Pythagoras and Plato

Pythagoras, Plato and their followers are often put together in accounts of ancient science. There are reasons why this should be so but there are also reasons why we should be suspicious of this association and a need to be aware of the differences as well as the similarities between the two schools of thought. Plato was certainly influenced to some extent by the Pythagoreans, though to what extent is still matter of debate. In the ancient world after Plato, there was a trend to attribute all of the work of the Pythagoreans to Pythagoras and to assimilate the Platonic and Pythagorean views together, sometimes attributing Plato's views to Pythagoras. In the modern world, there has been a tendency to group Pythagoras and Plato together on the basis that both were pioneers of the role of mathematics in science and both were allegedly interested in some form of numerology or number mysticism. The nature of the mathematization they envisage, however, has different manifestations and significantly different motives.

Pythagoras

Pythagoras was born in Samos *ca* 570 BCE and died *ca* 490 BCE. Around 530 BCE he relocated to Croton, which became a center for the Pythagoreans. Pythagoras himself wrote nothing and if his contemporaries wrote anything about him nothing of this has survived, so all we know of Pythagoras comes from significantly later sources. The "Pythagorean question" is how trustworthy these sources are for an attempt to reconstruct Pythagoras' own thought (Huffman 1999). It is possible to generate a picture of Pythagoras as someone who was an expert in mathematics and geometry, who proved Pythagoras' theorem, who made important discoveries on harmonics and mathematized harmonic theory, who in astronomy believed there was a harmony of the spheres, who believed the world was in some way constituted from numbers and that mathematics was the key to understanding the cosmos. However, it is clear that from the time of Plato and Aristotle onwards that many sources treated Pythagoras as a semi-divine or a divinely inspired figure. These sources glorified Pythagoras, often seeing him as the origin of philosophy and attributing to him many of the ideas of the later Pythagoreans, Plato and Aristotle (Burkert 1972; Huffman 1999; Dillon 2003). Many forged works were generated purporting to be by Pythagoras.

Since Walter Burkert's seminal *Lore and Science in Ancient Pythagoreanism* (1972) it has been held that to find out about Pythagoras, we must look to the earliest and least corrupt sources on him, which means looking at Plato and Aristotle. The picture that then emerges is radically different. It is of someone whose key beliefs were in the immortality of the soul and reincarnation and whose expertise was in the fate of the soul after death and in the nature of religious ritual. Pythagoras' major achievements are seen as the advocacy and founding a way of life based on stringent dietary regulations, strict self-discipline and the keen observance of religious ritual. It is notable though that while both Plato and Aristotle talk of presocratic natural philosophy, they do not give Pythagoras any significant role in this, nor do they recognize any Pythagorean cosmology prior to Philolaus (Plato, *Republic* 531a, 600a, *Sophist* 242c; Aristotle *Metaphysics* 1.5, *De Caelo* 2.13).

Pythagoras is not credited with a proof of Pythagoras' theorem, nor seen as a significant mathematician or geometer by Plato and Aristotle, and that is true of early histories of Greek mathematics as well. Pythagoras is not associated by Plato and Aristotle with any harmonic theory (Plato *Republic* 531a, *Timaeus* 35b; Aristotle *De Caelo* 2.9). It should be noted that one often-quoted tale of Pythagoras is that he discovered the mathematical ratios underpinning harmonics (2:1 as the octave, 3:2 as the fifth, 4:3 as the fourth etc.). He is alleged to have done so by listening to the sounds made by hammers in a smithy and finding the weights of these hammers to be in these integer ratios. However, the weight of a hammer has no such direct relationship to the note it will produce, so this tale must be false.

Burkert's approach, while a significant advance, does not entirely close the Pythagorean question. Plato did not claim to be a historian and when he mentions previous thinkers or theories he does so in philosophical, polemical or literary contexts which make it difficult to judge how accurate his representation is. Aristotle, though he did give historical surveys, is notorious for interpreting precious thinkers in terms of his own thought, sometimes producing serious distortions. There is evidence that Pythagoras was aware of something related to Pythagoras' theorem without having himself generated a proof, perhaps Pythagorean triples (integer side lengths for Pythagorean triangles such as 3, 4, 5), perhaps a significant diagram, perhaps the theorem but not the proof (Burkert 1972). Pythagoras clearly valued such knowledge highly as he is said to have sacrificed an oxen upon its discovery. Zhmud (1998) makes the best case for discovery by Pythagoras.

There is also evidence that Pythagoras valued mathematics in education. It may well be the case that the notion of the tetraktys can be traced back to Pythagoras. The tetraktys is the first four numbers and their sum is the Pythagorean perfect number, 10. They are often arranged in this manner:

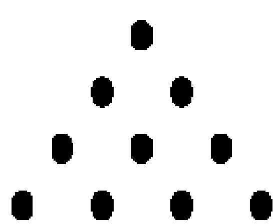

Figure 1

There is evidence that the tetraktys was in some way related to the harmony sung by the Sirens, which in turn may be related to the notion of the harmony of the spheres (Aristotle, *De Caelo* 2.9). Recent advances in our understanding of the relations between religion and science and between magic and science have rendered a position where Pythagoras was an important religious thinker but still interested in science more plausible. A parallel here is with scholarship on Empedocles. Historians who believed religion and science to be incompatible sorted Empedocles' fragments into two different works and assigned them to different parts of his life. More recently, historians who are happy with the compatibility of science and religion have made significant advances in piecing the fragments together in other ways producing a more coherent picture of Empedocles. A similar historiographical realignment has occurred in thinking about magic and science. We are not then forced to choose between a religious/ magical Pythagoras and a scientific Pythagoras, but may have a Pythagoras who combines all of these elements. The current state of the Pythagorean question then is that while Pythagoras was not the important mathematician, cosmologist and harmonic theorist of legend, he did have an interest in mathematical and related issues and that the tradition he fostered facilitated later Pythagoreans such as Philolaus and Archytas who developed many of the views that were later attributed to Pythagoras (Huffman 1999).

Before we move on to Philolaus and Archytas, who are considered to be Pythagoreans, Alkmaeon of Croton (dates uncertain but probably a generation younger than Pythagoras) has often been assumed to be a Pythagorean but is now generally reckoned not to have been so. The earliest evidence that he was a Pythagorean comes from Diogenes Laertius in the 3rd century CE. Aristotle wrote on the Pythagoreans, but wrote separately on Alkmaeon. He also contrasts Pythagorean views on opposites with those of Alkmaeon. The opposites which interest

Alkmaeon do not appear in the Pythagorean table of opposites and the critical limited/ unlimited Pythagorean pair did not interest Alkmaeon.

Philolaus

Philolaus of Croton lived from *ca* 470 to *ca* 385 BCE. Philolaus and Archytas were the most significant contributors to the Pythagorean tradition in the presocratic period. Philolaus wrote one book, *On Nature*, which if Pythagoras wrote nothing, is probably the first book of the Pythagorean tradition, of which a few fragments survive. He worked on astronomy, cosmology and cosmogony, on harmonic theory, medical theory and had an ontology of an unlimited which was determined by limiters (Huffman 1993).

With Philolaus we have the first surviving Pythagorean cosmology, as depicted in the diagram below:

