Data fusion to synthesise quantitative evidence, value and socio-economic factors

A framework and example of Dempster-Shafer theory

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Abstract – This paper presents a framework and example of how fuzzy data fusion processes can support decision making for energy efficiency in historic buildings. Dempster-Shafer (DS) theory is a framework of reasoning that deals with uncertainty, allowing one to combine evidence from different sources. DS theory can handle conflicting information, with the aim to provide a representation of the appropriateness and uncertainty for each option. The theory starts with a set of possibilities: for example, a range of retrofit options or energy-use schemes. Each one is assigned a degree of belief depending on how many evidence inputs contains the proposition and the subjective probability. DS theory incorporates hard data, e.g. energy models and economic estimates, and opinion, e.g. disruption to activities and changes in aesthetics. It is proposed that DS Theory and hard-soft data fusion algorithms provide an approach that can incorporate value and socio-economic aspects into decision making.

Keywords – decision making; data fusion; artificial intelligence; uncertainty; conflict resolution

1. INTRODUCTION

Making decisions on aspects of energy efficiency in historic buildings is complex [1]. There are several considerations, including technological feasibility, economy, impact on the historic character, etc. Each consideration is perceived differently by stakeholders – including managers, consultants, practitioners, occupants, and members of the public – based on their understanding of the issues and ability to confidently express an opinion. Trying to balance these inputs during a decision making process is subjective. These processes can be supported by using data fusion techniques.

Data fusion, or information fusion, is a method of combining input from multiple sources to produce information that is more consistent, accurate, and/or useful. It emerged from the need to combine sensors to improve military target tracking manufacturing precision [2]. In traditional approaches, the ‘sensors’ used were providing ‘hard’ data, i.e. quantitative information with associated precision and accuracy. More recently, interest in data fusion techniques that can incorporate ‘soft’ data has grown [3]. One technique capable of this is the Dempster-Shafer (DS) theory [4,5], also known as the theory of belief functions. It is a framework for combining evidence and dealing with uncertainty. The theory allows evidence from different sources to be combined resulting in a degree of belief that considers all available evidence. Due to its power as a decision making tool, DS theory has been applied in a range of fields including artificial intelligence [6], environmental impact [7] and building risk assessment [8].

Bayesian methods are commonly applied in energy retrofit decision making processes [9]. Bayesian approaches aim to assign a probability to each potential outcome. In contrast, DS theory allows for a probability to be assigned to a set of outcomes [10], e.g. the probability of one outcome or another. In this way, DS theory allows for more flexibility when assigning probabilities from evidence, and can be considered a generalization of Bayesianism. Bayesian methods require more information *a priori* to 'condition' the probabilities; DS theory does not rely on prior knowledge, making it particularly suited to situations in which it is difficult to collect or hypothesise probabilities [11].

This paper outlines the framework of Dempster-Shafer theory and introduces some of the metrics that can be used to assess the uncertainty of the output. Following this, an example is given of how the theory might be used as part of selecting a proposal from many to improve energy efficiency in a historic built context.

2. THEORY AND DEFINITIONS

2.1 DEMPSTER-SHAFER THEORY

The classical definition of Dempster-Shafer theory is given, followed by additional metrics to incorporate the degree of conflict and subsequent developments in uncertainty measures.

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$ be a finite nonempty set of mutually exclusive and exhaustive events, referred to as the frame of discernment (FOD). The power set of 2^N elements represents all possible combinations of the elements of the FOD and is denoted as follows:

$$2^{\Theta} = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_N\}, \ldots, \{\theta_1, \theta_2, \ldots, \theta_i\}, \ldots, \Theta\}, \tag{1}$$

in which Ø represents the null set.

A *mass function m* is defined as a mapping from the power set 2^{Θ} to the interval [0,1], which must meet the following criteria [4,5]:

$$\sum_{A \in \Theta} m(A) = 1. \tag{2}$$

If *m(A)* > 0, then A is a *focal element* for which there is supporting evidence. An *m = 0* represents no evidence for an element, while *m* = 1 represents complete certainty.

