Estimation Medhod	Elasticities		
							Comp. w	inc.	uncomp. w
van Soest, Woittiez & Kapteyn (1990)	Dutch Strategic Labour Market Research Survey 1985. # of obs: 331 participants 470 non- participants	H: Average weekly working hours W: Net hourly wage (incomes/ hours) Y: Not spesified	Wage rates predicted for non-participants using the sample selection technique	Linear	Convex (piecewise linear) Prog taxes	ML	1.02	-0.23	0.79

Chapter 2

Labour Supply and Progressive Income Taxation: An Empirical Study for Females Using Alternative Data Sets

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Abstract

It is well documented that elasticities from labour supply studies vary from one study to another(with or without taxes) and naturally from one country to another. A standard explanation has been that researchers have used different statistical methods and different data sets and it seems to be the case that there are no generally accepted robust way to estimate labour supply functions(with taxation). In this paper we use two different data sets for the same individuals. One is the survey data and the other is register based data provided by tax authorities. We use method developed by Burtless and Hausman[9] and we are able to overcome some of the main criticism expressed concerning the Hausman type of modelling. Special attention is given to constructing budget constraints for

all individuals and we are able to estimate model taking account factors usually neglected in similar kind of analysis, like individual tax deductions and local tax rates. We estimate models using register based income information and using survey based information. Both fixed preference model and random preference model are estimated. Results indicate that using survey information compensated wage elasticities are significantly higher than using register based information.

Keywords: Labour supply, Non-linear income taxation, Budget constraints, Ml-estimation

JEL classification: C21, C24, C25, H24, J22

2.1 Introduction

In the last few decades the estimation of the labour supply function has been one of the most active research areas in labour economics. A vast majority of the empirical work is based on the neo-classical labour supply model which isolates the wage rate (depends on hours of work) and exogenous or unearned incomes (does not depend on hours of work) as a main economic factors that determines the individual's allocation between labour supply and leisure time. It is well documented that estimated income and substitution effects vary considerable from one study to another and they are sensitive to economical and statistical assumptions. See e.g. Mroz[23]. There are no generally accepted robust ways to estimate labour supply functions and the problem gets even more complicated when we introduce some real life phenomena to empirical analysis, like non-linear taxes or fixed costs (see Cogan[10], just to mention few).

It is apparent that progressive income taxation, which causes discontinuities to the budget constraints individuals face, may have effects on labour supply behaviour. In some tax schedules marginal wages fall substantially as workers cross points of discontinuity in their budget sets (in other words, when workers move from one tax bracket to another) and it is important to examine the effects of such discontinuities on work incentives. This topic has been studied heavily during the last few decades and the results and methods varies substantially. Probably one of the few things on which all researchers agree is that estimation of the labour supply function in a robust and generally accepted way is a very difficult task to do and one of the sources which creates difficulties is non-linear income taxation.

In this paper we use the method developed by Burtless and Hausman[9] and Hausman[12][13]. This approach takes the whole budget constraint individuals face into account in estimation procedure and it has been criticised for different reasons. One of the main problems is that this approach requires a lot from the data and usually researchers do not have suitable income data available. Heckman[15] argues that in the most cases budget constraints cannot be measured

accurately enough. We argue that our data set overcomes some of these problems and we are able to construct the budget sets which corresponds the actual ones quite closely. It means that we are able to take into account different municipality (or local) income tax rates, which obviously is a improvement from the usually used assumption of constant tax rates. We also develop a method to calculate the tax deductions for all individuals which relates directly to the Heckman's criticism that researchers are unable to locate points where the marginal tax rate changes. To the best of our knowledge, there are no other studies which have have taken individually calculated deductions into account.

Question about taxation's incentive effects to labour supply is interesting from many different perspectives: theoretical and econometrical problems are demanding ones and it also has a great implications to the economic policy. For example, in 1980 the share of income tax from the total taxes collected in Finland was 31 per cent and in 1989 it was 33 per cent, which is our year of study in this paper. When translating these shares into the money terms we are dealing with a significant sums of money in Finnish economy. So, it is not a surprise that this problem has raised a lot of interest in Finland in recent years where the level of taxation is high and also the degree of progressivity is high. Question has also raised a lot purely political debate and keeping all above mentioned points in mind it is surprising that there are only few studies made concerning labour supply and taxation. One of this paper's aim is to provide new evidence concerning female labour supply in Finland.

One source of motivation to write this paper comes from the data sets which are available for us. The main data set is the Finnish Labour Force (LFS) survey for year 1989. LFS includes basic information from individuals work history, demographics etc. and it also includes a limited amount of information concerning earnings. Basically, corresponding data sets have mainly used in labour supply studies. Our alternative data set comes from the Tax Authorities. This tax registration data includes all the income information concerning corresponding individuals in the LFS plus their possible spouse's income information. Income data

sets includes all possible income sources(excluding those bank accounts which are not taxable) and thus it gives us a good opportunity to model individual's budget set accurately enough. Using this data set we have a possibility to do some interesting comparisons, because the LFS data, as mentioned earlier, includes wage and income information.

We use the maximum likelihood method proposed by Burtless and Hausman[9], where the whole budget constraint is taken into account in the estimation procedure. This means that the likelihood function takes into account the choice of hours over the entire exogenous tax schedule removing the endogeneity problem which is present in a simpler approaches. We also relax the assumption that all the variance in hours conditional on covariates is measurement error and we estimate our labour supply function allowing additional random term which tries to capture the heterogeneity of the preferences.

The remaining sections of the paper are as follows. In section 2.2 we shortly discuss some points of the labour supply theory when consumers face non-linear budget constraints. This gives us background and insight how we should proceed in the empirical part of our study. Following section describes the data used in a greater detail. In this section we go through how we have constructed the crucial variables used in the analysis. Section 2.4 describes the Finnish tax system and economy in 1989. Tax system affects to the shape of budget sets and our view is that giving some macroeconomic background may help us to put our results in context, so we describe shortly the economic situation in Finland in 1989. In the next section we discuss about our labour supply specification and section 2.6 introduce our econometrical approach. Section 2.7 presents the results and section 2.8 concludes.

2.2 Some remarks from the labour supply theory

Since the standard labour supply theory is well known and documented, we only shortly comment those points of it which we think are relevant for our empirical purposes.

It is usually assumed that individuals maximise the value of their utility function with respect to two constraints, namely budget and time constraints. One way to express the utility function is to assume that individual's utility depends on consumption c and leisure l . The Budget constraint can be written as $wh + y = pc$. w refers to (market) wage rate and y refers to exogenous (unearned) income. h is the hours worked and p is the price of consumption.¹ Time constraint can be written as $h + l = T$, where h refers to hours worked, l is the leisure time and T is the total amount of time available. Assuming that our utility function is strictly quasi-concave and because our constraints are linear, optimal allocation between leisure time and consumption can be found from the point where the ratio of marginal utility of the consumption and the marginal utility of the leisure equals the ratio of corresponding prices, i.e. $[(\partial u / \partial l) / (\partial u / \partial c)] = [w / p]$. In other words, the solution means that we have a unique tangency point between indifference curve and budget constraint for each wage rate. Obviously this is not the case under the non-linear budget constraint.

So, the neo-classical labour supply analysis gets complicated when the budget constraint becomes non-linear. In the case of linear income tax it is straightforward to derive Slutsky condition and see that linear income tax has only effect of a scale factor which dampens the substitution and income effects of a change in the wage rate compared to case of no taxation. Already this simply exercise shows, that estimating labour supply function without taking income taxation into account may lead to biased wage estimates.

In the presence of progressive income taxation the budget constraint becomes non-

¹Usually c denotes Hicksian composite commodity and thus p is set as unity.

linear, or strictly speaking piecewise linear with discontinuity points where the marginal tax rates change.² To the best of our knowledge, deriving comparative static results in this case is impossible. It is possible to derive the signs of income and wage elasticities conditional that the individual is on some linear segment. These conditional changes do not define the total effect, because the individual may change segment(or kink).

Let us illustrate this point by a simple example. Let us assume that our tax schedule generates convex piecewise linear budget constraint and our tax parameters are lump sum tax, exemption level, change of tax bracket limit, change of marginal tax rate and change of gross wage rate. In real life tax reforms usually more than one of these change simultaneously, but let us change them one at a time.

A change in a lump sum tax shifts our convex piecewise linear budget constraint upwards or downwards in a parallel way, so the only effect is the income effect. When we change the exemption level, it will affect exogenous income and all the virtual income components and all the points where the tax brackets changes. It will not affect marginal wages. When we change the limit of some tax bracket j , it will have the following effects: it will leave all lower segments unaffected. It will change the tax bracket limit j and it will increase all the virtual income components above the segment j . A change of the marginal tax rate (say it increases) for the bracket j will leave all the lower segments unaffected. It will change the slope of the segment j (decrease) and change the corresponding virtual income (increase). Finally, a change of the gross wage rate will change all the slopes and it will change all the tax bracket limits, but it has no effect on virtual income components.

Now, it is clear that changes in the tax parameters have different effects to different individuals depending their initial location in the budget constraint, i.e. reaction of different individuals with a different initial locations may even be op-

²In this context word progressivity means that the marginal tax rate increases with income, but it is constant within the tax bracket.

posite. We can also see that small changes in wage rate and virtual income may keep individual in the same segment (when predictions from the 'basic model' are valid) but there is always the likelihood that individual will change the segment. And finally we know that utility maximisation in the presence of the kink is compatible with many different marginal rates of substitution, so individual may as well stay at the kink.

Above example gives us an idea how we should model the income tax schedule in our empirical work and in fact, we will do so following a method developed by Hausman[12][13]. Because a lot of critique towards this method has been raised, and one of the reasons have been data sets used, we will next turn to describe our data set.

2.3 The data

A sample of married women of age between 25–60 is drawn from the Finnish Labour Force Survey (LFS) for the year 1989. It is a cross-section data and it includes individuals of age between 15 to 64. In the first stage the sample is drawn from the Finnish Population Census using geographical weights. After that the sample is drawn randomly by age and gender and the sample size is 7820 individuals. Before year 1993 the LFS was collected in every second year (independent cross-sections) and before year 1989 an substitute interviewees were allowed. Comparing the LFS data sets between the years should be done by some caution, because the method of collecting the data and some definitions have changed during this period. As an example, in 1989 an substitute interviewees were not allowed which means that the loss is bigger than in previous years but the number of 'not known' answers is respectively smaller and answers should also be more precise than before.

Income data corresponding those individuals in the LFS is drawn from the Tax Register Data and then merged with the LFS. The income information is not

based on the survey data and it includes approx. 70 variables on individuals earnings. Of course, it is very unlikely that someone's earnings are composed from all of these components. However, the data shows that individuals' earnings comes from the very different sources. Actually, for some individuals traditionally used income variables do not play any role at all. The income data also includes the same 70 variables for possible spouse, so all in all we have approx. 140 variables(if married) to construct the budget sets individuals face.³

2.3.1 Hours of work, wages and exogenous incomes

In the empirical labour supply analysis the following three variables are under special attention; namely hours of work, wage rate and unearned incomes. That is because the classical question usually asked in the labour supply studies is the following: what happens to the hours of work when wages and/or unearned incomes change? In order to answer this question we need to estimate the substitution and the income elasticities i.e. we need (ideally accurate) information on wages and unearned incomes in our deterministic part of the regression function. From the statistical point of view it is crucial that the hours of work variable (endogenous variable) varies enough around its mean. When studying a labour supply responses in Finland (or in any other Scandinavian country) we should especially examine this carefully.⁴

Our data includes information both from the regular weekly hours and from the hours worked in the survey week⁵. In this study we use the regular weekly hours.

³We would like to stress that the income data used in this study is exceptionally rich compared to some other data sets, for example to Panel Study Income Dynamics (PSID). Most of the variables have gone through several checks by the tax authorities.

⁴A widely shared view is that Scandinavian societies are highly unionised and the labour markets are not very flexible. This is partly true. If we look hours of work by gender, we see that male hours are much more concentrated than female hours. This partly reflect the fact that the male dominated unions may have different objects in negotiations than the female dominated unions.

⁵In the case of many individuals, worked hours in the survey week deviates from the regular

In addition, we take into account regular hours in the second job and we also take account the number of holidays.⁶ Unfortunately, we do not have any information for the overtime working hours. This is a clear drawback, because in many cases it is reasonable to assume that overtime hours are the flexible part in the labour supply decision. But, on the other hand, wage rate from the overtime work usually differs from the regular one, so incorporating that into a statistical analysis is not necessary a straightforward task to do.

The data set does not have the direct information on individuals' hourly wage, thus we have to construct it using the income and hours of work variables. This procedure means that the possible bias in the hours of work variable shifts also into the marginal hourly wage rate. Statistically it means that the dependent and the independent variable are negatively correlated. For example, if worked hours are smaller than their right value then the value of hourly wage becomes too high. We estimate the log wage equation using Heckman's method and the predicted values are used in the final analysis as a instrument for the hourly wage rate.

By definition unearned (exogenous) income does not depend on worked hours. In empirical work this definition is not always valid because, at least in the long run, it is quite difficult to say which income components depend on worked hours and which do not. The data set in hand gives us a good opportunity to construct different kinds of exogenous income variables. Unlike most studies made e.g. in UK, we do not have to use consumption information to evaluate this variable. We use the following income components when calculating the unearned incomes: interests(both taxable and non taxable),dividend payments, property incomes,

hours, so calculating the yearly hours from the survey hours lead to unrealistic, much too high or low, values. For example, using survey week hours summed up to yearly hours and adjusted to the number of holidays, we still found few individuals working over 6800 hours per year, which is clearly too much(18.6 hours per every day)

⁶From the data we see that the number of holidays varies considerably between individuals(from 0 to 40 working days(this includes special holidays negotiated by the trade unions and government)).

sales profits, regular non-taxable pensions, other regular subsidies etc. From all the components which are taxable we have subtracted the corresponding amount of taxes paid, so our variable measures net exogenous incomes. We have also taken into account spouse's net incomes. We argue that our data gives us enough detailed information to construct the budget sets which mimics the real ones in quite well.

2.3.2 Sample selection criteria's and some descriptive statistics

Our original sample size was 7825 individuals and from this we selected females. This left us with 4124 observations. For the empirical analysis we select married women aged 25–60. There are many reasons for this choice. First of all, looking the data shows that female labour supply is more flexible than male labour supply. It is still the case that women are secondary workers in most families. This means, a priori, taxation(or tax reform) is more likely to affect their labour supply behaviour. Secondly, we select above age group because of the following facts which characterise the society. Compared to, for example UK or US, Finnish young people start to work⁷ at much elderly age. The average graduation age from the university is around 26 and individuals graduating from vocational, trade schools etc. are usually over 20 years old. In addition, the data shows us that the proportion of females under 25 with regular incomes is low. The reason for the choice of the upper bound also reflects the features of Finnish labour markets. Retirement age is quite low compared to the some other European countries. It is common that women above 60 years old are on pension. As our objective in this paper is to study how income taxation affect hours of work, we want that the majority of our final sample are (at least potentially) active in the labour markets.

⁷We do not take into account summer jobs or other short term jobs students usually have between semesters.

Thirdly, we chose married⁸ women because this allows us to study the role of spouse's incomes in the labour supply decision.⁹ It is also the case that these individuals represent more or less the 'basic cases' and constructing their budget sets are much easier than for example, single mothers with children.¹⁰

In the second step we deleted some special groups like farmers and self-employed mainly because of the different tax legislation. After these selections the final sample size used in the empirical analysis is 2026 observations.

Features of the data

Next, we shortly comment basic features of our data. A comprehensive data appendix can be found from the appendices 2, 3 and 4. Let's first look at labour supply behaviour. The participation rate in the sample is 72 per cent. This figure is on the line with other years in the eighties and it is clearly higher than in, e.g. UK or US. Unemployment rate varies geographically and the figures are lowest in the Helsinki metropolitan area and highest in the East and North part of the country. One has to keep in mind that in 1989 Finland was experiencing an economic boom and the unemployment rate was lowest for many years, 3.2 per cent. Annual change in GDP(4.3 per cent) was above the historical trend. Wages increased rapidly and in general workers had a good possibilities to affect their working conditions.

Finland, like the other Scandinavian countries, is traditionally an highly unionised

⁸In principle, we should have also chosen the cohabiting cases but this leads to the following problem. First, we do not have any information on their partners incomes as we do in case of married couples. The amount of cohabiting cases is very small, less than half a percentage point.

⁹Unfortunately we do not have information on spouse's hours, so we cannot study intra-household labour supply decisions using this data set.

¹⁰This is not to say that this group is not worth of studying. Actually, it would be very useful exercise to do that because, a priori, this group's labour supply behaviour is probably the most sensitive one for the tax and benefit system reforms. The practical difficulty also arises if one likes to use LFS, because the number of single mothers with children is relatively small.

country and in 1987 75 (among participants) per cent belonged to some union. In 1989 the share declined to 71 percent, due to the well booming economy. It is well documented that individuals felt that it was useless to pay for the unions, because of no threat of unemployment.

As in the econometric part we will see that our main goal is to explain individuals optimal labour supply i.e. we will estimate parameters which also appears in the individuals' utility function. In that sense it is useful to see what the data says about the individuals desired labour supply. 17.4 per cent from the sample would like to work fewer hours and 7 per cent would like to work more hours than their current hours. From those women who work more than 30 hours per week 13 per cent would like to work as a part-timer for a change, and 6 per cent would like to be a permanent part-time workers. From those women who are part-time workers 66.4 per cent would like to stay as a part-timer. The most common reason for the part-time work is the will to take care of their children (36.2 per cent). In the sample the most preferred weekly working hours would be 30–34 hours per week which is clearly less than the average weekly hours.

It is also clear from the data (see appendix 3) that the blue-collar workers are more likely to have zero hours observations than the white-collar ones. Among women those who have two young child have the highest probability to be out of work.

When we divide the data to two different parts, namely for participants and non-participants, we can see the differences between these groups. It is quite obvious that the share who belongs to unions is higher in participants (71 per cent) than in non-participants (21 per cent). Participants are slightly younger than the non-participants and they also seem to have better educational background. 32 per cent of the non-participants have a child aged between 0 to 2 and respectively participants have proportionally more elderly children. It is also true that participants have more work experience. It is interesting to see that 86 per cent of the participants spouse's are working when the corresponding figure among

the non-participants is 68 per cent. So, it seems to be that likelihood of being non-participant increases if the spouse is also non-participant.

Non-participants have higher unearned incomes than the participants. One reason for this is that the non-participants received proportionally higher child benefits. When we take account the husbands net incomes we see that in this case unearned incomes are higher among the participants. Geographical location also seems to differ between these two groups. If you are non-participant you more likely live away from the Helsinki metropolitan area.

Let's take a closer look at the hours of work variable. As from the cross-tabulations can be seen blue-collar workers are more likely to have zero hours and also naturally union members are more likely to have positive number of hours. Among women who have children those who have two child have the highest probability to be out of work. For closer look see appendix 4.

2.4 Finnish income tax system and economy in 1989

Basically, the income tax system consists two parts: a progressive state income tax and a proportional local(municipal) income tax. In addition, individuals contribute to the National Pension Insurance(NP, 1.55 per cent from the taxable income) scheme and National Health Insurance(NH, 1.25 per cent from the taxable income) scheme, which are proportional to income changes. Roughly speaking, the tax liability in state tax and municipal(or local) tax is the same excluding the tax deduction system. A Further distinctive feature in Finnish tax system compared to some other European countries is that all individual are separate tax units. Husband's marginal tax rate does not affect wife's marginal tax rate.

In 1989 the state income tax schedule was composed of six marginal tax rates varying from 11 to 51 per cent. The following table shows the tax schedule for

the national tax in 1989.

Table 2.1. Income Tax Schedule.

State income tax schedule — 1989		
<i>taxable income</i>	<i>tax at lower bound</i>	<i>margin. tax rate</i>
36 000 – 51 000	50	11
51 000 – 63 000	1700	21
63 000 – 89 000	4220	26
89 000 – 140 000	10 980	32
140 000 – 250 000	27 300	37
250 000 –	68 000	44

To the best of our knowledge all previous studies have assumed fixed local tax rate in empirical specifications. We argue that this is an questionable assumption. For example, in 1989 the local tax varied from 14 per cent to 19.5 percent. We also have to keep in mind, that the local tax is paid from all labour incomes unlike in the case of state income tax where the exemption level varies from year to year. In this study we have calculated, using the additional information,¹¹ local tax rates to all individuals. For example, in 1989 the mean of the local tax rate was 16.38 per cent and the mean of calculated tax rate is 16.21 per cent. It is obvious that taking the variation in local tax rates into account when constructing the budget constraints lead to more precise procedure.

One other small innovation used in this study is that we have developed a formula to calculate the accepted tax deductions.¹² Previous studies have assumed a constant tax deductions(or no tax deductions at all, which actually is the same thing) for all individuals which means that authors have assumed independence between the tax deductions and the income level. This is clearly untrue, at least in Finnish tax systems. We use the tax function and the tax parameters to calculate

¹¹Statistic Finland deletes the municipality code from the data sets due to the legislation, but we can trace it using the income data set.

¹²We owe thanks to Ilpo Suoniemi who asked us to think this possibility.

the tax deductions individually. Of course, this procedure is not a perfect¹³, but it confirms the result that the high earners have also higher deductions and thus makes the actual tax system less progressive than it is usually thought to be. As an example, in 1989 estimated tax deductions varied for females from 0 FMK to 29 500 FMK. If we compare this information (by income quintiles) to one calculated by tax authorities, we get almost similar results. Again, taking tax deductions into account we get more precise picture about individuals' budget constraints.¹⁴ See further details from the appendix 4.

Labour supply decision is not only a microeconomic phenomenon. In addition for the individuals preferences some macroeconomic aspects also affects to the labour supply decision. For example, in the country like Finland it is much easier for the workers (or their unions) to negotiate about working conditions when the labour demand is relatively strong. Our sample year was time for the strong economic growth, low unemployment and so, *a priori* one could expect that labour market are closer to the state where individuals have a possibility to choose their amount of work time, as it is expected in the economic theory.

¹³For example, the tax legislation allows to shift deductions between couple in some special cases and tracing this using the given information is impossible.

¹⁴Basically we construct the tax function using the income information and the tax schedule. The procedure is roughly following: Because we know individuals labour income we also know her location in the tax schedule. From her labour incomes we deduct the minimum amount of tax that must be paid in that specific tax bracket. This means that we are left with the amount of income which exceeds that lower limit. Next we multiply this amount by the relevant marginal tax rate and we get the amount of taxes paid from it. Now, we can sum up the amount of tax paid at the lower bound of the tax bracket and the amount of tax paid above that bound. This leads us to the total amount of taxes paid. From this figure when we deduct the amount actually paid we are left with the accepted tax deductions.

