Part I
Preliminaries

Abstract A history of the loop antenna covering the period from the late 1800s to the present day precedes a chapter on the mathematical foundations used in this book. The history covers the experimental and the analytical (theoretical) history ending with nano-scaled rings used in meta-materials. Foundations covers the geometrical coordinate systems used in analysing the closed toroidal loop, useful expressions of Maxwell's equations, a detailed look at propagation parameters from the radio frequency (RF) through to the optical regimes, vector and scalar potentials with their respective equations particularly for a Green's perspective, the governing equations for the thick and thin wire loop, and three methods of coupling to a loop.

Chapter 1
General Introduction

Abstract Experimental work on loop antennas in the radio frequency (RF) region reaches back in time to the development of the first radiating structures. Almost immediately differences were identified between loops and linear dipoles, which showed that dipoles were to be much preferred over loops for AM broadcast applications. But with time, loops were found useful for FM at ultra-high frequencies (UHF) and for a few special needs. Unfortunately, loop antennas never attained as wide an acceptance in communications as that enjoyed by structures derived from dipoles. In 1999, loops seem to have a found a place in history. The development of meta-material structures, using small split-rings, suggests that loops may enjoy a renaissance as nano-scaled rings in the Microwave (MW), TeraHertz (THz) and Optical regions. This chapter covers the experimental and analytical history of RF loops and of nano-scaled rings.

1.1 A Motivation for the Study of Loops and Rings as Radiating Structures

The analytical equations describing the characteristic behaviour of circular loop antennas are considered solved problems by many people. This is not entirely true. The equations applied to thin-wire loops were indeed solved in the 1950's and 1960's, but even today the equations for thick wire loops have not been solved and remain on the cutting edge. It was only a few years ago that the equations for thin-wire lossy metal loops were solved. This has now taken metal loops down into the nano-scaled region, but dielectric loops are not yet solved either, and remain a hindrance in the advance of meta-material theory.

Another hindrance is that the classical derivation of the behaviour of loops comes from energising the loop at a given point on the periphery; that is by treating it as a radiator. It is, of course true that loop antennas as receivers are illuminated by plane wave, but that perspective was not important until meta-materials, especially meta-materials in the THz and optical regions. This is because meta-materials were thought to obtain their behaviour from the magnetic field penetrating the loop in a particular orientation of the plane wave. The subtle differences in behaviour have not been studied in much detail, a lack that is covered in this volume in Sections 2.6.3 and 9.2.

The nano-scaled thick ring has now become an item of some interest; it is a nano-structure that has led to some unexpected and rather curious behaviour. The question is whether there are more unknown and unexpected behaviours waiting to be found. Interestingly, the analytical theory we have about loops has been found deficient in answering these questions of nano-rings, and a thorough going examination of the dielectric ring in the THz and optical regions is now timely, and in fact, overdue.

1.2 The Experimental History of Loops

Much of the early knowledge of loop antennas was based more on experimentation and experience than on analytical theory. It is true that theory, early on, indicated that the small loop is the "dual" of the small linear dipole, in the sense that the fields radiated by the small loop have the same mathematical form as those radiated by a small linear dipole, except that the magnetic and electric fields are interchanged [1, pg 153 and 237]. This is because it is easy to guess the current distribution in a small loop (it must be a uniform distribution). As a result, small loops were, and often still are, thought of as magnetic field sensors [2, pg7-42]. Consequently loops in the period before the 1960's were used for reception, direction finding, and signal location. One does not find small loops used for transmission in the 1920s to the 1950s, the days of AM radio [3, 4, 5, 6]. Indeed, broadcasters used half wavelength dipoles on very tall towers, some 180 meters long at 833 kHz [4, 6, 7]. Why did they not use large loops?

Just as with small loops, the current distributions on half-wave and full-wave loops, in polygonal and circular configurations, can be easily guessed, and their radiation patterns calculated at their apparent resonant frequencies. Indeed, their lengths can be adjusted and matched to the source until standing waves appear, to ensure the expected resonance. Yet these large loops were not used, partly because of their areal size, partly because of a difficulty of coupling and impedance matching the loop to the transmitter [8], but mostly because of their radiation patterns.