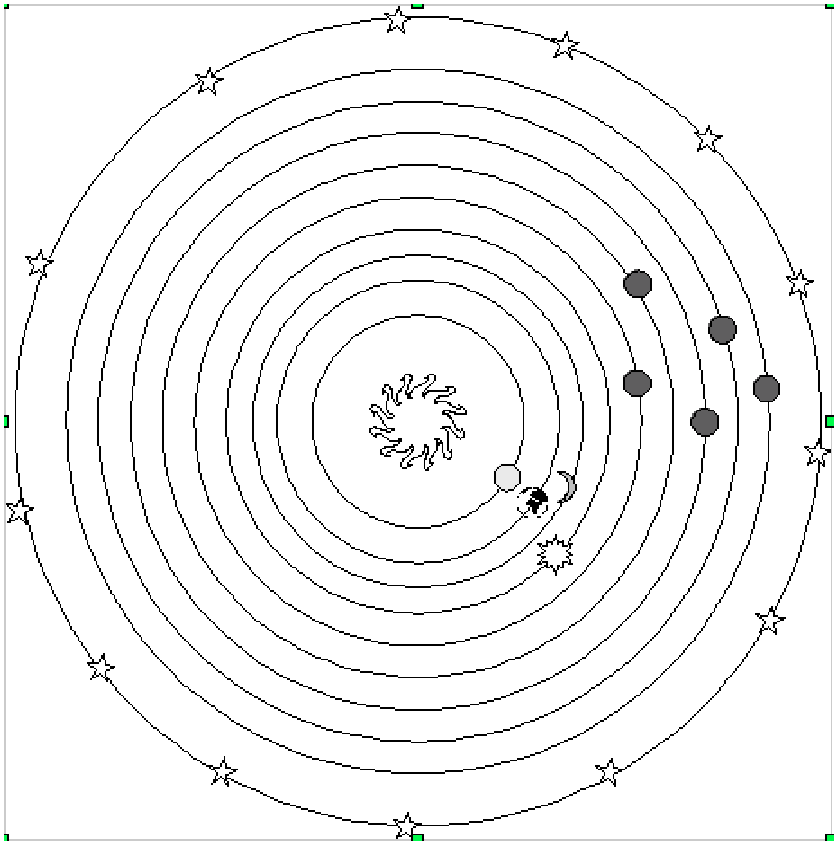

Figure 2

From the center moving outwards, there is a central fire, a counter-earth, the earth, the moon, the sun, the five naked eye planets (Mercury, Venus, Mars, Jupiter, Saturn) and the stars. This cosmology is notable for being one of the very few in antiquity to displace the earth from the center of the cosmos. Instead there is a central fire (not the sun) with the earth and sun orbiting around it. No explicit reason has survived as to why the earth was placed in motion. How well this model could account for the phenomena is still open to debate, as is whether accounting for the phenomena or some form of religious/ eschatological symbolism was its main function. It does seem that Philolaus replied to criticism that his model would not account for the phenomena by elaborating on his system to suggest how the criticisms might be met. In reply to the objection that we do not see the counter-earth or central fire Philolaus supposed a rotation of the earth such that we are always looking in the opposite direction. In reply to the objection that if the earth was in motion within his system, we would not see the sun and moon as we do, Philolaus replied that the earth's orbit was small in relation to the relevant distances so the effect would be negligible. This would suggest that both critics and Philolaus took his system seriously as a model (Huffman 1993). Aristotle is critical of Philolaus on slightly different lines, saying that the reason Philolaus supposed there to be 10 heavenly bodies was that 10 was the perfect Pythagorean number (Aristotle, *Metaphysics* 1.5, *De Caelo* 2.13).

One explanation of why there is a central fire in Philolaus' cosmology is his cosmogony. Fragment 7 has a hearth in the centre of the sphere as the first thing to be generated as the cosmos is fitted together. The cosmos comes to be from the unlimited and limiters. The outline of Philolaus' view of unlimited and limiters is clear enough even if we lack enough information to say much in detail about this view. It would seem that fire is first limited in the centre of the cosmos and then other unlimiteds are drawn in. There is some debate on exactly what these are, but it is likely they include breath, space and time. There is also debate concerning whether number is generated with the cosmos and whether the central fire is in some sense the number one or whether being the first unity, one describes the central fire. In common with other presocratic cosmogonies, limit and unlimited are seen as existing prior to the organization of the cosmos so there is no creation *ex nihilo*. The idea of the application of limit to the unlimited may be a comment on Anaximander, who had an initial unlimited which separated out into the elements rather than being limited (Huffman 1988, 1999, 2001).

Harmony

Philolaus' theory of music is a form of what is known as just intonation, that is it is based on ratios of small integers (Barker 1989). Whether Pythagoras, Philolaus or someone else discovered that the lengths of string required to generate musical notes have simple ratios or not, Philolaus produced the first theoretical account of this. If the length of a string is halved, then the note it sounds is an octave of the first note, and ratio of the string lengths is 2:1. If the ratio is 4:3 we get a musical fourth and if the ratio is 3:2 we get a musical fifth. Both of these notes sound harmonious when played with the original note. A fourth and a fifth make an octave (4/3 x 3/2 = 12/6 = 2/1). The difference between a fourth and a fifth is 9/8 (4/3 x 9/8 = 3/2). 9:8 is the ratio used to generate one whole tone. So if the root note is taken as 1, the first note in the scale will be 9/8. The second note will be 9/8 x 9/8 = 81/64. The Philolean semitone is generated by 256:243. The third note in the scale is 81/64 x 256/243 = 4/3 (a musical fourth). The following note is 4/3 x 9/8 = 3/2 (a musical fifth), then 3/2 x 9/8 = 27/16, then 27/16 x 9/8 = 243/128 and finally 243/128 x 256/243 = 2, which gives the octave as a ration of 2:1. Modern music uses something called twelve tone equal temperament ("12ET") where there are equal ratios between the twelve semi-tones making up an octave, that ratio being $\sqrt[12]{2}$ (the twelfth root of two). This table gives some sense of the differences:

The first row is Philolaus' notes expressed as ratios.

The second row gives the modern note names in the key of C major.

The third row are notes in 12ET expressed in "cents", where 1200 cents = one octave and 100 cents = one semi-tone.

The fourth row is Philolaus' note position expressed in cents.

1		9/8		81/64	4/3		3/2		27/16		243/128	2
C		D		E	F		G		A		B	C
0	100	200	300	400	500	600	700	800	900	1000	1100	1200
		203.91		407.82	498.04		701.96		905.87		1109.78	1200

The Philolean scale is mathematically very pure, using only powers of 2 and 3. The ratio 256/243 initially looks obscure, but in fact is $2^8/3^5$. Just intonation, like that of Philolaus, gives a purer sound to harmonies based on the fourth and the fifth, but it is rather inflexible and impractical. The advantages of 12ET are that it is easier to modulate (change key within a piece of music), easier to tune a range of different instruments to play together and that chords (three or more notes sounded together) sound rather better, at the cost of a little purity of some harmonies.

Medical

We have some information on Philolaus' medical views on the embryo and on disease. Philolaus conceived of the embryo as hot, as for him both womb and sperm are hot. On birth, we breathe in cool air. There are clear parallels here with the cosmogony of a central fire drawing in the unlimited and such parallels between birth and cosmogony are common among the presocratics. On disease, Philolaus considered blood, bile and phlegm to be hot and imbalances of hot, cold and nutrition to be cause of disease, a possible implication being that the appropriate cooling of the body is critical to health, in a parallel with the first breath cooling the new born infant.

Archytas

Archytas of Tarentum, 428-347 BCE, is important for his work in mathematics, cosmology and harmony theory and was also active as a political leader. If he wrote anything it has not survived as a whole. Only a few fragments have come down to us, though they are important ones. There were many works forged in Archytas' name in late antiquity (Huffman 2005).

Cosmology

The most famous argument that we have from Archytas is a thought experiment concerning the finite nature of space. Archytas imagines someone standing at the limit of a finite cosmos. Can this person take a staff and thrust it beyond the limit of the cosmos? If he can, and our intuition is that he can, then this is not the limit of space, and we have a new limit. Moreover, this thought experiment is replicable, that is wherever a new limit is supposed we can suppose someone thrusting a staff beyond it. Therefore space is unlimited.

This argument was much discussed in antiquity. The Stoics and Epicureans supported it and argued for an unlimited space, Plato and Aristotle argued in their own way for a limited space and the argument was much discussed by later commentators. Two replies to Archytas were that it is impossible to stand at the edge of the cosmos, or more subtly, outside the cosmos there is neither time nor space so there is nowhere to thrust the staff. Archytas' argument is less discussed nowadays as we have the conception of finite but unlimited space which has no edges (Huffman 2005).

Music theory

Archytas' work on harmonic theory builds on that of Philolaus and Archytas also had a theory of pitch (Barker 1989, Huffman 2005). According to Archytas, the pitch of a sound is related to how quickly it travels, a sound traveling more quickly being of higher pitch. Actually the speed of sound is a constant for a given medium, and it is frequency that is critical to pitch, how rapidly a string vibrates determining the frequency rather than the speed of the sound.

Archytas produced a variation on Philolaus' musical scale, using 9:8, 8:7 and 28:27 to determine the notes up to the fourth (9/8 x 8/7 x 28/27 = 4/3). This sort of scale is known as a diatonic and Archytas also worked on two other types of scale, the chromatic and the enharmonic. A chromatic scale includes all twelve semi-tones (which in 12ET would be equally spaced). The key ratios for Archytas' chromatic scale are 32:27, 243:224 and 28:27 (32/27 x 243/224 x 28/27 = 4/3). In the chromatic scale, $A^{\#} = B^{b}$. In an enharmonic scale this is not so, and what we would call $A^{\#}$ differs from B^{b}. The key ratios for Archytas' enharmonic scale are 5:4, 36:35, 28:27 (5/4 x 36/35 x 28/27 = 4/3). In contrast to Philolaus, who seems to be generating in some ways an ideal scale, Archytas seems to have been describing the scales in use during his time (Barker 1989, 50). He may be the target of Plato's criticism that the Pythagoreans search for audible harmonies when they should be considering which numbers are harmonious and why.

