Title: *Challenging inequity in mathematics education by making pedagogy more visible to learners*

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Abstract:

This paper reports on initial findings from the Visible Maths Pedagogy research project, a collaboration between an academic researcher and two teacher researchers (the paper's authors). The aim of the project was to explore the effects of making pedagogy more visible on students' success in school mathematics. We adopted a Participatory Action Research methodology to plan and evaluate five strategies used alongside 'progressive' teaching approaches to make the teacher's pedagogical rationale more visible to learners. Our findings show that students, particularly those from disadvantaged backgrounds, were initially prone to misinterpret the intentions of the teacher. However, the five strategies helped students gain a greater appreciation of the teacher's pedagogical rationale and how to respond appropriately. We discuss the implications of these findings for enabling all students to access the benefits of progressive teaching approaches and for opening up to scrutiny what it means to be a successful learner of mathematics.

Keywords:

Word count:

8565

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1. Introduction

This paper reports on the findings from the first year of the Visible Maths Pedagogy (VMP) research project, a collaboration between the three authors, which aimed to develop strategies that enable students to gain a greater appreciation of their teacher's pedagogical rationale. We also consider how these findings generate further insight into, and offer new contributions to, the debate about how best to challenge inequity in mathematics education.

In Section 2, we outline the rationale for the project, explaining how the idea for the project originated, and providing some background information about the school in which the project was situated. We present a theoretical framework in Section 3, which underpins the wider argument put forward in this paper, drawing on Bourdieu's theory of reproduction and notion of 'reflexive sociology' to contend that persistent inequities in mathematics education can be challenged by opening up what it means to be successful in mathematics to greater scrutiny. In Section 4, we detail how a Participatory Action Research methodology was applied to ensure the development and trialling of strategies, during two 'plan-teach-evaluate' cycles, was conducted in a collaborative, systematic and rigorous way. In Section 5, we describe the strategies we designed for making pedagogy more visible. In Section 6, we describe how coding schemes were used to carry out a thematic analysis on data collected from student surveys and interviews. We report on, and summarise, the findings from this analysis in Section 7, using the students' own words as evidence to support our results. In Section 8 we relate the findings back to the theoretical framework and highlight the significant contribution they make to the debate around empowering disadvantaged students and challenging inequities in mathematics education. We draw conclusions in Section 9 that we hope will inform future research in this field.

2. Background and rationale

The authors of this paper share a common concern about issues of equity and social justice in education. This has had a significant influence on the direction in which our classroom practice has developed and explains our interest in the wider context in which our research is situated. We share a commitment towards mathematics pedagogy that engages all learners and enables every student, regardless of her/his background, to become a successful learner of mathematics.

Unfortunately, these aspirations are not reflected by the current schooling system, in which a child's background to a large extent determines the level of mathematical success they experience. In the UK, and in many countries around the world, there is a strong and persistent association between a child's socio-economic background and their achievement and participation in school mathematics (Boaler, Altendorf, & Kent, 2011). Whilst differences in achievement between other groups, e.g. boys and girls, have narrowed in recent years, students' socio-economic backgrounds remain the most decisive factor in

determining their mathematical success (Jorgensen, 2016). Since school mathematics acts as a critical filter in regulating access to higher-paid jobs, these differences in achievement restrict the social mobility of children from disadvantaged backgrounds and serve to reproduce patterns of inequality in society from one generation to the next (Jorgensen, 2016).

There is no shortage of literature highlighting the damaging effects of transmission-based teaching (Foster, 2013; Grootenboer, 2013; Nardi & Steward, 2003) and setting, i.e. grouping students in classes according to their prior levels of attainment (Francis et al., 2017; Wilkinson & Penney, 2014), in mathematics classrooms. However, research findings that offer ideas for practitioners on how they can address these issues in their classrooms are limited (Wright, 2016, 2017). There is some compelling evidence that 'progressive' mathematics pedagogy can promote equitable outcomes, whilst enhancing the engagement, enjoyment and conceptual understanding of learners (Boaler, 1998; Foster, 2018; Skemp, 1972; Smith, Lee, & Newmann, 2001; Swan, 2006). Here we adopt Ernest's (1991) understanding of the word 'progressive' to refer to a range of teaching approaches focused on employing practical activities and personal exploration to help nurture the development of the individual learner. These approaches include using collaborative problem-solving approaches in mixed-attainment classes (Boaler, 2008), and using open-ended investigations, in which students make their own decisions and pose their own questions, to promote mathematical agency (Skovsmose, 2011). Hudson (2018) describes how a progressive and emancipatory approach to mathematics teaching that *"promotes critical thinking, creative reasoning, the generation of multiple solutions and of learning from errors and mistakes"* (p.388) leads to higher levels of engagement, enjoyment and fulfilment amongst learners.

However, some research findings from the US suggest progressive teaching approaches might not always lead to equitable outcomes. Lubienski (2004) describes how students from less wealthy backgrounds were further disadvantaged by the less-structured nature of progressive pedagogy, adopted as part of a 'reform'-oriented curriculum, as they were prone to misinterpret the intentions of the teacher. Similarly, Bernstein (2000) contends that students from working-class backgrounds, because of their upbringing, are generally less able to decipher the 'rules of the game' which determine success in the mathematics classroom. These include the 'recognition rules', i.e. identifying relevant meaning from the tasks set by the teacher, and the 'realisation rules', i.e. coming up with an appropriate or legitimate response.

These findings pose something of a dilemma for equity-minded teachers, including the authors, who are sympathetic towards progressive teaching approaches. Should we abandon such progressive pedagogies altogether and adopt more structured and teacher-led approaches in the mathematics classroom for the sake of achieving more equitable outcomes? Some have taken this route in advocating 'direct instruction' or 'explicit instruction' (Ewing, 2011). However, 'transmissionist' pedagogies, based on teacher-centred approaches, have been shown to accelerate students' disengagement from learning mathematics over the course of their schooling and to dissuade students from studying mathematics beyond compulsory level (Williams & Choudhury, 2016). Whilst direct and

explicit instruction claim to offer more than just rote learning based on the transmission of knowledge, Ewing (2011) argues that they present similar problems in terms of alienating, disengaging and disempowering learners, particularly those from marginalised communities.

