

# Decentralised sliding mode control for a class of nonlinear interconnected systems by static state feedback

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## Abstract

In this paper, a class of interconnected systems is considered, where the nominal isolated systems are fully nonlinear. A robust decentralised sliding mode control method is addressed based on the static state feedback. By local coordinate transformation and feedback linearization, the considered interconnected system is transformed to a new interconnected system in regular form. A composite sliding surface is proposed by using the system state variables, and the stability of the corresponding sliding mode is analysed. In light of a new proposed corresponding reachability condition, a robust decentralised sliding mode control is then designed to drive the system states to the sliding surface in finite time and maintain a sliding motion thereafter. Both uncertainties and interconnections are allowed to be unmatched and assumed bounded by nonlinear functions. The bounds on the uncertainties and interconnections have more general forms when compared with the existing works. A MATLAB simulation example is used to demonstrate the effectiveness of the proposed method.

**Key Words:** Decentralised control, Sliding mode control, Nonlinear interconnected systems, Static state feedback.

## I. INTRODUCTION

Increasingly complex industrial requirements coupled with rapid technological developments are producing engineering systems that are interconnected to form large-scale interconnected systems [1]. Such systems exist widely in multiple domains including process networks, power grids, offshore platforms, urban traffic networks as well as ecosystems [2]. Large-scale complex systems can require high on-line computing power to implement controllers. For example, in a centralized control implementation, all of the system information may be used to control the interconnected system. Such a large computational burden may not be attractive to industry as data transfer requirements are large and risks of cyber attack increase. Plant shut down may result from corruption or blockage of data transfer and the system robustness may be compromised [3]. This has motivated the development of decentralised control strategies in which only local subsystem information is needed for local control design. A decentralized control approach has the potential to improve the computational efficiency and the overall safety of the system. Hence the study of decentralized control is of significant importance for interconnected systems [4].

Decentralized control has been studied for many years and many results have been obtained relating to interconnected systems [5], [6], [7]. It is well known that uncertainties or modeling errors can seriously affect the performance of controlled systems. The uncertainties in subsystems will not only affect their own performance, but also affect the entire system's performance through the interactions among subsystems. In addition, the coupling between subsystems also brings difficulties to the stabilization of interconnected large-scale systems. Robust control schemes and adaptive control schemes are often designed to reject or reduce the effect of the uncertainties caused by the modeling errors within subsystems and the coupling among subsystems. Adaptive control can only be applied to some special systems, i.e. parametric uncertainties satisfying linear growth conditions [8], [9]. Alternative robust control methods, such as  $H_\infty$  control [10], [11], have strong robustness in dealing with large-scale operating regions even in the presence of disturbance scenarios, but the design process is complex and the synthesis can be difficult and time-consuming [12], [13].

As one of the classical robust control methods, sliding mode control [14] is widely considered to be an effective method to deal with uncertainties due to its excellent performance characteristics and relatively simple design process. Decentralized

sliding mode control has been extensively studied in large-scale systems. Based on state information, a decentralized sliding mode control is designed to deal with the matched uncertainties in [15]. A decentralised adaptive sliding mode control is designed for large-scale systems with unmatched perturbations in [16]. A type of decentralised output feedback sliding mode control is designed for a class of interconnected systems in [17], where only matched uncertainties and interconnections are considered. A class of interconnected systems is stabilized by an output feedback sliding mode controller with a more general structure for the interconnections and unmatched uncertainties in [18]. Note that the mentioned literatures require that the nominal isolated systems are linear. In practice, almost all systems are nonlinear and have nonlinear behaviour [19]. Moreover, the majority of control schemes for linear systems cannot be applied to nonlinear systems directly. These motivate the study of nonlinear interconnected systems with nonlinear nominal isolated systems and nonlinear interconnections.

Recently, the study of interconnected systems with nonlinear nominal isolated systems has received great attention and some interesting results have been achieved. Based on static output feedback, decentralized stabilization of nonlinear interconnected systems has been achieved in [20]. However, this output feedback method imposes higher requirements for the system model. The decentralized tracking control problem is studied for a class of interconnected stochastic nonlinear systems in [21], where the result obtained can only guarantee that the closed-loop system is practically stable. A smooth dynamic output feedback control is proposed for the case of multiple time-delay systems with nonlinear uncertainties in [22]. However, it is based on dynamic feedback, which increases the system dimension as well as the computational complexity. A decentralised state feedback control scheme is proposed for a class of time delay interconnected systems with matched interconnections, and a new term, called weak unmatched uncertainty, is introduced in [23]. It should be noted that high control gains are used in [23] to reject the effect of weak unmatched uncertainties and the matched interconnections, which in turn may lead to control input saturation. An adaptive dynamic programming technique is introduced for nonlinear large-scale systems in [24]. Although the nominal isolated systems is nonlinear in [24], only matched interconnections is considered. A multi-dimensional Taylor network decentralised control is investigated for large-scale stochastic nonlinear systems in [25], but only the tracking of specific signals has been considered. The knowledge that sliding mode control can deal with unmatched uncertainties to a certain extent [26] provides motivation for the design of a decentralised sliding mode control for nonlinear interconnected systems.

There are few results which apply sliding mode control methods to interconnected systems with nonlinear nominal isolated systems. A decentralised control strategy based on sliding mode techniques is proposed for a class of nonlinear interconnected systems in regular form in [27]. Although this method does not require the nominal isolated subsystems to be linearized or partially linearized, it is assumed that the system is in regular form, which requires transformation of the nonlinear systems; a constructive method to obtain the transformation matrix is not generally available. Further, the proposed control does not guarantee that the system states reach the sliding surface in finite time.

In this paper, a decentralised static sliding mode control strategy is proposed for a class of nonlinear interconnected systems. It is not required that the nominal isolated systems are linear or linearizable. Under the conditions that all the nominal isolated subsystems have uniform relative degree and the distribution is involutive, the class of nonlinear interconnected systems considered can be transformed to a convenient regular form representation using a local coordinate transformation and feedback linearization. In comparison with most of the existing control methods for interconnected systems [15], [16], [28], the nominal isolated subsystems of the interconnected system considered in this paper are fully nonlinear, which extends both the potential practical application value and theoretical significance. It should be mentioned that the regular form proposed greatly facilitates decentralised control design. In light of the regular form, the decentralized sliding mode control can accommodate matched and unmatched uncertainties very well. A set of sufficient conditions is developed to guarantee that the corresponding sliding motion is asymptotically stable by the Lyapunov approach, where the conservativeness of the interconnection terms is reduced compared with the results in [23], [27]. Meanwhile a new corresponding reachability condition is proposed for the nonlinear interconnected system, which can make the system states reach the designed sliding mode surface in finite time, while the existing reachability condition cannot guarantee this. The main theoretical contributions of this paper include: (i) a framework is proposed to transform the nonlinear interconnected system into a new regular form; (ii) a set of sufficient conditions is given to guarantee the large-scale interconnected system have asymptotically stable sliding motion; (iii) in light

of the proposed corresponding reachability condition, the decentralised sliding mode control can enable the system to reach the composite sliding surface in finite time.

The remainder of the paper is organized as follows. Section 2 formulates the problem and gives some definitions that will be used in the following sections. In Section 3, basic assumptions and stability analysis of the sliding mode are given. In Section 4, a decentralised sliding mode control is designed. In Section 5, a simulation example is presented to validate the proposed approach. Conclusions are given in Section 6.

## II. INTERCONNECTED SYSTEM ANALYSIS AND PROBLEM FORMULATION

Consider the nonlinear interconnected system described by:

$$\dot{x}_i = f_i(x_i) + g_i(x_i)(u_i + \phi_i(t, x_i)) + \xi_i(t, x_i) + \sum_{j=1}^n H_{ij}(t, x_j), \quad i = 1, 2, \dots, n \quad (1)$$

where  $x_i \in \chi_i \in R^{n_i}$ ,  $u_i \in R^{m_i}$  are the state and input of the  $i$ -th subsystem respectively,  $\chi_i$  is a neighborhood of the origin and  $x := \text{col}(x_1, \dots, x_n) \in \chi := \chi_1 \times \dots \times \chi_n$ ,  $f_i(x_i) := [f_{i1}(x_i), \dots, f_{in_i}(x_i)]^T \in R^{n_i}$ ,  $g_i(x_i) := [g_{i1}(x_i), \dots, g_{im_i}(x_i)] \in R^{n_i \times m_i}$  is the input distribution matrix,  $\phi_i(t, x_i) \in R^{m_i}$  and  $\xi_i(t, x_i) \in R^{n_i}$  represent the matched uncertainties and unmatched uncertainties respectively,  $\sum_{j=1}^n H_{ij}(t, x_j)$  denotes the unknown interconnections.

The following definitions are provided for ease of exposition.

*Definition 1:* [20] The system

$$\dot{x}_i = f_i(x_i) + g_i(x_i)(u_i + \phi_i(t, x_i)) + \xi_i(t, x_i), \quad i = 1, 2, \dots, n \quad (2)$$

is referred to as the  $i$ -th isolated subsystem of the interconnected system (1), and the system

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i, \quad i = 1, 2, \dots, n \quad (3)$$

is referred to as the  $i$ -th nominal isolated subsystem of the interconnected system (1).