Mathematics

Archytas demonstrated one very important property of what are known as superparticular ratios, that is ratios of the type where n+1:n. If p bears the same proportion to q as q does to r, then q is the mean proportional of p and r (if p:q :: q:r). This is important in music, as a double octave (4:1) can be split into two octaves with a mean proportional as 4:2 is the same proportion as 2:1. Archytas though proved that there is no mean proportional for numbers in superparticular ratios. This means that critical musical ratios, such as 3:2, 4:3 and 9:8 (which all have the form n+1:n) have no mean proportional and cannot be split in to two equal parts.

Archytas is famous for having provided a solution to the "Delian problem", that of doubling the volume of a cube (Heath 1921; Mueller 1997; Huffman 2005). What length is required for the sides if the volume of a cube is to be doubled? Archytas' solution built on an insight of Hippocrates of Chios. If L is the length of the original cube, it is possible to set up a series of ratios such that L:a :: a:b :: b:2L (L is in proportion to a, as a is to b, as b is to 2L). It is then possible to derive the relation $L:2L = L^3:a^3$. As $L^3:a^3$ is in the ratio of 1:2, a^3 is twice L^3, and the cube can be built with sides of length a. Archytas' solution, which is too complex to give in full here, involved constructing four similar triangles in the proportions suggested by Hippocrates by an imaginary rotation of triangles and calculation of their points of intersection.

At the beginning of his book on harmonics, Archytas praised the value of four disciplines, astronomy, geometry, "logistic" (calculation) and music (Huffman 2005). He also praised those who have practiced these disciplines before him. Of these disciplines, he takes calculation to be the key subject. It is hard to be precise on exactly what Archytas meant by logistic, though doubtless it is related to the notion that to know something is to know its relation to number, whether that be in terms of musical ratios, geometry or astronomy.

Plato

We have the great majority of the works of Plato, 428/427-328/327 BCE, though the interpretation of them is complex, as Plato wrote dialogues rather than treatises and it is far from clear how the views given to the characters in these dialogues relate to Plato's own views. Opinions on the nature and contribution of Plato' views on science and their subsequent influence have been widely varied and continue to be so (Lloyd 1968, 1991; Anton 1981; Gregory 2000; Johansen 2004). At least in part this is due to continuing disagreement about the nature of Plato's metaphysics but also involves the disputation of certain key passages where Plato mentions scientific topics. Plato has been accused of being anti-science, or at least antiphysical or empirical science, while others have seen him as an important pioneer of the role of mathematics in science and of an important tradition in ancient astronomy. Recent work on Plato has focused on the context and goals of investigation for Plato.

The key metaphysical issue is Plato's contrast between the particular things in the world about us and the forms. Exactly what Plato took the forms to be and whether he believed in them for his entire career are still hotly contested questions. One way of characterizing the difference between particulars and forms is like this. Particulars are perceptible, changing, material, and we can only have opinion about them. Forms are apprehended intellectually, are entirely unchanging, are immaterial and we can have knowledge about them. Forms are said to be, while particulars are in a state of becoming. In the middle books of the *Republic*, Plato develops the allegories of the sun, line and cave, where he emphasizes the ascent from the perception of particulars to the contemplation of forms as important for philosophy (Plato, *Republic* 509d ff.). It is here that he also make the notorious comment that astronomy should be pursued by means of problems as with geometry and we should set empirical considerations aside. Where does this leave science? Some commentators have taken the view that science for Plato only concerns forms and being and so excludes observation (Heath 1913, 135; Mueller 1981, 104; Knorr 1993, 399; Hetherington 1993, 85). Others have taken the view that science for Plato deals with the physical world of becoming, but as such science can never rise above the level of opinion (Cornford 1937, 29; Lee 1955, 311). In support, they cite the fact that Plato's Timaeus, his later work on the nature of the cosmos, calls itself an *eikos muthos*, a likely story.

More recent work has challenged these sorts of views on several levels. The characterization of forms and particulars given above has become known a the 'Two Worlds' view and has come under fire for being too rigid and formulaic and not being sufficiently subtle to capture Plato's concerns. Rather than viewing science or scientific disciplines as the sole province of one of these worlds, recent commentators have emphasized the notion of ascent such that disciplines begin by perceiving particulars and ascend to considering forms, thus giving an empirical role to each discipline (Gregory 2000). Commentators have also pointed out that in the middle books of the *Republic*, Plato also comments that having ascended to the contemplation of forms and learnt about justice, it is the duty of the guardians to return to govern the world about us (Lloyd 1991). So too then, knowledge of the relevant forms may help with scientific disciplines. The context of the comments about astronomy and observation in the middle books of the Republic have also be taken to be important. Plato is discussing how astronomy should be used in the philosophical education of the guardians of his ideal state. In this context it is not surprising that he calls for them to think about the nature of the heavens rather than carry out observations. Plato is making an educational point, not a point about how we investigate the heavens. If we turn to the *Timaeus*, at 47a ff. we find a eulogy to how sight can help with astronomy and as Vlastos has commented, the *Timaeus* is full of the language of observational astronomy (Vlastos 1975). On the nature of the *Timaeus'* account, exactly what Plato meant when he described it as an *eikos muthos*, which translates literally as "likely myth" but may be translated in several other ways, is a matter of ongoing debate (Burnyeat 2005; Betegh 2010). Some commentators emphasize that the thoroughgoing teleological account of the cosmos and all it contains show Plato's treatment of the natural world to be a serious one, especially as the place of humans in the cosmos and how humans should seek to improve themselves are also important themes in the *Timaeus* (Lloyd 1968). In his early work the *Phaedo*, Plato is critical of the *physiologoi*, the philosophers who have come before him who have carried out *historian peri phusis*, the investigation of nature. It is clear that he believes their explanations, based solely on material considerations, are inadequate. Does this mean that Plato dismisses the investigation of nature, or, in his later work the *Timaeus*, does he show how it should be carried out using teleological explanation? Any view of Plato's science must account for the brute fact that Plato wrote the *Timaeus*, a work which gives a full teleological account of the origins of the cosmos, the disposition of the cosmos, the nature and origins of the elements and the nature and origins of human beings. The *Timaeus* was possibly the most influential work on natural philosophy in the whole of antiquity.

Astronomy

The model of the heavens which Plato gives in the *Timaeus* is sometimes called a two-sphere model, though this is slightly misleading. The stars are arranged in a spherical pattern and rotate once a day on an axis which passes through the earth. The sun, moon, and five naked eye planets (Mercury, Venus, Mars, Jupiter and Saturn) have an additional circular motion, again centered on the earth, but with the axis offset from that of the stars. The moon takes a month to complete its extra circular motion, the sun, Mercury and Venus a year while the other planets have unspecified periods (Plato, *Timaeus* 38b ff.). The key innovation here is the notion of combinations of regular circular motion being combined to give an account of the motions of heavenly bodies. This principle will go unchallenged down to 1609 and Kepler's discovery that planetary orbits are ellipses around the sun.

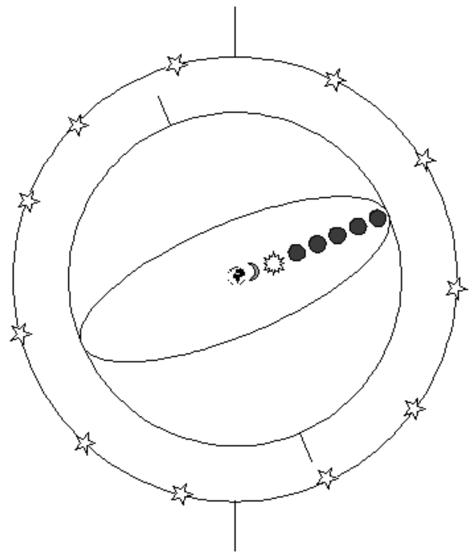

Figure 3

If the axes are offset by the angle of the ecliptic (approx 23.5 degrees) then this model will work well for the sun. Plato does not give any figure here, but says the angle is like that made by the arms of the Greek letter χ.