The alternative and, from our perspective, more desirable option is to embrace progressive teaching approaches whilst seeking ways of helping all students, particularly those from disadvantaged backgrounds, to gain an appreciation of the rationale behind them. This explains our focus for the Visible Maths Pedagogy (VMP) research project on the following research question:

> *To what extent can strategies designed to make pedagogy more visible to learners help students to recognise the intentions of the teacher, how to respond appropriately to progressive teaching approaches and how to realise success in school mathematics?*

The VMP project was a collaboration between the three authors (Pete, Tiago and Alba) carried out between 2017 and 2019. Pete taught mathematics for 15 years, mainly in comprehensive schools in relatively deprived areas in England, before becoming a teacher educator and researcher at UCL Institute of Education in 2011. His role as a tutor on an initial teacher education (ITE) programme included regular observations of student teachers' lessons, followed by one-to-one reflective discussions. He had become increasingly aware of how clearly many student teachers articulated the rationale behind the pedagogical choices they made during the lesson (observed during reflective discussions). However, this rationale was often hidden from students, leading them to misinterpret the teacher's intentions and respond inappropriately (observed during lessons). This heightened Pete's interest in exploring the impact on mathematics learners of making pedagogy more visible.

Tiago had completed the ITE programme that Pete worked on in 2016 and was in his second year of teaching. He contacted Pete in 2017 expressing an interest in engaging with research related to teaching mathematics and social justice, and Pete proposed they collaborate in developing a research project. Alba, who had been involved as a mentor on the same ITE programme and was in her fifth year of teaching, also expressed an interest in becoming involved in the study. The head teacher and the head of the mathematics department at Stoke Newington School, at which both Tiago and Alba were teaching, expressed strong support for the VMP project, which began in November 2017.

The school had a relatively high proportion of students classified as 'pupil premium' (approximately 30% in 2016-17, compared with a national average of 27%), which is an indicator of socio-economic disadvantage. The school's mathematics department, reflecting the broader educational philosophy of the school, favoured teaching approaches that could be described as 'progressive' and these were already in widespread use. They had recently incorporated a series of problem-solving activities into the scheme of work and were developing 'oracy' by encouraging students to express their reasoning through 'think-pair-share' (i.e. thinking about a problem individually, sharing ideas with a partner, then sharing with the whole class). The department had also resolved to move away from a rigid setting

structure and had recently introduced mixed attainment classes in Year 7 (the first year of secondary education for 11 and 12-year-olds) with a view to extending this in future years.

3. Theoretical framework

In order to explain the inequity that persists in mathematics education, and the relevance of visible pedagogy in challenging this situation, we present a theoretical framework that builds on the work of Bourdieu. In his theory of 'reproduction', Bourdieu argues that the primary function of schooling is to maintain inequitable power relations that exist between different groups in society (Bourdieu & Passeron, 1990). It does so implicitly by disguising the processes, such as setting that is commonplace in mathematics classrooms in England, which contribute towards reproducing the social order across generations (Francis et al., 2017). School is falsely presented as a meritocracy with success attributed to innate ability rather than the systemic advantage afforded to those in privileged positions. Bourdieu describes the social and cultural resources that are legitimised and more highly valued by the school as 'symbolic capital' (Black & Hernandez-Martinez, 2016). Bourdieu argues that middle-class parents are more likely to possess the social capital necessary to support their children in developing a mathematical 'habitus' that more closely aligns with the values of the school, e.g. by playing mathematical games involving counting. In this way, students from middle-class backgrounds tend to accumulate higher levels of 'mathematical capital', making them more likely to identify as learners of mathematics, and placing them in a stronger position to take advantage of opportunities presented within the mathematics classroom (Williams & Choudhury, 2016). This explains the strong and persistent association between socio-economic background and mathematical attainment highlighted earlier. Black and Hernandez-Martinez (2016) argue that mathematical capital is also closely linked to 'science capital', a form of symbolic capital that determines students' achievement in school science, since success in mathematics qualifications acts as a gatekeeper to scientific courses and careers.

This is a somewhat deterministic theory which suggests that being a mathematics teacher merely serves to endorse a system that is inherently exploitative. So, how is it possible to empower disadvantaged students whilst working within the constraints of the current schooling system? Some would argue that teachers should focus on helping disadvantaged students to realign their habitus with the middle-class values that are recognised by the school (Jorgensen, 2016). This might be achieved by demonstrating how mathematics can be made more meaningful, e.g. by solving problems that have greater relevance to students' real-life experiences, thus enhancing their levels of motivation and mathematical engagement (Boaler, 2008; Wright, 2016, 2017). However, Bourdieu contends that mathematical habitus exists for the sole purpose of enabling some students, primarily those from dominant groups in society, to succeed at the expense of others. Enabling some students from disadvantaged backgrounds to succeed merely helps them to join those from dominant groups in exploiting others, whilst providing further creditability to the myth that school is a genuinely meritocratic system (Bourdieu & Passeron, 1990).

In 'going beyond Bourdieu', Williams and Choudhury (2016) draw attention to the 'use value' of school mathematics, as well as its 'exchange value' (or 'mathematical capital' in

Bourdieu's terms). They argue that mathematics has more to offer learners, through its application to modelling and solving real-life problems, than merely providing a means to dominate others. This model helps explain conflicting pressures on equity-minded mathematics teachers who strive to enhance students' engagement, enjoyment and mathematical agency through adopting progressive teaching approaches, whilst being acutely aware of how the grades attained by their students serve to either restrict or broaden future life opportunities. For these reasons they might espouse entirely different pedagogical rationales across different teaching phases, adding to the confusion of disadvantaged students already prone to misinterpreting the intentions of the teacher.

Williams and Choudhury (2016) draw on Bourdieu's notion of 'reflexive sociology' to argue that, instead of enhancing the cultural capital of disadvantaged students so that they can compete with others, the focus should be on exposing and challenging common assumptions and myths that serve to disguise the arbitrary nature of mathematical capital. These include the claims that mathematical success can be attributed to innate ability or 'giftedness' and that the mathematics classroom offers a level playing field where all students can compete on equal terms. They argue that, by highlighting examples that contradict the dominant discourses in mathematics education, teachers can expose, and ultimately undermine, the arbitrary nature of mathematical capital and exploitative aspects of school mathematics.

We argue that equity-minded mathematics teachers can begin this process by making their pedagogical rationale more visible to learners. In formulating the VMP research project we anticipated that, through developing strategies which help students to recognise the intentions of the teacher and how to respond appropriately, we would open up what it means to be a successful learner of mathematics to greater scrutiny.