*Definition 2:* [29] System (3) is said to have uniform relative degree  $(r_{i1}, r_{i2}, \dots, r_{im_i})$  in domain  $\chi_i$  if there exists a series of functions  $h_{ij}(x_i) \in R^{m_i}$  such that for any  $x_i \in \chi_i$ :

- i)  $L_{g_{ij}} L_{f_i}^{k_i} h_{il}(x_i) = 0$  for all  $1 \leq j \leq m_i$ ,  $1 \leq l \leq m_i$ ,  $0 \leq k < r_i - 1$  and  $i = 1, 2, \dots, n$ .
- ii) A series of functions  $\Lambda_i(x_i) \in R^{m_i \times m_i}$  are nonsingular for  $i = 1, 2, \dots, n$ .

where

$$\Lambda_i(x_i) = \begin{bmatrix} L_{g_{i1}} L_{f_i}^{r_{i1}-1} h_{i1}(x_i) & \dots & L_{g_{im_i}} L_{f_i}^{r_{i1}-1} h_{i1}(x_i) \\ L_{g_{i1}} L_{f_i}^{r_{i2}-1} h_{i2}(x_i) & \dots & L_{g_{im_i}} L_{f_i}^{r_{i2}-1} h_{i2}(x_i) \\ \dots & \dots & \dots \\ L_{g_{i1}} L_{f_i}^{r_{im_i}-1} h_{im_i}(x_i) & \dots & L_{g_{im_i}} L_{f_i}^{r_{im_i}-1} h_{im_i}(x_i) \end{bmatrix} \quad (4)$$

where  $L_{f_i} h_{il}(x_i)$  denotes the derivative of  $h_{il}(x_i)$  along  $f_i$  defined by  $L_{f_i} h_{il}(x_i) := \sum_{s=1}^{n_i} \frac{\partial h_{il}}{\partial x_s} f_{is}(x)$  and  $L_{f_i}^k h_{il}(x_i)$  represents a recursion defined by  $L_{f_i}^k h_{il}(x_i) := \frac{\partial L_{f_i}^{k-1} h_{il}(x_i)}{\partial x} f_i(x)$  with  $L_{f_i}^0 h_{il}(x_i) := h_{il}(x_i)$ .

*Remark 1:* Note that  $r_i = \sum_{l=1}^{m_i} r_{il} \leq n_i$ . When  $r_i = n_i$ , system (3) can be linearized completely for  $i = 1, 2, \dots, n$ , and in this case, there is no zero dynamics. When  $r_i < n_i$ , system (3) can only be partially feedback linearized, and in this case, there are nonlinear zero dynamics. It should be pointed out that it is not required that  $r_i = n_i$  in this paper and thus it does not require that the nominal isolated subsystems are linearizable.

For simplicity, the distribution formed by the vector fields  $g_{i1}(x_i), g_{i2}(x_i), \dots, g_{im_i}(x_i)$  is denoted by:

$$G_i(x_i) := \text{span} \{g_{i1}(x_i), g_{i2}(x_i), \dots, g_{im_i}(x_i)\} \quad (5)$$

In this paper, it is assumed that the  $i$ -th nominal isolated subsystem of the interconnected system (1) has uniform relative degree  $(r_{i1}, r_{i2}, \dots, r_{im_i})$  and the distribution  $G_i(x_i)$  is involutive in domain  $\chi_i$  for  $i = 1, 2, \dots, n$ . For the convenience of analysis and design, the feedback linearization technique is applied to the system (3).

Let  $z_{il} = \begin{bmatrix} h_{il}(x_i) & L_{f_i}h_{il}(x_i) & \cdots & L_{f_i}^{r_{il}-1}h_{il}(x_i) \end{bmatrix}^T$ ,  $l = 1, 2, \dots, m_i$  with  $z_{il} \in R^{r_{il}}$ . From [29], there exist  $n_i - r_i$  functions  $z_{i(r_i+1)}, z_{i(r_i+2)}, \dots, z_{in_i}$  defined in  $\chi_i$  such that the Jacobian matrix of the mapping

$$T_i : x_i \rightarrow z_i := \text{col}(z_i^a, z_i^b) \quad (6)$$

is nonsingular, where  $z_i^a := \text{col}(z_{i1}, z_{i2}, \dots, z_{im_i}) \in R^{r_i}$  and  $z_i^b := \text{col}(z_{i(r_i+1)}, z_{i(r_i+2)}, \dots, z_{in_i}) \in R^{n_i - r_i}$ . Therefore,  $T_i$  in (6) defines a diffeomorphism.

It follows from [29] that in the new coordinate system  $z_i$ , system (3) can be described as:

$$\begin{aligned} \dot{z}_i^a &= \bar{A}_i z_i^a + \bar{B}_i(\alpha_i(z_i) + \Lambda_i(z_i) u_i) \\ \dot{z}_i^b &= \omega_i(z_i^a, z_i^b) \end{aligned} \quad (7)$$

where  $\bar{A}_i = \text{diag}\{\bar{A}_{i1}, \dots, \bar{A}_{il}, \dots, \bar{A}_{im_i}\} \in R^{r_i \times r_i}$ ,  $\bar{B}_i = \text{diag}\{\bar{B}_{i1}, \dots, \bar{B}_{il}, \dots, \bar{B}_{im_i}\} \in R^{r_i \times m_i}$ ,

$$\alpha_i(z_i) = \begin{bmatrix} L_{f_i}^{r_{i1}}h_{i1}(z_i) \\ L_{f_i}^{r_{i2}}h_{i2}(z_i) \\ \vdots \\ L_{f_i}^{r_{im_i}}h_{im_i}(z_i) \end{bmatrix} \quad \bar{A}_{il} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{r_{il} \times r_{il}} \quad \bar{B}_{il} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_{il} \times 1} \quad (8)$$

and  $\Lambda_i$  is defined in (4).

Design a state feedback

$$u_i = -\Lambda_i^{-1}(z_i)\alpha_i(z_i) + \Lambda_i^{-1}(z_i)\nu_i \quad (9)$$

where  $\nu_i \in R^{m_i}$  is the new input of the  $i$ -th subsystem for  $i = 1, 2, \dots, n$ .

By applying the feedback from (9) to system (7), the corresponding closed-loop system can be described as:

$$\begin{aligned} \dot{z}_i^a &= \bar{A}_i z_i^a + \bar{B}_i \nu_i \\ \dot{z}_i^b &= \omega_i(z_i^a, z_i^b) \end{aligned} \quad (10)$$

Consider the interconnected system (1). According to the above analysis, it follows from (10) that in the new coordinates  $z := \text{col}(z_1, \dots, z_n) \in Z := Z_1 \times \dots \times Z_n$  given in (6), system (1) can be described by:

$$\dot{z}_{i1}^a = A_i z_{i1}^a + B_i z_{i2}^a + \Xi_{i1}^a(t, z_i) + \Psi_{i1}^a(t, z) \quad (11a)$$

$$\dot{z}_i^b = \omega_i(z_{i1}^a, z_{i2}^a, z_i^b) + \Xi_i^b(t, z_i) + \Psi_i^b(t, z) \quad (11b)$$

$$\dot{z}_{i2}^a = \nu_i + \Xi_{i2}^a(t, z_i) + \Psi_{i2}^a(t, z) \quad (11c)$$

where  $z := \text{col}(z_1, \dots, z_n)$ ,  $z_i := \text{col}(z_{i1}^a, z_i^b, z_{i2}^a)$ ,  $z_{i1}^a := \text{col}(z_{i11}, z_{i12}, \dots, z_{i1(r_{i1}-1)}, \dots, z_{im_i1}, z_{im_i2}, \dots, z_{im_i(r_{im_i}-1)}) \in R^{r_i - m_i}$ ,  $z_{i2}^a = (z_{i1r_{i1}}, z_{i2r_{i2}}, \dots, z_{im_ir_{im_i}}) \in R^{m_i}$ ,  $\Psi_{i1}^a(t, z)$ ,  $\Psi_i^b(t, z)$ ,  $\Psi_{i2}^a(t, z)$  are the interconnections with appropriate dimensions,  $\Xi_{i1}^a(t, z_i)$ ,  $\Xi_i^b(t, z_i)$ ,  $\Xi_{i2}^a(t, z_i)$  are the uncertainties with appropriate dimensions. Taking into account the special structure of  $(\bar{A}_i, \bar{B}_i)$  in (10), form (11) can be obtained by matrix factorization to  $(\bar{A}_i, \bar{B}_i)$ , while  $(A_i, B_i)$  also has a similar structure to  $(\bar{A}_i, \bar{B}_i)$ .

In particular,  $A_i = \text{diag}\{A_{i1}, \dots, A_{il}, \dots, A_{im_i}\} \in R^{(r_i - m_i) \times (r_i - m_i)}$ ,  $B_i = \text{diag}\{B_{i1}, \dots, B_{il}, \dots, B_{im_i}\} \in R^{(r_i - m_i) \times m_i}$

with

$$A_{il} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{(r_l-1) \times (r_l-1)}, \quad B_{il} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{(r_l-1) \times 1} \quad (12)$$

*Remark 2:* Under the diffeomorphism  $T_i$  defined in (6) and  $T_i(0) = 0$ , the uncertainties and interconnections of the interconnected system (11) have a similar structure to those of the interconnected system (1). For example,  $\Psi_{i1}^a(t, z)$ ,  $\Psi_i^b(t, z)$  and  $\Psi_{i2}^a(t, z)$  are determined by  $\left[ \frac{\partial T_i}{\partial x_i} \right]_{x_i=T_i^{-1}(z_i)} \sum_{j=1}^n H_{ij}(t, T_j^{-1}(z_j))$ .

*Remark 3:* It is clear to see that the system (11) adopts a regular form as  $\nu_i$  is the system input, which has been transformed from the nonlinear interconnected system (1) by local coordinate transformation and feedback linearization. It should be noted that form (10) is directly studied in [23] with the non-square matrix  $\bar{B}_i \in R^{r_i \times m_i}$  as the input distribution matrix. Based on a high gain control approach, a decentralised state feedback control scheme is proposed for this form, which can only deal with weak unmatched uncertainties and matched interconnections [23]. Furthermore, the decentralised sliding mode control cannot be directly designed in the light of form (10). A new form (11) is developed in this study with nonsingular unit matrix as the new input distribution matrix, which not only facilitates the design of the sliding mode control, but also can deal with stronger unmatched uncertainties and interconnections. Note that form (11) can be obtained by a simple matrix factorization technique from (10).

The objective of this paper is to design a composite sliding surface under a set of conditions such that the reduced sliding mode is uniformly asymptotically stable. Then, for system (11), a decentralised sliding mode control

$$\nu_i = \nu_i(t, z_i) \quad (13)$$

will be designed such that the controlled interconnected system states can be driven to the pre-designed composite sliding surface in finite time and a sliding motion maintained thereafter.

*Remark 4:* For system (1), a constructive method to obtain the required regular form is not available [27]. In comparison with [27], the proposed controller design in this paper has more practical significance and application value in this regard. It will be shown that in the developed regular form (11), the method proposed in this paper can accommodate the case of unmatched uncertainties and interconnections.

