A graph theory approach for scenario aggregation for stochastic optimisation

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1 Supplementary Material

Summary of the mathematical models

The following mathematical formulations were taken directly from the literature. For continuity purposes in all the presented mathematical models, scenario/node (n) and time period (t) have been used here regardless of their definition in the original paper. The rest of the indexes, variables and parameters definitions were maintained.

1.1 Bio-based energy production supply chain by Medina-González et al. (2017)

Indexes	
act	Activities
tec	Technologies
s	Material state
f	Locations
e	Biomass supplier
p	Energy generation
m	Market site
t	Time period
n	Scenarios
Sets	
T_s and $\hat{T}_s$	Task that produce material s
E_{rm}	Suppliers e that provide raw materials
$\hat{E}_{prod}$	Suppliers e that provide production services
$\hat{E}_{tr}$	Suppliers e that provide transportation services
FP	Materials s that are final products
$A\hat{C}T$	Task act with variable input
ACT_{tec}	Task act that can be performed in technology tec
RM	Materials s that are raw materials
Parameters	
$As_{s,f,t}$	Biomass availability
$Dem_{s,f,t}$	Energy demand
$Distance_{f,f'}$	Distance between locations
$FCFJ_{tec,f,t}$	Fixed cost per unit tec
$FE^{limit}_{tec,f,k}$	Increment of capacity
rate	Discount rate
$invest^{MV}$	Investment required for medium voltage
M	Big positive number
$NormF_g$	Normalizing factor of damage category g
$Price_{s,f,t}$	Price of products s at market f in period t
$Price^{limit}_{tec,f,k}$	investment required of an increment of capacity
$Water_{s,n}$	Moisture for material s and scenario n
$Water^{max}_{tec,act}$	Maximum moisture for task act performed in technology tec
$\alpha_{s,act,tec}$	Mass fraction for production of material s
$\hat{\alpha}_{s,act,tec}$	Mass fraction for production of material s
$\beta_{tec,f}$	Minimum utilisation rate of technology
$\theta_{act,tec,f,f'}$	Capacity utilization rate of technology tec by task act whose origin is location f and destination location f'

$\rho^{tr}_{e,f,f',t}$	Unitary transportation costs from location f to location f' during period t
$\tau^{ut1}_{s,f,e,t}$	Unitary cost associated with task i performed in equipment j from location f and payable to external supplier e during period t
$\tau^{ut2}_{s,f,e,t}$	Unitary cost associated with handling the inventory of materials in location f and payable to external supplier e during period t
$\chi_{e,s.t}$	Unitary cost of raw materials offered by external supplier e in period t
$prob_n$	Probability of scenario n
Variables	
$EPurch_{e,t,n}$	Economic value of sales executed in period t during scenario n
$ESales_{t,n}$	Economic value of sales executed in period t and scenario n
$FAsset_{t,n}$	Investment on fixed assets in period t and scenario n
$FCost_{f,t,n}$	Fixed cost in facility f for period t and scenario n
$F_{tec,f,t,n}$	Total capacity technology tec during period t at location f and scenario n
$FE_{tec,f,t,n}$	Capacity increment of technology tec at location f during period t and scenario n
$HV_{s,n}$	Lower heating value for materials during scenario n
NPV_n	Economic metric for a deterministic case (just one scenario n)
$P_{act,tec,f,f',t,n}$	Specific activity of task act, by using technology tec during period t, whose origin is location f and destination is location f' and scenario n
$Profit_{f,t,n}$	Profit achieved in period for each facility f at time period t and scenario i
$Pv_{s,act,tec,f,t,n}$	Input/output material of material s for activity of task act with variable input/output, by using technology tec during period t in location f and scenario n
$Purch^{pr}_{e,t}$	Amount of money payable to supplier e in period t associated with production activities
$Purch^{rm}_{e,t}$	Amount of money payable to supplier e in period t associated with consumption of raw materials
$Purch^{tr}_{e,t}$	Amount of money payable to supplier e in period t associated with consumption of transport services
$Sales_{s,f,f',t,n}$	Amount of product s sold from location f in market f' in period t and scenario n
$S_{s,f,t,n}$	Amount of stock material s at location f in period t and scenario n
SoC_n	Surrogate social metric at each scenario i
ENPV	Expected net present value
ESoC	Expected social performance
Binary Variables	
$V_{tec,f,t}$	Technology installed at location f in period t
$Z_{f,f'}$	Facilities f and f' interconnected by a medium voltage line
$\xi_{tec,f,k,t}$	Variable to model the economies of scale technology tec in facility f at period t as a piecewise linear function

$$
\begin{aligned}
S_{s,f,t,n} - S_{s,f,t-1,n} = &\sum_{f'} \sum_{act \in T_s} \sum_{tec \in (TEC_{act} \cap TEC_{f'})} \alpha_{s,act,tec} P_{act,tec,f',f,t,n} \\
&- \sum_{f'} \sum_{act \in \hat{T}_s} \sum_{tec \in (TEC_{act} \cap TEC_{f'})} \alpha_{s,\hat{act},tec} P_{act,tec,f',f,t,n} \\
&+ \sum_{act \in (T_s \cap ACT)} \sum_{tec \in (TEC_{act} \cap TEC_{f'})} Pv_{s,act,tec,f,t,n} \qquad \forall\ s,\ f,\ t,\ n \\
&- \sum_{act \in (\widehat{T_s} \cap ACT)} \sum_{tec \in (TEC_{act} \cap TEC_{f'})} Pv_{s,act,tec,f,t,n}
\end{aligned} \tag{1.1.1}
$$