Plato’s model has the sun, moon and planets all moving in one plane. Viewed form the earth this means that sun, moon and planets will all follow one path across the havens. As a first approximation this is fine, but in fact the orbits of the moon and planets are all at small inclinations to the plane of the earth’s orbit around the sun. This means that the moon and planets actually move within a band around that of the motion of the sun, known as the zodiac, as was well known at the time. Mercury and Venus, as they have smaller orbits than the earth, are always seen relatively close to the sun, sometimes preceding it, sometimes following it. When Venus precedes the sun, it is seen low on the horizon just before sunrise, when it follows the sun it is seen low on the horizon just after sunset. Because in Plato’s model, the sun, Mercury and Venus all have uniform speeds, this phenomenon cannot be accounted for. Also problematic is planetary retrograde motion. Viewed from the earth, each of the naked eye planets appears periodically to reverse its course for a period before resuming its forward motion. We understand this as an effect of the relative motion between the earth and the planets against the background of the stars. However, if the earth is immobile, all the motions of the heavens must be merely apparent. Plato's combination of two regular circular motions cannot reproduce this phenomenon. A final major defect of Plato's system is that if sun, moon and earth are all permanently in the same plane, there will be full eclipses of the sun once a month and full eclipses of the moon every month, with no other type of eclipse. It is clear from a close reading of the *Timaeus* that Plato is aware of most, if not all of the problems (Gregory 2000a).

There are two ways of addressing these difficulties. Some scholars have argued that in order for the model to be in accord with the phenomena, we must drop the idea of regular circular motion, however insistent Plato may appear on this principle, and note that Plato says that Mercury and Venus are subject to a "contrary power" in relation to their movements relative to the sun (Cornford 1937; Knorr 1990). More recently it has been argued that it is improper to assume that every ancient theorist believes their model to be able to account for all of the phenomena. Simplicius states that Eudoxus was well aware of phenomena that could not be accounted for by his own model. On this view, Plato's model is a prototype, strong on principle (combinations of regular circular motion), better at explaining some phenomena than previous models but still with significant defects as we might expect from a prototype. Plato would have been aware of at least some of these defects (Gregory 2000a).

How influential this model was is again the subject of an ongoing controversy. Simplicius reports Plato as having asked others to work out which combinations of regular circular motions will save the phenomena. Simplicius' sources have been questioned and certainly there was a tendency in later antiquity to portray Plato as an architect of the sciences so we ought to be cautious (Mittlestrass 1962, 154; Vlastos 1975, 110; Zhmud 1999, 220). Some commentators have doubted whether Plato could have made such a remark on the grounds that either he did not believe in regular circular motion or that he was implacably opposed to observational astronomy. If Plato did hold to regular circular motion and consider his model to be a prototype with flaws, one can see why he would have asked for others to work out which circular motions would save the phenomena.

Eudoxus

Eudoxus of Knidus (*ca* 410-*ca* 347 BCE) was an important associate of Plato. We have very little of his original work, but we do have accounts of his astronomy preserved by later writers such as Simplicius (Simplicius, *De Caelo Commentary*). His work in astronomy can be seen as building on that of Plato (Gregory 2003). Where Plato used models involving two regular circular motions for the sun, moon and planets Eudoxus used three for the sun and moon and four for the planets.

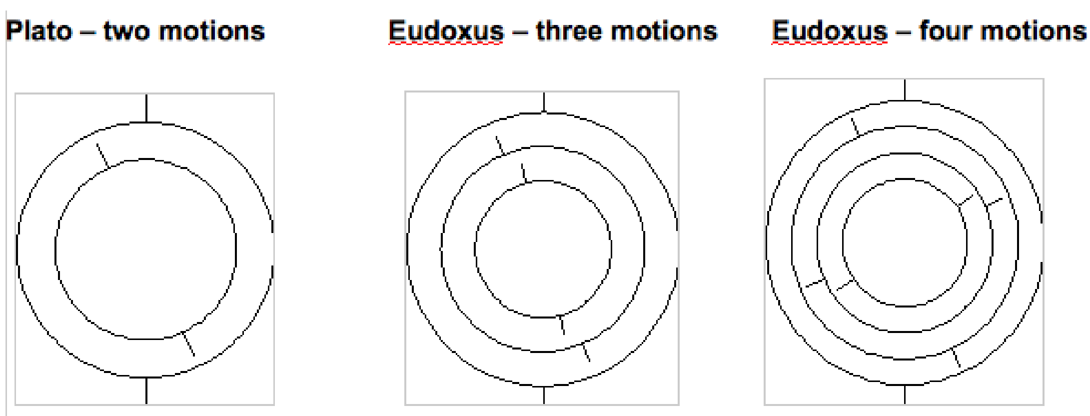

Figure 4

The third motion for the moon gives in motion in latitude through the zodiac and so gives a much better model. The third and fourth motions for the planets produce a figure known as the hippopede.

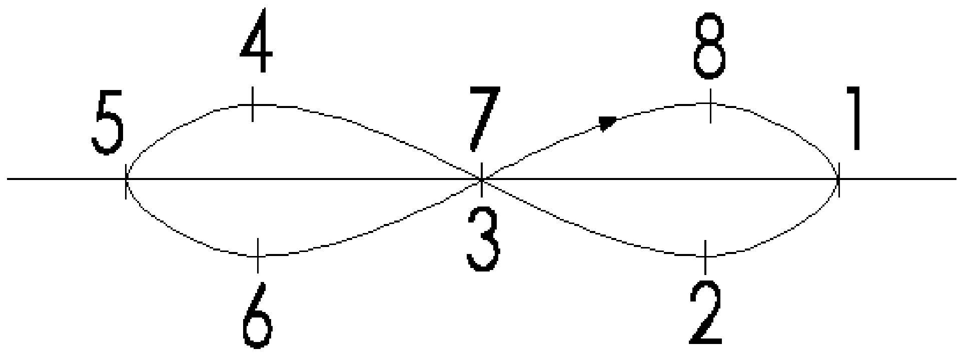

Figure 5

When this is combined with the other two motions, the result is like this:

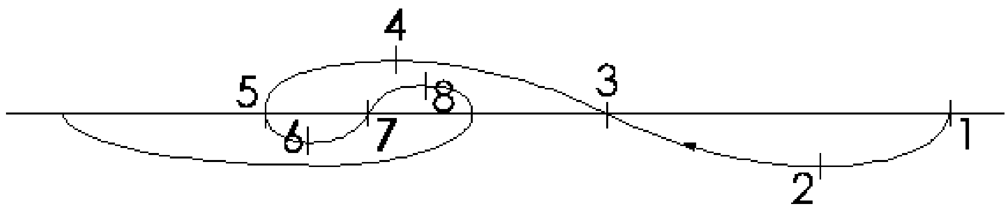

Figure 6

This allows Eudoxus to give some account of the retrograde motion of the planets, though the system is not flexible enough to give a full account (Mendell 1998; Yavetz 1998). Eudoxus also did important work in mathematics developing the theory of proportions such that it was able to cope with irrational numbers as well as integers and rational fractions. There has been debate about the interaction of Plato and Eudoxus, with the suggestion that the astronomy of the *Timaeus* was largely inspired by Eudoxus, the motivation for this supposition often being that Plato was not sufficiently interested in the physical world to produce such a sophisticated astronomy. There is, however, no evidence to support such a supposition.

Cosmology

Plato's astronomy is set within a broader cosmological picture. It is important to recognize here that the Greeks had no conception of gravity. They of course knew that heavy objects fall to earth and that the heavens have regular motions, but generated other ways to explain these phenomena. For Plato the cosmos was a living entity, though of a rather special sort. It has intelligence and self-motion, though Plato was adamant it did not have limbs, organs of sensation or organs of ingestion or excretion. It is a purely spherical, rotating entity (Plato, *Timaeus* 33b ff.). What is he attempting to capture with this model?

The motions of the heavens were perceived as orderly and regular with the stars having a certain set pattern (hence the “fixed stars”). The motions of the planets were intricate, due to the Greek belief in a central and stable earth. We understand many of the motions of the heavens as apparent motions, generated by the motion of the earth. If the earth is believed to be entirely motionless though, all of the motions of the heavens are real motions. Whereas we would explain the retrograde motion of the planets as an apparent effect generated by the relative motion of earth and planet against the star field, for the Greeks the planets really did reverse their motion. In addition to the cosmos having a soul, the individual planets have souls as well, so they can move relative to the cosmos.