4. Methodology, research design and data collection

The VMP project's research design was based on a Participatory Action Research (PAR) methodology, which entails a genuine collaboration between academics (in this case Pete) and practitioners (Tiago and Alba). PAR recognises that academics, with their research expertise, and teachers, with their detailed knowledge of the classroom, both play essential, but distinct, roles in the research process (Skovsmose & Borba, 2004). By involving teachers directly in the design and management of the research, PAR aims to develop a deeper understanding of 'theory-in-practice' and to generate knowledge of greater relevance to practitioners (Brydon-Miller & Maguire, 2009). Unlike much conventional mathematics education research, PAR is carried out in typical classroom situations, taking account of day-to-day challenges and constraints faced by teachers (Skovsmose, 2011). It offers a systematic approach based on clearly defined research processes, entailing reflecting critically on existing practice, which should not be taken as given, and relating this closely to theory. It involves articulating an alternative vision, trying out new approaches whilst taking account of constraints and opportunities, and evaluating these to help assess the feasibility of the alternative vision (Andersson & Valero, 2016).

During the first year of the VMP project we held nine research group meetings at the school. The initial meetings involved jointly agreeing the research design and structure of future meetings, identifying the students with whom we would try out the strategies, devising research tools for collecting data and evaluating their success. We agreed to maintain research journals to capture our experiences and inform discussions at future meetings. We took it in turns to present research literature identified by Pete as being relevant to the aims of the project (Boaler, 2008; Lubienski, 2004; Wright, 2016), relating this to existing practice and our own teaching experience. Subsequent research group meetings were used to collaboratively plan the strategies and evaluate their success after trying them out in the classroom.

We decided to work with two similar Year 7 mixed-attainment classes taught by Alba and Tiago and to carry out two 'plan-teach-evaluate' action research cycles with these classes. The focus of our planning was on developing strategies for making progressive pedagogies, already widely employed by the mathematics department, more visible. We identified one 'research lesson' for each cycle, in which the strategies would be trialled. In the first cycle, we decided to carry out a survey of all students at the end of the research lesson to evaluate the impact of these strategies. For the second cycle, we chose to conduct semi-structured interviews with six students (three from each class) within a day or two after the research lesson. The structure of the action research cycles is summarised in Table 1 (below).

Timescale	Action
1st/29th November 2017	Two initial research group meetings (agreed research design and overall structure; presented and discussed research literature)
January 2018	**Beginning of action research cycle 1**
16th January 2018	Research group meeting 1 (planned first research lessons)
31st January 2018	First research lessons (both video-recorded; based on Fraction Flags activity; 'advocating' and 'separating' strategies trialled)
31st January 2018	Student survey (administered immediately after research lessons)
7th February 2018	Research group meetings 2 and 3 (evaluated 'advocating' and 'separating' strategies; reviewed surveys and videos)
20th March 2018	Research group meeting 4 (overall evaluation of cycle 1)
May 2018	**Beginning of action research cycle 2**
9th May 2018	Research group meeting 5 (planned second research lessons)
16th May 2018	Second research lessons (both video-recorded; based on Ratio Flags activity; 'scribing', 'annotating' and 'classifying' strategies trialled)
17th/18th May 2018	Interviews conducted with 3 students in each class
13th June 2018	Research group meeting 6 (evaluated 'scribing', 'annotating' and 'classifying' strategies; reviewed interviews and videos)
2nd July 2018	Research group meeting 7 (overall evaluation of cycle 2)

Table 1: Structure of action research cycles

Given our interest in empowering disadvantaged students and challenging inequity, we selected disadvantaged students from those identified as 'pupil premium'. We agreed the questions to include in the surveys and interviews based on our interest in exploring

students' appreciation of, and willingness to engage with, the teacher's pedagogical rationale and what it means to be a successful mathematics learner. The survey and interview questions are included for reference in Appendix 1. We devised protocols for administering the surveys which included explaining beforehand the purpose of the research, how the data would be used, and that the survey was anonymous and optional. We decided that the interviews should be led by Tiago and Alba so that students would feel secure and answer questions candidly. This is in line with an 'empathetic' approach in which a relationship of trust between interviewer and interviewee is considered important to enable more meaningful representations of interviewees' views to emerge (Fontana & Frey, 2008). All six students and their parents were provided with an information leaflet about the research project before seeking their informed consent. Ethical approval for the research project was obtained from the UCL Institute of Education Research Ethics Committee.

We arranged to video record the four research lessons to provide evidence to use alongside the survey and interview responses in our evaluation of the strategies. The video recordings were not treated as data but were used as discussion prompts to facilitate critical reflection on practice (Geiger, Muir, & Lamb, 2016). The semi-structured interviews were audio recorded and the transcripts were used, along with the survey responses, as data for this study. The research group meetings were also audio-recorded and, at the time of writing, were still in the process of being analysed. This will enable a more detailed discussion of the research methodology to be reported in due course, including how it might contribute towards developing more participatory, collaborative and critical approaches to research in mathematics education. The focus of this paper is on what we can learn about students' appreciation of their teachers' pedagogical rationale from an analysis of the data from the surveys and interviews.

5. Strategies for making pedagogy more visible

In this section we describe the five strategies that we trialled during the first year of the research project which were designed to make the intentions of the teacher an explicit focus for discussion and consideration by learners. They were devised through discussions during research group meetings as follows. Alba and Tiago would present their initial plan for the research lesson (which was informed by weekly planning meetings with colleagues in the mathematics department) and highlight the progressive teaching approaches to be used. All three of us would then reflect critically on the rationale behind these teaching approaches, clarify their primary purposes, before considering how to make these more visible to students. The strategies included ways of adapting the original teaching approaches to make their purposes more explicit and planning additional discussions focusing on these purposes. In devising the strategies, we drew on the research literature presented during research group meetings (see Section 4) and other research findings relating to progressive pedagogy. For example, the 'advocating' strategy described below was informed by Boaler's (2008) ideas on allocating roles to facilitate group work and the 'classifying' strategy was inspired by Swan's (2006) typology of activities that encourage mathematical reasoning and discussion.

During the first research lesson (referred to as 'Fractions Flags') students were provided with a series of problems, involving calculating areas of different sections of various flags, which could be solved using different methods. Students were encouraged to work collaboratively using a 'think-pair-share' approach, i.e. thinking about the problems initially on their own, then discussing them with a partner, before sharing their ideas with the rest of the class. This progressive teaching approach was an example of those used routinely by Alba and Tiago prior to becoming involved in the project.

The 'advocating' strategy involved asking students to feed back their partner's ideas, rather than their own ideas, during the 'share' phase. It also included a follow-up whole-class discussion in which the teacher asked: *"Why do you think I asked you to present your partner's ideas rather than your own?"* The primary purpose of this strategy was to encourage students to appreciate the range of methods that could be used to solve such problems by sharing and engaging with each other's ideas.