$$
\sum_{s \in T_s} HV_{s,n} Pv_{s,act,tec,f,t,n} = \sum_{s \in \hat{T}_s} HV_{s,n} Pv_{s,act,tec,f,t,n} \qquad \forall\ act \in ACT,\ f,\ t,\ n \tag{1.1.2}
$$

$$
\begin{aligned}
&\sum_{s \in S_{act}} Water_{s,i} Pv_{s,act,tec,f,t,n} \leq Water^{m}_{act,tec} ax \sum_{s \in S\hat{_a}ct} PV_{s,act,tec,f,t,n} \\
&\qquad \forall\ n, \in \hat{N},\ tec,\ f,\ t,\ n
\end{aligned} \tag{1.1.3}
$$

$$
\sum_{k} \xi_{tec,f,k,t,n} FE^{limit}_{tec,f,k} = FE_{tec,f,t,n} \qquad \forall\ tec, \in TEC_f,\ f,\ t,\ n \tag{1.1.4}
$$

$$
\sum_{k} \xi_{tec,f,k,t,n} = V_{tec,f,t,n} \qquad \forall\ tec, \in TEC_f,\ f,\ t,\ n \tag{1.1.5}
$$

$$
F_{tec,f,t,n} = F_{tec,f,t-1,n} + FE_{tec,f,t,n} \qquad \forall\ tec, \in TEC_f,\ f,\ t,\ n \tag{1.1.6}
$$

$$
\beta_{tec,f} F_{tec,f,t1,n} \leq \sum_{f'} \sum_{act \in ACT_{tec}} \theta_{act,tec,f,f'} P_{act,tec,f,f',t,n} \qquad \forall\ tec, \in TEC_f,\ f,\ t,\ n \tag{1.1.7}
$$

$$
\sum_{f'} \sum_{act \in ACT_{tec}} \theta_{act,tec,f,f'} P_{act,tec,f,f',t,n} \leq F_{tec,f,t-1,n} \qquad \forall\ tec, \in TEC_f,\ f,\ t,\ n \tag{1.1.8}
$$

$$
\sum_{f'} \sum_{act \in \hat{T}_s} \sum_{tec \in te\hat{x}_{act}} P_{act,tec,f,f',t,n} \leq A_{s,f,t,n} \qquad \forall\ s \in RM,\ f \in E\ t,\ n \tag{1.1.9}
$$

$$
P_{act,tec,f,f',t,n} \leq M * Z_{f',f,n} \qquad \forall\ s \in FP,\ f \in M,\ f' \notin M\ t,\ n \tag{1.1.10}
$$

$$
\sum_{f'} \sum_{act \in T_s} \sum_{tec \in TEC_{act}} P_{act,tec,f,f',t,n} \leq Dem_{s,f,t,n} \qquad \forall\ s \in FP,\ f \in M,\ t,\ n \tag{1.1.11}
$$

$$
ESales_{f,t,n} = \sum_{s \in FP} \sum_{f' \in M} Sales_{s,f,f',t,n} Price_{s,f',t} \qquad \forall\ f \notin (M \cup E),\ t,\ n \tag{1.1.12}
$$

$$
FCost_{f,t,n} = \sum_{tec \in TEC_f} FCFJ_{j,f,t} F_{j,f,t,n} \qquad \forall\ f \notin (M \cup E),\ t,\ n \tag{1.1.13}
$$

$$
EPurch_{e,t,n} = Purch^{rm}_{e,t} + Purch^{tr}_{e,t} + Purch^{pr}_{e,t} \qquad \forall\ e,\ t,\ n \tag{1.1.14}
$$

$$
Purch^{rm}_{e,t,n} = \sum_{s \in RM} \sum_{f} \sum_{act \in \hat{T}_s} \sum_{tec \in I_{act}} P_{act,tec,f,f',t,n} X_{e,s,t} \qquad \forall\ f \in E_{rm},\ t,\ n \tag{1.1.15}
$$

$$
Purch^{tr}_{e,t,n} = \sum_{s \in TR} \sum_{tec} \sum_{f} \sum_{f'} P_{act,tec,f,f',t,n} \rho^{tr}_{e,f,f',t} \qquad \forall\ e \in \hat{E}_{rm},\ t,\ n \tag{1.1.16}
$$

$$
\begin{aligned}
Purch^{pr}_{e,t,n} = &\sum_{f} \sum_{act \notin Tr} \sum_{act \in \hat{T}_s} \sum_{tec} P_{act,tec,f,f',t,n} \tau^{ut1}_{act,tec,f,e,t} \\
&+ \sum_{s} \sum_{f \notin S \cup M} S_{s,f,t,n} \tau^{ut2}_{act,tec,f,e,t} \qquad \forall\ e \in \hat{E}_{prod},\ t,\ n
\end{aligned} \tag{1.1.17}
$$