For Plato, regular and orderly motion was characteristic of intelligence while irregularity was characteristic of matter on its own. In contrast, we take mechanism, in particular clockwork, to be a paradigm of regularity and contrast that with human frailty. That is an attitude of the seventeenth century and the rise of the mechanical philosophy and after though, and was not shared by the ancients (Furley 1987). Another important point in relation to this is that while we think of physical law in terms of equations and exceptionless laws, this has not always been so and there have been significant alternatives. In particular, in the ancient world it was possible to conceive of physical law as analogous to civil law. So for Plato there are courses which the planets ought to follow but no physical or mathematical necessity which forces them to follow those courses. However, for Plato intelligence always chooses what is good and so the world soul and planetary souls always choose to do what they ought to do. For Plato, the absolute regularity of the heavens is underpinned by the intelligence of the heavens.

Plato also incorporates music in his cosmology (Plato, *Timaeus* 36a ff.). Specifically, he uses the musical ratios in the scale advocated by Philolaus to determine the sizes of the orbits of the sun, moon and planets. This illustrates an important difference between Plato and modern cosmology. Where we would happily accept that the ratios of the orbits of the planets are an accidental matter, Plato's demiurge must have criteria for why he has set the orbits up like this as all he does is for the best. Here it is important to understand that this is a very early stage in the mathematization of the world. Although it may be evident to us that we should describe the world in terms of the equations familiar to modern physics, this has not always been so. If we accept this, it is easier to see why Plato attempts to incorporate not only mathematics but geometry and musical theory into his cosmology as well. Plato was far from being alone in this. As late as the seventeenth century, Johannes Kepler, famous for his three laws of planetary motion, was attempting to determine the nature of the elliptical orbits of a sun-centered cosmos in terms of the geometry of the Platonic solids and the ratios of musical theory. In this context, we can see why Plato chooses Philolaus' scale over those of Archytas. Philolaus' scale is musically the purest in some sense and mathematically the most elegant. As it does not have to be played, but just defines the ratios of the orbits, the practical problems with the scale are unimportant.

Cosmogony

Plato's account of cosmogony was highly influential in the ancient world. In the *Timaeus*, Plato gives an account of how the cosmos came into being. Before the cosmos, there was chaos.

Plato's craftsman god, the demiurge, then generated a cosmos from this chaos, at all times working with what would be best in mind (Plato, *Timaeus* 29e ff.). By imposing "number and form" on this chaos he generates the basic triangular particles and from these the "elements" of earth, water, air and fire (Plato, *Timaeus* 53d ff.). From the elements the earth, the solar system, animals and man are all generated in the best possible manner and there is also a process by which the heavens and humans are ensouled. Once the cosmos is formed, it can only be dissolved by the demiurge, who being entirely good will never have any inclination to do so. The key debate about Plato's account of cosmogony is whether it is meant to be taken literally or not. This debate began in antiquity and is still unresolved.

Some scholars say that we should take the cosmogony as a counter-factual analysis (Baltes 1996). The primordial chaos is what the world would be like if the demiurge was not constantly maintaining the cosmos in its present organized state. There was no point of generation for the cosmos, no point when it came to be, but it is in a permanent state of becoming and dependent on the demiurge. They point to many alleged inconsistencies in the account of cosmogony if it is taken literally, which are then not important on the allegorical view. They also point to very early interpretations of the *Timaeus* which took the cosmogony to be allegorical.

The literalists on the other hand argue that the inconsistencies are tolerable, especially as Plato has warned us that the account is difficult and we should not expect perfect consistency from it. With the ancient interpretation, they point out that Aristotle argued that either something came into being and then passed away at a later stage, or always has and always will exist. This argument was very influential in antiquity and the motivation for the early literalist view was to save Plato, who seemed to believe in a beginning but not an end for the cosmos, from criticism along these lines. The debate here is unlikely to be resolved swiftly or decisively as many aspects of the interpretation of the *Timaeus* are involved here. However, the two most recent studies of ancient cosmogony have both come out in favor of a literalist view, citing evidence both in the *Timaeus* and in Plato's other works (Gregory 2007, 2009; Sedley 2008).

One further important aspect of Plato's cosmology and cosmogony is that he is adamant that there is only one cosmos which has been well organized by the demiurge. This is in sharp contrast with the ancient atomists, who believed there to be many universes which came about by chance. A key part of the atomist account is that a vortex, which forms spontaneously from atoms moving in an infinite void, will organize matter "like to like" and so into a cosmos. Plato considers a cosmos to be a fitting and harmonious blend of hot and cold, dry and wet, soft and hard, etc., that is blends of opposites. He argued that if we only employ a like to like principle in cosmogony, this accounts for the grouping of like things but not of unlike things. Plato's argument for a unique cosmos by design is not merely theological. He attacks accounts of cosmogony that rely on chance by pointing out implausibilities in those accounts (Gregory 2007).

Matter.

Plato famously gives a new type of matter theory, geometrical atomism. In contrast to the earlier atomist theory of Leucippus and Democritus, which allowed all shapes and sizes of atoms, Plato will allow only two basic shapes, both triangular (Plato, *Timaeus* 53d ff.). In contrast to the theories of the Milesians, in which there was a basic substance such as water or air from which all the other substances are generated, Plato argues that we have no reason to believe earth, water, air or fire to be basic as we see each of then turning into the other. Instead he supposes there to be shapes more basic than the elements and which are below the level of perception. The justification of these shapes is that they are the best and most beautiful, which is why they were chosen by the Demiurge. These are the two shapes:

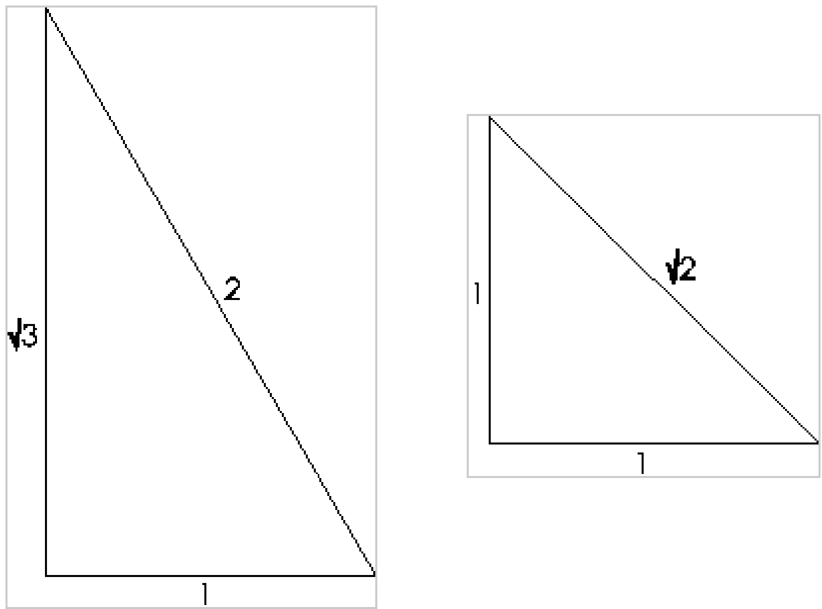

Figure 7

These two basic shapes are said to coalesce into two complexes:

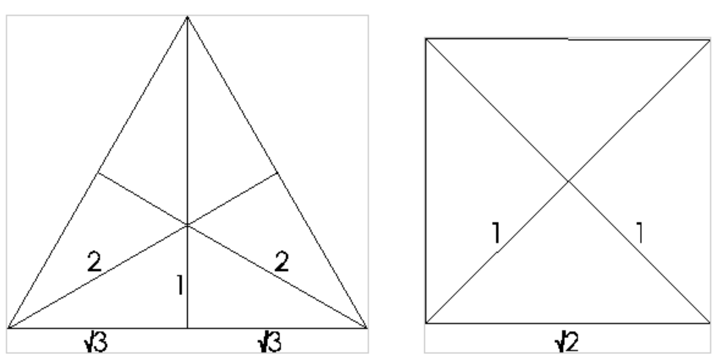

Figure 8

These more complex shapes in turn form up into solid bodies, the tetrahedron, the cube, the octahedron and the eikosahedron (twenty sided).