2.5 Labour supply specification and the optimal hours in the case of convex budget set

In many cases the micro-level data shows that there are quite a substantial amount of variety in individuals' behaviour. When this is the case we need to try to take this into account in the empirical work. But this is not an easy task to do and there are (almost) always a trade-off between statistical modelling and economic theory. For example, in this context sometimes our empirical models are used for policy analysis and this naturally requires simplicity and theory consistency. On the other hand data may require that the statistical model should be a flexible one. Stern[26] lists following general aspects which should be kept in mind when considering the choice of the functional form: Consistency with the utility theory, convenience in estimation, facility for incorporating in theoretical studies, ease of use in applied problems and flexibility in the type response it permits. In the older studies linear labour supply function(strictly speaking, linear respect to variables and parameters) was popular in empirical analysis, but it is quite obvious that this is an arguable choice as Blundell[4] mentions.¹⁵

In the empirical section we estimate the semi-log labour supply function of the

¹⁵He gives the following clarifying example. Consider an increase in the tax rate in a linear tax system. This reduces the hourly wage rate among the employed individuals and reduces the pay-off to every extra hours worked. Individual who are free to enter to the labour markets are less likely to do so after the change in tax rate. Those in work are expected to reduce their hours. This latter argument is only a prediction from the economic theory if those already in work are compensated for the loss in utility generated by the loss in leisure time. In the absence of this kind of compensation, the income effect generated by the loss in earned income may increase desired work effort. Result is the so-called backward bending labour supply curve. Blundell[4]

form¹⁶

$$h^* = \alpha \ln w + \beta y + \gamma z. \quad (2.1)$$

where w is the marginal hourly wage rate, y is the unearned income and z is the vector of demographic variables.¹⁷ The precise form of the labour supply function is left as an empirical choice.

2.5.1 Defining the optimal hours

In this subsection we shortly go through how the optimal labour supply can be defined in the presence of progressive income taxation using the standard microeconomic tools.

In general, we only need to know the form of the indirect utility function and this can be derived as a result from the restricted utility maximising problem or in the case of piecewise linear budget constraint given some linear segment.

$$c = v(w, y) = \max_{c, h} [u(c, h) : c - wh \leq y], \quad (2.2)$$

where c is the composite commodity and h is labour supply. As always, the relationship between labour supply function and indirect utility function follows from Roy's identity.¹⁸

¹⁶We also use the following functional form as a "check" for our empirical model

$$h^* = \alpha \ln w + \beta \left(\frac{y}{w} \right) + \gamma z.$$

¹⁷Above functional form can be generalised allowing demographic variables (*demographic translation*) enter into the hours equation through the parameters α and β as suggested by Pollack and Wales[24]. See also Blundell and Meghir[8] for model specification in the context of labour supply. Above functional form or its generalisations have been widely used in recent labour supply studies.

¹⁸In the general N-dimensional case Roy's identity raises the question about the integrability conditions, because the function must satisfy the conditions set for the Slutsky matrix. In

$$h = -\frac{dv/dw}{dv/dy}. \quad (2.3)$$

For all given indifference curves we treat y as a function of w , i.e.

$$c = c(w, y) = v(w, y(w)). \quad (2.4)$$

When we derivate the above equation and use the Roy's identity in the case of, for example, **semi-log labour supply function** we get

$$\frac{dy}{dw} + \beta y = -(\alpha \log w + \gamma), \quad (2.5)$$

which is the first degree linear differential equation and its solution is quite straightforward. It is easy to show that the indirect utility function has the following form.

$$v = v(w, y) = \frac{e^{\beta w}}{\beta} \left[y\beta + \alpha \log w + \gamma \right] - \frac{\alpha}{\beta} \int \frac{e^{\beta w}}{w} dw \quad (2.6)$$

where the last part is the standard exponential integral.¹⁹

It is also possible to derive the direct utility function from the indirect utility function in the both cases as result from the restricted minimisation problem, but we do not need these in estimation as we are dealing with the convex budget constraints(as we do not actually need the indirect utility functions either.)

this two dimensional case situation is quite easy, because all we need is that derivative of the compensated labour supply with respect to wage is nonnegative and the indirect utility function is monotonically increasing function with respect to wage and unearned income.

¹⁹If the labour supply function has the form presented in footnote 16, then we can derive the indirect utility function using the same procedure as above and it has the following form

$$V = \frac{w^{1+\beta}}{1+\beta} \left(\frac{y}{w} (1+\beta)^2 + \alpha \ln w + \gamma - \frac{\alpha}{1+\beta} \right). \quad (2.7)$$

In theory, it is important to derive the indirect and direct utility functions. First, If the estimated parameters fulfil the Slutsky condition, then we know that the used labour supply function is consistent with the theory of utility maximisation. Secondly, in the case of non-convex budget constraint we need the tangent condition between the indifference curve and the budget constraint and to do this we need to know the form of direct utility function. Thirdly, in the above situation we need tangent condition in our search algorithm.

Of course, in the case of linear budget constraint the optimal level of labour supply is easy to determine; we just have to equate the slope of the indifference curve to the slope of the budget constraint(which is w), and solve it for h . In the case of non-linear tax system we have many different wage rates and thus to find the optimal labour supply gets a bit more difficult.

Next we move to our empirical modelling and we show how the above mentioned procedure can be transformed into practise.

2.6 Derivation of the likelihood function under non-linear taxes

Practically in all countries majority of governments tax and transfer schemes, like progressive income taxation and mean-tested income programs, create non-linear budget sets and discontinuities in the labour supply schedules. Crucial feature in the traditional empirical consumer demand analysis is that the consumer is assumed to purchase any desired quantity at a constant price subject to a budget constraint, ie. budget constraint is assumed to be linear. For example, in the case of progressive income tax system the "price" (i.e. wage) is not a constant, it varies with hours of work. This means that consumers face many different

marginal wage rates.²⁰

Hausman[13] argues that ignoring non-linearities in empirical work is potential source of a mis-specification. How non-linearities then should be taken into account? The simplest solution is to use net wages as a regressor. But, it is apparent that net wage is correlated with the hours through the non-linear tax (and possible some transfer schedule) system causing the endogeneity problem.²¹

The Maximum Likelihood method proposed by Burtless and Hausman[9] is where the whole budget constraint is taken into account in the estimation procedure. The general principle in this approach is that the consumer chooses her most preferred labour supply point on each budget segment, determine the corresponding utility of that choice and then chooses the one that yields *maximum maximum* of utility across all segments (Hausman[13]). In labour supply context above means that the likelihood function takes into account the choice of hours over the entire exogenous tax schedule removing the endogeneity problem mentioned earlier.

Before going into the technical presentation, a few words about the so called Hausman–methodology is in place. MaCurdy, Green and Paarch[20] argued that a proper specification of the log-likelihood over the region of a convex interior kink(s) is only possible if the estimated coefficients fulfil the condition that the compensated substitution effect is positive. In other words, if the compensated substitution effect is not positive for some observations then they will have a negative probability of locating at the kink. In their study the data had a problems to

²⁰Kinked budget constraints are also present in many other demand applications, like in the case of rationing and block pricing.

²¹This procedure actually means that we linearise the budget constraint around the observed hours. The linearised and original budget constraints yield the same optimum if the data are generated by utility maximisation with globally convex preferences. Ordinary Least Squares can now be applied to the linearised data. However, using OLS can lead to inconsistent estimates, See e.g. Pudney[25], and therefore many researchers have applied Instrumental Variable method. Choice of instrument sets varies among the authors due to different exogeneity assumptions. See e.g. Blomquist[3] and MaCurdy et al[20].

meet that condition which means that the income coefficient must be constrained to be negative. As Blomquist[2] mentions, if the condition is legitimately met by the data then the estimation procedure imposes nothing on the data. There are no clear evidence that the data always fail to meet the condition mentioned above. Blundell et al.[5] avoid the problem by deleting the observations near the convex kink²² and then adding a selection term in a IV-regression to take the exclusion into account. This procedure is questionable if there are many kink points. First of all it means that we had to exclude considerable amount of observations from the analysis. Secondly, the definition of "near the kink" is unclear.

Relating to above discussion it is evident that this approach requires a lot from the data. Heckman[15] shows that in most of the cases the budget constraint is not accurately enough measured, but to estimate the model requires that the all kinks in the constraint for all individuals can be accurately determined. Moffit[22] writes following; *"It is difficult given the data available to accurately determine the exact location of the kinks... because insufficient data are available on deductions, filing status, tax avoidance and so on. As a result, the location of the kinks assumed for the analysis may be incorrect"*. As mentioned above, we argue that in this paper we can overcome these problems because of the data in hand and how we construct the budget constraints.

First assumption we will make is that preferences are non-stochastic i.e. any variation in preferences comes entirely from the observable personal attributes (we will relax this assumption later). Let the tax system be following. There are n linear tax segments and $n - 1$ kink points where the marginal tax rates changes (H_1, \dots, H_{n-1}) . Budget set is assumed to be convex. Zero hours and maximal hours are denoted H_0 and H_n correspondingly. In the case of linear budget constraint it is easy to find the optimal labour supply because there are only one marginal tax rate, but in the case of progressive income taxation there are many different marginal rates, so we have to develop an search algorithm to find the optimal solution.²³

²²In their application there is only one kink point.

²³Linearization method is sensitive for the two sources of bias. First is the simultaneity

Third, and in some sense most important assumption, is that we believe that observed labour market behaviour is the outcome of free rational choice subject to the constraints imposed by the income tax schedule. As Pudney[25] mentions, if one believes to the above mentioned assumption, there are good reasons to use statistical techniques that take complexity of the budget sets properly into account.

In presence of a convex budget set a quasi-concave utility function implies the existence of unique optimum given the maximisation. Now, we start by deriving desired labour supply on the first segment by substituting the corresponding net wage w_1 and unearned income y_i into the labour supply function. If desired hours h^* are less than equal to zero, then the individual is in the corner(zero hours). If desired hours lies between zero and H_1 (second kink point. Note that the zero hours is the first kink point) we have the unique optimal desired hours for the individual. If desired hours are above the second kink point we move to the second segment. Net wage is now w_2 and virtual income is y_2 . Now, desired hours are determined using these figures and if the desired hours is less than H_1 , then H_1 is the unique optimum. This can be seen applying the revealed preference argument. If $H_1 < h^* < H_2$, then unique optimum can be found from the second segment etc.

problem. Tax rate affects individuals work decision, but tax rate is also affected by the hours decision and the simple regression approach does not distinguish between these two causal links. At least one column of the matrix of observable characteristics is endogenous because it depends on the tax rate which depends on individual's gross wage and this is in turn function of hours decision. The result is non-zero correlation between the stochastic term and the deterministic term(one or more components of it), thus the term $N^{-1} \sum \beta \epsilon$ has a non-zero probability limit. The other source of the bias comes from the possible mis-specification of the regression function. Linearisation method assumes that if the individual is observed to be in one of the segments, then his or her optimal choice must also be in the **interior** point of that segment. But, the presence of the stochastic term means that observed and optimal choices can differ and thus lead to biased estimates.

2.6.1 A Standard approach

Like earlier in this paper we can write our general labour supply function as $h_i = h_i^*(w_i, y_i, z_i; \alpha, \beta, \gamma) + \epsilon_i$. In the statistical model we have to calculate the densities of h_i and this requires evaluation of the maximum utilities received on each linear segment of the budget set just like above described. More formally, we can now write the problem as

$$\begin{aligned}
 f(h_i) &= P[h_i = 0] + P[h_i > 0] * f(h_i \mid h_i > 0) + P[h_i = H_n] \\
 &= P[\text{ at zero }] \\
 &+ P[\text{ below kink 1 }] * f(h_i \mid \text{ below kink 1 }) \\
 &+ P[\text{ at kink 1 }] \\
 &+ P[\text{ above kink 1 }] * f(h_i \mid \text{ above kink 1 }) \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 &+ P[\text{ at maximum }]
 \end{aligned} \tag{2.8}$$

So, we can think that **observed** hours are generated by the following generalised Tobit-model²⁴

$$\begin{aligned}
 h &= 0 && \text{if } h^* + \epsilon = 0 \\
 h &= h^* + \epsilon && \text{if } 0 < h^* + \epsilon < H_n \\
 h &= H_n && \text{if } h^* + \epsilon \geq H_n
 \end{aligned}$$

and the corresponding Likelihood Function can now be written as

²⁴Note that we have dropped the subscript i .

$$L = \prod_{i \in A} \left[1 - \Phi \left(\frac{h^*}{\sigma_\varepsilon} \right) \right] \prod_{i \in B} \left[\frac{1}{\sigma_\varepsilon} \phi \left(\frac{h_i - h^*}{\sigma_\varepsilon} \right) \right] \prod_{i \in C} \left[1 - \Phi \left(\frac{H_n - h^*}{\sigma_\varepsilon} \right) \right]. \quad (2.9)$$

Where,

A is index set when $h = 0$

B is index set when $0 < h < H_n$

C is index set when $h \geq H_n$.

$\phi(\cdot)$ is Standardised Normal Density Function and $\Phi(\cdot)$ is Cumulative Normal.

The first part of the likelihood function correspond individuals whose observed hours are zero. The second part corresponds those individuals whose observed hours are in some of the segments or kink points and the third part corresponds those whose observed hours are at maximum.

At this stage we need to show how to find out the way to calculate the optimal supply of hours in the presence of kinked (convex) budget constraint.

The optimal supply of hours h^* can be found from the segment k ($k = 1, \dots, n$), if

$$H_{k-1} < h^*(w_k, y_k, z; \alpha, \beta, \gamma) < H_k \quad (2.10)$$

Intuition behind this calculation rule is following: after we have calculated the slope of the indifference curve from the direct utility function we replace consumption $c (= w_k h + y_k)$ by individuals income (calculated for all the segments) and after that we equate the slope of the indifference curve and the marginal wage w_k corresponding that segment. The algorithm iterate as long as this condition is satisfied (as explained earlier). If, for some individual we cannot find the solution we start to look if we can find it from some of the kink points.

Optimum h^* is found from the kink point H_k ($k = 1, \dots, n - 1$), if

$$h^*(w_k, y_k, z; \alpha, \beta, \gamma) \geq H_k \quad \text{and} \quad h^*(w_{k+1}, y_{k+1}, z; \alpha, \beta, \gamma) \leq H_k. \quad (2.11)$$

An other way to express this condition is that the optimum can be found from the H_K , if the slope of the indifference curve is bigger or equal than w_{k+1} and the the slope is smaller or equal than w_k .

For the completeness we can show that optimum can be found from the zero hours $h = 0$ if

$$h^*(w_1, y_1, z; \alpha, \beta, \gamma) \leq 0 \quad (2.12)$$

or correspondingly from the maximum hours $h = H_n$, if

$$h^*(w_n, y_n, z; \alpha, \beta, \gamma) \geq H_n. \quad (2.13)$$

The above formulation shows how we can calculate the optimal hours under the progressive income taxation when we know the following aspects: tax schedule, hourly wage, exogenous incomes and the shape of the labour supply function (or correspondingly the shape of the utility function).

Despite that h^* can be calculated quite easily in the convex case, the maximisation of the (log) likelihood function is not straightforward, because h^* is not a well behaving function respect to parameters. First of all, the log-likelihood function is not differentiable everywhere(kink points) and secondly there can be parameter values where the function becomes very flat. The latter can become serious problem if there are not enough variation between the budget sets. In fact, it is very likely that the likelihood function is not differentiable everywhere. Kendall and Stuart[17] have shown that ml-estimator is asymptotically consistent even if the likelihood function is not differentiable everywhere.

2.6.2 Derivation of the likelihood in the presence of two additive random terms

In the last section we did not make any specific assumptions about the stochastic specification i.e. we assumed that all variance in hours conditional on covariates is measurement error. This means that preferences are assumed to be non-stochastic; all variation in preferences is only due to observable personal characters(i.e. regressors). This is the usual procedure adapted by econometricians, at least in the cross-section studies. In the context of labour supply we might think that there exists different sources for stochastic disturbances to arise from. First, the usual measurement error interpretation implies that parameters to be estimated are same for All individuals, so there is only one utility maximising choice in the population(this is probably an questionable outcome). Secondly, there might exists randomness in preferences which are not captured by the variables we include to our regression function. As Moffit [21] argues it is reasonable to expect that at least some amount of the observed distribution of observations over the budget constraint is a result of heterogeneous preferences. It is quite natural to think that both of above aspects are relevant in the context of labour supply. Look Hausman [13] for the other possible sources of stochastic disturbances.

Above mentioned two different stochastic disturbances have different implications for the data. In the standard measurement error approach the observations should be distributed evenly over the whole budget constraint, as in the case of heterogeneity of preferences we should find clusters of observations at the kink points(in the case of convex budget constraint). In theory, the model which only includes heterogeneity term is possible, but not a very appropriate one for the most applications, because it is very rarely the case that all observations are clustered to the kinks. Although empirical evidence shows some clustering(especially in the cases of big changes in marginal tax rates), it is not usually strong enough to leave measurement error term without modelling(at least in our application).

One relatively easy way to proceed is to estimate the model with random preference term and then test if the variance of that term is different from zero. One further motivation for including the heterogeneity term is that usually in the cross-section studies one finds a large amount of unexplained variance and thus, there might be a role for this stochastic term.

Let us now write the labour supply function as follows

$$h = f(w, y, z, \eta; \alpha, \beta, \gamma) = h^* + \epsilon, \quad (2.14)$$

where the expression for the desired hours h^* (we use the semi-logarithmic form in the following expressions) now includes the additive random variable η as follows

$$h^* = \text{const.} + \alpha \ln w + \beta y + \gamma z + \eta \quad (2.15)$$

Maximum likelihood estimation of this model requires that we must specify the joint density of these two stochastic terms. We follow the usual assumption that these terms are normally distributed and they are independent, as follows:

$$\epsilon \sim N(0, \sigma_\epsilon^2) \quad (2.16)$$

$$\eta \sim N(0, \sigma_\eta^2) \quad (2.17)$$

Reason for the independence assumption is following: if we interpret the terms as heterogeneity and as measurement(or optimisation) error, there are no reason to expect them to be generated from a joint process (assumption is not needed in the estimation) as Moffit[21] mentions.

In this paper we assume that the heterogeneity term is additive random variable in the regression function. We do not find attractive to follow the work of Hausman [12] where he allowed the income parameter to vary across the individuals. The problem in this approach is that if the income parameter becomes too positive then the compensated wage elasticity becomes negative and we lose the

concavity of the indirect utility function. Hausman uses truncated normal distribution (forcing the income parameter to be nonpositive) to tackle this problem. In principle one can allow all the parameters to vary randomly but it would leave us with the very non-attractive and difficult model to estimate, as we will below.

Our next step is to construct the Likelihood function when we have two random terms. This is quite tricky because some level of labour supply is outcome from the two values of two random terms i.e. we have to take account all the possibilities which can produce this level of hours.

In this two random term model the algorithm to find optimal amount of labour supply can be constructed in the following way. Optimum h^* can be found from the segment k ($k=1, \dots, n$) if

$$H_{k-1} - \alpha \ln w_k - \beta y_k - \gamma z < \eta < H_k - \alpha \ln w_k - \beta y_k - \gamma z.$$

Above expression can be rewritten as

$$\eta_{kl} < \eta < \eta_{ku}.$$

Where

$$\eta_{kl} = H_{k-1} - \alpha \ln w_k - \beta y_k - \gamma z$$

$$\eta_{ku} = H_k - \alpha \ln w_k - \beta y_k - \gamma z.$$

Above the subindex l indicates to the lower limit of the segment k and respectively the subindex u indicates to the upper limit of the segment k .

In this case optimum labour supply h^* is located in the kink point H_k ($k = 1, \dots, n-1$), if

$$\eta_{ku} < \eta < \eta_{(k+1)l}.$$

Finally, for the completeness we can show that the utility is to be maximised at the zero hours H_0 and at the maximum hours H_n , if

$$\eta \leq \eta_{1l}$$

$$\eta \geq \eta_{nu}.$$

Thus, we can show that the optimal labour supply can be found from:

zero,	if	$\eta \leq \eta_{1l}$
1st segment,	if	$\eta_{1l} < \eta < \eta_{1u}$
1st kink,	if	$\eta_{1u} \leq \eta \leq \eta_{2l}$
2nd segment,	if	$\eta_{2l} \leq \eta \leq \eta_{2u}$
2nd kink,	if	$\eta_{2u} \leq \eta \leq \eta_{3l}$

:

nth segment,	if	$\eta_{nl} \leq \eta \leq \eta_{nu}$
nth kink,	if	$\eta \geq \eta_{nu}$

Now we can express the corresponding probabilities using the integrals. For example, the probability that the optimum is located on the second segment is

$$pr(h^* \text{ is on segment } 2) = \int_{\eta_{2l}}^{\eta_{2u}} \left(\frac{1}{\sigma_\eta} \right) \Phi \left(\frac{\eta}{\sigma_\eta} \right) d\eta. \quad (2.18)$$

Finally, the likelihood function can be shown(in appendix 7 we show how this can be done) to have a following form in the case of two additive random terms.

$$L(\alpha, \beta, \gamma, \sigma_\eta, \sigma_\epsilon : w_k, y_k, z)$$

$$\begin{aligned}
&= \prod_{i=1}^J \left\{ \left(\frac{1}{\sigma_\varepsilon} \right) \phi \left(\frac{h_i}{\sigma_\varepsilon} \right) \left[\Phi \left(\frac{(-\alpha \ln w_{i1} - \beta y_{i1} - \gamma z_i)}{\sigma_\eta} \right) \right] \right. \\
&+ \sum_{k=1}^n \left(\frac{1}{\sigma} \right) \phi \left(\frac{e_{ik}}{\sigma} \right) \left[\Phi \left(\frac{\eta_{iku} - \left(\frac{\sigma_\eta^2 e_{ik}}{\sigma^2} \right)}{(\sigma_\varepsilon \sigma_\eta / \sigma)} \right) - \Phi \left(\frac{\eta_{ikl} - \left(\frac{\sigma_\eta^2 e_{ik}}{\sigma^2} \right)}{(\sigma_\varepsilon \sigma_\eta / \sigma)} \right) \right] \\
&+ \sum_{k=1}^{n-1} \left(\frac{1}{\sigma_\varepsilon} \right) \phi \left(\frac{(h_i - H_{ik})}{\sigma_\varepsilon} \right) \left[\Phi \left(\frac{\eta_{i(k+1)l}}{\sigma_\eta} \right) - \Phi \left(\frac{\eta_{iku}}{\sigma_\eta} \right) \right] \\
&+ \left. \left(\frac{1}{\sigma_\varepsilon} \right) \phi \left(\frac{h_i - H_n}{\sigma_\varepsilon} \right) \left[1 - \Phi \left(\frac{H_n - \alpha \ln w_{in} - \beta y_{in} - \gamma z_i}{\sigma_\eta} \right) \right] \right\}, \quad (2.19)
\end{aligned}$$

where $e_{ik} = h_i - \alpha \ln w_{ik} - \beta y_{ik}$ and $\sigma^2 = \sigma_\varepsilon^2 + \sigma_\eta^2$.