$$
\begin{aligned}
FAsset_{t,n} &= \sum_j \sum_f \sum_k Price^{limit}_{tec,f,t} * \xi_{tec,f,k,t,n} \\
&\quad + \sum_f \sum_{f'} Invest^{MV} Distance_{f,f'} Z_{f,f',n} \qquad \forall\ t = 0,\ n
\end{aligned} \tag{1.1.18}
$$

$$
FAsset_{t,n} = \sum_j \sum_f \sum_k Price^{limit}_{tec,f,t} * \xi_{tec,f,k,t,n} \qquad \forall\ t \geq 1,\ n \tag{1.1.19}
$$

$$
Profit_{f,t,n} = ESales_{f,t,n} - (FCost_{f,t,n} + \sum_e EPurch_{e,f,t,n})X_{e,s,t} \qquad \forall\ f,\ t,\ n \tag{1.1.20}
$$

$$
NPV_n = \sum_f \sum_t ((Profit_{f,t,n} - FAsset_{f,t,n})/(1 + rate)^t) \qquad \forall\ n \tag{1.1.21}
$$

$$
ENPV_n = \sum_n NPV_i * Prob_n \tag{1.1.22}
$$

$$
\max ENPV \tag{1.1.23}
$$

1.2 Hybrid Biofuel Supply Chain by Akgul et al. (2011)

Indexes	
g,g'	Square regions that divide UK territory
i	Resource
m	Transport mode
p	Plant size intervals
n	Scenario
Sets	
BI	Set of biomass types (BI=$FI \cup CI \cup SI$)
CI	Set of first generation biomass co-products (straw)
FI	Set of first generation biomass types (wheat)
PI	Set of product types (biofuel)
SI	Set of second generation energy crops (miscanthus, SRC)
$Total_{i,g,g',l}$	Set of total transportation links allowed for each resource i via mode l between region g and g'
Parameters	
$ADD_{g,g',m}$	Actual delivery distance between regions g and g' via model l
ALD_g	Average local biomass delivery distance
A_g^s	Set-aside area available in region g
α	Operating period in a year
β	Fraction of straw recovered per unit of wheat cultivated
$BA_{i,g}^{min/max}$	Minimum/maximum availability of first generation biomass i($i \in FI$)in region g
CCF	Capital charge factor
$\gamma_{i',i}$	Biomass to biofuel conversion factor for biomass type i' ($i' \in BI$) to biofuel type i ($i \in PI$)
PCC_p	Investment cost of a plant of size p
$IMPC_{i,g*}$	Unit impost cost for importing resource i from foreign supplier g*
$PCap_p^{min/max}$	Minimum/maximum biofuel production capacity of a plant of size p
$Q_{i,l}^{min/max}$	Minimum/maximum flow rate of resource i via model
$SusF$	Maximum fraction of domestic first generation biomass allowed for biofuel production
$UCC_{i,g}$	Unit biomass cultivation cost of biomass type i in region g
$UPC_{i,p}$	Unit biofuel production cost from biomass type i at a plant of scale p
$UTC_{i,c}$	Unit transport cost of product i via mode l
UTC^*	Unit transportation cost for local biomass transfer
$Y_{i,g}$	Yield of second generation energy crop i ($i \in SI$) in region g
$DFFCC_t$	Discount rate
$DFPC_t$	Adjustment interest per time period
$target$	Minimum aceptable profit

Binary Variables	
$E_{p,g,t}$	1 if a biofuel production plant of size p is to be established in region g
$AVA_{p,g,t}$	1 if a product is send to region g at time t
Continuous variables	
$A_{i,g}$	Land occupied by second generation crop i($i \in SI$) in region g
$D_{i,g}$	Demand for resource i in region g
$Df_{i,p,g,t,n}$	Demand for biomass i at a plant of scale p located in region g
$Pf_{p,g,t,n}$	Biofuel production rate of biofuel i ($i \in PI$) at a plant of size p located in region g
$PT_{i,g,t,n}$	Production rate of resource i via mode l from refion g to g'
$Q_{i,g,g',l}$	Flow rate of resource i via mode l from region g to g'
TDC	Total daily cost of a biofuel supply chain network
TIC_t	Total investment cost of biofuel production facilities
$PC_{t,n}$	Production cost
$TPOC_{t,n}$	Total product outsourcing cost
$TC_{t,n}$	Transportation cost
$TP_{i,t,n}$	total prodcution of product i
$TotRev_{t,n}$	Profit at time period t ans scenario s
$ENPV$	Expected net present value
NPV_n	Net present value at scenario s
FCC_t	facilties capital costs
$TOC_{t,n}$	Total operating cost at time t and scenario s

$$\max ENPV \tag{1.2.1}$$

$$TotRev_{t,n} = \alpha * ((MPE(n) + 80) * \sum_{g} D(i,g,t,n) \qquad \forall\ t,\ n \tag{1.2.2}$$