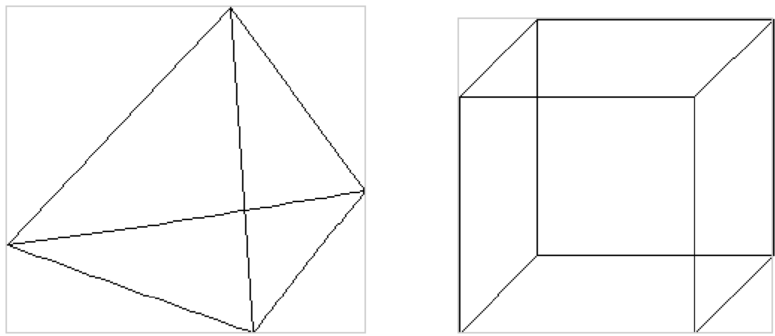

Figure 9

These are four of the Platonic solids, that is solids which are composed of identical faces. They are also the elements, the tetrahedron being fire, the cube being earth, the octahedron being air and the icosahedron water. The cube is reckoned to be earth on the basis of the stability of the cube and of earth, while the tetrahedron is reckoned to be fire on account of its sharp angles and supposed swift motion being able to cut things up as fire does. Fire, air and water are able to transmute into each other, as when the solids come apart into their triangles these triangles can form up again as any of these three solids. Earth is excluded from this having a different type of triangle as its basis. The theory of geometrical atomism has been seen as the basis for the use of equations in chemistry (Vlastos 1975). However, Plato does not use it in this way and his purpose rather seems to be in giving the Demiurge criteria for a choice of a small number of fundamental particles. Here again we can see Plato's predilection for explaining origins in terms of design rather than accident. Where the atomists had an infinite number of shapes and sizes for their particles, Plato supposes only two basic shapes, both mathematically well defined and both chosen by the demiurge because they are perceived to be good shapes.

Body and medicine

Plato's account of the body, disease and treatment is not much discussed but does occupy a significant part of the *Timaeus*. The account serves several important functions for Plato and was very influential for some traditions up to the 17^{th} century, the time of the scientific revolution. Like all else in the cosmos, humans are generated by the demiurge with the best in mind. Part of the scheme of their construction is to house the tripartite soul, with the intellectual faculties being housed as far as possible from the more base appetites of hunger and sex. The generation of living beings completes the account of the cosmos and makes the cosmos itself a proper whole. It is noteworthy that Plato describes the construction of humans in terms of the geometrical atomism he has set out.

Another important feature is that humans are a microcosm of the cosmos as a whole. While Plato does not originate the macrocosm/ microcosm language, it is clear that for him many aspects of the human body take on characteristics of the cosmos. It has the same sort of order, it acts on its contents as the cosmos acts on its contents (e.g., *Timaeus* 81ab), and our minds too have circuits like those of the heavens. This, although not original to Plato, was highly influential in neoplatonism and various magical traditions up until the seventeenth century.

One important aspect of human construction is that humans are a single, well designed species, not the result of a series of accidents, parallel to Plato's thinking on cosmogony. Plato may well have Empedocles in mind when he says that our heads were given means of locomotion, otherwise they would get stuck in ruts in the ground. Empedocles' account entails a rather nightmarish world where individual, dissociated body parts joining together accidentally until viable species are formed. How though, do some of those parts, like heads, move around?

Plato also emphasizes that body parts for humans need to be arranged not only in the correct order but also the right way round. Plato, as with his argument against the atomist cosmogony, adds further layers of implausibility to their account.

Plato's account of the human body also reveals something interesting about the relation of reason and necessity. The account of the *Timaeus* is set out in three stages. Firstly, we are given the works of reason, then the works of necessity while the account of the human body is a combination of reason and necessity. It is critical to Plato's account that while the demiurge is well intentioned and does everything as well as he can, that there are constraints on what he can do. The reason that the demiurge imposes cannot be absolute but is tempered in some way by "necessity." Exactly what Plato meant by reason and necessity has been the subject of much debate. Some have taken the view that nothing can act with perfect regularity in the physical world because of this compromise of reason with necessity, brute matter not behaving in a regular manner. What emerges from the description of humans is that the issue here may be more of an engineering consideration. How thick should he human skull be? For defense and long life, as thick as possible, for the best perception, as thin as possible. The demiurge does his best, but cannot instantiate both of these considerations so human skull thickness is a compromise (Cornford 1937; Morrow 1950; Gregory 2000; Johansen 2004).

In terms of medicine, Plato is very much in favour of good diet and regimen allied to gentle medical intervention rather than any radical use of drugs, purgatives and emetics, etc. Essentially he sees the body as a self-regulative system (as with the cosmos as a whole) which occasionally needs a little maintenance and help rather than vigorous medical intervention. Plato's account of aging is that the collections of triangles which constitute the elements of our body gradually lose their ability to cohere together, leading to a gradual running down of the bodies functions. Thus death in old age is seen as a perfectly natural occurrence for Plato.

Plato a Pythagorean?

Whether Plato was some form of Pythagorean has been much debated in the literature. In light of recent scholarship on Pythagoras, Philolaus and Archytas, the key initial question must be, what sort of Pythagoreanism have we in mind? It is now generally accepted, since the reliability of later sources has been determined, that it is unwise and misleading to consider Plato to be a Pythagorean but that there are certain important Pythagorean influences on Plato. Similarly Taylor's view that the *Timaeus* was essentially a Pythagorean work with its key views taken from the Pythagoreans is no longer considered a viable interpretation of that work (Taylor 1928).

There are some significant differences between Plato and the Pythagoreans on scientific issues. Firstly, the cosmology of the *Timaeus* is geometrical rather than arithmetical in nature. For the Pythagoreans, the cosmos is in some sense number, or is constituted out of numbers. This is an arithmetical cosmology, one that is based on number. For Plato in the *Timaeus*, the demiurge imposes specific geometrical shapes on the primordial chaos on order to generate a cosmos. These are the 1, 1, $\sqrt{2}$ and 1, $\sqrt{3}$, 2 triangles from which the cubes, tetrahedra, octahedra and eikosahedra of earth, fire, air and water are formed. It is these shapes which form the basis of Plato's cosmos, not numbers. Aristotle is critical of Plato for what he takes to be the arbitrary way in which Plato allows his analysis of the elements to end at triangles when it could have gone further to lines, numbers and points. Plato's geometrical atomism allows him to deal with irrational numbers such as $\sqrt{2}$ and $\sqrt{3}$ by treating nature as geometrical and having numbers apply to shapes. That is not so clear for the arithmetical conception of nature where every geometrical length ought to be expressible as the ratio of two natural numbers. If these numbers represent a length, then if we ask how long something is we count the number of monadic lengths involved. A problem for such a scheme comes with the discovery of the irrationality of the square root of two, for here we have a number/ length that cannot be expressed as a ratio of two natural numbers, or as a multiple of a monadic length.

Plato's use of musical theory in cosmology highlights some differences with the Pythagoreans as well as an evident Influence. Philolaus' scale uses the numbers 1, 2, 3 & 4 in various ratios to generate a musical scale. The rationale for this is that 1 + 2 + 3 + 4 = 10, the Pythagorean perfect number, or in other words the justification is numerological. So too Aristotle criticises the Pythagorean assumption that there are 10 celestial bodies (earth, moon, sun, 5 naked eye planets, counter-earth, central fire) on the basis that 10 is the perfect number and so there should be 10 celestial bodies (Aristotle: *Metaphysics* 1.5). Plato accepts there are seven visible heavenly bodies to (moon, sun, 5 naked eye planets) and has seven terms which generate his musical scale (1, 2, 3, 4, 8, 9, 27), which are the relative lengths of the soul stuff which the demiurge uses to fashion the orbits for these bodies (Plato, *Timaeus* 35b). Plato then generates a tone and semi-tone scale from these terms. The derivation is again geometric (dividing the soul stuff) rather than purely arithmetic as with the Pythagoreans. So while Philolaus has a numerological derivation of cosmology and of music, Plato has a cosmological derivation of music. It may also be important to note that in the *Timaeus* and subsequent works of Plato there is no mention of any audible harmony of the heavenly bodies. There is a harmony to the structure of the world soul, but no sound.

Kennedy

The most recent development on Plato and Pythagoras has been the work of Jay Kennedy (Kennedy 2010). He claims that Plato organized his work stichometrically, that is he was aware of the number lines in each of his works and in each of the parts of his works. He further claims that Plato divided each of his works into twelve parts and that Plato has means of indicating the transition from one twelfth to another. So Plato may make a reference to divine justice, or a speech may begin at a twelfth part of a work. Kennedy also claims that there is a harmonic organization to Plato's works based on a twelve note division of the octave. So each of the twelve parts represents a semi-tone. It is claimed that Plato writes predominantly of positive ideas at harmonious parts of the scale and predominantly of negative ideas at dissonant parts. The final claim is that this sort of analysis reveals Pythagorean doctrine and information about Pythagoras encoded in the text. Kennedy supports these theses with statistical analysis which certainly at first sight looks impressive in its breadth and claimed results.