### III. BASIC ASSUMPTIONS AND STABILITY ANALYSIS OF THE SLIDING MODE

In this section, a composite sliding surface will be designed. Then the stability of the corresponding sliding motion will be analysed. The following Assumption will be imposed on system (11).

*Assumption 1:* There exist known nonnegative continuous functions  $\mu_i(\cdot)$ ,  $\rho_i(\cdot)$ ,  $\alpha_{i1}(\cdot)$ ,  $\alpha_{i2}(\cdot)$ ,  $\beta_{i1}(\cdot)$ ,  $\beta_{i2}(\cdot)$ ,  $\gamma_{i1}(\cdot)$ ,  $\gamma_{i2}(\cdot)$ ,  $\alpha_{ij3}(\cdot)$ ,  $\alpha_{ij4}(\cdot)$ ,  $\alpha_{ij5}(\cdot)$ ,  $\beta_{ij3}(\cdot)$ ,  $\beta_{ij4}(\cdot)$ ,  $\beta_{ij5}(\cdot)$ ,  $\gamma_{ij3}(\cdot)$ ,  $\gamma_{ij4}(\cdot)$ ,  $\gamma_{ij5}(\cdot)$ , such that

$$\|\Xi_{i2}^a(t, z_i)\| \leq \mu_i(z_i) \quad (14a)$$

$$\|\Xi_{i1}^a(t, z_i)\| \leq \rho_i(z_i) = \alpha_{i1}(z_i) \|z_{i1}^a\| + \beta_{i1}(z_i) \|z_{i2}^a\| + \gamma_{i1}(z_i) \|z_i^b\| \quad (14b)$$

$$\|\Xi_i^b(t, z_i)\| \leq \alpha_{i2}(z_i) \|z_{i1}^a\| + \beta_{i2}(z_i) \|z_{i2}^a\| + \gamma_{i2}(z_i) \|z_i^b\| \quad (14c)$$

$$\|\Psi_{i1}^a(t, z)\| \leq \sum_{j=1}^n (\alpha_{ij3}(z_j) \|z_{j1}^a\| + \beta_{ij3}(z_j) \|z_{j2}^a\| + \gamma_{ij3}(z_j) \|z_j^b\|) \quad (15a)$$

$$\|\Psi_{i2}^a(t, z)\| \leq \sum_{j=1}^n (\alpha_{ij4}(z_j) \|z_{j1}^a\| + \beta_{ij4}(z_j) \|z_{j2}^a\| + \gamma_{ij4}(z_j) \|z_j^b\|) \quad (15b)$$

$$\|\Psi_i^b(t, z)\| \leq \sum_{j=1}^n (\alpha_{ij5}(z_j) \|z_{j1}^a\| + \beta_{ij5}(z_j) \|z_{j2}^a\| + \gamma_{ij5}(z_j) \|z_j^b\|) \quad (15c)$$

*Remark 5:* Assumption 1 is made on the uncertainties and interconnections in the nonlinear interconnected system (11). Meanwhile, Remark 2 shows that the bounds on the uncertainties and interconnections in the interconnected system (1) have a similar structure to those in the interconnected system (11). The terms  $\Xi_{i1}^a(t, z_i)$  and  $\Xi_i^b(t, z_i)$  represent unmatched uncertainties, while  $\Psi_{i1}^a(t, z)$  and  $\Psi_i^b(t, z)$  represent unmatched interconnections. It should be noted that only matched uncertainties and interconnections are discussed in [27] and [23] respectively, while unmatched uncertainties and interconnections are also considered in this paper. Furthermore, the bounds on the uncertainties and interconnections are nonlinear and have more general forms.

It is clear that  $(A_{il}, B_{il})$  in (12) have the Brunovsky standard form [30], and according to the controllability criterion,  $(A_i, B_i)$  are controllable. It follows that there exist  $K_i \in R^{m_i \times (r_i - m_i)}$  such that the matrices  $(A_i - B_i K_i)$  are stable. **Therefore, there exist positive definite symmetric matrices  $Q_i, P_i$  such that**

$$(A_i - B_i K_i)^T P_i + P_i (A_i - B_i K_i) + Q_i \leq 0 \quad (16)$$

*Remark 6:* It should be noted that matrix inequalities (16) can be transformed into the following standard linear matrix inequalities (LMI). Given  $Q_i > 0$ , find matrices  $P_i$  and  $K_i$  such that:

$$\begin{bmatrix} A_i^T P_i + P_i A_i - F_i^T - F_i & I_i \\ I_i & -Q_i^{-1} \end{bmatrix} \leq 0 \quad (17)$$

where  $F_i = P_i B_i K_i$  and  $I_i$  are unit matrices.

Choose the composite sliding surface

$$\sigma(z) = \text{col}(\sigma_1(z_1), \dots, \sigma_n(z_n)) = 0 \quad (18)$$

where  $\sigma_i(z_i) = z_{i2}^a + K_i z_{i1}^a$  and  $K_i$  satisfies (16).

During sliding motion  $\sigma_i(z_i) = 0$  which implies

$$z_{i2}^a = -K_i z_{i1}^a \quad (19)$$

Based on the designed sliding surface (18) and (19), it follows from the structure of system (11) that the corresponding sliding motion will be determined by (11). Then the sliding mode dynamics can be described by:

$$\begin{aligned} \dot{z}_{i1}^a &= (A_i - B_i K_i) z_{i1}^a + T_i^{as}(\cdot) + \Upsilon_i^{as}(\cdot) \\ \dot{z}_i^b &= \Omega_i(\cdot) + T_i^{bs}(\cdot) + \Upsilon_i^{bs}(\cdot) \end{aligned} \quad (20)$$

where  $T_i^{as}(t, z_{11}^a, z_1^b, \dots, z_{n1}^a, z_n^b) = \Psi_{i1}^a(t, z)|_{z_{i2}^a = -K_i z_{i1}^a}$ ,  $\Upsilon_i^{as}(t, z_{i1}^a, z_i^b) = \Xi_{i1}^a(t, z_i)|_{z_{i2}^a = -K_i z_{i1}^a}$ ,  $T_i^{bs}(t, z_{11}^a, z_1^b, \dots, z_{n1}^a, z_n^b) = \Psi_i^b(t, z)|_{z_{i2}^a = -K_i z_{i1}^a}$ ,  $\Upsilon_i^{bs}(t, z_{i1}^a, z_i^b) = \Xi_i^b(t, z_i)|_{z_{i2}^a = -K_i z_{i1}^a}$ ,  $\Omega_i(z_{i1}^a, z_i^b) = \omega_i(z_{i1}^a, z_{i2}^a, z_i^b)|_{z_{i2}^a = -K_i z_{i1}^a}$ .

*Assumption 2:* There exist  $C^1$  functions  $V_i^b(t, z_{i1}^a, z_i^b) : R \times R^{r_i - m_i} \times R^{n_i - r_i} \mapsto R^+$  such that

$$\begin{aligned} c_{i1} \|z_{i1}^a\|^2 + c_{i2} \|z_i^b\|^2 &\leq V_i^b(t, z_{i1}^a, z_i^b) \leq c_{i3} \|z_{i1}^a\|^2 + c_{i4} \|z_i^b\|^2 \\ \frac{\partial V_i^b}{\partial t} + \left( \frac{\partial V_i^b}{\partial z_{i1}^a} \right)^T (A_i - B_i K_i) z_{i1}^a + \left( \frac{\partial V_i^b}{\partial z_i^b} \right)^T \Omega_i(z_{i1}^a, z_i^b) &\leq -c_{i5} \|z_{i1}^a\|^2 - c_{i6} \|z_i^b\|^2 \\ \left\| \frac{\partial V_i^b}{\partial (z_{i1}^a, z_i^b)} \right\| &\leq c_{i7} \|z_{i1}^a\| + c_{i8} \|z_i^b\| \end{aligned} \quad (21)$$

where  $c_{i1}, \dots, c_{i8}$  are positive constants,  $\frac{\partial V_i^b}{\partial (z_{i1}^a, z_i^b)} = \begin{bmatrix} \left( \frac{\partial V_i^b}{\partial z_{i1}^a} \right)^T & \left( \frac{\partial V_i^b}{\partial z_i^b} \right)^T \end{bmatrix}$ ,  $K_i$  satisfies (16) and  $\Omega_i(z_{i1}^a, z_i^b)$  is given in (20).

*Remark 7:* Assumption 2 implies that the nominal isolated subsystems of (20) are asymptotically stable. Such conditions

are derived from the converse Lyapunov uniform asymptotic stability theorem (see e.g. [31]).

From Assumption 1,  $T_i^{as}(\cdot)$ ,  $\Upsilon_i^{as}(\cdot)$ ,  $T_i^{bs}(\cdot)$  and  $\Upsilon_i^{bs}(\cdot)$  in (20) satisfy

$$\|T_i^{as}(t, z_{11}^a, z_1^b, \dots, z_{n1}^a, z_n^b)\| \leq \sum_{j=1}^n (\varepsilon_{ij1}(z_{j1}^a, z_j^b) \|z_{j1}^a\| + \varepsilon_{ij2}(z_{j1}^a, z_j^b) \|z_j^b\|) \quad (22a)$$

$$\|T_i^{bs}(t, z_{11}^a, z_1^b, \dots, z_{n1}^a, z_n^b)\| \leq \sum_{j=1}^n (\varepsilon_{ij5}(z_{j1}^a, z_j^b) \|z_{j1}^a\| + \varepsilon_{ij6}(z_{j1}^a, z_j^b) \|z_j^b\|) \quad (22b)$$

$$\|\Upsilon_i^{as}(t, z_{i1}^a, z_i^b)\| \leq \varepsilon_{i3}(z_{i1}^a, z_i^b) \|z_{i1}^a\| + \varepsilon_{i4}(z_{i1}^a, z_i^b) \|z_i^b\| \quad (23a)$$

$$\|\Upsilon_i^{bs}(t, z_{i1}^a, z_i^b)\| \leq \varepsilon_{i7}(z_{i1}^a, z_i^b) \|z_{i1}^a\| + \varepsilon_{i8}(z_{i1}^a, z_i^b) \|z_i^b\| \quad (23b)$$

where

$$\varepsilon_{ij1}(z_{j1}^a, z_j^b) = \{\alpha_{ij3}(z_j) + \beta_{ij3}(z_j) \|K_j\|\}|_{z_{j2}^a = -K_j z_{j1}^a}$$

$$\varepsilon_{ij2}(z_{j1}^a, z_j^b) = \gamma_{ij3}(z_j)|_{z_{j2}^a = -K_j z_{j1}^a}$$

$$\varepsilon_{i3}(z_{i1}^a, z_i^b) = \{\alpha_{i1}(z_j) + \beta_{i1}(z_j) \|K_i\|\}|_{z_{i2}^a = -K_i z_{i1}^a}$$

$$\varepsilon_{i4}(z_{i1}^a, z_i^b) = \gamma_{i1}(z_i)|_{z_{i2}^a = -K_i z_{i1}^a}$$

$$\varepsilon_{ij5}(z_{j1}^a, z_j^b) = \{\alpha_{ij5}(z_j) + \beta_{ij5}(z_j) \|K_j\|\}|_{z_{j2}^a = -K_j z_{j1}^a}$$

$$\varepsilon_{ij6}(z_{j1}^a, z_j^b) = \gamma_{ij5}(z_j)|_{z_{j2}^a = -K_j z_{j1}^a}$$

$$\varepsilon_{i7}(z_{i1}^a, z_i^b) = \{\alpha_{i2}(z_i) + \beta_{i2}(z_i) \|K_i\|\}|_{z_{i2}^a = -K_i z_{i1}^a}$$

$$\varepsilon_{i8}(z_{i1}^a, z_i^b) = \gamma_{i2}(z_i)|_{z_{i2}^a = -K_i z_{i1}^a}$$