In above likelihood the first term corresponds the probability that an individual's worked hours are zero. Second term corresponds that the optimum lies in some segment and third term corresponds the case that optimum can be found from some of the kink points. Last term corresponds the case that individual works maximum amount of hours.

2.7 Results

One of our motivation to write this paper is to estimate labour supply function with progressive income taxation when an alternative set of income information is available. Does it really make any difference whether one is using survey information or archive data? The difference between the data sources may become a crucial one (as suggested by Heckman) when one uses more advanced(or complicated) methods to model taxes. Actually, it is undesirable to estimate labour supply models using Hausman type approach if the amount of budget set variables is limited. As far as we know there are no other studies made which try to compare survey and tax register data in this context and thus this exercise may shed some light to the already vast literature of taxes and labour supply.

We estimate labour supply function for Finnish married females aged 25–60 years. We will estimate models with differently constructed budget sets and we use semi-logarithmic labour supply function. Basic differences between the models are how

we have constructed the exogenous income variable and do we use "subjective" wage measure or wage measure constructed from the tax data. In both cases wage equations are estimated and predictions are used in the final analysis.

The presentation of our results follow the sequential nature of the estimation strategy. First of all, the wage equation is estimated using Heckman's[14] correction method and it is presented in the appendix 5.²⁵ Results show the familiar age-wage effect, i.e. that up to a certain age wage increases and then decreases. Education(Educ10-Educ15 are dummy variables indicating the number of completed years of education. Reference group is individuals with less than 10 years of education.) has a positive effect on wage rate and the effect gets stronger with years of education. Working experience(measured in years) increases wage rate up to a certain experience and then start to decrease it. Tenure variable(number of years with the same employer) shows the similar quadratic shaped effect. Individuals who have a permanent job get higher wage, as is expected and also individuals who are white-collar workers. Especially individuals in managerial positions seem to earn more than the others. It is also evident that individuals living in south of Finland earns more.

Let us now turn into the participation equation results.²⁶It is estimated by a simple probit-method and the results are shown in appendix 6. Properties of the parameter estimates are broadly plausible. Age affects negatively on labour supply participation. It becomes more likely to participate when years of education increases. In the case of children results show the familiar outcome: if woman's youngest child(aged 0-3) the likelihood to participate decreases significantly as it seems to decrease(but not significantly) also when the youngest child is aged 4-6. If the youngest child is 7-9 or 10+ years old, then the likelihood to participate increases and in the latter case it is statistically significant. Those women whose spouse's are working are also more likely to work and it is also the case when one

²⁵We only present wage equation for the wage rate obtained from the tax register data. Results for the wage equation using the survey data are available from the author upon request.

²⁶Participation equation is estimated because we estimate labour supply function also only for the participants.

lives in the southern part of Finland.

Estimates for the labour supply function are presented in the table 2.2 below. Table shows results for the 2 different models. In both models semi-logarithmic functional form is used.²⁷ In equations 1, when constructing the exogenous variable we have not included spouse's net incomes but in equations 2 it is included. Both models presented below includes all individuals(2037 obs.) and we utilise the likelihood function presented in equation 9, except that we have dropped the last part because we do not observe any individuals whose labour supply is in the upper limit.

²⁷We also estimated the same two labour supply functions using the functional form presented in footnote 16. Obtained results we similar and are available from the author.

Table 2.2. Labour Supply Functions: Estimates for the Full Sample

Labour Supply Functions		
<i>Variables</i>	<i>Model 1</i>	<i>Model 2</i>
Constant	-2.57905 (0.55650)	-2.63026 (0.56407)
Ln W	0.37046 (0.12135)	0.38249 (0.12374)
Exog. inc	-0.00045 (0.00022)	-0.00018 (0.00010)
Cdum1	-0.33917 (0.09948)	-0.35012 (0.0995)
Cdum2	-0.00487 (0.10437)	0.06751 (0.10428)
Cdum3	0.09616 (0.10050)	0.11315 (0.10042)
Cdum4	0.14310 (0.07690)	0.15722 (0.07672)
Age	0.16118 (0.02484)	0.163992 (0.02423)
Age*Age	-0.00227 (0.00028)	-0.00230 (0.00028)
Sosio	0.19945 (0.09521)	0.19008 (0.09607)
Nkids	-0.08419 (0.03235)	-0.100989 (0.03538)
σ_ϵ^2	0.98208 (0.01907)	0.98340 (0.01910)
Ln L	-2669.61	-2672.38

Note: In both models above, dependent variable(yearly hours) is divided by 1000. In model 1 the exogenous income variable contains only own exogenous income components(net) and it is divided by 100. In model 2 the exogenous income variable includes also husbands net incomes and it is divided by 1000.

Estimates for the net wage term satisfy theoretical expectations and it is also

precisely estimated in all cases. Exogenous income variable has negative sign in both models and it is statistically insignificant. Indicator variables indicate that age of the youngest child in home affects for desired labour supply. Presence of 0–3 years old children reduces the desired labour supply and the effect is significant in both cases. If the youngest child is older than ten years, then there is tendency to want more work(measured in hours). Age increases the labour supply up to a saturation point and number of children in the household reduces the desired labour supply.

Results are consistent between both models. Both models above show a negligible income effect²⁸ but a reasonably large uncompensated wage effect and thus also the level of compensated wage elasticities are reasonable large. In model 1 the compensated wage elasticity(among workers) is 0.213, which is clearly smaller than the result got by Ilmakunnas[16] for Finnish married females using 1987 data. She estimated labour supply function only for participants using linear labour supply function and her estimate for the compensated wage elasticity was 0.29. Kuismanen[18] estimated labour supply function for married males using also only participants and linear labour supply function and his estimate for the compensated labour supply was 0.08. In case of Sweden, Blomquist and Hansson–Brusewitz(1990) obtain wage elasticities for females that vary from 0.34 to 0.75 using roughly the similar kind of statistical approach. Taking husbands net incomes into account seems to have almost no effect at all and the estimate for the compensated wage elasticity is 0.207.²⁹

It is interesting shortly to comment how compensated elasticities vary across different demographic groups. We use model one as our reference model. When we divide our sample by age we see that the age group 25–35 have highest elasticity, 0.26 per cent and it monotonically decreases with age and it is only 0.14 per cent among the individuals above 55 years. Elasticities also vary between industries. Using the Statistics Finland’s classification we divided individuals to the eight

²⁸All elasticities are calculated using the mean values.

²⁹Changing the functional form(see footnote 16) does not change the picture essentially. Estimated elasticities are slightly higher than previously, 0.229 and 0.222 respectively.

categories. From these categories individuals who worked in private sector services or in education and research have highest mean elasticities, 0.25 and 0.27 respectively. All the other group's (See appendix 3 for definitions) elasticities are near the overall mean except in case of manufacturing workers, whose elasticity is 0.17. It is a bit surprising that number of children do not have a great effect on elasticities. Mean elasticity for females with no child is 0.20 and it increases monotonically with the number of children.

2.7.1 Results for alternative budget sets

But do we still get similar kind results after we have changed some crucial features of budget sets, namely using subjective wage measure³⁰, not taking tax deductions into account and using constant local tax rate. Table 2.3 shows results for both models when we have used different methods to construct the budget sets.

³⁰For subjective wage measure we mean the wage rate calculated from the survey data.

Table 2.3. Alternative Budget Sets

Estimates for Alternative Budget Sets				
<i>Model</i>	<i>Variab.</i>	<i>Sub. Wage</i>	<i>No Deductions</i>	<i>Const. L. Tax.</i>
Model 1	Ln W	0.49776**	0.26881**	0.45521**
	Exog.	-0.00039**	-0.00044**	-0.00022*
	Comp. E	0.280	0.157	0.251
Model 2	Ln W	0.50138**	0.27012**	0.45937**
	Exog.	-0.00016**	-0.00031*	-0.00017**
	Comp. E	0.277	0.159	0.254

*Note: Both models are scaled in a similar way than mentioned in the note for table 2.2 * and ** refer to a statistical significance at 10% and 5% level respectively.*

Results presented above are quite interesting. When using predictions for subjective wage measure we get a higher compensated wage elasticities than using wage measure from the archive data. This might be because there are less variation in subjective wage measure, but it is very hard to say exact reason for this result.

Results show that consistently the compensated elasticities are lower when individually calculated tax deductions are neglected than those presented in table 2.2. First of all, tax deductions affect the location of kink points i.e. point where marginal tax rate changes. Our approach assumes that individuals rationally choose their labour supply behaviour given the budget set they face and this is directly related to Heckman's[15] criticisms that the approach in question to model taxes is insufficient if we are not able to measure the budget sets accurately enough. Calculation of individual tax deductions indicate that it may be a significant factor when defining individuals yearly earnings. For example, the maximum amount of deduction in our case is 29 500FMK, which is a one fifth of that person's yearly incomes. It is not a very difficult task for a individual itself to calculate her possible deductions, so if we neglect them from our analysis we contradict our earlier made assumptions. Taking deductions into account means

higher(compared to no deductions case) net yearly earnings which might also be a one possible explanation for our result.

When using a constant municipal tax rate(17%) compensated elasticities are slightly higher than when using calculated municipal tax rates. Using common local tax rate to all individuals means that there is less variation in the measure of net hourly wage but it is very hard to find an exact justification why the elasticities are higher. It can be that one of the reasons is that when using constant local tax rates we again violate the assumptions made about individuals' behaviour.

2.7.2 Results for the participants

In his well cited paper Mroz[23] got interesting results using different economical and statistical assumptions when estimating the labour supply function for for US females. One of his results, broadly speaking, was that tobit-specification exaggerate both wage and income elasticities.

We estimated the two models presented above only for participants. Before that we again estimated wage equations only for the participants and also participation equation. Results for the latter are presented in appendix 6. In the labour supply function we took sample selection into account by adding selection term into it. Table 2.4 shows results for participants. In these models we have used the tax register based wage rates and we also have taken into account individual tax deductions and different local tax rates.

Table 2.4. Results for the Participants

Estimates for participants.		
<i>Variab.</i>	<i>Model1</i>	<i>Model2</i>
Ln W	0.176916**	0.12126**
Exog.	-0.00037	-0.000169
σ_ϵ^2	0.38994**	0.38942**
Comp. E	0.0953	0.094

*Note: Both models are scaled in a similar way than mentioned in the note for table 2.2. * and ** refer to a statistical significance at 10% and 5% level respectively.*

Our results indicate that when estimating the model only for workers we get lower compensated wage elasticities. Results for the other exogenous variables are similar in nature than those presented in table 2.2 and thus are not shown here. Compensated elasticities are around 0.10 for both models and thus our results are in line with Mroz[23].

2.7.3 Results for the random preference model

In above, all variance in hours conditional on covariates is measurement error i.e. preferences are assumed to be non-stochastic. This is the usual procedure adapted by econometricians, at least in the cross-sectional studies. In the context of labour supply it is reasonable to expect that at least some amount of the observed distribution of observations over the budget constraint is a result of heterogeneous preferences and to study this we have derived a likelihood function (see appendix 7.) which takes this additional stochastic term into account.

This model is estimated only for the workers and the results are presented for the models 1 ³¹ This model appears to be quite difficult to estimate and we used

³¹Results for the model 2 do not differ significantly from those presented above and are available from the author upon request.

different starting values in estimation to be sure that the solution is a global one³².

Table 2.5. Results for the Random Preference Model

Random Preference Model	
<i>Variab.</i>	<i>Model1</i>
Ln W	0.19747**
Exog.	-0.00011*
σ_{ϵ}^2	0.38120**
σ_{η}^2	0.01078
Comp. E	0.106

*Note: The model above is scaled in a similar way than mentioned in the note for table 2.2. * and ** refer to a statistical significance at 10% and 5% level respectively.*

From the above table we immediately realise that the additive random preference term does not have any explanatory power. According to our result random variation is totally captured by the traditional optimization error term ϵ and its estimates are almost equal to those presented in table 2.5. It is also the case that adding the additional random term had no effect on the other explanatory variables. Compensated elasticities are almost same in their magnitudes as those presented in table 2.4.

How this result relates to the other studies? In her study for finnish females Ilmakunnas[16] got similar results. On the other hand, Hausman[12] and Blomquist[1] find that the additional random term was a significant one, but we have to remember that their specification was different from ours, as mentioned in section 2.6.2.

³²We had a lots of convergence problems when we tried to estimate the model for the whole sample thus it is safer to present the results for the subsample of workers only.

When we look our data we see that there are no strong bunching(it is difficult to provide a picture, because the kink point vary from one individual to another)around kink points and in this light our result is intuitive because the heterogeneity of preferences tends to generate bunching. Our case is different, for example, from the UK one where there seem to be a significant bunching in the earnings distribution at National Insurance thresholds. See for example, Duncan[11] and Blundell et al.[5] These two different random terms have different implications how the data should look. If there exist bunching, then the heterogeneous term should have explanatory power and on the other hand if there exist no bunching, then the term should have very little explanatory power if all, which is our case.

2.8 Conclusions

In this paper we have examined the sensitivity of the labour supply estimates to the use of differently constructed budget sets for all the individuals in the data set. We use two alternative data sets. One is familiar survey type of data and the other is tax register data from the Finnish tax authorities. This is especially interesting when we use Hausman-type of methodology to take progressive income taxation into account. We also estimate models for the whole sample and sub-sample of workers.

Section 2 points out some theoretical aspects when dealing with piecewise linear budget constraints and in section 3 and 4 we describe in detail our data and Finnish tax system. Again, these two aspects are important for the empirical application. Section 5 discuss our choice of labour supply function and we also develop an algorithm a la Hausman [12] to estimate these models. In the following section we derive likelihood function in the presence of two additive random terms.

The results presented here shows that, at least in our application, differently constructed budget set give different results. Our benchmark model is the one

where we take individual tax deductions and local tax rates into account. These two factors are usually neglected in other studies. Our results indicate that if we neglect the tax deductions we get lower compensated wage elasticities than when we take them into account. Using constant local tax rates seem, *ceteris paribus*, to lead lower elasticities. Interesting finding is that when we use hourly wage rate calculated from the survey data we find that it will lead to a significantly higher elasticities than when we use tax register data.

Elasticities for the whole sample and sub-sample vary also. We find that elasticities for the sub-sample of workers are clearly lower than for the whole sample. It is also evident that introducing the additional term to capture the heterogeneous preferences does not have any significant contribution to the model.

Our results indicate that decreasing marginal tax rates would have a reasonable effect on Finnish females desired labour supply. One has keep in mind our results are for the year when the Finnish economy was booming and there are some evidence available that these results do not hold in recession years 1991 and 1993, see Kuusmanen [19].

In this paper we assume that individuals can work as many hours they desire and if they are not working that is because they do not want to work. These assumptions are probably a debatable ones and there exists evidence that taking different groups into account when estimating labour supply function might have some effect to the estimates. See e.g. Blundell et al. [6][7]. We do think that it would be a useful exercise to expand these models to take also taxation into account.

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Appendix 1: Definitions of the variables

union=1, if the respondent is a member of an union

age=Age of the respondent

age2= Age squared

educ10=1, if the respondent has 10 years of education. Otherwise zero.

educ12=1, if the respondent has 11-12 years of education. Otherwise zero.

educ14=1, if the respondent has 13-14 years of education. Otherwise zero.

educ15=1, if the respondent has 15+ years of education. Otherwise zero.

ueduc=1, if the respondent has university degree from the following fields:

Technology,business,law,natural science and social sciences

nchild=Number of dependent children.

cdum1,...,cdum4= Dummy variables for the youngest child. Age groups are 0-3,4-6,7-9 and 10+.

schild=Number of children aged 0-3.

cchild=Number of children aged 4-6.

bchild=Number of children aged 7-9.

exp= Working experience

exp2= Exp. squared

tenure= Duration of the current job

tenure2= Square of tenure

pjob=1, if respondent has a permanent job

phusb=1, if respondent's husband is working

stat=1, if the respondent is a white-collar worker and 0 if a blue-collar worker.

socio=1, if the respondent is a upper white-collar worker. Otherwise zero

hwage= Hourly wage rate.

shwage= Subjective Hourly wage rate.

exo= Unearned income.

exo+hnet= Unearned income+husband's net incomes.

south=South Finland.

west=West Finland.

east=East Finland.

middle=Middle Finland.

north=North Finland.

lapl=Lapland.

Appendix 2: Descriptive Statistics

Descriptive statistics: participants and Non-participants		
<i>Variables</i>	<i>Participants</i>	<i>Non-participants</i>
Hours	1855.58(560.10)	
union	0.71(0.45)	0.21(0.41)
age	41.29(8.47)	43.13(11.61)
educ10	0.30(0.46)	0.26(0.41)
educ12	0.18(0.39)	0.16(0.36)
educ14	0.05(0.21)	0.06(0.24)
educ15	0.07(0.26)	0.06(0.24)
cdum1	0.13(0.34)	0.32(0.46)
cdum2	0.12(0.33)	0.06(0.24)
cdum3	0.12(0.32)	0.05(0.21)
cdum4	0.25(0.44)	0.10(0.30)
workexp	19.50(9.32)	17.30(11.60)
jobdur	8.60(8.34)	
permjob	0.77(0.41)	
phusb	0.86(0.33)	0.68(0.46)
hwage	48.81(23.99)	
shwage	44.28(19.35)	
exo	5525.66(14028.9)	6935.43(11183.8)
exo+hnet	84590.67(61197.5)	77666.32(46447.6)
south	0.25(0.44)	0.21(0.41)
west	0.16(0.36)	0.13(0.33)
east	0.19(0.39)	0.24(0.43)
middle	0.14(0.34)	0.15(0.36)
north	0.18(0.39)	0.18(0.38)
lapl	0.08(0.27)	0.08(0.27)

For the definitions of the variables see appendix 1.

Appendix 3: Cross-tabulations

Cross-Tabulation of Female Hours vs. Number of Children								
n. of kids	0	1-1000	1001-1500	1501-2000	2001-2200	2201-3000	3000+	Total
0	264(0.13)	38(0.02)	40(0.02)	223(0.11)	146(0.07)	67(0.03)	16(0.008)	794(0.39)
1	97(0.05)	25(0.01)	42(0.02)	171(0.08)	83(0.04)	47(0.02)	10(0.005)	475(0.23)
2	132(0.06)	36(0.02)	47(0.02)	187(0.09)	96(0.05)	42(0.02)	9(0.004)	549(0.27)
3	59(0.03)	8(0.003)	12(0.02)	42(0.02)	17(0.008)	23(0.01)	2(0.001)	103(0.05)
4+	25(0.01)	2(0.001)	4(0.002)	10(0.004)	8(0.004)	7(0.003)	0(0.00)	56(0.03)
Total	577(0.28)	109(0.05)	145(0.07)	633(0.31)	350(0.17)	186(0.09)	37(0.02)	2037

Cross-Tabulation of Female Hours vs. Status								
Status	0	1-1000	1001-1500	1501-2000	2001-2200	2201-3000	3000+	Total
0	437(0.21)	55(0.03)	53(0.03)	171(0.08)	176(0.09)	90(0.04)	25(0.01)	1007(0.49)
1	140(0.07)	54(0.02)	92(0.04)	412(0.20)	174(0.08)	96(0.05)	12(0.006)	1030(0.51)
Total	577(0.28)	109(0.05)	145(0.07)	633(0.31)	350(0.17)	186(0.09)	37(0.02)	2037
NOTE: 1=White-collar worker, 0=Blue-Collar worker								

Cross-Tabulation of Female Hours vs. Union Membership								
Union	0	1-1000	1001-1500	1501-2000	2001-2200	2201-3000	3000+	Total
0	448(0.22)	50(0.02)	51(0.02)	135(0.07)	77(0.04)	77(0.04)	26(0.01)	864(0.42)
1	129(0.06)	59(0.03)	94(0.05)	498(0.27)	273(0.13)	109(0.05)	11(0.005)	1173(0.58)
Total	577(0.28)	109(0.05)	145(0.07)	633(0.31)	350(0.17)	186(0.09)	37(0.02)	2037
NOTE: 1=Union member, 0=Not a member								

Educ.	0	1-1000	1001-1500	1501-2000	2001-2200	2201-3000	3000+	Total
1	263(0.13)	45(0.02)	54(0.03)	219(0.11)	162(0.08)	81(0.04)	19(0.01)	843(0.41)
2	243(0.12)	54(0.03)	70(0.03)	327(0.16)	155(0.08)	85(0.04)	17(0.01)	951(0.47)
3	71(0.03)	10(0.004)	21(0.01)	87(0.04)	33(0.01)	20(0.01)	1(0.00)	243(0.12)
Total	577(0.28)	109(0.05)	145(0.07)	633(0.31)	350(0.17)	186(0.09)	37(0.02)	2037
NOTE: 1= Less than 9 years of education, 2=9-13 years of education 3= More than 13 years of education.								

Cross-Tabulation of Female Hours vs. Occupation								
Occup.	0	1-1000	1001-1500	1501-2000	2001-2200	2201-3000	3000+	Total
0	341	2	4	22	13	3	1	386(0.19)
1	53	33	33	124	101	60	8	412(0.20)
2	43	20	20	116	47	33	13	295(0.15)
3	12	6	11	24	10	7	2	72(0.04)
4	23	10	5	68	21	6	2	135(0.07)
5	6	5	4	50	13	2	0	80(0.04)
6	15	8	26	75	37	10	2	173(0.08)
7	65	11	27	120	84	50	7	361(0.18)
8	22	14	15	34	24	15	2	126(0.06)
Total	577(0.28)	109(0.05)	145(0.07)	633(0.31)	350(0.17)	186(0.09)	37(0.02)	2037
NOTE: 0=unspecified, 1=Manufacturing, 2= Retail and Catering, 3=Transport industry, 4= Banking and Insurance, 5=Public Sector, 6= Education and Research, 7= Health and Social Services, 8=Other Private Sector Services								

Appendix 4: Estimated Tax Deductions and Locations on the Budget Constraint

Calculated Tax Deductions in 1989: Group Means.