The set of mass values associated with a single piece of evidence is called a body of evidence (BOE), often denoted *m*(-) [12]. Each BOE is a subset of the power set 2^{Θ} meeting (2), in which each A $\in$ *m*(-) has an associated non-zero mass value *m*.

Two independent mass functions, m_1 and m_2, can be combined with Dempster's rule of combination to produce a *joint mass* $m_{1,2}$ defined as [4,5]:

$$m_{1,2}(A) = (m_1 \oplus m_2)(A) = \frac{1}{1-k} \sum_{B \cap C = A} m_1(B) m_2(C), \tag{3}$$

where A, B, and C are non-unique elements of the BOE, and k is used to for the *degree of conflict* between m_1 and m_2, defined as:

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C). \tag{4}$$

Conflict occurs when elements B and C do not have any intersecting events. The normalisation process ensures that $m_{1,2}$ meets the criteria in (2).

2.2 EVALUATING THE BELIEF INTERVAL

For each element of a set, the upper and lower bounds of a probability interval can be defined. This interval contains the precise probability of a set, and is bounded by two non-additive continuous associated measures for a set A called the belief function *Bel*(*A*) and plausibility *Pl*(*A*), defined respectively as [4,5]:

$$Bel(A) = \sum_{B|B \cap A} m(B). \tag{5}$$

$$Pl(A) = \sum_{B|B \cap A \neq \emptyset} m(B). \tag{6}$$

The belief *Bel*(*A*) is the sum of the masses $m_{i \ldots N}$ that are subsets of set A, i.e. those that directly provide evidence for that set. The plausibility *Pl*(*A*) is the sum of the intersecting masses to set A, i.e. those that *could* provide evidence for that set but cannot be further subdivided into component scenarios. The size of the interval [*Bel*(*A*), *Pl*(*A*)] characterises the confidence of the probability, not the certainty of a claim. The Pignistic function [13] represents the extent to which we fail to disbelieve A, defined as:

$$BetP(A) = \sum_{B \subseteq \Theta, B \neq \emptyset} m(B) \frac{|A \cap B|}{|B|} \tag{7}$$

3. FRAMEWORK FOR APPLYING THE DS-THEORY

A framework for applying DS theory in decision making is presented in Figure 1. The application of DS theory to decision making process first involves collecting evidence. This can take many forms, comprising both 'hard' data (e.g. models, experimental trials), and 'soft' data (e.g. testimonies, surveys). Other types of input are also possible.

These inputs are converted into bodies of evidence, each of which is a set of mass functions satisfying (2). It is not necessary that every element of 2^{Θ} is addressed by each BOE. A unique feature of DS theory is the ability to analyse

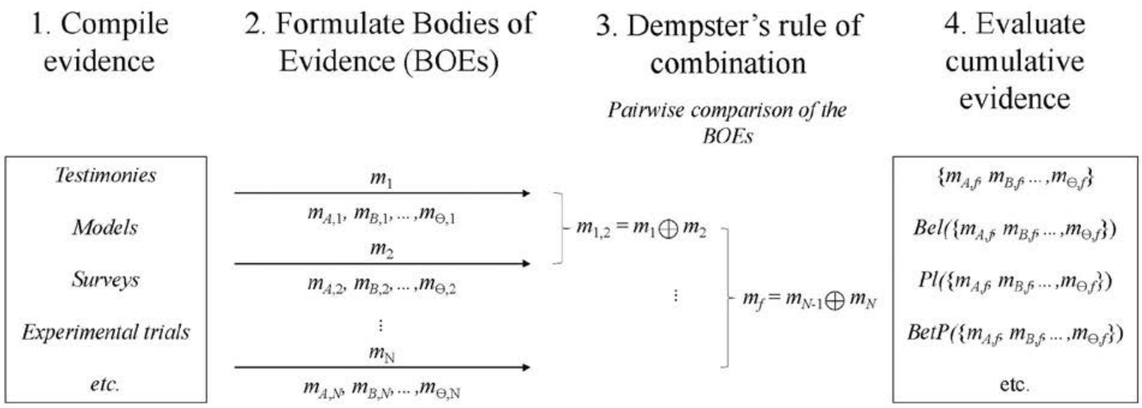

Figure 1. Framework for applying Dempster-Shafer theory to decision making processes.

incomplete information. More realistically, each BOE will address a small subset of the frame of discernment ($\oplus$). Once the BOEs have been formulated, they are combined in pairs through Dempster's rule of combination. The order in which the BOEs are combined is irrelevant, as the final set of cumulative mass functions m_f will be the same. m_f can then be evaluated by comparing the mass, belief, plausibility, and Pignistic functions.