**Theorem 1:** Under Assumptions 1-2, the sliding mode dynamics (20) of system (11) are asymptotically stable if  $M^T + M > 0$  where the matrix  $M = [m_{ij}]_{2n \times 2n}$  is defined by

$$m_{ij} = \begin{cases} c_{i5} + \lambda(Q_i)_{\min} - c_{i7}\varepsilon_{i3} - c_{i7}\varepsilon_{i7} - 2\|P_i\|\varepsilon_{i3} - 2\varepsilon_{ij1}\|P_i\| - c_{i7}\varepsilon_{ij1} - c_{i7}\varepsilon_{ij5} & 1 \leq i = j \leq n \\ c_{(i-n)6} - c_{(i-n)8}\varepsilon_{(i-n)8} - c_{(i-n)8}\varepsilon_{(i-n)4} - c_{(i-n)8}\varepsilon_{(i-n)(j-n)6} - c_{(i-n)8}\varepsilon_{(i-n)(j-n)2} \\ -2\varepsilon_{ij1}\|P_i\| - c_{i7}\varepsilon_{ij1} - c_{i7}\varepsilon_{ij5} & n+1 \leq i = j \leq 2n \\ -c_{(i-n)8}\varepsilon_{(i-n)(j-n)6} - c_{(i-n)8}\varepsilon_{(i-n)(j-n)2} & 1 \leq i, j \leq n, i \neq j \\ -2\varepsilon_{(i-n)j2}\|P_{(i-n)}\| - c_{(i-n)7}\varepsilon_{(i-n)j2} - c_{(i-n)7}\varepsilon_{(i-n)j6} - c_{(i-n)8}\varepsilon_{(i-n)j5} - c_{(i-n)8}\varepsilon_{(i-n)j1} & n+1 \leq i, j \leq 2n, i \neq j \\ -2\varepsilon_{(i-n)j2}\|P_{(i-n)}\| - c_{(i-n)7}\varepsilon_{(i-n)j2} - c_{(i-n)7}\varepsilon_{(i-n)j6} - c_{(i-n)8}\varepsilon_{(i-n)j5} - c_{(i-n)8}\varepsilon_{(i-n)j1} & i > n, 1 \leq j \leq n, i \neq j+n \\ -c_{(i-n)7}\varepsilon_{(i-n)4} - c_{(i-n)8}\varepsilon_{(i-n)7} - 2\|P_{(i-n)}\|\varepsilon_{(i-n)4} - c_{(i-n)7}\varepsilon_{(i-n)8} - c_{(i-n)8}\varepsilon_{(i-n)3} & i > n, 1 \leq j \leq n, i = j+n \\ -2\varepsilon_{i(j-n)2}\|P_i\| - c_{i7}\varepsilon_{i(j-n)2} - c_{i7}\varepsilon_{i(j-n)6} - c_{i8}\varepsilon_{i(j-n)5} & j > n, 1 \leq i \leq n, j = i+n \\ -c_{i8}\varepsilon_{i(j-n)1} - c_{i7}\varepsilon_{i4} - c_{i8}\varepsilon_{i7} - 2\|P_i\|\varepsilon_{i4} - c_{i7}\varepsilon_{i8} - c_{i8}\varepsilon_{i3} & j > n, 1 \leq i \leq n, j = i+n \\ -2\varepsilon_{i(j-n)2}\|P_i\| - c_{i7}\varepsilon_{i(j-n)2} - c_{i7}\varepsilon_{i(j-n)6} - c_{i8}\varepsilon_{i(j-n)5} - c_{i8}\varepsilon_{i(j-n)1} & j > n, 1 \leq i \leq n, j \neq i+n \end{cases} \quad (24)$$

where  $\lambda(\cdot)_{\min}$  represents the minimum eigenvalue of  $(\cdot)$ .

**Proof:** For system (20), consider the Lyapunov function candidate

$$V(t, z) = \sum_{i=1}^n V_i^b(t, z_{i1}^a, z_i^b) + \sum_{i=1}^n (z_{i1}^a)^T P_i z_{i1}^a \quad (25)$$

where  $V_i^b(t, z_{i1}^a, z_i^b)$  is given by Assumption 2, and  $P_i$  satisfies (16).

Then, the time derivative of  $V(t, z)$  along the trajectories of system (20) is given by:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^n \left\{ \frac{\partial V_i^b}{\partial t} + \frac{\partial V_i^b}{\partial (z_{i1}^a, z_i^b)} \begin{bmatrix} \dot{z}_{i1}^a \\ \dot{z}_i^b \end{bmatrix} \right\} + \sum_{i=1}^n \left\{ -(z_{i1}^a)^T Q_i z_{i1}^a + 2(z_{i1}^a)^T P_i (\mathbf{T}_i^{as} + \Upsilon_i^{as}) \right\} \\ &= \sum_{i=1}^n \left\{ \frac{\partial V_i^b}{\partial t} + \left[ \begin{pmatrix} \frac{\partial V_i^b}{\partial z_{i1}^a} \\ \frac{\partial V_i^b}{\partial z_i^b} \end{pmatrix}^T \begin{pmatrix} \frac{\partial V_i^b}{\partial z_{i1}^a} \\ \frac{\partial V_i^b}{\partial z_i^b} \end{pmatrix} \right] \begin{bmatrix} (A_i - B_i K_i) z_{i1}^a + \mathbf{T}_i^{as} + \Upsilon_i^{as} \\ \Omega_i(z_{i1}^a, z_i^b) + \mathbf{T}_i^{bs} + \Upsilon_i^{bs} \end{bmatrix} \right\} + \sum_{i=1}^n \left\{ -(z_{i1}^a)^T Q_i z_{i1}^a + 2(z_{i1}^a)^T P_i (\mathbf{T}_i^{as} + \Upsilon_i^{as}) \right\}\end{aligned}\quad (26)$$

where  $Q_i = -(A_i - B_i K_i)^T P_i - P_i (A_i - B_i K_i)$  has been used for convenience.

Hence, (26) can be expressed as:

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^n \left\{ \frac{\partial V_i^b}{\partial t} + \left( \frac{\partial V_i^b}{\partial z_{i1}^a} \right)^T (A_i - B_i K_i) z_{i1}^a + \left( \frac{\partial V_i^b}{\partial z_i^b} \right)^T \Omega_i(z_{i1}^a, z_i^b) + \left\| \frac{\partial V_i^b}{\partial (z_{i1}^a, z_i^b)} \right\| \left\| \begin{bmatrix} \mathbf{T}_i^{as} + \Upsilon_i^{as} \\ \mathbf{T}_i^{bs} + \Upsilon_i^{bs} \end{bmatrix} \right\| \right\} \\ &\quad + \sum_{i=1}^n \left\{ -(z_{i1}^a)^T Q_i z_{i1}^a + 2(z_{i1}^a)^T P_i (\mathbf{T}_i^{as} + \Upsilon_i^{as}) \right\}\end{aligned}\quad (27)$$

Then, from Assumptions 2 and (27), the following inequalities can be obtained:

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^n \left\{ -c_{i5} \|z_{i1}^a\|^2 - c_{i6} \|z_i^b\|^2 + (c_{i7} \|z_{i1}^a\| + c_{i8} \|z_i^b\|) (\|\mathbf{T}_i^{as}\| + \|\Upsilon_i^{as}\|) \right. \\ &\quad \left. + (c_{i7} \|z_{i1}^a\| + c_{i8} \|z_i^b\|) (\|\mathbf{T}_i^{bs}\| + \|\Upsilon_i^{bs}\|) \right\} \\ &\quad + \sum_{i=1}^n \left\{ -\lambda(Q_i)_{\min} \|z_{i1}^a\|^2 + 2 \|z_{i1}^a\| \|P_i\| \|\mathbf{T}_i^{as}\| + 2 \|z_{i1}^a\| \|P_i\| \|\Upsilon_i^{as}\| \right\}\end{aligned}\quad (28)$$

From the inequalities (22) and (23), it follows that

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^n \left\{ \{-c_{i5} - \lambda(Q_i)_{\min} + c_{i7}\varepsilon_{i3} + c_{i7}\varepsilon_{i7} + 2\|P_i\|\varepsilon_{i3}\} \|z_{i1}^a\|^2 + \{-c_{i6} + c_{i8}\varepsilon_{i8} + c_{i8}\varepsilon_{i4}\} \|z_i^b\|^2 \right. \\ &\quad \left. + \{c_{i7}\varepsilon_{i4} + c_{i8}\varepsilon_{i7} + 2\|P_i\|\varepsilon_{i4} + c_{i7}\varepsilon_{i8} + c_{i8}\varepsilon_{i3}\} \|z_{i1}^a\| \|z_i^b\| \right\} \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \{2\varepsilon_{ij1} \|P_i\| + c_{i7}\varepsilon_{ij1} + c_{i7}\varepsilon_{ij5}\} \|z_{i1}^a\| \|z_{j1}^a\| + \sum_{i=1}^n \sum_{j=1}^n \{2\varepsilon_{ij2} \|P_i\| + c_{i7}\varepsilon_{ij2} + c_{i7}\varepsilon_{ij6}\} \|z_{i1}^a\| \|z_j^b\| \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \{c_{i8}\varepsilon_{ij6} + c_{i8}\varepsilon_{ij2}\} \|z_i^b\| \|z_j^b\| + \sum_{i=1}^n \sum_{j=1}^n \{c_{i8}\varepsilon_{ij5} + c_{i8}\varepsilon_{ij1}\} \|z_i^b\| \|z_{j1}^a\| \\ &= -\frac{1}{2} Y (M + M^T) Y^T\end{aligned}\quad (29)$$

where  $Y = [\|z_{11}^a\|, \|z_{21}^a\|, \dots, \|z_{n1}^a\|, \|z_1^b\|, \|z_2^b\|, \dots, \|z_n^b\|]$ .

