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Wolf-Rayet Stars

VI. The Nature of the Optical and Infrared Continua

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Summary. Scanner spectrophotometry between 3300 and 11100 Å of Wolf-Rayet stars is combined with infrared photometry between 1.6 and 11.3 μ to produce composite energy distributions. The dereddened stellar continua are subtracted from these distributions yielding "difference spectra" of the infrared excesses. These spectra are matched by either a free-free emission continuum or a blackbody-like continuum. WN stars show only free-free emission whereas only WC stars show dust. Physical parameters of the various emitting regions are derived. New red and near-infrared spectra of WC 9 stars have been obtained and evidence is found for visual extinction by circumstellar dust around the star Ve 2–45. Interpretation of the excesses in WC 9 stars as thermal emission by graphite grains yields estimates of the radii and dust masses of the circumstellar shells. The evolutionary status of the unusual WC 9 star Ve 2–45 is briefly discussed.

Key words: Wolf-Rayet stars — infrared photometry — free-free — dust grains

I. Introduction

As part of an ongoing photometric investigation of a variety of hot stars, infrared observations in the 2.2–11.3 μ region have been made of 23 Wolf-Rayet stars, comprising 10 of the WN sequence and 13 of the WC. The aims of the infrared photometric programme were principally: (1) to see whether dust forms about population I Wolf-Rayet stars, as it appears to do in the vicinity of Of stars (Cohen and Barlow, 1973) and near emission-line nuclei of planetary nebulae, including some nuclei whose spectra have Wolf-Rayet characteristics (Cohen and Barlow, 1974a); and (2) to seek systematic differences between WN and WC stars at long wavelengths. By combining the infrared data with existing (and some unpublished) spectrophotometry between .33 μ and 1.1 μ, and the near-infrared measurements of Allen *et al.* (1972) (ASH hereinafter) the composite energy distributions have been constructed over a very large range of wavelengths. The brightest stars at 2 μ were selected from the 1.6 and 2.2 μ survey by ASH, and an attempt was made to obtain a sample well-distributed in spectral type. Particular attention was given to the rare WC 9 stars, following the discovery of the long wavelength brightness of Ve 2–45 (ASH).

Previous optical scanner measurements have been brought up to date and additional stars were measured in the 0.8 to 1.1 μ region to provide the flux distribution over as wide a wavelength range as possible. Optical spectra were obtained of the WC 9 stars in order to assess their spectral characteristics and to attempt to separate interstellar and circumstellar extinction.

II. The Observations

a) Infrared Photometry

The infrared photometry was acquired with the 152 cm UM/UCSD Mt. Lemmon instrument and the Lick Observatory 305 cm reflector, using associated multifilter photometers with liquid helium cooled bolometers. Sky-chopping methods were used with 11″ beams separated by 15″ at Mt. Lemmon, and 4″ beams separated by 8″ at Lick. Most of the observations were made between April and July 1973, and a few in late 1973. Table 1 presents the data at wavelengths of 2.2, 3.6, 4.8, 8.6, 10 and 11.3 μ, together with the previous 1.6 and 2.2 μ data of ASH. Stars are listed by their running number in the catalogue of Roberts (1962)

Table 1. Infrared Photometry

MR No.	Star	Type	H	K	[2.2]	[3.6]	[4.8]	[8.6]	[10]	[11.3]
4	HD 16523	WC 6	8.4	7.8		>7.5			>4.6	
5	HD 17638	WC 6	8.2	7.6	7.5 (.25)	>7.3			>4.7	
6	HD 50896[a])	WN 5	6.2	5.9	5.8 (.05)	5.3 (.1)	4.8 (.3)		3.85 (.25)	
7	HD 56925[a])	WN 5			>9.1				>4.8	
72	HD 157504	WC 6 + OB			6.5 (.1)	5.8 (.1)				
82	AS 320	WC 9	7.2	6.0	6.2 (.05)	4.6 (.05)	4.0 (.2)	3.0 (.3)	3.3 (.1)	3.0 (.3)
83	HD 165688	WN 8	6.6	6.3	6.3 (.05)	5.5 (.1)	5.2 (.2)	4.4 (.25)	3.9 (.15)	4.15 (.3)
84	HD 165763	WC 5	7.0	6.5	6.4 (.05)	5.9 (.1)			4.85 (.3)	
85	HD 168206	WC 8 + B 0	6.2	5.4	5.5 (.05)	4.4 (.05)	4.1 (.2)	3.8 (.1)	3.3 (.15)	3.4 (.2)
90	HDE 313643	WC 9	6.3	4.9	5.0 (.05)	3.1 (.05)	2.3 (.1)	1.4 (.15)		1.4 (.1)
99	HD 190918	WN 4.5 + O 9.5 Ia	6.4	6.3	6.3 (.05)	6.1 (.1)	5.6 (.2)		5.1 (.3)	
100	HD 191765	WN 6	6.5	6.2	6.3 (.1)	5.6 (.15)			3.9 (.25)	
101	HD 192103	WC 8	7.1	6.7	6.7 (.1)	6.25 (.1)	>4.9		5.0 (.3)	
102	HD 192163[a])	WN 6	6.1	5.7	5.7 (.05)	4.9 (.05)	4.6 (.2)		3.8 (.15)	3.4 (.1)
103	HD 192641	WC 7 + Be	6.7	6.0	6.1 (.1)	5.2 (.1)	4.3 (.2)	3.3 (.1)	3.6 (.15)	3.25 (.1)
104	HD 193077	WN 5 + OB	6.9	6.7					>4.5	
106	HD 193576	WN 5 + O 6	6.5	6.2	6.2 (.1)	5.8 (.1)	5.5 (.2)	5.1 (.3)	5.1 (.3)	4.7 (.25)
107	HD 193793	WC 7 + O 5	5.3	4.9	5.2 (.05)	4.7 (.05)	4.6 (.2)	3.7 (.2)	3.0 (.1)	3.2 (.1)
109	HD 195177	WC 5 + OB	8.0	7.3	7.5 (.2)	7.2 (.2)			>3.4	
111	AS 422	WN 5	6.7	6.2	6.2 (.1)	5.5 (.1)	5.5 (.2)	4.9 (.2)	5.1 (.3)	5.1 (.3)
112	[b])	WC 5	6.1	5.5	5.6 (.05)	5.0 (.05)	4.8 (.15)	4.5 (.2)	4.5 (.2)	4.15 (.2)
	LS 15	WC 9	8.2	7.2		6.0 (.1)	5.4 (.25)	4.5 (.25)	4.0 (.2)	
	AS 374	WN 8	7.9	7.4	7.4 (.2)	7.1 (.1)	6.7 (.3)		4.9 (.2)	

Columns 4 and 5 are data from ASH; columns 6–11 are the data of the present paper in the form of mean magnitudes with bracketed estimated total errors.

[a]) Star excites a ring nebula: HD 50896 (anon.), HD 56925 (NGC 2359), HD 192163 (NGC 6888).

[b]) This star is not BD + 40° 4243 as Roberts' (1962) catalogue states. The Wolf-Rayet star is the Np component of a fairly close pair, the brighter component of which is the BD star.

and by name. LS 15 is noted by Smith (1968a). Mean magnitudes are given in Table 1, together with estimated "total errors" in parentheses. These errors include photometric statistics, and various systematic effects arising from variations in sky transmission or detector response in a given night, as well as small uncertainties in the magnitudes of calibration stars. If no detection was achieved, an upper limit of 3 standard deviations of the mean is tabulated.

It should be noted that the filters of effective wavelength "10 μ" have a very broad bandpass (6 μ), hence $\lambda_{\rm eff}$ is conspicuously a function of the observed spectrum. For example, if the Rayleigh-Jeans region of a blackbody is observed ($F_\lambda \propto \lambda^{-4}$), $\lambda_{\rm eff}$ is 9.6 μ, while for an optically thin free-free spectrum ($F_\lambda \propto \lambda^{-2}$), $\lambda_{\rm eff}$ is 10.0 μ. In plotting the energy distributions, this should be borne in mind since small, spurious apparent features can be introduced in the 10 μ region when broad 10 μ data is compared with adjacent narrow-band data.

