# Physical conditions in the planetary nebula Abell 30

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# ABSTRACT

We have analysed optical spectra of two of the hydrogen-deficient knots (J1 and J3) in the born-again planetary nebula Abell 30, together with ultraviolet (UV) spectra of knots J3 and J4. We determine electron temperatures in the knots based on several diagnostics. The [O III] nebular-to-auroral transition ratio yields temperatures of the order of 17 000 K. The weak temperature-dependence of the ratios of helium lines  $\lambda$ 4471,  $\lambda$ 5876 and  $\lambda$ 6678 is used to derive a temperature of 8850 K for knot J3 and 4600 K for knot J1. Ratios of O II recombination lines, which directly measure the temperature in the coldest regions of the knots, are used to derive temperatures of 2500 K for knot J3, and just 500 K for knot J1.

We calculate abundances both from collisionally excited lines and from the well-observed recombination spectra of C, N, O and Ne ions. The forbidden line abundances agree well with previous determinations, but the recombination line abundances are several hundred times higher. These results confirm the scenario proposed by Harrington & Feibelman, in which the knots contain a cold core highly enriched in heavy elements. Forbidden lines are almost entirely emitted by the hot outer part of the knot, while recombination lines are emitted predominantly from the cold core. The C/O ratios we derive imply that the knots are oxygen-rich, contrary to theoretical predictions for born-again nebulae.

Key words: ISM: abundances - planetary nebulae: individual: Abell 30.

#### **1 INTRODUCTION**

Abell 30 consists of a large (~120 arcsec across) spherical shell of low surface brightness, with several bright clumps of material concentrated within 10 arcsec of the central star. The knots were first discovered by Jacoby (1979) and independently by Hazard et al. (1980), and were found to be extremely hydrogen-deficient. Knots J1 and J3 are collinear with the central star (Borkowski et al. 1993), and according to Jacoby & Chu (1989) are polar knots, while J2 and J4 are in the equatorial plane. An evolutionary scenario to account for the hydrogen-deficient knots was proposed by Iben et al. (1983), who suggested that in some cases, after the central star of a planetary nebula (CSPN) has become a white dwarf it might experience a final thermal pulse. When this happens, most of the hydrogen left in the star is incorporated into the helium-burning shell and burned.

Two long-standing problems in the study of planetary nebulae are (i) the discrepancy between ionic abundances measured from optical recombination lines (ORLs) and those measured from collisionally excited lines (CELs); and (ii) the discrepancy between temperatures measured from the hydrogen Balmer jump and those measured from [O III] forbidden lines. PN ORL abundances are found to be higher than CEL abundances by factors ranging from near unity to over 20 (see for example Liu et al. 2000, 2001), while Balmer jump is strong evidence that these two phenomena are related (Liu et al. 2001; Liu 2002). In the most extreme case, Hf 2-2, ORL abundances are a factor of 80 higher than CEL abundances while  $T_{\rm BJ} = T_{\rm [OIII]}/10$  (Liu 2002). Several explanations have been offered to explain these discrepancies, including temperature fluctuations (Peimbert 1967), density fluctuations (Viegas & Clegg 1994), abundance inhomogeneities (Torres-Peimbert, Peimbert & Peña 1990) and the presence of shock-waves (Peimbert, Sarmiento & Fierro 1991). In a recent study of the planetary nebula NGC 6153, for which ORL abundances are a factor of 10 higher than CEL abundances, Liu et al. (2000) suggested that the discrepancy might be caused by the presence of metal-rich knots within the nebula. Empirical two-component models of NGC 6153 were reasonably successful in reproducing the observed ORL and CEL line intensities, and also the observed Balmer jump and [OIII] forbidden line temperatures. The posited knots in NGC 6153 would be unseen due to the much higher surface brightness of the main nebula, but could be of similar origin to those in Abell 30.

temperatures are invariably lower than [OIII] temperatures. There

Jacoby & Ford (1983) determined ionic abundances within knots J3 and J4 of Abell 30, including a recombination line abundance for  $C^{2+}/H^+$  from C II  $\lambda$ 4267 relative to H $\beta$ . The carbon abundance thus derived was almost half that of hydrogen, far higher than abundances of other heavy elements derived from CELs. They considered this abundance questionable: Barker (1982) had earlier proposed

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that the  $\lambda$ 4267 line in many nebulae might be enhanced by charge transfer, dielectronic recombination, line blending or resonance fluorescence. Jacoby & Ford also noted that such a huge carbon abundance would make it hard to maintain the high temperatures implied by standard diagnostics. Later analysis of ultraviolet (UV) spectra of Abell 30 by Harrington & Feibelman (1984) found that UV carbon CELs were very strong, implying that the carbon abundance is indeed high. Their results suggested the presence of extreme temperature and abundance inhomogeneities within the knots of Abell 30. From the carbon emission line ratios C II  $\lambda$ 4267/C III]  $\lambda$ 1908 and C III  $\lambda$ 4650/C IV  $\lambda$ 1549, they found evidence for a much lower temperature in the knot than given by the standard [O III] line ratios, and suggested that the very high abundances of CNO coolants within the knots had given rise to a very cool (~1000 K) but still ionized core. Then, the ORL emission would come predominantly from the cool core, while the CEL emission would come from the much hotter outer regions of the knot.

Guerrero & Manchado (1996) determined CEL abundances within knots J1–4, and found significantly higher helium abundances in the polar knots J1 and J3 than in the equatorial knots J2 and J4. They established the rates of conversion of hydrogen into helium, and found that between 75 and 95 per cent of the original hydrogen has been burned into helium. Oxygen and nitrogen abundances relative to the initial hydrogen abundance were found to be an order of magnitude lower than typical values in PNe.

In this work we present an analysis of long slit optical spectra and *Hubble Space Telescope* Faint Object Spectrograph (FOS) UV spectra of knots J1 and J3 of Abell 30. We determine the temperature in the core of the knots directly, from the ratio of oxygen recombination lines, and confirm the existence of cold cores. We determine abundances using both CELs and ORLs for carbon, nitrogen, oxygen and neon and show that the cold cores contain very high abundances of heavy elements. The presence of cold CNO-rich cores in the knots of Abell 30 lends support to the scenario proposed by Liu et al. (2000, 2001) and Liu (2002) to explain the abundance and temperature discrepancies in planetary nebulae.

## **2 SPECTROSCOPIC OBSERVATIONS**

### 2.1 Optical spectra

Abell 30 was observed using the double-armed ISIS spectrograph mounted on the 4.2-m William Herschel Telescope (WHT) at the Observatorio del Roque de los Muchachos, on La Palma, Spain, on the night of 2000 February 20. The spectrograph slit was aligned such that the central star and the polar knots J1 and J3 were all observed. The slit width used was 0.82 arcsec, which should have been sufficient to catch all the flux from the knots, which are approximately 0.3 arcsec across (Borkowski et al. 1993). Spectra covering wavelengths from 3400 to 5200 Å and 5500 to 7000 Å were taken. Data were reduced using standard procedures in the MIDAS package LONG92.<sup>1</sup> They were bias-subtracted, flat-fielded and wavelengthcalibrated using a Cu–Ne lamp for the red spectra and a Cu–Ar lamp for the blue spectra. The observations were flux calibrated by comparison with observations of the standard star Feige 34.

Hydrogen emission from the faint surrounding nebula is clearly visible in our long-slit spectra. When measuring hydrogen emission from the knots, the contribution from the outer nebula was subtracted using a spectrum extracted from a region outside the knots covering the same number of rows on the CCD chip as the knots spectra. The

<sup>1</sup> MIDAS is developed and distributed by the European Southern Observatory.

measured H $\beta$  flux from the knots was  $1.24 \times 10^{-16}$  erg cm<sup>-2</sup> s<sup>-1</sup> for J1 and  $1.49 \times 10^{-16}$  erg cm<sup>-2</sup> s<sup>-1</sup> for J3. The contribution from the background for other lines is negligible.

Lines in the spectra of knots J1 and J3 were identified and measured by fitting Gaussian profiles. A list of all the lines observed is given in Table 1. Line fluxes are conventionally normalized to  $H\beta =$ 100 in nebular studies, but because in this case  $H\beta$  is so weak, line fluxes are normalized such that  $I(H\beta) = 1$  to avoid large numbers.

#### 2.2 Ultraviolet spectra

Knots J3 and J4 of Abell 30 were observed by the FOS aboard *HST* on 1994 December 8, as part of GO program 5690, and these observations were obtained by us from the *HST* archive. Observations were taken using gratings G130H, G190H and G270H, which together cover wavelengths from 1100 to 3300 Å. The spectral resolutions are 1, 1.47 and 2.09 Å, respectively. All observations were taken using a round aperture with diameter 0.86 arcsec, which should have captured all the flux from the knots, based on their dimensions given by Borkowski et al. (1993). Standard pipeline reduction yields files in the Flexible Image Transport System (FITS) format containing a wavelength array and a flux array, and these were combined in MIDAS.