As students presented ideas, the 'separating' strategy involved the teacher recording summary notes in one of two unlabelled columns in a table already drawn on the board. Comments related to reasoning, i.e. making some attempt to explain why particular procedures were employed, were recorded in one column and those involving working-out, i.e. simply recounting the calculations performed in applying these procedures, in the other. The rationale for separating the comments was not revealed and, when the table was complete, the teacher prompted a discussion around two questions: *"What is the difference between the statements in the two columns of the table?"* and *"Why do you think I separated reasoning and working-out in the table?"* The primary purpose of this strategy was to help students to appreciate the distinction between the two so that they could respond appropriately when asked to give reasons for their answers (a common question in school mathematics tests and examinations).

The remaining three strategies, referred to as 'scribing', 'annotating' and 'classifying', were employed during the second research lesson ('Ratio Flags'). This lesson was similar to Fraction Flags, but this time the problems involved finding the ratios between areas of different sections of the flags.

The 'scribing' strategy involved the teacher writing down on the board exactly what students said during whole class discussions (whether correct or incorrect). It included a follow-up discussion in which the teacher asked students what they thought the reasons were for doing this. Its primary purposes were to enable students to recognise errors and misconceptions for themselves and identify ambiguities in the language used by students.

The 'annotating' strategy involved the teacher reviewing solutions recorded on the board through discussion with students and adding notes on how they might be improved. Students were then asked to annotate their own solutions to the same problem in their books. Its primary purpose was to draw further attention to errors and misconceptions and consider ways of improving the way the solution was written, e.g. using more precise language and providing reasons as well as working-out.

The 'classifying' strategy involved students sorting a series of ratio problems into three types they had encountered during the lesson. The teacher discussed the rationale for using this strategy with students in advance, i.e. highlighting the importance of being able to identify and classify different types of problems. After students had sorted the problems, there was a whole-class discussion focusing on the differences between them prompted by examples of solutions displayed on the board. The primary purpose was to help students recognise key features of unfamiliar problems so they could select an appropriate method to apply.

6. Data analysis

This paper focuses on the analysis of data collected from the responses to the student surveys and audio-recordings of interviews conducted with the six targeted students during the first year of the research project. The recordings were transcribed with names of students, and others referred to, replaced by pseudonyms. A thematic analysis was carried out using a combination of deductive and inductive coding (Fereday & Muir-Cochrane, 2006), with separate coding schemes used to analyse the surveys and interviews. Since the project involves applying theory to practice, the initial coding schemes were derived from the research literature. These schemes were modified through familiarisation, i.e. reading and re-reading the surveys and transcripts to ensure that the codes were appropriate to the data. Further changes were made during the coding process itself as it became apparent that some codes were redundant or duplicated, and that other codes were required to take account of unanticipated responses. The final coding themes, with more detailed descriptions of the codes, are included for reference in Appendix 2.

The student surveys were treated differently to the interviews as they were more structured in nature, with each student being invited to make a written response to all five questions. 45 students completed the surveys (23 in Alba's class and 22 in Tiago's class). This included almost all students present with the exception of two students in Alba's class (one took up the option of leaving the survey blank and one refused to engage with the survey) and one student in Tiago's class (who only arrived towards the end of the lesson). The coding was informed by the research literature but was largely derived inductively by reading and re-reading the data, categorising the responses according to initial codes, then extending and modifying the codes until all responses accorded with the coding scheme. Some responses were assigned two or more different codes. For the third question, "*How do you know [how well you've done in today's maths lesson]?",* it seemed appropriate from the responses to group the codes into four types relating to: students' dispositions towards learning; students' judgements about their work output; students' judgements about their level of understanding; and students' perceptions about how others saw them. The thematic analysis was carried out by comparing and contrasting responses assigned the same code and considering the number of responses assigned to each code.

The final coding scheme for the interviews comprised six broad groups:

1) References to teaching approaches considered to be 'progressive' (including 'problem-solving', 'collaborative work' and using 'misconceptions' as learning opportunities).

These codes were generated from the literature advocating progressive teaching approaches (e.g. Boaler, 2008; Swan, 2006).

2) References to strategies developed to make pedagogy more visible to learners (including 'scribing', 'annotating' and 'classifying' described above). These codes draw on the literature related to visible pedagogy (Bernstein, 2000; Lubienski, 2004).

3) Descriptions by students of their experiences of learning mathematics (including experiencing 'satisfaction', 'challenge' and 'familiarity' with the lesson content).

4) Reference to students' dispositions towards learning mathematics (including exhibiting 'enjoyment', 'empathy with others' and 'shared responsibility' for others' learning). Codes in groups 3/4 were derived from the literature around students' relationships with mathematics (Black, Mendick, & Solomon, 2009; Boaler, 2008).

5) Students' sense of their teacher's pedagogic rationale that relates to recognition rules (the extent to which they identify relevant meaning from the activities set by the teacher including articulating a 'primary', i.e. a reason articulated by the teachers, other 'valid' or 'invalid' purpose).

6) Students' articulation of the ways they experience success that relates to realisation rules (including why they consider themselves to be 'successful' or 'unsuccessful'). Codes in groups 5/6 were derived from Bernstein (2000).

NVivo software was used to code interview transcripts as this allows multiple coding of text and facilitates the comparison of text assigned similar or related codes. A thematic analysis was carried out by selecting all extracts from the six interviews assigned the same code, then reading and re-reading this text to draw out initial themes. We considered other codes assigned to these extracts and looked for patterns in the coding, allowing us to explore 'commonalities', 'differences' and 'relationships' between codes and to enable further themes to be identified from the data (Gibson & Brown, 2009). In the analysis of the interviews, we chose not to compare the number of references for each code as this can be reductive, misleading and lead to an impoverishment of the data (Gibson & Brown, 2009). Instead we focused on reading and re-reading extracts from the text to avoid the original meaning being lost.

7. Findings

In this section we report the findings from the thematic analyses of the surveys (from the first cycle) and the interviews (conducted at the end of the second cycle). The themes that emerged are then related back to the research question and theoretical framework in Section 8 to generate further meaning (Kvale & Brinkmann, 2009).

7.1 Findings from cycle 1

In the surveys, students were asked four questions relating to their perceived success in the mathematics lesson and their understanding of the teacher's pedagogical rationale. There was no noticeable difference in the pattern of responses between the two classes, so the 45 responses have been aggregated in the analysis below. The numbers in brackets represent

the number of students responding in the manner described, exemplified by individual responses from students.