Very recently, Hackwell *et al.* (1974) and Gehrz and Hackwell (1974) have published infrared photometry of a number of Wolf-Rayet stars. There are several discrepancies between their magnitudes and ours for stars common to both investigations. Most of these may be resolved by paying due regard to the relevant errors of photometry, and by noting that the filters used by the Wyoming group have different effective wavelengths and bandpasses from ours, which makes an appreciable difference in magnitudes when observing the steeply rising spectra of WC 9 stars. Furthermore, there are some differences in the magnitudes adopted for calibration stars (cf. Cohen and Barlow, 1974). Essentially, however, the two sets of photometry are in agreement.

b) Optical Spectrophotometry and Spectroscopy

Scanner measurements of the continuous energy distributions of Wolf-Rayet stars obtained at Mt. Wilson were reported originally by Kuhi (1966) and later revised (Kuhi, 1968) because of changes in the fundamental calibration of Vega (Hayes, 1970). Further changes were made by Smith and Kuhi (1970) to take account of an over-correction for interstellar reddening applied by Kuhi in the 1966 paper. Since these multiple revisions are scattered throughout the literature it was decided to present the final corrected version here along with the infrared data so that the entire measured spectral range would be conveniently accessible in one place. The original data were also normalized to the m_v's given by Roberts (1962) which results in a considerable error for WC stars and a somewhat smaller error for WN stars due to the contamination of broad band measurements by emission lines (Pyper, 1966). Consequently the Mt. Wilson data have been renormalized via secondary standards to 0.0 mag at λ 5556 for Vega [which corresponds to $F_\nu = 3.46 \times 10^{-20}$ erg cm^{-2} s^{-1} Hz^{-1} (according to Oke and Schild, 1970)] and fluxes are given only for those wavelengths for which the measurements can be considered to be essentially line-free. These data are given in Table 2a and b. In addition the wavelength region from 0.80 to 1.11 μ was observed with the Wampler prime focus spectrum scanner (Wampler, 1966) using the 305 cm reflector at Lick Observatory in August and December of 1967. Exit slits of 96 Å and 128 Å were used for wavelengths shorter and longer than 9344 Å respectively with a refrigerated ITT FW 118 dual channel photomultiplier system with S-1 photocathode sensitivity. These measurements were corrected for atmospheric extinction

Table 2. (a) Mt. Wilson measurements: WN stars

λ	HD 4004	9974	50896	186943	187282	190918	191765	192163	193077	193576	193928	211853	214419	219460
	WN5	WN3	WN5	WN4+B	WN4	WN4.5+O9.5Ia	WN6	WN6	WN5+OB	WN5+O6	WN6+OB	WN6+BO:I	WN7+O7	WN4.5+BO
3320	11.07	10.33	6.73	10.73	10.22	7.28	8.32	7.38	8.44	8.90	11.51	9.46	9.64	10.94
3411	11.02	10.33	6.50	10.74	10.24	6.96	8.26	7.43	8.52	8.89	11.45	9.59	9.57	10.90
3571	10.95	10.35	6.59	10.68	10.34	6.92	8.29	7.40	8.42	8.88	11.40	9.47	9.45	10.89
3636	10.89	10.42	6.63	10.68	10.34	6.98	8.23	7.38	8.43	8.78	11.19	9.45	9.43	10.75
4150	10.72	10.39	6.63	10.51	10.38	6.78	8.33	7.30	8.17	8.46	10.72	9.23	9.22	10.37
4255	10.72	10.46	6.68	10.49	10.41	6.78	8.14	7.39	8.14	8.40	10.70	9.21	9.17	10.33
4426	10.66	10.42	6.70	10.48	10.41	6.84	8.21	7.37	8.17	8.36	10.55	9.20	9.16	10.26
4786	10.36	10.49	6.77	10.40	10.33	6.83	8.12	7.34	8.07	8.23	10.26	9.02	9.03	10.05
5000	10.23	10.42	6.82	10.34	10.40	6.81	8.00	7.18	8.01	8.11	10.07	9.01	8.93	9.92
5130	10.11	10.42	6.76	10.29	10.42	6.81	7.97	7.14	8.99	8.04	9.95	8.99	8.81	9.79
5263	10.12	10.42	6.84	10.30	10.46	6.76	7.99	7.18	7.90	8.03	9.88	9.00	8.85	9.84
5360	10.26	10.49	6.88	10.34	10.43	6.84	8.03	7.31	7.93	8.03	9.85	8.96	8.86	9.80
5556	10.11	10.52	6.86	10.27	10.38	6.79	8.00	7.25	7.92	8.00	9.68	8.90	8.68	9.82
5950	—	10.53	6.93	10.26	10.41	6.83	7.94	7.28	7.90	7.87	9.46	8.85	8.68	9.64
6800	—	10.65	7.01	10.22	10.45	6.94	7.88	7.27	7.88	7.64	9.07	8.80	8.55	9.42
8400	—	10.77	716	10.16	10.63	6.98	7.83	7.30	7.81	7.55	8.59	8.67	8.35	9.19
9882	—	10.91	7.26	10.28	10.73	7.08	7.78	7.41	7.88	7.53	8.43	8.75	8.35	9.10
10680	—	10.80	7.28	10.11	10.77	7.13	7.68	7.43	7.84	7.55	8.39	8.76	8.36	9.09
11100	—	—	7.31	—	—	7.20	7.75	7.43	7.92	7.58	8.25	8.76	8.16	8.95

Table 2. (b) Mt.Wilson measurements: WC stars

λ	HD 16523	17638	165763	168206	192103	192641	193793
	WC 6	WC 6	WC 5	WC 8 + BO	WC 8	WC 7 + Be	WC 7 + O 5
3320	10.86	11.68	8.08	10.69	8.45	8.66	7.62
3508	11.00	11.79	8.14	10.54	8.64	8.61	7.80
3658	10.83	11.49	8.02	10.47	8.47	8.57	7.53
4938	10.45	10.92	8.09	9.71	8.32	8.17	7.07
5195	10.37	10.79	8.08	9.50	8.25	8.11	6.97
5359	10.22	10.67	8.05	9.45	8.23	8.14	6.91
5531	10.32	10.70	8.11	9.37	8.33	8.16	6.97
5992	10.17	10.51	8.14	9.19	8.22	8.05	6.91
8000	9.94	9.94	8.14	9.43	8.25	7.87	6.65
10526	10.03	9.85	8.31	8.24	8.36	7.95	6.66
11136	—	9.70	8.36	8.11	8.48	8.18	6.65

Table 2. (c) Lick measurements

λ	HD 4004	6327	9974	16523	17638	50896	AS 320	165688
	WN 5	W(He)	WN 3	WC 6	WC 6	WN 5	WC 9	WN 6
8000	9.54	10.99	10.71	9.93	9.94	7.05	10.51	8.51
8580	9.39	10.86	10.70	9.88	9.77	7.03	10.20	8.50
8764	9.33	10.88	10.67	9.81	9.86	7.08	10.09	8.44
10680	9.26	10.54	11.01	9.93	9.82	7.27	9.67	8.15
11100	9.20	10.39	10.98	9.93	9.83	7.24	9.61	8.12

λ	165763	168206	169010	313643	186943	187282	190918	191765
	WC 5	WC 8 + BO	WC 5	WC 9	WN 4 + B	WN 4	WN 4.5 + O 9.5 Ia	WN 6
8000	8.14	8.44	10.96	10.37	10.09	10.63	6.95	7.82
8580	8.05	8.31	10.50	10.10	10.02	10.62	6.93	7.78
8764	8.06	8.19	10.40	10.03	10.02	10.59	6.90	7.70
10680	8.24	7.99	10.13	9.67	10.05	10.72	7.09	7.75
11100	8.27	8.12	10.28	9.66	10.22	10.87	7.07	7.81

λ	192103	192163	192641	193077	193576	228766	193793	193928
	WC 8	WN 6	WC 7 + Be	WN 5 + OB	WN 5 + OB	WN 7 + O	WC 7 + O 5	WN 6 + OB
8000	8.25	7.28	7.87	7.82	7.59	8.69	6.65	8.69
8580	8.19	7.21	7.81	7.74	7.54	8.62	6.45	8.70
8764	8.15	7.14	7.72	7.69	7.45	8.55	6.47	8.56
10680	8.26	7.18	7.86	7.76	7.46	8.82	6.54	8.34
11100	8.36	7.22	8.03	7.93	7.64	8.73	6.72	8.39

λ	195177	197406	MR 114	211564	211853	213049	MR 119	219460
	WC 5 + OB	WN 7	WN 5 + OB	WN 3	WN 6 + BO:I	WC 5:	WN 8	WN 4.5 + BO
8000	10.31	9.70	11.09	11.29	8.68	10.70	9.74	9.25
8580	10.01	9.59	10.93	11.19	8.64	10.39	9.49	9.11
8764	9.99	9.59	10.89	11.13	8.50	10.45	9.40	9.09
10680	9.84	9.75	10.78	11.27	8.53	10.41	9.13	9.04
11100	9.72	9.62	10.63	11.33	8.71	10.55	9.22	9.22

Note to Table 2c: The standard deviation for stars brighter than 10.0 at λ 8000 is 0.01 mag for λλ 8000, 8580 and 8764 and 0.02 mag for λλ 10680, 11100; for stars fainter than 10.0 the corresponding numbers are ~2 times larger.