Small dipoles and small loops (those with circumference under $.2\lambda$) share the same omni-directional radiation pattern in the azimuthal direction (at $\theta = \pi/2$, when the loop lies in the *xy* plane), but these patterns diverge as the dipole and loop grow larger with respect to operating wavelength. While the dipole pattern remains roughly omni-directional, and therefore an optimal pattern for tall towers, a large loop (where circumference $\approx \lambda$) mutates toward a thick lobe facing normal to the surface of the plane of the loop (that is, toward $\theta = 0$ and $\theta = \pi$). So if the plane of the loop were placed horizontally to the earth, it would radiate uselessly straight up and straight down; and if placed at right angles to the earth, two of the four directions would have reduced transmission. The omni-directional pattern of the dipole would be preferred.

It wasn't until FM radio began to challenge the dominance of AM, first in the very high frequency (VHF) band and later in the 1950s in the ultra-high frequency (UHF) band for television that loops became an alternative to dipoles. By that time the importance of shaping radiation patterns by design significantly affected the choice of antenna configuration, and the loop was one among many that were developed and measured. Some of the others, for example, were stacked dipole arrays, the Vee, the Bow-tie, and the Rhombic [9]. The Quad, a rectangular, square or circular loop in Yagi-Uda format with reflectors and a driving loop, was also studied for UHF purposes in some detail [10]. The loop for the Quad was designed to have no gaps in the periphery in order to eliminate regions of voltage, since it was used in its first rendition high in the Andes of Ecuador where gaps suffered coronal discharges and affected transmission.

It is difficult to find anything more than basic theory in the literature for these UHF loops. The earliest useful loop appears to have been a square design by Andrew Alford [11] in 1940. The square is broken into four sections, each as long as a quarter wavelength, and energised at four equidistant points in order to ensure a uniform current distribution around the loop. The goal was to produce an omni-directional pattern with low radiation in the vertical direction.

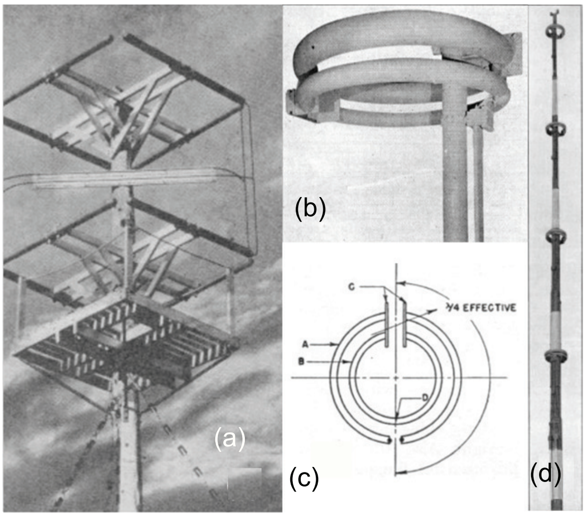

Fig. 1.1 Evolution of the Alford Loop in the 1940s [11]. (a) Sheldorf's extension of the Alford Loop. The Alford Loop is the top square. (b) The eventual loop with (c) its schematic, showing it as a split-ring; probably the first split-ring design. (d) is the implementation as a set of loops on a pole. Courtesy November 1942 QST; copyright ARRL

In 1942, M. W. Scheldorf at General Electric developed a stacked set of circular loops [12], quite by trial and error it seems, starting with two stacked square Alford loops with each arm of length 0.25λ. This evolved into a nested pair of squares, twisted 90^o with respect to each other, with one arm on each square missing, as shown in Fig. 1.1. The resulting pattern was omni-directional as needed. Finally the squares evolved into the two stacked circular loops, looking much like a folded dipole shaped into circular form, where the two circles are connected using flat plates at the 180^o point opposite the driving gap on the outside circle. The practical example given had diameter 33 inches, a circumference about 0.16λ for the operating frequency of 46 MHz. This was an extraordinary decrease in the size of the loop from the full wavelength, suddenly making the loop manageable in terms of areal size for communications. These were then stacked on a long pole to provide an omni-directional field. Note that the final design was, indeed, a split-ring at MHz frequencies, operating sub-wavelength, probably the first useful split-ring that appears in the literature.