7.1.1 Level of success

Firstly, students were asked to respond to the question: "How well do you think you've done in today's maths lesson? On a scale of 1 (Not well at all) to 5 (Very well indeed)." This was followed up by the question: "How do you know?"

In general, students held positive perceptions of their level of success with more than half responding that they did 'very well' (6) or 'quite well' (18), and 15 saying they did 'OK'. Only 5 students said 'not well' and just 1 student said 'not well at all'.

Most students attributed the level of their success to their output, e.g. the amount of work completed (12), getting answers correct or incorrect (9), answering the teacher's questions (4):

> *I completed most of the questions in a reasonable amount of time.*

Others attributed their level of success to finding the work easy (7) or difficult (4), their participation (9), behaviour (8), effort (2), and working well with others (5).

7.1.2 The 'separating' strategy

Next, students were asked to respond to the question "What do you think was the purpose of separating 'reasoning' and 'working-out' in the table?"

It was noticeable how the vast majority of students misinterpreted the intentions of the teacher in using this strategy, despite these being the focus of a discussion during the lesson. Only 3 students (out of 45) gave a response that closely reflected the primary purpose for using 'separating', i.e. to help students appreciate the difference between reasoning and working out:

> *I think it was separated to show how to do it and why they did it.*

> *So that we understand the difference between the two.*

Another 6 students responded by describing this difference, rather than the rationale behind the strategy, suggesting only a partial awareness of the pedagogical rationale:

> *The purpose of separating reasoning and working-out is that working out shows how to do it but reasoning explains how to do it.*

A further 16 students gave responses that suggested a reason for 'separating' that did not reflect its primary purpose:

> *So we can understand better how to use the methods.*

> *So the teacher can see your working out and how you got it.*

The remaining 20 students either did not provide an answer or gave a response that suggests they did not understand the question:

> *Because they are two different ways of working out.*

I don't know what this means.

7.1.3 The 'advocating' strategy

Finally, students were asked to respond to the question "Why do you think the teacher asked you to explain your partner's thinking and not your own?"

There appeared to be a greater appreciation of the teacher's rationale for using this strategy, with just over a third of responses (14 out of 45) suggesting reasons that closely reflected the primary purpose, i.e. to encourage students to share and engage with each other's ideas and methods:

> *So that we understand other's point [of] view.*

> *Because you can get two different perspective[s] and it may help you finalise your idea.*

However, a significant majority (28 out of 45) of students either misinterpreted or misunderstood the teachers' intentions in using this strategy with just under half the responses (21 out of 45) suggesting that students viewed 'advocating' as simply a means of enforcing listening:

> *To see if you're listening to your friend.*

> *Make sure you're listening and learning.*

There were 3 (out of 45) responses that hinted at a partial understanding of the teachers' rationale, as well as a means of enforcing listening:

> *So you can get more ideas and so you can improve your listening skill.*

> *So we can listen better and know each other's ways of working stuff out.*

The remaining 7 students either left the question blank or responded in such a way to suggest they hadn't understood the question.

7.2 Findings from cycle 2

The interviews were conducted by Alba and Tiago with six students: Ennis, Keira, Sophia from Alba's class and Mary, Marcus, Neal from Tiago's class (all pseudonyms). All six students were selected by the teachers from those identified as 'pupil premium', an indicator of socio-economic disadvantage (see Section 4). Students were asked to articulate the reasons why they thought the teacher used each strategy. They were also asked about the extent to which they enjoyed the lesson, considered themselves successful, and what they attributed this to.

The responses of the six students suggested that, in general, they were beginning to develop a greater awareness of the teachers' rationale for using particular strategies. As a group, they were able to articulate some, but not all, of the primary reasons we identified for using the three strategies. Every student managed to correctly identify a primary purpose for at least one strategy, but none correctly identified primary purposes for all three.

7.2.1 The 'scribing' strategy

For the 'scribing' strategy, four of the students (Marcus, Sophia, Mary and Keira) articulated reasons that closely reflected one of its primary purposes, i.e. for students to identify and address errors and misconceptions.

Marcus and Sophia described how the strategy can help bring errors out into the open so they can be corrected:

> *To maybe see like where we go … because it's better if you say it out than keep it in because the teacher could help you and try and improve from wrong to right (Marcus).*

> *Cos then you can compare the correct and incorrect answers together and see, like, where you went wrong, and how you, you know, changed the answer to get the correct one. (Sophia)*

Mary explained how the strategy contrasted favourably with more conventional teaching approaches:

> *… it was nice to like write down it, and then look at our mistakes … then we can, like, fix the mistakes. … But usually people will like … if we make a mistake, and then they just change it. They just tell you it's this, but they don't tell you why.*

Keira articulated most clearly how the strategy enables students to identify the errors themselves:

> *And by writing everything we've said, that will help, not just like the person, it will show everyone like where it went wrong. Instead of like you telling us, and that, we can learn from our mistakes.*

The same four students also suggested other reasons that might be valid in different contexts, but which were not considered primary in this instance. Marcus described how the strategy enables students to look back at their work later to help revise for a test:

> *It helps them when they're in a test and maybe, before the test, they revise. And they see that working out … maybe in the test, and they can remember it without having to think or skip to the next question.*

Similarly, the responses of the other two students demonstrated only a partial appreciation of the primary purpose, by identifying other potentially valid reasons. Ennis referred to learning from errors, but not to students identifying these themselves:

> *So you help where they've gone wrong. And they can like learn from their mistakes and like write down the actual answer.*

Neal didn't mention errors or misconceptions at all, instead referring to a focus on the method rather than the answer:

> *But then it doesn't really matter if it's correct or not … their working out might be correct, but it's just that they maybe done something wrong at the end.*

Other primary purposes we considered were not mentioned by the students. These include challenging the assumption that the teacher only ever writes down correct or perfect solutions, enabling students to verbalise their thinking by making appropriate use of informal and mathematical language, and highlighting ambiguity in the language used by students.

7.2.2 The 'annotating' strategy

The pattern of responses was similar for the 'annotating' strategy, with three students (Neal, Sophia and Keira) describing how it can draw attention to errors or misconceptions and enable students to improve their solutions.

> *Well, it just kind of helped me like to know what to do next time … Because if the person had like a long way, you could annotate it and make it like a shorter way or easier way to do it. (Neal)*

These three students, along with Ennis and Mary, also articulated other potentially valid reasons not considered primary in this instance, e.g. helping to pick out key information in the question, providing useful tips and serving as a useful reference point for revision at a later date.