4. EXAMPLE

4.1 INTRODUCTION

The utility of DS theory in supporting decision making processes is demonstrated with a simple scenario incorporating a hard and soft data input, which is worked through the framework given in Section 3. A third piece of evidence is provided to demonstrate the process for a multi-stage combination.

4.2 SCENARIO

An energy appraisal has been conducted for a museum that is housed in a historic building with protected status. Three options are considered to reduce the energy consumption of the building:

1) Retrofit with visible change to the historic fabric (option A);
2) Retrofit with minimal visible change to the historic fabric (option B);
3) Change environment conditions specifications to reduce HVAC load (option C)

4.3 COMPILE EVIDENCE

Two methods of evaluation are used to try and select the best option:

- A building surveyor with significant experience makes a detailed assessment of the building and its function (soft data);
- An energy consumption model is created by a consultant company and run under various stochastic scenarios (hard data).

The surveyor's report specified that they are somewhat confident that a retrofit would be effective. They meet with the building management to discuss the survey results. During this, they mention a similar project they recently worked on where changing the environment only provided a small savings in energy. Due to this, they are quite sceptical of this being an effective technique.

The modelling exercise has the following outputs: to meet financial targets an energy consumption target of 40 % reduction is set. The model did not have the ability to estimate the savings in a scenario where a bespoke non-visible retrofit was undertaken. Under the various scenarios, 80 % of iterations met the target reduction in energy consumption.

4.4 FORMULATE BODIES OF EVIDENCE

In this scenario, we assume that one of the options explored will be more effective than the others (i.e. the null set is 0). Let m_1 represent the surveyor's body of evidence. To translate their evidence to a quantitative scale, let us map confidence to a numeric scale: absolutely confident (100 %), very confident (80 %), somewhat confident (60 %), not very confident (40 %), not confident at all (20 %), don't know/not sure (0 %). Therefore, the surveyor is 60 % certain that a retrofit option would be effective. Unfortunately, their report did not break this down between the visible and non-visible options. Let us equate 'sceptical' with 'not confident at all'. The BOE m_1 from the surveyor would be comprised of the following elements: visible *or* minimally visible change: $m_1\{A,B\} = 0.6$; change environmental specifications: $m_1\{C\} = 0.2$; any option: $m_1\{A,B,C\} = 0.2$. The final element represents that no further preference was expressed in the surveyor's evidence that distinguishes between the three options, and accounts for the remaining certainty to meet the criteria in (2). It is implied that all other possible outcomes (e.g. visible change *only* {A}, minimally visible change *only* {B}, etc.) have $m = 0$.

The modelling exercise provided information on option A. Based on the model output and the consumption reduction target of 40 %, the model predicts that this will be achieved in 80 % of scenarios. To this end, the model BOE m_2 can be expressed as: visible change: $m_2\{A\} = 0.8$; any option: $m_2\{A,B,C\} = 0.2$. In a similar manner, the second element represents the unexpressed indifference between the three options when taking a modelling approach.

4.5 DEMPSTER'S RULE OF COMBINATION

It is useful to set up an 'intersection tableau' for computational purposes (Table 1) [14]. Let m_3 denote the combination between the surveyor's evidence m_1 and the model output m_2. In this case, there is one non-intersecting element, so $k = m_1\{C\} \oplus m_2\{A\} = (0.2)(0.8) = 0.16$.