Hence, due to  $M^T + M > 0$ , Theorem 1 follows.

*Remark 8:* It should be noted noted that for  $M \in R^{2n \times 2n}$ , Theorem 1 has presented a set of sufficient conditions under which the sliding mode (20) of the nonlinear interconnected system (11) is asymptotically stable even in the presence of unmatched uncertainties and interconnections.

#### IV. DECENTRALISED SLIDING MODE CONTROL DESIGN

This section aims to design a decentralised sliding mode control such that system (11) can be driven to the sliding surface (18) in finite time.

*Lemma 1:* For the nonlinear interconnected system (11) with the sliding surface given in (18), if

$$\sum_{i=1}^n \sigma_i^T(z_i) \dot{\sigma}_i(z_i) \leq -\eta \sum_{i=1}^n \|\sigma_i(z_i)\| \quad (30)$$

where  $\eta$  is a small positive constant, then the system (11) can reach the sliding surface (18) in finite time and maintain a sliding motion thereafter.

**Proof:** For the nonlinear interconnected system, composite switching function  $\sigma(z) = \text{col}(\sigma_1(z_1), \dots, \sigma_n(z_n))$ , the following equation holds:

$$\sigma^T(z) \dot{\sigma}(z) = \sum_{i=1}^n \sigma_i^T(z_i) \dot{\sigma}_i(z_i) \quad (31)$$

Furthermore,

$$\sum_{i=1}^n \|\sigma_i(z_i)\| = \|\sigma_1(z_1)\| + \dots + \|\sigma_n(z_n)\| \geq \|\sigma(z)\| = \sqrt{\|\sigma_1(z_1)\|^2 + \dots + \|\sigma_n(z_n)\|^2} \quad (32)$$

From (31) and (32),

$$\sigma^T(z) \dot{\sigma}(z) = \sum_{i=1}^n \sigma_i^T(z_i) \dot{\sigma}_i(z_i) \leq -\eta \sum_{i=1}^n \|\sigma_i(z_i)\| \leq -\eta \|\sigma(z)\| \quad (33)$$

Hence, Lemma 1 follows since a so-called  $\eta$  reachability condition is satisfied [32].

*Remark 9:* In the existing research concerning nonlinear interconnected systems, the corresponding reachability condition is given by: (see, [33])

$$\sum_{i=1}^n \frac{\sigma_i^T(z_i) \dot{\sigma}_i(z_i)}{\|\sigma_i(z_i)\|} < 0 \quad (34)$$

Condition (34) cannot guarantee that the control drives the system states to the sliding surface in finite time. However, the reachability condition proposed in (30) in this paper can guarantee that the system states can be driven to the sliding surface in finite time and also maintain a sliding motion thereafter.

For the interconnected system (11), the following control law is proposed

$$\begin{aligned} \nu_i = & -K_i A_i z_{i1}^a - K_i B_i z_{i2}^a - \mu_i(z_i) \text{sgn}(\sigma_i(z_i)) - \rho_i(z_i^a, z_{i2}^a, z_i^b) \|K_i\| \text{sgn}(\sigma_i(z_i)) \\ & - \Phi_i \text{sgn}(\sigma_i(z_i)) - \eta \text{sgn}(\sigma_i(z_i)) \end{aligned} \quad (35)$$

where  $K_i$  is given in (16) and

$$\begin{aligned} \Phi_i = & \sum_{j=1}^n \{(\alpha_{ji4}(z_i) + \|K_j\| \alpha_{ji3}(z_i)) \|z_{i1}^a\| + (\beta_{ji4}(z_i) + \|K_j\| \beta_{ji3}(z_i)) \|z_{i2}^a\| \\ & + (\gamma_{ji4}(z_i) + \|K_j\| \gamma_{ji3}(z_i)) \|z_i^b\|\} \end{aligned} \quad (36)$$

*Theorem 2:* Under Assumptions 1-2, the control (35) is able to drive the system (11) to the sliding surface (18) in finite time and maintain a sliding motion on it thereafter.

**Proof:** From (11) and (18), for  $i = 1, 2, \dots, n$ ,

$$\begin{aligned} \dot{\sigma}_i(z_i) &= \dot{z}_{i2}^a + K_i \dot{z}_{i1}^a \\ &= \nu_i + \Psi_{i2}^a(t, z) + \Xi_{i2}^a(t, z_i) + K_i (A_i z_{i1}^a + B_i z_{i2}^a + \Psi_{i1}^a(t, z) + \Xi_{i1}^a(t, z_i)) \end{aligned} \quad (37)$$

Substituting (35) into (37),

$$\begin{aligned} \sum_{i=1}^n \sigma_i^T(z_i) \dot{\sigma}_i(z_i) &= \sum_{i=1}^n \sigma_i^T(z_i) \{ -\mu_i(z_i) \text{sgn}(\sigma_i(z_i)) - \rho_i(z_i) \|K_i\| \text{sgn}(\sigma_i(z_i)) \\ & - \Phi_i \text{sgn}(\sigma_i(z_i)) - \eta \text{sgn}(\sigma_i(z_i)) + \Psi_{i2}^a(t, z) + \Xi_{i2}^a(t, z_i) \\ & + K_i \Psi_{i1}^a(t, z) + K_i \Xi_{i1}^a(t, z_i) \} \end{aligned} \quad (38)$$

From Assumptions 1 and the definition of  $\Phi_i$  in (36), the following inequality can be obtained:

$$\begin{aligned}
\sum_{i=1}^n \sigma_i^T(z_i) \dot{\sigma}_i(z_i) &\leq \sum_{i=1}^n \{-\mu_i(z_i) \|\sigma_i(z_i)\| - \rho_i(z_i) \|K_i\| \|\sigma_i(z_i)\| + \mu_i(z_i) \|\sigma_i(z_i)\| \\
&\quad + \rho_i(z_i) \|K_i\| \|\sigma_i(z_i)\| - \eta \|\sigma_i(z_i)\| \} + \sum_{i=1}^n \{-\Phi_i \|\sigma_i(z_i)\| \\
&\quad + \sum_{j=1}^n \{(\alpha_{ij4}(z_j) + \|K_i\| \alpha_{ij3}(z_j)) \|z_{j1}^a\| + (\beta_{ij4}(z_j) + \|K_i\| \beta_{ij3}(z_j)) \|z_{j2}^a\| \\
&\quad + (\gamma_{ij4}(z_j) + \|K_i\| \gamma_{ij3}(z_j)) \|z_j^b\| \} \|\sigma_i(z_i)\| \} \\
&= -\eta \sum_{i=1}^n \|\sigma_i(z_i)\|
\end{aligned} \tag{39}$$

where  $\sum_{i=1}^n \sum_{j=1}^n \Phi_i = \sum_{i=1}^n \sum_{j=1}^n \Phi_j$  has been used to generate the result.

Hence, from Lemma 1, Theorem 2 follows.

*Remark 10:* Theorems 1 and 2 together guarantee the uniform asymptotic stability of the closed-loop system formed by applying the control (35) to the system (11), irrespective of the uncertainties and the interconnections within the subsystems. Meanwhile, it should be noted that from the relationship between systems (1) and (11), it is straightforward to rewrite the control (9) and (35) in terms of the  $x$  coordinates to stabilize system (1) with  $x_i = T_i^{-1}(z_i)$  and  $T_i$  defined in (6). It should be noted that the results of this paper are local. A method to estimate the stability region has been studied in [31], [34], [35].

*Remark 11:* There are few results which apply sliding mode control methods to the nonlinear interconnected system described by (1). Since the nominal isolated system in (1) is completely nonlinear and has unmatched uncertainties and interconnections, these characteristics make the design of the control challenging. Based on the local coordinate transformation and feedback linearization, the considered interconnected system (1) is transformed to a new interconnected system (11), which is in regular form. In light of the regular form, a composite sliding surface (18) is proposed by using the system state variables. Then a set of sufficient conditions is developed to guarantee that the corresponding sliding mode dynamics (20) is asymptotically stable even in the presence of unmatched uncertainties and interconnections. Meanwhile in light of the proposed new corresponding reachability condition (30), the decentralised sliding mode control law (35) can drive the system (11) to the sliding surface (18) in finite time and maintain a sliding motion on it thereafter.

## V. SIMULATION EXAMPLE

This section aims to demonstrate the validity of the proposed method by a simulation example based on the MATLAB software.