Taxable Income mk.	Deductions
36000 – 51000	187.28
51000 – 63000	436.07
63000 – 89000	1222.67
89000 – 140000	2413.13
140000 – 250000	4797.89
250000 –	3646.65

Individuals Location in Tax Brackets: Participants

<i>Segment</i>	<i>%</i>
1	5.8
2	3.4
3	31.0
4	50.0
5	8.1
6	0.8

Appendix 5: Wage Equation

Wage Equation. Dependent variable: ln hwage.		
<i>Variables</i>	<i>Coefficient</i>	<i>Standard Error</i>
Constant	2.83342	0.2511
Age	0.01753	0.0135
Age2	-0.00017	0.0001
Educ10	0.06881	0.0024
Educ12	0.19534	0.0297
Educ14	0.27270	0.0469
Educ15	0.51690	0.0469
Exp	0.01659	0.0053
Exp2	-0.00027	0.0001
Tenure	0.02410	0.0038
Tenure2	-0.00045	0.0001
Pjob	0.04720	0.0299
Husb	0.00760	0.0290
Stat	0.10338	0.0241
Socio	0.23919	0.0366
Nchild	-0.03065	0.0104
South	0.15898	0.0222
Exo+hnet	3.95e-07	1.67e-07
Occ. dummies	Yes	
Ln L	-1221.91	

NOTE:The selection index is a function of the individual, geographical and demand side variables. The selectivity effect was statistically significant. Reference group for occupation is manufacturing workers.

For the definitions of the variables see appendix 1.

Appendix 6: Participation Equation

Participation Probability Index.		
<i>Variables</i>	<i>Coefficient</i>	<i>Standard Error</i>
Constant	1.59701	0.30336
Age	-0.06109	0.00632
Educ10	0.31018	0.84777
Educ12	0.46843	0.10564
Educ14	0.61320	0.16594
Educ15	0.77089	0.15803
Exp	0.10879	0.01243
Exp2	-0.00154	0.00027
Cdum1	-0.70759	0.12232
Cdum2	-0.12581	0.13538
Cdum3	0.01337	0.13469
Cdum4	0.22237	0.10371
Husb	0.34048	0.08732
Exog	-0.16679	0.00021
South	0.16679	0.08155
Ln L	- 936.40	

For the definitions of the variables see appendix 1.

Appendix 7: Derivation of the Likelihood in Case of Two Additive Random Terms.

In this appendix we show how to derive the density function of h when $0 < h < H_n$. η_{kl} and η_{ku} are as in text. η and ε are normally distributed random variables with mean 0 and variance σ_η^2 and σ_ε^2 respectively. We assume that these random variables are uncorrelated. Let us denote the density function of η and the density function of ε by $f(\eta)$ and $p(\varepsilon)$ respectively.

Let us start by deriving the distribution function of h .

$$\begin{aligned}
 G(x) &= pr(h \leq x) \\
 &= pr(h^* = 0 \text{ and } \varepsilon < x - 0) \\
 &+ \sum_{k=1}^n pr(h^* = h_k^* \text{ and } \varepsilon < x - h_k^*) \\
 &+ \sum_{k=1}^{n-1} pr(h^* = H_k \text{ and } \varepsilon < x - H_k) \\
 &+ pr(h^* = H_n \text{ and } \varepsilon < x - H_n) \\
 &= \int_{-\infty}^{\eta_{1l}} f(\eta) d\eta \left[\int_{-\infty}^x p(\varepsilon) d\varepsilon \right] \\
 &+ \sum_{k=1}^n \int_{\eta_{kl}}^{\eta_{ku}} f(\eta) \left[\int_{-\infty}^{x - \alpha \ln w_k - \beta(y_k/w_k) - \gamma z - \eta} p(\varepsilon) d\varepsilon \right] d\eta \\
 &+ \sum_{k=1}^{n-1} \int_{\eta_{ku}}^{\eta_{(k+1)l}} f(\eta) d\eta \left[\int_{-\infty}^{x - H_k} p(\varepsilon) d\varepsilon \right] \\
 &+ \int_{\eta_{nl}}^{\infty} f(\eta) d\eta \left[\int_{-\infty}^{x - H_n} p(\varepsilon) d\varepsilon \right]
 \end{aligned}$$

In the next step we differentiate the distribution function with respect of x , thus we get

$$\begin{aligned}
 g(x) &= G'(x) \\
 &= p(x) \int_{-\infty}^{\eta_{1l}} f(\eta) d\eta \\
 &+ \sum_{k=1}^n \int_{\eta_{kl}}^{\eta_{ku}} f(\eta) p(x - \alpha \ln w_k - \beta(y_k/w_k) - \gamma z - \eta) d\eta
 \end{aligned}$$

$$\begin{aligned}
& + \sum_{k=1}^{n-1} \int_{\eta_{ku}}^{\eta_{(k+1)l}} f(\eta) p(x - H_k) \\
& + p(x - H_n) \int_{\eta_{nu}}^{\infty} f(\eta) d\eta.
\end{aligned}$$

Now, let us use the following notation $x = h_i$

$$e_{ik} = h_i - \alpha \ln w_{ik} - \beta(y_{ik}/w_k) - \gamma z$$

$$\sigma^2 = \sigma_\varepsilon^2 + \sigma_\eta^2$$

Now the second term is

$$\int_{\eta_{kl}}^{\eta_{ku}} f(\eta) p(e_{ik} - \eta) d\eta$$

where

$$f(\eta) = \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left[\frac{-\eta^2}{2\sigma_\eta^2}\right]$$

$$p(e_{ik} - \eta) = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp\left[\frac{-(e_{ik} - \eta)^2}{2\sigma_\varepsilon^2}\right]$$

\Rightarrow

$$\int_{\eta_{kl}}^{\eta_{ku}} f(\eta) p(e_{ik} - \eta) d\eta = \frac{1}{2\pi\sigma_\eta\sigma_\varepsilon} \int_{\eta_{kl}}^{\eta_{ku}} \exp\left\{-\frac{1}{2}\left[\frac{\eta^2}{\sigma_\eta^2} + \frac{(e_{ik} - \eta)^2}{\sigma_\varepsilon^2}\right]\right\} d\eta. \quad (2.20)$$

Let us denote the following

$$Q = \frac{\eta^2}{\sigma_\eta^2} + \frac{(e_{ik} - \eta)^2}{\sigma_\varepsilon^2}$$

After some algebra we can show that Q is

$$Q = \frac{(\sigma_\varepsilon^2 + \sigma_\eta^2)}{\sigma_\varepsilon^2 \sigma_\eta^2} \left[\eta - \frac{\sigma_\eta^2 e_{ik}}{\sigma_\varepsilon^2 + \sigma_\eta^2} \right]^2 + \frac{e_{ik}^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}.$$

Now, Q can be substituted back, so we get

$$\begin{aligned}
& \int_{\eta_{kl}}^{\eta_{ku}} f(\eta) p(e_{ik} - \eta) \\
& = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{e_{ik}^2}{\sigma_\varepsilon^2 + \sigma_\eta^2}\right) \frac{1}{\sqrt{2\pi}\sigma_\eta\sigma_\varepsilon} \int_{\eta_{kl}}^{\eta_{ku}} \exp\left\{-\frac{1}{2} \frac{\sigma_\varepsilon^2 + \sigma_\eta^2}{\sigma_\varepsilon^2 \sigma_\eta^2} \left[\eta - \frac{\sigma_\eta^2 e_{ik}}{\sigma_\varepsilon^2 + \sigma_\eta^2}\right]^2\right\} d\eta \quad (2.21)
\end{aligned}$$

Now, we multiply above by $\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}$

\Rightarrow

$$\frac{1}{\sqrt{2\pi}\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} \exp\left(-\frac{1}{2} \frac{e_{ik}^2}{(\sigma_\epsilon^2 + \sigma_\eta^2)}\right) \frac{1}{\sqrt{2\pi}} \sqrt{(1/\sigma_\epsilon^2) + (1/\sigma_\eta^2)} \int_{\eta_{kl}}^{\eta_{ku}} \exp\left\{-\frac{1}{2} \frac{(\sigma_\epsilon^2 + \sigma_\eta^2)}{\sigma_\eta^2 \sigma_\epsilon^2} \left[\eta - \frac{\sigma_\eta^2 e_{ik}}{\sigma_\epsilon^2 + \sigma_\eta^2}\right]^2\right\} d\eta. \quad (2.22)$$

After some algebra we can show that the above formula can be written as

$$\frac{1}{\sigma} \phi\left(\frac{e_{ik}}{\sigma}\right) \left\{ \Phi\left[\frac{\eta_{ku} - \frac{\sigma_\eta^2 e_{ik}}{\sigma^2}}{\sigma_\epsilon \sigma_\eta / \sigma}\right] - \Phi\left[\frac{\eta_{kl} - \frac{\sigma_\eta^2 e_{ik}}{\sigma^2}}{\sigma_\epsilon \sigma_\eta / \sigma}\right] \right\}.$$

which is same as the second term in the Likelihood function shown in the text. First, third and fourth terms are much easier to derive and can be done using the same logic as above and the complete likelihood has the following form.

$$\begin{aligned} & L(\alpha, \beta, \gamma, \sigma_\eta, \sigma_\epsilon : w_k, y_k, z) \\ &= \prod_{i=1}^J \left\{ \left(\frac{1}{\sigma_\epsilon}\right) \phi\left(\frac{h_i}{\sigma_\epsilon}\right) \left[\Phi\left(\frac{(-\alpha \ln w_{i1} - \beta(y_{i1}/w_{i1}) - \gamma z_i)}{\sigma_\eta}\right) \right] \right. \\ &+ \sum_{k=1}^n \left(\frac{1}{\sigma}\right) \phi\left(\frac{e_{ik}}{\sigma}\right) \left[\Phi\left(\frac{\eta_{iku} - \left(\frac{\sigma_\eta^2 e_{ik}}{\sigma^2}\right)}{(\sigma_\epsilon \sigma_\eta / \sigma)}\right) - \Phi\left(\frac{\eta_{ikl} - \left(\frac{\sigma_\eta^2 e_{ik}}{\sigma^2}\right)}{(\sigma_\epsilon \sigma_\eta / \sigma)}\right) \right] \\ &+ \sum_{k=1}^{n-1} \left(\frac{1}{\sigma_\epsilon}\right) \phi\left(\frac{(h_i - H_{ik})}{\sigma_\epsilon}\right) \left[\Phi\left(\frac{\eta_{i(k+1)l}}{\sigma_\eta}\right) - \Phi\left(\frac{\eta_{iku}}{\sigma_\eta}\right) \right] \\ &+ \left.\left(\frac{1}{\sigma_\epsilon}\right) \phi\left(\frac{(h_i - H_n)}{\sigma_\epsilon}\right) \left[1 - \Phi\left(\frac{H_n - \alpha \ln w_{in} - \beta(y_{in}/w_{in}) - \gamma z_i}{\sigma_\eta}\right) \right] \right\} \quad (2.23) \end{aligned}$$

where $e_{ik} = h_i - \alpha \ln w_{ik} - \beta(y_{ik}/w_{ik})$ and $\sigma^2 = \sigma_\epsilon^2 + \sigma_\eta^2$.

Chapter 3

Piece-wise Linear or Differentiable Budget Constraint? Estimation of Labour Supply Function in Presence of Non-linear Income Taxation

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Abstract

Various estimation approaches have been used in recent labour supply research in the presence of non-linear income taxation. Different techniques and data sets have produced a wide range of income and substitution elasticities. The Maximum Likelihood approach developed by Hausman[9] has been the most popular choice in recent studies, but after MaCurdy et al.[13] suggested that this method implicitly imposes restrictions that generate a positive Slutsky effect researchers have tried to develop alternative methods. In their seminal paper MaCurdy et al. suggested a new approach which utilises a differentiable budget constraint approach to approximate piecewise linear marginal tax rate function. In this study we utilise register data provided by tax authorities and thus it gives us a good possibilities to construct a detailed budget constraints for all the individuals in our sample. We estimate labour supply function using both methods. Our results support the view that if one is able to mimic actual budget set closely and the

degree of progression is high then these two methods are likely to give similar results. On the other hand, if the above mentioned factors are not present then the differentiable budget constraint approach is likely to be better choice.

Keywords: Labour supply, Income taxation, Piece-wise linear budget constraint, Differentiable budget constraint

JEL classification: C21, H24, J22

3.1 Introduction

It is a well known fact that economic theory gives us a little prediction power on how taxation affects labour supply. For example, introducing a different kind of welfare systems may create a non-convex budget sets and thus certain areas of the budget constraint cannot correspond the utility-maximising points. In many cases we do not even know if an increase in marginal tax rate will increase or decrease supplied hours. So, it seems to be that only empirical research, despite its flaws, can give us the needed information concerning tax and benefit systems' behavioural effects.

In empirical work we have to take a stand on how to deal budget sets. If one is willing to model income tax and benefit systems when estimating the labour supply function, then this evidently leads to a complicated econometrical models. For example, when we are modelling non-linear budget sets, then different kind of stochastic assumptions, unobserved wages, discontinuities, non-convexities etc. has to be incorporated to the model. On the other hand, these advanced methods seem to generate a dispute among researchers, some claims that nothing is achieved by modelling budget sets accurately and instead one should use simpler and more robust approaches.

But, it is not clear that the simpler methods are allways more robust. Our aim in this paper is to compare two different approaches to model labour supply behaviour when progressive income taxation is taken into account. We start by a conventional piece-wise linear method. In this approach we try to mimic the actual budget constraint as well as possible and then it is fully taken into account in estimation procedure. This means that the likelihood function takes into account the choice of hours over the entire exogenous tax schedule removing the endogeneity problem which is present in the simpler approaches, like in linearisation approach, as an example. This method has been criticised for various reasons. First, to mimic the budget constraints requires lot from the data. Secondly, it is questionable that econometricians and individuals really know the actual shape

of the budget constraints.

To avoid above mentioned problems but still allow the convex shape of the budget constraint we follow MaCurdy et al[13] to construct a differentiable budget constraint. The central idea in this technique is to approximate the actual piece-wise linear constraint by using continuous smooth polynomial function because it is unlikely that individuals know exact shape of their budget sets. Technically this means mimicing the tax schedule by fitting a polynomial function to the marginal tax rates and then integrating this function we can get a differentiable relation which approximates the amount of total taxes paid by individuals. This method is much easier to estimate since a purely continuous distribution describes the hours of work decision.

What we find is that a differentiable budget constraint is able to approximate piece-wise linear one quite closely and the estimation results do not differ significantly between these two approaches. This result is on line with the study by Macurdy and Flood[12] but in conflict with the original Macurdy et al. study. Our preliminary conclusion is that in the case of high degree of progressivity these methods are likely to produce similar outcomes because then the polynomial function is simply capable to produce almost identical constraint compared to the piece-wise linear one.

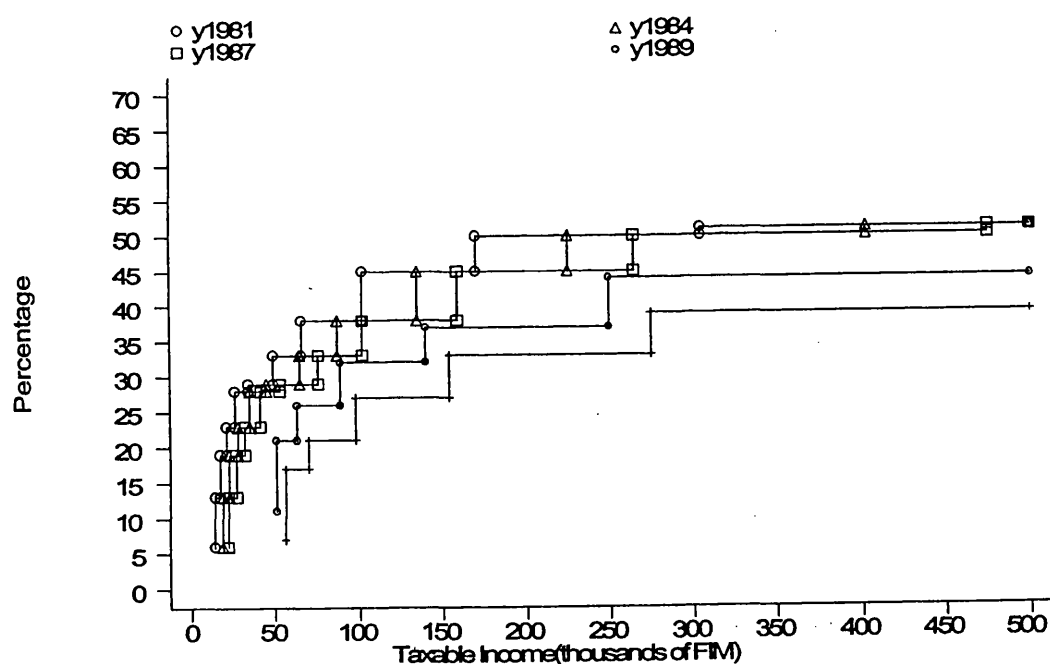
The set up of this paper is as follows. In section 3.2 we motivate this study by presenting the basic ideas behind the two approaches mentioned above. After that we go through the econometrics behind the two models and in section 3.4 we discuss about MaCurdy's critique towards the piece-wise linear approach. Section 3.5 discusses about empirical specifications and section 3.6 presents the estimation results. Section 3.7 finally concludes the paper.

3.2 Budget sets

We start this section by presenting shortly and non-technically the idea behind the piecewise-linear budget constraint. After that we proceed to discuss about its properties and present an alternative method to model non-linear budget constraint developed by MaCurdy et al.[13] This discussion serves as background for the next section where we will discuss the econometrics of these models.

Figure 3.1 below shows the state marginal tax rates¹ in Finland in years 1981,1984,1987, 1989,1991 and 1993.

Figure 3.1. Marginal tax rates in Finland



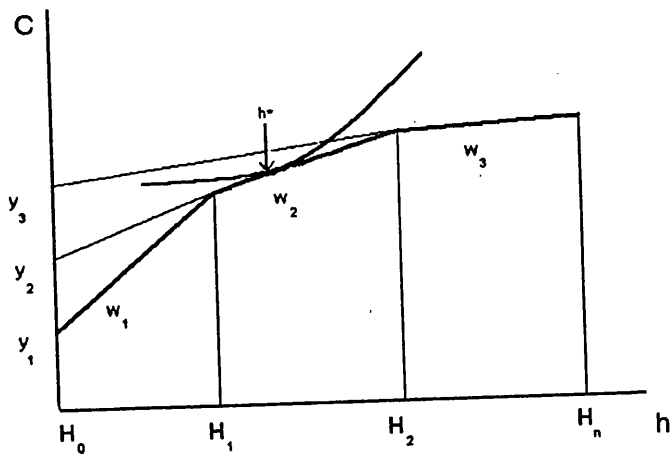
From the above figure we see the common feature that is also reported in many other studies(see e.g. Atkinson[1]), namely that the tax reforms carried out in

¹Note that we are only considering the federal income taxation in the picture. In top of that comes proportional municipality taxes and other social security contributions, so that the total income tax becomes much higher.

80's cut top marginal tax rates significantly and widened the tax brackets. In years 1981, 1984 and 1987 the top marginal tax rate was 51 percentage point. In 1989 a income tax reform was carried out and the top marginal tax rate was reduced to 44 percentage points. More significantly, in 1987 the top marginal rate was taxed after above 475 000 FIM in 1987 money terms and in 1989 the top marginal rate was taxed after 250 000 FIM in 1989 money terms(or 223 100 FIM in 1987 money terms). So, the tax reform was especially favourable for high income earners.

In the 1989 tax reform the tax base was broadened and the tax deduction system was simplified significantly. This means that the budget sets of the individuals were much simpler in 1987 than in 1989. Figure 3.1 tells us, *ceteris paribus*, how many piecewise-linear segments the consumer is facing. Figure 3.2 shows the piecewise-linear budget set when consumer faces three strictly increasing marginal tax rates.

Figure 3.2. Piece-wise linear budget constraint



The income tax system in figure 3.2 consists three tax brackets and thus three marginal tax rates; $t_1, t_2, \text{ and } t_3$. Inside the tax brackets the marginal tax rate is constant but it increases with the income. As we can see from the figure the budget constrained faced by individuals is piece-wise linear. As usual c is the composite commodity and h is labour supply(hours).

Marginal tax rate t_1 leads to the after-tax net wage $w_1 = (1 - t_1)w$ and this corresponds to the first segment in the figure above. Correspondingly the net wage rate in the second segment is $w_2 = (1 - t_2)w$ etc. $H_0, H_1 \text{ and } H_2$ are kink points where the marginal tax rate changes and H_n stands for the upper limit of labour supply. y_1 is the exogenous income component and thus does not depend on hours supplied.² Note that this component is directly observed from the data. y_2 and y_3 are called 'virtual income' terms and must be calculated recursively in the following way:

$$y_i = y_{i-1} + (w_{i-1} - w_i)H_{i-1}$$

Thus, geometrically we just extend a given budget segment to the vertical axis. Now, the crucial thing to realise is that these components cannot be observed directly from the data. In above picture, the optimal amount of hours, h^* , is on the second segment and this would also be the optimum in the case of linear budget constraint(w_2, y_2) and thus the name virtual income.

For simplicity we assume that the non-labour income is not taxed and E (can also vary between individuals) stands for the tax deductions. In this case the taxable income I_i is earned income (wh) less the deductions and the kink points can be calculated as

$$H_i = (I_i + E)/w$$

If we assume that consumers preferences can be presented using the quasi-concave

²This assumption is questionable, at least in the long run, and we will discuss more about it below.

utility function, $u(c, h)$. This function is non-decreasing with respect to c and non-increasing with respect to h . The budget set in figure 2 is convex. These properties guarantee that there exists a unique global maximum for the utility maximisation problem.

But what are the pros and cons of this approach? First of all, this method allows us to construct³ budget sets which recognise all the institutional features of tax and social security system. Piecewise-linear method also treats the marginal tax rate endogenously in the estimation procedure. It also allows us to incorporate different stochastic assumptions, like allowing the randomness in hours to arise from measurement (optimisation) and/or unobserved individual preferences. We can introduce fixed costs of working into the model quite straightforwardly and the treatment of unobserved wages can be done in different ways. MaCurdy et al.[13] includes an excellent discussion how unobserved wages should be calculated (for the details, see their article). As far as we know, their method has not been used in empirical studies (MaCurdy et al. did not use it in their empirical application either).