$$\begin{aligned} NPV_n = \sum_{t} TotRev_{t,n} * DFPC_t - FCC_t - DFFCC_t \\ - ((TOC_{t,n} + TPOC_{t,n}) * DFPC_t)) \qquad \forall\ n \end{aligned} \tag{1.2.3}$$

$$ENPV = \sum_{n} NPV_n/|N| \tag{1.2.4}$$

$$TOC_{t,n} = (TC_{t,n} + PC_{t,n}) \qquad \forall\ t,\ n \tag{1.2.5}$$

$$FCC_t = \sum_{g,p} Y_{p,g,t} * PCC_p * 10^6 \qquad \forall\ t \tag{1.2.6}$$

$$\begin{aligned} PC_{t,n} = \alpha * \sum_{i,g} UPC_{i,t} * PT_{i,g,t,n} \\ + \sum_{i,p,g} UPC_{i,p,t} * Df_{i,p,g,t,n} * \gamma_i \qquad \forall\ t,\ n \end{aligned} \tag{1.2.7}$$

$$\begin{aligned} TPOC_{t,n} = \alpha * (IMPC * Q_{i,g,m,r,t,n} * IMPCR_t + IMPCE_n * Q_{i,g,m,r,t,n}) \\ \forall\ t,\ n \end{aligned} \tag{1.2.8}$$

$$\begin{aligned} TC_{t,n} = \alpha * \sum_{i,g,r,m} UTC_{i,m} * Q_{i,g,m,r,t,n} * ADD_{g,m,r} \\ + UTCL * \sum_{i,g} PT_{i,g,t,n} * LD_{g,g'} \qquad \forall\ t,\ n \end{aligned} \tag{1.2.9}$$

$$\sum_{i} Df_{i,p,g,t,n} * \gamma_i = Pf_{p,g,t,n} \qquad \forall\ p,\ g,\ t,\ n \tag{1.2.10}$$

$$\sum_{p} Df_{i,p,g,t,n} = D_{i,g,t,n} \qquad \forall\ i,\ g,\ t,\ n \tag{1.2.11}$$

$$TP_{i,t,n} = \sum_{g} PT_{i,g,t,n} \qquad \forall\ i,\ t,\ n \tag{1.2.12}$$

$$PT_{i,g,t,n} + \sum_{m,r} Q_{i,r,m,g,t,n} = D_{i,g,t,n} + \sum_{m,r} Q_{i,r,m,g,t,n} \qquad \forall\ i,\ g,\ t,\ n \tag{1.2.13}$$

$$PT_{i,g,t,n} = \sum_{p} Pf_{p,g,t,n} \qquad \forall\ g,\ t,\ n \tag{1.2.14}$$

$$AVA_{p,g,t} * PCAP_p^{MIN}/365,000 \leq Pf_{p,g,t,n} \qquad \forall\ p,\ g,\ t,\ n \tag{1.2.15}$$

$$PF_{p,g,t,n} \leq AVA_{p,g,t} * PCAP_p^{MAX}/365,000 \qquad \forall\ n \tag{1.2.16}$$

$$PT_{p,g,t,n} = \leq 0.65 * PT_{p,g,t,n} \qquad \forall\ p,\ g,\ t,\ n \tag{1.2.17}$$

$$\sum_{p,t} Y_{p,g,t} \leq 1 \qquad \forall\ g \tag{1.2.18}$$

$$AVA_{p,g,t} = AVA_{p,g,t-1} + Y_{p,g,t} \qquad \forall\ p,\ g,\ t \tag{1.2.19}$$

$$del_n \geq target - NPV(n) \qquad \forall\ n \tag{1.2.20}$$

$$RF = \sum_{n} del_n/|N| \qquad \forall\ t,\ n \tag{1.2.21}$$

$$\tag{1.2.22}$$

1.3 Water distribution network by Medina-González et al. (2018)

Indexes	
b	Number of industrial storage tanks
h	Agricultural sinks
j	Domestic sinks
k	Number of natural sources of water
l	Location for the storage tanks
m	Tributaries
g	Location of artificial ponds
t	Set of time periods
u	Set of industrial tanks
w	Location of industrial artificial ponds
n	Scenarios
Parameters	
A_g^a	Collection area in location g for artificial ponds a
A_g^{max}	Maximum capacity of artificial ponds a in location g
A_l^s	Collection area in location l for storage tanks s
A_k^{ROW}	Area of collection for runoff water for natural source k
A_k^{DPW}	Area of collection for direct precipitation for natural source k
AI_w^{max}	Maximum capacity of industrial artificial ponds AI in location w
ACS	Cost of water for agricultural use
ATN_g	Depth of artificial ponds in location g
ATS_l	Height of storage in location l
$CTAA$	Treatment cost for rainwater for agricultural use
$CTAI$	Treatment cost for rainwater for industrial use
$CTFP$	Treatment cost for water purchased with domestic use
$CTND$	Treatment cost for natural sources with domestic use
$CTNA$	Treatment cost for natural sources with agricultural use
$CTNI$	Treatment cost for natural sources with industrial use
$CTAD$	Treatment cost for rainwater for domestic use
$CTPA$	Treatment cost for regeneration of wastewater for agricultural use
$CTPE$	Treatment cost for regeneration of wastewater for final disposal
$CTRP$	Treatment cost for water purchased with agricultural use
$CTQP$	Treatment cost for eater purchased with industrial use
$D_{h,t,n}^{as}$	Agricultural users h demands in time t and scenario n
$D_{u,t,n}^{di}$	Industrial users u demands in time t and scenario n
$D_{j,t,n}^{ds}$	Domestic users j demands in time t and scenario n
$DPWV_{k,t}$	Water collected from direct precipitation in natural sources k in time t
DSC	Water sale cost for domestic use
ISC	Cost of water for industrial use
$KF_{l,t}$	Factor to take into account the annualized investment for storage tanks in location l in time t