Consider the nonlinear interconnected system formed of the two subsystems described by:

$$\dot{x}_1 = \begin{bmatrix} -\sin x_{12} \\ x_{11} \\ x_{12}^2 + x_{13}^2 \\ x_{13} \\ 2x_{11}x_{12} - 4x_{15} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 2 + \sin x_{15} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} (u_1 + \phi_1(t, x_1)) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \xi_{14} \\ \xi_{15} \end{bmatrix}}_{\xi_1(t, x_1)} + \underbrace{\begin{bmatrix} H_{111}(t, x_1) + H_{121}(t, x_2) \\ 0 \\ 0 \\ H_{141}(t, x_1) + H_{124}(t, x_2) \\ 0 \end{bmatrix}}_{\sum_{j=1}^2 H_{1j}(t, x_j)} \tag{40}$$

$$\dot{x}_2 = \begin{bmatrix} 0.1 \sin x_{23} - x_{21} \\ -x_{21}^2 - 3x_{22} \\ x_{21}^2 x_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\xi_2} (u_2 + \phi_2(t, x_2)) + \underbrace{\begin{bmatrix} 0 \\ \xi_{22}(t, x_2) \\ 0 \end{bmatrix}}_{\xi_2} + \underbrace{\begin{bmatrix} 0 \\ H_{212}(t, x_1) + H_{222}(t, x_2) \\ 0 \end{bmatrix}}_{\sum_{j=1}^2 H_{2j}(t, x_j)} \tag{41}$$

where  $x := \text{col}(x_1, x_2)$ ,  $x_1 := \text{col}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15})$ ,  $x_2 := \text{col}(x_{21}, x_{22}, x_{23})$ ,  $u_1 \in R^2$ ,  $u_2 \in R$  in the domain  $x \in \chi = \{(x_1, x_2) | |x_{15} - x_{12}^2| \leq \frac{1}{8}, |x_{22}| \leq \frac{1}{2}, |x_{23}| \leq 1\}$  and the uncertainties and interconnections satisfy

$$\|\phi_1\| \leq |\sin(x_{11} + x_{12} + x_{14})| + |x_{13}| \tag{42}$$

$$\|\xi_{14}\| \leq 0.05\sqrt{x_{12}^2 + x_{14}^2} + 0.05\sqrt{x_{11}^2 + x_{13}^2} + 0.1\|x_{15} - x_{12}^2\| \quad (43)$$

$$\|\xi_{15}\| \leq 0.1\sqrt{x_{11}^2 + x_{13}^2} + 0.1\|x_{15} - x_{12}^2\| \quad (44)$$

$$\|H_{111}(t, x_1) + H_{121}(t, x_2)\| \leq |\sin(x_{13})|\|x_{15} - x_{12}^2\| + 0.5\|x_{21}\| \quad (45)$$

$$\|H_{114}(t, x_1) + H_{124}(t, x_2)\| \leq 0.49|x_{12}|\sqrt{x_{11}^2 + x_{13}^2} + 0.1\|x_{15} - x_{12}^2\| + 0.1\|x_{21}\| + 0.1\|x_{22}\| \quad (46)$$

$$\|\phi_2(t, x_2)\| \leq |\sin(x_{21})| + |x_{22}| \quad (47)$$

$$\|\xi_{22}(t, x_2)\| \leq 0.1|x_{21}| + 0.1|x_{22}| \quad (48)$$

$$\|H_{212}(t, x_1) + H_{222}(t, x_2)\| \leq 0.1\sqrt{x_{11}^2 + x_{13}^2} + 0.1\|x_{15} - x_{12}^2\| + 0.1|x_{21}| + 0.1|x_{22}| \quad (49)$$

Let

$$h_1(x_1) = \begin{bmatrix} h_{11}(x_1) \\ h_{12}(x_1) \end{bmatrix} = \begin{bmatrix} x_{12} \\ x_{14} \end{bmatrix}, \quad h_2(x_2) = x_{21}$$

By direct computation, the nominal isolated subsystem of the first subsystem (40) has uniform relative degree  $r_1 = (r_{11}, r_{12}) = (2, 2)$ , and the nominal isolated subsystem of the second subsystem (41) has relative degree  $r_2 = 2$ .

The coordinate transformation  $z_i = T_i(x)$  for  $i = 1, 2$  is given by

$$T_1 = \begin{cases} z_1^a = \begin{bmatrix} z_{11} \\ z_{12} \\ z_{13} \\ z_{14} \end{bmatrix} = \begin{bmatrix} x_{12} \\ x_{11} \\ x_{14} \\ x_{13} \end{bmatrix} \\ z_1^b = z_{15} = x_{15} - x_{12}^2 \end{cases} \quad (50)$$

$$T_2 = \begin{cases} z_2^a = \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix} = \begin{bmatrix} x_{21} \\ -x_{21} + 0.1 \sin x_{23} \end{bmatrix} \\ z_2^b = z_{23} = x_{22} \end{cases} \quad (51)$$

Then consider the state feedback

$$u_i = -\Lambda_i^{-1}(z_i)\alpha_i(z_i) + \Lambda_i^{-1}(z_i)\nu_i \quad i = 1, 2 \quad (52)$$

where

$$\Lambda_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 + \sin(z_{15} + z_{11}^2) \end{bmatrix}, \quad \Lambda_2 = 0.1 \cos \{\arcsin \{10(z_{21} + z_{22})\}\} \quad (53)$$

$$\alpha_1 = \begin{bmatrix} -\sin z_{11} \\ z_{11}^2 + z_{14}^2 \end{bmatrix}, \quad \alpha_2 = -z_{22} + 0.1z_{21}^2 z_{23} \cos \{\arcsin \{10(z_{21} + z_{22})\}\} \quad (54)$$

In the new coordinates  $z$  given in (50)-(51), by applying (52) to the nonlinear interconnected system (40)-(41), the closed-loop system is described by:

$$\begin{aligned} \dot{z}_{11}^a &= A_1 z_{11}^a + B_1 z_{12}^a + \Xi_{11}^a(t, z_1) + \Psi_{11}^a(t, z) \\ \dot{z}_{12}^a &= \nu_1 + \Xi_{12}^a(t, z_1) + \Psi_{12}^a(t, z) \\ \dot{z}_1^b &= -4(z_{15} + z_{11}^2) + \Xi_1^b(t, z_1) \end{aligned} \quad (55)$$

$$\begin{aligned} \dot{z}_{21}^a &= A_2 z_{21}^a + B_2 z_{22}^a \\ \dot{z}_{22}^a &= \nu_2 + \Xi_{22}^a(t, z_2) \\ \dot{z}_2^b &= -3z_{23} - z_{21}^2 + \Xi_2^b(t, z_2) + \Psi_2^b(t, z) \end{aligned} \quad (56)$$

where  $z := \text{col}(z_1, z_2) \in \mathbf{Z} = \{(z_1, z_2) | |z_{15}| \leq \frac{1}{8}, |\arcsin\{10(z_{21} + z_{22})\}| \leq 1, |z_{23}| \leq \frac{1}{2}\}$ ,  $z_1 := \text{col}(z_{11}^a, z_{12}^b, z_{12}^a)$ ,  $z_{11}^a := \text{col}(z_{11}, z_{13})$ ,  $z_{12}^a := \text{col}(z_{12}, z_{14})$ ,  $z_1^b := z_{15}$ ,  $z_2 := \text{col}(z_{21}^a, z_{22}^b, z_{22}^a)$ ,  $z_{21}^a := z_{21}$ ,  $z_{22}^a := z_{22}$ ,  $z_2^b := z_{23}$ ,  $A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $A_2 = 0$  and  $B_2 = 1$ . Meanwhile, the uncertainties and interconnections satisfy

$$\|\Xi_{11}^a(t, z_1)\| \leq \rho_1(z_{11}^a, z_{12}^a, z_1^b) = \underbrace{0.05}_{\chi_{11}} \|z_{11}^a\| + \underbrace{0.05}_{\beta_{11}} \|z_{12}^a\| + \underbrace{0.1}_{\gamma_{11}} \|z_1^b\| \quad (57)$$

$$\|\Psi_{11}^a(t, z)\| \leq \underbrace{0.49|z_{11}|}_{\beta_{113}} \|z_{12}^a\| + \underbrace{0.1}_{\gamma_{113}} \|z_1^b\| + \underbrace{0.1}_{\chi_{123}} \|z_{21}^a\| + \underbrace{0.1}_{\gamma_{123}} \|z_2^b\| \quad (58)$$

$$\|\Xi_{12}^a(t, z_1)\| \leq \mu_1 = |\sin(z_{11} + z_{12} + z_{13})| + |z_{14}| \quad (59)$$

$$\|\Psi_{12}^a(t, z)\| \leq \underbrace{|\sin(z_{14})|}_{\gamma_{114}} \|z_1^b\| + \underbrace{0.5}_{\alpha_{124}} \|z_{21}^a\| \quad (60)$$

$$\|\Xi_1^b(t, z_1)\| \leq \underbrace{0.1}_{\beta_{12}} \|z_{12}^a\| + \underbrace{0.1}_{\gamma_{12}} \|z_1^b\| \quad (61)$$

$$\|\Xi_{22}^a(t, z_2)\| \leq \mu_2 = |\sin(z_{21})| + |z_{23}| \quad (62)$$

$$\|\Xi_2^b(t, z_2)\| \leq \underbrace{0.1}_{\alpha_{22}} \|z_{21}^a\| + \underbrace{0.1}_{\gamma_{22}} \|z_2^b\| \quad (63)$$

$$\|\Psi_2^b(t, z)\| \leq \underbrace{0.1}_{\beta_{215}} \|z_{12}^a\| + \underbrace{0.1}_{\gamma_{215}} \|z_1^b\| + \underbrace{0.1}_{\alpha_{225}} \|z_{21}^a\| + \underbrace{0.1}_{\gamma_{225}} \|z_2^b\| \quad (64)$$

Choose  $K_1 = I_2$ ,  $K_2 = I_1$ ,  $Q_1 = 4I_2$ ,  $Q_2 = 4I_1$  and

$$P_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad P_2 = 2 \quad (65)$$

Then choose  $V_1^b = z_{11}^2 + z_{13}^2 + z_{15}^2$  and  $V_2^b = z_{21}^2 + z_{23}^2$ . By calculation, it follows that in the considered domain in  $z \in \mathbf{Z}$ , Assumption 2 is satisfied with

$$\begin{aligned} c_{11} &= 1, c_{12} = 1, c_{13} = 1, c_{14} = 1, c_{15} = 1, c_{16} = 8, c_{17} = 2, c_{18} = 2 \\ c_{21} &= 1, c_{22} = 1, c_{23} = 1, c_{24} = 1, c_{25} = 1, c_{26} = 6, c_{27} = 2, c_{28} = 2 \end{aligned} \quad (66)$$

Then, when the sliding motion takes place, inequalities (22)-(23) are satisfied with  $\varepsilon_{111} = 0.49|z_{11}|$ ,  $\varepsilon_{112} = \varepsilon_{121} = \varepsilon_{122} = \varepsilon_{13} = \varepsilon_{14} = \varepsilon_{17} = \varepsilon_{18} = \varepsilon_{27} = \varepsilon_{28} = \varepsilon_{215} = \varepsilon_{216} = \varepsilon_{225} = \varepsilon_{226} = 0.1$ ,  $\varepsilon_{115} = \varepsilon_{116} = \varepsilon_{125} = \varepsilon_{126} = \varepsilon_{23} = \varepsilon_{24} = \varepsilon_{211} = \varepsilon_{212} = \varepsilon_{221} = \varepsilon_{222} = 0$ .