Two exposures were taken with each grating, and these were combined, except for G190H on J3, for which there is only one exposure. The observations taken using grating G130H were rebinned by a factor of five to improve the signal to noise ratio, while those using grating G190H were rebinned by a factor of 2. Observations using grating G270H were not rebinned. Lines were identified and measured using MIDAS by fitting Gaussian profiles. A list of all lines is given in Table 2. The observed fluxes are given in columns 2 (J3) and 5 (J4), while column 3 gives the fluxes from knot J3 dereddened and normalized to H $\beta = 1$ , from the optical spectra.

A strong, broad feature at ~2140 Å that appears in the UV spectra of both knots is probably due to sky emission (Lyons et al. 1993). The [O II] line at  $\lambda$ 2470 is also affected by sky emission, being much stronger than would be expected given the O<sup>+</sup>/H<sup>+</sup> abundance implied by the  $\lambda\lambda$ 3726, 3729 optical lines.

# **3 NEBULAR ANALYSIS**

#### 3.1 Extinction

Greenstein (1981) first noted the unusual extinction towards Abell 30. Instead of the usual 2200-Å feature in the extinction curve, increased extinction is seen at 2600 Å, and UV extinction is much lower than implied by a standard galactic curve. Greenstein determined an extinction curve by dereddening spectra of the central star to match a blackbody of 200 000 K. Later, Jeffery (1995) found a similar curve by dereddening to match a model atmosphere with a temperature of 114 000 K. Harrington (1996) followed an approach which avoids the uncertainties of model comparisons by instead comparing the spectrum of the central star of Abell 30 with a dereddened spectrum of the central star of Abell 78, which has very similar properties to those of A30, but has normal interstellar extinction. Over their common spectral range, Harrington's curve is very similar to that of Greenstein. Greenstein's curve covers visual wavelengths as well as UV, therefore we use it to deredden both our optical and UV line fluxes.

Previous studies have suggested that while the spectrum of the central star is reddened, the knots are not (Guerrero & Manchado 1996). We have measured the extinction in the knots using hydrogen and helium line ratios, and we find that extinction in the knots is

 Table 1.
 Observed line fluxes.

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	Knot J1			Knot J3								
$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	Ion	$\lambda_0$	Mult	Lower term	Upper term	$g_1$	<i>g</i> <sub>2</sub>
3426.37	21.14	33.49	3426.45	9.442	14.96	[Ne v]	3425.86	F1	2p2 3P	2p2 1D	5	5
3444.32	3.395	5.335	3444.19	5.638	8.858	Ош	3444.07	V15	3p 3P	3d 3P*	5	5
3479.44	1.527	2.369	*	*	*	HeI	3478.97	V43	2p 3P*	15d 3D	9	15
3635.09	0.636	0.934	*	*	*	Heı	3634.25	V28	2p 3P*	8d 3D	9	15
3665.15	0.459	0.665	*	*	*	NeII	3664.07	V1	3s 4P	3p 4P*	6	4
3694.99	0.502	0.718	3694.40	0.972	1.390	NeII	3694.21	V1	3s 4P	3p 4P*	6	6
	*	*	3705.17	0.663	0.944	Heı	3705.02	V25	2p 3P*	7d 3D	9	15
3710.43	0.233	0.332	*	*	*	Ne II	3709.62	V1	3s 4P	3p 4P*	4	2
3713.79	0.790	1.121	3713.12	1.205	1.709	Ne II	3713.08	V5	3s 2P	3p 2D*	4	6
3715.53	0.386	0.547	*	*	*	OIII	3715.08	V14	3p 3P	3d 3D*	5	7
3726.77	9.235	13.03	3726.19	9.767	13.78	[O II]	3726.03	F1	2p3 4S*	2p3 2D*	4	4
3729.51	5.687	7.923	3728.92	5.735	7.990	[O II]	3728.82	F1	2p3 4S*	2p3 2D*	4	6
3750.48	0.578	0.807	3749.77	0.437	0.609	Оп	3749.48	V3	3s 4P	3p 4S*	6	4
3755.55	1.196	1.666	3754.91	0.989	1.378	Ош	3754.70	V2	3s 3P*	3p 3D	3	5
	*	*	3757.19	0.252	0.351	Ош	3757.24	V2	3s 3P*	3p 3D	1	3
3760.65	1.694	2.354	3760.06	1.305	1.814	Ош	3759.87	V2	3s 3P*	3p 3D	5	7
	*	*	3777.41	0.382	0.526	Nell	3777.14	VI	3s 4P	3p 4P*	2	4
2024 75	0.504	* 0.705	3819.89	0.802	1.086	Hel	3819.62	V22	2p 3P*	6d 3D	9	15
3834.75	0.584	0.785	* 00	<b>a</b> ( 102	25 46	Hell	3834.89	4.18	4f+ 2F*	18g+ 2G	32	÷
3869.50	29.50	39.00	3868.92	26.83	35.46		3868.75		2p4 3P	2p4 ID	5	5
3889.42	5.125	0.707	3888.80	5.495	/.191	Hel	3888.03	He1	28 33 46± 35*	2p 3P*	9	9
3924.10	0.552	0./13	2064.95	~ 0.402	0 (27	Hell	3923.48	4.15	41 ' 2F*'	15g ' 2G	32	~ 2
3903.30	0.509	0.047	3904.83	0.495	0.027	He I [No III]	3904.73		28 15 2p4 2D	4p 1P** 2n4 1D	1	5 5
<i>4026 80</i>	9.027	2.606	4026.21	1.002	2 270		4026.21	1°1 V19	2p4 3r 2p 2D*	2p4 1D 5d 2D	0	15
4020.89	2.092	2.000	4020.31	0.272	2.370	Nu	4020.21	V 10 V 20b	2p 3F*	JU JD 4f 2[5]	9	13
4042.17	0.024	0.773	4041.47	0.272	0.337		4041.51	V07	3d 2E	412[3]	8	*
4000.14	0.000	0.744	4060.59	0.389	0.462		4060.00	V 97	/f 3F*	5g 3G	5	7
4069.66	0.538	0.500	4069.10	0.215	0.204	Сш	4068.92	V16	4f 3F*	5g 3G	7	7
4070 37	0.550	0.005	4069.00	0.275	0.545	Оп	4069.62	V10	3n 4D*	3d 4F	2	4
4070.63	1 179	1 457	4070.05	1 127	1 393	Оп Оп	4069.82	V10	3p 4D*	3d 4F	4	6
4071.01	0.702	0.868	4070.42	0.365	0.451	Сш	4070.26	V16	4f 3F*	5g 3G	9	11
4072.91	1.982	2.448	4072.32	1.515	1.871	Оп	4072.16	V10	3n 4D*	3d 4F	6	8
4076.61	2.078	2.564	4076.03	1.511	1.864	Оп	4075.86	V10	3p 4D*	3d 4F	8	10
4079.59	0.310	0.382	4079.01	0.133	0.164	Оп	4078.84	V10	3p 4D*	3d 4F	4	4
4084.65	0.340	0.418	4084.06	0.195	0.240	Оп	4083.90	V48b	3d 4F	4f G4*	6	8
4085.86	0.306	0.378	4085.27	0.197	0.243	Оп	4085.11	V10	3p 4D*	3d 4F	6	6
4087.90	0.328	0.404	4087.31	0.150	0.185	Оп	4087.15	V48c	3d 4F	4f G3*	4	6
4090.04	1.042	1.283	4089.45	0.613	0.755	Оп	4089.29	V48a	3d 4F	4f G5*	10	12
4098.15	2.355	2.893	4097.50	1.777	2.183	Оп	4097.26	V48b	3d 4F	4f G4*	8	10
	*	*	*	*	*	Оп	4097.25	V20	3p 4P*	3d 4D	2	4
	*	*	*	*	*	N III	4097.33	V1	3s 2S	3p 2P*	2	4
4103.98	1.002	1.231	*	*	*	Неп	4100.04	4.12	4f <sup>+</sup> 2F*	12g <sup>+</sup> 2G	32	*
	*	*	*	*	*	Ηδ	4101.73	Ηδ	2p+ 2P*	6d <sup>+</sup> 2D	8	72
4110.58	0.409	0.502	*	*	*	Оп	4110.78	V20	3p 4P*	3d 4D	4	2
4120.10	0.642	0.787	4119.42	0.388	0.475	Оп	4119.22	V20	3p 4P*	3d 4D	6	8
4144.86	0.276	0.337	4144.39	0.500	0.611	Heı	4143.76	V53	2p 1P*	6d 1D	3	5
4154.06	0.706	0.861	*	*	*	Оп	4153.30	V19	3p 4P*	3d 4P	4	6
	*	*	4169.28	0.293	0.356	Оп	4169.22	V19	3p 4P*	3d 4P	6	6
4187.85	0.273	0.332	4186.72	0.761	0.925	Сш	4186.90	V18	4f IF*	5g IG	1	9
4200.51	0.435	0.528	4200.14	0.489	0.594	Неп	4199.83	4.11	4f <sup>+</sup> 2F*	11g <sup>+</sup> 2G	32	*
1005 50	÷	*	4219.84	0.547	0.662	Nell	4219.37	V52a	3d 4D	4f 2[4]*	8	8
4237.72	0.380	0.459	4237.07	0.301	0.364	N II N	4236.91	V48a	3d 3D*	4f 1[3]	3	5
4242.11	0.435	0.525	*	*	*	N II	4241.78	V48b	3d 3D*	4f 1[4]	10	9
4267.95	5.693	0.851	4267.36	4.699	5.655		4267.15	VO	3d 2D	4f 2F*	10	14
42/6.3/	0.642	0.772	42/5.92	1.056	1.270		4276.28	V6/D	30 4D	4I F3*	0	0
	т Ф	*	*	~ ~	т *	01	4276.62	V 53C	3d 4P	41 D1*	4	4
	*	*	*	~ ÷	*		4276.71	V 53C	30 4P	41 D1*	4	2
1277 02	0016	1 017		0 520	0 625		4210.13	V0/D	30 4D	41 F3* 4f E3*	0	ð 1
4211.92	0.840	1.017	4277.73 *	0.528	0.033		4211.43	V0/C V67h	30 4D	41 FZ* Af E2*	2	4
1282 10	0.042	0.052	1281 60	0 102	0.125		4211.09	V0/D V52h	34 4D	41 F3* /f D2*	0	0
4202.10	0.045	0.052	4201.00	0.105	0.123		4201.32	v 330 V67a	3d 4D	41 D2" Af E9*	0	0 6
4203.70	0.472	0.307	4203.24	0.529	0.399		4202.90	V670	34 4D	41 ΓΔ" /f Ε?*	4	1
+204.33	0.028	0.034	+204.01	0.200	0.307	0 II	+203.13	v0/C	5u 4D	<b>H</b> Γ2 <sup>*</sup>	4	4