In 1943, George Levy summarised the current state of loop antennas used on aircraft by the military in the Proceedings of the IRE [13]. The preferred "low-impedance" loop carried anywhere from 4 to 20 turns, the high-impedance carried 20 to 70 turns and both were significantly smaller than a wavelength. The loop, mounted on the fuselage outside the aircraft, was kept close to the receiver, so that a capacitor in the receiver could resonate with the loop.The theory applied simple notions of LC resonance and the article spent most of its effort on simplifying the circuit analysis of the loop, receiver and transmission line and on the radiation pattern of the loop rather than on intricate development of behaviour from first principles.

Loop antennas do not appear at all in the rather lengthy *Radio Antenna Engineering* reference by Edmund A. Laport in 1952 [9]. That may be due to the fact that Radio Corporation of America (RCA) strongly supported AM, rather than FM [14]. But even in an historical review of antennas that appeared in the Proceedings of the IRE in 1962, there is little mention of loops antennas except in casual reference [15]. The 1960 edition of the American Radio Relay League's (ARRL) antenna reference book for amateur radio operators describes various useful notes about small, half and full wavelength circular loops in the section under Antenna Fundamentals, but only one loop antenna appears elsewhere in the book, fashioned somewhat like the Levy loop, noted above for aircraft, but many fewer turns [16].

In 1962, James B. McKinley began building and experimenting with closed loop antennas in the 20 meter amateur radio band. These evolved into single capacitor circular loops and then multiple capacitor, polygonal loop antennas. He designed each to carry a high Q resonance and he learned experimentally how to match these loops for a 1:1 standing wave ratio on the transmission line. The circumference of his loops were substantially smaller than the operating wavelength, on the order of 0.4λ (see Fig. 1.2). The principle idea was simply that the inductance of the loop resonated with the capacitor and under certain conditions would attain a very high Q, high currents and excellent radiation. It is clear from his calculations that when the loop was divided into N sections, by spacing the capacitors evenly around the periphery, he thought each section went into resonance separately; the sectional

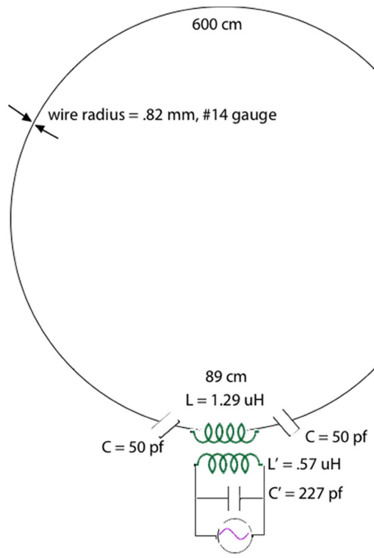

Fig. 1.2 Single capacitor loop antenna design by McKinley from 1963. Redrawn from plans [17].

L resonated with the sectional C. When the loop was energised, he would short an individual capacitor and draw long arcs, showing a very high voltage gradient across the capacitors. The values of the total L and C needed for loop resonance were determined using the standard resonance formula, $f = 1/\sqrt{L_T C_T}$, and the free space impedance $\sqrt{L_T/C_T} = 377$ ohms. The values of the individual capacitors where then given by $C = C_T N$ and $L = L_T/N$. The design eventually evolved various use antennas, consisting of many nested hexagons and arrays of hexagons (see Fig. 1.3).

In an electronics article [18] published in 1967, Kenneth Patterson described the development of a hexagonal loop at the US military's Aberdeen Proving Grounds for use in Vietnam. The loop resonated with a set of two adjustable capacitors and then impedance matched to the transmission line using an additional capacitor. The "Army Loop", as it came to be called in amateur radio venues achieved a certain notoriety after it appeared in a March 1968 article in QST [19], due to a very strong resonance it exhibited for a small areal size. The circumference was 0.16λ for a design frequency of 3.98 MHz. The article is thought by subsequent amateur authors to be the first of the single capacitor loops that eventually came to be known as a very small, efficient, high-Q antenna for distance communications. That loop and variations on that design began appearing in the ARRL's Antenna Book in about 1988 [20] and has appeared in every edition since then.