Table 1. Intersection tableau for Dempster's rule of combination in the Scenario set out in Section 4.1

	m_1		
m_2	$m_1\{A,B\} = 0.6$	$m_1\{C\} = 0.2$	$m_1\{A,B,C\} = 0.2$
$m_2\{A\} = 0.8$	$m_3\{A\} = (0.6)(0.8)/k = 0.571$	*Non-intersecting*	$m_3\{A\} = (0.2)(0.8)/k = 0.190$
$m_2\{A,B,C\} = 0.2$	$m_3\{A,B\} = (0.6)(0.2)/k = 0.143$	$m_3\{C\} = (0.2)(0.2)/k = 0.048$	$m_3\{A,B,C\} = (0.2)(0.2)/k = 0.048$

4.6 EVALUATE CUMULATIVE EVIDENCE

From the cumulative evidence m_3 calculated in Section 4.4, a summary of the functions can be compiled (Table 2).

Table 2. Summary of cumulative functions

Scenario		m_3	*Bel*	*Pl*	*Bel interval (probability range)*	*|Bel interval|*	*BetP*
Visible change	{A}	0.761	0.761	0.952	[0.761, 0.952]	0.191	0.849
Environmental specifications	{C}	0.048	0.048	0.096	[0.048, 0.096]	0.048	0.064
Visible *or* minimally visible change	{A,B}	0.143	0.904	0.952	[0.904, 0.952]	0.048	0.175
Any option	{A,B,C}	0.048	1.00	1.00	[1.00, 1.00]	0	0.048
TOTAL	-	1.00	2.71	3.00	-	-	1.14

m_3 represents the combined mass functions from our two bodies of evidence: the surveyor and the model. In this section, all elements refer to m_3. We can see that the ordered magnitudes are: $m_3\{A\} > m_3\{A,B\} > m_3\{C\} > m_3\{A,B,C\}$.

This demonstrates that the combined evidence supports the visible retrofit {A} most strongly, followed by one of the retrofit options {A,B}. There is no strong case for environment specifications {C}, since $|m_3\{C\}| = |m_3\{A,B,C\}|$, i.e. the cumulative evidence for changing environment specifications is equal to the evidence supporting any option.

It is important to note that Bel and Pl do not sum to 1. The belief represents the masses of evidence that directly supports an element, i.e. the minimum amount of confidence we have in it. As the visible option{A} is a component that supports belief that the set {A,B} (either retrofit option) has a higher Belief; Belief({A,B}) = 0.143 + 0.761 = 0.952.

The plausibility represents the masses that *could* support an element. For example, the plausibility of any option having the strongest case is 1.00 since all other elements are subsets of it, therefore supporting it: Pl({A,B,C}) = 0.761 + 0.048 + 0.143 + 0.048 = 1.00.

The Belief interval is the range of probabilities that an element is the ideal option according to the evidence provided. The size of the interval represents how certain we are of that probability. For example, the Belief interval for {A,B,C} is 0, since we defined that the null hypothesis = 0. It could have been allowed that a body of evidence does not support any of the options i.e. there is a non-zero null hypothesis.

Although there is less evidence directly supporting ‘either retrofit option’ A or B ($m\{A,B\} < m\{A\}$), they have the same plausibility. This means that they are equally plausible according to the full set of possibilities. The Belief interval for either retrofit {A,B} is smaller than a visible retrofit {A}, which means that we are more confident in one of the retrofit options being appropriate than we are that the visible option is appropriate. This is only because they have the same plausi-bility; having identically-sized Belief intervals does not mean they are equally plausible or confident. Based on the Pignistic functions, we most significantly fail to disbelieve in the visible option{A}.

From this assessment, it can be concluded that a retrofit with visible changes is most strongly directed supported by the surveyor and the models, but it is equally plausible that either of the retrofit options could be appropriate. We fail to disbelieve the former, but we have more confidence in stating the probability of the latter.

4.7 A MULTI-STEP COMBINATION EXTENSION

4.7.1 Scenario

A second opinion is sought from a curator of the museum. They are certain that the historic elements of the building are a reason that many people visit. They are concerned that a retrofit with visible changes will affect the visitor experience and ultimately income. The curator doesn't have experience with changing environmental specifications but heard from a colleague that it had been effective in many cases to reduce energy consumption.