The fundamental assumption behind this model is that the observed market behaviour is outcome of free rational choice subject to the piece-wise linear budget constraint. In other words, we assume that the econometrician and, probably more crucially, individuals have the perfect knowledge of their budget sets. Obviously this means that the econometrician should be able to measure all the budget set variables without error and that the individuals knows these variables before making any decision about the supplied hours. It is hard to argue that all above mentioned assumptions are actually met in the applied work.

Most of the studies carried out have assumed that all the other variables except the hours are exogenous. At least in the long run it is natural to think that variables like gross wage and especially nonlabour income are endogenous. This leads us to question of static versus dynamic labour supply analysis. By nature

³Pudney[15] shows how difficult it is actually to construct the budget sets accurately

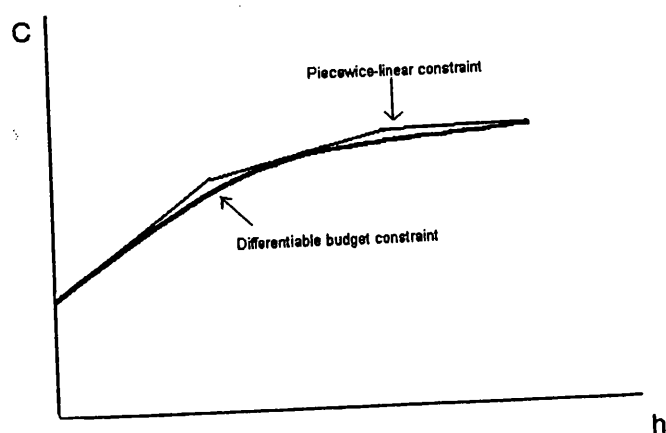
labour supply is a dynamic phenomena. People do plan their future and thus transfer income between periods. So, one can question that is it relevant to use elasticities calculated from the cross-sections to predict behavioural responses to tax reforms.

Probably the most serious claim against the piece-wise linear modelling is the So-called MaCurdy-critique. MaCurdy et. al. claimed "that the this method requires the satisfaction of parametric restrictions that constraint the signs of estimated substitution and income effects" MaCurdy et. al.[13] This claim has arisen a lot of attention lately and for example Blomquist[3][4] has written that MaCurdy's claim is not generally true. Section 3.4 is devoted to this topic.

Above we mentioned that it is questionable that neither econometrician or individuals in the sample know exactly the shape of their budget constraints. So, it is natural to ask if we could develop a method which approximates the real budget sets closely but at the same time would carry with it less restrictive assumptions and would be easier to estimate. In their influential paper MaCurdy et al.[13] develops new approach which uses a differentiable function to approximate marginal tax function. The idea sounds complicated, but as figure 3.3 on the next page shows it turns out to be a quite simple and intuitive one.

Figure 3.3 is identical to the figure 3.2 in all respect except that we have replaced the piecewise-linear budget constraint for differentiable budget constraint. We can think that individuals do not know the exact shape of their budget set, but they do have a view about its approximate shape. For example, most likely individuals do not exactly know that after how many hours they move from one tax brackets to another or what their accepted tax deductions might be.

Figure 3.3. Differentiable budget constraint



Differentiable budget constraint approach is also easier to apply in practise because a purely continuous distribution describes the supplied hours for all individual whose hours are strictly positive. If we assume that the constraint is convex and consumers preferences are strictly quasi-concave then we can be sure that we can find a tangent point and this point is going to be a unique one. In addition, because we have "smoothened out" All the segments and kinks we are left with a "one continuous non-linear segment", we do not need write the tax algorithm into the likelihood function.

3.3 Econometrics behind the piece-wise linear and differentiable budget constraint

3.3.1 Piece-wise linear method

Measurement/optimisation error approach

In the case of piecewise linear budget constraint the economic theory predicts bunching of observations(hours of work) at kink points or just below of them. This is because if individuals increase their hours they will move to an upper tax bracket(or, for example, in social security system they move to the point where the credit is taxed away) where they will face higher marginal tax rate. Blundell, Duncan and Meghir[6] provides an example where they find bunching. In chapter 2 we showed that in Finnish case we did not find this kind of phenomenon. As Moffit[14] states, that if the observations are distributed evenly across the budget sets it provides a reason to introduce measurement error term into the model. Naturally, there are also other reasons to introduce measurement error, like reporting errors in hours.

We first start with a basic model where we can write our general labour supply function as $h_i = h_i^*(w_i, y_i, z_i; \alpha, \beta, \gamma) + \epsilon_i$. α, β and γ are parameters to be estimated. Vector z includes individual characteristics(e.g. socio-economical and demographic variables.) and the variables w and y represent the marginal wage and virtual income variables correspondingly.

In the statistical model we have to calculate the densities of h_i and this requires evaluation of the maximum utilities received on each linear segment of the budget set. More formally, we can now write the problem as

$$\begin{aligned}
f(h_i) &= P[h_i = 0] + P[h_i > 0] * f(h_i | h_i > 0) + P[h_i = H_n] \\
&= P[\text{ at zero }] \\
&+ P[\text{ below kink } 1] * f(h_i | \text{ below kink } 1) \\
&+ P[\text{ at kink } 1] \\
&+ P[\text{ above kink } 1] * f(h_i | \text{ above kink } 1) \\
&\cdot \\
&\cdot \\
&\cdot \\
&+ P[\text{ at maximum }]
\end{aligned} \tag{3.1}$$

The optimal supply of hours h^* can be found from the segment k ($k = 1, \dots, n$), if

$$H_{k-1} < h^*(w_k, y_k, z; \alpha, \beta, \gamma) < H_k \tag{3.2}$$

Intuition behind this calculation rule is following: after we have calculated the slope of the indifference curve from the direct utility function we replace consumption c ($= w_k h + y_k$) by individuals income (calculated for all the segments) and after that we equate the slope of the indifference curve and the marginal wage w_k corresponding that segment. The algorithm iterate as long as this condition is satisfied (as explained earlier). If, for some individual we cannot find the solution we start to look if we can find it from some of the kink points.

Optimum h^* is found from the kink point H_k ($k = 1, \dots, n - 1$), if

$$h^*(w_k, y_k, z; \alpha, \beta, \gamma) \geq H_k \quad \text{and} \quad h^*(w_{k+1}, y_{k+1}, z; \alpha, \beta, \gamma) \leq H_k. \tag{3.3}$$

An other way to express this condition is that the optimum can be found from the H_K , if the slope of the indifference curve is bigger or equal than w_{k+1} and the

the slope is smaller or equal than w_k .⁴

The above formulation shows how we can calculate the optimal hours under the progressive income taxation when we know the following aspects: tax schedule, hourly wage, exogenous incomes and the shape of the labour supply function (or correspondingly the shape of the utility function).

Despite that h^* can be calculated quite easily in the convex case, the maximisation of the (log) likelihood function is not straightforward, because h^* is not a well behaving function respect to parameters. First of all, the log-likelihood function is not differentiable everywhere (kink points) and secondly there can be parameter values where the function becomes very flat. The latter can become serious problem if there are not enough variation between the budget sets.

Unobserved heterogeneity approach

In the last section we did not make any specific assumptions about the stochastic specification i.e. we assumed that all variance in hours conditional on covariates is measurement error.⁵ This means that preferences are assumed to be non-

⁴For the completeness we can show that optimum can be found from the zero hours $h = 0$ if

$$h^*(w_1, y_1, z; \alpha, \beta, \gamma) \leq 0 \quad (3.4)$$

or correspondingly from the maximum hours $h = H_n$, if

$$h^*(w_n, y_n, z; \alpha, \beta, \gamma) \geq H_n. \quad (3.5)$$

⁵It is important to realise that in the literature measurement error is interpreted in two different ways. The older interpretation is that the *positive* observed hours is measured with error. In this case one must choose the density function which ensures that reported hours of work are always positive with a feasible ϵ . The second interpretation is the optimisation error which reflects to the degree to which individuals' actual hours of work deviate from their desired hours. Thus, it is possible to observe that some individuals are not working even their desired hours are strictly positive because a realisation of ϵ causes measured hours to be non-positive. Most studies made are consistent with the latter interpretation.

stochastic; all variation in preferences is only due to observable personal characters(i.e. regressors). This is the usual procedure adapted by econometricians, at least in the cross-section studies. In the context of labour supply we might think that there exists different sources for stochastic disturbances to arise from. First, the usual measurement error interpretation implies that parameters to be estimated are same for all individuals, so there is only one utility maximising choice in the population(this is probably an questionable outcome). Secondly, there might exists randomness in preferences which are not captured by the variables we include to our regression function. As Moffit [14] argues it is reasonable to expect that at least some amount of the observed distribution of observations over the budget constraint is a result of heterogeneous preferences. It is quite natural to think that both of above aspects are relevant in the context of labour supply. Look Hausman [10] for the other possible sources of stochastic disturbances.

Above mentioned two different stochastic disturbances have different implications for the data. In the standard measurement error approach the observations should be distributed evenly over the whole budget constraint, as in the case of heterogeneity of preferences we should find clusters of observations at the kink points(in the case of convex budget constraint). In theory, the model which only includes heterogeneity term is possible, but not a very appropriate one for the most applications, because it is very rarely the case that all observations are clustered to the kinks. Although empirical evidence shows some clustering(especially in the cases of big changes in marginal tax rates), it is not usually strong enough to leave measurement error term without modelling(at least in our application). One relatively easy way to proceed is to estimate the model with random preference term and then test is the variance of that term is different from zero. One further motivation for including the heterogeneity term is that usually in the cross-section studies one finds a large amount of unexplained variance and thus, there might be a role for this stochastic term.

Let us now write the deterministic part of the labour supply function as follows

$$h^* = \zeta + \alpha \ln w + \beta y + \gamma z, \quad (3.6)$$

which again is the expression for the desired hours with the above given notation.

In principle we can introduce the unobserved heterogeneity in a many different ways. The most common solution is to allow α, β or ζ to vary across individuals. As we will see below, it is practically impossible to allow more than one parameter to vary across individuals because we would have to deal with the multiple integrals.

Maximum likelihood estimation of this model requires that we must specify the joint density of these two stochastic terms. We follow the usual assumption that these terms are normally distributed and they are independent, as follows:

$$\varepsilon \sim N(0, \sigma_\varepsilon^2) \quad (3.7)$$

$$\eta \sim N(0, \sigma_\eta^2) \quad (3.8)$$

Reason for the independence assumption is following: if we interpret the terms as heterogeneity and as measurement(or optimisation) error, there are no reason to expect them to be generated from a joint process (assumption is not needed in the estimation) as Moffit[14] mentions.

Our next step is to construct the Likelihood function when we have two random terms. This is quite tricky because some level of labour supply is outcome from the two values of two random terms i.e. we have to take account all the possibilities which can produce this level of hours.

In this two random term model the algorithm to find optimal amount of labour supply can be constructed in the following way. Optimum h^* can be found from the segment $k(k=1, \dots, n)$ if

$$H_{k-1} - \alpha \ln w_k - \beta y_k - \gamma z < \eta < H_k - \alpha \ln w_k - \beta y_k - \gamma z.$$

Above expression can be rewritten as

$$\eta_{kl} < \eta < \eta_{ku}.$$

Where

$$\eta_{kl} = H_{k-1} - \alpha \ln w_k - \beta y_k - \gamma z$$

$$\eta_{ku} = H_k - \alpha \ln w_k - \beta y_k - \gamma z.$$

Above the subindex l indicates to the lower limit of the segment k and respectively the subindex u indicates to the upper limit of the segment k.

In this case optimum labour supply h^* is located in the kink point H_k ($k = 1, \dots, n-1$), if

$$\eta_{ku} < \eta < \eta_{(k+1)l}.$$

Finally, for the completeness⁶ we can show that the utility is to be maximized at the zero hours H_0 and at the maximum hours H_n , if

$$\eta \leq \eta_{1l}$$

$$\eta \geq \eta_{nu}.$$

We can express the corresponding probabilities using the integrals. For example,

⁶Thus, we can show that the optimal labour supply can be found from:

zero,	if	$\eta \leq \eta_{1l}$
1st segment,	if	$\eta_{1l} < \eta < \eta_{1u}$
1st kink,	if	$\eta_{1u} \leq \eta \leq \eta_{2l}$
2nd segment,	if	$\eta_{2l} \leq \eta \leq \eta_{2u}$
2nd kink,	if	$\eta_{2u} \leq \eta \leq \eta_{3l}$
\vdots		
nth segment,	if	$\eta_{nl} \leq \eta \leq \eta_{nu}$
nth kink,	if	$\eta \geq \eta_{nu}$

the probability that the optimum is located on the second segment is

$$pr(h^* \text{ is on segment 2}) = \int_{\eta_{2l}}^{\eta_{2u}} \left(\frac{1}{\sigma_\eta} \right) \Phi \left(\frac{\eta}{\sigma_\eta} \right) d\eta. \quad (3.9)$$

For further technical details see chapter 2.

3.3.2 Differentiable budget constraint

The technique for constructing differentiable budget constraints was first introduced by MaCurdy et al.[13] and our presentation will follow it with suitable modifications to take account Finnish tax system. Intuition behind this method is to approximate the tax schedule by fitting a function to the marginal tax rate and then integrating this marginal tax function we get a differentiable relation approximating the amount of total taxes paid as a function of taxable income.

Let us introduce some new notation; denote $I(h)$ for taxable income at h hours of work and $M[I(h)]$ for marginal tax rate. Now, the simplified tax schedule presented in figure 3.2 can be presented in the following way:

$$\begin{aligned} M[I(h)] &= t_1 \text{ [from } I(H_0) \text{ to } I(h_1)] \\ &= t_2 \text{ [from } I(H_1) \text{ to } I(h_2)] \\ &= t_3 \text{ [above } I(h_2)] \end{aligned}$$

To approximate the marginal tax rate schedule the function must fit a step function presented above closely and it still should be differentiable at the step points (ie. kink points). Macurdy et al. suggested a following kind of approximation

$$\overline{M[I(h)]} = \sum_{i=1}^K [\Phi_i(I(h)) - \Phi_{i+1}(I(h))] * p_i(I(h)), \quad (3.10)$$

where $\Phi_i(I(h))$ denote the cumulative normal distribution function evaluated at the income level I with mean μ_i and variance σ_i^2 . The idea is that the difference $\Phi_i(I(h)) - \Phi_{i+1}(I(h))$ takes value of one over the range where t_i is relevant and zero elsewhere. Now, we can control this by adjusting the mean and the variance. Adjusting the mean we control the moment when the value of one begins and ends, and adjusting the variances we can control how quickly this happens. The trade-off here is the smoothness of transition against the precision. $p_i(I(h))$ are the polynomials in income. For example, in Finnish case in 1989 there were 7 tax brackets, so we can set $K = 7$ and p_i is the marginal tax rate t_i associated with the i th tax bracket.

To see how the above presented generalisation works let us go back to our simplified tax schedule presented in figure 3.2. In this case we have three marginal tax rates $t_1 < t_2 < t_3$, so we have "three segments to smooth out". We can now write our approximation function using the above notation for this problem as

$$\overline{M[I(h)]} = [\Phi_1(I(h)) - \Phi_2(I(h))] * t_1 + [\Phi_2(I(h)) - \Phi_3(I(h))] * t_2 + \Phi_3(I(h)) * t_3 \quad (3.11)$$

So, the first segment has a height t_1 (can be thought as a flat line with a height t_1 .) and in the figure 3.2 corresponding taxable income is from $I(H_0)$ to $I(H_1)$. This feature is captured by parameterising $\Phi_1(I(h))$ with mean $\mu_1 = I(H_0)$ and correspondingly $\Phi_2(I(h))$ with mean $\mu_2 = I(H_1)$. The first distribution function $I(H_0)$ takes value of one above the income level $I(H_0)$ and zero elsewhere and the second distribution function $I(H_1)$ takes value of zero below the income level $I(H_1)$ and then switches to one above it. So the difference of these functions is one between $I(H_0)$ and $I(H_1)$ and zero elsewhere and correspondingly for all other ranges. So, we can control the switch from zero to one (and vice versa) by adjusting the means. How quickly these switches will take place depends on the values given to the variances.

In 1989 Finnish marginal tax rates varied from zero to 44 percent including 7 tax brackets. Local tax rates varied from 14 percent to 19.5 percent and these taxes are paid from all labour incomes unlike in the case of state income tax where the tax exemption level was 36 000 FIM. In top of these all individuals have to contribute to the National Pension Insurance and to the National Health Insurance Schemes, which were proportional to the income changes. To put above described tax system into the described framework concerning differentiable budget constraints we get the following three parts. Part one is valid from zero income up to 36000 FIM and the tax rate for this range is equal to the individuals local tax rate t_l . In the second part income ranges from 36000 FIM up to 250 000 FIM where the tax rate is the local tax rate plus the monotonically increasing marginal tax rate $t_i, i = 1, \dots, 7$. The third part is for the incomes over 250 000 FMK. In this case the tax rate is the local tax rate plus the federal tax rate of 44 percent.

So, according to given information we can write the approximation for the marginal tax rate function in the Finnish case as:

$$\begin{aligned} \overline{M[I(h)]} &= \left[\Phi_1 \left(\frac{(I(h)) - \mu_1}{\sigma_1} \right) - \Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) \right] * t_l \\ &+ \left[\Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) - \Phi_3 \left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) \right] * F(I) \\ &+ \Phi_3 \left[\left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) \right] * (t_l + 0.44), \end{aligned} \quad (3.12)$$

where $F(I)$ is a polynomial in taxable income which approximate the increasing tax rates from 36000 FIM to 250000 FIM.

3.4 MaCurdy's critique

In their paper MaCurdy et al.[13] asked the following question: "*if one estimates the labour supply function by maximum likelihood methods using the piecewise-*

linear approach, is the estimated function restricted to satisfy certain properties?"

In this section we try to give an intuition behind their claim. Technical details can be found from their article.

As we saw from the previous section, utility maximisation is the basis for deriving the formulas for the likelihood function in the Piece-wise linear case(as in the case of differentiable budget constraints, but we will come to this point later). Economic theory says that we should see some bunching near the kink points where the marginal tax rates changes. For these individuals optimal labour supply will be below the kink when evaluated at the marginal tax rate after the kink and above the kink when evaluated at the marginal tax rate before the kink. Now, when marginal tax rate(s) increase(s), individuals will stay at the kink if their optimal hours at the higher marginal tax rate would be below the kink(s). In theory, this to be true the labour supply parameters α and β have to be restricted in a way that the compensated wage elasticity is positive. In statistical terminology this means that if the above condition is invalidated we do not have a properly defined statistical model, because we would get negative densities in the kink points.

Because above argument has to be true for all kink points for all individuals in the sample. In many cases we have a significant number of individuals in the sample and they all face many kink points at different level of hours then the above argument basically states that Slutsky condition must be fulfilled globally. In the most simple case, where we have a linear taxation no such constraints are imposed because we do not have any kink points in budget sets. All individuals will face linear budget constraints.

But, using the same logic as above the differentiable budget constraint may also impose some restrictions to the estimated parameters. As we mentioned above we approximate the marginal tax function around a kink point using a step function. Adjusting the variances we can make our approximation react more or less sharply to the change in marginal tax rate. At the extreme, giving very low values to the

variances, we make the function change very abruptly. This corresponds exactly to the piecewise linear case and thus the same restrictions are imposed to it. If we prefer a very smooth approximation, then we make the budget constraints to be continuous and no restrictions are imposed.

Blomquist[3][4] writes that MaCurdy's claim is not always true. He mentions that in all maximum likelihood estimation we specify the data-generating mechanism and if the estimated parameter values are inconsistent with it we do not know how to interpret the results. He basically concludes that a bad choice of functional form or bad data may lead to a "negative Slutsky effect".

Empirical evidence from this subject is mixed. Some authors, like Blundell, Duncan and Meghir[6] avoid this problem by excluding those observations which are "near" the kink. In their 1990 study using the U.S. data they find above mentioned constraint binding. On the other hand, when using the Swedish data MaCurdy and Flood[12] do not find any such restrictions. In this paper we will estimate labour supply function for using both methods and study if we can find any differences between these two approaches.

3.5 Empirical specifications

3.5.1 Piecewise-linear budget set models

In this subsection we will shortly go through different empirical models and present the corresponding likelihood functions.⁷ We start by the (technically) simplest approach where we do not allow any individual heterogeneity and the error term is interpreted to be optimising or measurement error. Observed hours h may then deviate from the desired hours h^* by the amount of the opti-

⁷We do not derive these models here, because that would need a substantial amount of space and in chapter 2 we already showed the required technique. Other references are, for example, Appendix 1. in Blomquist[2] and Pudney[15].

mising or measurement error ε , thus $h = h^* + \varepsilon$. We assume that $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and that $E(\varepsilon|h^*) = 0$.

Keeping in mind the discussion in the last section, it is natural to think that **observed** hours are generated by the following generalised Tobit-model⁸

$$\begin{aligned} h &= 0 && \text{if } h^* + \varepsilon = 0 \\ h &= h^* + \varepsilon && \text{if } 0 < h^* + \varepsilon < H_n \\ h &= H_n && \text{if } h^* + \varepsilon \geq H_n \end{aligned}$$

and the corresponding Likelihood Function can now be written as

$$L = \prod_{i \in A} \left[1 - \Phi \left(\frac{h^*}{\sigma_\varepsilon} \right) \right] \prod_{i \in B} \left[\frac{1}{\sigma_\varepsilon} \phi \left(\frac{h_i - h^*}{\sigma_\varepsilon} \right) \right] \prod_{i \in C} \left[1 - \Phi \left(\frac{H_n - h^*}{\sigma_\varepsilon} \right) \right]. \quad (3.13)$$

Where,

A is index set when $h = 0$

B is index set when $0 < h < H_n$

C is index set when $h \geq H_n$.

$\phi(\cdot)$ is Standardised Normal Density Function and $\Phi(\cdot)$ is Cumulative Normal.

The first part of the likelihood function corresponds individuals whose observed hours are zero. The second part corresponds those individuals whose observed hours are in some of the segments or kink points and the third part corresponds those whose observed hours are at maximum.

As we know, the stochastic specification is important when we face non-linear budget constraints and the error term has a more specific interpretation in these models as discussed in chapter 2. The most important drawback in this traditional model is its restrictiveness to the labour supply responses. For example, according

⁸Note that we have dropped the subscript i .

theory a change in the marginal tax rate in the case of convex budget set would have identical zero effect on the labour supply for all individuals not located on that segment (when we do not take income effects into account). In other words, change in slopes of the other segments do not have any behavioural effects.

It might be the case that that wage and income effects are not same for all individuals so there might be many different utility maximising choices i.e. we might find some preference heterogeneity. Discussion concerning this topic can be found from chapter 1 or from Moffit[14]. Introducing a source of heterogeneity can be done in a several different ways. We shortly go through the most usual cases.