By direct computation,

$$M + M^T = \begin{bmatrix} 8.4 - 5.88|z_{11}| & -0.8 & -3.6 - 1.96|z_{11}| & -1.2 \\ -0.8 & 9.2 & -1.2 & -1.6 \\ -3.6 - 1.96|z_{11}| & -1.2 & 14.8 & -0.4 \\ -1.2 & -1.6 & -0.4 & 11.2 \end{bmatrix} \quad (67)$$

which is symmetric positive definite in  $z \in \mathbf{Z}$ .

Hence the conditions of Theorem 1 hold.

Based on the parameter selections above, the control (35) is well defined with  $\eta_1 = 0.01$ ,  $\eta_2 = 0.01$  and  $\Phi_1$ ,  $\Phi_2$  given by:

$$\Phi_1 = 0.49|z_{11}|\|z_{12}^a\| + (|\sin(z_{14})| + 0.1)\|z_1^b\| \quad (68)$$

$$\Phi_2 = 0.6\|z_{21}^a\| + 0.1\|z_2^b\| \quad (69)$$

For simulation purposes,  $\sigma_i / (\|\sigma_i\| + \beta_i)$  is used to replace  $\text{sgn}(\sigma_i)$  in order to reduce the chattering with  $\beta_1 = \beta_2 = 0.001$ . The time response of the system states is shown in Fig.1. The time response of the system control signals is shown in Fig.2. The time response of the sliding functions is shown in Fig.3. It can be seen that all subsystems can be stabilized even in the presence of unmatched uncertainties  $\xi_i$  and interconnections  $\sum_{j=1}^n H_{ij}(t, x_j)$ . It also shows that the nonlinear interconnected systems can be driven to the sliding surface (18) in finite time. The simulation results show that the proposed decentralised sliding mode control is effective. The obtained results also validate the approach of transforming system (40)-(41) into system (55)-(56) by local coordinate transformation and feedback linearization.

*Remark 12:* All the simulation parameters have been given.  $h_i(x_i)$  are chosen to get the uniform relative degree  $r_i$  based on [29].  $\eta_i$  are small positive constants to guarantee the reachability condition (35). For convenience,  $Q_i = \alpha_i I_{n_i}$  with  $\alpha_i$  are positive constants. Then  $P_i$  are positive definite symmetric matrices to guarantee  $M^T + M > 0$  defined in (24). Finally,  $K_i$  can be obtained by solving the LMI (17) since  $P_i$  and  $Q_i$  have been chosen.

*Remark 13:* It should be noted that the above example has the following characteristics: (i) the nominal isolated systems are fully nonlinear; (ii) the uncertainties  $\xi_i$  are nonlinear and cannot be bounded by a linear function of  $x_i$ ; (iii) the interconnections  $\sum_{j=1}^n H_{ij}(t, x_j)$  are also nonlinear and cannot be bounded by a linear function of  $x_j$ ; (iv) the uncertainties  $\xi_i$  and interconnections  $\sum_{j=1}^n H_{ij}(t, x_j)$  are both unmatched. Therefore, existing decentralised state feedback schemes ([36], [37], [38], [39]) cannot apply to this nonlinear interconnected systems (40)-(41). Furthermore, the schemes of [27] cannot be applied unless the system (40)-(41) is in the required regular form in [27]. This shows that the results of this paper are valuable.

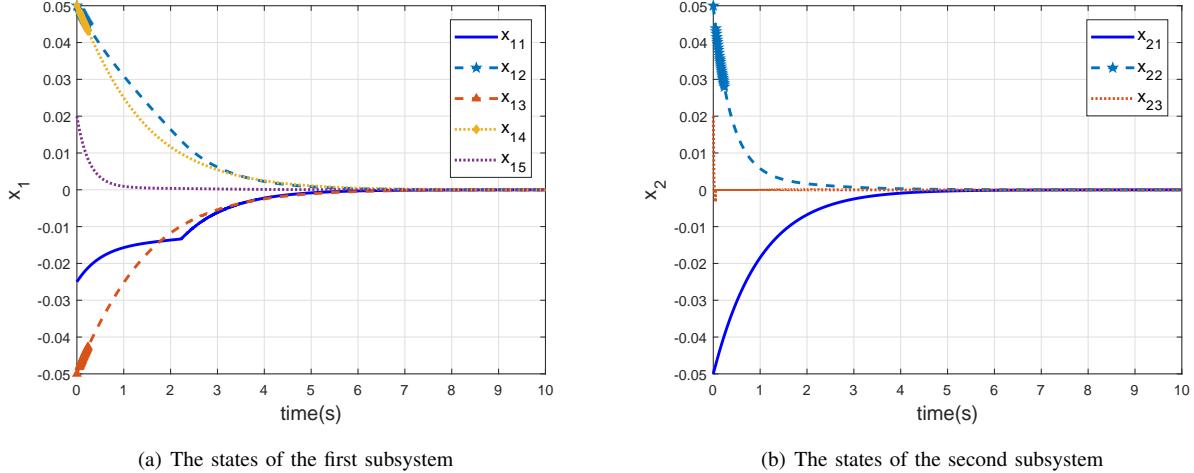
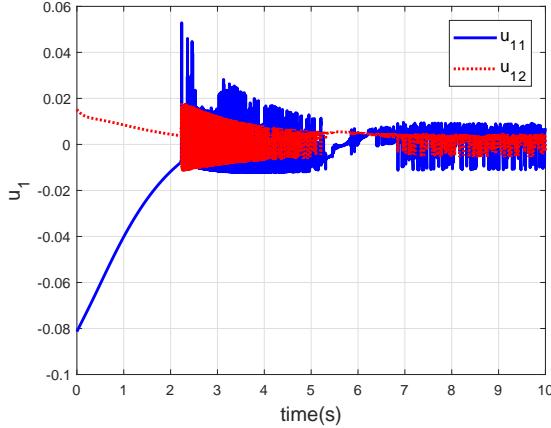
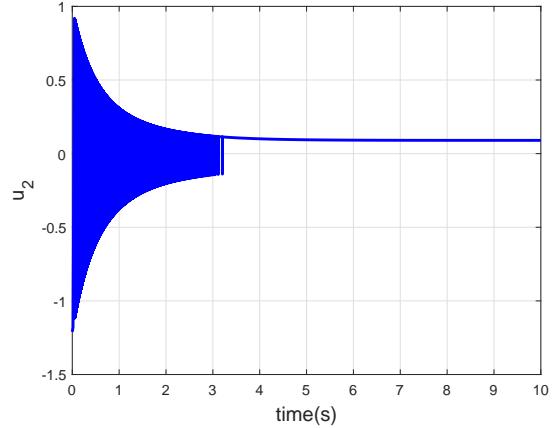


Fig. 1: The time response of the system states

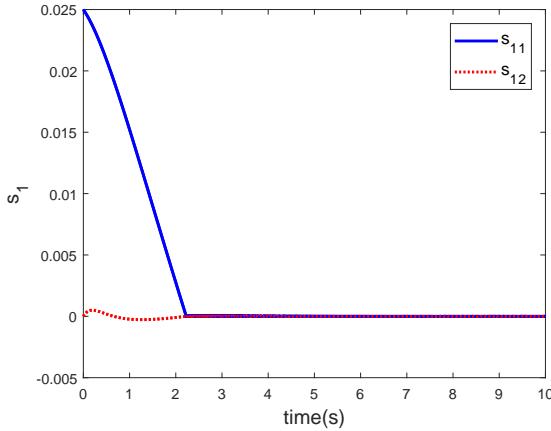


(a) The control signals of the first subsystem

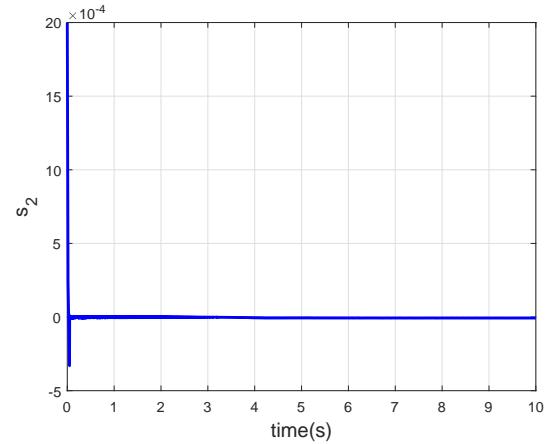


(b) The control signals of the second subsystem

Fig. 2: The time response of the system control signals



(a) The sliding functions of the first subsystem



(b) The sliding functions of the second subsystem

Fig. 3: The time response of the sliding functions

To further test the proposed decentralised sliding mode control, the decentralised state feedback control (DSFC) scheme proposed in [23] will be compared with the proposed method in this paper. The main control parameters in [23] are given:

$$K_1 = \begin{bmatrix} 10 & 20 & -5 & -3 \\ 2 & 10 & 2 & 3 \end{bmatrix}, P_1 = \begin{bmatrix} 2.27 & 0.02 & 0.10 & 0.14 \\ 0.02 & 0.05 & -0.04 & -0.06 \\ 0.10 & -0.04 & 1.14 & 0.14 \\ 0.14 & -0.06 & 0.14 & 0.16 \end{bmatrix} \quad (70)$$

$$K_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.50 & 0.50 \\ 0.50 & 1.00 \end{bmatrix}$$