The ARRL reference also describes a variant on the Army Loop with design data for 38 loops [22, 23, page 5-11]. All of them exhibit a ratio of circumference to incident or driving wavelength $(2\pi b/\lambda)$ between .06 and .40. This is true of the Schel-

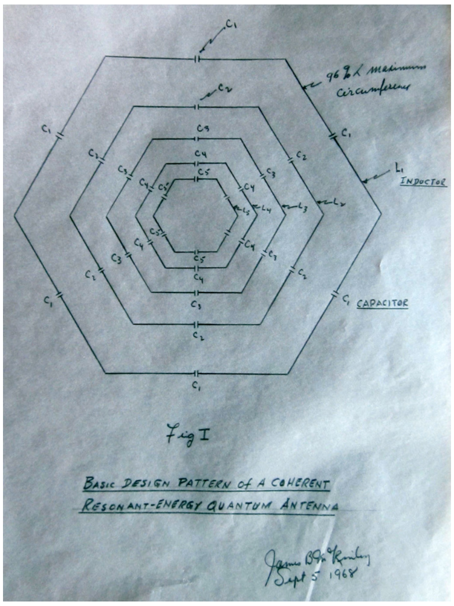

Fig. 1.3 Nested loop antenna design by McKinley from 1968. [17]

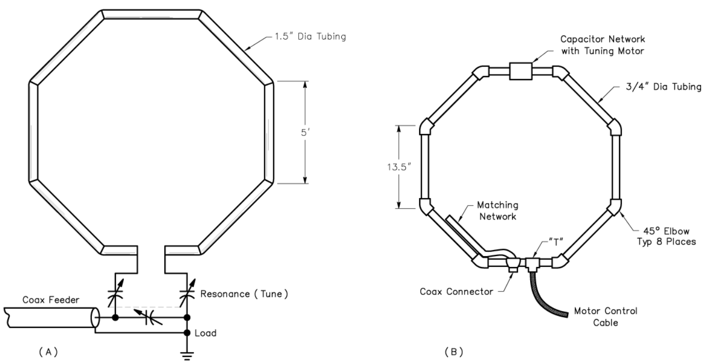

Fig. 1.4 A Simplified Diagram of the Army Loop from 1968 and of the Hart Loop from 1986 described in the ARRL Antenna Book. [21]. Courtesy ARRL Antenna Book , 18th Edition, 1998

dorf loop of 1942 (above) and all of the McKinley loops of 1962+ (also above) as well. In the later literature this region is called the "sub-wavelength" region, because the circumference is smaller than 0.50λ. All exhibit high quality Qs, sometimes as high as several thousand. The high Q was for amateurs, either a blessing, because of the consequent amplification of the signal, or a curse, because the bandwidth is small, requiring an antenna retune and impedance rematch when looking for new contacts.

These sub-wavelength resonances are described in detail in Section 7.5.

1.3 The Analytical History of Loops

Large circular closed loops, those with circumferences on the order of the incident or driving wavelength, present complications that are not present in analytical studies of small loops. This is because the current distribution is difficult to guess.

Generally, in small loop analyses, the wire is assumed to be very thin compared with wavelength and the current distribution is assumed to be the same at every point around the periphery of the loop, assumptions which give a uniform magnetic field and, for a closed circular loop, an omni-directional radiation pattern. However, a large closed circular loop can handle many resonant modes and trying to guess which of these is dominant under different circumstances is not straightforward. The simplistic guess is the condition $2\pi b = n\lambda$, where b is the radius of the loop and n is an integer. This is the "Bohr" condition for loops, which comes from Quantum Mechanics [24]. But such a loop can handle many resonant modes and Pocklington [25] in 1897, considering a closed loop illuminated by a plane wave, assumed for the current an infinite series of resonance modes in a Fourier Series. Later solutions for the current based on his approach do show that very thin loops indeed resonate under the Bohr conditions, but a thick wire is not likely to do so for two reasons: (1) toroidal, and less likely radial, currents are more likely to occur in thicker wires and (2) even in metals at low frequencies, which exhibit skin-effect, surface filaments of current on the inside circumference of the loop (at $r = b - a$) travel a shorter distance than those on the outside surface ($r = b + a$), where b is the average radius of the loop (through the middle of the wire) and a is the radius of the wire. Most charge follows a path on the inner radius, thus blue-shifting the resonances slightly. Nevertheless, these assumptions taken together as the "thin-wire" approximation, show a blue-shift in loops as thin as a loop to wire radius of 64.