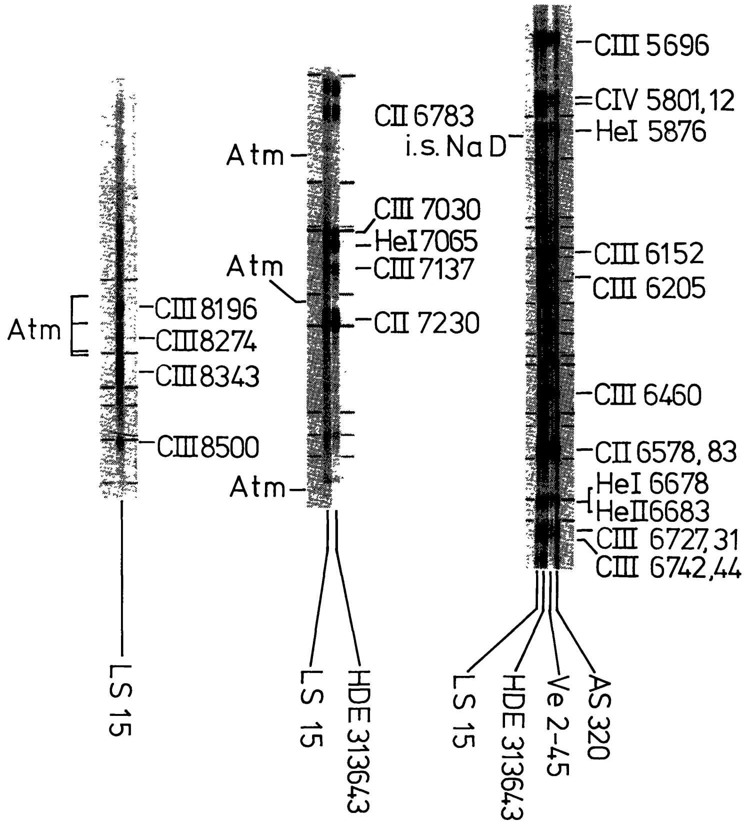

Fig. 1. Red and near-infrared spectra of WC 9 stars

using mean extinction coefficients and converted to absolute fluxes using the Hayes (1970) calibration. The results are given in Table 2c and because of their greater accuracy should be used in preference to the Mt. Wilson data for this wavelength range.

Optical spectra of the WC 9 stars were obtained with the Varo image tube at the 50.8 cm camera of the coudé spectrograph at Lick Observatory. The wavelength range λ 5600 to λ 8600 was covered in a series of plates obtained on U.T. May 3 and June 2, 3 1974 at a dispersion of 33 Å/mm. Major emission line identifications were made from the list of identified lines in HD 164720 and HD 184378 already published by Smith and Aller (1971) and are indicated in Fig. 1. Intensity tracings were also obtained in order to estimate the strengths of the interstellar NaD lines.

III. Composite Energy Distributions

a) Reddening Corrections

Reddening corrections were determined from narrowband and scanner measurements as follows. The values of A_V (total extinction at V) originally determined by Kuhi (1966) were revised by 0.44 mag to take into account the over-correction for reddening pointed out by Smith and Kuhi (1970). These A_V's were then plotted against the observed colour index $[v - F_\nu(\lambda 8000)]$ to define a reddening relation (v is a narrowband filter with effective $\lambda = 5160$ Å). The scanner equivalent of v was used wherever possible since it is somewhat more emission-line-free than Smith's v; however this difference is small in general and the two measurements can used interchangeably. A second order least-squares fit to A_V as a function of $[v - F_\nu(\lambda 8000)]$ was then used to estimate A_V for the stars measured in 1967 (see Fig. 2).

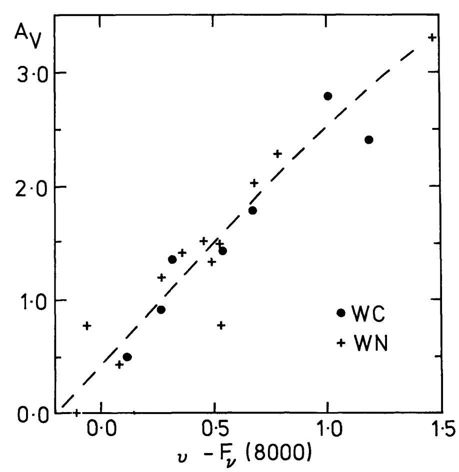

Fig. 2. The relation between A_V and the $[v - F_\nu(8000)]$ colour index for Wolf-Rayet stars. The dashed curve represents the quadratic least squares fit to the plotted points which was used for extrapolation to the WC 9 stars

Table 3. Dereddened Fluxes (F_ν as relative monochromatic mag). (a) Mt. Wilson measurements: WN stars

λ	4004	9974	50896	186943	187282	190918	191765	192163
3320	6.51	9.66	6.73	8.72	9.92	5.98	6.08	4.85
3411	6.55	9.67	6.50	8.77	9.96	5.68	6.06	4.95
3571	6.64	9.72	6.59	8.78	9.11	5.69	6.18	5.02
3636	6.66	9.80	6.63	8.82	9.13	5.77	6.16	5.04
4150	6.87	9.83	6.63	8.81	9.28	5.68	6.34	5.17
4255	6.95	9.91	6.68	8.83	9.33	5.70	6.29	5.31
4426	7.00	9.88	6.70	8.87	9.36	5.79	6.42	5.35
4786	7.03	10.00	6.77	8.93	9.38	5.88	6.49	5.50
5000	7.09	9.96	6.82	8.96	9.50	5.91	6.46	5.44
5130	7.08	9.98	6.76	8.96	9.55	5.94	6.48	5.47
5263	7.17	9.99	6.84	9.00	9.62	5.92	6.54	5.55
5360	7.39	10.07	6.88	9.08	9.61	6.02	6.62	5.72
5556	7.38	10.12	6.86	9.07	9.60	6.01	6.66	5.74
5950	—	10.17	6.93	9.16	9.70	6.12	6.72	5.90
6800	—	10.35	7.01	9.31	9.86	6.35	6.87	6.13
8400	—	10.55	7.16	9.51	10.21	6.56	7.10	6.48
9882	—	10.75	7.26	9.79	10.41	6.76	7.23	6.79
10680	—	10.66	7.28	9.68	10.49	6.85	7.20	6.89
11100	—	—	7.31	—	—	6.94	7.31	6.93
A_V (mag)	2.73	0.40	0.00	1.20	0.78	0.78	1.34	1.51

Table 3. (a) (continued)

λ	193077	193576	193928	211853	214419	219460
3320	6.07	5.53	6.01	6.97	6.27	7.15
3411	6.19	5.58	6.05	7.15	6.26	7.18
3571	6.18	5.69	6.20	7.12	6.26	7.30
3636	6.23	5.65	6.09	7.14	6.30	7.23
4150	6.17	5.61	6.07	7.13	6.37	7.17
4255	6.18	5.61	6.15	7.15	6.38	7.20
4426	6.27	5.66	6.13	7.20	6.46	7.22
4786	6.33	5.77	6.26	7.20	6.56	7.28
5000	6.38	5.79	6.29	7.30	6.60	7.31
5130	6.41	5.84	6.29	7.34	6.57	7.27
5263	6.37	5.85	6.32	7.39	6.67	7.39
5360	6.44	5.91	6.39	7.40	6.74	7.42
5556	6.50	5.98	6.39	7.41	6.66	7.55
5950	6.60	6.03	6.46	7.49	6.84	7.57
6800	6.81	6.12	6.59	7.68	7.03	7.71
8400	7.04	6.45	6.80	7.86	7.25	7.96
9882	7.30	6.70	7.08	8.14	7.52	8.17
10680	7.34	6.83	7.22	8.23	7.64	8.29
11100	7.35	6.92	7.17	8.27	7.50	8.20
A_V (mag)	1.42	2.02	3.29	1.49	2.02	2.27