Marcus did not articulate a valid reason for annotating (despite the question being rephrased several times) and appeared to confuse annotation with the written solution:

> *Was it about simplifying it? … I can't … I don't really know.*

7.2.3 The 'classifying' strategy

Again, the responses for the 'classifying' strategy followed a similar pattern with three students (Neal, Mary and Ennis) articulating its primary purpose, i.e. to recognise which procedures to apply to solve problems.

> *And like, instead of going through tons and tons of methods, it would just be like 'I know how to do this, we just do that method'. (Mary)*

Keira and Sophia both articulated a partially valid reason, i.e. that it helps students appreciate that there are different types of problems but did not explain clearly how this might be useful in solving problems in future.

> *It, like, shows you that we know the difference between each of the questions, and they're not all just like the same. (Sophia)*

Marcus articulated a less valid purpose, i.e. to see whether students know how to sort problems in case such a task comes up in a test (which is unlikely).

7.2.4 Preoccupation with testing

A recurring theme across all six interviews was the students' preoccupation with testing, with each student at some point associating the strategies with the need to prepare for tests. This was apparent in Marcus' second response on the 'scribing' strategy (see Section

7.2.1) and in the responses of Mary and Ennis on the 'annotating' and 'classifying' strategies respectively (shown below).

> *It's one day before a test … if you write in your book, then you can, when you go home, you can revise better. (Mary)*

> *So you can like know which type is which, if you're on your test. (Ennis)*

7.2.5 Enjoyment

When students were asked whether they enjoyed the lesson, all six claimed that they did, but gave a variety of reasons for doing so. Mary, Marcus and Keira attributed their enjoyment to finding the work easy or being familiar with the content. Mary enjoyed solving easier problems and claimed not to like the harder ones:

> *The simplifying ones I didn't like. But the other ones I did like, because when … you think you understand it … and it's easy.*

In contrast, Neal seemed to enjoy some of the more challenging problems:

> *I liked … the questions were … well, some of them were easy, but then some of them were tricky, I did like a lot of working out for it.*

Keira, Ennis, Neal and Sophia appeared to enjoy the opportunity to engage with progressive pedagogies, particularly collaborative working and problem-solving.

> *I liked the partner work when you had to answer the starter. … yeah, that was fun. … I just like explaining myself to someone, cos that's what I like to do instead of trying to figure it out by myself. (Sophia)*

> *Because it's like a variety, you can like learn more ways of doing it. (Ennis)*

7.2.6 Level of success

When asked how well they thought they did in the lesson, all six students described themselves as being successful. Sophia, Ennis, Mary and Marcus appeared to associate success primarily with completing a large number of questions correctly without necessarily experiencing challenge.

> *Cos, at the end of the classroom, you went through it, and you told us the answers … and, yeah, I got them right. (Sophia)*

> *Because I got most of the questions right, and … like, I understood how to do it. And I wasn't really stuck, except for the last one (Ennis)*

In contrast, Neal and Keira seemed to associate success with engaging with the progressive teaching strategies used by the teacher.

> *Well, I did pretty good … because me and [another student] … we did our own separate question. And then after we just worked it out together, after, to see if how we got the same answer, and then what method we did and see what's the easier method. (Neal)*

I think I did really good, because I was, like, annotating in my work … when you were telling us to annotate … everything you did, like, I was doing as well. (Keira)

7.3 Summary of findings

It was apparent from the surveys and interviews that the vast majority of students, including some disadvantaged students, perceived themselves as being successful in mathematics lessons. Most students exhibited high levels of enjoyment in both lessons in which the strategies were trialled as part of the project. Increased levels of enjoyment and engagement, and a higher proportion of students experiencing success, are commonly argued to be the benefits of employing progressive pedagogies in mathematics teaching (Boaler, 2008; Wright, 2016, 2017). The Visible Maths Pedagogy research project therefore provides further evidence to highlight the benefits of progressive pedagogies for enhancing the enjoyment and fulfilment of mathematics learners. Given Bourdieu's contention (highlighted in Section 3) that working-class students are less likely to accumulate 'mathematical capital' and identify themselves as mathematics learners (Williams & Choudhury, 2016), increases in students' engagement with mathematics are seen as essential for addressing the under-performance of disadvantaged students.

It was noticeable that, during the first cycle, significantly more students attributed their success in mathematics to characteristics associated with conventional teaching approaches, such as completing large numbers of questions correctly, than with those associated with progressive approaches, such as working well with others and developing deeper understanding. There were however indications that, over the first year of the project, students began to develop broader notions of success. At the end of the second cycle, two out of the six students attributed their success in the lesson to their engagement with progressive pedagogies, whilst the other four articulated a more conventional notion of success. Four of the students attributed their enjoyment of the lesson to having the opportunity to engage with progressive pedagogies, particularly collaborative working and problem-solving, whilst only three attributed this to being able to complete the work easily. Throughout the course of the project, students continued to refer to preparing for and doing well in tests, often in relation to the use of progressive pedagogies. This resonates with claims that progressive teaching approaches can help disadvantaged students to achieve more highly in traditional modes of testing (Boaler, 1998; Smith, Lee, & Newmann, 2001).

It should be noted that the responses from the surveys and interviews are not easily comparable as the survey was given to all students in the two classes whilst the interviews involved only six students. Indeed, in evaluating the first and second action research cycles, it was recognised by the research group that it would have been preferable to use both surveys and interviews for the two research lessons (and this is our intention for future cycles). However, these six students were drawn from those identified as disadvantaged who, according to the theoretical framework (Bernstein, 2000; Lubienski, 2004), generally find it more difficult to relate to progressive pedagogies than others. Hence the broader notions of success and enjoyment articulated by some disadvantaged students appear to be particularly significant.

It was also noticeable that, initially (demonstrated by the survey responses in cycle 1), a significant majority of students misinterpreted the reasons why their teachers adopted progressive pedagogies, reflecting Lubienski's (2004) findings. It was common for students to misrecognise the teachers' intentions as attempts to make them more compliant in their behaviour. However, it appeared from the interview responses (in cycle 2) that some disadvantaged students began to exhibit greater appreciation of their teachers' rationale for using progressive teaching approaches. For each strategy trialled in the second cycle, at least one of its primary purposes was recognised and clearly articulated by half of the six students interviewed.

The five strategies referred to above involved teachers making the rationale behind their progressive teaching approaches explicit, primarily through discussion of this rationale with students during the research lessons. It might be argued therefore that this finding merely illustrates the students' capacity to accurately repeat what their teachers had told them earlier. Even if this were the case, it would represent a significant improvement on existing practice in that the strategies encouraged teachers to share with students what routinely remains hidden, i.e. their pedagogical rationale. However, it should be noted that the interviews were semi-structured in nature, i.e. students were asked follow-up questions to encourage them to expand on their ideas, and they were held one or more days after the research lessons. The detailed accounts given by students therefore suggest a genuine appreciation of the rationale behind the teaching approaches rather than an ability to recall this from memory.