Hallen [26] extended Pocklington's method using a driven, rather than illuminated, closed loop, finding Fourier solutions for both the current and the impedance, but the coefficients showed a singularity, making the series only "quasi-convergent" and thus limiting the results to small loops. In the mid-century, Storer [27, 28] attempted to solve the convergence problem, but was successful only up to a point; he truncated the series at mode 4 and estimated the rest of the series with a closed function. This method leads to some inaccuracies [29]. Wu [30] discovered the weaknesses in Storer's method and solved the convergence problem completely, allowing for an infinite sum in the Fourier series. These results have been noted in all of the major antenna reference books and in the literature since then, for example [31, 32, 33, 2, 34, 35, 1].

In 1963 a US patent was granted on papers filed in 1958 by Julius Herman [36]. It contains a summary of $\vec{E}$ and $\vec{H}$ fields for the closed loop derived under the assumption of an infinite Fourier series representation for the current. The loop is embedded in material media with loss characteristics. Herman was able also to solve the Fourier coefficients and formulate a theory of input impedance using a delta function voltage source, the same approach used by Storer and Wu. Herman also added lumped and distributed impedances around the periphery of the loop. The point of the patent was to devise means of controlling the radiation patterns using these distributed impedances. Unfortunately, Herman never published except in this patent. Since he did not cite any of the above authors, the work apparently is entirely his own.

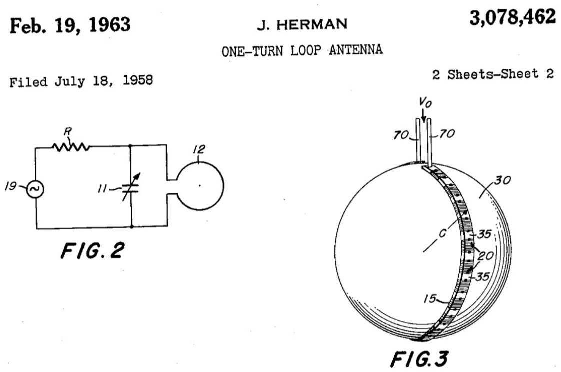

Fig. 1.5 A portion of the cover page to Herman's patent.

In 1964, Iizuka [31] extended the Storer theory to include a loading of multiple impedances and driving sources spread around the periphery of the loop. In particular, if only one driving source is used, his work provides the input impedance of the multiply loaded loop. The results are presented in matrix form and the impedances, in particular, form a diagonal matrix. He focused his examples specifically on negative resistive loads. Unfortunately, he used the Storer truncation method on the series and his printed results are therefore a bit inaccurate.

In 1964, Harrington [37] developed a general solution for loaded wire antennas as scatterers using generalised impedances in matrix representation and in 1965 with Ryerson [38] reproduced the Storer/Wu theory and then extended it to the scattering of multiply loaded loops using his new method, noting Iizuka's previous work. However, instead of a driven loop, they instead used a plane wave and were therefore able to derive the radar cross sections. Then, in a 1968 paper written with Mautz [39], Harrington identified the Fourier coefficients of the infinite series as impedances, and as a result, recognised that the input impedance of the loop was the same as that of an infinite network of *parallel* impedances. One of their figures shows the real and imaginary parts of the DC mode impedance and the first three frequency mode impedances. The DC mode is inductive over the region $0 < b/\lambda < 0.40$ (circumference < 2.5), while the remaining modes are in some regions capacitive and in other regions inductive. With these results, Harrington and Mautz produced gain patterns for transmitting closed loops and loops loaded with a single capacitor. They also produced forward and backward bi-static scattering plots for the closed and capacitively loaded loop. Harrington and Mautz do not appear to have recognised the sub-wavelength resonance noticed by the amateur community.

In 1966, Iizuka [40] extended the Storer/Wu model to two loops separated in space along the z-axis with planes perpendicular to the z coordinate. Both loops were driven at $\phi = 0$. He solves the two coupled integral equations by splitting the voltage generators into symmetric and anti-symmetric parts and reduces the two integrals to one for solution. This result yields the self and mutual impedances of the two loops. In 1967, Thiele [41] noticed the sub-wavelength resonance when studying loops with a small finite gap. His work was experimental, focusing on the loop as a scatterer and relying on Harrington's 1966 paper.