Table 3. (b) Mt. Wilson measurements: WC stars

λ	16523	17638	165763	168206	192103	192641	193793
3320	7.88	7.67	7.26	6.05	6.93	6.39	5.23
3508	8.15	7.95	7.36	6.09	7.18	6.53	5.51
3658	8.08	7.77	7.26	6.17	7.06	6.47	5.32
4938	8.37	8.11	7.52	6.46	7.25	6.58	5.40
5195	8.41	8.15	7.54	6.44	7.25	6.61	5.38
5359	8.35	8.15	7.54	6.53	7.27	6.71	5.41
5531	8.52	8.28	7.61	6.56	7.41	6.79	5.53
5992	8.57	8.35	7.70	6.68	7.40	6.82	5.62
8000	8.89	8.52	7.85	6.79	7.71	7.07	5.81
10526	9.38	8.98	8.13	7.23	8.03	7.46	6.14
11136	—	8.92	8.20	7.20	8.18	7.74	6.18
A_V (mag)	1.78	2.40	0.49	2.78	0.91	1.36	1.43

The agreement with Smith's reddening corrections is quite reasonable except for HD 195177 which is not nearly so red as Smith's observations would suggest. Finally in a few cases for which we have no scanner measurements reddening corrections were determined from Smith's (1968a) narrowband measurements and

Table 3. (c) Lick measurements

λ	HD 4004	6327	9974	16523	17638	50896
8000	7.93	10.31	10.46	8.89	8.52	7.05
8580	7.95	10.25	10.47	8.94	8.51	7.03
8764	7.95	10.29	10.45	8.91	8.64	7.08
10680	8.29	10.13	10.86	9.30	8.97	7.27
11100	8.30	10.01	10.84	9.35	9.04	7.24
A_V (mag)	2.73	1.16	0.43	1.78	2.40	0.00

λ	AS 320	165688	165763	168206	169010
8000	8.19	6.51	7.85	6.79	8.54
8580	8.13	6.72	7.79	6.85	8.34
8764	8.10	6.72	7.81	6.78	8.32
10680	8.27	6.95	8.07	7.00	8.67
11100	8.32	7.00	8.11	7.21	8.93
A_V (mag)	3.93	3.39	0.49	2.78	4.10

λ	313643	186943	187282	190918	191765	192103
8000	8.01	9.88	10.17	6.49	7.03	7.71
8580	7.99	9.39	10.21	6.52	7.08	7.71
8764	8.00	9.41	10.19	6.50	7.02	7.69
10680	8.25	9.62	10.44	6.81	7.27	7.94
11100	8.34	9.83	10.61	6.81	7.37	8.04
A_V (mag)	4.01	1.20	0.78	0.78	1.34	0.91

λ	192163	192641	193077	193576	228766	193793
8000	6.39	7.07	6.98	6.40	7.72	5.81
8580	6.41	7.09	6.99	6.48	7.75	5.71
8764	6.36	7.03	6.97	6.43	7.71	5.75
10680	6.64	7.38	7.26	6.74	8.24	6.04
11100	6.72	7.59	7.46	6.98	8.19	6.25
A_V (mag)	1.51	1.36	1.42	2.02	1.65	1.43

λ	193928	195177	197406	MR 114	211564	211853
8000	6.75	7.84	8.46	9.30	10.58	7.80
8580	6.97	7.81	8.49	9.34	10.56	7.86
8764	6.89	7.87	8.53	9.35	10.52	7.74
10680	7.17	8.36	9.01	9.70	10.84	8.00
11100	7.31	8.34	8.93	9.63	10.94	8.22
A_V (mag)	3.29	4.18	2.10	3.03	1.20	1.49

λ	213049	MR 119	219460
8000	9.19	7.34	7.91
8580	9.04	7.35	7.92
8764	9.15	7.34	7.94
10680	9.50	7.69	8.24
11100	9.71	7.88	8.47
A_V (mag)	2.51	3.66	2.27

colour excesses in $(b-v)$. A standard interstellar reddening curve (Whitford, 1958) was then used to obtain the dereddened fluxes which are presented in Table 3 along with the values of A_V.

Because of the large infrared excesses and the large values of A_V for the WC 9 stars ($A_V \gtrsim 4.0$ mag) an attempt was also made to separate circumstellar from interstellar reddening and extinction using the interstellar NaD lines. These lines unfortunately occur on top of a complicated blend of He I, C II and C III lines which makes the equivalent width measurement extremely difficult because of the consequent uncertainty in the location of the continuum. In fact only D_1 can be measured with confidence. However Binnendijk (1952) has noted that $D_2 \simeq 1.21 \times D_1$ for strong lines so

Table 4. (a) Physical parameters of free-free emission regions

Star	Spectral type	Optically thick	T_e (°K)	r (kpc)	$N_e^2 R_s^3$ (cm^{-3})	N_e (cm^{-3})	R_s (cm)
HD 50896	WN 5	yes, > 10 μ	13000	1.59	1.7 (60)	1.0 (11)	5.5 (12)
191765	WN 6	no	9000	1.91	1.5 (60)	1.0 (11)	5.3 (12)
192163	WN 6	yes, 8 μ	7800	1.45	1.9 (60)	6.2 (10)	7.9 (12)
193793	WC 7 + O 5	no	6000	1.66	2.4 (60)	1.0 (11)	6.2 (12)
193576[a])	WN 5 + O 6	yes, 6 μ	6000	2.29	8.7 (59)	9.2 (10)	4.7 (12)
165688[a])	WN 6	yes, 10 μ	6000	2.00	9.4 (59)	4.0 (10)	8.4 (12)
165763[a])	WC 5	yes, 6 μ	14000	2.63	6.5 (60)	2.3 (11)	5.0 (12)
192103[a])	WC 8	no	7200	2.89	2.4 (60)	1.0 (11)	6.2 (12)

[a]) Less reliable decomposition.
If no reasonable estimate can be made of the wavelength at which the difference spectrum becomes optically thick, N_e is taken as 1.0 (11) cm^{-3}.

Table 4. (b) Physical parameters of dust emission regions[b])

Star	Spectral type	T (°K)	r (kpc)	R_s (cm)	R_{black} (cm)
HD 192641	WC 7 + Be	1100	2.09	4.2 (13)	2.6 (14)
HD 168206	WC 8 + BO:	1400	1.74	2.7 (13)	1.6 (14)
AS 320	WC 9	1100	2.79	7.2 (13)	3.2 (14)
HDE 313643	WC 9	940	3.01	1.9 (14)	3.6 (14)
HD 192103[a])	WC 8	1500	2.89	1.8 (13)	1.4 (14)
LS 15[a])	WC 9	1200	3.55	4.3 (13)	2.2 (14)

[a]) Less reliable decomposition.
[b]) Assuming an optically thick dust shell [see Eq. (5)].

Table 5. Magnitudes and distances of WC 9 stars

Star	v	A_V	A_V	v_0	r (kpc)
Ve 2–45	13.45	6.54	5.88	7.00	1.91
HDE 313643	12.36	4.37	3.93	7.99	3.01
AS 320	12.28	4.46	4.01	7.82	2.79
LS 15	12.59	4.24	3.81	8.35	3.55

that $D = \dfrac{D_1 + D_2}{2} = 1.10 \times D_1$ can be used to determine the average NaD line strength. This can then be used with the formula for the average distance derived by Beals and Oke (1953) to estimate the distance. In addition the total extinction can be estimated from Binnendijk's (1952) relation between colour excess, distance [converted to $(B-V)$] and NaD line strength along with the assumption that $A_V \simeq 3\, E_{B-V}$. Admittedly this is a chain of reasoning with very weak links. Nevertheless it is instructive to see how far it can be pushed. For all WC 9 stars in our sample (except Ve 2–45) we find values of A_V equal to or larger than the A_V's derived in Table 5. Considering the gross nature of the calculations (i.e. the error in A_V is at least 1 mag) we can treat the two values as being in essential agreement. That is to say, there seems to be no circumstellar component following a non-grey extinction law. This is consistent with the existence of WC 9 stars with small amounts of reddening (e.g. HD 157451 and 164270 for which $A_V = 1.38$ and 1.26 mag respectively; Smith, 1968b). However for Ve 2–45 the value of A_V derived from the interstellar D lines is 3.6 mag smaller than the value in Table 5 derived from the colour thus implying a substantial amount of circumstellar extinction. The implications of this result will be discussed further in Section V below.