We first start by assuming that the heterogeneity term is additive random variable in the regression function. If we write our deterministic part of the labour supply function as

$$h^* = \zeta + \alpha \ln w + \beta y + \eta, \quad (3.14)$$

where η follows some probability distribution. So, we do not estimate η for all individuals separately because this would require estimation of more parameters than we have observations. We estimate parameters of the distribution function of η . Each persons η is considered to be a random drawing from this distribution.⁹

⁹For example, if we assume that η follows normal distribution with $(0, \sigma_\eta^2)$ and it is uncorrelated with ϵ we can derive the following formula for the likelihood function. Technical details are presented in chapter 2.

$$\begin{aligned} & L(\alpha, \beta, \gamma, \sigma_\eta, \sigma_\epsilon : w_k, y_k, z) \\ &= \prod_{i=1}^J \left\{ \left(\frac{1}{\sigma_\epsilon} \right) \phi \left(\frac{h_i}{\sigma_\epsilon} \right) \left[\Phi \left(\frac{(-\alpha \ln w_{i1} - \beta y_{i1} - \gamma z_i)}{\sigma_\eta} \right) \right] \right. \\ &+ \sum_{k=1}^n \left(\frac{1}{\sigma} \right) \phi \left(\frac{e_{ik}}{\sigma} \right) \left[\Phi \left(\frac{\eta_{iku} - \left(\frac{\sigma_\eta^2 e_{ik}}{\sigma^2} \right)}{(\sigma_\epsilon \sigma_\eta / \sigma)} \right) - \Phi \left(\frac{\eta_{ikl} - \left(\frac{\sigma_\eta^2 e_{ik}}{\sigma^2} \right)}{(\sigma_\epsilon \sigma_\eta / \sigma)} \right) \right] \\ &+ \sum_{k=1}^{n-1} \left(\frac{1}{\sigma_\epsilon} \right) \phi \left(\frac{(h_i - H_{ik})}{\sigma_\epsilon} \right) \left[\Phi \left(\frac{\eta_{i(k+1)l}}{\sigma_\eta} \right) - \Phi \left(\frac{\eta_{iku}}{\sigma_\eta} \right) \right] \end{aligned}$$

Other commonly used possibilities is to allow the substitution effect to vary across the individuals or to allow income effect to vary across individuals. Actually, nothing restricts us to allow, for example, that both effects can vary, but as already can be seen from the above formulation this will lead to an untractable estimation problem because we would have to evaluate multiple integrals numerically. Note that in above presented case we do not have to restrict the choice of the distribution function but in these present two cases we might have to do so. Obviously this depends from the choice of the functional form.¹⁰ Interesting reader may consult Moffit[14], Blomquist[2], MaCurdy et al.[13] and chapter 2 for the technical details.

3.5.2 Differentiable budget set models

As we saw in the section 4 we need to approximate the variation in the marginal tax rate over the taxable income interval from 36 000 FIM to 250 000 FIM. Remember that below 36 000 FIM an individual faces only the local tax and the social security contributions and above 250 000 FIM she faces the highest marginal tax rate 44 percent plus the above mentioned components. Approximation is done

$$+ \left(\frac{1}{\sigma_\epsilon} \right) \phi \left(\frac{h_i - H_n}{\sigma_\epsilon} \right) \left[1 - \Phi \left(\frac{H_n - \alpha \ln w_{in} - \beta y_{in} - \gamma z_i}{\sigma_\eta} \right) \right] \Bigg\}, \quad (3.15)$$

where $e_{ik} = h_i - \alpha \ln w_{ik} - \beta y_{ik}$ and $\sigma^2 = \sigma_\epsilon^2 + \sigma_\eta^2$.

In the above likelihood the first term corresponds the probability that an individual's worked hours are zero. Second term corresponds that the optimum lies in some segment and third term corresponds the case that optimum can be found from some of the kink points. The last term corresponds the case that individual works maximum amount of hours.

¹⁰Linear labour supply function provides an intuitive example. In this case we can present the Slutsky condition as $\alpha - \beta h > 0$ and if the labour supply is very close to the zero then α must be positive. Usually researchers have used a truncated normal distribution to force the wage effect to be positive. Also in the random income effect model we might get results which do not fulfil the Slutsky condition. Namely, when α is small then in the case when hours are large we need negative β . This can be guaranteed by choosing the truncated normal distribution over the negative values.

by running the following Ordinary Least Squares regression

$$t_i = \xi_0 + \xi_1 * I_i + \xi_2 * I_i^2 + \xi_3 * I_i^3, \quad (3.16)$$

Where t_i is the marginal *federal* tax rate. What we do is that we create an variable which increases by 100 FIM starting at 36 000 FIM and ending at 250 000 FIM. In other words we create 2140 equally spaced combinations (t_i, I_i) . After estimation, we can use the estimates from this third-degree polynomial approximation to specify the equation 3.16 given above.

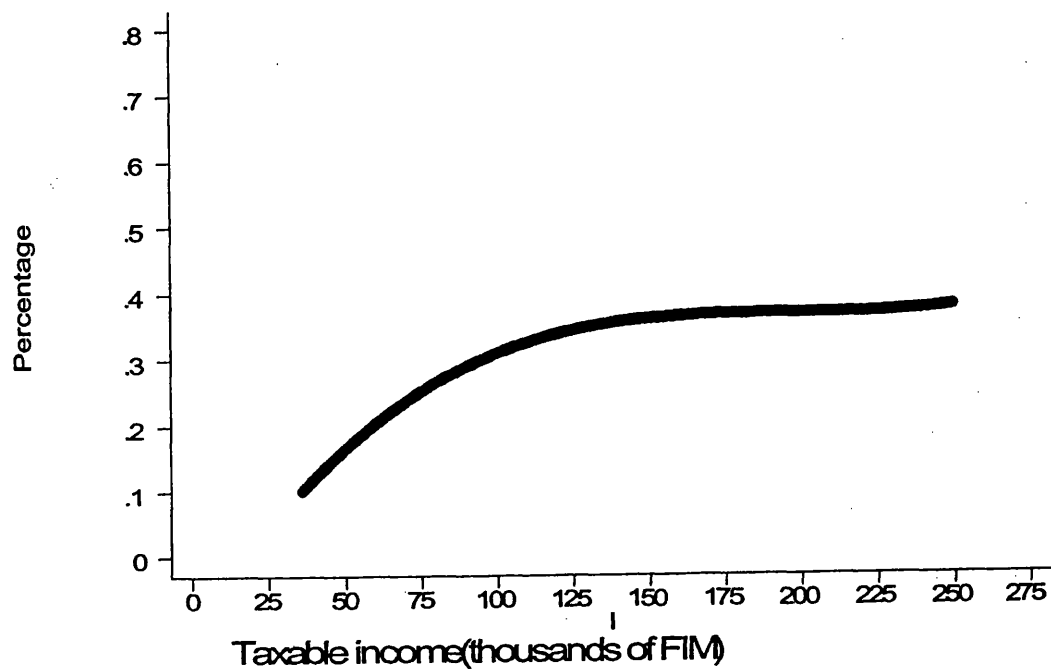
$$H(I) = -1219 + t_l + 7.46 * 10^{-6} * I - 3.80 * 10^{-11} * I^2 + 6.46 * 10^{-17} * I^3, \quad (3.17)$$

where t_i is the local tax rate. Our model explains 95 percent of the variance in marginal tax rates.¹¹

Figure 4.4 shows the approximation for the Finnish marginal tax function for the year 1989. Note, that the local tax rates and social security contributions are not included into the figure.

¹¹We did a lot of experimenting using other combinations of (t_i, I_i) , but the above produced the best approximation.

Figure 3.4. Approximated marginal tax function



Plugging the formula 3.14 into the marginal tax rate function and integrating it with respect to the income we can derive formula which approximates the amount

of total taxes paid.¹²

$$T(I(h)) = \int \overline{M[I(h)]} dI \quad (3.18)$$

$$\begin{aligned} &= \left[t_l \Phi_1 \left(\frac{(I(h)) - \mu_1}{\sigma_1} \right) - t_l \Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) \right] \\ &+ \left[\xi_0 \int \Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) dI + \xi_1 \int \Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) I dI \right. \\ &+ \xi_2 \int \Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) I^2 dI + \xi_3 \int \Phi_2 \left(\frac{(I(h)) - \mu_2}{\sigma_2} \right) I^3 dI \left. \right] \\ &- \left[\xi_0 \int \Phi_3 \left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) dI + \xi_1 \int \Phi_3 \left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) I dI \right. \\ &+ \xi_2 \int \Phi_3 \left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) I^2 dI + \xi_3 \int \Phi_3 \left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) I^3 dI \left. \right] \\ &+ \left[(t_l + 0.44) \int \Phi_3 \left[\left(\frac{(I(h)) - \mu_3}{\sigma_3} \right) dI \right] \right]. \quad (3.19) \end{aligned}$$

Above presented differentiable approach is more straightforward than the kink one since a purely continuous distribution describes the hours of work. In the last section we showed how to construct the tax algorithm needed in the estimation. Intuition behind this differentiable approach follows the idea presented by Hall[8].

¹²From equation 3.19 it is relatively easy derive analytical function of the Φ'_i s, the corresponding density functions ϕ'_i s and the powers of I. To derive the following expression we need to integrate over the Cumulative Normal Distributions with respect to I. For technical details see for example, Dudewicz and Mishra[7]. Using these expressions one can derive the explicit form for the $\overline{M[I(h)]}$:

$$\begin{aligned} \int \Phi dI &= I\Phi + \phi \\ \int I\Phi dI &= \frac{1}{2}I^2\Phi - \frac{1}{2}\Phi + \frac{1}{2}I\phi \\ \int I^2\Phi dI &= \frac{1}{3}I^3\Phi + \frac{2}{3}\phi + \frac{1}{3}I^2\phi \\ \int I^3\Phi dI &= \frac{1}{4}I^4\Phi - \frac{3}{4}\Phi + \frac{3}{4}I\phi + \frac{1}{4}I^3\phi \end{aligned}$$

We can think that the marginal wage rate and virtual income as a function of working hours. Hall's idea was to linearise the actual non-linear budget constraint at the observed hours. The implied slope of this linearised constraint is the marginal wage rate and the intercept of the vertical axis is the virtual income. In other words, utility maximisation implies a solution for hours of work which can be written in the form of implicit equation, $h = f[w(h), y(h)]$. By applying the Implicit Function Theorem to this equation, we can solve this implicit equation for h and hence derive the labour supply function which applies in the nonlinear case.

3.6 Estimation results

3.6.1 Data

A sample of married women of age between 25–60 is drawn from the Finnish Labour Force Survey (LFS) for the year 1989. It is a cross-section data and it includes individuals of age between 15 to 64. In the first stage the sample is drawn from the Finnish Population Census using geographical weights. After that the final sample is drawn randomly by age and gender. In 1989 data, the final sample size is 7820 individuals.

Income data corresponding those individuals in the LFS is drawn from the Tax Register Data and then merged with the LFS. The income information is not based on the survey¹³ data and it includes approx. 70 variables on individuals earnings. Of course, it is very unlikely that someone's earnings are composed from all of these components. However, the data shows that individuals' earnings comes from the very different sources. Actually, for some individuals traditionally used income variables do not play any role at all. The income data also includes

¹³LFS data set also includes some information about individuals financial situation. See also Kuusmanen[11].

the same 70 variables for possible spouse, so all in all we have approx. 140 variables(if married) to construct the budget sets individuals face. Detailed information how crucial variables like working hours, wages and exogenous incomes are calculated can be found from Kuismanen[11].

Original sample size was 7825 individuals and from this we selected females. After this we were left with 4124 observations. For the empirical analysis we selected married women aged 25–60. We also deleted some groups like farmers and self-employed mainly because of the very different tax and social security legislation. The final sample size used in this analysis is 2037 observations.

In below we will only shortly comment the main features of the data and A comprehensive data description can be found from the appendices 2, 3 and 4 of chapter 2. The participation rate in the selected sample is 72 per cent. Unemployment rates vary geographically and the figures are lowest in the Helsinki Metropolitan area and highest in the east and north part of the country and the average unemployment rate 3.2 per cent.¹⁴ Blue-collar workers are more likely to have zero hours observations than the white-collar ones and women who have two young children have the highest probability to be out of work. Participants are slightly younger than the non-participants and they are also better educated. It also seems to be the case that the likelihood of being non-participant increases if the spouse is also non-participant. In section 2 we have described the Finnish income tax system and more comprehensive description can be found from the chapter 2.

3.6.2 Results

As mentioned previously, our main goal in this paper is to estimate similarly specified labour supply functions using the piece-wise linear and differentiable approach. This is because we are interested that will these two methods produce

¹⁴As it is well known Finland experienced severe recession in early 90's and the unemployment rate rose dramatically

similar results and thus are we able to conclude that, at least using our data, the piece-wise linear method will produce sensible results.¹⁵

Our data set does not have direct information on individuals' hourly wage, thus we have to construct it using the income and the hours of work variables. This means that the possible bias in the hours of work variable shifts also into the marginal hourly wage rate. For example, if worked hours are smaller than the right value then hourly wage rate becomes too high. To get rid of this bias we estimate the log-wage equation using Heckman's method and the predicted values are used in the final analysis as instrument for the hourly wage rate¹⁶. For working hours we use regular reported weekly working hours also taking into account regular hours in the second job. When calculating the exogenous income term we took into account the following components: Interests(both taxable and nontaxable), dividend payments, sales profits, regular untaxable pensions, other regular subsidies etc. From all the components which are taxable we have subtracted the corresponding amount taxes paid i.e. our constructed variable measures net exogenous incomes. For details see chapter 2.

As mentioned earlier our aim is to study whether the choice of how differently constructed budget sets affect to the estimation results. For this reason we have estimated exactly the same labour supply functions using same data in both models. We have chosen the semilogarithmic labour supply function with measurement/optimisation approach to our representative model. This due to the fact that we have previously estimated labour supply functions using the same data as here with different functional specifications and with different unobserved heterogeneity assumptions and our chosen specification seemed to work well. On top of that these models are not the most straightforward to estimate one after another.

¹⁵Obviously this is not to say that the differentiable method is the right one when estimating labour supply functions in the presence of non-linear income taxation. In the differentiable budget constraint approach we need less restrictive assumptions in background than in piece-wise linear approach and thus we can "test" if these assumptions are important or not.

¹⁶For the results see appendix 5 in chapter 1

Table 3.1. Results for the labour supply functions

Maximum Likelihood Estimates		
Asymptotic Standard Errors in Parenthesis		
<i>Variables</i>	<i>Piecewise linear</i>	<i>Differentiable</i>
	<i>budget constraint</i>	<i>budget constraint</i>
Constant	-2.57905 (0.55650)	-2.70210 (0.55474)
Ln W	0.37046 (0.12135)	0.39121 (0.12444)
Exog. inc	-0.00045 (0.00022)	-0.00045 (0.00020)
Cdum1	-0.33917 (0.09948)	-0.33823 (0.0990)
Cdum2	-0.00487 (0.10437)	-0.00482 (0.10431)
Cdum3	0.09616 (0.10050)	0.09601 (0.10049)
Cdum4	0.14310 (0.07690)	0.14310 (0.07688)
Age	0.16118 (0.02484)	0.16144 (0.02488)
Age*Age	-0.00227 (0.00028)	-0.00229 (0.00028)
Sosio	0.19945 (0.09521)	0.19912 (0.09567)
Nkids	-0.08419 (0.03235)	-0.09011 (0.03239)
σ_ϵ^2	0.98208 (0.01907)	0.96998 (0.01918)
Ln L	-2669.61	-2675.56

Note: In both models above, dependent variable(yearly hours) is divided by 1000. The exogenous income variable contains only own exogenous income components(net) and it is divided by 100. For the definitions of variables see appendix 1 in chapter 2.

As can be seen from the above table, our results between two estimated models are almost identical. For example, income elasticities are identical in both models. In the case of compensated wage elasticity the differentiable approach gives a slightly higher elasticity, 0.215 compared to the piece-wise linear approach, 0.21, but practically speaking policy elasticities are same in both models. If we look the other covariates situation does not change at all. Thus, it can be concluded that at least in our case (same data, same specifications) both approaches seem to work similar fashion.

What is the evidence from other studies? As far as we know there are two similar studies made earlier. In the MaCurdy et al.[13] paper they found that in the case of piecewise linear approach estimates implied larger responses than differentiable approach. They used U.S. data from 1976 (PSID; 1017 prime-aged males). This data set is almost the same as in Hausman's[9] original study but the findings differ. Even in the piece-wise linear case MaCurdy et al. finds much more modest labour supply responses than Hausman.

The other similar study by MaCurdy and Flood[12] is more likely to be better comparison to our paper due to the similar tax and other institutional systems in Sweden. Their analysis deals with male labour supply and the data set is drawn from the Swedish Household Market and Non-market Activities Survey(HUS) for the year 1984. Results indicated that Differentiable and piece-wise linear approaches produced identical results. Authors concluded that results might depend on the degree of progressivity in a way that in the case of high degree of progressivity methods are likely to produce similar results and when the tax system consist only few tax brackets then differentiable approach might function better. Their results are very similar to results by Blomquist[2] and Blomquist and Hansson-Brusewitz[5] who also used Swedish data with high degree of progressivity.

It is evident that our result is not a proof that these methods work similarly in the presence of high degree of progressivity, but together with the above mentioned

studies it gives a strong indication that this might be the case. If we go back to the section 3.3 it can be seen that introducing a very high degree of progressivity to piece-wise linear case, then the shape of budget constraint actually becomes more and more like differentiable constraint.

3.7 Conclusions

Sophisticated Maximum-likelihood approaches have become standard tools for analysing the labour supply disincentive effects of income tax systems. On the other hand, these ML approaches have generated a discussion among the researcher of these method's robustness. To analyse Finnish tax system we have estimated labour supply function with nonlinear income taxation using two different ML approaches. First we consider so-called piece-wise linear approach, which has been the most popular procedure in recent years. Our second approach is so-called differentiable function method, which approximates the actual piecewise linear function as closely as possible using polynomial function.

One advantage of piecewise linear approach is that it allows us to model carefully all the institutional characteristics which affects to the shape of the budget sets. Obviously, this approach is a data intensive one. It has been argued that individuals do not know the exact shape of their budget constraints and thus differentiable approach could be a suitable substitute.

Our results indicate that proxying tax schedules by smooth continuous functions produces similar results with the piecewise linear approach. For example, the mean compensated wage elasticities are almost the same in both models (approximately 0.21). Our conclusion is that because of the high degree of progressivity these two approaches actually generates a very similar budget constraints and because our functional specification and data set is exactly the same one in both approaches, then it is no surprise that similar results appear. For example, Macurdy and Flood[12] found when using Swedish data that these two methods produced

almost identical results. Their conclusion was also that this result might depend on the degree of progressivity. As an example their results are very similar to the results got by Blomquist[2] and Blomquist and Hansson–Brusewitz[5] who also used Swedish data from the 70's and 80's. This Finding is not necessary valid when the tax system includes only few tax brackets. Results by Macurdy at al.[13] shows that in the case of U.S. data piece–wise linear approach yields a larger labour supply responses than piece–wise linear approach.

As nearly all studies and also this one has generated probably more new questions than answers. For example, we have spoken about the degree of progressivity but have not given any specific answer when the degree of of progressivity is in that high level that it is same which one method to use. To answer this question needs much more empirical work and it depends on so many factors that to give the precise answer is most likely impossible task to do.

In addition to these two methods presented here there are also other relevant approaches to model labour supply. For example, we have all the way assumed that, for example, exogenous income is really an exogenous determinant of labour supply. Instrumental variable procedure would permit one to determine the appropriateness of the exogeneity assumptions maintained in our analysis. This is an interesting topic in this context and hopefully it will be on the research agenda in the future.

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Chapter 4

Labour Supply and Income Tax Changes: A Simulation Exercise for Finland

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Abstract

It is well known that the estimation of labour supply function is complicated by the fact that the budget constraint individuals face is non-linear. This non-linearity may arise for variety of reasons - the structure of the tax/benefit scheme, overtime rates etc. Non-linearities also cause problems when it comes to interpreting the policy implications of the estimates. In this study we use well-structured econometric labour supply model which mimics the actual budget constraints as closely as possible to analyse different income tax regimes and systems to labour supply. On top of the empirically specified labour supply model we have constructed first time in Finland a behavioural microsimulation model and our aim is to contribute to the Finnish tax debate by simulating different kinds of tax reform suggestions. Our simulation results show that none of the proposed reforms are self-financing. Revenue neutral proportional tax system do not have major effects on labour supply. Biggest behavioural responses are achieved if we reduce the marginal tax rates from the lower end of the state income tax schedule.

Keywords: Microsimulation, Labour supply, Progressive income taxation, Tax

reforms

JEL classification: H24, J22, C31

4.1 Introduction

The purpose of this paper is to contribute to the ongoing income tax debate in Finland using the behavioural microsimulation approach. As far as we know, there are no other studies which have used the behavioural microsimulation approach to analyse various tax proposals. Our aim is to go through some of the most frequently proposed income tax reforms in Finland and discuss their implications from various points of view.

As the current tax debate in Finland shows, income tax and benefit systems create more debate than many other economic subjects due to many different reasons. Viewpoints are often politically or philosophically oriented and may relate to things like the equity-efficiency dilemma. Other sources of debate can be more practical ones, like taxation's incentive problems a la poverty traps etc. Partly due to the above mentioned factors a significant amount of research also in many other fields than economics has been devoted to tax issues. Particularly, in the case of income taxation research has mainly concentrated on the determinants of individual (or family) labour supply decisions. Majority of these studies, and rightly so, have concentrated on the difficult topic of how to estimate the labour supply function, either taking into account real life complications or not. Most of the studies report summary elasticities, but as it is well known, this information is not enough to answer interesting questions concerning the different tax reform proposals.

However, to answer these interesting questions is not a straightforward task to do. As mentioned above, summary elasticities do not contain the information needed to study the behavioural consequences of different tax reforms. This is due to the fact that the tax and benefit systems almost without an exception give rise to non-convex budget constraints, so that marginal tax rates differ across individuals. The other aspect is that the income distribution before and after the reforms are not similar. As Hausman[8] has written, in circumstances mentioned above a change in either the gross wage rate or some parameter of the tax system may

cause individual to shift from one segment of the budget constraint to another and these movements cannot be captured without detailed knowledge of the labour supply curve for each individual in the sample to be used for tax reform analysis.