	Knot J1			Knot J3								
$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	Ion	$\lambda_0$	Mult	Lower term	Upper term	$g_1$	<i>g</i> <sub>2</sub>
4286 49	0 352	0.423	4285 97	0.232	0.281	0 п	4285 69	V78b	3d 2F	4f F3*	6	8
4200.42	0.352	0.425	4203.57	0.232	0.261	Оп Оп	4205.05	V55	3d 4P	4f G3*	6	8
4293.01	0.071	0.450	4292.49	0.239	0.200	Оп Оп	4292.21	V78c	3d 2F	4f F2*	6	6
4295.61	0.456	0.547	4295.09	0.551	0.566	О II	4294 78	V53h	3d 4P	4f D2*	4	6
4304 53	0.450	1 1 27	4303.90	0.933	1 1 1 8	Оп Оп	4303.82	V53a	3d 4P	4f D3*	6	8
4318.02	0.461	0.551	4317 59	0.366	0.438	О II	4317.14	V2	3s 4P	3n 4P*	2	4
4320.31	0.401	0.412	4319.86	0.500	0.450	Оп Оп	4319.63	V2	3s 4P	3p 4P*	4	6
4520.51	*	*	4325.91	0.177	0.175	Оп Оп	4325 76	$v^2$	3s 4P	3p 4P*	2	2
4339.50	0.824	0.982	4338.87	0.573	0.682	Неп	4338.67	4.10	$4f^+ 2F^*$	$10^{9}$ 2G	32	*
4341.28	1 469	1.751	4340.55	0.547	0.652	Ην	4340.47	Ην	$2n^+ 2P^*$	$5d^+ 2D$	8	50
4343.01	0.492	0.586	4342.24	0.540	0.644	Оп	4342.00	V77	3d 2F	4f 2[5]*	8	10
4346.08	0.778	0.927	4345.64	0.175	0.208	О II	4345.56	V2	3s 4P	3p 4P*	4	2
	*	*	*	*	*	Оп	4345.55	V65c	3d 4D	4f G3*	8	8
4350.31	0.685	0.815	4349.61	0.622	0.740	О II	4349.43	V2	3s 4P	3n 4P*	6	6
4364.02	4.616	5 486	4363.38	3.668	4 360	[0 m]	4363.21	F2	2n2.1D	2p2.1S	5	1
4367.90	1 299	1.544	4366.99	0.579	0.688	<u>О</u> п	4366.89	V2	3s 4P	3p 4P*	6	4
4370.52	0.592	0.703	*	*	*	Neп	4369.86	V56	3d 4F	4f 0[3]*	4	6
4372.70	0.315	0.373	*	*	*	Оп	4371.62	V76b	3d 2F	4f G4*	8	10
4380.12	2.207	2.615	4379.48	1.652	1.958	Νш	4379.11	V18	4f 2F*	5g 2G	14	18
4388.83	0.488	0.577	4387.90	0.478	0.555	HeI	4387.93	V51	2p 1P*	5d 1D	3	5
4392.85	0.686	0.812	4391.99	0.311	0.369	Neп	4391.99	V55e	3d 4F	4f 2[5]*	10	12
	*	*	*	*	*	NeII	4392.00	V55e	3d 4F	4f 2[5]*	10	10
4398.84	0.371	0.439	4398.62	0.293	0.347	NeII	4397.99	V57b	3d 4F	4f 1[4]*	6	8
4409.91	0.759	0.896	4409.19	0.777	0.917	Neп	4409.30	V55e	3d 4F	4f 2[5]*	8	10
4413.79	0.326	0.385	4414.85	0.533	0.629	NeII	4413.22	V65	3d 4P	4f 0[3]*	6	8
4415.83	0.561	0.662	*	*	*	Оп	4414.90	V5	3s 2P	3p 2D*	4	6
4417.83	0.393	0.464	4417.17	0.361	0.426	Оп	4416.97	V5	3s 2P	3p 2D*	2	4
4429.46	0.437	0.515	4428.91	0.512	0.603	Ne II	4428.64	V60c	3d 2F	4f 1[3]*	6	8
	*	*	*	*	*	NeII	4428.52	V61b	3d 2D	4f 2[3]*	6	8
4432.09	0.205	0.242	4431.46	0.390	0.459	NeII	4430.94	V61a	3d 2D	4f 2[4]*	6	8
	*	*	4434.82	0.599	0.704	NI?	4435.11		3p 2P*	3d 2P	2	2
4442.55	0.118	0.139	*	*	*	ΝII	4442.02	V55a	3d 3P*	4f 2[3]	3	5
4448.60	0.314	0.367	4448.26	0.355	0.416	Оп	4448.19	V35	3p' 2F*	3d' 2F	8	8
4454.25	0.601	0.702	*	*	*				1			
4459.62	0.398	0.464	*	*	*	Νп	4459.94	V21	3p 3D	3d 3P*	3	1
4467.05	0.307	0.357	4466.60	0.247	0.287	Оп	4466.42	V86b	3d 2P	4f D2*	4	6
4472.33	3.934	4.567	4471.67	3.902	4.530	Heı	4471.50	V14	2p 3P*	4d 3D	9	15
4476.39	0.375	0.434	*	*	*							
	*	*	4491.53	0.238	0.274	Оп	4491.23	V86a	3d 2P	4f D3*	4	6
4531.98	0.233	0.265	4530.71	0.293	0.333	NII	4530.41	V58b	3d 1F*	4f 2[5]	7	9
4542.51	0.813	0.920	4541.82	0.841	0.952	Неп	4541.59	4.9	4f <sup>+</sup> 2F*	9g+ 2G	32	*
	*	*	4602.60	0.623	0.690	Оп	4602.13	V92b	3d 2D	4f F3*	4	6
4610.61	0.838	0.925	4609.70	0.379	0.419	Оп	4609.44	V92a	3d 2D	4f F4*	6	8
	*	*	4624.93	0.265	0.291	[Arv]	4625.53		3p2 1D	3p2 1S	5	1
4635.02	0.363	0.397	4634.27	0.194	0.212	N III	4634.14	V2	3p 2P*	3d 2D	2	4
4640.24	1.055	1.153	4638.99	0.630	0.689	Оп	4638.86	V1	3s 4P	3p 4D*	2	4
	*	*	4640.77	0.544	0.594	N III	4640.64	V2	3p 2P*	3d 2D	4	6
4642.48	2.150	2.347	4641.94	1.479	1.615	Оп	4641.81	V1	3s 4P	3p 4D*	4	6
	*	*	4641.97	0.039	0.042	N III	4641.84	V2	3p 2P*	3d 2D	4	4
4649.00	0.474	0.517	4647.55	0.352	0.384	Сш	4647.42	V1	3s 3S	3p 3P*	3	5
4650.04	2.560	2.787	4649.26	2.306	2.510	Оп	4649.13	V1	3s 4P	3p 4D*	6	8
4651.59	0.761	0.829	4650.38	0.211	0.230	Сш	4650.25	V1	3s 3S	3p 3P*	3	3
4651.83	0.158	0.172	4650.97	0.677	0.737	Оп	4650.84	V1	3s 4P	3p 4D*	2	2
4653.06	0.095	0.103	4651.60	0.070	0.077	Сш	4651.47	V1	3s 3S	3p 3P*	3	1
4659.15	0.302	0.327	4658.61	0.136	0.147	C IV	4658.64	V8	5f 2F*	6g 2G	14	18
4662.39	0.932	1.010	4661.75	0.778	0.843	Оп	4661.63	V1	3s 4P	3p 4D*	4	4
	*	*	4669.40	0.178	0.193	Оп	4669.27	V89b	3d 2D	4f D2*	4	6
	*	*	4673.87	0.116	0.125	Оп	4673.73	V1	3s 4P	3p 4D*	4	2
4677.04	0.661	0.713	4676.38	0.594	0.641	Оп	4676.24	V1	3s 4P	3p 4D*	6	6
4686.59	34.92	37.63	4685.92	28.93	31.17	HeII	4685.68	3.4	3d <sup>+</sup> 2D	4f <sup>+</sup> 2F*	18	32
4713.27	2.017	2.165	4711.35	0.388	0.416	Heı	4713.17	V12	2p 3P*	4s 3S	9	3
	*	*	*	*	*	[Ar IV]	4711.37	F1	3p3 4S*	3p3 2D*	4	6
4715.08	1.409	1.498	4714.97	1.883	2.002	[Ne IV]	4714.17	F1	2p3 2D*	2p3 2P*	6	4
	*	*	*	*	*	[Ne IV]	4715.66	F1	2p3 2D*	2p3 2P*	6	2