Another possible explanation of the infrared excesses in the late WC stars is an anomalous reddening law. However, let us compare the values of M_V derived for the four WC 9 stars discussed in this paper, and the slightly reddened WC 9 star HD 164270, for a value of $R = A_V/E_{B-V}$ equal to 5. For all five stars, distances were estimated from the interstellar D lines. For $R = 3$, HD 164270 has M_V in essential agreement with those derived for the other four stars. But if $R = 5$, there is a discrepancy of 3–4 mag between HD 164270 and the other stars. Therefore, we reject the hypothesis of anomalous reddening.

b) The Energy Distributions

The long wavelength data have been combined with the dereddened scanner photometry and 1.6 and 2.2 μ measurements to produce composite energy distributions (λF_λ in W cm^{-2}) for the stars. The reddening corrections used in the near infrared are $A_{1.6}/A_V = 0.14$, $A_{2.2}/A_V = 0.10$, $A_{3.6}/A_V = 0.03$ and the values of A_V are those given above in Table 3. The dereddened scanner data for a given star from 0.33 μ to 1.11 μ define, usually without ambiguity, a black body curve from beyond the peak on into the Rayleigh-Jeans region. At wavelengths $\gg 1$ μ, this curve has been extrapolated by a line of $\lambda F_\lambda \propto 1/\lambda^3$. Photometric points between 1.1 μ and 11.3 μ have been connected by smooth curves to define the "observed" energy distribution of the star. It is then possible to construct the spectrum of the excess infrared radiation by subtracting the Rayleigh-Jeans tail from the smooth "observed" curve yielding a "difference spectrum". This process clearly produces an appreciable scatter at short wavelengths ($\lambda \sim 1$ μ), where the difference between the two curves is very small. However, it provides a consistent and unbiased method of extracting the infrared component of an energy distribution. A smooth curve is then drawn through the point-by-point "difference spectrum".

Figure 3 displays the results of this procedure when applied to the data for seven stars: HD 192163, 193793, 165763, 168206, HDE 313643, LS 15 and AS 320. For comparison, curves corresponding to a blackbody and to optically thin free-free emission are also shown. For several stars, the decomposition of the energy distribution produces an easily recognizable difference spectrum: either a blackbody (HD 168206), or an optically thin free-free curve (HD 193793), or a free-free curve which becomes optically thick at some wavelength between about 4 and 10 μ (HD 165763). Unfortunately not all stars for which composite λF_λ curves may be drawn can be reduced to such simply recognized elements. Nevertheless, the spectra of six stars (HD 50896, 168206, 191765, 192163, 192641 and 193793) may be decomposed reasonably satisfactorily and of five others

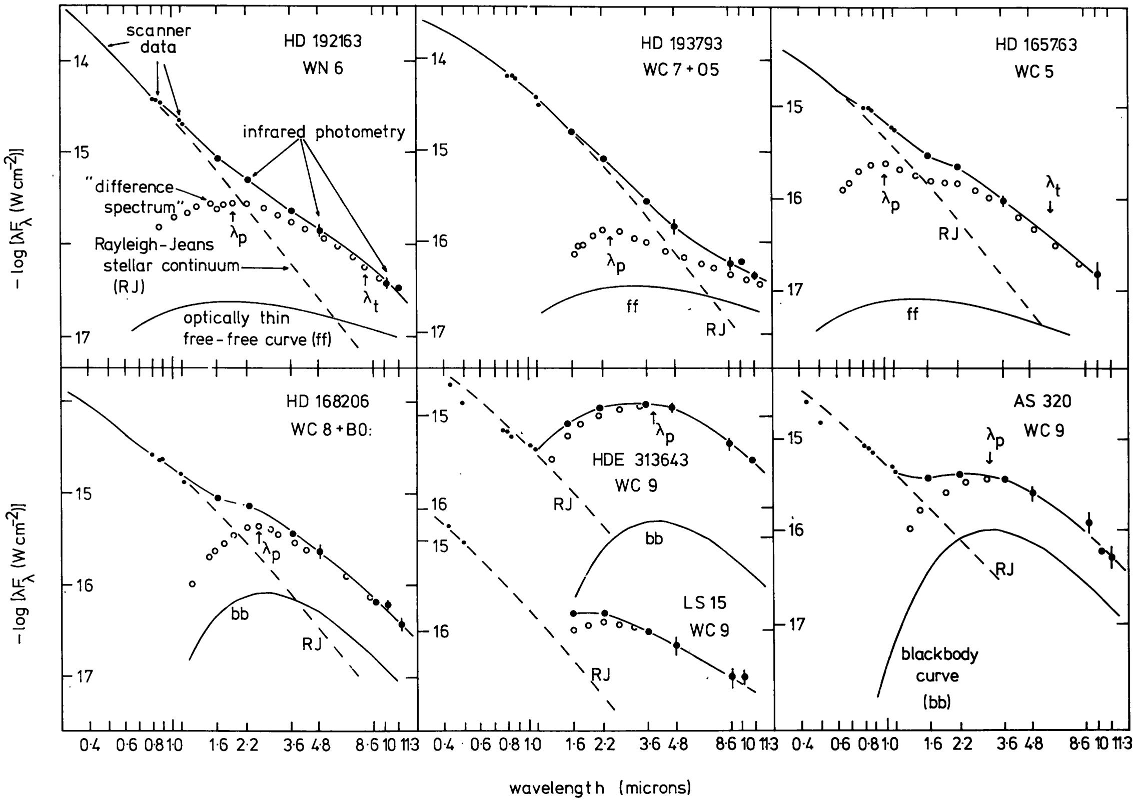

Fig. 3. λF_λ plots for the composite energy distributions of seven Wolf-Rayet stars, showing the method of decomposing the spectra using the dereddened scanner data. The short wavelength scanner data is represented by a smooth curve, while filled circles represent the photometry between 8000 and 11 000 Å (small circles) and the longest wavelength observations (large circles). 1σ error bars are shown for the 2–11 μ points. The adopted wavelengths of the peaks for the infrared difference spectrum are indicated (λ_p) as are the points where free-free curves become optically thick (λ_t)

with less confidence (HD 165763, 165688, 190918, 192103 and 193576). In addition to these eleven stars, the difference spectra of the three WC 9 stars (HDE 313643, LS 15 and AS 320), for which scanner data are available only between 0.8 and 1.1 μ, seem well matched by blackbody curves at long wavelengths but are broader than Planck curves at ~1–2 μ. No scanner data are yet available for the remaining three stars for which infrared photometry exists (AS 422, AS 374 and MR 112).

Although many Wolf-Rayet stars are members of binary systems this poses no problem when reducing spectra by the above method. Typically the other component of all known Wolf-Rayet binaries is an OB star of temperature comparable to that of the Wolf-Rayet. Longward of 1 μ, the energy distributions of both the Wolf-Rayet and the OB star are in their Rayleigh-Jeans regions; consequently the binary produces a λF_λ curve with a Rayleigh-Jeans slope also. This slope is defined by the scanner measurements of the binary and, when subtracted from the "observed λF_λ" curve, the resulting difference spectrum correctly indicates the infrared excess radiation.

All stars for which photometry is available beyond 3.5 μ have substantial excess radiation in the infrared (over the extrapolated Rayleigh-Jeans tails): among the WN stars, only free-free excesses appear to be present; among the WC stars both free-free and blackbody excesses are found, although blackbodies occur only in types WC 7–9 of the present sample.

A possible source of error in defining the infrared excess is in the assumption that the photometric points may be connected by a smooth curve. If substantial line emission occurs in some broad bandpass filters this assumption is invalid. Kuhi (1968) has investigated the near-infrared spectra of Wolf-Rayets and finds that in WN stars only a small fraction (10–30%) of the total flux is carried by emission lines whereas in the WC stars this line flux is a substantial fraction (30–50%) of the total. This percentage decreases with increasing wavelength. In the WC stars the emission line spectra of C IV, III and II are extremely well developed and may be expected to contribute something to the broadband 1.6 and 2.2 μ fluxes. However, extrapolating from the data shortward of 1.1 μ we can estimate that this contribution is at most 20% and is very likely to be considerably less. In the 10 μ region there are no significant features either in emission or absorption. Consequently the above interpolation should be valid for all wavelengths.