4.7.2 Formulating bodies of evidence

We already have the combined evidence from the surveyor and the model m_3: $m_3\{A\} = 0.761$, $m_3\{C\} = 0.048$, $m_3\{A,B\} = 0.143$, $m_3\{A,B,C\} = 0.048$. We can produce from the curator a third *independent* body of evidence m_4: $m_4\{C\} = 0.2$, $m_4\{A,B,C\} = 0.8$. Put another way, the curator thinks changing environmental specifications is a reasonable option to explore but is not confident that it will be appropriate. No opinions on retrofit options were provided.

4.7.3 Dempster's rule of combination

Another intersection tableau is created for $m_5 = m_3 \oplus m_4$ (Table 3), which has a $k = (0.8)*(0.761 + 0.143) = 0.723$ [1-k = 0.277]. It is important to note that partially due to reduced evidence provided by the curator, there is more conflict.

Table 3. Intersection tableau for Dempster's rule of combination combining the first two bodies of evidence with that representing the curator

m_4	m_3			
	$m_3\{A\} = 0.761$	$m_3\{C\} = 0.048$	$m_3\{A,B\} = 0.143$	$m_3\{A,B,C\} = 0.048$
$m_4\{C\} = 0.8$	*Non-intersecting*	$m_5\{C\} = 0.139$	*Non-intersecting*	$m_5\{C\} = 0.139$
$m_4\{A,B,C\} = 0.2$	$m_5\{A\} = 0.549$	$m_5\{C\} = 0.035$	$m_5\{A,B\} = 0.103$	$m_5\{A,B,C\} = 0.035$

The same metrics produced in the original scenario can be calculated for the new combined evidence (Table 4).

Table 4. Summary of cumulative functions

Scenario		m_5	*Bel*	*Pl*	*Bel interval (probability range)*	*|Bel interval|*	*BetP*
Visible change	{A}	0.549	0.549	0.687	[0.549, 0.687]	0.138	0.612
Environmental specifications	{C}	0.313	0.313	0.348	[0.313, 0.348]	0.035	0.325
Visible *or* minimally visible change	{A,B}	0.103	0.562	0.687	[0.562, 0.687]	0.125	0.126
Any option	{A,B,C}	0.035	1.00	1.00	[1.00, 1.00]	0	0.035
TOTAL		1.00	2.42	2.72	-	-	1.10

Now, the mass of a visible retrofit {A} has been reduced by the curator's support for changing environment conditions {C}. Since no evidence for options A or B was given {A,B}, the visual retriofit {A} still has the same (but reduced) plausibility as either retrofit option {A,B}. We are now almost equally confident in the probability that a visible retrofit and either of the retrofit options is most appropriate. As further evidence was given, the ambiguous mass supporting any of the options is reduced.

In contrast to quantitative evidence typically used to make retrofit decisions, the metrics used herein have specific benefits:

- a single piece of evidence can support multiple outcomes in a non-discrete manner;
- the Beliefs and Belief intervals represent the full range of potential probabilities, providing additional discussion beyond averaged metrics;
- no knowledge on the probability of each option is needed, since the Belief functions are formulated directly from the evidence provided.

5. FUTURE WORK

The example scenario included a limited number of bodies of evidence. There is no limit to the number of steps in Dempster's rule of combination, meaning that any number of evidences can be considered. With more bodies of evidence, it would become important to explore metrics that evaluate the degree of conflict between bodies. Future work will explore more complex scenarios (e.g. more bodies of evidence with a greater number of options and fuzziness) and conflict metrics.

6. CONCLUSION

Dempster-Shafer theory was applied to a simple decision making scenario, which demonstrated its ability to combine inputs from a variety of sources. This is especially pertinent for the historic environment in which complex issues must be addressed with a combination of quantitative and qualitative means. Data fusion techniques, such as Dempster-Shafer theory, are not meant to replace existing decision making processes. Hard-soft data fusion algorithms provide a tool that can support decision making by synthesising a diverse range of heritage conservation considerations into a cohesive output.

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