There is evidence therefore that the research project achieved significant success during the first year in relation to its aim of helping students recognise the intentions of the teacher and how to respond appropriately to progressive teaching approaches and realise success. At the same time, however, it was clear that disadvantaged students continued to misinterpret the teachers' intentions on some occasions suggesting that there is still more work to be done in this area.

8. Discussion

We now refer back to the theoretical framework to consider how the research project findings might inform the debate around empowering disadvantaged students and challenging inequities in mathematics education. The preoccupation of students with preparing for tests, evident in the findings, illustrates the appreciation students have for the 'exchange value' of school mathematics (Williams & Choudhury, 2016). The research project highlighted the potential of making pedagogy more visible for enhancing disadvantaged students' understanding of progressive teaching approaches and how these can bolster their success in tests.

At the same time, developing a greater appreciation of the rationale behind progressive pedagogies, which are associated with 'high epistemic quality' (Hudson, 2018) and with students gaining 'mathematical power' (Gutstein, 2006) and agency (Skovsmose, 2011), has the potential to heighten students' awareness of the 'use value' of mathematics (Williams & Choudhury, 2016). Students' developing appreciation of the value of collaborating with

others in problem-solving activities, apparent in the findings, suggests a willingness to strive for alternative, more emancipatory, notions of mathematical success which encourage taking greater responsibility for others' learning (Boaler, 2008) and assigning value to communal effort, solidarity and trust amongst learners (Radford, 2012). This reflects the key aim emerging from the theoretical framework (which draws on Bourdieu's notion of 'reflexive sociology') of opening up what it means to be successful in learning mathematics to greater scrutiny (Williams & Choudhury, 2016).

The misinterpretation of the teacher's pedagogical rationale as seeking compliant behaviour, evident in the findings, resonates with Bernstein's (2000) theory of 'pedagogic discourse'. Bernstein highlights how the 'regulative discourse', i.e. rules relating to maintaining social order, always dominate the 'instructional discourse', i.e. rules governing the discursive order, in influencing pedagogy. He argues that a socially-just pedagogy would require teachers to develop 'relational' authority, based on explaining reasons behind pedagogical choices and negotiating with students, rather than 'positional' authority, whereby teachers revert to exploiting their position of power over learners.

This resonates with Brousseau's (1984) notion of 'didactic tension' in which teachers' attempts to make desirable mathematical behaviour more explicit can result in the required behaviour being exhibited without necessarily leading to greater understanding. The notion of 'didactic contract' appears to be relevant here, i.e. the system of rules and mutual obligations (often implicit) that regulate the actions of students and teachers involved in the generation, or passing on, of mathematical knowledge (Brousseau, 1997). Pepin (2014) highlights how some (particularly disadvantaged) students resist or merely go along with their obligations under the didactic contract. She argues that identifying with contractual obligations is largely dependent on the environment in which a child is brought up, i.e. closely linked to Bourdieu's notion of cultural capital (Jorgensen, Gates, & Roper, 2014).

There is evidence that the research project helped (particularly disadvantaged) students to better appreciate the intentions of the teacher and to broaden their understanding of mathematical success to include responding more appropriately to progressive teaching approaches. This suggests that making pedagogy more visible to leaners has the potential to enhance teachers' relational authority and lessen the didactic tension existing between teachers and learners, both associated with the empowerment of disadvantaged students.

9. Conclusion

Despite the best efforts of mathematics education researchers over many years, there remains a persistent association between socio-economic background and mathematical achievement. The findings from the first year of the Visible Maths Pedagogy research project concur with Lubienski's (2004) findings that many students, particularly those from disadvantaged backgrounds, are prone to misinterpret the teachers' intentions when adopting progressive teaching approaches. However, the project has also highlighted strategies that teachers might successfully use to make progressive pedagogy more visible to learners and to help disadvantaged students appreciate how to respond appropriately to progressive teaching approaches in order to achieve success in school mathematics.

We have discussed how these findings might inform the debate about addressing inequities in mathematics education. We have highlighted the potential of making pedagogy more visible for enabling disadvantaged students to access benefits associated with progressive teaching approaches, including being better prepared for high-stakes tests as well as gaining more powerful forms of mathematical knowledge (Gutstein, 2006; Hudson, 2018; Skovsmose, 2011). We have employed a theoretical framework, drawing on Bourdieu's notion of 'reflexive sociology', to suggest how making pedagogy more visible can open up what it means to be successful in learning mathematics to greater scrutiny (Williams & Choudhury, 2016).

We are encouraged by the apparent potential of the strategies developed during the first year of the Visible Maths Project to offer hope to those with both a firm commitment towards equitable mathematics teaching and a strong belief in progressive teaching approaches. In the second and third years of the project we intend to further explore and refine effective strategies for making pedagogy more visible to learners and to assess the longer-term impact of such strategies on students' views of success in the mathematics classroom. We aim to build on the participatory action research methodology already employed to generate a research model that can be used by others in advancing more equitable approaches to teaching mathematics. We intend to further develop and refine data collection and evaluation methods that can be used by teachers in researching their own practice (including teacher-initiated student surveys, interviews and video-stimulated reflection). We plan to test the efficacy of our research model to be employed on a wider scale, e.g. across a whole mathematics department.

Declaration of interest statement

There are no potential conflicts of interest in relation to this submitted manuscript.

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Appendix 1: Survey and interview questions

Survey questions

Students were asked to respond in writing to the following five questions in the survey. Note question 1 was included to facilitate allocation of pseudonyms to students (although this was subsequently not felt necessary in the analysis). Questions 4 and 5 relate to the 'separating' and 'advocating' strategies respectively.

1. *Are you male or female?* (students were invited to tick the appropriate box)

2. How well do you think you've done in today's maths lesson? (students were asked to indicate on a scale of 1 to 5)

3. *How do you know?* (students were provided with space to explain)

4. What do you think was the purpose of separating 'reasoning' and 'working-out' in the table? (students were provided with space to explain)

5. Why do you think the teacher asked you to explain your partner's thinking and not your own? (students were provided with space to explain)

Interview questions

Teacher researchers posed the following six initial questions to students during the interviews. Follow-up questions were used to encourage students to expand on their ideas, e.g. *'That's interesting, tell me more about …'*. Note questions 3, 4 and 5 relate to the rationale for using the 'scribing', 'annotating' and 'classifying' strategies respectively.