IV. Physical Parameters

Once the difference spectrum is recognized as a free-free curve, or as a blackbody, we may proceed to derive some physical parameters of the emitting region.

a) Free-free Spectra

For optically thin free-free emission we may write,

$$N_e N_i R_S^3 = 1.46\,(30)\,F_{\rm diff}(\lambda)\,\lambda^2 r^2\,T_e^{1/2} Z^{-2} \cdot \exp(C_2/\lambda T_e)\,\mathrm{cm}^{-3}\,, \tag{1}$$

where $F_{\mathrm{diff}(\lambda)}$ W cm^{-2} μ^{-1}, is the intensity of the difference spectrum at wavelength λ (microns), r in cm is the distance of the star, R_S in cm the radius of the emitting region, $C_2 = 14400$ μ °K and N_e, N_i, T_e and Z have their usual meanings (the Gaunt factor in the infrared is taken as unity).

If the difference spectrum indicates that the emission has become optically thick at some wavelength λ_t (microns) then we also have

$$N_e N_i R_S = \frac{7.3\,(34)\,T_e^{1/2}}{\lambda_t^3 Z^2 (1 - e^{-C_2/\lambda_t T_e})}\,\mathrm{cm}^{-5}\,. \tag{2}$$

We require N_i in terms of N_e and a value for Z. The WN stars are grossly hydrogen deficient (e.g. see Smith, 1973) so that $N(\mathrm{H})/N(\mathrm{He}) = 0$ for WN 4–6 stars, and $N(\mathrm{H})/N(\mathrm{He}) = 1$ for WN 3, 7 and 8. We assume that He is only singly ionized in the cool WN 7 and 8 stars, but is twice ionized in the earlier WN's. This leads to $Z = 1$, $N_i = N_e$ for WN 7, 8; $Z = 2$, $2N_i = N_e$ for WN 4–6 and $\bar{Z} = 1.5$, $1.5 N_i = N_e$ for WN 3 stars. Since there is a lack of similar information for WC stars, we adopt $Z = 1$, $N_i = N_e$.

A free-free emission spectrum has a maximum, indicative of T_e: if the peak of λF_λ occurs at λ_p, then $\lambda_p T_e = 14400$ (λ_p in μ, T_e in °K). Consequently we derive T_e immediately from the difference spectrum. Unfortunately it is possible to see a spectral turnover due to optical thickness only for HD 50896, 165688 and 192163 and less precisely for HD 165763 and 193576. In general, then, $N_e^2 R_S^3$ may be obtained but not $N_e^2 R_S$. To derive these quantities, the values $F_{\mathrm{diff}}(\lambda)$ of the free-free spectrum at several wavelengths (on the optically thin portion) are substituted into (1) and the average value of $N_e^2 R_S^3$ is found. If a turnover can be distinguished, the wavelength λ_t is approximated and $N_e^2 R_S$ results from (2). Hence we solve for N_e and R_S.

Table 4a presents the derived values of T_e, $N_e^2 R_S^3$, N_e and R_S, and the distance used in the computation. Distances are taken from Smith (1968b) and revised according to the new absolute magnitude scale for Wolf-Rayet stars (Smith, 1973). More reliably reduced difference spectra are indicated together with a note as to the presence of an optical thickness turnover. If the spectrum remains optically thin, N_e has been taken as 10^{11} cm^{-3} (cf. Munch, 1950) and this yields a value of R_S.

b) Blackbody Spectra

If the peak of a blackbody curve in λF_λ space is at λ_p microns, then the temperature is given by $\lambda_p T = 3670$ μ °K. The temperatures obtained for the blackbody components in Wolf-Rayet stars (see Table 4b) are all reasonable for dust grains, especially graphite grains which can condense at relatively high temperatures (Gilman, 1969; Hackwell, 1971) (1500–2000 °K). The reduced spectra of the WC stars which have cool blackbody-like components show no indications of spectral features near 10 μ (e.g. as silicate materials would produce). If thermal emission by dust grains were responsible for the excesses in WC stars then one may infer that either (1) the grains are optically thick in the 1–11 μ region (no features would then be seen) or (2) the infrared emission is optically thin and the grain material has no resonances within this wavelength range.

It is noteworthy that the WN stars show only free-free emission whereas cool blackbody components occur in the latest WC type stars. Even the energy distribution of AS 374, the sole WN 8 star in our sample, shows no evidence for a blackbody excess when an attempt is made to reduce its spectrum using Pyper's (1966) dereddened *UBV* data. This suggests that whatever the composition of the dust grains, carbon is likely to be a constituent, and that graphite grains satisfy all of the observational requirements. For graphite, we may exclude case (1) since it would imply enormous circumstellar extinction at V (in contrast to the $A_V \approx 0$ deduced in Section IIIa) and we should then not expect to see these WC stars at short wavelengths. Graphite has the merit of having no resonances in the infrared and case (2) would seem to apply. Consequently, the flux, $F_{\mathrm{diff}}(\lambda)$ is given by

$$F_{\rm diff}(\lambda) = \pi B_\lambda(T)\,(1 - e^{-\tau_\lambda})\,(R_s/r)^2 \tag{3}$$

and the optical depth at infrared wavelengths

$$\tau_\lambda = N\,Q_{\rm abs}(\lambda)\,\pi \boldsymbol{a}^2\,, \tag{4}$$

where T is the dust temperature, r the distance of the star, N the column density of grains/cm^2 through the infrared geometrically thin region where dust grains occur, $\boldsymbol{a}$ is the radius of a grain (assuming all grains to be spherical and of the same size), and $Q_{\mathrm{abs}}(\lambda)$ is the absorption efficiency factor.

If we make the unrealistic assumption that the shell is optically thick in the infrared, we have

$$F_\lambda = \pi B_\lambda(T)\,(R_s/r)^2\,. \tag{5}$$

F_λ has been measured from the difference spectrum at several wavelengths and the average value of (R_s/r)

obtained from (5). Table 4b presents the deduced parameters for WC stars. This value of R_s may be directly compared with the distance ($R_{\rm black}$) from a given WC star that a hypothetical black sphere must have to radiate at the observed temperature. Taking T^* as 30000 °K for WC 7, 8 and 9 stars we calculate the values of $R_{\rm black}$ in column 6 of Table 4b. We note in passing that for HD 192641 there is no way to determine whether the dust component comes from the WC star or its companion Be star (cf. Allen, 1973).

It is worthy of note that the difference spectra are well-matched by blackbody curves at long wavelengths (~10 μ) yet lie above these curves in the near infrared (~1.6 μ). One may infer either that dust grains are present within a very narrow range of temperature and that some free-free emission may occur around 1.6 μ, or that we are observing a shell of grains with a well-defined outer edge but that hotter material (which is emitting in the 1–2 μ region), perhaps recently ejected from the star, creates a diffuse inner edge.

c) Momentum Flux

Kuan and Kuhi (1975) have shown that for stars showing a classical P Cygni emission-line profile and having roughly the same temperature there is a clear separation on a diagram of $(\varrho v^2)_0$, the momentum flux at the base of the envelope, vs. the [2.2]–[10] colour index, between stars with free-free and with dust infrared excesses. The critical value of ϱv^2 at which this separation occurs also shifts to lower values for increasing temperature. The implication is that dust grains either cannot form from ejected material or cannot survive from the time of formation if the critical value of ϱv^2 is exceeded. It would be of considerable interest to see whether the same conclusion applied to WR stars. However several additional problems manifest themselves, chiefly the lack of a suitable model for the atmosphere of the star and its expanding envelope. A rough estimate can still be obtained by assuming that the conditions in the envelope are the same for each sequence; i.e. we can take $T_e = 30000$ °K and $N_e = 4 \times 10^{11}$ cm^{-3} [as obtained by Castor and Nussbaumer (1972) for C III line formation in γ^2 Vel] for all the WC stars and $T_e = 50000$ °K and $N_e = 10^{12}$ cm^{-3} [as obtained by Castor and Van Blerkom (1970) for He II line formation] for the WN stars. This approach of course ignores completely the change in physical conditions and stratification that must occur with the likely change in temperature due to the different spectral subclasses. In addition we have estimated a value of velocity v from the C III 5696 half-width at half-intensity for the WC stars and from the He II 4686 line for the WN stars. The line-width data come from the unpublished Atlas of WR Spectra (Smith and Kuhi, 1974) or from more recent spectra (e.g. for AS 374) obtained at Lick Observatory. In all cases the actual line width

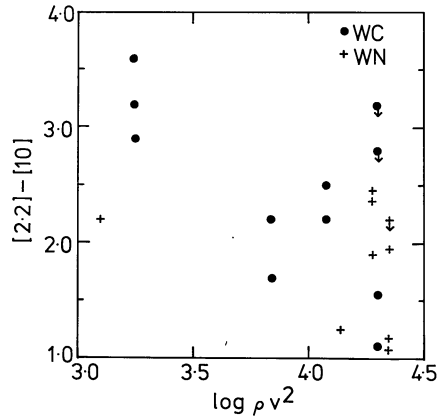

Fig. 4. The plot of momentum flux against [2.2]–[10] colour index. Arrows denote upper limits. Logϱv^2 increases systematically from WC 9 to WC 5. No such systematic increase exists for the WN stars. The isolated WN star at the left of the diagram is of spectral type WN 8

measurements were used when available; otherwise the width for the same spectral type was used taking into account the distinction between WN binary and "single" stars. This method does not strictly lead to the same quantity $(\varrho v^2)_0$ plotted by Kuan and Kuhi but the value of ϱv^2 that we do obtain should give a reasonable impression of the behavior of the momentum flux at some "representative" point in the envelope.