This theory was not extended again until the second decade of the current century when McKinley et al [42, 43] recognised that the model impedances identified by Harrington in 1964 could be construed as series resonant *RLC* circuits. This allowed them to calculate the resistance, inductance and capacitance of each individual mode and the total R, L and C values of the closed loop over the region $0 < 2\pi b/\lambda < 2.5$. They then extended the theory to metallic materials, using the index of refraction as the basis for characterisation. This allowed computations beyond 100 GHZ into the optical region. Finally, following Iizuka's method, McKinley extended their results to multiply-loaded loops at low frequency and to multiply-loaded nano-rings in the optical region [44].

Several authors have developed analytical expressions for the near and far field radiation patterns of loops. Rao [32] in 1968 used the Storer theory to calculate the radiation patterns of the far-field, which are good to 2.5λ. Martin [45] in 1960 developed a direct integration procedure for far-zone vector potentials leading to closed form solutions for most current distributions on the loop. In 1996, Werner [46] finally developed an exact integration procedure for the near-zone vector potentials. The last is covered in this volume. In 1997, Li et al. [47] published an alternative vector analysis employing the dyadic Green's functions.

1.4 The Recent History of Nano-Scaled Rings

A surprising development in loops occurred at the turn of the 21st century in the micro-wave (MW) region where metals still conduct well. Veselago [48] in 1968 had suggested that negative index of refraction materials could physically exist and, if constructed, would show novel optical effects. Such a material would be "left-handed", in the sense that the vectors $\vec{E}, \vec{H}$, and $\vec{k}$ would follow a left-handed rule of orientation rather than a "right-handed" rule. This result would create strange Doppler effects, Vasilov-Cerenkov effects, and negative refraction behaviour at boundaries with right-handed materials.

Three seminal papers form the core of efforts to design and build such negative index materials: the works by Pendry et al [49], Smith et al. [50], and Shelby et al. [51]. After examining and calculating the permeability of various configurations of metallic cylinders and rolled metallic sheets, Pendry's group decided on the flat disk, nested split-ring configuration shown in Fig. 1.6, as the fundamental building block of a 3D structure, which they calculated would show a region of negative permeability at 13.5 GHz. In 2000 they suggested it could be used to create a perfect lens [52]. The Smith group included a small cylinder, just above the ring, to add a

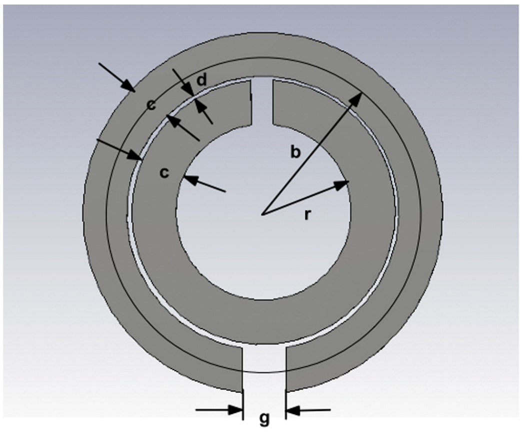

Fig. 1.6 A redrawing of the of the split rings designed by Pendry for the 3D structure [49]. $a = 10.0$ mm; $c = 1.0$ mm; $d = 0.1$ mm; $l = 2$ mm; $r = 2$ mm. Consequently, the middle radius of the outside ring is $b = r + d + 1.5c = 3.6$ mm.

negative permittivity to the structure and thus create the required negative index of refraction. The Shelby group built the 3D structure shown in Fig. 1.7 using nested square loops, and showed that it did indeed yield negative index phenomena (see Fig. 1.8). This effect came to be the identifying mark of "meta-materials", human-made electromagnetic structures for which there is no counter-part in nature.