At the time of writing, this research is at a fairly early stage. Kennedy has published one paper and there is a further book to come. If his claims are shown to be true, they will revolutionize our understanding of both Pythagoras and Plato. It would be fair to say that this is a big 'if' and that scholars have so far reacted with some caution to Kennedy's theses. One important issue at stake here is our understanding of how and why Plato wrote his works. Whether Kennedy's claims about the organization of Plato's works withstand tight scrutiny will be the critical question. While he claims impressive accuracy for his results, accuracy is not the same as statistical significance. If I claim Plato uses the letter α at each of the twelfth points of his works, that will doubtless be very accurate but given the profusion of α in the rest of his work it will not be statistically significant. The real test for Kennedy's work will be running proper tests for statistical significance on the claims he has made.

Bibliography

Pythagoras and the early Pythagoreans

Barker, A. D. *Greek Musical Writings, Vol. II: Harmonic and Acoustic Theory*. Cambridge: Cambridge University Press, 1989.

—. "Ptolemy's Pythagoreans, Archytas, and Plato's conception of mathematics." *Phronesis* 39.2 (1994): 113–135.

—. "Archytas Unbound: A Discussion of Carl A. Huffman, *Archytas of Tarentum*." *Oxford Studies in Ancient Philosophy* 31 (2006): 297–321.

Barnes, J. *The Presocratic Philosophers*. London: Routledge, 1982.

Burkert, W. "Platon oder Pythagoras? Zum Ursprung des Wortes 'Philosophia'." *Hermes* 88 (1960): 159–177.

—. "Hellenistische Pseudopythagorica." *Philologus* 105 (1961): 16–43, 226–246.

—. *Lore and Science in Ancient Pythagoreanism*. Trans. E. L. Minar, Jr. Cambridge, Mass.: Harvard University Press, 1972.

Guthrie, W. K. C. *A History of Greek Philosophy*. Vol. 1. Cambridge: Cambridge University Press, 1962.

Huffman, C. A. "The Authenticity of Archytas Fr. 1." *Classical Quarterly* 35.2 (1985): 344–348.

—. "The Role of Number in Philolaus' Philosophy." *Phronesis* 33 (1988): 1–30.

—. *Philolaus of Croton: Pythagorean and Presocratic*. Cambridge: Cambridge University Press, 1993.

—. “Limite et illimité chez les premiers philosophes grecs.” In *La Fêlure du Plaisir: Études sur le Philèbe de Platon, Vol. II: Contextes*, ed. M. Dixsaut, 11–31. Paris: Vrin, 1999.

—. “The Philolaic Method: The Pythagoreanism behind the *Philebus*.” In *Essays in Ancient Greek Philosophy VI: Before Plato*, ed. A. Preus, 67–85. Albany: State University of New York Press, 2001.

—. *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*. Cambridge: Cambridge University Press, 2005.

—. “Philolaus and the Central Fire.” In *Reading Ancient Texts, Volume I: Presocratics and Plato, Essays in Honour of Denis O'Brien*, ed. Suzanne Stern-Gillet and Kevin Corrigan, 57–94. Leiden: Brill, 2007.

—. “Another Incarnation of Pythagoras.” *Ancient Philosophy* 28 (2008): 201–225.

Kahn, C.H., “Pythagorean Philosophy before Plato.” In Mourelatos 1993, 161–185.

—. *Pythagoras and the Pythagoreans*. Indianapolis: Hackett, 2001.

Mourelatos, Alexander P. D. *The Pre-Socratics: a collection of critical essays.* Princeton: Princeton University Press, 1993.

Mueller, Ian. “Greek arithmetic, geometry and harmonics: Thales to Plato.” In *Routledge History of Philosophy Vol. I: From the Beginning to Plato*, ed. C. C. W. Taylor, 271–322. London: Routledge, 1997.

Philip, J. A. *Pythagoras and Early Pythagoreanism*. Toronto: University of Toronto Press, 1966.

Riedweg, Christoph. *Pythagoras: His Life, Teaching and Influence*. Ithaca and London: Cornell University Press, 2005.

Zhmud, L. “‘All is number?’” *Phronesis* 34 (1989): 270–292.

—. “Plato as Architect of Science.” *Phronesis* 43.3 (1998): 210–244.

—. “Some Notes on Philolaus and the Pythagoreans.” *Hyperboreus* 4 (1998): 1–17.

—. “Eudemus' History of Mathematics.” In *Eudemus of Rhodes*, ed. W. W. Fortenbaugh and I. Bodnar, 263–306. New Brunswick: Transaction, 2002.

—. *Wissenschaft, Philosophie und Religion im frühen Pythagoreismus*. Berlin: Akademie Verlag, 1997.

—. Review of Riedweg (2002), *Ancient Philosophy* 23 (2003): 416–420.

Plato

Anton, J.P. ed., *Science and the Sciences in Plato*. New York: Eidos, 1980.

Artmann B. and L. Schäfer. “On Plato’s 'Fairest triangles' *Timaeus* 54a.” *Historia Mathematica*, 20 (1993): 255–264.

Baltes, M. “*Gegonen* Platon, *Tim*. 28 B 7. Ist die Welt real entstanden oder nicht?” In *Polyhistor: Studies in the History and Historiography of Ancient Philosophy Presented to Jaap Mansfeld on His Sixtieth Birthday*, ed. K. Algra, *et al*., 76–96. Leiden: Brill, 1996.

Betegh, G. “What Makes a Myth Eikos?” In *One Book, the Whole Universe: Plato’s Timaeus Today*, ed. R.D. Mohr and B. Sattler, 213–224. Las Vegas, Zurich, Athens: Parmenides, 2010.

Brisson, L. & F.W. Meyerstein. *Inventing the Universe: Plato's Timaeus, the Big Bang, and the Problem of Scientific Knowledge*. Albany: SUNY, 1995.

Bulmer-Thomas, I. “Plato's Astronomy.” *Classical Quarterly* 34 (1984): 107–112.

Burnet, J. *Platonis Opera* 5 vols. Oxford: Clarendon Press, 1903–1910.

Burnyeat, M.B. “Eikos Muthos.” *Rhizai* 2.2 (2005): 143–165.

Bury, R.G. *Plato’s Timaeus*. Cambridge, Mass.: Loeb, 1929.

Calvo, T. and L. Brisson, ed., *Interpreting the Timaeus and Critias*. Sankt Augustin: Academia, 1997.

Cavagnaro, E. “The Timaeus of Plato and the Erratic Motion of the Planets.” In Calvo and Brisson 1997, pp. 351–362.

Cornford, F.M. *Plato’s Cosmology* London: Routledge & Kegan Paul, 1937.

Crombie, I.M. “Cosmology and Theory of Nature.” In *An Examination of Plato’s Doctrines*, vol. 2: *Plato on Knowledge and Reality*, 153–246. London: Routledge & Kegan Paul, 1963.

Dicks, D.R. *Early Greek Astronomy to Aristotle*. Bristol: Thames and Hudson, 1970.

Furley, D.J. *The Greek Cosmologists*, vol. 1: *The Formation of the Atomic Theory and Its Earliest Critics* Cambridge: Cambridge University Press, 1987.

Gregory, A. D. “Astronomy and Observation in Plato's Republic.” *Studies in History and Philosophy of Science* 27 (1996): 451–471.

—. *Plato’s Philosophy of Science* London: Duckworth, 2000.

—. “Plato and Aristotle on Eclipses.” *Journal for the History of Astronomy* 31 (2000): 245–259.

—. “Eudoxus, Callippus and the Astronomy of the Timaeus.” In *Ancient Approaches to Plato's Timaeus*. *Bulletin of the Institute of Classical Studies, Supplement* 78, ed. R.W. Sharples and A. Sheppard, 5–28. London: Institute of Classical Studies, 2003.

—. *Ancient Greek Cosmogony*, London: Duckworth, 2007.

—. “Plato on Order from Chaos.” *Proceedings of the Biennial Conference on Greek Studies*, ed. E. Close, G. Couvalis, G. Frazis, M. Palakisoglu, and M. Tsianikas, 47–54. Adelaide: Flinders University Press, 2009.

Hackforth, R. “Plato’s Cosmogony *Timaeus* 27d ff.” *Classical Quarterly*, n.s. 9 (1959): 17–22.