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Table 1 – continued

	Knot J1			Knot J3								
$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	Ion	$\lambda_0$	Mult	Lower term	Upper term	$g_1$	<i>g</i> 2
4725.48	4.111	4.363	4725.47	1.995	2.118	[Ne IV]	4724.15	F1	2p3 2D*	2p3 2P*	4	4
	*	*	*	*	*	[Ne IV]	4725.62	F1	2p3 2D*	2p3 2P*	4	2
4741.10	0.634	0.670	4740.55	0.308	0.325	[Ar IV]	4740.17	F1	3p3 4S*	3p3 2D*	4	4
4860.17	1.253	1.318	4859.49	1.243	1.307	Неп	4859.32	4.8	4f <sup>+</sup> 2F*	8g <sup>+</sup> 2G	32	*
4862.35	1.000	1.000	4861.41	1.000	1.000	$H\beta$	4861.33	$H\beta$	2p+ 2P*	4d+ 2D	8	32
4922.85	1.164	1.169	4922.08	1.217	1.222	Heı	4921.93	V48	2p 1P*	4d 1D	3	5
4925.36	0.258	0.253	4925.05	0.332	0.326	Оп	4924.53	V28	3p 4S*	3d 4P	4	6
4959.84	67.03	65.71	4959.07	61.59	60.38	[O III]	4958.91	F1	2p2 3P	2p2 1D	3	5
	*	*	4995.23	0.295	0.285	ΝII	4994.36	V24	3p 3S	3d 3P*	3	3
5007.76	203.2	193.1	5006.99	186.7	177.4	[O III]	5006.84	F1	2p2 3P	2p2 1D	5	5
5016.56	1.384	1.310	5015.83	1.277	1.209	Heı	5015.68	V4	2s 1S	3p 1P*	*	1
	*	*	5664.84	0.299	0.243	ΝII	5666.63	V3	3s 3P*	3p 3D	3	5
	0.691	0.563	5677.83	0.509	0.415	ΝII	5679.56	V3	3s 3P*	3p 3D	5	7
	*	*	5753.64	0.374	0.302	[N II]	5754.60	F3	2p2 1D	2p2 1S	5	1
5874.75	17.62	13.97	5874.16	15.77	12.50	Heı	5875.66	V11	2p 3P*	3d 3D	9	15
	*	*	6298.79	1.196	0.888	[O I]	6300.34	F1	2p4 3P	2p4 1D	5	5
	*	*	6362.26	0.368	0.270	[O I]	6363.78	F1	2p4 3P	2p4 1D	3	5
	*	*	6460.54	0.696	0.502	Сп	6461.95		4f 2F*	6g 2G	14	18
6547.38	5.911	4.202	6546.69	5.050	3.590	[N II]	6548.10	F1	2p2 3P	2p2 1D	3	5
6560.47	9.984	7.078	6559.66	7.886	5.591	Неп	6560.10	4.6	4f <sup>+</sup> 2F*	6g <sup>+</sup> 2G	32	*
	*	*	*	*	*	Ηα	6562.77	Hα	$2p^{+} 2P^{*}$	3d <sup>+</sup> 2D	8	18
6582.59	17.70	12.49	6581.94	14.68	10.36	[N II]	6583.50	F1	2p2 3P	2p2 1D	5	5
6677.32	5.450	3.785	6676.67	5.068	3.519	Heı	6678.16	V46	2p 1P*	3d 1D	3	5
	*	*	6683.17	0.218	0.151	HeII	6683.20	5.13	5g <sup>+</sup> 2G	13h+ 2H*	50	*

Table 2. HST FOS line fluxes.

	Knot J3			]	Knot J4	
$\lambda_{obs}$	$F(\lambda)$	$I(\lambda)$	$\lambda_{obs}$	$F(\lambda)$	Ion	$\lambda_0$
	$(10^{-16} \mathrm{erg}\mathrm{cm}^{-2}\mathrm{s}^{-1})$	$(I(\mathrm{H}\beta)=1)$		$(10^{-16} \text{ erg cm}^{-2} \text{ s}^{-1})$		
1483.60	24	32	*	*	N IV]	1483.32/1486.50
1548.57	44	57	1548.47	77	C IV	1548.20
1551.62	45	58	1552.71	49	CIV	1550.78
1640.00	170	210	1639.10	38	Неп	1640.42
	*	*	1772.23	4.3	?	?
1905.95	24	37	1905.44	21	С Ш]	1906.68
1908.28	23	35	1907.64	10	CIII]	1908.73
2297.86	5.7	12	*	*	СШ	2297.58
2423.42	84	190	2422.74	70	[Ne IV]	2422.51/2425.15
2470.85	6.2	14	*	*	[O II]	2471.05/2471.12
2511.74	4.3	9.5	*	*	Неп	2511.96
2733.31	6.0	12	*	*	Неп	2734.11
2836.54	4.8	8.7	*	*	ОШ	2836.31
3048.26	3.8	5.6	*	*	ОШ	3047.10
3133.89	15	20	3132.52	1.6	ОШ	3132.79
3188.75	1.6	2	*	*	Heı	3187.74
3203.75	9.1	11	3202.57	1.9	Неп	3203.10

comparable to that of the central star. The use of appropriate directly measured temperatures when calculating theoretical line ratios results in consistent determination of the logarithmic extinction at H $\beta$ , c(H $\beta$ ) from all the diagnostics used.

The extinction to the central star was determined by dereddening its optical spectrum with the Greenstein extinction curve to match the continuum emission of a blackbody with a temperature of 10<sup>5</sup> K.  $c(H\beta)$  was found from this method to be 0.60, which agrees fairly well with previous determinations. Cohen et al. (1977) derived  $c(H\beta) = 0.44$ , while Jeffery (1995) found  $A_v = 1.18$  for the central star, which is equivalent to  $c(H\beta) = 0.61$ . The extinction to the knots was estimated from  $I(\text{H}\alpha)/I(\text{H}\beta)$ . In our spectra, the He II Pickering series line at  $\lambda$ 4860 is resolved from H $\beta$ , but in the lower resolution red spectra, H $\alpha$  is blended with He II  $\lambda$ 6560 emission. This is corrected for using the observed flux of the He II 4686 line – at the temperatures derived from helium lines in the knots (see Section 3.2),  $I(\lambda$ 6560) = 0.13  $I(\lambda$ 4686) (Storey & Hummer 1995). After subtraction of the helium contribution, H $\alpha$ /H $\beta$  is found to be 5.44 (J1) and 4.13 (J3). The intrinsic H $\alpha$ /H $\beta$ ratio depends on the temperature, and adopting the electron temperature determined from helium line ratios in the following section, i.e. 4600 K in J1 and 8850 K in J3, gives c(H $\beta$ ) = 1.02 to J1 and 0.64 to J3. Using the lower temperatures given by O II recombination lines,  $c(H\beta) = 0.60$  in J1 and 0.45 in J3.