$KF_{n,t}$	Factor to take into account the annualized investment for artificial ponds in location n in time t
M	Large number
P_t	Precipitation over time period t
P^total	Annual precipitation
$PCSTD$	Unit cost of transport from storage tank l to domestic sink j
$PCASD$	Unit cost of pumping from artificial pond n to domestic sink j
$PCSTA$	Unit cost of pipeline and pumping from storage tank in location l to agricultural sink h
$PCASA$	Unit cost of transport water from artificial pond in location n to agricultural sink h
$PCSTI$	Unit cost of transport water from industrial storage tank in location b to industrial sink h
$PCASI$	Unit cost of transport water from industrial artificial ponds in location w to industrial sink u
$PCND$	Unit costs for transport from natural sources k to domestic main
$PCNA$	Unit costs for transportation of water from natural sources k to agricultural main
$PCNI$	Unit cost of water transportation from natural sources k to industrial main
$PCTW$	Unit water transportation cost from treatment plant to agricultural sink h
$PCTI$	Unit water transportation cost from industrial treatment plant to agricultural sink h
PFP	Unit water transportation cost from external water vendor to domestic users j
PQP	Unit water transportation cost from external water vendor to industrial users u
PRP	Unit water transportation cost from external water vendor to agricultural users h
PSC	Water sale cost for water purchased sent to users
$p^g_{k,t}$	Water collected from direct precipitation and runoff water in sources k at time t
$r_{m,k,t}$	Segregated flow rate from the tributaries m to natural sources k over time period t
$ROWV_{k,t}$	Runoff water collection in natural sources k over time period t
S_l^{max}	Maximum capacity of storage tanks s in location l
SI_b^{max}	Maximum capacity of industrial storage tanks si in location b
$VP_{l,t}$	Factor to consider the value of investment for storage tank in location l and time t
$VP_{g,t}$	Factor to consider the value of investment for artificial ponds in location g and time t
A	Fixed cost for storage tank
B	Variable cost for storage tank
C	Fixed cost for artificial ponds
D	Variable cost for artificial ponds

Variables	
$G_{k,t,n}$	Existing water in natural sources k in time t and scenario n
$g^{d}_{k,t,n}$	Segregated flow rate from the natural source k to main domestic d in time t an scenario n
$g^{a}_{k,t,n}$	Segregated flow rate from the natural sources k to main agricultural a in time t and scenario n
$g^{i}_{k,t,n}$	Segregated flow rate from the natural source k to main industrial i in time t and scenario n
$v^{g}_{k,t}$	Water losses in natural sources k in time t
$Drop^{g}_{,k,t,n}$	Water that exceeds the maximum capacity of natural sources k in time t and scenario n
$s^{in}_{l,t,n}$	Water obtained from rainfall sent to storage tanks s in location l in time t and scenario n
$si^{in}_{b,t,n}$	Water obtained from rainfall sent to industrial storage tanks si in location b and time t and scenario n
$S_{l,t,n}$	Existing water in storage tanks s in location l in time t and scenario n
$SI_{b,t,n}$	Existing water in industrial storage tanks SI in location b in time t and scenario n
$s^{out,a}_{l,h,t,n}$	Segregated flow rate from storage tanks s in location l sent to agricultural users h in time t and scenario n
$s^{out,d}_{l,j,t,n}$	Segregated flow rate from storage tanks s in location l sent to domestic users j in time t and scenario n
$si^{out,i}_{b,u,t,n}$	Segregated flow rate from industrial storage tanks si in location b sent to industrial users u in time t and scenario n
$A_{g,t,n}$	Existing water in artificial ponds a in location g at time t and scenario n
$a^{in}_{g,t,n}$	Water obtained from rainfall sent to artificial ponds a in location g and time t and scenario n
$ai^{in}_{w,t,n}$	Water obtained from rainfall sent to artificial industrial ponds ai in location w and time t and scenario n
$AI_{w,t,n}$	Existing water in industrial artificial ponds ai in location w and time t and scenario n
$a^{out,a}_{g,h,t,n}$	Segregated flow rate from artificial ponds a in location g to agricultural users h in time t and scenario n
$a^{out,d}_{g,j,t,n}$	Segregated flow rate from artificial ponds a in location g sent to domestic users j in time t and scenario n
$ai^{out,i}_{g,u,t,n}$	Segregated flow rate from industrial artificial ponds ai in location g sent to industrial users u in time t and scenario n
$f_{j,t,n}$	Segregated flow rate sent from the domestic main to the domestic users j in time t and scenario n
$r_{h,t,n}$	Segregated flow rate sent from the agricultural main to the agricultural users h in time t and scenario n
$q_{u,t,n}$	Segregated flow rate sent from the industrial main to the industrial users u in time t and scenario n
$Fpch_{j,t,n}$	Segregated flow rate of water purchased sent to domestic users j in time t and scenario n
$cw^{d}_{j,t,n}$	Water consumed and losses in domestic sinks j in time t and scenario n