The outer loop in the Pendry configuration has a circumference of 22.6 mm and a calculated expected resonance was 13.5 GHz, about 22 mm. It resonates therefore at the first mode $n = 1$. Indeed, neither of their two rings resonates in the sub-wavelength region. On the other hand, the nested rings studied by Smith's group does generate a sub-wavelength resonance, at 4.845 GHz where the circumference of the outer ring is 0.25λ; they note this in the text. The Shelby group makes a point of saying that the unit cell dimension of the configuration of square loops that actually demonstrates the negative index effect is a factor 6 less than the resonant

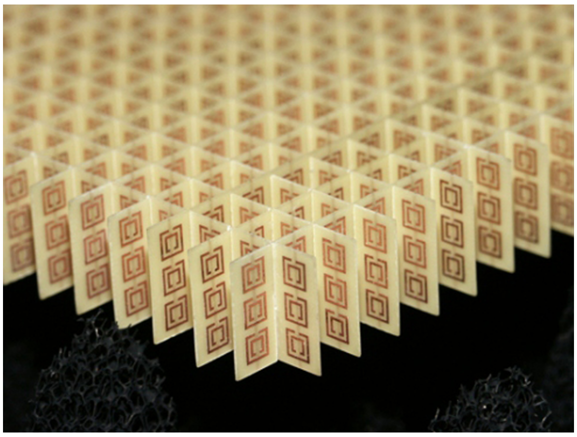

Fig. 1.7 A Shelby like meta-material structure using flat disk square loops. From Public Domain, NASA; Wikipedia, "Metamaterial", accessed Oct 2018.

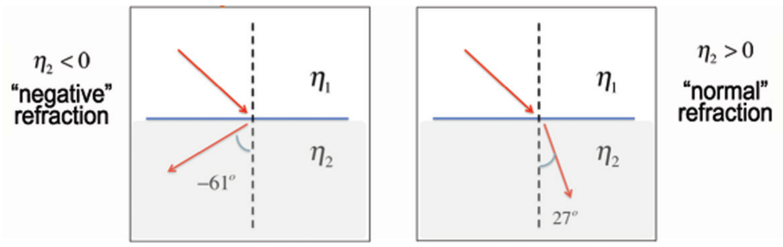

Fig. 1.8 Effects of the negative Index of refraction on incident waves. The waves bend away from the normal at a negative angle, instead of toward the normal at a positive angle. From [51].

wavelength, which puts it on the edge of the sub-wavelength region. Consequently, it may be supposed that the reason for the behaviour of early meta-materials is this very high Q resonance, due to the resonant effect of the inductance of the loop and of the gap capacitance. Indeed, all of these authors identified the cause of their resonances as an LC pair created by the inductance of loop and the capacitance of the gap, together with the coupling capacitance between the loops.

In 2004, Shamonin et al [53] developed a model that used an infinite number of lumped *LC* circuits in a circular transmission line configuration, where a lumped capacitance was taken for the gap. This allowed them to establish transmission line differential equations relating the voltage and current around the ring. They were able to show how the first, second and third harmonic resonances varied with gap widths. The *RLC* values per unit length are inputs to the theory.

A few years later, Zhou and Chui [54] claimed more accurate results, finding that Shamonin had left out important capacitances of the system. They used instead a "Quasi-static Approximation (QSA)", which, they claimed, takes into account the inductive and capacitive effects of the system completely and discovers the *RLC* elements as a result of calculation. QSA is an assumption specifically meant for structures that are small compared with the incident wavelength. The assumption makes their analysis inherently weaker than that used by Storer and Wu when applied to rings that are large compared to wavelength. Hence their method is analytically strong for small $2\pi b/\lambda$ and weaker for, say, $2\pi b > 0.6\lambda$.

A large number of papers have appeared since then, applying LC circuit models to circular and square rings of varying sizes, attempting to understand the origin of the resonance [55, 56, 57, 58, 59, 60]. In 2017, McKinley [44] identified and characterised the sub-wavelength resonance of the single capacitor loop at low frequencies. The rigorous extension of capacitance to gaps, as in the single gap nano-ring at high-frequencies up to the optical region, has not been covered yet in the literature but appears here in Section 9.3. These gaps behave capacitively in nano-rings, and follow marginally the flat-plate capacitor model that many workers were using at the time.

Nano-rings have been found useful for high-definition imaging [61], concentrating energy using tiny Fresnel lenses [62], tuning antennas in the THz region [59], controlling loop resonances with dielectric materials [63], shaping the radiation beam pattern [64], emitting single photons [65], bio-sensing [66], capturing light more effectively in solar cells [67], super-scattering [68], and super-directivity [69]. Might not many more uses for these rings be found once we probe ring behaviour more fully? It is only reasonable to expect that a detailed analytical model, describing all magnetic and harmonic resonances of the ring, of various thicknesses, with and without gaps, applicable at all frequencies, will shed light on new applications.

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