For J3, the UV observations allow a determination of  $c(H\beta)$  from the He II  $\lambda 1640/\lambda 4686$  ratio. The observed value is 3.94, while the predicted value at 8850 K (the temperature derived below from He I lines) is 6.40, giving  $c(H\beta) = 0.55$ , which agrees well with the value derived from the other diagnostics.

We adopt  $c(H\beta) = 0.60$  to deredden the spectra of both knots.

#### 3.2 Temperatures and densities

The temperature structure of the knots in A30 has been suggested to be complex. To explain the observed ratios of carbon ORLs to CELs, Harrington & Feibelman (1984) proposed cool, carbon-rich cores in the knots. The large abundances of CNO coolants relative to He would result in material which is very cool ( $\sim$ 1000 K) but still highly ionized, as the heat input by photoionization would be balanced by radiation in the infrared fine-structure lines of the heavy elements. We have used several temperature diagnostics to study the temperature structure of knots J1 and J3.

The electron densities of the knots were measured from the [O II] lines at  $\lambda\lambda 3726$ , 3729, and found to be 2800 cm<sup>-3</sup> and 3200 cm<sup>-3</sup> for knots J1 and J3, respectively. Adopting this density, the [O III]  $(\lambda4959 + \lambda5007)/\lambda4363$  ratio gives temperatures of 17960 and 16680 K, respectively.

Lines of HeI are well observed in our long-slit spectra. The ratios of the strong lines at  $\lambda$ 4471,  $\lambda$ 5876 and  $\lambda$ 6678 are moderately dependent on temperature:  $\lambda 5876/\lambda 4471$  varies from  $\sim 2.5$  at 20 000 K to  $\sim$ 3.5 at 5000 K, while  $\lambda$ 6678/ $\lambda$ 4471 varies from  $\sim$ 0.6 to  $\sim 0.9$  over the same temperature range (Smits 1996). Collisional excitation from the metastable 2s 3S level can be important in enhancing the intensities of these lines, and the predicted ratios given in Smits (1996) were corrected for this effect using formulae from Kingdon & Ferland (1995), which are derived using a 29-state quantal calculation of HeI extending to n = 5. We use the average of the temperatures implied by these two line ratios to derive temperatures of 4600 K for knot J1 and 8850 K for knot J3. Although the lines are well detected, with fluxes accurate to within 5 per cent, the shallow slope of the temperature dependence of the line ratios means the error on these temperature measurements is of the order of ±2000 K.

Some O II recombination line ratios are temperature-sensitive. For example, the ratio of the lines at  $\lambda$ 4649 and  $\lambda$ 4089 varies from approximately 2 to 6 between 1000 and 20 000 K (Storey 1994; Liu et al. 1995). These lines are weak but fairly well detected in our spectra, with errors on the flux of the order of 5 per cent for  $\lambda$ 4649 and 20 per cent for  $\lambda$ 4089. We use their ratio to derive temperatures of 500 K for J1 and 2100 K for J3. The ratio of the lines at  $\lambda$ 4075 and  $\lambda$ 4089 varies by a factor of 2 between 500 K and 20 000 K, and the observed values of this ratio (given in Table 3) also imply very low temperatures, of 400 K for J1 and 2800 K for J3. The errors on these values are of the order of  $\pm$ 2000 K.

The three O II lines at  $\lambda 4089$ ,  $\lambda 4075$  and  $\lambda 4649$  are the highest J-value quartet transitions from the 3d–4f, 3p–3d and 3s–3p configurations respectively. Their emission can only be produced by recombination from the  $2p^2$   ${}^{3}P_2$  level of O<sup>2+</sup>. Other O II line ratios that are temperature-sensitive include  $\lambda 4072/\lambda 4089$  and  $\lambda 4414/\lambda 4089$ . However, the observed values of these ratios imply much higher temperatures, of the order of 5–20 kK. Liu (2002) observes a similar phenomenon for other nebulae, and suggests that one possibility is that the  $2p^2$   ${}^{3}P_2$  level of the O<sup>2+</sup> ion is underpopulated relative to its statistical equilibrium value. Temperatures measured from two

Table 3. Derived electron temperatures in Abell 30.

	Lines	Ratio	Temperature (K)
J1	[O III] (λλ4959+5007)/λ4363	47.18	17 960
	He1 (λ5876/λ4471)	3.059	4900
	He I (λ6678/λ4471)	0.829	4300
	Оп (\\ 4649/\\ 4089)	2.172	500
	Оп (\\ 4075/\\ 4089)	1.998	400
J3	[O III] (λλ4959+5007)/λ4363	54.54	16680
	He I (λ5876/λ4471)	2.759	9240
	He1 (λ6678/λ4471)	0.777	8450
	Оп (\\(\lambda 4649/\)\(\lambda 4089)	3.325	2100
	Оп (\\ 4075/\\ 4089)	2.470	2800
J3	O II ( $\lambda$ 4049/ $\lambda$ 4869) O II ( $\lambda$ 4075/ $\lambda$ 4089) [O III] ( $\lambda$ 4959+5007)/ $\lambda$ 4363 He I ( $\lambda$ 5876/ $\lambda$ 4471) He I ( $\lambda$ 6678/ $\lambda$ 4471) O II ( $\lambda$ 4649/ $\lambda$ 4089) O II ( $\lambda$ 4075/ $\lambda$ 4089)	2.172 1.998 54.54 2.759 0.777 3.325 2.470	400 16 680 9240 8450 2100 2800

lines that both originate via recombination from this level would not be affected, but temperatures measured from two lines that originate via recombinations from different levels would then be unreliable.

The different temperatures derived here from forbidden lines, He I recombination lines and O II recombination lines are exactly what would be expected if the knots contain very cool, CNO-rich, ionized cores. Essentially all CEL emission would come from the warmer outer layers, with most of the ORL emission coming from the cooler cores. Helium emission would come from both components if helium is not enriched in the core. Therefore, in the presence of a cold ionized core, one would expect to find that  $T_{\rm [OIII]} > T_{\rm HeI} > T_{\rm OII}$  (Liu 2002). The derived temperatures follow this relation.

#### 3.3 Ionic abundances from CELs

Abundances relative to hydrogen were derived from CELs for ionic species of C, N, O and Ne. The temperature implied by the [O III] nebular to auroral line ratio and the density implied by the [O II]  $\lambda 3726/\lambda 3729$  ratio were adopted for all species in both knots. Transition probabilities and collision strengths were taken from the references listed in Table 4. The results are given in Table 5. For species in common, the results agree fairly well with the abundances previously derived for J1 and J3 by Guerrero & Manchado (1996).

#### 3.4 Ionic abundances from ORLs

Because of the strong power-law dependence of recombination line fluxes ( $\propto T_e^{-1}$ ), ORL emission is expected to originate almost entirely from the coolest regions of the knots. Temperatures measured from O II recombination lines should be representative of the temperatures of the knot cores, and we therefore derive ORL abundances of heavy elements using temperatures of 500 K for knot J1 and 2500 K for J3, the means of the values derived from  $I(\lambda 4649)/I(\lambda 4089)$ and  $(\lambda 4075)/I(\lambda 4089)$ . Helium abundances are determined using temperatures of 4600 K for J1 and 8850 K for J3, from the means of the helium temperature diagnostics described in Section 3.2. We adopt the electron densities derived from [O II] forbidden lines; 2800 and 3200 cm<sup>-3</sup> for J1 and J3, respectively. The resulting ORL abundances are given in Table 13, later, while the abundances for individual ions are discussed below.

# 3.4.1 $He^+/H^+$ and $He^{2+}/H^+$

Ionic and total abundances relative to hydrogen derived from helium recombination lines are given in Table 6. The He<sup>+</sup>/H<sup>+</sup> abundance was derived from the  $\lambda$ 4471,  $\lambda$ 5876 and  $\lambda$ 6678 lines, averaged with weights of 1:3:1, roughly proportional to their observed intensity ratios. Effective recombination coefficients were taken from

Ion	Transition probabilities	Collision strengths
Сш	Keenan, Feibelman & Berrington (1992), Fleming et al. (1996)	Keenan et al. (1992)
CIV	Wiese, Smith & Glennon (1966)	Gau & Henry (1977)
Νп	Nussbaumer & Rusca (1979)	Stafford et al. (1994)
N IV	Nussbaumer & Storey (1979), Fleming et al. (1995)	Mendoza (1983)
Оп	Zeippen (1982)	Pradhan (1976)
Ош	Nussbaumer & Storey (1981)	Aggarwal (1983)
O IV	Nussbaumer & Storey (1982)	Zhang, Graziani & Pradhan (1994)
Ne III	Mendoza (1983)	Butler & Zeippen (1994)
Neıv	Zeippen (1982)	Giles (1981)
Neiv	Froese Fischer & Saha (1985)	Lennon & Burke (1994)
Ar IV	Mendoza & Zeippen (1982)	Zeippen, Le Bourlot & Butler (1987)

Table 4. Atomic data references.