1. Did you enjoy yesterday's [or Wednesday's] lesson? Why?

2. Did you notice anything different about yesterday's [or Wednesday's] lesson?

3. Why do you think I wrote down exactly what students said even if it wasn't correct?

4. How was it useful to you to annotate the answers?

5. Why did I ask you to sort the problems into three types?

6. How well did you do in today's lesson? How do you know?

Appendix 2: Coding schemes

Coding scheme and categorised responses for student surveys

*Note the 'Number of responses' refers to the number to which each code was applied.

Code/category	Description	Number of responses*
Question 1: How well do you think you've done in today's maths lesson?		
1	Not well at all	1
2	Not well	5
3	OK	15
4	Quite well	18
5	Very well	6
Question 2: How do you know?		
The following categories are grouped as 'Students' dispositions towards learning':		
Good behaviour	Student attributes success to their own good behaviour.	2
High Effort	Student attributes success to their own high levels of effort.	2
Participation	Student attributes success to their own high levels of participation.	7
Working with others	Student attributes success to working well with other students.	5
Lack of participation	Student attributes lack of success to their own low levels of participation.	2
Poor behaviour	Student attributes lack of success to their own poor behaviour.	6
The following categories are grouped as 'Students' judgements about their work output':		
Answered questions	Student attributes success to answering teachers' questions correctly.	4
Correct answers	Student attributes success to getting correct answers to questions completed independently.	9

Work completed	Student attributes success to completing a large amount of work.	12
The following categories are grouped as 'Students' perceptions about how others saw them':		
Teacher approval	Student attributes success to the approval they receive from the teacher.	5
Lack of ability	Student attributes lack of success to their own lack of ability.	1
The following categories are grouped as 'Students' judgements about their level of understanding':		
Found work easy	Student attributes success to finding the work easy.	7
Found work difficult	Student attributes lack of success to finding the work difficult.	4
Needed help	Student attributes lack of success to needing to ask others for help.	1
Question 3: What do you think was the purpose of separating 'reasoning' and 'working-out' in the table?		
Articulates difference	Student refers to the difference between the two without referring to teachers' rationale for using the table.	6
Improves layout	Student refers to table helping students to improve the layout of their work.	7
Understand difference	Student refers to table helping students to see the difference between the two.	3
Understand method	Student refers to table helping students to understand the method.	9
Not understood	Student is not able to articulate a valid reason for separating the two or does not understand the question.	20
Question 4: Why do you think the teacher asked you to explain your partner's thinking and not your own?		
Understand other's ideas	Student refers to strategy helping them to understand each other's ideas and methods.	7

Check partner's understanding	Student refers to strategy as a means of checking their partner understood the work.	2
Engage with other's ideas	Student refers to strategy as encouraging them to engage with each other's ideas and methods.	1
Express your ideas	Student refers to strategy as encouraging them to express their ideas better.	2
Check listening to partner	Student refers to strategy as a means of checking they are listening to their partner.	24
Share ideas with others	Student refers to strategy as encouraging them to share their ideas and methods with each other.	6
Test your recollection	Student refers to strategy as a means of checking their recollection of the method.	1
Not understood	Student is not able to articulate a valid reason for explaining partner's thinking or does not understand the question.	7

Coding scheme for student interviews

*Note the 'Number of references' refers to the number of extracts of text to which each code was applied. The number in brackets refers to the number of interviewees for whom each code was applied to at least one response.

Code	Description	Number of references (sources)*
Teaching approaches (progressive pedagogy)		
Rich questioning	Student describes teachers' use of questioning to draw out understanding.	1 (1)
Problem-solving	Student describes the use of open-ended problems during the lesson.	12 (5)
Connecting topics	Student makes connections between different mathematical topics.	2 (2)
Focus on methods	Student refers to focusing on methods for solving problems rather than correct answers.	8 (4)
Explanation	Student refers to explaining and justifying solutions.	6 (3)

Collaborative work	Student refers to working collaboratively with other students.	11 (3)
Misconceptions	Student describes the use of misconceptions and errors to develop understanding.	9 (5)
Teaching strategies (visible pedagogy)		
Scribing	Student refers to features of the 'scribing' strategy (teacher writes down exactly what students say even if incorrect).	22 (6)
Annotating	Student refers to features of the 'annotating' strategy (teacher or student annotates a solution already displayed).	41 (6)
Classifying	Student refers to features of the 'classifying' strategy (students sort problems into different types).	31 (6)
Distinctions	Student refers to features of the 'distinctions' strategy (teacher draws out distinctions between reasoning and working-out by using a two-column table).	6 (2)
Students' experiences of learning		
Satisfaction	Student expresses satisfaction with their learning during the lesson.	10 (5)
Challenge	Student describes experiencing challenge during the lesson.	7 (3)
Lack of challenge	Student describes experiencing a lack of challenge during the lesson.	1 (1)
Frustration	Student expresses frustration with their learning during the lesson.	1 (1)
Fear/anxiety	Student describes experiencing fear or anxiety during the lesson.	2 (2)
Familiarity	Student refers to being familiar with the content of the lesson.	7 (3)
Confusion	Student describes experiencing confusion during the lesson.	4 (2)
Dispositions towards learning mathematics		
Enjoyment	Student articulates reasons for enjoying the lesson.	29 (6)

Lack of enjoyment	Student articulates reasons for not enjoying the lesson.	4 (1)
Perseverance	Students refers to their own perseverance during the lesson.	1 (1)
Motivation	Students refers to their own motivation during the lesson.	1 (1)
Empathy with others	Student exhibits empathy towards other learners.	11 (4)
Shared responsibility	Student describes sharing responsibility for others' learning.	8 (3)
Recognition rules		
Notices strategy	Student recalls the main strategy used by the teacher during the lesson.	9 (6)
Fails to notice strategy	Student does not recall the main strategy used by the teacher during the lesson.	7 (3)
Primary purpose	Student articulates a primary purpose (as identified by the teacher) for using the teaching approach in question.	21 (6)
Valid purpose	Student articulates a valid purpose (not identified as primary by the teacher) for using the teaching approach in question.	57 (6)
Invalid purpose	Student articulates an invalid purpose for using the teaching approach in question.	18 (4)
Realisation rules		
Successful	Student articulates reasons why they consider themselves successful during lesson.	23 (6)
Unsuccessful	Student articulates reasons why they consider themselves unsuccessful during lesson.	6 (2)
Appropriate behaviour	Student refers to behaviour which they consider appropriate during lesson.	5 (3)