$int_{j,t,n}^{in}$	Wastewater sent from site j to treatment plant in time t and scenario n
$int_{t,n}^{out}$	Wastewater sent to treatment plant in time t and scenario n
$cw_{t,n}^{tp}$	Water reclaimed in domestic treatment plant and sent to final disposal in time t and scenario n
$int_{h,t,n}^{out,a,g}$	Water reclaimed in industrial treatment plant and sent to agricultural sinks h in time t and scenario n
$rpch_{h,t,n}$	Segregated flow rate of water purchased sent to agricultural users h in time t and scenario n
$inti_{h,t,n}^{out,i}$	Water reclaimed in industrial treatment plant and sent to agricultural sink h in time t and scenario n
$qpch_{u,t,n}$	Segregated flow rate of water purchased sent to industrial users u in time t and scenario n
$cw_{u,t,n}^{di}$	Water consumed and losses in industrial sink u in time t and scenario n
$inti_{u,t,n}^{in}$	Wastewater sent from site u to treatment plant in time t and scenario n
$inti_{t,n}^{out}$	Wastewater sent to treatment plant in time t and scenario n
$cwi_{t,n}^{tp}$	Water reclaimed in industrial plant and sent to final disposal in time t and scenario n
$ZagS_{l,t,n}$	Variable for installing storage tanks in location l in time t and scenario n
$ZagA_{g,t,n}$	Variable for installing artificial ponds in location g at time t and scenario n
$CostS_{l,n}$	Cost of storage tank in location l at scenario n
$CostA_{l,n}$	Cost of artificial ponds in location n at scenario n
$ARS_{l,n}$	Area occupied by the storage tank in location l at scenario n
$ARL_{g,n}$	Area occupied by the artificial ponds in location g at scenario n
$APA_{g,n}$	Total area occupied by artificial ponds in industrial location g at scenario n
$WaterSales$	Total profit from water sales
$TreatCost$	Total cost associated to treatment processes
$StorCost$	Total cost for water storage tasks
$PipingCost$	Total cost associated to piping of water
Binary Variables	
$ZS_{l,t}$	1 if a storage tank has been installed at location l and time t; 0 otherwise
$ZA_{g,t}$	1 if a artificial pond has been installed at location g and time t; 0 otherwise
$ZAI_{w,t}$	1 if a artificial industrial pond has been installed at location w and time t; 0 otherwise
$ZSI_{b,t}$	1 if a artificial industrial pond has been installed at location b and time t; 0 otherwise

$$G_{k,t,s} - G_{k,t-1,n} = \sum_{m} + p^{g}_{k,t} - g^{d}_{k,t,n} - g^{a}_{k,t,n} - g^{i}_{k,t,n} - v^{g}_{k,t} - Drop^{g}_{k,t,n} \quad \forall\ k,\ t,\ n \tag{1.3.1}$$

$$p^{g}_{k,t} = ROWV_{k,t} + DPWV_{k,t} \quad \forall\ k,\ t \tag{1.3.2}$$

$$ROWV_{k,t} = P_t * A^{ROW}_{k} * 0.14 \quad \forall\ k,\ t \tag{1.3.3}$$

$$DPWV_{k,t} = P_t * A^{DPW}_{k} * 0.14 \quad \forall\ k,\ t \tag{1.3.4}$$

$$S_{l,t,n} - S_{l.t-1,n} = s^{in}_{l,t,n} - \sum_{j} s^{out,d}_{l,j,t,n} - \sum_{h} s^{out,a}_{l,h,t,n} \quad \forall\ l,\ t,\ n \tag{1.3.5}$$

$$SI_{b,t,n} - SI_{b,t-1,n} = si^{in}_{b,t,n} - \sum_{u} si^{out,i}_{b,u,t,n} \quad \forall\ b,\ t,\ n \tag{1.3.6}$$

$$A_{g,t,n} - A_{g,t-1,n} = a^{in}_{g,t,n} - \sum_{j} a^{out,d}_{g,j,t,n} - \sum_{h} a^{out,a}_{g,h,t,n} \quad \forall\ g,\ t,\ n \tag{1.3.7}$$

$$AI_{w,t,n} - AI_{w,t-1,n} = ai^{in}_{w,t,n} - \sum_{j} a^{out,i}_{w,u,t,n} \quad \forall\ w,\ t,\ n \tag{1.3.8}$$