Table 5. Ionic abundances from CELs.

$X^{i+}/H^+$	Lines	J1	J3
$C^{2+}/H^{+}$	λλ1906, 1908		7.01 (-4)
$C^{3+}/H^{+}$	λλ1548, 1551		7.66 (-4)
$N^+/H^+$	λλ6548, 6584	6.52 (-5)	6.24 (-5)
$N^{3+}/H^{+}$	λλ1483, 1486		4.66 (-4)
$O^+/H^+$	λλ3726, 3729	1.57 (-4)	2.05 (-4)
$O^{2+}/H^{+}$	λλ4959, 5007	1.41(-3)	1.52(-3)
$O^{3+}/H^{+}$	λλ1401, 1404		1.75(-2)
$Ne^{2+}/H^+$	λλ3868, 3967	5.95 (-4)	6.55 (-4)
$Ne^{3+}/H^+$	λλ4715, 4726	3.97 (-3)	5.12 (-3)
$Ne^{3+}/H^+$	λ2423		6.13 (-3)
$Ne^{4+}/H^+$	λ3426	4.28 (-4)	2.37(-4)
$Ar^{3+}/H^{+}$	λλ4711, 4740	3.37 (-6)	1.84 (-6)

Table 6. He/H abundance ratios, by number.

$\mathrm{He}^{i+}/\mathrm{H}^{+}$	Line	<b>J</b> 1	J3
He <sup>+</sup> /H <sup>+</sup>	He I λ4471	8.560	8.766
$He^+/H^+$	He 1 λ5876	8.545	8.562
$He^+/H^+$	He 1 λ6678	8.678	8.692
$He^+/H^+$	Mean	8.575	8.629
$He^{2+}/H^+$	Не II λ4686	2.203	3.015
He/H		10.78	11.64

Brocklehurst (1972). For the three He I lines used, the difference between Brocklehurst's results and the more recent calculations by Smits (1996) is no more than 1.5 per cent. Case A was assumed for the  $\lambda\lambda$ 4471, 5876 triplet lines, and Case B for the  $\lambda$ 6678 singlet line. Contributions due to collisional excitation from the 2s <sup>3</sup>S metastable level were corrected for using formulae from Kingdon & Ferland (1995). At the low temperatures prevailing in J1, collisional excitation effects are negligible, but in J3 they contribute 2.0, 5.2 and 2.5 per cent of the  $\lambda$ 4471,  $\lambda$ 5876 and  $\lambda$ 6678 line fluxes, respectively. The He<sup>2+</sup>/H<sup>+</sup> abundance was derived from the He II  $\lambda$ 4686 line, using effective recombination coefficients from Storey & Hummer (1995).

Several other He I lines were observed in our spectra, and in Table 7 we compare their observed fluxes relative to that of the  $\lambda$ 4471 line to those predicted by Brocklehurst (1972). The agreement between theory and observations is generally good for the 2p <sup>1</sup>P°-*n*d <sup>1</sup>D and 2p <sup>3</sup>P°-*n*d <sup>3</sup>D series. However, the intensities of the 2s <sup>1</sup>S-*n*p <sup>1</sup>P° lines relative to  $\lambda$ 4471 are 40–50 per cent lower than predicted. If this was caused by self-absorption from the metastable 2s <sup>1</sup>S level, then the 2p <sup>1</sup>P°-*n*s <sup>1</sup>S series (lines at 7281, 5047 and

**Table 7.** Comparison of dereddened He I lines strengths with predictions from Brocklehurst (1972) and Smits (1996) for  $T_e = 4600$  K (J1) and 8850 K (J3).

n	$\lambda_{o}$	I <sub>obs</sub>	I pred	I pred	I <sub>obs</sub>	I pred	I pred
		J1	B72	S96	J3	B72	S96
			2p <sup>1</sup> P <sup>o</sup> -n	d 1D serie	S		
3	6678.16	0.829		0.861	0.777	0.809	0.789
4	4921.93	0.256		0.269	0.270	0.273	0.267
5	4387.93	0.126		0.120	0.123	0.127	0.122
6	4143.76	0.074			0.135	0.071	
			$2p^{3}P^{o}-n$	d <sup>3</sup> D serie	s		
3	5875.66	3.060	3.051	2.997	2.760	2.820	2.734
4	4471.50	1.000	1.000	1.000	1.000	1.000	1.000
5	4026.21	0.571	0.456	0.448	0.523	0.470	0.462
6	3819.62	*	0.249		0.240	0.261	
7	3705.02	*	0.153		0.208	0.161	
8	3634.25	0.135	0.100		*	0.106	
			$2s^{1}S-np$	<sup>1</sup> P <sup>o</sup> serie	s		
3	5015.68	0.287		0.488	0.267	0.555	0.551
4	3964.73	0.142			0.138	0.219	
			$2s^{3}S-n$	p <sup>3</sup> P series			
3	3888.65	1.469	1.854	1.809	1.587	2.178	2.162

4438 Å) would be enhanced. Unfortunately, this series is not seen in our spectra. Liu et al. (2000) found that in NGC 6153, the 2s  $^{1}$ S-*n*p  $^{1}$ P° series was a factor of 2–3 weaker than predicted, but the expected enhancement of the 2p <sup>1</sup>P<sup>o</sup>-ns <sup>1</sup>S series was not seen. A similar phenomenon was also found in the galactic bulge PNe M 1-42 and M 2-36 (Liu et al. 2001). The predicted intensities assume Case B, where the He I Lyman series is optically thick, and for the 2s  ${}^{1}S-np$   ${}^{1}P^{0}$  lines the predicted intensities relative to He I  $\lambda$ 4471 are a factor of 50 lower if Case A is assumed instead, so the observed intensities could be explained by a departure from pure Case B recombination towards Case A. Such a departure could arise through the destruction of He I Lyman series photons by dust. The core of Abell 30 is known to be very dusty (Cohen & Barlow 1974; Harrington 1996). Another possibility is that Case B is inappropriate simply due to the small physical size, and hence low optical depth, of the knots.

# $3.4.2 C^{2+}/H^+$

The  $C^{2+}/H^+$  ratio was derived using only the  $\lambda$ 4267 line, taking effective recombination coefficients from Davey et al. (2000). Only one other C II line ( $\lambda$ 6462) was detected in the spectrum of J3, and no others were seen in J1. The  $C^{2+}/H^+$  abundances thus derived are

**Table 8.**  $C^{2+}/H^+$  abundances from ORLs.

λο		J1		J3
	I <sub>obs</sub>	$\mathrm{C}^{2+}/\mathrm{H}^+$	I <sub>obs</sub>	$\mathrm{C}^{2+}/\mathrm{H}^+$
4267.15	6.851	0.376	5.655	0.413
6461.95	*	*	0.502	0.361

colossal (Table 8), nearly half that of hydrogen and several hundred times higher than the value derived from the UV  $\lambda\lambda$  1906, 1909 CELs of C III]. The C<sup>2+</sup> abundance derived from  $\lambda$ 4267 is not sensitive to case: the recombination coefficient varies by only 1 per cent between Case A and Case B recombination (Davey et al. 2000)

To explain the frequently observed discrepancy between  $C^{2+}$ abundances derived from  $\lambda 4267$  and the  $\lambda \lambda 1906$ , 1909 lines, Barker (1982) considered several possible mechanisms for enhancing the  $\lambda 4267$  line, including charge transfer, dielectronic recombination, blending and resonance fluorescence. Jacoby & Ford considered the high carbon abundance they derived for J3 to be questionable in the light of these possibilities, and suggested that it could be too high by a factor of up to 10.

The 3d–4f  $\lambda$ 4267 line is mainly fed by 4f <sup>2</sup>F°–*ng* <sup>2</sup>G transitions. The other C II line seen in the spectrum of J3 is the 4f <sup>2</sup>F°–6g <sup>2</sup>G  $\lambda$ 6462 line. The ratio of this line to  $\lambda$ 4267 is predicted by recombination theory (Davey et al. 2000) to be 0.104, which is in reasonable agreement with the observed value of 0.089. It would be useful to observe more high-excitation C II lines to further confirm the reliability of abundances based on  $\lambda$ 4267, but none the less this result indicates that no unknown process in addition to recombination is populating the 4f <sup>2</sup>F° level and leading to an overestimated  $\lambda$ 4267 C<sup>2+</sup>/H<sup>+</sup> abundance ratio. 4f–ng transitions have also been detected in NGC 6153, M1–42 and M2–36 (Liu et al. 2000, 2001), and in each case the observed intensities are in very good agreement with the predictions of recombination theory.