Figure 4 shows a wide separation between the WC 9 stars and the rest of the WC's which reflects again their different infrared energy distributions, the WC 9 stars having proportionally more energy in their infrared component relative to the stellar flux. The WN stars show little change in [2.2]–[10] with ϱv^2 although the single WN 8 star is well separated from the others. The diagram perhaps reflects the inhomogeneity of the WN stars which do not define any clearcut sequence of spectral characteristics (such as line-widths) as do the WC stars. The WC 9 stars however do provide strong support for the idea that dust grains form from the ejected material since it is difficult to see how any primeval grains could have survived for so long in the vicinity of a hot star without having been blown away or destroyed. This also presents difficulties in the formation of grains in the first place but that is an outstanding problem for the theoretician.

V. WC9 Stars

There are several principal points we wish to discuss for WC 9 stars (including Ve 2–45 for which we have no new infrared photometric data): the coudé spectra and evidence for grain formation or companion stars; the distances and A_V; and more realistic estimates of the sizes of the inferred dust shells using the optical properties of graphite and the energy balance.

As already mentioned the optical spectra are illustrated in Fig. 1. We note that lines from ions of C II to C IV are present along with those of He I and II and that there are no major differences among the WC 9 stars. However the ratio of emission line intensity to that of the continuum seems somewhat lower in Ve 2–45 than in the others and hence might indicate the presence of an early-type companion. A rough estimate of this ratio suggests that the luminosity of the companion must be ~ 2.2 times that of the WR star at λ 6000. Since Humphreys and Ney (1974) have suggested that a late type companion may be responsible for the infrared radiation of this star the excess optical continuum radiation could equally well be due to the late type star. However this can be immediately ruled out on observational grounds. Late K or early M stars contributing sufficient continuum to "drown" the emission lines would also contribute their own absorption line spectrum; in particular the infrared Ca II triplet should be present. Late M stars, on the other hand, should show very strong molecular bands of TiO in the observed wavelength region. However no trace of any absorption feature is found (other than interstellar and terrestrial lines) and consequently one can definitely rule out the presence of a late type companion. This conclusion applies to the other WC 9 stars as well because their spectra also show no late type absorption features.

The total absence of stellar absorption features rules out the presence of various molecules of carbon such as C_2 or C_3 which might be formed in the process of forming the hypothesized graphite grains. The over-abundance of carbon in WC stars (Castor and Nussbaumer, 1972), the fact that no WN star possesses a blackbody-like difference spectrum, the lack of infrared spectral features and the relatively high temperature of the dust emission strongly suggest graphite as a constituent of the grains. If, as suggested above, the grains are actually formed from the ejected material then one might expect simpler compounds of carbon to form along the road to graphite particles. However the lack of either C_2 or C_3 suggests that these molecules are not formed in sufficient quantity to present any appreciable optical depth at any given time to allow them to be detected. Alternatively they may indeed be formed in large numbers but their life-time is so short that no large optical depth can be built up. This is in marked contrast to the usual carbon stars (such as R CrB, Humphreys and Ney, 1974), in which C_2, C_3 and CN bands clearly dominate the spectrum. The observed infrared excess in R CrB is similar to that in the WC 9 stars but is much broader and flatter, suggesting a geometrically thick shell with a temperature gradient. In addition its photospheric temperature is much lower and hence would be considerably more favorable to the presence of such molecules in very large numbers.

An attempt to find circumstellar extinction by the indirect use of the interstellar NaD lines was described above. The inherent uncertainties in this method probably amount to at least ± 1 magnitude and we are unable, therefore, to preclude the possibility of a few tenths of a magnitude of circumstellar reddening for all stars but Ve 2–45. It is of interest to estimate the distance from the WC 9 stars of their associated dust grains in the light of this modest amount of local extinction, for, as discussed above, it is unrealistic to hypothesize grains which are optically thick in the infrared region.

We note that if Δm_c is the circumstellar visual extinction due to the dust shell then

$$\Delta m_c = 1.086\, N \pi \boldsymbol{a}^2\, Q_{\mathrm{ext}}(V)\,, \tag{6}$$

where N is the column density of grains cm^{-2} through the dust shell, and all grains are assumed to be spherical and of radius $\boldsymbol{a}$. $Q_{\mathrm{ext}}(V)$ is the visual extinction efficiency factor. We shall adopt $\Delta m_c = 0.5$ mag for all the WC 9 stars except Ve 2–45, for which we use 3.6 mag. Due to the likelihood of graphite being a major constituent of the dust shell, we shall compute the shell sizes for spherical graphite grains of radii 0.01 and 0.1 μ for which $Q_{\mathrm{ext}}(V)$ is 0.1 and 3.3 respectively. Equation (6) yields the product $N\pi\boldsymbol{a}^2$. Infrared optical depths are then calculated from Eq. (4) and shell radii $R_g(\boldsymbol{a})$ from (3), denoting the previous "R_s" by $R_g(\boldsymbol{a})$ in the present calculation. To illustrate the various derived values of the radius of the dust shell under different assumptions, we have evaluated $R_g(\boldsymbol{a})$ for the four WC 9 stars. Table 6 brings together the various values of shell radius alluded to in the text. The optical depths at the wavelengths 3.6 and 10 μ for the three stars with 0.5 mag circumstellar extinction, are 0.013 and 0.002 ($\boldsymbol{a} = 0.1$ μ) or 0.031 and 0.006 ($\boldsymbol{a} = 0.01$ μ), emphasizing the unrealistic nature of an assumption of optical thickness at long wavelengths.

For these small grains, Q_{abs} is a rather strong function of λ, and the values of R_g (.01) and R_g (.1) in Table 6 are the averages of R_g determined at several wavelengths. Yet the observations strongly suggest a Planckian shape for $F_{\mathrm{diff}}(\lambda)$ at wavelengths beyond about 3 μ. We wish to retain the idea that graphite is a component of the grains. Consequently, we must argue that the grains are sufficiently large that they are almost grey from about 2 to 10 μ, when the convolution of $Q_{\mathrm{abs}}(\lambda)$ and a blackbody function is still sensibly Planckian. This requires grains with radii on the order of 1 μ. Table 6 includes R_g computed for 1 μ grains.

We may easily derive the value of $R_g(a)$ for any given star if Δm_c is not 0.5 mag; if $\Delta m_c = 0.5x$, then $R_g(a, 0.5x) = R_g(a, 0.5)\, x^{-1/2}$ [as Eq. (3) demonstrates].