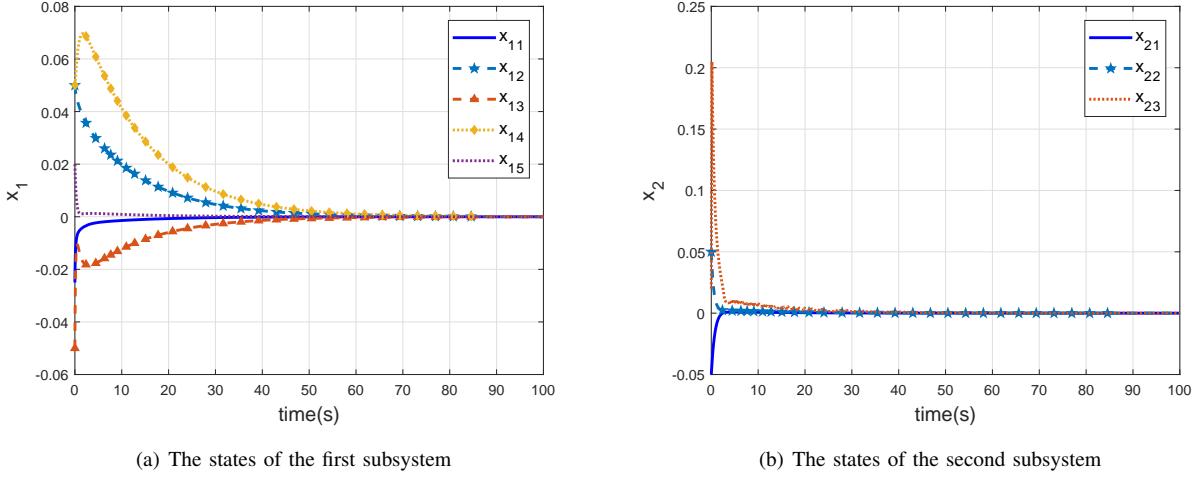


Fig. 4: The time response of the system states by using DSFC proposed in [23]

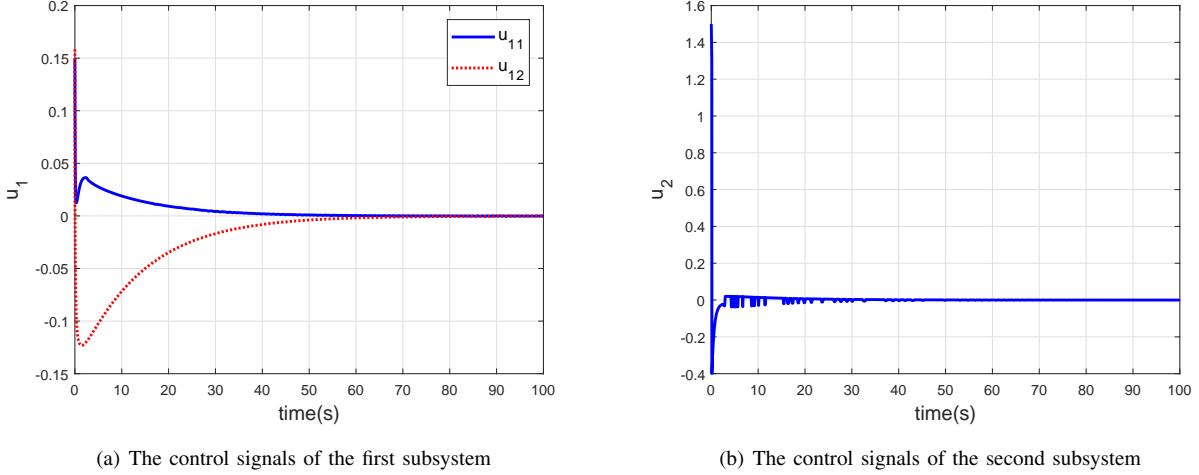


Fig. 5: The time response of the system control signals by using DSFC proposed in [23]

The time response of the system states is shown in Fig.4 by using DSFC proposed in [23]. The time response of the system control signals is shown in Fig.5 by using DSFC proposed in [23]. It should be noted that the unmatched interconnections  $\Psi_{11}^a(t, z)$  has not been considered in [23], but the method proposed in this paper can deal with the unmatched interconnections well. Meanwhile, the unmatched uncertainties  $\Xi_{11}^a(t, z_1)$  has been required to satisfy a so called weakly unmatched condition in [23], however stronger unmatched uncertainties can also be dealt in this paper. Comparing with Fig.1 and Fig.4, the proposed approach in this paper has a faster response speed due to it considers the unmatched uncertainties and the unmatched interconnections explicitly.

In order to further test the robustness of the proposed decentralised sliding mode control, the unmatched uncertainties  $\xi_{14}$  and interconnections  $H_{114}(t, x_1) + H_{124}(t, x_2)$  of the system (40)-(41) will be increased, while the other terms remain unchanged. Specifically, the conditions (43) and (46) are transformed into:

$$\|\xi_{14}\| \leq 0.2\sqrt{x_{12}^2 + x_{14}^2} + 0.2\sqrt{x_{11}^2 + x_{13}^2} + 0.4\|x_{15} - x_{12}^2\| \quad (71)$$

$$\|H_{114}(t, x_1) + H_{124}(t, x_2)\| \leq 1.96|x_{12}|\sqrt{x_{11}^2 + x_{13}^2} + 0.4\|x_{15} - x_{12}^2\| + 0.4\|x_{21}\| + 0.4\|x_{22}\| \quad (72)$$

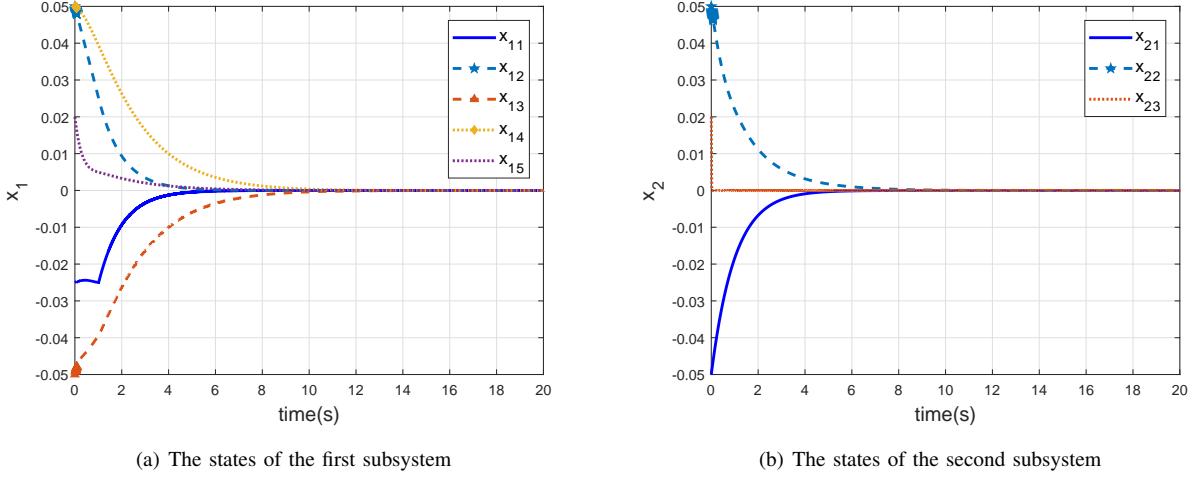


Fig. 6: The time response of the system states by using the method proposed in this paper under the new conditions (71)-(72)

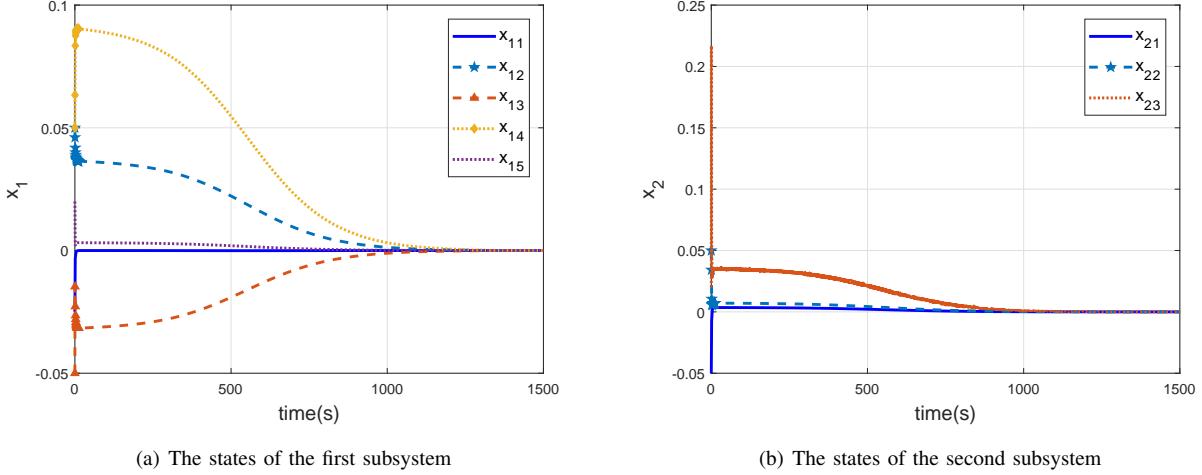


Fig. 7: The time response of the system states by using DSFC proposed in [23] under the new conditions (71)-(72)

The time response of the system states is shown in Fig.6 by using proposed approach in this paper under the new conditions (71)-(72). The time response of the system states is shown in Fig.7 by using DSFC proposed in [23] under the new conditions (71)-(72). Comparing with Fig.6 and Fig.7, it is easy to see that although the unmatched interconnections and uncertainties increase, the proposed method still has a better performance.

## VI. CONCLUSION

A robust decentralized control design method has been proposed for a class of interconnected large-scale systems with nonlinear nominal isolated systems. The uncertainties and interconnections of the systems are all nonlinear with nonlinear bounds. The boundary of uncertainties and interconnections is assumed to comprise known functions. These have been employed to reject the effects of the uncertainty and interconnections on the system to enhance robustness. A sliding mode control has been designed to ensure that the system states reach the designed sliding surface in finite time. The obtained results can be applied to a class of interconnected systems with uniform relative degree and involutive distribution. A numerical example is given to show the effectiveness of the proposed decentralised control scheme. Future work will focus on experimental testing and the practical application of the proposed method. In the aspect of theory, decentralized output feedback sliding mode control will be further considered.

## VII. DECLARATION OF CONFLICTING INTERESTS

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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