Heath, T.L. *Aristarchos of Samos*. 1913. Reprint, New York: Dover, 1981.

J.L Heiberg, ed. *Simplicius. Aristotelis in De Caelo Commentaria*. Berlin: Akademie der Wissenschaften, 1894.

Hetherington, N.S. *Encyclopedia of Cosmology: Historical, Philosophical and Scientific Foundations of Modern Cosmology*. New York: Garland, 1993.

—. "Plato and Eudoxus: Instrumentalists, Realists or Prisoners of Themata?" *Studies in History and Philosophy of Science* 27 (1996): 271–289.

Johansen, T.K. *Plato's Natural Philosophy: A Study of the Timaeus-Critias*. Cambridge: Cambridge University Press, 2004.

Kennedy, J.B. "Plato's Forms, Pythagorean Mathematics, and Stichometry." *Apeiron* 43 (2010): 1–31.

Keyt, D. "The Mad Craftsman of the *Timaeus*." *Philosophical Review* 80 (1971): 230–235.

Knorr, W. "The Interaction of Mathematics and Philosophy in Antiquity." In *Infinity and Continuity in Ancient and Medieval Thought*, ed. N. Kretzman. Ithaca: Cornell, 1982.

—. "Plato and Eudoxus on the Planetary Motions." *Journal for the History of Astronomy* 21 (1990): 313–329.

Kraut R., ed. *The Cambridge Companion to Plato*. Cambridge: Cambridge University Press, 1992.

Lasserre, F. *The Birth of Mathematics in the Age of Plato*. London: Hutchinson, 1964.

—. *Die Fragmente des Eudoxus von Knidos*. Berlin: De Gruyter, 1966.

Lee, H.P.D. *The Republic*. London: Penguin, 1955.

—. 1965 *Plato: Timaeus and Critias*. London: Penguin, 1965.

Lennox, J.G. "Plato's Unnatural Teleology." In *Platonic Investigations*, ed. D.J. O'Meara, 195–218. Washington, DC: The Catholic University of America Press, 1985.

Lloyd, D.R. “Symmetry and asymmetry in the construction of the ‘elements’ in the Timaeus.” *Classical Quarterly* 56 (2006): 459–474.

—. “The Chemistry of Platonic Triangles: Problems of interpretation in the Timaeus.” *HYLE – International Journal for Philosophy of Chemistry* 13 (2007): 99–118

—. “Symmetry and Beauty in Plato.” *Symmetry* 2 (2010): 455–465.

Lloyd, G.E.R. “Plato as a Natural Scientist.” *Journal of Hellenic Studies* 28 (1968): 78–92.

—. *Early Greek Science to Aristotle*. London: Chatto and Windus, 1970.

—. “Saving the Appearances.” *Classical Quarterly* 88 (1978): 202–222. See also Lloyd 1991, pp. 248–253.

—. *The Revolutions of Wisdom*. Berkeley: California U. Press, 1987.

—. “Plato and Archytas in the Seventh Letter.” *Phronesis* 35 (1990): 159–174.

—. “Plato on Mathematics and Nature, Myth and Science.” In *Methods and Problems in Greek Science: Selected Papers*, 333–351. Cambridge: Cambridge University Press, 1991.

Mendell, H. “Reflections on Eudoxus, Callippus and their Curves: Hippopedes and Callippopedes.” *Centaurus* 40 (1998): 177–275.

Mittlestrass, J. *Die Rettung der Phänomene.* Berlin: de Gruyter, 1962.

Mohr, R.D. *The Platonic Cosmology*. Leiden: Brill, 1985.

Morrow, G.R. “Necessity and Persuasion in Plato’s *Timaeus*." *Philosophical Review* 59 (1950): 147–160; repr. in *Studies in Plato’s Metaphysics*, ed. R.E. Allen, 421–437. London: Routledge & Kegan Paul, 1965.

Mourelatos, A.P.D. “Astronomy and Kinematics in Plato's Project of Rationalist Explanation.” *Studies in the History and Philosophy of Science* 12 (1981): 1–32.

Mueller, Ian. “Ascending to Problems: Astronomy and Harmonics in Republic VII.” In Anton 1980, 103–122.

—. “Platonism and the Study of Nature.” In *Method in Ancient Philosophy*, ed. J. Gentzler, 67–90. Oxford: Oxford University Press, 1998.

Parry, R.D. “The Unique World of the Timaeus.” *Journal for the History of Philosophy* 17 (1979): 1–10.

—. “The Intelligible World-Animal in Plato's Timaeus.” *Journal for the History of Philosophy* 29 (1991): 13–32.

Patterson, R. “The Unique Worlds of the *Timaeus*.” *Phoenix* 35 (1981): 105–119.

Reale, G. “Plato's Doctrine of the Origin of the World, with Special Reference to the Timaeus.” In Calvo and Brisson 1997, pp. 149–164.

Robinson, T.M. “Understanding the Timaeus.” *Proceedings of the Boston Area Colloquium on Ancient Philosophy* 2 (1986): 103–119.

—. “Aristotle, the Timaeus and contemporary Cosmology.” *Philosophical Inquiry* 15 (1993): 48–58.

De Santillana, G. “Eudoxus and Plato.” *Isis* 32 (1949): 248–262.

Sedley, D. *Creationism and its Critics in Antiquity*. Berkeley: University of California Press, 2008.

Shorey, P. “Platonism and the History of Science.” *Proceedings of the American Philosophical Association* 46 (1927): 159–182.

Skemp, J.B. *The Theory of Motion in Plato’s Later Dialogues*. Cambridge: Cambridge University Press, 1942.

Smith, N.D., ed. *Plato: Critical Assessments*, 4 vols. London: Routledge, 1998.

Sorabji, R. *Time, Creation and the Continuum*. London: Duckworth, 1983.

Tarán, L. "The Creation Myth in Plato's *Timaeus*." In *Essays in Ancient Greek Philosophy*, ed. J. Anton and G. Kustas, 372–407. Albany: State University of New York Press, 1971.

Taylor, A.E. *A Commentary on Plato's Timaeus*. London: Oxford University Press, 1928.

Vallejo, A. "No, It's Not a Fiction." In Calvo and Brisson 1997, 141–148.

Visentainer, J. "A Potential Infinity of Triangle Types: On the Chemistry of Plato's *Timaeus*." *HYLE – International Journal for Philosophy of Chemistry* 4 (1998): 117–128.

Vlastos, G. "Disorderly Motion in Plato's *Timaeus*." *Classical Quarterly* 33 (1939): 71–83; repr. in *Studies in Plato's Metaphysics*, ed. R.E. Allen, 379–399. London: Routledge & Kegan Paul, 1965; repr. in *Studies in Greek Philosophy*, vol. 2: *Socrates, Plato, and Their Tradition*, ed. G. Vlastos, 247–264. Princeton: Princeton University Press, 1995.

—. "Creation in the *Timaeus*: Is it a Fiction?" In *Studies in Plato's Metaphysics*, ed. R.E. Allen, 401–419. London: Routledge & Kegan Paul, 1965; repr. in *Studies in Greek Philosophy*, vol. 2: *Socrates, Plato, and Their Tradition*, ed. G. Vlastos, 265–279. Princeton: Princeton University Press, 1995.

—. "Plato's Supposed Theory of Irregular Atomic Figures." *Isis* 58 (1967): 204–209; repr. in *Platonic Studies*, 2nd ed., ed. G. Vlastos, 366–373. Princeton: Princeton University Press, 1981.

—. *Plato's Universe*. Seattle: University of Washington Press, 1975.

—. "The Role of Observation in Plato's Conception of Astronomy." In Anton 1981, pp. 1–22.

Wright, M.R. *Cosmology in Antiquity* London: Routledge, 1995.

Wright, M.R., ed. *Reason and Necessity: Essays on Plato's Timaeus*. London/ Swansea: Duckworth/ The Classical Press of Wales, 2000.

Yavetz, I. “On the Homocentric Spheres of Eudoxus.” *Archive for History of the Exact Sciences* 52 (1998): 221–278.

Zeyl, D. *Plato: Timaeus*. Indianapolis: Hackett, 2000.

Zhmud, L. “Die Beziehungen Zwischen Philosophie und Wissenschaft in der Antike.” *Sudhoffs Archiv* 78 (1994): 1–13.

—. *Wissenschaft, Philosophie und Religion im Fruhen Pythagoreismus*. Berlin: Akademie Verlag, 1997.

—. “Plato as 'Architect of Science'.” *Phronesis* 43 (1999): 211–244.