Another calculation which may be made more realistic by assuming that graphite grains are present in the shell is that of the "black sphere distance". Specifically, we have

$$\int_0^\infty Q_{\mathrm{ext}}(\lambda, \boldsymbol{a})\, B_\lambda(T^*) \left(\frac{R^*}{R_{ss}}\right)^2 d\lambda = \int_0^\infty Q_{\mathrm{ext}}(\lambda, \boldsymbol{a})\, B_\lambda(T)\, d\lambda\,, \tag{7}$$

Table 6. Deduced parameters for WC 9 stars

Star	Δm_c	T	R_s	R_{black}	$R_g(.01)$	$R_g(.1)$	$R_g(1)$	$R_{ss}(.01)$	$R_{ss}(.1)$	$M_{\text{dust}}(.01)$	$M_{\text{dust}}(.1)$	$M_{\text{dust}}(1)$	L_{bb}/L^*	Flux ratio
Ve 2–45	3.6	920	5.5 (14)	3.7 (14)	2 (15)	3 (15)	6 (14)	8.1 (15)	2.4 (15)	1 (−6)	7 (−7)	3 (−7)	0.83	7.4
HDE 313643	—	940	1.9 (14)	3.6 (14)	2 (15)	3 (15)	3 (14)	8.2 (15)	2.6 (15)	1 (−7)	9 (−8)	1 (−8)	0.04	0.8
AS 320	—	1100	7.2 (13)	3.2 (14)	6 (14)	9 (14)	9 (13)	6.9 (15)	2.1 (15)	2 (−8)	1 (−8)	1 (−9)	0.01	0.2
LS 15	—	1200	4.3 (13)	2.2 (14)	4 (14)	7 (14)	8 (13)	2.9 (15)	8.4 (14)	1 (−8)	8 (−9)	8 (−10)	0.06	0.15

The assumptions made are:
R_s optically thick thermal emission at long wavelengths.
R_{black} grains with black absorption properties come into radiative equilibrium at temperature T near a 30000 °K star, of $R^* = 10\ R_\odot$.
$R_g(\boldsymbol{a})$ graphite grains of radius $\boldsymbol{a}$ produce both the infrared thermal emission and Δm_c mag visual extinction
$R_{ss}(\boldsymbol{a})$ graphite grains of radius $\boldsymbol{a}$ come into radiative equilibrium at T near a 30000 °K star, of $R^* = 10\ R_\odot$.
All radii are in cm; M is in $M_\odot$.

where R^* and T^* are the stellar radius and temperature respectively, R_{ss} the shell radius, T the grain temperature, and $Q_{\text{ext}}(\lambda, \boldsymbol{a})$ the extinction efficiency factor for graphite. In the absence of appreciable shell extinction at V, it seems most unlikely that multiple scattering by the grains is important and we shall take the single-scattering situation in which $Q_{\text{ext}}(\lambda, \boldsymbol{a})$ is replaced by $Q_{\text{abs}}(\lambda, \boldsymbol{a})$. Table 6 shows the results of this calculation, with grains of radius .01 μ or .1 μ. (The larger grains exist in a more distant shell than the smaller principally due to their great ability to radiate energy away.) All the above calculations were performed using Mie scattering theory and optical constants for graphite due to Taft and Phillips (1965).

We may also derive the mass of circumstellar dust grains from these calculations since this is

$$M_{\text{dust}}(\boldsymbol{a}) = \frac{4\pi}{3} R_g^2(\boldsymbol{a})\, N \frac{4\pi}{3} \boldsymbol{a}^3 \varrho\,, \tag{8}$$

where ϱ is the density of graphite, taken as 3 g cm^{-3}, and N, $\boldsymbol{a}$ and $R_g(\boldsymbol{a})$ have their previous meaning. These dust masses are given in Table 6, for the three sizes of grain.

It is interesting to note that if we form the ratios of λF_λ at the peak of the blackbody excess to that at V for the four WC 9 stars, then these ratios are essentially proportional to the derived dust masses. The final column of Table 6 presents the flux ratios.

Another question about the dust emission of the WC 9 stars relates to the fraction of total stellar energy radiated by the blackbody-like component. This fraction has been evaluated for the three WC 9 stars observed by us in the infrared, and Ve 2–45 using the photometry to 10 μ by ASH. Table 6 includes as its penultimate column the radio of the luminosity in the dust component (L_{bb}) to the total stellar luminosity (L_*). Each luminosity was obtained by normalizing a blackbody of temperature 30000 °K to the dereddened data on the stellar continua (see Fig. 3), and of temperature T, the grain temperature to the difference spectrum.

These luminosity ratios emphasize the difference between the physical parameters of the dust shell of Ve 2–45 and those of the other three WC 9 stars. This difference also exists in the values of $R_g(\boldsymbol{a})$ and $M_{\text{dust}}(\boldsymbol{a})$, in the sense that Ve 2–45 possesses both a larger and more massive shell. It should be noted that our estimate of $\Delta m_c = 0\overset{m}{.}5$ represents a maximal value for the three WC 9 stars. Lowering the value of Δm_c serves to increase slightly both the shell radii and masses.

Reviewing the various estimates of shell radii in Table 6, we note that R_s is totally unrealistic, and both R_{black} and its graphite counterpart R_{ss} are highly dependent on an integrated energy balance using a unique grain temperature. The difference spectra do suggest that grains hotter than this temperature are observed in the near-infrared. However, R_g depends on the longer wavelength properties of the difference spectrum (near the peak of the dust component) and is directly related to an independent property, Δm_c. Consequently, of these several estimates, R_g would appear to be the most reliable.

VI. Conclusions

It is appropriate to compare the physical parameters of the region in which the infrared component of the free-free emission arises with the canonical picture of the envelopes of Wolf-Rayet stars. The work of Castor and Van Blerkom (1970) shows that for a WN star representative parameters are $T_e = 50000$ °K, $N_e = 10^{12}$ cm^{-3} and $R_s \simeq 3 \times 10^{12}$ cm. Table 4a suggests that the infrared emission derives from a cooler and less dense region of somewhat greater extent than the line emitting region. All models for Wolf-Rayet stars have a temperature and electron density decreasing with increasing radius. In addition Eq. (1) indicates that the infrared flux is strongly weighted by the cooler outer parts of the envelope. Consequently our derived parameters are qualitatively consistent with the existing models for WN stars. A similar conclusion can be drawn for the WC stars having free-free emission by comparison with the model of Castor and Nussbaumer (1972) for γ^2 Vel for which $T_e = 22000$ °K, $N_e = 4 \times 10^{11}$ cm^{-3} and $R_s = 4 \times 10^{12}$ cm.

A consequence of the free-free emission model is that bound-free emission should be present at the Balmer discontinuity (for those stars having an appreciable amount of hydrogen) or at the Pickering discontinuity (for those stars deficient in hydrogen). For example, taking the two stars for which this discontinuity is expected to be the largest (i.e. HD 50896 WN 5 and HD 165763 WC 5) we find that the bound-free emission contributes only 10% and 25% respectively to the total flux at λ 3643. Because of the extremely rich emission line spectrum of WC 5 stars it is impossible to locate the continuum and therefore the bound-free discontinuity cannot be recognized. For the WN star the detection of a 10% discontinuity would be somewhat easier but still difficult because of the presence of emission lines. Considering the assumptions involved in such calculations and the errors of absolute flux measurements we feel that there is no conflict between existing observations and our interpretation of the infrared excess as free-free emission.

For the WC stars having dust shells it should be noted that the characteristic shell sizes, particularly for the WC 9 stars, are far larger than the much hotter regions in which emission lines arise (see Tables 4b and 6). Therefore dust grains are not required to exist in a hostile environment; in fact they are found only around the coolest WC stars (WC 8 and WC 9; the dust component in the WC 7 star may be due to the Be companion).

Only one WC star, HD 192103, has a difference spectrum that could equally well be fit by either free-free or dust emission and consequently it has been included in both Table 4a and b.

The findings of Section V suggest the following evolutionary scheme for the WC 9 stars. In the early stages of the Wolf-Rayet phase a WC 9 star is surrounded by a compact, low mass shell of gas and dust of its own making which produces very little visual extinction. As the star evolves and continues to undergo mass loss, the shell expands outwards and the total mass in grains increases causing the visual optical depth to increase appreciably. On this basis, Ve 2–45 with its large circumstellar extinction and ratio of infrared to total luminosity would be relatively an evolved object.

The nuclei of some planetary nebulae are classified as WC stars and it is of interest to note that dust apparently exists around these objects, although at much cooler temperatures (Cohen and Barlow, 1974). For example, the WC 6 nucleus of NGC 6751, and Campbell's hydrogen envelope star (BD + 30° 3639) of type WC 9 both have large infrared excesses interpreted as thermal emission by cool ($\sim$200 °K) dust grains. However, no WN star is found as the nucleus of a planetary nebula but rather these stars occur in ring nebulae. The nebula M 1–67, containing a WN 8 star, although regarded as a planetary nebula in the past, is almost certainly not one (Cohen and Barlow, 1975). Among the WN stars in our sample, three lie within ring nebulae (see Table 1) but none shows infrared excesses attributable to dust. However, the physical conditions within a ring nebula are very different from those in a planetary—in particular, the mass of gas within a ring nebula is typically much greater than the mass of the exciting star. It would appear that the mass is predominantly interstellar material swept up by the large momentum flux of the star. Therefore, one would not necessarily expect that dust would form in the outflow as has been suggested for the case of planetary nebulae by Cohen and Barlow (1974). The evolutionary status of WN stars in ring nebulae remains to be established.

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