$$\sum_{k} g^{d}_{k,t,n} = \sum_{j} f_{j,t,n} \quad \forall\ t,\ n \tag{1.3.9}$$

$$\sum_{k} g^{a}_{k,t,n} = \sum_{h} r_{h,t,n} \quad \forall\ t,\ n \tag{1.3.10}$$

$$\sum_{k} g^{i}_{k,t,n} = \sum_{u} q_{u,t,n} \quad \forall\ t,\ n \tag{1.3.11}$$

$$D^{ds}_{j,t,n} = f_{j,t,n} + \sum_{l} s^{out,d}_{l,j,t,n} + \sum_{g} a^{out,d}_{g,j,t,n} + Fpch_{j,t,n} \quad \forall\ j,\ t,\ n \tag{1.3.12}$$

$$D^{ds}_{j,t,n} = cw^{d}_{j,t,n} + int^{in}_{j,t,n} \quad \forall\ j,\ t,\ n \tag{1.3.13}$$

$$\sum_{j} int^{in}_{j,t,n} = int^{out}_{t,n} + cw^{tp}_{t,n} \quad \forall\ t,\ n \tag{1.3.14}$$

$$int^{out}_{t,n} = \sum_{h} int^{out,ag}_{h,t,n} \quad \forall\ t,\ n \tag{1.3.15}$$

$$D^{as}_{h,t,n} = r_{h,t,n} + \sum_{l} s^{out,a}_{l,h,t,n} + \sum_{g} a^{out,a}_{g,j,t,n} + rpch_{j,t,n} + int^{out,ag}_{h,t,n} + inti^{out,i}_{h,t,n} \quad \forall\ h,\ t,\ n \tag{1.3.16}$$

$$D^{di}_{u,t,n} = q_{u,t,n} + \sum_{b} si^{out,i}_{b,u,t,n} + \sum_{w} ai^{out,i}_{w,u,t,n} + qpch_{u,t,n} \quad \forall\ u,\ t,\ n \tag{1.3.17}$$

$$D^{di}_{u,t,n} = cw^{di}_{u,t,n} + inti^{in}_{u,t,n} \quad \forall\ u,\ t,\ n \tag{1.3.18}$$

$$\sum_{u} inti^{in}_{u,t,n} = inti^{out}_{t,n} + cw^{tp}_{t,n} \quad \forall\ t,\ n \tag{1.3.19}$$

$$inti^{out,i}_{t,n} = \sum_{h} inti^{out,i}_{h,t,n} \quad \forall\ t,\ n \tag{1.3.20}$$

$$S^{max}_{l} \geq S_{l,t,n} \quad \forall\ l,\ t,\ n \tag{1.3.21}$$

$$A^{max}_{g} \geq A_{g,t,n} \quad \forall\ g,\ t,\ n \tag{1.3.22}$$

$$S^{max}_{l} \geq s^{in}_{l,t,n} \quad \forall\ l,\ t,\ n \tag{1.3.23}$$

$$A^{max}_{g} \geq a^{in}_{g,t,n} \quad \forall\ g,\ t,\ n \tag{1.3.24}$$

$$SI^{max}_{b} \geq SI_{b,t,n} \quad \forall\ b,\ t,\ n \tag{1.3.25}$$

$$AI^{max}_{w} \geq AI_{w,t,n} \quad \forall\ w,\ t,\ n \tag{1.3.26}$$

$$SI^{max}_{b} \geq si^{in}_{b,t,n} \quad \forall\ b,\ t,\ n \tag{1.3.27}$$

$$AI^{max}_{w} \geq ai^{in}_{w,t,n} \quad \forall\ w,\ t,\ n \tag{1.3.28}$$

$$\tag{1.3.29}$$

$$\sum_{t} ZS_{l,t} \leq 1 \qquad \forall\ l \tag{1.3.30}$$

$$\sum_{t} ZA_{g,t} \leq 1 \qquad \forall\ g \tag{1.3.31}$$

$$\sum_{t} ZSI_{b,t} \leq 1 \qquad \forall\ b \tag{1.3.32}$$

$$\sum_{t} ZAI_{w,t} \leq 1 \qquad \forall\ w \tag{1.3.33}$$

$$\begin{aligned} CostS_{l,n} = {} & (\sum_{t} KF_{l,t} * VP_{l,t} * ZS_{l,t}) * A \\ & + (\sum_{t} KF_{l,t} * VP_{l,t} * ZagS_{l,t,n}) * B \qquad \forall\ l,\ n \end{aligned} \tag{1.3.34}$$

$$\begin{aligned} CostA_{g,n} = {} & (\sum_{t} KF_{g,t} * VP_{g,t} * ZA_{g,t}) * C \\ & + (\sum_{t} KF_{g,t} * VP_{g,t} * ZagA_{g,t,n}) * D \qquad \forall\ g,\ n \end{aligned} \tag{1.3.35}$$

$$ZagS_{l,t,n} \leq S_l^{max} + M * (1 - ZS_{l,t}) \qquad \forall\ l,\ t,\ n \tag{1.3.36}$$

$$ZagS_{l,t,n} \geq S_l^{max} - M * (1 - ZS_{l,t}) \qquad \forall\ l,\ t,\ n \tag{1.3.37}$$