# $3.4.3 N^{2+}/H^+$

Seven N II lines were observed from one or both knots, and the  $N^{2+}/H^+$  ionic abundances derived from them are given in Table 9. Multiplet V3 recombination coefficients are fairly case-insensitive, with the Case B value being about 20 per cent higher. All the 3d–4f transitions are also case insensitive. Abundances were derived assuming Case A for singlets and Case B for triplets, using the recombination coefficients of Escalante & Victor (1990). The adopted  $N^{2+}/H^+$  abundance ratios are the values found by averaging the re-

**Table 9.**  $N^{2+}/H^+$  abundances from ORLs.

$\lambda_0$	Mult		J1	J3		
		I <sub>obs</sub>	$\mathrm{N}^{2+}/\mathrm{H}^+$	I <sub>obs</sub>	$N^{2+}/H^{+}$	
5666.63	V3	*	*	0.243	0.207	
5679.56 V 3 3s <sup>3</sup> P <sup>o</sup> -3	V3 p <sup>3</sup> D	0.563 <b>1.173</b>	0.228 <b>0.23</b>	0.415 <b>0.892</b>	0.190 <b>0.20</b>	
	3	3d–4f tran	sitions			
4041.31	V39b	0.775	0.235	0.337	0.156	
4236.91,4237.05	V48a	0.459	0.225	0.364	0.238	
4241.24,.78	V48b	0.525	0.210	*	*	
4442.02	V55a	0.139	0.256	*	*	
4530.41	V58b	0.265	0.120	0.333	0.227	
Sum		2.163	0.20	1.034	0.20	

sults from multiplet V3 with the co-added 3d–4f transition values, and are 0.22 for J1 and 0.20 for J3.

## $3.4.4 O^{2+}/H^+$

Many OII recombination lines are seen from both knots. The resulting  $O^{2+}/H^+$  ionic abundance ratios are given in Table 10. The

**Table 10.**  $O^{2+}/H^+$  abundances from ORLs.

1.	Mult		T1			
×0	with	I <sub>obs</sub>	$O^{2+}/H^+$	I <sub>obs</sub>	O <sup>2+</sup> /H <sup>+</sup>	
4638.86	V1	1.153	1.234	0.689	0.696	
4641.81	V1	2.347	0.996	1.615	0.647	
4649.13	V1	2.787	0.622	2.510	0.529	
4650.84	V1	0.172	0.184	0.737	0.745	
4661.63	V1	1.010	0.847	0.843	0.667	
4673.73	V1	*	*	0.125	0.638	
4676.24	V1	0.713	0.712	0.641	0.604	
V 1 3s	<sup>4</sup> P–3p <sup>4</sup> D <sup>0</sup>	8.409	0.75	7.232	0.61	
4317.14	V2	0.551	0.752	0.438	0.591	
4319.63	V2	0.412	0.520	0.213	0.266	
4325.76	V2	*	*	0.175	1.182	
4345.56	V2	0.927	1.273	0.208	0.283	
4349.43	V2	0.815	0.444	0.740	0.399	
4366.89	V2	1.544	1.972	0.688	0.870	
V 2 3s	5 <sup>4</sup> P–3p <sup>4</sup> P <sup>0</sup>	4.578	0.87	2.464	0.49	
3749.48 V 3 3r	V3 2P0-3d 2P	0.807 1.620	0.926 0.93	0.609	0.651	
4414 90	V5	0.662	2 381	*	*	
4416.97	V5 V5	0.464	3 013	0.426	2 009	
V 5 3s	$^{2}P-3p^{2}D^{0}$	1.207	2.66	1.279	2.00	
4069.62	V10	0.910	0.869	0.870	0.857	
4069.89	V10	1.457	0.872	1.393	0.860	
4072.16	V10	2.448	0.967	1.871	0.763	
4075.86	V10	2.564	0.701	1.864	0.526	
4078.84	V10	0.382	0.992	0.164	0.440	
4085.11	V10	0.378	0.799	0.243	0.531	
V 10 3	p <sup>4</sup> Dº–3d <sup>4</sup> F	8.460	0.83	6.658	0.68	
4153.30	V19	0.861	0.990	*	*	
4169.22	V19	*	*	0.356	1.267	
V 19 3	p *P°–3d *P	2.290	0.99	2.781	1.27	
4110.78	V20	0.502	1.880	*	*	
4119.22 V 20 2	V20	0.787	0.798	0.475	0.507	
V 20 3	p ·P · - 30 ·D	2.508	1.03	1.1/0	0.51	
4924.55 V 28 3	v <sup>4</sup> S⁰–3d <sup>4</sup> P	0.255 0.471	0.535 <b>0.54</b>	0.326	0.726	
	r ~	3d-4f tran	sitions			
4089.29	V48a	1.283	0.712	0.755	0.516	
4083.90	V48b	0.418	0.817	0.240	0.573	
4097.26 <sup>a</sup>	V48b	2.893	2.431	2.183	2.298	
4087.15	V48c	0.404	0.826	0.185	0.465	
4303.82	V53a	1.127	1.332	1.118	1.627	
4294.78	V53b	0.682	1.324	0.651	1.554	
4281-83	V53b, V67c	0.653	1.072	0.831	1.075	
4276	V53c,V67b	0.772	1.365	1.270	2.764	
4291-92	V55, V78c	0.523	1.139	0.776	2.080	
4277	V67c	1.017	2.432	0.635	1.870	
4371.62	V76b	0.373	2.096	*	*	
4342.00	V77	0.586	0.590	0.644	0.799	
4285.69	V78b	0.423	1.237	0.281	1.011	
4491.23	V86a	*	*	0.274	1.363	
4466.42	V86b	0.357	1.940	0.287	1.926	
4609.44	V92a	0.925	1.197	0.419	0.667	
4602.13	V92b	*	*	0.690	2.760	
sum		9.543	1.10	9.056	1.18	

<sup>*a*</sup>Excluded from co-added 3d–4f intensity: affected by blending with  $N \pi$  lines excited by secondary Bowen flourescencing.

**Table 11.**  $Ne^{2+}/H^+$  abundances from ORLs.

λο	J1		J3	
	I <sub>obs</sub>	$Ne^{2+}/H^+$	I <sub>obs</sub>	Ne <sup>2+</sup> /H <sup>+</sup>
3694	0.665	0.174	*	*
3664	0.718	0.439	1.390	0.954
3777	0.332	0.219	*	*
3710	*	*	0.526	0.390
V 1 3s <sup>4</sup> P-3p <sup>4</sup> P <sup>0</sup>	2.684	0.25	6.630	0.71

effective recombination coefficients used are from Storey (1994) for 3s–3p transitions, and from Liu et al. (1995) for 3p–3d and 3d–4f transitions. Case A is assumed for doublets and Case B for quartets. Of the multiplets observed, only V19 and V28 are strongly sensitive to case. The mean  $O^{2+}/H^+$  abundances derived from these multiplets are (0.8 ± 0.2) for J1 and (1.0 ± 0.2) for J3, which are in agreement within the errors with the average values derived from the other 3–3 multiplets, (1.2 ± 0.2) for J1, and (0.9 ± 0.2) for J3.

The mean  $O^{2+}/H^+$  abundance ratios derived by averaging with equal weight the values from individual 3–3 multiplets with the co-added 3d–4f values are (1.08 ± 0.19) for J1 and (0.91 ± 0.15) for J3. We adopt these values as the  $O^{2+}/H^+$  recombination line abundance.

# 3.4.5 Ne<sup>2+</sup>/H<sup>+</sup>

The Ne<sup>2+</sup>/H<sup>+</sup> abundance ratios presented in Table 11 are derived using the line fluxes of the V1 multiplet, using atomic data from Kisielius et al. (1998). The abundances derived are not casesensitive: the recombination coefficient differs by less than 1 per cent between Case A and Case B.

#### 3.4.6 Higher ionization species

Several optical recombination lines due to triply ionized species are seen in our spectra, as well as one line ( $\lambda$ 4658) due to quadruply ionized carbon. We derive C<sup>3+</sup>/H<sup>+</sup> ratios from  $\lambda$ 4650 (V1) and  $\lambda$ 4187 (V18) lines strengths, and the N<sup>3+</sup>/H<sup>+</sup> ratio from  $\lambda$ 4379 (V18). The results are given in Table 12. Effective recombination coefficients for all three ions were taken from Péquignot, Petitjean & Boisson (1991). For both knots, we adopt the mean of the two abundances as the C<sup>3+</sup>/H<sup>+</sup> recombination line abundance.

Our adopted ORL ionic abundances for C, N, O and Ne ions are summarized in Table 13.