So, when tax laws and different kinds of rules of transfer programs introduce censoring and truncation and when sub-populations differ in behaviour, then models and calculations of average behaviour become inadequate to evaluate the impact of policy changes. At this point one usually turns to microsimulation. Microsimulation may be viewed as an attempt to model and simulate the whole distribution of policy target variables, not only their mean values. For example, in many cases we are interested to analyse the impact of an income tax change on the whole distribution, who gains and who loses. One of the main advantages of microsimulation is that it allows us to deal with heterogeneous behaviour; all individuals (or firms) do not behave as the average economic agent.

The data in our simulations is the same which has been the basis of our empirical work, see chapter 2 and chapter 3 in this thesis. In this paper we do not concentrate on statistical matters and thus many important and technically demanding subjects, like integrability conditions and functional form issues, will not be dealt in this paper. Interested reader should consult Blundell and Meghir[5] and Blundell et al.[3].

The paper proceeds as follows. In section 4.2 we want to outline the reasons why this simulation exercise is worthwhile to do. In this section we also describe our data source and go through some theoretical remarks on labour supply which are essential for the rest of the paper and for the general discussion. We have devoted a reasonably large space for justifying why microsimulation is a good tool to use in tax reform calculations. This is because the lack of this kind of behavioural analysis and majority of the calculations presented have been based on examples dealing with the representative individual or family. We also say some words concerning the simulation procedure.

Section 4.4 presents the results. First we will go through some basic calculations

between different tax systems, like proportional taxation versus progressive taxation, Lump-sum taxation versus progressive taxation etc. After that we'll do some purely hypothetical calculations to get some touch how labour supply reacts in our simulation framework. The next subsection is devoted to the reforms which try to mimic those reforms presented in public debate as closely as possible. Basically we simulate three different types of reforms. The first one is a simple one percentage point reduction in the marginal tax rates in all tax brackets. The second one studies the effects if we reduce the marginal tax rates at the lower end of tax schedule and in the final simulation we reduce the top marginal tax rates.

Our results suggest that if we would change our tax system to a proportional income taxation in a revenue neutral way, then the marginal tax rate would be 28 per cent. It has to be kept in mind that this is a sample dependent result. Changing to proportional tax system does not have major effect on labour supply. The current progressive tax system creates deadweight loss and its magnitude is estimated to be approximately 15 per cent of the tax revenue. An interesting finding is that tax reductions and tax increases do not have a symmetrical labour supply responses. Also, behavioural effects are biggest in the case of a progressive income tax system. Reducing the marginal tax rates from the tax schedule's lower end has biggest labour supply effects and none of the reforms suggested are self-financing.

4.2 Motivation of the simulations

It is natural to ask why are we doing this simulation exercise and can we somehow contribute valuable information for the current tax debate in Finland? First, answer to the latter part of the question is yes, we do believe that our study will provide some fruitful ingredients to the general discussion and hopefully it will also generate more discussion concerning disincentive effects of income taxation. Our answer to the first part of the question presented above is a longer one and in below we will try to provide answer for it.

In the late 80's and early 90's a fashionable topic in economics and in general economical debate was the disincentive effects of the income tax systems. After the tax reforms in the US (in early and mid 80's) the debate arrived to continental Europe and to UK. Many countries like UK, Germany, France etc. followed the US example and simplified their tax systems and reduced their (top) marginal tax rates. In Appendix 3 we utilise the figures calculated by Maki and Viren[13] and present taxable incomes and marginal tax rates in 19 countries in 1995. Taxable incomes are converted to Finnish currency which makes the comparisons easy. Already from these figures we see that income tax systems vary considerably. For example, in Ireland and Sweden there are only two marginal tax rates and in Spain there are 17 marginal tax rates, in some countries income tax is paid from all income and in some countries there exist a tax threshold etc. If we would have compared the situation 25 years ago, then differences would have been even larger.

Taking into account that Finland is a typical Nordic welfare state where the size of the public sector is relatively large and the tax burden is high(see appendix 2), it is a surprise that this tax discussion was mild one here at the late 80's. One reason for this was the long lasting boom in Finnish economy and loosely stated it was felt those days that we do not have a need for any major tax reforms. In the late 80's only minor changes to the tax schedules and tax rules were done regarding taxes on labour income. At the very beginning of the 90's Finnish economical situation started to deteriorate rapidly and the recession to come was an unseeingly severe. Unemployment rate rose from app.4 per cent (1989) to 19 per cent (1993) and at that time many Finnish economists argued that the only thing which will lower the unemployment rate is a rapid growth of the economy and especially the export sector will be a key player in improving employment. At that time tax and benefit systems were seen as a secondary topics. The annual growth of Finnish economy has indeed been a very rapid since 1994; approximately 4,5 per cent per year. But, despite of that, the decline of unemployment has been more subdued than expected and at the moment the

real unemployment rate is still approximately 14 per cent.¹ Naturally this has started a hectic debate why unemployment rate still stays at that high level. The reasons suggested are familiar ones: unions, inflexible labour markets and especially the income tax system.

During the last year or so the role of income taxation and social security payments (either paid by employer or employee) have been the hot topic and sometimes it seems that these factors are the only ones which matters. Practically all discussants agree that the tax burden is too high, but at that point the consensus usually ends. It is not a surprise that according to the political status reform suggestions differ from each others; some have stressed that marginal tax rates should be cut mainly from the low income earners and others have stated they should be cut equally throughout the tax schedule. Some have even stated that we should abandon the progressive nature of the income taxation and move to the proportional tax system. Others have suggested that we should reduce the number of tax brackets to simplify income tax schedule etc.

A common denominator to all of these suggestions is that no calculations of the behavioural effects on labour supply and tax revenues have been presented by the proposal makers. Our aim is to try to provide such information concerning the implications of suggested reforms using the behavioural microsimulation model. We will go through the main proposals made in public debate and discuss practical implications of the reforms.

4.2.1 Data source and some theoretical remarks

In seeking data on actual families to analyse tax and benefit changes, the most appropriate source, at least in the Finnish case, is the Labour Force Survey

¹The official unemployment rate (from Labour Force Survey) is approximately 9.5 per cent. Official definition of unemployment (definition changed to correspond EU-definition) do not take into account persons who are participants in public sectors programs etc. Also ministry of labour's statistics shows higher unemployment rates than the official one.

data (LFS). It covers a great deal of information concerning the labour market activity. We are also able to merge register based income information from the tax authorities' data base for all the individuals in the LFS.² For more detailed information see Chapter 2.

In our analyses we use sample of a married women of age 25-60 drawn from the LFS for the year 1989. Chapter 2 describes more detail the data and its properties. The final sample size is 2037 individuals and it is exactly the same one used in our econometrical work.

But, what are then the theoretical justifications for lowering marginal tax rates? The answer is that if individuals will face a lower tax rate on **additional** income they will be willing to work more, *ceteris paribus*. Shortly stated, this is the incentive (or substitution) effect of a tax change. But, this is not the only effect because also the net incomes received by the individuals will rise. If leisure time is a normal good then higher net incomes may decrease the willingness to work more and this is called income effect. Thus, any change in tax or benefit system will create a complicated set of income and substitution effects and as Blundell and Walker[6] have stated, it is sometimes hard to distinguish whether people are referring to the total effect (income and substitution effects) of tax changes or to the substitution effect alone.

If one is referring to economic efficiency of the tax system then only the substitution effect is a relevant factor. That is because the *marginal* tax rate distorts the individuals economic decisions by creating a wedge between the wage rate an employer is willing to pay for an extra hour of work and the net wage rate the employee receives. Thus, the higher the marginal tax rate the bigger is the wedge. A positive marginal tax rate may make exchanges (hours of work for an hour's worth of pay) between agents unattractive while in the absence of the tax wedge they would be mutually beneficial. From the above we can conclude that lower marginal tax rates are to be preferred to higher ones.

²So, we avoid the problem that individuals do not fully report their incomes in surveys.

However, efficiency is not the only dimension of taxation. The other important dimension is the equity. For equity we mean the objective of re-distributing incomes (from poor to rich). The tax system is re-distributive if the average tax rate faced by the rich is higher than the average rate on the poor.³ So, re-distribution from rich to poor inevitable implies work disincentive effects because of the high marginal tax rates. This is the so called *equity-efficiency trade-off*. Ongoing Finnish tax debate indicates that the efficiency costs from an equitable tax system are regarded high and both the marginal and the average tax rates should be lowered to reduce efficiency costs.

In some statements it has been argued that tax rates in Finland are at so high level that tax increases do not yield more tax revenues or in other words, tax reductions yield revenue increases. This is the so-called Laffer-curve effect. The flatter is the compensated labour supply curve (the higher is the compensated labour supply elasticity) the more concave is the government's budget constraint (\approx Laffer-curve). The government's problem is to choose a point from that constraint. So, government which put no weight on the degree of inequality will choose a solution where tax rate is zero. Thus, the most **efficient** tax system is no tax system at all. The other extreme is that if government cares only about equality it should raise as much money as possible since this benefits most the poor who have so low incomes that the required high marginal tax rates do not have any effect on them, see Tuomala[15]. Because the labour supply elasticity determines the concavity of the government budget constraint, it follows that the higher is the labour supply elasticity the lower will be the optimal marginal tax rate. Despite our aim is not to study optimal taxation in this paper the above reasoning is good to keep in mind when studying different tax and benefit systems.

³In other words, average tax rate should be an increasing function of income and that the marginal tax rate must be higher than the average rate.

4.2.2 Rationale for microsimulation

Despite the fact that, for example, budget proposals or tax reforms have allocational and distributional consequences, it is still a common practise that these effects are almost always ignored, at least in Finland, in policy discussion. Typically, only some macroeconomic measures are presented and these figures usually represent the first-round cash impacts and thus ignore any responses individuals may make. Actually, this is a bit illogical because the real purpose of many policy reforms is to create incentives for individuals to change their behaviour, so in that respect it seems odd if we do not try to include behavioural aspects into our analysis. Obviously, we do not claim that there always are such big behavioural effects.

It seems to be the case that if those who have suggested tax reforms have some calculations to support their views these are usually based on hypothetical individual/family examples. This is not necessarily the best way to proceed in this context because we know that the diversity of individual circumstances is a very import aspect in this kind of exercises. More preferably, the analysis should be based on the actual circumstances of a representative sample of individuals. Too often discussion concerning important policy issues is limited to simple calculations on a hypothetical individual who has average earnings, one or two children, married, lives in her own flat and has X amount mortgages etc. It has been shown that such calculations can be highly misleading, see e.g. Atkinson [1].

Two examples

In this subsection we shortly go through, using standard microeconomic tools, why the knowledge of labour supply function or the location of hypothetical individual is insufficient to analyse the outcomes from various tax reforms. Consider an individual who has exogenous income y and gross wage w . In a very simplified world without taxation her desired hours of work h^* are determined by the labour

supply function

$$h^* = h(w, y). \quad (4.1)$$

This labour supply function is derived from her utility function and budget constraint. Now, income tax is introduced to the economy (to finance for example public health care system) and individuals have to pay it at the rate t on all income above the tax threshold A . This reform means that individuals will face non-linear budget constraints. Let us assume that the tax she is paying is a positive amount, then we can write her labour supply function to be as

$$h^* = h[(1 - t)w, y + tA]. \quad (4.2)$$

Note that we have written above equation in a way that individual pays tax on all her income and is reimbursed with a lump sum amount tA for the tax she was not obliged to pay on the first A units of income. The effect of changing the tax rate on the individual's desired working hours is⁴

$$\frac{\partial h^*}{\partial t} = -h_1 w + h_2 A. \quad (4.3)$$

Now, we can substitute the Slutsky-Hicks equation ($h_1 = h_1^c + h_2 h$) into equation 4.3, so it can be written as

$$\frac{\partial h^*}{\partial t} = -[h_1^c w + h_2 (wh - A)] \quad (4.4)$$

where h^c is the compensated labour supply function. After some rearranging above equation can be written in the following form

⁴ h_1 is the first partial derivative of h etc.

$$\frac{\partial h^*}{\partial t} = -w \left[h_1^c + h_2 \left(h - \frac{A}{w} \right) \right] \quad (4.5)$$

From the above equation we can see that the effect of the tax on labour supply cannot be predicted from the knowledge of income and substitution effects alone but it depends on how close the individual's hours are to those that she would have to work to reach the tax threshold.⁵ Specifically, for individuals near to the tax threshold the effect of raising the tax rate will be almost purely a substitution effect and there will be a reduction in the hours of work even if the income elasticity is negative. Because in our example individuals not paying taxes will be unaffected by the increase in the tax rate we see that the aggregate effect of the tax change will thus depend on the distribution of individuals around the tax threshold (and on the shape of the labour supply function).

Another example which illustrates the importance of knowing the shape of budget constraints and individuals' distribution on it can also be shown using the above framework. Consider the labour supply responses when the exemption level (A) is changed. For taxpayers the effect of this is given by

$$\frac{\partial h^*}{\partial A} = t * h_2 \quad (4.6)$$

and assuming that leisure time is a normal good, then hours will decline. For those individuals who previously worked less than A/w , there will be no effect on their supplied hours. But, for those individuals who chose to work right up to the tax threshold, but do not pay taxes, hours of work are given by

$$h^* = \frac{A}{w} \quad (4.7)$$

and thus for small changes in A we got

⁵Note that the argument we are using in this simple example with only one kink point is also valid for more general and realistic cases with many kink points.

$$\frac{\partial h^*}{\partial A} = \frac{1}{w} \quad (4.8)$$

We can now express equation 4.6 in percentage terms

$$\frac{A}{h^*} \frac{\partial h^*}{\partial A} = \left[\frac{tA}{y + tA} \right]^\epsilon \quad (4.9)$$

where ϵ is the income elasticity of labour supply. Equation 4.8 becomes

$$\frac{A}{h^*} \frac{\partial h^*}{\partial A} = 1 \quad (4.10)$$

Because the income elasticity of labour supply is usually small in absolute magnitude, then if y is close to zero we see that the overall effect in equation 4.9 is likely to be small(in percentage terms), while the effect in equation 10 is large. Thus, a substantially small proportion of individuals located around the tax threshold(kink point) is needed for the positive effect(eq.10) to offset the negative effect in(eq.9) on aggregate. Whichever dominates, the overall effect of the change in tax allowance will depend on the distribution of individuals around the tax threshold.

4.2.3 Simulation procedure

When calculating optimal labour supply by a tax simulation, we need a behavioural model of labour supply (minimum requirement is the knowledge of the marginal wage rate and the exogenous income terms). Using our familiar notation this model can be written using the standard notation as in equation 4.1. Before going to actual simulations we need to estimate the above function econometrically. In this study we use the behavioural model (see appendix 1.) estimated in chapter 2. Estimating the labour supply function in the presence of non-linear taxation is a topic of its own and in this paper we are not going go

into that discussion. An excellent overview concerning the estimation of labour supply functions is provided by Blundell and McCurdy[2].

However, we should mention that it would be practical if the estimated labour supply function used in simulations satisfies some requirements, see also Stern[14]. First, the functional form should be parsimonious but at the same time also flexible. It would also be practical, if we could find an explicit algebraic form for the indirect utility function; this allows a straightforward comparisons of utility levels (if needed). Finally, preference parameters should be theory-consistent for most of the data points.

The general purpose of the simulation procedure is to solve a series of constrained optimisation problems. Each individual in the sample faces a budget constraint which is non-linear because of the income tax(benefit) system. It is possible that budget constraint includes a range of kinks and discontinuities, but in our case it will be a basic piecewise linear constraint for all individuals.

In this case the budget constraint can be separated into a series of linear segments, each of which can be described by the following form

$$I = [1 - t(h)wh] + y(h) \quad (4.11)$$

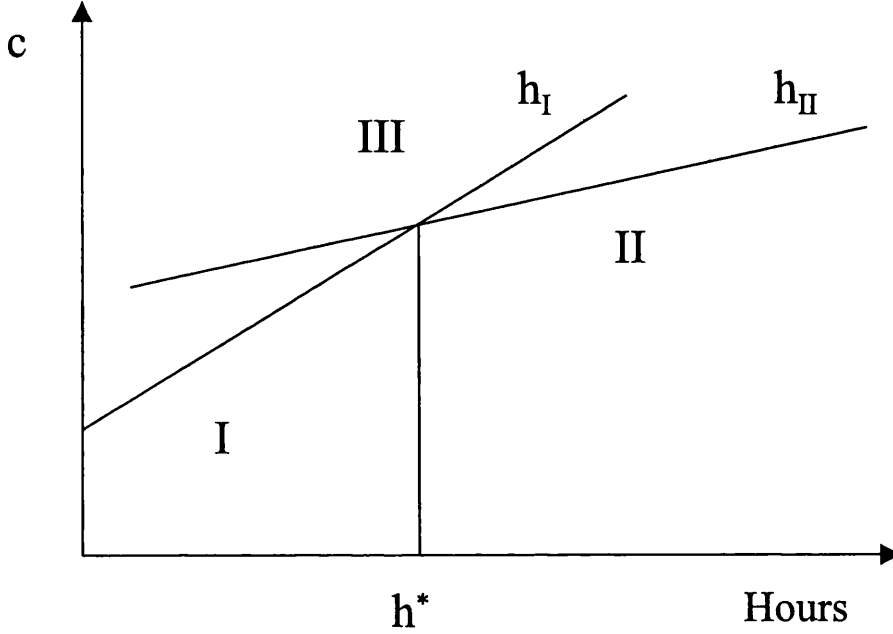
where I represents net income and $t(h)$ and $y(h)$ represent functions for the marginal tax rate and virtual income respectively. Now, when we know the gross wage rate, the values of functions $t(h)$ and $y(h)$ depend on working hours.⁶

In the actual simulation, the algorithm proceeds by identifying the locally optimal choice (hours) from the desired one for all linear segments of the budget constraint. In the case when the optimal solution is calculated to be within the range of hours over which the linear segment is defined, it is said to be feasible.

⁶Observing the change in marginal tax rates and virtual income across adjacent budget segments it is possible to distinguish between convex and non-convex kink points and discontinuities.

After all feasible local optimums have been calculated, the global optimum is established by calculating which of the feasible local optimums yield maximum utility. We can describe this procedure with the help of the figure 4.1 shown below.

Figure 4.1: A sketch of the simulation procedure



If the optimal solution h_1 for the lower budget segment, I , lies above the feasible range ($h_1 > h^*$) and the optimal solution h_2 for the upper budget segment, II , lies below the feasible range ($h_2 < h^*$), then the local optimum must be at the intersection of I and II . This is the case when feasible labour supply can be found at kink point, III .⁷

For non-participants in the sample, the budget constraint is modelled using a predicted gross wage rate from an estimated wage equation, see chapter 2. This

⁷If the indirect utility is only implicitly available then we can calculate the level of utility using the inverse demand function $w = w(h, y)$ to yield so called support wage. By duality, the substitution of the support wage into the indirect utility function at the optimal labour supply is sufficient to calculate the level of direct utility at the kink point. In more complicated situations (discontinuities, non-convexities) the algorithm compares all local optimums for the complete budget constraint and then returns the global optimum as the *maximum maximorum of utilities*.

allows us to generate a complete set of budget constraints under both the baseline tax system and reform system for all sample data points.⁸

4.3 Simulation results

Before going to the effects of income tax changes we will go through quickly the income tax system in year 1989, which will be our "baseline" solution and all the results shown below will be percentage changes from that baseline solution (except when stated otherwise).

The income tax system consists two parts: a progressive state income tax and a proportional local(municipal) income tax. In addition, individuals contribute to the National Pension Insurance(NP, 1.55 per cent from the taxable income) scheme and National Health Insurance(NH, 1.25 per cent from the taxable income) scheme, which are proportional to income changes. Roughly speaking, the tax liability in state tax and municipal(or local) tax is the same excluding the tax deduction system. A further distinctive feature in the Finnish tax system compared to some other European countries is that all individuals are separate tax units. Husband's marginal tax rate does not affect wife's marginal tax rate.

In 1989 the state income tax schedule was composed of six marginal tax rates varying from 11 to 44 per cent. The following table shows the tax schedule for the state tax in 1989.

⁸In the case of non-participants the gross wage that they would face if they work and the estimated stochastic error term of their preferences are both unknown. For participants these problems do not occur since the wage rate is observed and the stochastic term is taken to be the estimated residual.

Table 4.1. Income Tax Schedule.

State income tax schedule — 1989		
<i>taxable income</i>	<i>tax at lower bound</i>	<i>margin. tax rate</i>
36 000 – 51 000	50	11
51 000 – 63 000	1700	21
63 000 – 89 000	4220	26
89 000 – 140 000	10 980	32
140 000 – 250 000	27 300	37
250 000 –	68 000	44

In 1989 the municipality tax rate varied from 14 per cent to 19.5 percent. We have developed a formula to calculate state tax deduction for all persons in the sample, see chapter 2. Estimated tax deductions varied from 0 FIM to 29 500 FIM.

4.3.1 Some preliminary results

In this subsection we will study the "properties" of our simulation model. We use our estimated labour supply function to calculate the labour supply reactions and deadweight losses between different income tax systems. In all calculations we use the simulation results from 1989 progressive income schedule as our baseline and results are presented as percentage changes from that baseline. We want to stress that all reforms are made using the 1989 case as a baseline and absolute values are not very informative in today's respect but the direction of changes is. It also has to be kept in mind that we do not argue that our behavioural model underlying the simulation results are definitely valid in today's world(or in the 1989 world either), but it still provides us together with the simulation framework the best available tools to analyse income tax reforms in Finland.

Our analysis is of partial equilibrium type and thus only the supply side effects

can be derived. So, we are not able to answer any demand side questions like what will be the effect of lowering the social security and pension contributions paid by the employers. It also has to be remembered that our calculations are based on a representative sample of females. A fair amount of empirical research supports the view that female labour supply is more flexible than male labour supply and this also seems to be the case in Finland, see Kuismanen[11].

We start comparing the actual income tax system to the proportional tax system yielding the same tax revenue, to the lump-sum tax system with same the tax revenue and to the no-tax case. There are several ways to calculate deadweight losses but in this study we are not going to discuss them, it is a topic of its own. For a good presentation see for example Hausman[7]. We use the definition, $CV - T$, where CV stands for the compensated variation and T denotes the tax collected at individual's optimum position.⁹

In a standard case the deadweight loss increases as the marginal tax rate increases, so it is interesting to calculate the average deadweight loss for a sample of Finnish females between progressive income tax system and proportional income tax system. Results are shown in table 4.2 below.

⁹Compensating variation is the lump-sum income necessary to increase individual's utility to the level that would be obtained if there were no taxes. Another possibility to define deadweight loss is $CV - T_c$, where t_c is the tax that would be collected at the compensated optimum. Results did not differ significantly whichever method we used.