$$ZagS_{l,t,n} \leq M * (ZS_{l,t}) \qquad \forall\ l,\ t,\ n \tag{1.3.38}$$

$$ZagA_{g,t,n} \leq A_g^{max} + M * (1 - ZA_{g,t}) \qquad \forall\ g,\ t,\ n \tag{1.3.39}$$

$$ZagA_{g,t,n} \geq A_g^{max} - M * (1 - ZA_{g,t}) \qquad \forall\ g,\ t,\ n \tag{1.3.40}$$

$$ZagA_{g,t,n} \leq M * (ZA_{g,t}) \qquad \forall\ g,\ t,\ n \tag{1.3.41}$$

$$S_l^{max} = ARS_{l,n} * ATS_l \qquad \forall\ l,\ n \tag{1.3.42}$$

$$A_g^{max} = ARL_{g,n} * ATN_g \qquad \forall\ g,\ n \tag{1.3.43}$$

$$APA_g = \sum_{t} ZA_{g,t} * A_g^a \qquad \forall\ g \tag{1.3.44}$$

$$\begin{aligned} WaterSales = {} & (\sum_{n} Prob_n * \\ & ((\sum_{k,t} g_{k,t}^{d,n} + \sum_{l,j,t} s_{l,j,t,n}^{out,d} + \sum_{g,j,t} a_{g,j,t,n}^{out,d}) * DSC \\ & + (\sum_{k,t} g_{k,t,n}^{a} + \sum_{l,h,t} s_{l,h,t,n}^{out,a} + \sum_{g,h,t} a_{g,h,t,n}^{out,a} + \sum_{h,t} int_{h,t,s}^{out,ag}) * ASC \\ & + (\sum_{k,t} g_{k,t,n}^{i} + \sum_{b,u,t} si_{b,u,t,n}^{out,i} + \sum_{q,y,t} ai_{w,u,t,n}^{out,i} + \sum_{h,t} inti_{h,t,s}^{out,i}) * ISC \\ & + (\sum_{j,t} fpch_{j,t,n} + \sum_{h,t} rpch_{h,t,n} + \sum_{u,t} qpch_{u,t,n}) * PSC)) \end{aligned} \tag{1.3.45}$$

$$\begin{aligned} StorCost = {} & (\sum_{n} Prob_n * (\sum_{l} CostS_{l,n} + \sum_{g} CostA_{g,n} \\ & + \sum_{b} CostSI_{b,n} + \sum_{w} CostAI_{w,n})) \end{aligned} \tag{1.3.46}$$

$$
\begin{aligned}
TreatCost = (\sum_n Prob_n * \\
&(\sum_{k,t} g^d_{k,t,n} CTND + \sum_{k,t} g^a_{k,t,n} CTNA + \sum_{k,t} g^i_{k,t,n} CTNI \\
&+ (\sum_{l,j,t} s^{out,d}_{l,j,t,n} + \sum_{g,j,t} a^{out,d}_{g,j,t,n}) * CTAD \\
&+ (\sum_{l,h,t} s^{out,a}_{l,h,t,n} + \sum_{g,h,t} a^{out,a}_{g,h,t,n}) * CTAA \\
&+ (\sum_{b,u,t} si^{out,i}_{b,u,t,n} + \sum_{w,u,t} ai^{out,i}_{w,u,t,n}) * CTAI \\
&+ (\sum_{h,t} int^{out,ag}_{h,t,n} + \sum_{h,t} inti^{out,i}_{h,t,n}) * CTPA \\
&+ (\sum_{j,t} fpch_{j,t,n} CTFP + \sum_{h,t} rpch_{h,t,n} CTRP + \sum_{u,t} qpch_{h,t,n} CTQP) \\
&+ (\sum_t cw^{tp}_{t,n} + \sum_t cw^{tpi}_{t,n}) * CTPE))
\end{aligned}
\tag{1.3.47}
$$

$$
\begin{aligned}
PipingCost = (\sum_n Prob_n * (\sum_{l,j,t} s^{out,d}_{l,j,t,n} PCSTD + \sum_{g,j,t} a^{out,d}_{g,j,t,n} PCASD \\
&+ \sum_{l,h,t} s^{out,a}_{l,h,t,n} PCSTA + \sum_{g,h,t} a^{out,a}_{g,h,t,n} PCASA \\
&+ \sum_{b,u,t} si^{out,i}_{b,u,t,n} PCSTI + \sum_{g,h,t} ai^{out,i}_{g,u,t} PCASI \\
&+ \sum_{k,t} g^d_{k,t,n} PCND + \sum_{k,t} g^a_{k,t,n} PCNA + \sum_{k,t} g^i_{k,t,n} PCNI \\
&+ \sum_{h,t} int^{out,ag}_{h,t,n} PCTW + \sum_{j,t} fpch_{j,t,n} PFP \\
&+ \sum_{h,t} rpch_{h,t,n} PRP + \sum_{u,t} qpch_{u,t} PQP \\
&+ \sum_{h,t} inti^{out,i}_{h,t} PCTI))
\end{aligned}
\tag{1.3.48}
$$

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