Table 12.  $C^{3+}/H^+$ ,  $C^{4+}/H^+$  and  $N^{3+}/H^+$  abundances from ORLs.

			J1		J3	
λο	Mult	I <sub>obs</sub>	$C^{3+}/H^+$ (×10 <sup>-2</sup> )	I <sub>obs</sub>	$C^{3+}/H^+$ (×10 <sup>-2</sup> )	
4187	V18	0.323	3.033	0.291	4.706	
4649	V1	0.517	8.797	0.384	3.396	
$\lambda_{o}$	Mult	I <sub>obs</sub>	$C^{4+}/H^+$ (×10 <sup>-3</sup> )	I <sub>obs</sub>	$C^{4+}/H^+$ (×10 <sup>-3</sup> )	
4658	V8	0.327	4.358	0.127	2.200	
$\lambda_{o}$	Mult	I <sub>obs</sub>	$N^{3+}/H^+$ (×10 <sup>-2</sup> )	I <sub>obs</sub>	$N^{3+}/H^+$ (×10 <sup>-2</sup> )	
4379	V18	2.615	6.411	1.958	6.237	

Table 13. Ionic abundances from ORLs, by number.

$X^{i+}/H^+$	<b>J</b> 1	J3
He <sup>+</sup> /H <sup>+</sup>	8.57	8.62
$He^{2+}/H^+$	2.20	3.02
$C^{2+}/H^{+}$	0.38	0.41
$C^{3+}/H^{+}$	5.92 (-2)	4.05 (-2)
$C^{4+}/H^{+}$	4.36 (-3)	2.20 (-3)
$N^{2+}/H^{+}$	0.22	0.20
$N^{3+}/H^{+}$	6.41 (-2)	6.24 (-2)
$O^{2+}/H^{+}$	1.08	0.91
$Ne^{2+}/H^+$	0.25	0.71

Table 14.	Abundances derived from
UV lines in	n knot J4.

Line	$X^{i+}/He$
$\begin{array}{c} C^{2+} 1906/8 \\ C^{3+} 1548/51 \\ O^{3+} 1401/4 \\ Ne^{3+} 2423 \end{array}$	5.2 (-4) 2.6 (-4) 1.4 (-2) 2.8 (-3)

#### 3.5 Abundances in knot J4

From CELs observed in the FOS spectra of knot J4, we determined abundances of  $C^{2+}$ ,  $C^{3+}$ ,  $N^+$ ,  $O^{3+}$  and  $Ne^{3+}$  relative to helium. We assumed that the temperature, density and  $He^{2+}/He^+$  ratio in this knot were the same as those found for knot J3 (Sections 3.2 and 3.4.1). Guerrero & Manchado (1996) found that the electron densities in knots J2 and J4 are an order of magnitude lower than those in J3 and J4, but the abundances derived are fairly insensitive to the adopted density, changing by only 10 per cent at most between the two values. The ionic abundances found for knot J4 are a factor of 3–7 higher relative to helium than those found for knot J3 (Table 5), and are shown in Table 14. This agrees with previous analyses which have shown a chemical segregation between the polar knots and the equatorial ring (Guerrero & Manchado 1996; Jacoby & Ford 1983).

#### 3.6 Total elemental abundances

Total elemental abundances derived from ORLs and CELs are given in Table 15. They were calculated from the ionic abundances using the ionization correction scheme of Kingsburgh & Barlow (1994) to correct for unseen ionization stages. The scale is logarithmic, with  $\log N(H)$  set to 12.

No  $O^+/H^+$  abundance is available from ORLs. To derive a total O/H abundance, we assume that the  $O^{2+}/O^+$  ratio derived from CELs is applicable to ORLs.

**Table 15.** Elemental abundances in units such that  $\log N(\mathrm{H}) = 12.0$ .

	J	1	J	3
Ion	ORLs	CELs	ORLs	CELs
He	13.03		13.07	
С	11.65		11.66	9.22
N	11.49	8.88	11.43	8.90
0	12.15	9.26	12.10	9.32
Ne	11.51	9.70	11.99	9.78
Ar		7.45		7.22

For nitrogen, the CEL abundance in J1 is derived from the N<sup>+</sup>/H<sup>+</sup> abundance. The ionization correction factor is given by O/O<sup>+</sup>, which has a value of 11.63. For J3, the abundance is derived by assuming that N<sup>2+</sup>/H<sup>+</sup> is half way between N<sup>+</sup>/H<sup>+</sup> and N<sup>3+</sup>/H<sup>+</sup>.

Only  $N^{2+}$  and  $N^{3+}$  ORLs are seen, and to derive a total N/H abundance from ORLs, we assume that  $N^+/N = O^+/O$ . This correction amounts to only a few per cent.

# **4 DISCUSSION**

## 4.1 Knots with cool, CNO-rich cores

The extremely low temperatures measured from oxygen recombination lines, together with the extremely high heavy element abundances relative to hydrogen, show that the two polar knots J1 and J3 analysed here must contain some very cool but still ionized material. Virtually all the flux from recombination lines would originate in this cool core, while the forbidden line flux would originate in the outer parts of the knot, where the hydrogen-poor material may be mixing with the hydrogen-rich material of the nebula. The very low temperatures in the core of the knots are assumed to result from the strong cooling due to the enhanced heavy elements. Preliminary photoionization modelling using the current observations data of the knots of Abell 30 has shown that models containing a cool ionized core surrounded by a warmer shell can successfully reproduce most of the observed spectrum, although the abundances required do not correspond closely to those determined empirically (Ercolano et al., in preparation).

## 4.2 C/O and N/O ratios

Previous determinations of PN abundances from both ORLs and CELs have found that the discrepancy factor between ORL and CEL abundances is the same for each element (Liu et al. 2000, 2001). Thus, C/O and N/O ionic ratios, as long as both ionic abundances are derived from one type of line or the other, should be reliable. For Abell 30's knots the discrepancies are broadly similar for C, N and O, ranging from 2.44 dex to 2.89 dex – factors of 300–700. Neon has rather lower discrepancies, the ORL abundance being higher than the CEL value by a factor of 65 for J1 and 160 for J3. This difference may be caused by the weakness of the lines measured and the small number observed.

An important result is that the C/O ratio in the knots is less than unity, whether derived from ORLs or CELs. The values derived are 0.29 for J1 from CELs, and 0.36 and 0.79 for J3, measured from CELs and ORLs respectively. Cool, ionized knots similar to those described here are proposed to exist in the planetary nebulae NGC 6153, M1–42 and M2–36 (Liu et al. 2000, 2001), and for these nebulae the C/O ratios derived from ORLs are also less than unity, at 0.6, 0.15 and 0.76 respectively, implying that their knots could have a similar origin to those in Abell 30.

#### 4.3 Origin of the knots

The common explanation for the knots found in Abell 30 and similar objects is the 'born-again' scenario, in which the central star of a PN which has reached the white dwarf stage experiences a very late helium flash, causing a further ejection of highly processed material into the old planetary nebula (Iben et al. 1983). Abell 58 is another hydrogen-deficient PN, morphologically quite similar to Abell 30, which contains a hydrogen deficient knot (Guerrero & Manchado 1996), and in this case the knot was ejected in a nova-

like explosion in 1916, presumed to be a late helium flash. There are some difficulties with the born-again scenario, however: the theory predicts that the ejected material will be carbon-rich (Iben et al. 1983), whereas in the case of Abell 30 the C/O ratio is less than unity for both knots.

Borkowski et al. (1993) found from their *HST* images that the two polar knots, J1 and J3, are collinear with the central star to within 5 arcmin. Such a degree of collimation is hard to explain in a single-star scenario, and Harrington (1996) suggested that the knots may result from a bipolar jet, suggesting an accretion disc within a binary system.

It may be that some hydrogen-deficient PNe are related to classical novae. DQ Her is an old nova, the shell of which has many properties in common with the knots of Abell 30: it shows strong recombination line emission, and the strength of the Balmer jump implies a temperature of only 500 K (Williams et al. 1978). Abundance determinations from recombination lines by Petitjean, Boisson & Péquignot (1990) show that heavy element abundances in DQ Her's ejecta are very high relative to helium, and its C/O ratio is 0.36.

#### 4.4 Implications for other nebulae

Our confirmation that very cold ionized regions can exist within planetary nebulae is very important. For some nebulae showing extreme discrepancies between ORL and CEL abundances, temperatures measured from helium and oxygen recombination lines follow the same patterns seen here (Liu 2002), and this strongly suggests that cold, ionized, metal-rich regions also exist in these nebulae, explaining the differences between observed ORL and CEL abundances as well as the different temperatures from different diagnostics. These discrepancies, rather than implying large uncertainties in abundance determinations, may instead reflect the fact that CELs and ORLs sample different parts of a nebula, with very different physical conditions.

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