Table 4.2. Simulation results: Different income tax systems.

Baseline: Progressive income tax			
<i>Tax system</i>	<i>%- change in average hours of work(1)</i>	<i>Participation rate</i>	<i>Deadweight loss(2)</i>
Prog.tax(baseline)	-	0.72	14.8%
No tax	13.3%	0.76	-
Lump-sum tax	17.0%	0.77	0
Prop. tax (0.28)	2.5%	0.72	4.8%

Note: (1) *Percentage changes in average hours of work relative to baseline simulation (progressive tax system)*

(2) *Deadweight loss is calculated as percentage of tax revenue*

As can be seen from the table the progressive income tax decreases hours of work by approximately 13 per cent compared to the no tax case and the deadweight loss of the progressive income tax is 15 per cent of tax revenue. Naturally the NO TAX case does not create any deadweight loss. Our calculations show that the proportional tax rate to collect the same tax revenue as in the baseline case would be 28 per cent in our sample.¹⁰ If the proportional tax had been used then the deadweight loss would be approximately 5 per cent according to our calculations and labour supply would have been 2.5 per cent higher than in the actual progressive case. Calculations imply that that moving to proportional tax system does not increase labour force participation (only 4 cases) and the labour supply effects comes mainly from the upper end of the income distribution. Losers and winners can be easily identified. Winners are the high income earners and

¹⁰According to calculations of the Taxpayers Association of Finland, the average worker paid 27 per cent marginal tax rate in year 1996 for the extra income.

relatively the better is the situation after changing to proportional system the higher are the incomes. By symmetry, the biggest losers are the very low income earners.

In practise, a lump-sum tax is not that interesting for the real life purposes but it provides us a benchmark case of a tax system which creates zero deadweight loss. As known, lump-sum tax has only an income effect and if leisure time is a normal good, as according to our results it is, then labour supply should be higher than in no tax case, which is also confirmed by our results. It is interesting to note that labour force participation increases by 5 percentage points and the percentage change in average hours of work is 17 per cent. The individuals entering labour force are willing to work relatively few hours per year (all would like to work less than 350 hours) and thus only some 2.5 per cent of the increased hours is explained by these new entrants and the rest is explained by those already working but also they are willing to increase their hours of work.

Related to the Finnish tax debate it is interesting to calculate out how labour supply reacts when we change marginal tax rates by the same percentage points throughout the tax schedule. From the baseline case we decrease and increase the marginal tax rate by 2,4 and 6 percentage points in turn. For example, in the case of 2 percentage point reduction in the marginal tax rate, we have modified the tax schedule in the following way (table 4.3).

Table 4.3. Example for the reformed Tax Schedule.

Baseline and reformed tax schedule				
<i>taxable income</i>	<i>tax at lower bound-baseline</i>	<i>margin. tax rate-baseline</i>	<i>Tax at lower bound-reformed</i>	<i>Margin. tax rate-reformed</i>
36 000 – 51 000	50	11	50	9
51 000 – 63 000	1700	21	1400	19
63 000 – 89 000	4220	26	3680	24
89 000 – 140 000	10 980	32	9920	30
140 000 – 250 000	27 300	37	25 220	35
250 000 –	68 000	44	63 720	42

In the reform shown above we have only changed the state income tax schedule and all other components were left unchanged. The rest five reforms were made following exactly the same procedure. Results from these calculations are shown in table 4.4.

Table 4.4 The effects of a change in the progressive income tax rate

Relative to Baseline						
<i>Percentage</i>						
<i>change in</i>	-6	-4	-2	2	4	6
<i>Mean hour</i>	7.5%	6.1%	4.0%	-1.0%	-3.1%	-5.0%
<i>Mean tax revenue</i>	-31.3%	-19.0%	-8.9%	7.8%	15.4%	22.7%

Then first and most important result is that the increases in hours are not big enough to compensate for the reduction in tax revenues. Even this result is expected it is still important, because as we mentioned in section 4.2 some discussants have assumed that such a tax reduction would increase tax revenues in Finland. Secondly, it is also interesting to note that reactions are not symmetrical. Percentage changes in mean hours and mean tax revenue relative to the baseline are bigger when decreasing the marginal tax rates. The main reason for this is that tax reductions lead more people to enter the labour market than tax increases lead people to step out.

Comparing the above results to the case where a proportional income tax system is used is quite interesting. Below, we use our proportional income tax system with 28 per cent tax rate as a benchmark. We simulate labour supply reactions after changing the tax rate to 22, 24, 26, 30, 32 and 34 per cent respectively. Note that these changes are not that small, for example a reduction in the marginal tax rate from 28 to 26 per cent represents a 7.2 per cent reduction in the tax rate. Similarly, increasing the marginal tax rate from 28 to 34 per cent represents a 21.5 per cent increase in the tax rate. Results are shown in the table below.

Table 4.5. The effect of a change in the proportional income tax rate

Baseline tax rate(0.28)						
<i>Percentage</i>						
<i>change in</i>	22%	24%	26%	30%	32%	34%
Mean hour	4.1%	1.8%	0.5%	-2.8%	-5.2%	-8.3%
Mean tax revenue	-27.2%	-17.0%	-7.6%	7.4%	13.4%	18.2%

Although results are not directly comparable with the previous exercise, simulations indicate that labour supply and tax revenue reactions are smaller under the proportional tax system. This relates to our previous discussion about shape of the budget constraint and individuals distribution on it. In the case of non-linear budget sets small changes in marginal tax rates may lead to jumps from one budget segment to another. As in the basic calculations, labour force participation effects are very moderate.

4.3.2 Results from the tax reform proposals

One percentage point reduction in the state income tax schedule

The most frequently suggested reform is that marginal tax rates should be reduced by one percentage point throughout the tax schedule. The rationale behind this suggestion is its simplicity and its acceptability. It has also been suggested that this moderate reform will open the way for more "radical" reforms on the future.

Table 4.6. New tax schedule, -1%

Reformed tax schedule: (-1%)		
<i>taxable income</i>	<i>tax at lower bound</i>	<i>margin. tax rate</i>
36 000 – 51 000	50	10
51 000 – 63 000	1550	20
63 000 – 89 000	3950	25
89 000 – 140 000	10 450	31
140 000 – 250 000	26 260	36
250 000 –	65 860	43

When comparing the above tax schedule to the baseline case, we see that the difference between them is small, indeed. So are the differences between the results. Increase in the mean hours is only 1.8 per cent from the baseline and the reduction in mean tax revenue is approximately 5 per cent. Again, this reform has only a limited effect on labour force participation. All in all, if we only focus to labour supply effects, then reducing the marginal tax rates by one percentage point won't make any major difference compared to the baseline.

Reducing the marginal tax rate at the lower end of the tax schedule

It has been suggested in the Finnish discussion that the biggest effects to labour supply are achieved when marginal tax rates are cut from the bottom of the tax schedule. The intuition behind this is that individuals who do not earn that much are willing to increase their hours of work when their net wage increases. In other words, for them the substitution effect dominates more than for high income earners.¹¹ Also, if entering to the labour markets, then the new net wage

¹¹Obviously, this is an empirical question and a common belief is that income effect starts to dominate after some point of labour income.

might be higher than the reservation wage even if this was not the case before the reform.

Figure 4.2: Simplified example of the reform

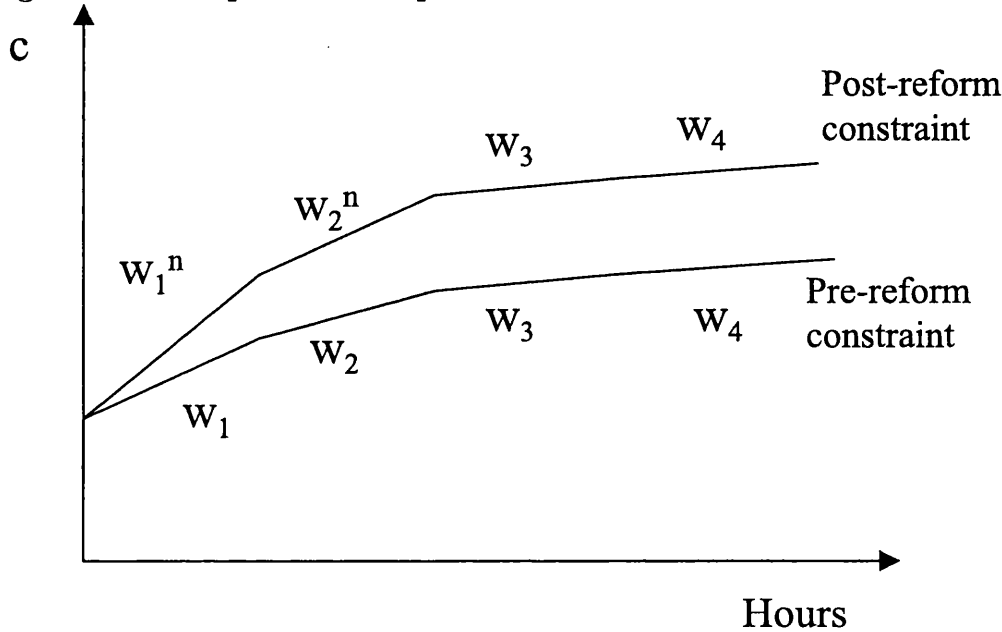


Figure 4.2 represents the simplified case of only four tax brackets, but our example can be generalised also to the more realistic situation except that graphs come easily quite messy. In figure two lowest marginal tax rates have been reduced and the new net wages are w_1^n and w_2^n instead of w_1 and w_2 respectively. Note that the two highest marginal tax rates w_3 and w_4 respectively are exactly same before and after the reform. But as can be seen from the figure, even we reduce the only the two lowest marginal tax rates, all individuals' net incomes will increase. In other words, reducing the marginal tax rates at the lower end of tax schedule does not only reduce the amount of taxes paid by low income earners as it has been many times wrongly stated in the public discussion. This can also be seen from the table 4.7 below which shows the actual implemented reform.

Table 4.7. Lower marginal tax rates reduced

Reformed tax schedule:		
<i>taxable income</i>	<i>tax at lower bound</i>	<i>margin. tax rate</i>
36 000 – 51 000	50	7
51 000 – 63 000	1100	17
63 000 – 89 000	3140	25
89 000 – 140 000	9640	32
140 000 – 250 000	25 960	37
250 000 –	66 660	44

In the above tax schedule we have reduced marginal tax rates in the three lowest segments. In the two lowest segments, the reduction is 4 percentage points and in the third segment it is one percentage points. From the column "tax at the lower bound" we see that compared to the baseline case these figures are lower throughout the tax schedule.

Results from this simulation are quite interesting. First, this reform seems to have a reasonably large effect on labour force participation, it increases by 4 percentage points. As above, individuals who would be willing to enter the labour markets are willing to work quite a few hours per year. Percentage change in mean hours relative to the baseline case is 8.8 per cent and the main response comes from individuals whose net incomes are in the lowest three deciles. It is also the case that their working hours are located at the lower end of the hours distribution.

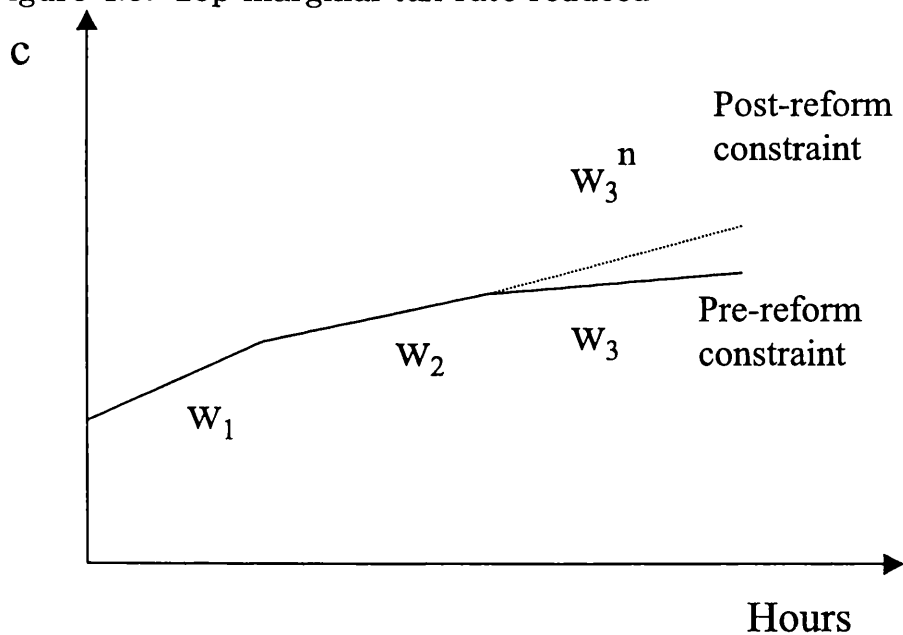
This reform had only a small impact on labour supply of the individuals whose incomes were above the median. Although this reform seems to have some attractive features the percentage loss in mean tax revenue is still approximately 12 per cent.

Lowering the top marginal tax rates

In our third reform we want to study what are the labour supply effects when top marginal tax rate is reduced. In the baseline case the top marginal tax rate is 44 per cent and the second highest marginal tax rate is 37 per cent if the taxable income is over 140 000 FIM. Now, we have changed tax schedule in a way that the highest marginal tax is going to be 35 per cent for all individual whose taxable income exceeds 140 000 FIM.

The idea behind this reform can be clarified by looking the the simplified tax system presented in the figure 4.3 below.

Figure 4.3: Top marginal tax rate reduced



This reform does not, of course, have any effect on those individuals whose labour supply is on the first segment. Those individuals whose optimal hours were on the second segment and especially close to the kink point may increase their hours of work. Marginal tax rates at the two lowest segments are not affected by the reform.

This reform is related to the discussion that highest marginal tax rates are at

a level that it prevents individuals to increase their hours of work and that the deadweight losses for the whole economy are simply too big. It has also been stated that exactly those high income earners are the driving force in our economy and we should create incentives for them to stay and work in Finland. This has become a more and more popular topic because of the current Information Technology boom which has created an unseen amount of new wealth to some individuals.

Table 4.8. Introducing top marginal tax rate–35%

Reformed state income tax schedule (top marg. rate 35%)		
<i>taxable income</i>	<i>tax at lower bound</i>	<i>margin. tax rate</i>
36 000 – 51 000	50	11
51 000 – 63 000	1700	21
63 000 – 89 000	4220	26
89 000 – 140 000	10 980	32
140 000 – 250 000	27 300	35

In table 4.8 the reformed tax schedule consists of five tax brackets instead of six and the highest marginal tax rate is 9 percentage point lower than in the baseline case.

Our results from this simulation are the following ones. First, cutting the top marginal tax rates does not have any effect on labour force participation. Secondly, the percentage change in mean hours is 4.5 per cent and the biggest labour supply changes are found from the three highest income deciles. This reform improves high income earners' position relative to rest of the population, because their after reform net incomes increase. An interesting aspect is that the loss in tax revenue is relatively large, approximately 13 per cent relative to the baseline. This is due to the fact that tax revenue losses are big in the three highest income deciles.

4.4 Discussion and conclusions

Before summarising and discussing our results it is worthwhile to comment some limitations in our analysis. First, it is a partial equilibrium analysis and only supply side effects can be taken into account. We have not been able to answer questions like what happens to labour demand if employers social security and pension contributions were reduced. Luckily, there is some Finnish evidence concerning this question. Holm, Honkapohja and Koskela[9] and Honkapohja, Koskela and Uusitalo[10] have calculated that reducing the above mentioned contributions will increase labour demand and this effect is strongest when it is done in low salary occupations like in the service sector. This result in a way matches with our analysis and thus it is likely that the biggest effects will be achieved if marginal tax rates and employers social security contributions of low income earners are reduced simultaneously.

Secondly, labour supply is a dynamic phenomenon but our analysis is based on the assumption of no intertemporal effects. We would need to estimate a dynamic labour supply function to get an estimate for the intertemporal elasticity of substitution before we could do dynamic simulations. This is a topic for further research. In empirical work it has been found that estimates of the intertemporal elasticities of substitution are usually quite small, see e.g. Blundell and Walker[4]. This does not mean that dynamic effects are necessary negligible because variations in preferences and changes in life situations may be important since reservation wages for women are sensitive to the demographic changes. So, it might be the case that intertemporal labour supply may not be as sensitive as labour supply in one period, but obviously this is an empirical question.

Thirdly, in our analysis tax unit is an individual and not a household. Unlike many other countries, we have an independent income taxation and so this choice is legitimate. Of course, even in independent tax system household behaviour matters. Unfortunately, our data does not allow to study this matter and we had to leave it outside our analysis. At this point, we also have to comment

the most frequently stated criticism against this kind of research, namely that most of the people work standard hours and that is decided collectively between unions and employers, at least in a country like Finland. One has to keep in mind that we have estimated the desired labour supply and we can only answer questions concerning what would be individual's desired reaction in different kind of reforms, but this is as far as we can proceed using econometrics.

Despite the limitations of our analysis we think that this work has something to give to the Finnish tax debate. At least, it is the first microsimulation study which takes behavioural responses into account and thus serves as a basis for hopefully forthcoming similar studies. One purpose of writing this paper was to indicate why microsimulation studies are needed and what are the advantages compared to the "representative individual or household" case and cash effect studies. This is done in sections 4.2 and 4.3.

Our main findings are the following ones. If we would move from progressive income taxation to proportional taxation in a revenue neutral way then the marginal tax rate would be 28 per cent (for to our sample) and the labour supply effects of this change are reasonably small. In efficiency terms the current tax system could be improved upon because its estimated deadweight loss is approximately 15 per cent of tax revenue, whereas in the case of proportional tax it is approximately 5 per cent. The biggest effects in terms of labour supply are achieved by reforms which cut marginal taxes at the low end of the income distribution.

Our results indicate that none of the reforms we did are self-financing. Increases in labour supply are too moderate to offset reduced tax revenues. Although this result seems a very simple one, it is still quite important due to the following fact. In general discussion a variety of reforms have been suggested and some have claimed that these reforms are definitely self-financing ones. It has even been suggested that some tax cuts could increase tax revenues, thus we would be on decreasing side of the Laffer-curve. This is not the case according to our results. Our conclusion is that if we would like to lower income taxes then we

would also have to think how we can reduce public expenditures.

Finally, many other types of reforms also could have been possible to analyse, but we assume that the ones we did represent a good portfolio of reforms. Many other possible scenarios will be placed between the ones we analysed and then the outcomes are very likely to be somewhere between our results. Needless to say, a further work is needed to get a better picture of the labour supply responses in Finland.

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Appendix 1: Labour Supply Equation, Wage Equation and Variable Definitions

Labour Supply Function

Labour Supply Functions		
<i>Variables</i>	<i>Coefficients</i>	<i>Asymp. Standard Errors</i>
Constant	-2.57905	(0.55650)
Ln W	0.37046	(0.12135)
Exog. inc	-0.00045	(0.00022)
Cdum1	-0.33917	(0.09948)
Cdum2	-0.00487	(0.10437)
Cdum3	0.09616	(0.10050)
Cdum4	0.14310	(0.07690)
Age	0.16118	(0.02484)
Age*Age	-0.00227	(0.00028)
Sosio	0.19945	(0.09521)
Nkids	-0.08419	(0.03235)
σ_e^2	0.98208	(0.01907)
Ln L	-2669.61	

Note: In both models above, dependent variable(yearly hours) is divided by 1000. The exogenous income variable contains only own exogenous income components(net) and it is divided by 100.

Wage Equation

Wage Equation. Dependent variable: ln hwage.		
<i>Variables</i>	<i>Coefficient</i>	<i>Standard Error</i>
Constant	2.83342	0.2511
Age	0.01753	0.0135
Age2	-0.00017	0.0001
Educ10	0.06881	0.0024
Educ12	0.19534	0.0297
Educ14	0.27270	0.0469
Educ15	0.51690	0.0469
Exp	0.01659	0.0053
Exp2	-0.00027	0.0001
Tenure	0.02410	0.0038
Tenure2	-0.00045	0.0001
Pjob	0.04720	0.0299
Husb	0.00760	0.0290
Stat	0.10338	0.0241
Socio	0.23919	0.0366
Nchild	-0.03065	0.0104
South	0.15898	0.0222
Exo+hnet	3.95e-07	1.67e-07
Occ. dummies	Yes	
Ln L	-1221.91	

NOTE: The selection index is a function of the individual, geographical and demand side variables. The selectivity effect was statistically significant. Reference group for occupation is manufacturing workers.

Definitions of the variables

union=1, if the respondent is a member of an union

age=Age of the respondent

age2= Age squared

educ10=1, if the respondent has 10 years of education. Otherwise zero.

educ12=1, if the respondent has 11-12 years of education. Otherwise zero.

educ14=1, if the respondent has 13-14 years of education. Otherwise zero.

educ15=1, if the respondent has 15+ years of education. Otherwise zero.

ueduc=1, if the respondent has university degree from the following fields:

Technology,business,law,natural science and social sciences

nchild=Number of dependent children.

cdum1,...,cdum4= Dummy variables for the youngest child. Age groups are 0-3,4-6,7-9 and 10+.

schild=Number of children aged 0-3.

cchild=Number of children aged 4-6.

bchild=Number of children aged 7-9.

exp= Working experience

exp2= Exp. squared

tenure= Duration of the current job

tenure2= Square of tenure

pjob=1, if respondent has a permanent job

phusb=1, if respondent's husband is working

stat=1, if the respondent is a white-collar worker and 0 if a blue-collar worker.

socio=1, if the respondent is a upper white-collar worker. Otherwise zero

hwage= Hourly wage rate.

shwage= Subjective Hourly wage rate.

exo= Unearned income.

exo+hnet= Unearned income+husband's net incomes.

south=South Finland.

west=West Finland.

Appendix 2: Share of Taxes on Income, Wealth etc. of the GDP

Share of Taxes, Wealth etc. of the GDP				
<i>Country</i>	1998	1997	1996	1995
EU-15	13.7	13.2	13	12.6
Belgium	17.7	17.1	16.7	16.7
Denmark	29.4	30.1	30.6	30.4
Germany	11.5	11.2	11.5	11.2
Greece	8.5	7.9	7.1	7.4
Spain	10.3	10.5	10.3	10.1
France	11.5	9.5	8.9	8.5
Ireland	13.8	14.3	14.2	13.7
Italy	14.4	16.1	15.3	14.7
Netherlands	12.2	12.4	12.9	12.4
Austria	13.8	13.5	13.1	12
Portugal	10.4	10.2	10	9.3
Finland	18.9	18.5	18.9	17.5
United Kingdom	16.5	15.1	14.9	15

Source: Eurostat